ELECTROMAGNETIC INDUCTION

29

29.1. **IDENTIFY:** Altering the orientation of a coil relative to a magnetic field changes the magnetic flux through the coil. This change then induces an emf in the coil.
SET UP: The flux through a coil of N turns is $\Phi = NBA \cos\phi$, and by Faraday's law the magnitude of the induced emf is $\mathcal{E} = d\Phi/dt$.
EXECUTE: **(a)** $\Delta\Phi = NBA = (50)(1.20\ \text{T})(0.250\ \text{m})(0.300\ \text{m}) = 4.50\ \text{Wb}$
(b) $\mathcal{E} = d\Phi/dt = (4.50\ \text{Wb})/(0.222\ \text{s}) = 20.3\ \text{V}$
EVALUATE: This induced potential is certainly large enough to be easily detectable.

29.2. **IDENTIFY:** $\mathcal{E} = \left|\dfrac{\Delta\Phi_B}{\Delta t}\right|$. $\Phi_B = BA\cos\phi$. Φ_B is the flux through each turn of the coil.
SET UP: $\phi_\text{i} = 0°$. $\phi_\text{f} = 90°$.
EXECUTE: **(a)** $\Phi_{B,\text{i}} = BA\cos 0° = (6.0\times10^{-5}\ \text{T})(12\times10^{-4}\ \text{m}^2)(1) = 7.2\times10^{-8}\ \text{Wb}$. The total flux through the coil is $N\Phi_{B,\text{i}} = (200)(7.2\times10^{-8}\ \text{Wb}) = 1.44\times10^{-5}\ \text{Wb}$. $\Phi_{B,\text{f}} = BA\cos 90° = 0$.
(b) $\mathcal{E} = \left|\dfrac{N\Phi_\text{i} - N\Phi_\text{f}}{\Delta t}\right| = \dfrac{1.44\times10^{-5}\ \text{Wb}}{0.040\ \text{s}} = 3.6\times10^{-4}\ \text{V} = 0.36\ \text{mV}.$
EVALUATE: The average induced emf depends on how rapidly the flux changes.

29.3. **IDENTIFY** and **SET UP:** Use Faraday's law to calculate the average induced emf and apply Ohm's law to the coil to calculate the average induced current and charge that flows.
(a) EXECUTE: The magnitude of the average emf induced in the coil is $|\mathcal{E}_\text{av}| = N\left|\dfrac{\Delta\Phi I_B}{\Delta t}\right|$. Initially,
$\Phi_{B\text{i}} = BA\cos\phi = BA$. The final flux is zero, so $|\mathcal{E}_\text{av}| = N\dfrac{|\Phi_{B\text{f}} - \Phi_{B\text{i}}|}{\Delta t} = \dfrac{NBA}{\Delta t}$. The average induced current is
$I = \dfrac{|\mathcal{E}_\text{av}|}{R} = \dfrac{NBA}{R\Delta t}$. The total charge that flows through the coil is $Q = I\Delta t = \left(\dfrac{NBA}{R\Delta t}\right)\Delta t = \dfrac{NBA}{R}$.
EVALUATE: The charge that flows is proportional to the magnetic field but does not depend on the time Δt.
(b) The magnetic stripe consists of a pattern of magnetic fields. The pattern of charges that flow in the reader coil tell the card reader the magnetic field pattern and hence the digital information coded onto the card.
(c) According to the result in part (a) the charge that flows depends only on the change in the magnetic flux and it does not depend on the rate at which this flux changes.

29.4. **IDENTIFY** and **SET UP:** Apply the result derived in Exercise 29.3: $Q = NBA/R$. In the present exercise the flux changes from its maximum value of $\Phi_B = BA$ to zero, so this equation applies. R is the total resistance so here $R = 60.0\ \Omega + 45.0\ \Omega = 105.0\ \Omega$.
EXECUTE: $Q = \dfrac{NBA}{R}$ says $B = \dfrac{QR}{NA} = \dfrac{(3.56\times10^{-5}\ \text{C})(105.0\ \Omega)}{120(3.20\times10^{-4}\ \text{m}^2)} = 0.0973\ \text{T}.$
EVALUATE: A field of this magnitude is easily produced.

29.5. **IDENTIFY:** Apply Faraday's law.
SET UP: Let $+z$ be the positive direction for $\vec{A}$. Therefore, the initial flux is positive and the final flux is zero.
EXECUTE: **(a)** and **(b)** $\mathcal{E} = -\dfrac{\Delta\Phi_B}{\Delta t} = -\dfrac{0-(1.5\ \text{T})\pi(0.120\ \text{m})^2}{2.0\times10^{-3}\ \text{s}} = +34\ \text{V}$. Since $\mathcal{E}$ is positive and $\vec{A}$ is toward us, the induced current is counterclockwise.
EVALUATE: The shorter the removal time, the larger the average induced emf.

29.6. **IDENTIFY:** Apply Eq.(29.4). $I = \mathcal{E}/R$.

SET UP: $d\Phi_B/dt = AdB/dt$.

EXECUTE: **(a)** $\mathcal{E} = \dfrac{Nd\Phi_B}{dt} = NA\dfrac{d}{dt}(B) = NA\dfrac{d}{dt}\left((0.012\ \text{T/s})t + (3.00\times10^{-5}\ \text{T/s}^4)t^4\right)$

$\mathcal{E} = NA\left((0.012\ \text{T/s}) + (1.2\times10^{-4}\ \text{T/s}^4)t^3\right) = 0.0302\ \text{V} + (3.02\times10^{-4}\ \text{V/s}^3)t^3.$

(b) At $t = 5.00$ s, $\mathcal{E} = 0.0302\ \text{V} + (3.02\times10^{-4}\ \text{V/s}^3)(5.00\ \text{s})^3 = 0.0680\ \text{V}$. $I = \dfrac{\mathcal{E}}{R} = \dfrac{0.0680\ \text{V}}{600\ \Omega} = 1.13\times10^{-4}\ \text{A}.$

EVALUATE: The rate of change of the flux is increasing in time, so the induced current is not constant but rather increases in time.

29.7. **IDENTIFY:** Calculate the flux through the loop and apply Faraday's law.

SET UP: To find the total flux integrate $d\Phi_B$ over the width of the loop. The magnetic field of a long straight wire, at distance r from the wire, is $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: **(a)** When $B = \dfrac{\mu_0 i}{2\pi r}$, into the page.

(b) $d\Phi_B = BdA = \dfrac{\mu_0 i}{2\pi r}Ldr.$

(c) $\Phi_B = \int_a^b d\Phi_B = \dfrac{\mu_0 iL}{2\pi}\int_a^b \dfrac{dr}{r} = \dfrac{\mu_0 iL}{2\pi}\ln(b/a).$

(d) $\mathcal{E} = \dfrac{d\Phi_B}{dt} = \dfrac{\mu_0 L}{2\pi}\ln(b/a)\dfrac{di}{dt}.$

(e) $\mathcal{E} = \dfrac{\mu_0(0.240\ \text{m})}{2\pi}\ln(0.360/0.120)(9.60\ \text{A/s}) = 5.06\times10^{-7}\ \text{V}.$

EVALUATE: The induced emf is proportional to the rate at which the current in the long straight wire is changing

29.8. **IDENTIFY:** Apply Faraday's law.

SET UP: Let $\vec{A}$ be upward in Figure 29.28 in the textbook.

EXECUTE: **(a)** $|\mathcal{E}_{\text{ind}}| = \left|\dfrac{d\Phi_B}{dt}\right| = \left|\dfrac{d}{dt}(B_\perp A)\right|$

$|\mathcal{E}_{\text{ind}}| = A\sin 60^\circ\left|\dfrac{dB}{dt}\right| = A\sin 60^\circ\left|\dfrac{d}{dt}\left((1.4\ \text{T})e^{-(0.057\text{s}^{-1})t}\right)\right| = (\pi r^2)(\sin 60^\circ)(1.4\ \text{T})(0.057\ \text{s}^{-1})e^{-(0.057\text{s}^{-1})t}$

$|\mathcal{E}_{\text{ind}}| = \pi(0.75\ \text{m})^2(\sin 60^\circ)(1.4\ \text{T})(0.057\ \text{s}^{-1})e^{-(0.057\text{s}^{-1})t} = (0.12\ \text{V})\ e^{-(0.057\ \text{s}^{-1})t}.$

(b) $\mathcal{E} = \frac{1}{10}\mathcal{E}_0 = \frac{1}{10}(0.12\ \text{V})$. $\frac{1}{10}(0.12\ \text{V}) = (0.12\ \text{V})\ e^{-(0.057\ \text{s}^{-1})t}$. $\ln(1/10) = -(0.057\ \text{s}^{-1})t$ and $t = 40.4$ s.

(c) $\vec{B}$ is in the direction of $\vec{A}$ so Φ_B is positive. B is getting weaker, so the magnitude of the flux is decreasing and $d\Phi_B/dt < 0$. Faraday's law therefore says $\mathcal{E} > 0$. Since $\mathcal{E} > 0$, the induced current must flow *counterclockwise* as viewed from above.

EVALUATE: The flux changes because the magnitude of the magnetic field is changing.

29.9. **IDENTIFY** and **SET UP:** Use Faraday's law to calculate the emf (magnitude and direction). The direction of the induced current is the same as the direction of the emf. The flux changes because the area of the loop is changing; relate dA/dt to dc/dt, where c is the circumference of the loop.

(a) EXECUTE: $c = 2\pi r$ and $A = \pi r^2$ so $A = c^2/4\pi$

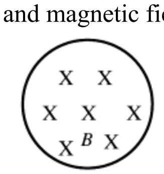
$\Phi_B = BA = (B/4\pi)c^2$

$|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = \left(\dfrac{B}{2\pi}\right)c\left|\dfrac{dc}{dt}\right|$

At $t = 9.0$ s, $c = 1.650\ \text{m} - (9.0\ \text{s})(0.120\ \text{m/s}) = 0.570\ \text{m}$

$|\mathcal{E}| = (0.500\ \text{T})(1/2\pi)(0.570\ \text{m})(0.120\ \text{m/s}) = 5.44\ \text{mV}$

(b) SET UP: The loop and magnetic field are sketched in Figure 29.9.



Take into the page to be the positive direction for $\vec{A}$. Then the magnetic flux is positive.

Figure 29.9

EXECUTE: The positive flux is decreasing in magnitude; $d\Phi_B/dt$ is negative and $\mathcal{E}$ is positive. By the right-hand rule, for $\vec{A}$ into the page, positive $\mathcal{E}$ is clockwise.

EVALUATE: Even though the circumference is changing at a constant rate, dA/dt is not constant and $|\mathcal{E}|$ is not constant. Flux ⊗ is decreasing so the flux of the induced current is ⊗ and this means that *I* is clockwise, which checks.

29.10. **IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

SET UP: The flux through a coil is $\Phi = NBA\cos\phi$ and the induced emf is $\mathcal{E} = d\Phi/dt$.

EXECUTE: **(a)** and **(c)** The magnetic flux is constant, so the induced emf is zero.

(b) The area inside the field is changing. If we let *x* be the length (along the 30.0-cm side) in the field, then $A = (0.400\text{ m})x$. $\Phi_B = BA = (0.400\text{ m})x$

$$\mathcal{E} = d\Phi/dt = B\,d[(0.400\text{ m})x]/dt = B(0.400\text{ m})dx/dt = B(0.400\text{ m})v$$

$$\mathcal{E} = (1.25\text{ T})(0.400\text{ m})(0.0200\text{ m/s}) = 0.0100\text{ V}$$

EVALUATE: It is not a large *flux* that induces an emf, but rather a large *rate of change* of the flux. The induced emf in part (b) is small enough to be ignored in many instances.

29.11. **IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

SET UP: The flux through a coil is $\Phi = NBA\cos\phi$ and the induced emf is $\mathcal{E} = d\Phi/dt$.

EXECUTE: **(a)** $\mathcal{E} = d\Phi/dt = d[A(B_0 + bx)]/dt = bA\,dx/dt = bAv$

(b) clockwise

(c) Same answers except the current is counterclockwise.

EVALUATE: Even though the coil remains within the magnetic field, the flux through it increases because the strength of the field is increasing.

29.12. **IDENTIFY:** Use the results of Example 29.5.

SET UP: $\mathcal{E}_{\text{max}} = NBA\omega$. $\mathcal{E}_{\text{av}} = \dfrac{2}{\pi}\mathcal{E}_{\text{max}}$. $\omega = (440\text{ rev/min})\left(\dfrac{2\pi\text{ rad/rev}}{60\text{ s/min}}\right) = 46.1\text{ rad/s}$.

EXECUTE: **(a)** $\mathcal{E}_{\text{max}} = NBA\omega = (150)(0.060\text{ T})\pi(0.025\text{ m})^2(46.1\text{ rad/s}) = 0.814\text{ V}$

(b) $\mathcal{E}_{\text{av}} = \dfrac{2}{\pi}\mathcal{E}_{\text{max}} = \dfrac{2}{\pi}(0.815\text{ V}) = 0.519\text{ V}$

EVALUATE: In $\mathcal{E}_{\text{max}} = NBA\omega$, ω must be in rad/s.

29.13. **IDENTIFY:** Apply the results of Example 29.5.

SET UP: $\mathcal{E}_{\text{max}} = NBA\omega$

EXECUTE: $\omega = \dfrac{\mathcal{E}_{\text{max}}}{NBA} = \dfrac{2.40\times10^{-2}\text{ V}}{(120)(0.0750\text{ T})(0.016\text{ m})^2} = 10.4\text{ rad/s}$

EVALUATE: We may also express ω as 99.3 rev/min or 1.66 rev/s.

29.14. **IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

SET UP: The flux through a coil is $\Phi = NBA\cos\phi$ and the induced emf is $\mathcal{E} = d\Phi/dt$.

EXECUTE: The flux is constant in each case, so the induced emf is zero in all cases.

EVALUATE: Even though the coil is moving within the magnetic field and has flux through it, this flux is not *changing*, so no emf is induced in the coil.

29.15. **IDENTIFY** and **SET UP:** The field of the induced current is directed to oppose the change in flux.

EXECUTE: **(a)** The field is into the page and is increasing so the flux is increasing. The field of the induced current is out of the page. To produce field out of the page the induced current is counterclockwise.

(b) The field is into the page and is decreasing so the flux is decreasing. The field of the induced current is into the page. To produce field into the page the induced current is clockwise.

(c) The field is constant so the flux is constant and there is no induced emf and no induced current.

EVALUATE: The direction of the induced current depends on the direction of the external magnetic field and whether the flux due to this field is increasing or decreasing.

29.16. **IDENTIFY:** By Lenz's law, the induced current flows to oppose the flux change that caused it.

SET UP and **EXECUTE:** The magnetic field is outward through the round coil and is decreasing, so the magnetic field due to the induced current must also point outward to oppose this decrease. Therefore the induced current is counterclockwise.

