25

CURRENT, RESISTANCE, AND ELECTROMOTIVE FORCE

25.1. **IDENTIFY:** $I = Q/t$.

SET UP: $1.0\text{ h} = 3600\text{ s}$

EXECUTE: $Q = It = (3.6\text{ A})(3.0)(3600\text{ s}) = 3.89\times10^{4}\text{ C}$.

EVALUATE: Compared to typical charges of objects in electrostatics, this is a huge amount of charge.

25.2. **IDENTIFY:** $I = Q/t$. Use $I = n|q|v_{\text{d}}A$ to calculate the drift velocity v_{d}.

SET UP: $n = 5.8\times10^{28}\text{ m}^{-3}$. $|q| = 1.60\times10^{-19}\text{ C}$.

EXECUTE: **(a)** $I = \dfrac{Q}{t} = \dfrac{420\text{ C}}{80(60\text{ s})} = 8.75\times10^{-2}\text{ A}$.

(b) $I = n|q|v_{\text{d}}A$. This gives $v_{\text{d}} = \dfrac{I}{nqA} = \dfrac{8.75\times10^{-2}\text{ A}}{(5.8\times10^{28})(1.60\times10^{-19}\text{ C})(\pi(1.3\times10^{-3}\text{ m})^2)} = 1.78\times10^{-6}\text{ m/s}$.

EVALUATE: v_{d} is smaller than in Example 25.1, because I is smaller in this problem.

25.3. **IDENTIFY:** $I = Q/t$. $J = I/A$. $J = n|q|v_{\text{d}}$

SET UP: $A = (\pi/4)D^2$, with $D = 2.05\times10^{-3}\text{ m}$. The charge of an electron has magnitude $+e = 1.60\times10^{-19}\text{ C}$.

EXECUTE: **(a)** $Q = It = (5.00\text{ A})(1.00\text{ s}) = 5.00\text{ C}$. The number of electrons is $\dfrac{Q}{e} = 3.12\times10^{19}$.

(b) $J = \dfrac{I}{(\pi/4)D^2} = \dfrac{5.00\text{ A}}{(\pi/4)(2.05\times10^{-3}\text{ m})^2} = 1.51\times10^{6}\text{ A/m}^2$.

(c) $v_{\text{d}} = \dfrac{J}{n|q|} = \dfrac{1.51\times10^{6}\text{ A/m}^2}{(8.5\times10^{28}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})} = 1.11\times10^{-4}\text{ m/s} = 0.111\text{ mm/s}$.

EVALUATE: **(a)** If I is the same, $J = I/A$ would decrease and v_{d} would decrease. The number of electrons passing through the light bulb in 1.00 s would not change.

25.4. **(a) IDENTIFY:** By definition, $J = I/A$ and radius is one-half the diameter.

SET UP: Solve for the current: $I = JA = J\pi(D/2)^2$

EXECUTE: $I = (1.50\times10^{6}\text{ A/m}^2)(\pi)[(0.00102\text{ m})/2]^2 = 1.23\text{ A}$

EVALUATE: This is a realistic current.

(b) IDENTIFY: The current density is $J = nqv_{\text{d}}$

SET UP: Solve for the drift velocity: $v_{\text{d}} = J/nq$

EXECUTE: Since most laboratory wire is copper, we use the value of n for copper, giving

$v_{\text{d}} = (1.50\times10^{6}\text{ A/m}^2)/[(8.5\times10^{28}\text{ el/m}^3)(1.60\times10^{-19}\text{ C}) = 1.1\times10^{-4}\text{ m/s} = 0.11\text{ mm/s}$

EVALUATE: This is a typical drift velocity for ordinary currents and wires.

25.5. **IDENTIFY** and **SET UP:** Use Eq. (25.3) to calculate the drift speed and then use that to find the time to travel the length of the wire.

EXECUTE: **(a)** Calculate the drift speed v_{d}:

$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{4.85\text{ A}}{\pi\left(1.025\times10^{-3}\text{ m}\right)^2} = 1.469\times10^{6}\text{ A/m}^2$$

$$v_{\text{d}} = \frac{J}{n|q|} = \frac{1.469\times10^{6}\text{ A/m}^2}{\left(8.5\times10^{28}/\text{m}^3\right)\left(1.602\times10^{-19}\text{ C}\right)} = 1.079\times10^{-4}\text{ m/s}$$

$$t = \frac{L}{v_{\text{d}}} = \frac{0.710\text{ m}}{1.079\times10^{-4}\text{ m/s}} = 6.58\times10^{3}\text{ s} = 110\text{ min.}$$

(b) $v_d = \dfrac{I}{\pi r^2 n|q|}$

$t = \dfrac{L}{v_d} = \dfrac{\pi r^2 n|q|L}{I}$

t is proportional to r^2 and hence to d^2 where $d = 2r$ is the wire diameter.