EVALUATE: Careful! Lenz's law does not say that the induced current flows to oppose the magnetic flux. Instead it says that the current flows to oppose the *change* in flux.

29.17. **IDENTIFY** and **SET UP:** Apply Lenz's law, in the form that states that the flux of the induced current tends to oppose the change in flux.

EXECUTE: **(a)** With the switch closed the magnetic field of coil A is to the right at the location of coil B. When the switch is opened the magnetic field of coil A goes away. Hence by Lenz's law the field of the current induced in coil B is to the right, to oppose the decrease in the flux in this direction. To produce magnetic field that is to the right the current in the circuit with coil B must flow through the resistor in the direction *a* to *b*.

(b) With the switch closed the magnetic field of coil A is to the right at the location of coil B. This field is stronger at points closer to coil A so when coil B is brought closer the flux through coil B increases. By Lenz's law the field of the induced current in coil B is to the left, to oppose the increase in flux to the right. To produce magnetic field that is to the left the current in the circuit with coil B must flow through the resistor in the direction b to a.
(c) With the switch closed the magnetic field of coil A is to the right at the location of coil B. The current in the circuit that includes coil A increases when R is decreased and the magnetic field of coil A increases when the current through the coil increases. By Lenz's law the field of the induced current in coil B is to the left, to oppose the increase in flux to the right. To produce magnetic field that is to the left the current in the circuit with coil B must flow through the resistor in the direction b to a.
EVALUATE: In parts (b) and (c) the change in the circuit causes the flux through circuit B to increase and in part (a) it causes the flux to decrease. Therefore, the direction of the induced current is the same in parts (b) and (c) and opposite in part (a).

29.18. **IDENTIFY:** Apply Lenz's law.
SET UP: The field of the induced current is directed to oppose the change in flux in the primary circuit.
EXECUTE: **(a)** The magnetic field in A is to the left and is increasing. The flux is increasing so the field due to the induced current in B is to the right. To produce magnetic field to the right, the induced current flows through R from right to left.
(b) The magnetic field in A is to the right and is decreasing. The flux is decreasing so the field due to the induced current in B is to the right. To produce magnetic field to the right the induced current flows through R from right to left.
(c) The magnetic field in A is to the right and is increasing. The flux is increasing so the field due to the induced current in B is to the left. To produce magnetic field to the left the induced current flows through R from left to right.
EVALUATE: The direction of the induced current depends on the direction of the external magnetic field and whether the flux due to this field is increasing or decreasing.

29.19. **IDENTIFY** and **SET UP:** Lenz's law requires that the flux of the induced current opposes the change in flux.
EXECUTE: **(a)** Φ_B is $\odot$ and increasing so the flux Φ_{ind} of the induced current is $\otimes$ and the induced current is clockwise.
(b) The current reaches a constant value so Φ_B is constant. $d\Phi_B/dt = 0$ and there is no induced current.
(c) Φ_B is $\odot$ and decreasing, so Φ_{ind} is $\odot$ and current is counterclockwise.
EVALUATE: Only a change in flux produces an induced current. The induced current is in one direction when the current in the outer ring is increasing and is in the opposite direction when that current is decreasing.

29.20. **IDENTIFY:** Use the results of Example 29.6. Use the three approaches specified in the problem for determining the direction of the induced current. $I = \mathcal{E}/R$.
SET UP: Let $\vec{A}$ be directed into the figure, so a clockwise emf is positive.
EXECUTE: **(a)** $\mathcal{E} = vBl = (5.0\text{ m/s})(0.750\text{ T})(1.50\text{ m}) = 5.6\text{ V}$
(b) (i) Let q be a positive charge in the moving bar, as shown in Figure 29.20a. The magnetic force on this charge is $\vec{F} = q\vec{v} \times \vec{B}$, which points *upward*. This force pushes the current in a *counterclockwise* direction through the circuit.
(ii) Φ_B is positive and is increasing in magnitude, so $d\Phi_B/dt > 0$. Then by Faraday's law $\mathcal{E} < 0$ and the emf and induced current are counterclockwise.
(iii) The flux through the circuit is increasing, so the induced current must cause a magnetic field out of the paper to oppose this increase. Hence this current must flow in a *counterclockwise* sense, as shown in Figure 29.20b.
(c) $\mathcal{E} = RI$. $I = \dfrac{\mathcal{E}}{R} = \dfrac{5.6\text{ V}}{25\ \Omega} = 0.22\text{ A}$.
EVALUATE: All three methods agree on the direction of the induced current.

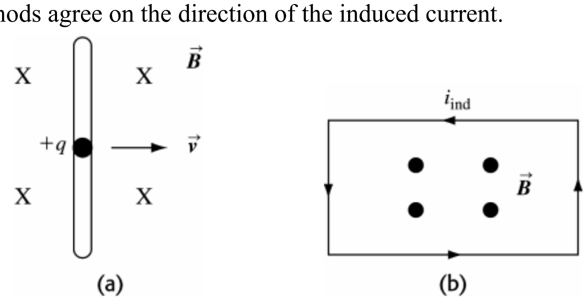


Figure 29.20

29.21. **IDENTIFY:** A conductor moving in a magnetic field may have a potential difference induced across it, depending on how it is moving.
SET UP: The induced emf is $\mathcal{E} = vBL\sin\phi$, where ϕ is the angle between the velocity and the magnetic field.

EXECUTE: **(a)** $\mathcal{E} = vBL \sin\phi = (5.00\text{ m/s})(0.450\text{ T})(0.300\text{ m})(\sin 90°) = 0.675\text{ V}$

(b) The positive charges are moved to end *b*, so *b* is at the higher potential.

(c) $E = V/L = (0.675\text{ V})/(0.300\text{ m}) = 2.25\text{ V/m}$. The direction of $\vec{E}$ is from, *b* to *a*.

(d) The positive charge are pushed to *b*, so *b* has an excess of positive charge.

(e) (i) If the rod has no appreciable thickness, $L = 0$, so the emf is zero. (ii) The emf is zero because no magnetic force acts on the charges in the rod since it moves parallel to the magnetic field.

EVALUATE: The motional emf is large enough to have noticeable effects in some cases.

29.22. **IDENTIFY:** The moving bar has a motional emf induced across its ends, so it causes a current to flow.

SET UP: The induced potential is $\mathcal{E} = vBL$ and Ohm's law is $\mathcal{E} = IR$.

EXECUTE: (a) $\mathcal{E} = vBL = (5.0\text{ m/s})(0.750\text{ T})(1.50\text{ m}) = 5.6\text{ V}$

(b) $I = \mathcal{E}/R = (5.6\text{ V})/(25\ \Omega) = 0.23\text{ A}$

EVALUATE: Both the induced potential and the current are large enough to have noticeable effects.

29.23. **IDENTIFY:** $\mathcal{E} = vBL$

SET UP: $L = 5.00\times10^{-2}$ m. 1 mph = 0.4470 m/s.

EXECUTE: $v = \dfrac{\mathcal{E}}{BL} = \dfrac{1.50\text{ V}}{(0.650\text{ T})(5.00\times10^{-2}\text{ m})} = 46.2\text{ m/s} = 103\text{ mph}.$

EVALUATE: This is a large speed and not practical. It is also difficult to produce a 5.00 cm wide region of 0.650 T magnetic field.

29.24. **IDENTIFY:** $\mathcal{E} = vBL.$

SET UP: $1\text{ mph} = 0.4470\text{ m/s}$. $1\text{ G} = 10^{-4}\text{ T}$.

EXECUTE: **(a)** $\mathcal{E} = (180\text{ mph})\left(\dfrac{0.4470\text{ m/s}}{1\text{ mph}}\right)(0.50\times10^{-4}\text{ T})(1.5\text{ m}) = 6.0\text{ mV}$. This is much too small to be noticeable.

(b) $\mathcal{E} = (565\text{ mph})\left(\dfrac{0.4470\text{ m/s}}{1\text{ mph}}\right)(0.50\times10^{-4}\text{ T})(64.4\text{ m}) = 0.813\text{ mV}$. This is too small to be noticeable.

EVALUATE: Even though the speeds and values of *L* are large, the earth's field is small and motional emfs due to the earth's field are not important in these situations.

29.25. **IDENTIFY** and **SET UP:** $\mathcal{E} = vBL$. Use Lenz's law to determine the direction of the induced current. The force F_{ext} required to maintain constant speed is equal and opposite to the force F_I that the magnetic field exerts on the rod because of the current in the rod.

EXECUTE: **(a)** $\mathcal{E} = vBL = (7.50\text{ m/s})(0.800\text{ T})(0.500\text{ m}) = 3.00\text{ V}$

(b) $\vec{B}$ is into the page. The flux increases as the bar moves to the right, so the magnetic field of the induced current is out of the page inside the circuit. To produce magnetic field in this direction the induced current must be counterclockwise, so from *b* to *a* in the rod.

(c) $I = \dfrac{\mathcal{E}}{R} = \dfrac{3.00\text{ V}}{1.50\ \Omega} = 2.00\text{ A}$. $F_I = ILB\sin\phi = (2.00\text{ A})(0.500\text{ m})(0.800\text{ T})\sin 90° = 0.800\text{ N}$. $\vec{F}_I$ is to the left. To keep the bar moving to the right at constant speed an external force with magnitude $F_{ext} = 0.800$ N and directed to the right must be applied to the bar.

(d) The rate at which work is done by the force F_{ext} is $F_{ext}v = (0.800\text{ N})(7.50\text{ m/s}) = 6.00\text{ W}$. The rate at which thermal energy is developed in the circuit is $I^2R = (2.00\text{ A})(1.50\ \Omega) = 6.00\text{ W}$. These two rates are equal, as is required by conservation of energy.

EVALUATE: The force on the rod due to the induced current is directed to oppose the motion of the rod. This agrees with Lenz's law.

29.26. **IDENTIFY:** Use Faraday's law to calculate the induced emf. Ohm's law applied to the loop gives *I*. Use Eq.(27.19) to calculate the force exerted on each side of the loop.

SET UP: The loop before it starts to enter the magnetic field region is sketched in Figure 29.26a.

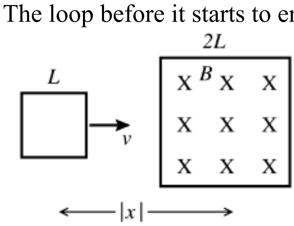


Figure 29.26a

EXECUTE: For $x < -3L/2$ or $x > 3L/2$ the loop is completely outside the field region. $\Phi_B = 0$, and $\dfrac{d\Phi_B}{dt} = 0.$

Thus $\mathcal{E} = 0$ and $I = 0$, so there is no force from the magnetic field and the external force *F* necessary to maintain constant velocity is zero.

SET UP: The loop when it is completely inside the field region is sketched in Figure 29.26b.

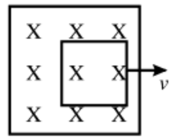


Figure 29.26b

EXECUTE: For $-L/2 < x < L/2$ the loop is completely inside the field region and $\Phi_B = BL^2$.

But $\dfrac{d\Phi_B}{dt} = 0$ so $\mathcal{E} = 0$ and $I = 0$. There is no force $\vec{F} = I\vec{l} \times \vec{B}$ from the magnetic field and the external force F necessary to maintain constant velocity is zero.

SET UP: The loop as it enters the magnetic field region is sketched in Figure 29.26c.

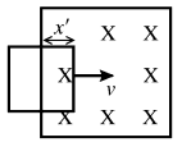


Figure 29.26c

EXECUTE: For $-3L/2 < x < -L/2$ the loop is entering the field region. Let x' be the length of the loop that is within the field.

Then $|\Phi_B| = BLx'$ and $\left|\dfrac{d\Phi_B}{dt}\right| = Blv$. The magnitude of the induced emf is $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = BLv$ and the induced current is $I = \dfrac{|\mathcal{E}|}{R} = \dfrac{BLv}{R}$. Direction of I: Let $\vec{A}$ be directed into the plane of the figure. Then Φ_B is positive. The flux is positive and increasing in magnitude, so $\dfrac{d\Phi_B}{dt}$ is positive. Then by Faraday's law $\mathcal{E}$ is negative, and with our choice for direction of $\vec{A}$ a negative $\mathcal{E}$ is counterclockwise. The current induced in the loop is counterclockwise.

SET UP: The induced current and magnetic force on the loop are shown in Figure 29.26d, for the situation where the loop is entering the field.

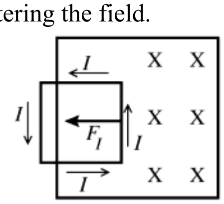


Figure 29.26d

EXECUTE: $\vec{F}_I = I\vec{l} \times \vec{B}$ gives that the force $\vec{F}_I$ exerted on the loop by the magnetic field is to the left and has magnitude $F_I = ILB = \left(\dfrac{BLv}{R}\right)LB = \dfrac{B^2L^2v}{R}$.

The external force $\vec{F}$ needed to move the loop at constant speed is equal in magnitude and opposite in direction to $\vec{F}_I$ so is to the right and has this same magnitude.

SET UP: The loop as it leaves the magnetic field region is sketched in Figure 29.26e.

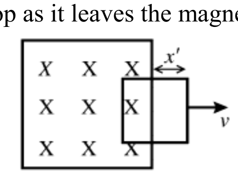


Figure 29.26e

EXECUTE: For $L/2 < x < 3L/2$ the loop is leaving the field region. Let x' be the length of the loop that is outside the field.

Then $|\Phi_B| = BL(L - x')$ and $\left|\dfrac{d\Phi_B}{dt}\right| = BLv$. The magnitude of the induced emf is $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = BLv$ and the induced current is $I = \dfrac{|\mathcal{E}|}{R} = \dfrac{BLv}{R}$. Direction of I: Again let $\vec{A}$ be directed into the plane of the figure. Then Φ_B is positive and decreasing in magnitude, so $\dfrac{d\Phi_B}{dt}$ is negative. Then by Faraday's law $\mathcal{E}$ is positive, and with our choice for direction of $\vec{A}$ a positive $\mathcal{E}$ is clockwise. The current induced in the loop is clockwise.

SET UP: The induced current and magnetic force on the loop are shown in Figure 29.26f, for the situation where the loop is leaving the field.

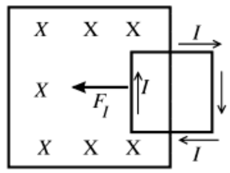


Figure 29.26f

EXECUTE: $\vec{F}_I = I\vec{l} \times \vec{B}$ gives that the force $\vec{F}_I$ exerted on the loop by the magnetic field is to the left and has magnitude $F_I = ILB = \left(\frac{BLv}{R}\right)LB = \frac{B^2L^2v}{R}$.

The external force $\vec{F}$ needed to move the loop at constant speed is equal in magnitude and opposite in direction to $\vec{F}_I$ so is to the right and has this same magnitude.

(a) The graph of F versus x is given in Figure 29.26g.

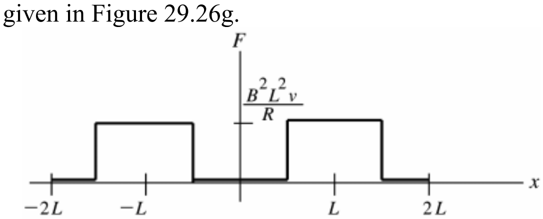


Figure 29.26g

(b) The graph of the induced current I versus x is given in Figure 29.26h.

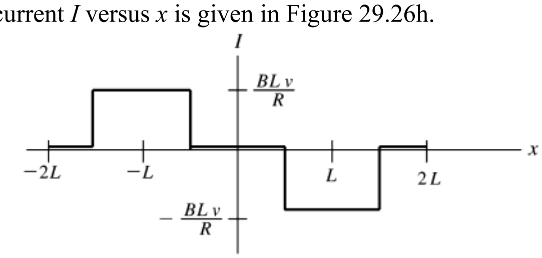


Figure 29.26h

EVALUATE: When the loop is either totally outside or totally inside the magnetic field region the flux isn't changing, there is no induced current, and no external force is needed for the loop to maintain constant speed. When the loop is entering the field the external force required is directed so as to pull the loop in and when the loop is leaving the field the external force required is directed so as to pull the loop out of the field. These directions agree with Lenz's law: the force on the induced current (opposite in direction to the required external force) is directed so as to oppose the loop entering or leaving the field.

29.27. **IDENTIFY:** A bar moving in a magnetic field has an emf induced across its ends.

SET UP: The induced potential is $\mathcal{E} = vBL \sin \phi$.

EXECUTE: Note that $\phi = 90°$ in all these cases because the bar moved perpendicular to the magnetic field. But the effective length of the bar, $L \sin \theta$, is different in each case.

(a) $\mathcal{E} = vBL \sin \theta = (2.50\text{ m/s})(1.20\text{ T})(1.41\text{ m}) \sin(37.0°) = 2.55\text{ V}$, with a at the higher potential because positive charges are pushed toward that end.

(b) Same as (a) except $\theta = 53.0°$, giving 3.38 V, with a at the higher potential.

(c) Zero, since the velocity is parallel to the magnetic field.

(d) The bar must move perpendicular to its length, for which the emf is 4.23 V. For $V_b > V_a$, it must move upward and to the left (toward the second quadrant) perpendicular to its length.

EVALUATE: The orientation of the bar affects the potential induced across its ends.

29.28. **IDENTIFY:** Use Eq.(29.10) to calculate the induced electric field E at a distance r from the center of the solenoid. Away from the ends of the solenoid, $B = \mu_0 nI$ inside and $B = 0$ outside.

(a) SET UP: The end view of the solenoid is sketched in Figure 29.28.

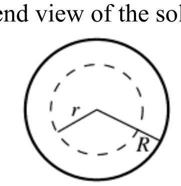


Figure 29.28

Let R be the radius of the solenoid.

Apply $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ to an integration path that is a circle of radius r, where $r < R$. We need to calculate just the magnitude of E so we can take absolute values.

EXECUTE: $\left|\oint \vec{E} \cdot d\vec{l}\right| = E(2\pi r)$

$\Phi_B = B\pi r^2$, $\left|-\frac{d\Phi_B}{dt}\right| = \pi r^2 \left|\frac{dB}{dt}\right|$

$\left|\oint \vec{E} \cdot d\vec{l}\right| = \left|-\frac{d\Phi_B}{dt}\right|$ implies $E(2\pi r) = \pi r^2 \left|\frac{dB}{dt}\right|$

$E = \frac{1}{2} r \left|\frac{dB}{dt}\right|$

$B = \mu_0 n I$, so $\frac{dB}{dt} = \mu_0 n \frac{dI}{dt}$

Thus $E = \frac{1}{2} r \mu_0 n \frac{dI}{dt} = \frac{1}{2}(0.00500\text{ m})(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(900\text{ m}^{-1})(60.0\text{ A/s}) = 1.70 \times 10^{-4}\text{ V/m}$

(b) r = 0.0100 cm is still inside the solenoid so the expression in part (a) applies.

$E = \frac{1}{2} r \mu_0 n \frac{dI}{dt} = \frac{1}{2}(0.0100\text{ m})(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(900\text{ m}^{-1})(60.0\text{ A/s}) = 3.39 \times 10^{-4}\text{ V/m}$

EVALUATE: Inside the solenoid E is proportional to r, so E doubles when r doubles.

29.29. **IDENTIFY:** Apply Eqs.(29.9) and (29.10).

SET UP: Evaluate the integral if Eq.(29.10) for a path which is a circle of radius r and concentric with the solenoid. The magnetic field of the solenoid is confined to the region inside the solenoid, so $B(r) = 0$ for $r > R$

EXECUTE: **(a)** $\frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi r_1^2 \frac{dB}{dt}$.

(b) $E = \frac{1}{2\pi r_1}\frac{d\Phi_B}{dt} = \frac{\pi r_1^2}{2\pi r_1}\frac{dB}{dt} = \frac{r_1}{2}\frac{dB}{dt}$. The direction of $\vec{E}$ is shown in Figure 29.29a.

(c) All the flux is within $r < R$, so outside the solenoid $E = \frac{1}{2\pi r_2}\frac{d\Phi_B}{dt} = \frac{\pi R^2}{2\pi r_2}\frac{dB}{dt} = \frac{R^2}{2r_2}\frac{dB}{dt}$.

(d) The graph is sketched in Figure 29.29b.

(e) At $r = R/2$, $\mathcal{E} = \frac{d\Phi_B}{dt} = \pi(R/2)^2 \frac{dB}{dt} = \frac{\pi R^2}{4}\frac{dB}{dt}$.

(f) At $r = R$, $\mathcal{E} = \frac{d\Phi_B}{dt} = \pi R^2 \frac{dB}{dt}$.

(g) At $r = 2R$, $\mathcal{E} = \frac{d\Phi_B}{dt} = \pi R^2 \frac{dB}{dt}$.

EVALUATE: The emf is independent of the distance from the center of the cylinder at all points outside it. Even though the magnetic field is zero for $r > R$, the induced electric field is nonzero outside the solenoid and a nonzero emf is induced in a circular turn that has $r > R$.

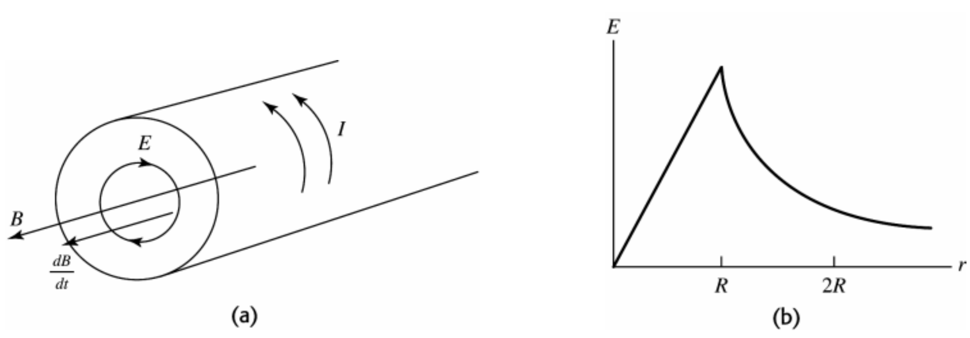


Figure 29.29

29.30. **IDENTIFY:** Use Eq.(29.10) to calculate the induced electric field E and use this E in Eq.(29.9) to calculate $\mathcal{E}$ between two points.

(a) SET UP: Because of the axial symmetry and the absence of any electric charge, the field lines are concentric circles.

(b) See Figure 29.30.

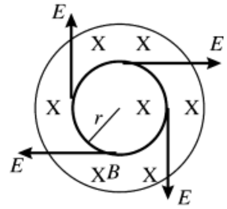


Figure 29.30

$\vec{E}$ is tangent to the ring. The direction of $\vec{E}$ (clockwise or counterclockwise) is the direction in which current will be induced in the ring.

EXECUTE: Use the sign convention for Faraday's law to deduce this direction. Let $\vec{A}$ be into the paper. Then Φ_B is positive. B decreasing then means $\frac{d\Phi_B}{dt}$ is negative, so by $\mathcal{E} = -\frac{d\Phi_B}{dt}$, $\mathcal{E}$ is positive and therefore clockwise. Thus $\vec{E}$ is clockwise around the ring. To calculate E apply $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ to a circular path that coincides with the ring.

$\oint \vec{E} \cdot d\vec{l} = E(2\pi r)$

$\Phi_B = B\pi r^2$; $\left|\frac{d\Phi_B}{dt}\right| = \pi r^2 \left|\frac{dB}{dt}\right|$

$E(2\pi r) = \pi r^2 \left|\frac{dB}{dt}\right|$ and $E = \frac{1}{2} r \left|\frac{dB}{dt}\right| = \frac{1}{2}(0.100 \text{ m})(0.0350 \text{ T/s}) = 1.75\times10^{-3} \text{ V/m}$

(c) The induced emf has magnitude $\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = E(2\pi r) = (1.75\times10^{-3} \text{ V/m})(2\pi)(0.100 \text{ m}) = 1.100\times10^{-3} \text{ V}$. Then $I = \frac{\mathcal{E}}{R} = \frac{1.100\times10^{-3} \text{ V}}{4.00\ \Omega} = 2.75\times10^{-4} \text{ A}$.

(d) Points a and b are separated by a distance around the ring of πr so $\mathcal{E} = E(\pi r) = (1.75\times10^{-3} \text{ V/m})(\pi)(0.100 \text{ m}) = 5.50\times10^{-4} \text{ V}$

(e) The ends are separated by a distance around the ring of $2\pi r$ so $\mathcal{E} = 1.10\times10^{-3}$ V as calculated in part (c).

EVALUATE: The induced emf, calculated from Faraday's law and used to calculate the induced current, is associated with the induced electric field integrated around the total circumference of the ring.

29.31. IDENTIFY: Apply Eq.(29.1) with $\Phi_B = \mu_0 niA$.

SET UP: $A = \pi r^2$, where $r = 0.0110$ m. In Eq.(29.11), $r = 0.0350$ m.

EXECUTE: $|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d}{dt}(BA)\right| = \left|\frac{d}{dt}(\mu_0 niA)\right| = \mu_0 nA\left|\frac{di}{dt}\right|$ and $|\mathcal{E}| = E(2\pi r)$. Therefore, $\left|\frac{di}{dt}\right| = \frac{E2\pi r}{\mu_0 nA}$.

$\left|\frac{di}{dt}\right| = \frac{(8.00\times10^{-6} \text{ V/m})2\pi(0.0350 \text{ m})}{\mu_0(400 \text{ m}^{-1})\pi(0.0110 \text{ m})^2} = 9.21 \text{ A/s}.$

EVALUATE: Outside the solenoid the induced electric field decreases with increasing distance from the axis of the solenoid.

29.32. IDENTIFY: A changing magnetic flux through a coil induces an emf in that coil, which means that an electric field is induced in the material of the coil.

SET UP: According to Faraday's law, the induced electric field obeys the equation $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$.

EXECUTE: **(a)** For the magnitude of the induced electric field, Faraday's law gives

$$E2\pi r = d(B\pi r^2)/dt = \pi r^2\, dB/dt$$

$$E = \frac{r}{2}\frac{dB}{dt} = \frac{0.0225 \text{ m}}{2}(0.250 \text{ T/s}) = 2.81\times10^{-3} \text{ V/m}$$

(b) The field points toward the south pole of the magnet and is decreasing, so the induced current is counterclockwise.

EVALUATE: This is a very small electric field compared to most others found in laboratory equipment.

29.33. IDENTIFY: Apply Faraday's law in the form $|\mathcal{E}_{\text{av}}| = N\left|\frac{\Delta\Phi_B}{\Delta t}\right|$.

SET UP: The magnetic field of a large straight solenoid is $B = \mu_0 nI$ inside the solenoid and zero outside. $\Phi_B = BA$, where A is 8.00 cm^2, the cross-sectional area of the long straight solenoid.

EXECUTE: $|\mathcal{E}_{\text{av}}| = N\left|\dfrac{\Delta\Phi_B}{\Delta t}\right| = \left|\dfrac{NA(B_\text{f} - B_\text{i})}{\Delta t}\right| = \dfrac{NA\mu_o nI}{\Delta t}$.

$$\mathcal{E}_{\text{av}} = \frac{\mu_0(12)(8.00\times10^{-4}\,\text{m}^2)(9000\ \text{m}^{-1})(0.350\ \text{A})}{0.0400\ \text{s}} = 9.50\times10^{-4}\ \text{V}.$$

EVALUATE: An emf is induced in the second winding even though the magnetic field of the solenoid is zero at the location of the second winding. The changing magnetic field induces an electric field outside the solenoid and that induced electric field produces the emf.

29.34. **IDENTIFY:** Apply Eq.(29.14).

SET UP: $\epsilon = 3.5\times10^{-11}$ F/m

EXECUTE: $i_\text{D} = \epsilon\dfrac{d\Phi_E}{dt} = (3.5\times10^{-11}\ \text{F/m})(24.0\times10^3\ \text{V}\cdot\text{m/s}^3)t^2$. $i_\text{D} = 21\times10^{-6}$ A gives $t = 5.0$ s.

EVALUATE: i_D depends on the rate at which Φ_E is changing.

29.35. **IDENTIFY:** Apply Eq.(29.14), where $\epsilon = K\epsilon_0$.

SET UP: $d\Phi_E/dt = 4(8.76\times10^3\ \text{V}\cdot\text{m/s}^4)t^3$. $\epsilon_0 = 8.854\times10^{-12}$ F/m.

EXECUTE: $\epsilon = \dfrac{i_D}{(d\Phi_E/dt)} = \dfrac{12.9\times10^{-12}\ \text{A}}{4(8.76\times10^3\ \text{V}\cdot\text{m/s}^4)(26.1\times10^{-3}\ \text{s})^3} = 2.07\times10^{-11}$ F/m. The dielectric constant is

$K = \dfrac{\epsilon}{\epsilon_0} = 2.34.$

EVALUATE: The larger the dielectric constant, the larger is the displacement current for a given $d\Phi_E/dt$.

29.36. **IDENTIFY** and **SET UP:** Eqs.(29.13) and (29.14) show that $i_\text{C} = i_\text{D}$ and also relate i_D to the rate of change of the electric field flux between the plates. Use this to calculate dE/dt and apply the generalized form of Ampere's law (Eq.29.15) to calculate B.

(a) EXECUTE: $i_\text{C} = i_\text{D}$, so $j_\text{D} = \dfrac{i_\text{D}}{A} = \dfrac{i_\text{C}}{A} = \dfrac{0.280\ \text{A}}{\pi r^2} = \dfrac{0.280\ \text{A}}{\pi(0.0400\ \text{m})^2} = 55.7\ \text{A/m}^2$

(b) $j_\text{D} = \epsilon_0\dfrac{dE}{dt}$ so $\dfrac{dE}{dt} = \dfrac{j_\text{D}}{\epsilon_0} = \dfrac{55.7\ \text{A/m}^2}{8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 6.29\times10^{12}\ \text{V/m}\cdot\text{s}$

(c) SET UP: Apply Ampere's law $\oint\vec{B}\cdot d\vec{l} = \mu_0(i_\text{C} + i_\text{D})_{\text{encl}}$ (Eq.(28.20)) to a circular path with radius $r = 0.0200$ m. An end view of the solenoid is given in Figure 29.36.

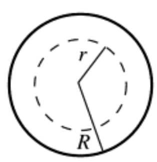


Figure 29.36

By symmetry the magnetic field is tangent to the path and constant around it.

EXECUTE: Thus $\oint\vec{B}\cdot d\vec{l} = \oint Bdl = B\int dl = B(2\pi r)$.

$i_\text{C} = 0$ (no conduction current flows through the air space between the plates)

The displacement current enclosed by the path is $j_\text{D}\pi r^2$.

Thus $B(2\pi r) = \mu_0(j_\text{D}\pi r^2)$ and $B = \frac{1}{2}\mu_0 j_\text{D} r = \frac{1}{2}(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(55.7\ \text{A/m}^2)(0.0200\ \text{m}) = 7.00\times10^{-7}$ T

(d) $B = \frac{1}{2}\mu_0 j_\text{D} r$. Now r is $\frac{1}{2}$ the value in (c), so B is $\frac{1}{2}$ also: $B = \frac{1}{2}(7.00\times10^{-7}\ \text{T}) = 3.50\times10^{-7}$ T

EVALUATE: The definition of displacement current allows the current to be continuous at the capacitor. The magnetic field between the plates is zero on the axis ($r = 0$) and increases as r increases.

29.37. **IDENTIFY:** $q = CV$. For a parallel-plate capacitor, $C = \dfrac{\epsilon A}{d}$, where $\epsilon = K\epsilon_0$. $i_\text{C} = dq/dt$. $j_\text{D} = \epsilon\dfrac{E}{dt}$.

SET UP: $E = q/\epsilon A$ so $dE/dt = i_\text{C}/\epsilon A$.

EXECUTE: **(a)** $q = CV = \left(\dfrac{\epsilon A}{d}\right)V = \dfrac{(4.70)\epsilon_0(3.00\times10^{-4}\ \text{m}^2)(120\ \text{V})}{2.50\times10^{-3}\ \text{m}} = 5.99\times10^{-10}$ C.

(b) $\frac{dq}{dt} = i_C = 6.00 \times 10^{-3}$ A.

(c) $j_D = \epsilon \frac{dE}{dt} = K\epsilon_0 \frac{i_C}{K\epsilon_0 A} = \frac{i_C}{A} = j_C$, so $i_D = i_C = 6.00 \times 10^{-3}$ A.

EVALUATE: $i_D = i_C$, so Kirchhoff's junction rule is satisfied where the wire connects to each capacitor plate.

29.38. **IDENTIFY** and **SET UP:** Use $i_C = q/t$ to calculate the charge q that the current has carried to the plates in time t. The two equations preceeding Eq.(24.2) relate q to the electric field E and the potential difference between the plates. The displacement current density is defined by Eq.(29.16).

EXECUTE: **(a)** $i_C = 1.80 \times 10^{-3}$ A

$q = 0$ at $t = 0$

The amount of charge brought to the plates by the charging current in time t is

$q = i_C t = (1.80 \times 10^{-3}\ \text{A})(0.500 \times 10^{-6}\ \text{s}) = 9.00 \times 10^{-10}$ C

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A} = \frac{9.00 \times 10^{-10}\ \text{C}}{(8.854 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.00 \times 10^{-4}\ \text{m}^2)} = 2.03 \times 10^5\ \text{V/m}$$

$V = Ed = (2.03 \times 10^5\ \text{V/m})(2.00 \times 10^{-3}\ \text{m}) = 406$ V

(b) $E = q/\epsilon_0 A$

$$\frac{dE}{dt} = \frac{dq/dt}{\epsilon_0 A} = \frac{i_C}{\epsilon_0 A} = \frac{1.80 \times 10^{-3}\ \text{A}}{(8.854 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.00 \times 10^{-4}\ \text{m}^2)} = 4.07 \times 10^{11}\ \text{V/m}\cdot\text{s}$$

Since i_C is constant dE/dt does not vary in time.

(c) $j_D = \epsilon_0 \frac{dE}{dt}$ (Eq.(29.16)), with ϵ replaced by ϵ_0 since there is vacuum between the plates.)

$j_D = (8.854 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.07 \times 10^{11}\ \text{V/m}\cdot\text{s}) = 3.60\ \text{A/m}^2$

$i_D = j_D A = (3.60\ \text{A/m}^2)(5.00 \times 10^{-4}\ \text{m}^2) = 1.80 \times 10^{-3}$ A; $i_D = i_C$

EVALUATE: $i_C = i_D$. The constant conduction current means the charge q on the plates and the electric field between them both increase linearly with time and i_D is constant.

29.39. **IDENTIFY:** Ohm's law relates the current in the wire to the electric field in the wire. $j_D = \epsilon \frac{dE}{dt}$. Use Eq.(29.15) to calculate the magnetic fields.

SET UP: Ohm's law says $E = \rho J$. Apply Ohm's law to a circular path of radius r.

EXECUTE: **(a)** $E = \rho J = \frac{\rho I}{A} = \frac{(2.0 \times 10^{-8}\ \Omega\cdot\text{m})(16\ \text{A})}{2.1 \times 10^{-6}\ \text{m}^2} = 0.15$ V/m.

(b) $\frac{dE}{dt} = \frac{d}{dt}\left(\frac{\rho I}{A}\right) = \frac{\rho}{A}\frac{dI}{dt} = \frac{2.0 \times 10^{-8}\ \Omega\cdot\text{m}}{2.1 \times 10^{-6}\ \text{m}^2}(4000\ \text{A/s}) = 38\ \text{V/m}\cdot\text{s}$.

(c) $j_D = \epsilon_0 \frac{dE}{dt} = \epsilon_0(38\ \text{V/m}\cdot\text{s}) = 3.4 \times 10^{-10}\ \text{A/m}^2$.

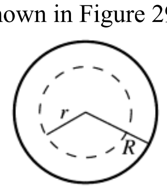
(d) $i_D = j_D A = (3.4 \times 10^{-10}\ \text{A/m}^2)(2.1 \times 10^{-6}\ \text{m}^2) = 7.14 \times 10^{-16}$ A. Eq.(29.15) applied to a circular path of radius r gives $B_D = \frac{\mu_0 I_D}{2\pi r} = \frac{\mu_0(7.14 \times 10^{-16}\ \text{A})}{2\pi(0.060\ \text{m})} = 2.38 \times 10^{-21}$ T, and this is a negligible contribution.

$B_C = \frac{\mu_0 I_C}{2\pi r} = \frac{\mu_0}{2\pi}\frac{(16\ \text{A})}{(0.060\ \text{m})} = 5.33 \times 10^{-5}$ T.

EVALUATE: In this situation the displacement current is much less than the conduction current.

29.40. **IDENTIFY:** Apply Ampere's law to a circular path of radius $r < R$, where R is the radius of the wire.

SET UP: The path is shown in Figure 29.40.



$$\oint \vec{B} \cdot d\vec{l} = \mu_0\left(I_C + \epsilon_0 \frac{d\Phi_E}{dt}\right)$$

Figure 29.40

EXECUTE: There is no displacement current, so $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_C$

The magnetic field inside the superconducting material is zero, so $\oint \vec{B}\cdot d\vec{l} = 0.$ But then Ampere's law says that $I_C = 0;$ there can be no conduction current through the path. This same argument applies to any circular path with $r < R$, so all the current must be at the surface of the wire.

EVALUATE: If the current were uniformly spread over the wire's cross section, the magnetic field would be like that calculated in Example 28.9.

29.41. **IDENTIFY:** A superconducting region has zero resistance.

SET UP: If the superconducting and normal regions each lie along the length of the cylinder, they provide parallel conducting paths.

EXECUTE: Unless some of the regions with resistance completely fill a cross-sectional area of a long type-II superconducting wire, there will still be no total resistance. The regions of no resistance provide the path for the current.

EVALUATE: The situation here is like two resistors in parallel, where one has zero resistance and the other is non-zero. The equivalent resistance is zero.

29.42. **IDENTIFY:** Apply Eq.(28.29): $\vec{B} = \vec{B}_0 + \mu_0\vec{M}.$

SET UP: For magnetic fields less than the critical field, there is no internal magnetic field. For fields greater than the critical field, $\vec{B}$ is very nearly equal to $\vec{B}_0$.

EXECUTE: **(a)** The external field is less than the critical field, so inside the superconductor $\vec{B} = 0$ and $\vec{M} = -\dfrac{\vec{B}_0}{\mu_0} = -\dfrac{(0.130\ T)\hat{i}}{\mu_0} = -(1.03\times 10^5\ \text{A/m})\hat{i}.$ Outside the superconductor, $\vec{B} = \vec{B}_0 = (0.130\ T)\hat{i}$ and $\vec{M} = 0.$

(b) The field is greater than the critical field and $\vec{B} = \vec{B}_0 = (0.260\ \text{T})\hat{i},$ both inside and outside the superconductor.

EVALUATE: Below the critical field the external field is expelled from the superconducting material.

29.43. **IDENTIFY:** Apply $\vec{B} = \vec{B}_0 + \mu_0\vec{M}.$

SET UP: When the magnetic flux is expelled from the material the magnetic field $\vec{B}$ in the material is zero. When the material is completely normal, the magnetization is close to zero.

EXECUTE: **(a)** When $\vec{B}_0$ is just under $\vec{B}_{c1}$ (threshold of superconducting phase), the magnetic field in the material must be zero, and $\vec{M} = -\dfrac{\vec{B}_{c1}}{\mu_0} = -\dfrac{(55\times 10^{-3}\ \text{T})\hat{i}}{\mu_0} = -(4.38\times 10^4\ \text{A/m})\hat{i}.$

(b) When $\vec{B}_0$ is just over $\vec{B}_{c2}$ (threshold of normal phase), there is zero magnetization, and $\vec{B} = \vec{B}_{c2} = (15.0\ \text{T})\hat{i}.$

EVALUATE: Between B_{c1} and B_{c2} there are filaments of normal phase material and there is magnetic field along these filaments.

29.44. **IDENTIFY** and **SET UP:** Use Faraday's law to calculate the magnitude of the induced emf and Lenz's law to determine its direction. Apply Ohm's law to calculate *I*. Use Eq.(25.10) to calculate the resistance of the coil.

(a) EXECUTE: The angle ϕ between the normal to the coil and the direction of $\vec{B}$ is $30.0°$.

$|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = (N\pi r^2)(dB/dt)$ and $I = |\mathcal{E}|/R.$

For $t < 0$ and $t > 1.00$ s, $dB/dt = 0\ |\mathcal{E}| = 0$ and $I = 0.$

For $0 \le t \le 1.00$ s, $dB/dt = (0.120\ \text{T})\ \pi \sin \pi t$

$|\mathcal{E}| = (N\pi r^2)\pi(0.120\ \text{T})\sin \pi t = (0.9475\ \text{V})\sin \pi t$

R for wire: $R_w = \dfrac{\rho L}{A} = \dfrac{\rho L}{\pi r^2};\ \rho = 1.72\times 10^{-8}\ \Omega\cdot\text{m},\ r = 0.0150\times 10^{-3}\ \text{m}$

$L = Nc = N2\pi r = (500)(2\pi)(0.0400\ \text{m}) = 125.7\ \text{m}$

$R_w = 3058\ \Omega$ and the total resistance of the circuit is $R = 3058\ \Omega + 600\ \Omega = 3658\ \Omega$

$I = |\mathcal{E}|/R = (0.259\ \text{mA})\sin \pi t.$ The graph of *I* versus *t* is sketched in Figure 29.44a.

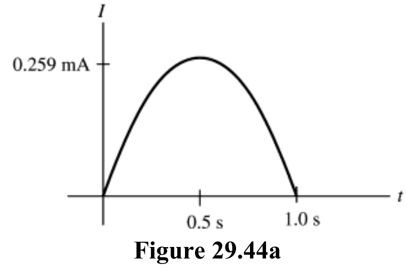
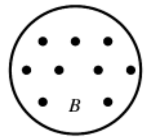


Figure 29.44a

(b) The coil and the magnetic field are shown in Figure 29.44b.



B increasing so Φ_B is $\odot$
and increasing. Φ_B is $\otimes$
so I is clockwise

Figure 29.44b

EVALUATE: The long length of small diameter wire used to make the coil has a rather large resistance, larger than the resistance of the 600-Ω resistor connected to it in the circuit. The flux has a cosine time dependence so the rate of change of flux and the current have a sine time dependence. There is no induced current for $t < 0$ or $t > 1.00$ s.

29.45. **IDENTIFY:** Apply Faraday's law and Lenz's law.

SET UP: For a discharging RC circuit, $i(t) = \frac{V_0}{R}e^{-t/RC}$, where V_0 is the initial voltage across the capacitor. The resistance of the small loop is $(25)(0.600\text{ m})(1.0\ \Omega/\text{m}) = 15.0\ \Omega$.

EXECUTE: **(a)** The large circuit is an RC circuit with a time constant of $\tau = RC = (10\ \Omega)(20\times10^{-6}\text{ F}) = 200\ \mu\text{s}$. Thus, the current as a function of time is $i = \big((100\text{ V})/(10\ \Omega)\big)\,e^{-t/200\ \mu\text{s}}$. At $t = 200\ \mu\text{s}$, we obtain $i = (10\text{ A})(e^{-1}) = 3.7$ A.

(b) Assuming that only the long wire nearest the small loop produces an appreciable magnetic flux through the small loop and referring to the solution of Exercise 29.7 we obtain $\Phi_B = \int_c^{c+a}\frac{\mu_0 ib}{2\pi r}dr = \frac{\mu_0 ib}{2\pi}\ln\left(1+\frac{a}{c}\right)$. Therefore, the emf induced in the small loop at $t = 200\mu\text{s}$ is $\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\mu_0 b}{2\pi}\ln\left(1+\frac{a}{c}\right)\frac{di}{dt}$.

$\mathcal{E} = -\frac{(4\pi\times10^{-7}\text{ Wb/A}\cdot\text{m}^2)(0.200\text{ m})}{2\pi}\ln(3.0)\left(-\frac{3.7\text{ A}}{200\times10^{-6}\text{s}}\right) = +0.81\text{ mV}$. Thus, the induced current in the small loop is $i' = \frac{\mathcal{E}}{R} = \frac{0.81\text{ mV}}{15.0\ \Omega} = 54\mu\text{A}$.

(c) The magnetic field from the large loop is directed out of the page within the small loop.The induced current will act to oppose the decrease in flux from the large loop. Thus, the induced current flows counterclockwise.

EVALUATE: **(d)** Three of the wires in the large loop are too far away to make a significant contribution to the flux in the small loop–as can be seen by comparing the distance c to the dimensions of the large loop.

29.46. **IDENTIFY:** A changing magnetic field causes a changing flux through a coil and therefore induces an emf in the coil.

SET UP: Faraday's law says that the induced emf is $\mathcal{E} = -\frac{d\Phi_B}{dt}$ and the magnetic flux through a coil is defined as $\Phi_B = BA\cos\phi$.

EXECUTE: In this case, $\Phi_B = BA$, where A is constant. So the emf is proportional to the *negative* slope of the magnetic field. The result is shown in Figure 29.46.

EVALUATE: It is the rate at which the magnetic field is *changing*, not the field's magnitude, that determines the induced emf. When the field is constant, even though it may have a large value, the induced emf is zero.

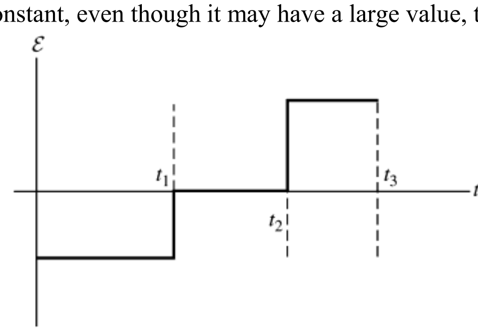


Figure 29.46

29.47. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: Let the flux through the loop due to the current be positive.

EXECUTE: **(a)** $\Phi_B = BA = \frac{\mu_0 i}{2a}\pi a^2 = \frac{\mu_0 i\pi a}{2}$.

(b) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = iR \Rightarrow -\dfrac{d}{dt}\left(\dfrac{\mu_0 i\pi a}{2}\right) = -\dfrac{\mu_0\pi a}{2}\dfrac{di}{dt} = iR \Rightarrow \dfrac{di}{dt} = -i\dfrac{2R}{\mu_0\pi a}$

(c) Solving $\dfrac{di}{i} = -dt\dfrac{2R}{\mu_0\pi a}$ for $i(t)$ yields $i(t) = i_0 e^{-t(2R/\mu_0\pi a)}$.

(d) We want $i(t) = i_0(0.010) = i_0 e^{-t(2R/\mu_0\pi a)}$, so $\ln(0.010) = -t(2R/\mu_0\pi a)$ and

$t = -\dfrac{\mu_0\pi a}{2R}\ln(0.010) = -\dfrac{\mu_0\pi(0.50\text{ m})}{2(0.10\ \Omega)}\ln(0.010) = 4.55\times10^{-5}$ s.

EVALUATE: **(e)** We can ignore the self-induced currents because it takes only a very short time for them to die out.

29.48. **IDENTIFY:** A changing magnetic field causes a changing flux through a coil and therefore induces an emf in the coil.

SET UP: Faraday's law says that the induced emf is $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$ and the magnetic flux through a coil is defined as $\Phi_B = BA\cos\phi$.

EXECUTE: In this case, $\Phi_B = BA$, where A is constant. So the emf is proportional to the *negative* slope of the magnetic field. The result is shown in Figure 29.48.

EVALUATE: It is the rate at which the magnetic field is *changing*, not the field's magnitude, that determines the induced emf. When the field is constant, even though it may have a large value, the induced emf is zero.

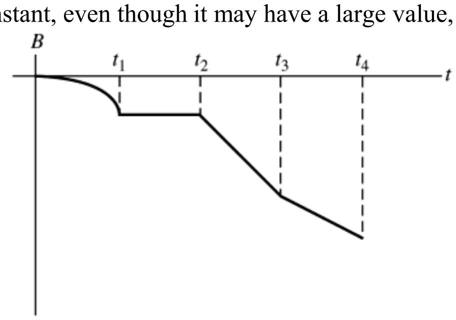


Figure 29.48

29.49. **(a) IDENTIFY:** (i) $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right|$. The flux is changing because the magnitude of the magnetic field of the wire decreases with distance from the wire. Find the flux through a narrow strip of area and integrate over the loop to find the total flux.

SET UP:

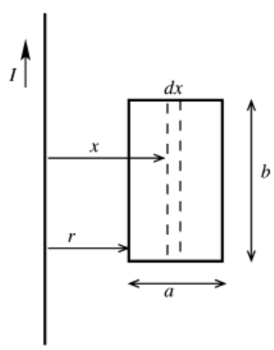


Figure 29.49a

Consider a narrow strip of width dx and a distance x from the long wire, as shown in Figure 29.49a. The magnetic field of the wire at the strip is $B = \mu_0 I/2\pi x$. The flux through the strip is $d\Phi_B = Bb\ dx = (\mu_0 Ib/2\pi)(dx/x)$

EXECUTE: The total flux through the loop is $\Phi_B = \int d\Phi_B = \left(\dfrac{\mu_0 Ib}{2\pi}\right)\int_r^{r+a}\dfrac{dx}{x}$

$$\Phi_B = \left(\frac{\mu_0 Ib}{2\pi}\right)\ln\left(\frac{r+a}{r}\right)$$

$$\frac{d\Phi_B}{dt} = \frac{d\Phi_B}{dt}\frac{dr}{dt} = \frac{\mu_0 Ib}{2\pi}\left(-\frac{a}{r(r+a)}\right)v$$

$$|\mathcal{E}| = \frac{\mu_0 Iabv}{2\pi r(r+a)}$$

(ii) **IDENTIFY:** $\mathcal{E} = Bvl$ for a bar of length l moving at speed v perpendicular to a magnetic field B. Calculate the induced emf in each side of the loop, and combine the emfs according to their polarity.

SET UP: The four segments of the loop are shown in Figure 29.49b.

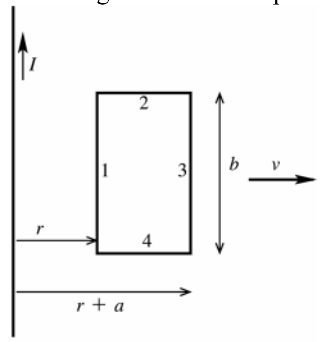


Figure 29.49b

EXECUTE: The emf in each side of the loop is $\mathcal{E}_1 = \left(\dfrac{\mu_0 I}{2\pi r}\right)vb$,

$$\mathcal{E}_2 = \left(\frac{\mu_0 I}{2\pi r(r+a)}\right)vb, \quad \mathcal{E}_2 = \mathcal{E}_4 = 0$$

Both emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ are directed toward the top of the loop so oppose each other. The net emf is

$$\mathcal{E} = \mathcal{E}_1 - \mathcal{E}_2 = \frac{\mu_0 Ivb}{2\pi}\left(\frac{1}{r} - \frac{1}{r+a}\right) = \frac{\mu_0 Iabv}{2\pi r(r+a)}$$

This expression agrees with what was obtained in (i) using Faraday's law.

(b) (i) **IDENTIFY** and **SET UP:** The flux of the induced current opposes the change in flux.

EXECUTE: $\vec{B}$ is $\otimes$. Φ_B is $\otimes$ and decreasing, so the flux Φ_{ind} of the induced current is $\otimes$ and the current is clockwise.

(ii) **IDENTIFY** and **SET UP:** Use the right-hand rule to find the force on the positive charges in each side of the loop. The forces on positive charges in segments 1 and 2 of the loop are shown in Figure 29.49c.

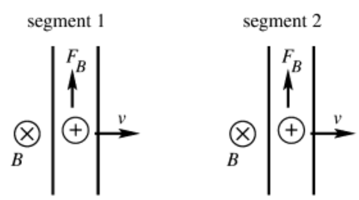


Figure 29.49c

EXECUTE: B is larger at segment 1 since it is closer to the long wire, so F_B is larger in segment 1 and the induced current in the loop is clockwise. This agrees with the direction deduced in (i) using Lenz's law.

(c) EVALUATE: When $v = 0$ the induced emf should be zero; the expression in part (a) gives this. When $a \to 0$ the flux goes to zero and the emf should approach zero; the expression in part (a) gives this. When $r \to \infty$ the magnetic field through the loop goes to zero and the emf should go to zero; the expression in part (a) gives this.

29.50. **IDENTIFY:** Apply Faraday's law.

SET UP: For rotation about the *y*-axis the situation is the same as in Examples 29.4 and 29.5 and we can apply the results from those examples.

EXECUTE: **(a)** Rotating about the *y*-axis: the flux is given by $\Phi_B = BA\cos\phi$ and

$$\mathcal{E}_{\max} = \left|\frac{d\Phi_B}{dt}\right| = \omega BA = (35.0 \text{ rad/s})(0.450 \text{ T})(6.00\times10^{-2} \text{ m}) = 0.945 \text{ V}.$$

(b) Rotating about the *x*-axis: $\dfrac{d\Phi_\text{B}}{dt} = 0$ and $\mathcal{E} = 0$.

(c) Rotating about the *z*-axis: the flux is given by $\Phi_B = BA\cos\phi$ and

$$\mathcal{E}_{\max} = \left|\frac{d\Phi_B}{dt}\right| = \omega BA = (35.0 \text{ rad/s})(0.450 \text{ T})(6.00\times10^{-2} \text{ m}) = 0.945 \text{ V}.$$

EVALUATE: The maximum emf is the same if the loop is rotated about an edge parallel to the *z*-axis as it is when it is rotated about the *z*-axis.

29.51. **IDENTIFY:** Apply the results of Example 29.4, so $\mathcal{E}_{\max} = N\omega BA$ for *N* loops.

SET UP: For the minimum ω, let the rotating loop have an area equal to the area of the uniform magnetic field, so $A = (0.100 \text{ m})^2$.

EXECUTE: $N = 400$, $B = 1.5 \text{ T}$, $A = (0.100 \text{ m})^2$ and $\mathcal{E}_{\max} = 120 \text{ V}$ gives

$\omega = \mathcal{E}_{\max}/NBA = (20 \text{ rad/s})(1 \text{ rev}/2\pi \text{ rad})(60 \text{ s}/1 \text{ min}) = 190 \text{ rpm}.$

EVALUATE: In $\mathcal{E}_{\max} = \omega BA$, ω is in rad/s.

29.52. **IDENTIFY:** Apply the results of Example 29.4, generalized to N loops: $\mathcal{E}_{\max} = N\omega BA$. $v = r\omega$.

SET UP: In the expression for $\mathcal{E}_{\max}$, ω must be in rad/s. $30\text{ rpm} = 3.14\text{ rad/s}$

EXECUTE: **(a)** Solving for A we obtain $A = \dfrac{\mathcal{E}_0}{\omega NB} = \dfrac{9.0\text{ V}}{(3.14\text{ rad/s})(2000\text{ turns})(8.0\times 10^{-5}\text{ T})} = 18\text{ m}^2$.

(b) Assuming a point on the coil at maximum distance from the axis of rotation we have

$$v = r\omega = \sqrt{\frac{A}{\pi}}\,\omega = \sqrt{\frac{18\text{ m}^2}{\pi}}\,(3.14\text{ rad/s}) = 7.5\text{ m/s}.$$

EVALUATE: The device is not very feasible. The coil would need a rigid frame and the effects of air resistance would be appreciable.

29.53. **IDENTIFY:** Apply Faraday's law in the form $\mathcal{E}_{\text{av}} = -N\dfrac{\Delta\Phi_B}{\Delta t}$ to calculate the average emf. Apply Lenz's law to calculate the direction of the induced current.

SET UP: $\Phi_B = BA$. The flux changes because the area of the loop changes.

EXECUTE: **(a)** $\mathcal{E}_{\text{av}} = \left|\dfrac{\Delta\Phi_B}{\Delta t}\right| = B\left|\dfrac{\Delta A}{\Delta t}\right| = B\dfrac{\pi r^2}{\Delta t} = (0.950\text{ T})\dfrac{\pi(0.0650/2\text{ m})^2}{0.250\text{ s}} = 0.0126\text{ V}.$

(b) Since the magnetic field is directed into the page and the magnitude of the flux through the loop is decreasing, the induced current must produce a field that goes into the page. Therefore the current flows from point a through the resistor to point b.

EVALUATE: Faraday's law can be used to find the direction of the induced current. Let $\vec{A}$ be into the page. Then Φ_B is positive and decreasing in magnitude, so $d\Phi_B/dt < 0$. Therefore $\mathcal{E} > 0$ and the induced current is clockwise around the loop.

29.54. **IDENTIFY:** By Lenz's law, the induced current flows to oppose the flux change that caused it.

SET UP: When the switch is suddenly closed with an uncharged capacitor, the current in the outer circuit immediately increases from zero to its maximum value. As the capacitor gets charged, the current in the outer circuit gradually decreases to zero.

EXECUTE: **(a)** (i) The current in the outer circuit is suddenly increasing and is in a counterclockwise direction. The magnetic field through the inner circuit is out of the paper and increasing. The magnetic flux through the inner circuit is increasing, so the induced current in the inner circuit is clockwise (a to b) to oppose the flux increase. (ii) The current in the outer circuit is still counterclockwise but is now decreasing, so the magnetic field through the inner circuit is out of the page but decreasing. The flux through the inner circuit is now decreasing, so the induced current is counterclockwise (b to a) to oppose the flux decrease.

(b) The graph is sketched in Figure 29.54.

EVALUATE: Even though the current in the outer circuit does not change direction, the current in the inner circuit does as the flux through it changes from increasing to decreasing.

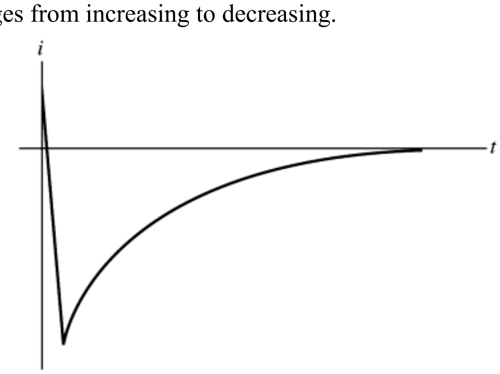


Figure 29.54

29.55. **IDENTIFY:** Use Faraday's law to calculate the induced emf and Ohm's law to find the induced current. Use Eq.(27.19) to calculate the magnetic force F_I on the induced current. Use the net force $F - F_I$ in Newton's 2nd law to calculate the acceleration of the rod and use that to describe its motion.

(a) SET UP: The forces in the rod are shown in Figure 29.55a.

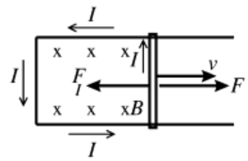


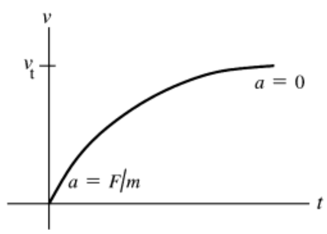
Figure 29.55a

EXECUTE: $|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = BLv$

$I = \frac{BLv}{R}$

Use $\mathcal{E} = -\frac{d\Phi_B}{dt}$ to find the direction of I: Let $\vec{A}$ be into the page. Then $\Phi_B > 0$. The area of the circuit is increasing, so $\frac{d\Phi_B}{dt} > 0$. Then $\mathcal{E} < 0$ and with our direction for $\vec{A}$ this means that $\mathcal{E}$ and I are counterclockwise, as shown in the sketch. The force F_I on the rod due to the induced current is given by $\vec{F}_I = I\vec{l} \times \vec{B}$. This gives $\vec{F}_I$ to the left with magnitude $F_I = ILB = (BLv/R)LB = B^2L^2v/R$. Note that $\vec{F}_I$ is directed to oppose the motion of the rod, as required by Lenz's law.

EVALUATE: The net force on the rod is $F - F_I$, so its acceleration is $a = (F - F_I)/m = (F - B^2L^2v/R)/m$. The rod starts with $v = 0$ and $a = F/m$. As the speed v increases the acceleration a decreases. When $a = 0$ the rod has reached its terminal speed v_t. The graph of v versus t is sketched in Figure 29.55b.



(Recall that a is the slope of the tangent to the v versus t curve.)

Figure 29.55b

(b) EXECUTE: $v = v_t$ when $a = 0$ so $\frac{F - B^2L^2v_t/R}{m} = 0$ and $v_t = \frac{RF}{B^2L^2}$.

EVALUATE: A large F produces a large v_t. If B is larger, or R is smaller, the induced current is larger at a given v so F_I is larger and the terminal speed is less.

29.56. **IDENTIFY:** Apply Newton's 2nd law to the bar. The bar will experience a magnetic force due to the induced current in the loop. Use $a = dv/dt$ to solve for v. At the terminal speed, $a = 0$.

SET UP: The induced emf in the loop has a magnitude BLv. The induced emf is counterclockwise, so it opposes the voltage of the battery, $\mathcal{E}$.

EXECUTE: **(a)** The net current in the loop is $I = \frac{\mathcal{E} - BLv}{R}$. The acceleration of the bar is $a = \frac{F}{m} = \frac{ILB\sin(90°)}{m} = \frac{(\mathcal{E} - BLv)LB}{mR}$. To find $v(t)$, set $\frac{dv}{dt} = a = \frac{(\mathcal{E} - BLv)LB}{mR}$ and solve for v using the method of separation of variables:

$$\int_0^v \frac{dv}{(\mathcal{E} - BLv)} = \int_0^t \frac{LB}{mR}dt \rightarrow v = \frac{\mathcal{E}}{BL}(1 - e^{-B^2L^2t/mR}) = (10\text{ m/s})(1 - e^{-t/3.1\text{ s}})$$

The graph of v versus t is sketched in Figure 29.56. Note that the graph of this function is similar in appearance to that of a charging capacitor.

(b) Just after the switch is closed, $v = 0$ and $I = \mathcal{E}/R = 2.4$ A, $F = ILB = 2.88$ N and $a = F/m = 3.2\text{ m/s}^2$.

(c) When $v = 2.0$ m/s, $a = \frac{[12\text{ V} - (1.5\text{ T})(0.8\text{ m})(2.0\text{ m/s})](0.8\text{ m})(1.5\text{ T})}{(0.90\text{ kg})(5.0\ \Omega)} = 2.6\text{ m/s}^2$.

(d) Note that as the speed increases, the acceleration decreases. The speed will asymptotically approach the terminal speed $\frac{\mathcal{E}}{BL} = \frac{12\text{ V}}{(1.5\text{ T})(0.8\text{ m})} = 10$ m/s, which makes the acceleration zero.

EVALUATE: The current in the circuit is counterclockwise and the magnetic force on the bar is to the right. The energy that appears as kinetic energy of the moving bar is supplied by the battery.

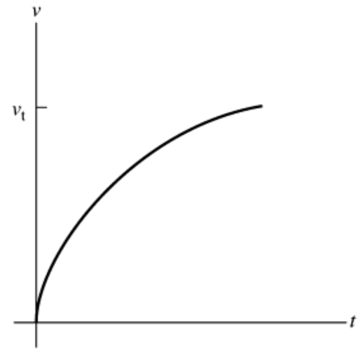


Figure 29.56

29.57. **IDENTIFY:** Apply $\mathcal{E} = BvL$. Use $\sum \vec{F} = m\vec{a}$ applied to the satellite motion to find the speed v of the satellite.

SET UP: The gravitational force on the satellite is $F_g = G\frac{mm_E}{r^2}$, where m is the mass of the satellite and r is the radius of its orbit.

EXECUTE: $B = 8.0\times10^{-5}$ T, $L = 2.0$ m. $G\frac{mm_E}{r^2} = m\frac{v^2}{r}$ and $r = 400\times10^3\ \text{m} + R_E$ gives $v = \sqrt{\frac{Gm_E}{r}} = 7.665\times10^3$ m/s.

Using this v in $\mathcal{E} = vBL$ gives $\mathcal{E} = (8.0\times10^{-5}\ \text{T})(7.665\times10^3\ \text{m/s})(2.0\ \text{m}) = 1.2$ V.

EVALUATE: The induced emf is large enough to be measured easily.

29.58. **IDENTIFY:** The induced emf is $\mathcal{E} = BvL$, where L is measured in a direction that is perpendicular to both the magnetic field and the velocity of the bar.

SET UP: The magnetic force pushed positive charge toward the high potential end of the bullet.

EXECUTE: **(a)** $\mathcal{E} = BLv = (8\times10^{-5}\ \text{T})(0.004\ \text{m})(300\ \text{m/s}) = 96\ \mu\text{V}$. Since a positive charge moving to the east would be deflected upward, the top of the bullet will be at a higher potential.

(b) For a bullet that travels south, $\vec{v}$ and $\vec{B}$ are along the same line, there is no magnetic force and the induced emf is zero.

(c) If $\vec{v}$ is horizontal, the magnetic force on positive charges in the bullet is either upward or downward, perpendicular to the line between the front and back of the bullet. There is no emf induced between the front and back of the bullet.

EVALUATE: Since the velocity of a bullet is always in the direction from the back to the front of the bullet, and since the magnetic force is perpendicular to the velocity, there is never an induced emf between the front and back of the bullet, no matter what the direction of the magnetic field is.

29.59. **IDENTIFY:** Find the magnetic field at a distance r from the center of the wire. Divide the rectangle into narrow strips of width dr, find the flux through each strip and integrate to find the total flux.

SET UP: Example 28.8 uses Ampere's law to show that the magnetic field inside the wire, a distance r from the axis, is $B(r) = \mu_0 Ir/2\pi R^2$.

EXECUTE: Consider a small strip of length W and width dr that is a distance r from the axis of the wire, as shown in Figure 29.59. The flux through the strip is $d\Phi_B = B(r)W\,dr = \frac{\mu_0 IW}{2\pi R^2}r\,dr$. The total flux through the rectangle is

$$\Phi_B = \int d\Phi_B = \left(\frac{\mu_0 IW}{2\pi R^2}\right)\int_0^R r\,dr = \frac{\mu_0 IW}{4\pi}.$$

EVALUATE: Note that the result is independent of the radius R of the wire.

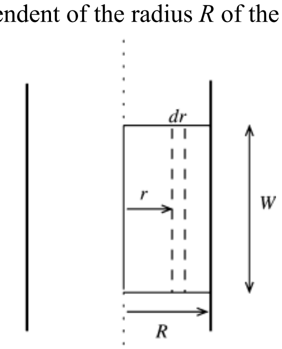


Figure 29.59

29.60. **IDENTIFY:** Apply Faraday's law to calculate the magnitude and direction of the induced emf.

SET UP: Let $\vec{A}$ be directed out of the page in Figure 29.50 in the textbook. This means that counterclockwise emf is positive.

EXECUTE: **(a)** $\Phi_B = BA = B_0\pi r_0^2(1-3(t/t_0)^2 + 2(t/t_0)^3)$.

(b) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -B_0\pi r_0^2\dfrac{d}{dt}(1-3(t/t_0)^2+2(t/t_0)^3) = -\dfrac{B_0\pi r_0^2}{t_0}(-6(t/t_0)+6(t/t_0)^2)$. $\mathcal{E} = -\dfrac{6\,B_0\pi r_0^2}{t_0} = \left(\left(\dfrac{t}{t_0}\right)^2 - \left(\dfrac{t}{t_0}\right)\right)$. At $t = 5.0\times10^{-3}\text{s}$, $\mathcal{E} = -\dfrac{6B_0\pi(0.0420\text{ m})^2}{0.010\text{ s}}\left(\left(\dfrac{5.0\times10^{-3}\text{ s}}{0.010\text{ s}}\right)^2 - \left(\dfrac{5.0\times10^{-3}\text{ s}}{0.010\text{ s}}\right)\right) = 0.0665\text{ V}$. $\mathcal{E}$ is positive so it is counterclockwise.

(c) $I = \dfrac{\mathcal{E}}{R_{\text{total}}} \Rightarrow R_{\text{total}} = r + R = \dfrac{\mathcal{E}}{I} \Rightarrow r = \dfrac{0.0665\text{ V}}{3.0\times10^{-3}\text{ A}} - 12\ \Omega = 10.2\ \Omega$.

(d) Evaluating the emf at $t = 1.21\times10^{-2}$ s and using the equations of part (b), $\mathcal{E} = -0.0676$ V, and the current flows clockwise, from *b* to *a* through the resistor.

(e) $\mathcal{E} = 0$ when $0 = \left(\left(\dfrac{t}{t_0}\right)^2 - \left(\dfrac{t}{t_0}\right)\right)$. $1 = \dfrac{t}{t_0}$ and $t = t_0 = 0.010$ s.

EVALUATE: At $t = t_0$, $B = 0$. At $t = 5.00\times10^{-3}$ s, $\vec{B}$ is in the $+\hat{k}$ direction and is decreasing in magnitude. Lenz's law therefore says $\mathcal{E}$ is counterclockwise. At $t = 0.0121$ s, $\vec{B}$ is in the $+\hat{k}$ direction and is increasing in magnitude. Lenz's law therefore says $\mathcal{E}$ is clockwise. These results for the direction of $\mathcal{E}$ agree with the results we obtained from Faraday's law.

29.61. **(a)** and **(b)** **IDENTIFY** and **Set Up:**

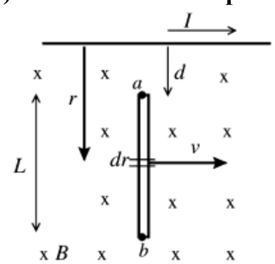


Figure 29.61a

The magnetic field of the wire is given by $B = \dfrac{\mu_0 I}{2\pi r}$ and varies along the length of the bar. At every point along the bar $\vec{B}$ has direction into the page. Divide the bar up into thin slices, as shown in Figure 29.61a.

EXECUTE: The emf $d\mathcal{E}$ induced in each slice is given by $d\mathcal{E} = \vec{v}\times\vec{B}\cdot d\vec{l}$. $\vec{v}\times\vec{B}$ is directed toward the wire, so $d\mathcal{E} = -vB\,dr = -v\left(\dfrac{\mu_0 I}{2\pi r}\right)dr$. The total emf induced in the bar is

$$V_{ba} = \int_a^b d\mathcal{E} = -\int_d^{d+L}\left(\frac{\mu_0 Iv}{2\pi r}\right)dr = -\frac{\mu_0 Iv}{2\pi}\int_d^{d+L}\frac{dr}{r} = -\frac{\mu_0 Iv}{2\pi}[\ln(r)]_d^{d+L}$$

$$V_{ba} = -\frac{\mu_0 Iv}{2\pi}(\ln(d+L) - \ln(d)) = -\frac{\mu_0 Iv}{2\pi}\ln(1+L/d)$$

EVALUATE: The minus sign means that V_{ba} is negative, point *a* is at higher potential than point *b*. (The force $\vec{F} = q\vec{v}\times\vec{B}$ on positive charge carriers in the bar is towards *a*, so *a* is at higher potential.) The potential difference increases when *I* or *v* increase, or *d* decreases.

(c) **IDENTIFY:** Use Faraday's law to calculate the induced emf.

SET UP: The wire and loop are sketched in Figure 29.61b.

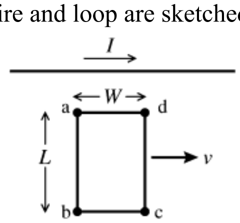


Figure 29.61b

EXECUTE: As the loop moves to the right the magnetic flux through it doesn't change. Thus $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = 0$ and $I = 0$.

EVALUATE: This result can also be understood as follows. The induced emf in section *ab* puts point *a* at higher potential; the induced emf in section *dc* puts point *d* at higher potential. If you travel around the loop then these two induced emf's sum to zero. There is no emf in the loop and hence no current.

29.62. **IDENTIFY:** $\mathcal{E} = vBL$, where v is the component of velocity perpendicular to the field direction and perpendicular to the bar.

SET UP: Wires A and C have a length of 0.500 m and wire D has a length of $\sqrt{2(0.500\text{ m})^2} = 0.707$ m.

EXECUTE: Wire A: $\vec{v}$ is parallel to $\vec{B}$, so the induced emf is zero.

Wire C: $\vec{v}$ is perpendicular to $\vec{B}$. The component of $\vec{v}$ perpendicular to the bar is $v\cos 45°$.
$\mathcal{E} = (0.350\text{ m/s})(\cos 45°)(0.120\text{ T})(0.500\text{ m}) = 0.0148\text{ V}.$

Wire D: $\vec{v}$ is perpendicular to $\vec{B}$. The component of $\vec{v}$ perpendicular to the bar is $v\cos 45°$.
$\mathcal{E} = (0.350\text{ m/s})(\cos 45°)(0.120\text{ T})(0.707\text{ m}) = 0.0210\text{ V}.$

EVALUATE: The induced emf depends on the angle between $\vec{v}$ and $\vec{B}$ and also on the angle between $\vec{v}$ and the bar.

29.63. **(a) IDENTIFY:** Use the expression for motional emf to calculate the emf induced in the rod.

SET UP: The rotating rod is shown in Figure 29.63a.

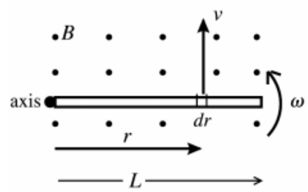


Figure 29.63a

The emf induced in a thin slice is $d\mathcal{E} = \vec{v} \times \vec{B} \cdot d\vec{l}$.

EXECUTE: Assume that $\vec{B}$ is directed out of the page. Then $\vec{v} \times \vec{B}$ is directed radially outward and $dl = dr$, so $\vec{v} \times \vec{B} \cdot d\vec{l} = vB\, dr$

$v = r\omega$ so $d\mathcal{E} = \omega Br\, dr$.

The $d\mathcal{E}$ for all the thin slices that make up the rod are in series so they add:

$$\mathcal{E} = \int d\mathcal{E} = \int_0^L \omega Br\, dr = \tfrac{1}{2}\omega BL^2 = \tfrac{1}{2}(8.80\text{ rad/s})(0.650\text{ T})(0.240\text{ m})^2 = 0.165\text{ V}$$

EVALUATE: $\mathcal{E}$ increases with ω, B or L^2.

(b) No current flows so there is no IR drop in potential. Thus the potential difference between the ends equals the emf of 0.165 V calculated in part (a).

(c) SET UP: The rotating rod is shown in Figure 29.63b.

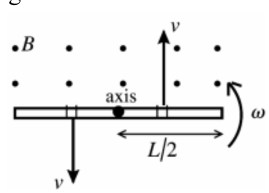


Figure 29.63b

EXECUTE: The emf between the center of the rod and each end is $\mathcal{E} = \tfrac{1}{2}\omega B(L/2)^2 = \tfrac{1}{4}(0.165\text{ V}) = 0.0412\text{ V}$, with the direction of the emf from the center of the rod toward each end. The emfs in each half of the rod thus oppose each other and there is no net emf between the ends of the rod.

EVALUATE: ω and B are the same as in part (a) but L of each half is $\tfrac{1}{2}L$ for the whole rod. $\mathcal{E}$ is proportional to L^2, so is smaller by a factor of $\tfrac{1}{4}$.

29.64. **IDENTIFY:** The power applied by the person in moving the bar equals the rate at which the electrical energy is dissipated in the resistance.

SET UP: From Example 29.7, the power required to keep the bar moving at a constant velocity is $P = \dfrac{(BLv)^2}{R}$.

EXECUTE: **(a)** $R = \dfrac{(BLv)^2}{P} = \dfrac{[(0.25\text{ T})(3.0\text{ m})(2.0\text{ m/s})]^2}{25\text{ W}} = 0.090\ \Omega.$

(b) For a 50 W power dissipation we would require that the resistance be decreased to half the previous value.

(c) Using the resistance from part (a) and a bar length of 0.20 m,

$$P = \frac{(BLv)^2}{R} = \frac{[(0.25\text{ T})(0.20\text{ m})(2.0\text{ m/s})]^2}{0.090\ \Omega} = 0.11\text{ W}.$$

EVALUATE: When the bar is moving to the right the magnetic force on the bar is to the left and an applied force directed to the right is required to maintain constant speed. When the bar is moving to the left the magnetic force on the bar is to the right and an applied force directed to the left is required to maintain constant speed.

29.65. **(a) IDENTIFY:** Use Faraday's law to calculate the induced emf, Ohm's law to calculate I, and Eq.(27.19) to calculate the force on the rod due to the induced current.

SET UP: The force on the wire is shown in Figure 29.65.

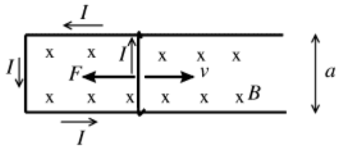


Figure 29.65

EXECUTE: When the wire has speed v the induced emf is $\mathcal{E} = Bva$ and the induced current is $I = \mathcal{E}/R = \dfrac{Bva}{R}$

The induced current flows upward in the wire as shown, so the force $\vec{F} = I\vec{l} \times \vec{B}$ exerted by the magnetic field on the induced current is to the left. $\vec{F}$ opposes the motion of the wire, as it must by Lenz's law. The magnitude of the force is $F = IaB = B^2a^2v/R$.

(b) Apply $\sum \vec{F} = m\vec{a}$ to the wire. Take $+x$ to be toward the right and let the origin be at the location of the wire at $t = 0$, so $x_0 = 0$.

$\sum F_x = ma_x$ says $-F = ma_x$

$$a_x = -\frac{F}{m} = -\frac{B^2a^2v}{mR}$$

Use this expression to solve for $v(t)$:

$$a_x = \frac{dv}{dt} = -\frac{B^2a^2v}{mR} \text{ and } \frac{dv}{v} = -\frac{B^2a^2}{mR}dt$$

$$\int_{v_0}^{v} \frac{dv'}{v'} = -\frac{B^2a^2}{mR}\int_0^t dt'$$

$$\ln(v) - \ln(v_0) = -\frac{B^2a^2t}{mR}$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{B^2a^2t}{mR} \text{ and } v = v_0e^{-B^2a^2t/mR}$$

Note: At $t = 0$, $v = v_0$ and $v \to 0$ when $t \to \infty$

Now solve for $x(t)$:

$$v = \frac{dx}{dt} = v_0e^{-B^2a^2t/mR} \text{ so } dx = v_0e^{-B^2a^2t/mR}dt$$

$$\int_0^x dx' = \int_0^t v_0e^{-B^2a^2t/mR}dt'$$

$$x = v_0\left(-\frac{mR}{B^2a^2}\right)\left[e^{-B^2a^2t'/mR}\right]_0^t = \frac{mRv_0}{B^2a^2}\left(1 - e^{-B^2a^2t/mR}\right)$$

Comes to rest implies $v = 0$. This happens when $t \to \infty$.

$t \to \infty$ gives $x = \dfrac{mRv_0}{B^2a^2}$. Thus this is the distance the wire travels before coming to rest.

EVALUATE: The motion of the slide wire causes an induced emf and current. The magnetic force on the induced current opposes the motion of the wire and eventually brings it to rest. The force and acceleration depend on v and are constant. If the acceleration were constant, not changing from its initial value of $a_x = -B^2a^2v_0/mR$, then the stopping distance would be $x = -v_0^2/2a_x = mRv_0/2B^2a^2$. The actual stopping distance is twice this.

29.66. **IDENTIFY:** Since the bar is straight and the magnetic field is uniform, integrating $d\mathcal{E} = \vec{v} \times \vec{B} \cdot d\vec{l}$ along the length of the bar gives $\mathcal{E} = (\vec{v} \times \vec{B}) \cdot \vec{L}$

SET UP: $\vec{v} = (4.20 \text{ m/s})\hat{i}$. $\vec{L} = (0.250 \text{ m})(\cos 36.9°\hat{i} + \sin 36.9°\hat{j})$.

EXECUTE: **(a)** $\mathcal{E} = (\vec{v} \times \vec{B}) \cdot \vec{L} = (4.20 \text{ m/s})\hat{i} \times ((0.120 \text{ T})\hat{i} - (0.220 \text{ T})\hat{j} - (0.0900 \text{ T})\hat{k}) \cdot \vec{L}$.

$$\mathcal{E} = \left((0.378 \text{ V/m})\hat{j} - (0.924 \text{ V/m})\hat{k}\right) \cdot \left((0.250 \text{ m})(\cos 36.9°\hat{i} + \sin 36.9°\hat{j})\right).$$

$$\mathcal{E} = (0.378 \text{ V/m})(0.250 \text{ m})\sin 36.9° = 0.0567 \text{ V}.$$

(b) The higher potential end is the end to which positive charges in the rod are pushed by the magnetic force. $\vec{v} \times \vec{B}$ has a positive y-component, so the end of the rod marked + in Figure 29.66 is at higher potential.

EVALUATE: Since $\vec{v}\times\vec{B}$ has nonzero $\hat{j}$ and $\hat{k}$ components, and $\vec{L}$ has nonzero $\hat{i}$ and $\hat{j}$ components, only the $\hat{k}$ component of $\vec{B}$ contributes to $\mathcal{E}$. In fact, $|\mathcal{E}| = |v_x B_z L_y| = (4.20\text{ m/s})(0.0900\text{ T})(0.250\text{ m})\sin 36.9° = 0.0567\text{ V}$.

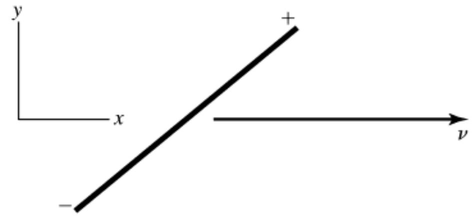


Figure 29.66

29.67. IDENTIFY: Use Eq.(29.10) to calculate the induced electric field at each point and then use $\vec{F} = q\vec{E}$.

SET UP:

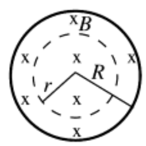


Figure 29.67a

Apply $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ to a concentric circle of radius r, as shown in Figure 29.67a. Take $\vec{A}$ to be into the page, in the direction of $\vec{B}$.

EXECUTE: B increasing then gives $\dfrac{d\Phi_B}{dt} > 0$, so $\oint \vec{E}\cdot d\vec{l}$ is negative. This means that E is tangent to the circle in the counterclockwise direction, as shown in Figure 29.67b.

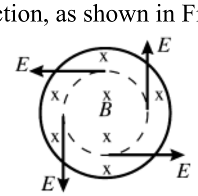


Figure 29.67b

$$\oint \vec{E}\cdot d\vec{l} = -E(2\pi r)$$

$$\frac{d\Phi_B}{dt} = \pi r^2 \frac{dB}{dt}$$

$-E(2\pi r) = -\pi r^2 \dfrac{dB}{dt}$ so $E = \tfrac{1}{2} r \dfrac{dB}{dt}$

point a The induced electric field and the force on q are shown in Figure 29.67c.

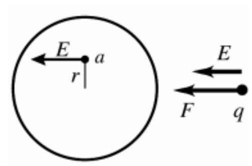


Figure 29.67c

$F = qE = \tfrac{1}{2} q r \dfrac{dB}{dt}$

$\vec{F}$ is to the left

($\vec{F}$ is in the same direction as $\vec{E}$ since q is positive.)

point b The induced electric field and the force on q are shown in Figure 29.67d.

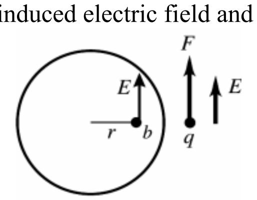


Figure 29.67d

$F = qE = \tfrac{1}{2} q r \dfrac{dB}{dt}$

$\vec{F}$ is toward the top of the page.

point c $r = 0$ here, so $E = 0$ and $F = 0$.

EVALUATE: If there were a concentric conducting ring of radius r in the magnetic field region, Lenz's law tells us that the increasing magnetic field would induce a counterclockwise current in the ring. This agrees with the direction of the force we calculated for the individual positive point charges.

29.68. IDENTIFY: A bar moving in a magnetic field has an emf induced across its ends. The propeller acts as such a bar.

SET UP: Different parts of the propeller are moving at different speeds, so we must integrate to get the total induced emf. The potential induced across an element of length dx is $d\mathcal{E} = vBdx$, where B is uniform.

EXECUTE: **(a)** Call x the distance from the center to an element of length dx, and L the length of the propeller. The speed of dx is $x\omega$, giving $d\mathcal{E} = vBdx = x\omega Bdx$. $\mathcal{E} = \int_0^{L/2} x\omega Bdx = \omega BL^2/8$.

(b) The potential difference is zero since the potential is the same at both ends of the propeller.

(c) $\mathcal{E} = 2\pi\left(\dfrac{220 \text{ rev}}{60 \text{ s}}\right)(0.50\times10^{-4}\text{ T})\dfrac{(2.0\text{ m})^3}{8} = 5.8\times10^{-4}\text{ V} = 0.58\text{ mV}$

EVALUATE: A potential difference of about $\frac{1}{2}$ mV is not large enough to be concerned about in a propeller.

29.69. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: The electric field region is sketched in Figure 29.69.

EXECUTE: $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$. If $\vec{B}$ is constant then $\dfrac{d\Phi_B}{dt} = 0$, so $\oint \vec{E}\cdot d\vec{l} = 0$. $\int_{abcda} \vec{E}\cdot d\vec{l} = E_{ab}L - E_{cd}L = 0$. But $E_{cd} = 0$, so $E_{ab}L = 0$. But since we assumed $E_{ab} \neq 0$, this contradicts Faraday's law. Thus, we can't have a uniform electric field abruptly drop to zero in a region in which the magnetic field is constant.

EVALUATE: If the magnetic field in the region is constant, then the integral $\oint \vec{E}\cdot d\vec{l}$ must be zero.

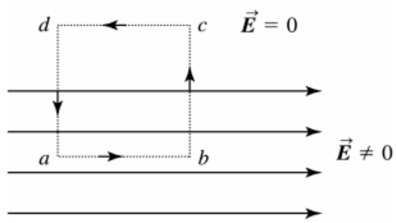


Figure 26.69

29.70. **IDENTIFY** and **SET UP:** At the terminal speed v_t, the upward force F_I exerted on the loop due to the induced current equals the downward force of gravity: $F_I = mg$. Use Eq.(29.6) to find the induced emf in the side of the loop that is totally within the magnetic field. There is no induced emf in the other sides of the loop.

EXECUTE: $\mathcal{E} = Bvs$, $I = Bvs/R$ and $F_I = IsB - B^2s^2v/R$

$$\frac{B^2s^2v_t}{R} = mg \text{ and } v_t = \frac{mgR}{B^2s^2}$$

$$m = \rho_m V = \rho_m(4s)\pi(d/2)^2 = \rho_m \pi d^2$$

$$R = \frac{\rho L}{A} = \frac{\rho_R 4s}{\frac{1}{4}\pi d^2} = \frac{16\rho_R s}{\pi d^2}$$

Using these expressions for m and R gives $v_t = 16\rho_m\rho_R g/B^2$

EVALUATE: We know $\rho_m = 8900\text{ kg/m}^3$ (Table 14.1) and $\rho_R = 1.72\times10^{-8}\ \Omega\cdot\text{m}$ (Table 25.1). Taking $B = 0.5$ T gives $v_t = 9.6$ cm/s.

29.71. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: **(a)** The magnetic field region is sketched in Figure 29.71.

EXECUTE: **(b)** $\oint \vec{B}\cdot d\vec{l} = 0$ (no currents in the region). Using the figure, let $\vec{B} = B_0\hat{i}$ for $y < 0$ and $B = 0$ for $y > 0$. $\int_{abcde} \vec{B}\cdot d\vec{l} = B_{ab}L - B_{cd}L = 0$ but $B_{cd} = 0$. $B_{ab}L = 0$, but $B_{ab} \neq 0$. This is a contradiction and violates Ampere's Law.

EVALUATE: We often describe a magnetic field as being confined to a region, but this result shows that the edges of such a region can't be sharp.

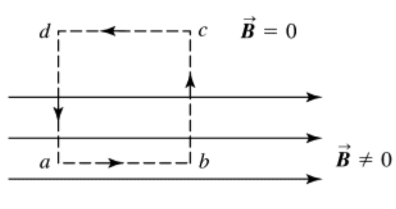


Figure 29.71

29.72. **IDENTIFY** and **SET UP:** Apply Ohm's law to the dielectric to relate the current in the dielectric to the charge on the plates. Use Eq.(25.1) for the current and obtain a differential equation for $q(t)$. Integrate this equation to obtain $q(t)$ and $i(t)$. Use $E = q/\epsilon A$ and Eq.(29.16) to calculate j_D.

EXECUTE: **(a)** Apply Ohm's law to the dielectric: The capacitor is sketched in Figure 29.72.

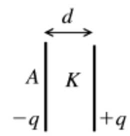


Figure 29.72

$$i(t) = \frac{v(t)}{R}$$

$$v(t) = \frac{q(t)}{C} \text{ and } C = K\frac{\epsilon_0 A}{d}$$

$$v(t) = \left(\frac{d}{K\epsilon_0 A}\right)q(t)$$

The resistance R of the dielectric slab is $R = \rho d/A$. Thus $i(t) = \frac{v(t)}{R} = \left(\frac{q(t)d}{K\epsilon_0 A}\right)\left(\frac{A}{\rho d}\right) = \frac{q(t)}{K\epsilon_0 \rho}$. But the current $i(t)$ in the dielectric is related to the rate of change dq/dt of the charge $q(t)$ on the plates by $i(t) = -dq/dt$ (a positive i in the direction from the + to the – plate of the capacitor corresponds to a decrease in the charge). Using this in the above gives $-\frac{dq}{dt} = \left(\frac{1}{K\rho\epsilon_0}\right)q(t)$. $\frac{dq}{q} = -\frac{dt}{K\rho\epsilon_0}$. Integrate both sides of this equation from $t = 0$, where $q = Q_0$, to a later time t when the charge is $q(t)$. $\int_{Q_0}^{q}\frac{dq}{q} = -\left(\frac{1}{K\rho\epsilon_0}\right)\int_0^t dt$. $\ln\left(\frac{q}{Q_0}\right) = -\frac{t}{K\rho\epsilon_0}$ and $q(t) = Q_0 e^{-t/K\rho\epsilon_0}$. Then

$i(t) = -\frac{dq}{dt} = \left(\frac{Q_0}{K\rho\epsilon_0}\right)e^{-t/K\rho\epsilon_0}$ and $j_C = \frac{i(t)}{A} = \left(\frac{Q_0}{AK\rho\epsilon_0}\right)e^{-t/K\rho\epsilon_0}$. The conduction current flows from the positive to the negative plate of the capacitor.

(b) $E(t) = \frac{q(t)}{\epsilon A} = \frac{q(t)}{K\epsilon_0 A}$

$$j_D(t) = \epsilon\frac{dE}{dt} = K\epsilon_0\frac{dE}{dt} = K\epsilon_0\frac{dq(t)/dt}{K\epsilon_0 A} = -\frac{i_C(t)}{A} = -j_C(t)$$

The minus sign means that $j_D(t)$ is directed from the negative to the positive plate. $\vec{E}$ is from + to – but dE/dt is negative (E decreases) so $j_D(t)$ is from – to +.

EVALUATE: There is no conduction current to and from the plates so the concept of displacement current, with $\vec{j}_D = -\vec{j}_C$ in the dielectric, allows the current to be continuous at the capacitor.

29.73. **IDENTIFY:** The conduction current density is related to the electric field by Ohm's law. The displacement current density is related to the rate of change of the electric field by Eq.(29.16).

SET UP: $dE/dt = \omega E_0 \cos\omega t$

EXECUTE: **(a)** $j_C(\text{max}) = \frac{E_0}{\rho} = \frac{0.450 \text{ V/m}}{2300\ \Omega\cdot\text{m}} = 1.96\times10^{-4}\text{ A/m}^2$

(b) $j_D(\text{max}) = \epsilon_0\left(\frac{dE}{dt}\right)_{\text{max}} = \epsilon_0\omega E_0 = 2\pi\epsilon_0 f E_0 = 2\pi\epsilon_0(120\text{ Hz})(0.450\text{ V/m}) = 3.00\times10^{-9}\text{ A/m}^2$

(c) If $j_C = j_D$ then $\frac{E_0}{\rho} = \omega\epsilon_0 E_0$ and $\omega = \frac{1}{\rho\epsilon_0} = 4.91\times10^7$ rad/s

$$f = \frac{\omega}{2\pi} = \frac{4.91\times10^7\text{ rad/s}}{2\pi} = 7.82\times10^6\text{ Hz.}$$

EVALUATE: **(d)** The two current densities are out of phase by $90°$ because one has a sine function and the other has a cosine, so the displacement current leads the conduction current by $90°$.

29.74. **IDENTIFY:** A current is induced in the loop because of its motion and because of this current the magnetic field exerts a torque on the loop.

SET UP: Each side of the loop has mass $m/4$ and the center of mass of each side is at the center of each side. The flux through the loop is $\Phi_B = BA\cos\phi$.

EXECUTE: **(a)** $\vec{\tau}_g = \sum \vec{r}_{cm} \times m\vec{g}$ summed over each leg.

$$\tau_g = \left(\frac{L}{2}\right)\left(\frac{m}{4}\right)g\sin(90° - \phi) + \left(\frac{L}{2}\right)\left(\frac{m}{4}\right)g\sin(90° - \phi) + (L)\left(\frac{m}{4}\right)g\sin(90° - \phi)$$

$\tau_g = \frac{mgL}{2}\cos\phi$ (clockwise).

$\tau_B = \left|\vec{\tau} \times \vec{B}\right| = IAB\sin\phi$ (counterclockwise).

$I = \frac{\mathcal{E}}{R} = \frac{BA}{R}\frac{d}{dt}\cos\phi = -\frac{BA}{R}\frac{d\phi}{dt}\sin\phi = \frac{BA\omega}{R}\sin\phi$. The current is going counterclockwise looking to the $-\hat{k}$ direction.

Therefore, $\tau_B = \frac{B^2A^2\omega}{R}\sin^2\phi = \frac{B^2L^4\omega}{R}\sin^2\phi$. The net torque is $\tau = \frac{mgL}{2}\cos\phi - \frac{B^2L^4\omega}{R}\sin^2\phi$, opposite to the direction of the rotation.

(b) $\tau = I\alpha$ (I being the moment of inertia). About this axis $I = \frac{5}{12}mL^2$. Therefore,

$$\alpha = \frac{12}{5}\frac{1}{mL^2}\left[\frac{mgL}{2}\cos\phi - \frac{B^2L^4\omega}{R}\sin^2\phi\right] = \frac{6g}{5L}\cos\phi - \frac{12B^2L^2\omega}{5mR}\sin^2\phi.$$

EVALUATE: **(c)** The magnetic torque slows down the fall (since it opposes the gravitational torque).
(d) Some energy is lost through heat from the resistance of the loop.

29.75. **IDENTIFY:** Apply Eq.(29.10).
SET UP: Use an integration path that is a circle of radius r. By symmetry the induced electric field is tangent to this path and constant in magnitude at all points on the path.
EXECUTE: **(a)** The induced electric field at these points is shown in Figure 29.75a.
(b) To work out the amount of the electric field that is in the direction of the loop at a general position, we will use the geometry shown in Figure 29.75b. $E_{loop} = E\cos\theta$ but $E = \frac{\mathcal{E}}{2\pi r} = \frac{\mathcal{E}}{2\pi(a/\cos\theta)} = \frac{\mathcal{E}\cos\theta}{2\pi a}$. Therefore,

$E_{loop} = \frac{\mathcal{E}\cos^2\theta}{2\pi a}$. But $\mathcal{E} = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi r^2\frac{dB}{dt} = \frac{\pi a^2}{\cos^2\theta}\frac{dB}{dt}$, so $E_{loop} = \frac{\pi a^2}{2\pi a}\frac{dB}{dt} = \frac{a}{2}\frac{dB}{dt}$. This is exactly the value for a ring, obtained in Exercise 29.30, and has no dependence on the part of the loop we pick.

(c) $I = \frac{\mathcal{E}}{R} = \frac{A}{R}\frac{dB}{dt} = \frac{L^2}{R}\frac{dB}{dt} = \frac{(0.20\text{ m})^2(0.0350\text{ T/s})}{1.90\ \Omega} = 7.37 \times 10^{-4}$ A.

(d) $\mathcal{E}_{ab} = \frac{1}{8}\mathcal{E} = \frac{1}{8}L^2\frac{dB}{dt} = \frac{(0.20\text{ m})^2(0.0350\text{ T/s})}{8} = 1.75 \times 10^{-4}$ V. But there is potential drop $V = IR = -1.75 \times 10^{-4}$ V, so the potential difference is zero.

EVALUATE: The magnitude of the induced emf between any two points equals the magnitudes of the potential drop due to the current through the resistance of that portion of the loop.

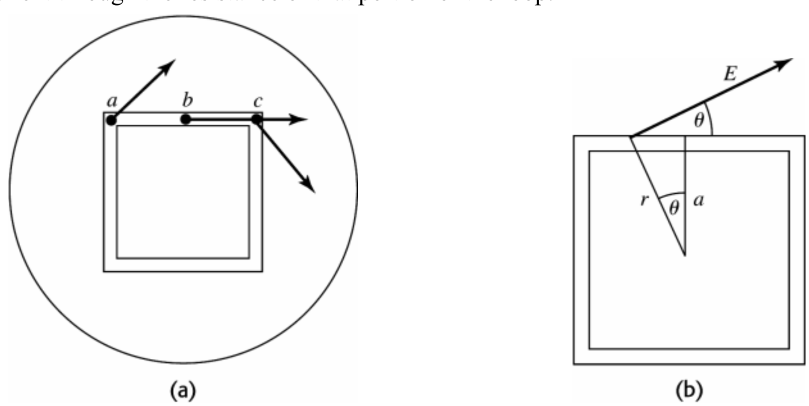


Figure 29.75

29.76. **IDENTIFY:** Apply Eq.(29.10).
SET UP: Use an integration path that is a circle of radius r. By symmetry the induced electric field is tangent to this path and constant in magnitude at all points on the path.
EXECUTE: **(a)** The induced emf at these points is shown in Figure 29.76.
(b) The induced emf on the side ac is zero, because the electric field is always perpendicular to the line ac.

(c) To calculate the total emf in the loop, $\mathcal{E} = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = L^2\frac{dB}{dt}$. $\mathcal{E} = (0.20\text{ m})^2(0.035\text{ T/s}) = 1.40 \times 10^{-3}$ V.

(d) $I = \frac{\mathcal{E}}{R} = \frac{1.40\times10^{-3}\text{ V}}{1.90\ \Omega} = 7.37\times10^{-4}\text{ A}$

(e) Since the loop is uniform, the resistance in length *ac* is one quarter of the total resistance. Therefore the potential difference between *a* and *c* is $V_{ac} = IR_{ac} = (7.37\times10^{-4}\text{ A})(1.90\ \Omega/4) = 3.50\times10^{-4}\text{ V}$ and the point *a* is at a higher potential since the current is flowing from *a* to *c*.

EVALUATE: This loop has the same resistance as the loop in Challenge Problem 29.75 and the induced current is the same.

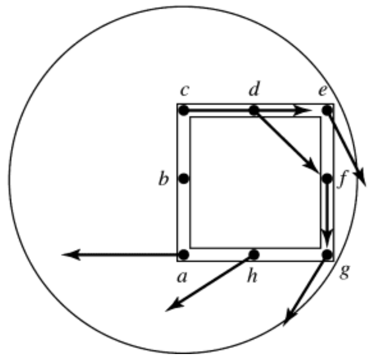


Figure 29.76

29.77. IDENTIFY: The motion of the bar produces an induced current and that results in a magnetic force on the bar.

SET UP: $\vec{F}_B$ is perpendicular to $\vec{B}$, so is horizontal. The vertical component of the normal force equals $mg\cos\phi$, so the horizontal component of the normal force equals $mg\tan\phi$.

EXECUTE: **(a)** As the bar starts to slide, the flux is decreasing, so the current flows to increase the flux, which means it flows from *a* to *b*. $F_B = iLB = \frac{LB}{R}\mathcal{E} = \frac{LB}{R}\frac{d\Phi_B}{dt} = \frac{LB}{R}B\frac{dA}{dt} = \frac{LB^2}{R}(vL\cos\phi) = \frac{vL^2B^2}{R}\cos\phi.$ At the terminal speed the horizontal forces balance, so $mg\tan\phi = \frac{v_tL^2B^2}{R}\cos\phi$ and $v_t = \frac{Rmg\tan\phi}{L^2B^2\cos\phi}$.

(c) $i = \frac{\mathcal{E}}{R} = \frac{1}{R}\frac{d\Phi_B}{dt} = \frac{1}{R}B\frac{dA}{dt} = \frac{B}{R}(vL\cos\phi) = \frac{vLB\cos\phi}{R} = \frac{mg\tan\phi}{LB}$.

(d) $P = i^2R = \frac{Rm^2g^2\tan^2\phi}{L^2B^2}$.

(e) $P_g = Fv\cos(90° - \phi) = mg\left(\frac{Rmg\tan\phi}{L^2B^2\cos\phi}\right)\sin\phi$ and $P_g = \frac{Rm^2g^2\tan^2\phi}{L^2B^2}$.

EVALUATE: The power in part (e) equals that in part (d), as is required by conservation of energy.

29.78. IDENTIFY: Follow the steps indicated in the problem.

SET UP: The primary assumption throughout the problem is that the square patch is small enough so that the velocity is constant over its whole area, that is, $v = \omega r \approx \omega d$.

EXECUTE: **(a)** $\omega \to$ clockwise, $B \to$ into page. $\mathcal{E} = vBL = \omega dBL$. $I = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}A}{\rho L} = \frac{\omega dBA}{\rho}$. Since $\vec{v}\times\vec{B}$ points outward, A is just the cross-sectional area tL. Therefore, $I = \frac{\omega dBLt}{\rho}$ flowing radially outward since $\vec{v}\times\vec{B}$ points outward.

(b) $\vec{\tau} = \vec{d}\times\vec{F}$ and $\vec{F}_B = I\vec{L}\times\vec{B} = ILB$ pointing counterclockwise. So $\tau = \frac{\omega d^2B^2L^2t}{\rho}$ pointing out of the page (a counterclockwise torque opposing the clockwise rotation).

(c) If $\omega \to$ counterclockwise and $B \to$ into page, then $I \to$ inward radially since $\vec{v}\times\vec{B}$ points inward. $\tau \to$ clockwise (again opposing the motion). If $\omega \to$ counterclockwise and $B \to$ out of the page, then $I \to$ radially outward. $\tau \to$ clockwise (opposing the motion)

The magnitudes of I and τ are the same as in part (a).

EVALUATE: In each case the magnetic torque due to the induced current opposes the rotation of the disk, as is required by conservation of energy.