$t = (6.58\times10^3\ \text{s})\left(\dfrac{4.12\ \text{mm}}{2.05\ \text{mm}}\right)^2 = 2.66\times10^4\ \text{s} = 440\ \text{min}.$

(c) EVALUATE: The drift speed is proportional to the current density and therefore it is inversely proportional to the square of the diameter of the wire. Increasing the diameter by some factor decreases the drift speed by the square of that factor.

25.6. **IDENTIFY:** The number of moles of copper atoms is the mass of 1.00 m^3 divided by the atomic mass of copper. There are $N_A = 6.023\times10^{23}$ atoms per mole.

SET UP: The atomic mass of copper is $63.55\ \text{g/mole}$, and its density is $8.96\ \text{g/cm}^3$. Example 25.1 says there are 8.5×10^{28} free electrons per m^3.

EXECUTE: The number of copper atoms in $1.00\ \text{m}^3$ is

$$\frac{(8.96\ \text{g/cm}^3)(1.00\times10^6\ \text{cm}^3/\text{m}^3)(6.023\times10^{23}\ \text{atoms/mole})}{63.55\ \text{g/mole}} = 8.49\times10^{28}\ \text{atoms/m}^3.$$

EVALUATE: Since there are the same number of free electrons/m^3 as there are atoms of copper/m^3, the number of free electrons per copper atom is one.

25.7. **IDENTIFY** and **SET UP:** Apply Eq. (25.1) to find the charge *dQ* in time *dt*. Integrate to find the total charge in the whole time interval.

EXECUTE: **(a)** $dQ = I\ dt$

$Q = \int_0^{8.0\ \text{s}} \left(55\ \text{A} - (0.65\ \text{A/s}^2)t^2\right)dt = \left[(55\ \text{A})t - (0.217\ \text{A/s}^2)t^3\right]_0^{8.0\ \text{s}}$

$Q = (55\ \text{A})(8.0\ \text{s}) - (0.217\ \text{A/s}^2)(8.0\ \text{s})^3 = 330\ \text{C}$

(b) $I = \dfrac{Q}{t} = \dfrac{330\ \text{C}}{8.0\ \text{s}} = 41\ \text{A}$

EVALUATE: The current decreases from 55 A to 13.4 A during the interval. The decrease is not linear and the average current is not equal to (55A + 13.4 A) / 2.

25.8. **IDENTIFY:** $I = Q/t$. Positive charge flowing in one direction is equivalent to negative charge flowing in the opposite direction, so the two currents due to Cl^- and Na^+ are in the same direction and add.

SET UP: Na^+ and Cl^- each have magnitude of charge $|q| = +e$

EXECUTE: **(a)** $Q_{\text{total}} = (n_{\text{Cl}} + n_{\text{Na}})e = (3.92\times10^{16} + 2.68\times10^{16})(1.60\times10^{-19}\ \text{C}) = 0.0106\ \text{C}$. Then

$I = \dfrac{Q_{\text{total}}}{t} = \dfrac{0.0106\ \text{C}}{1.00\ \text{s}} = 0.0106\ \text{A} = 10.6\ \text{mA}.$

(b) Current flows, by convention, in the direction of positive charge. Thus, current flows with Na^+ toward the negative electrode.

EVALUATE: The Cl^- ions have negative charge and move in the direction opposite to the conventional current direction.

25.9. **IDENTIFY:** The number of moles of silver atoms is the mass of $1.00\ \text{m}^3$ divided by the atomic mass of silver. There are $N_A = 6.023\times10^{23}$ atoms per mole.

SET UP: For silver, $\text{density} = 10.5\times10^3\ \text{kg/m}^3$ and the atomic mass is $M = 107.868\times10^{-3}\ \text{kg/mol}$.

EXECUTE: Consider 1 m^3 of silver. $m = (\text{density})V = 10.5\times10^3\ \text{kg}$. $n = m/M = 9.734\times10^4$ mol and the number of atoms is $N = nN_A = 5.86\times10^{28}$ atoms. If there is one free electron per atom, there are 5.86×10^{28} free electrons/m^3. This agrees with the value given in Exercise 25.2.

EVALUATE: Our result verifies that for silver there is approximately one free electron per atom. Exercise 25.6 showed that for copper there is also one free electron per atom.

25.10. **(a) IDENTIFY:** Start with the definition of resisitivity and solve for *E*.

SET UP: $E = \rho J = \rho I/\pi r^2$

EXECUTE: $E = (1.72\times10^{-8}\ \Omega\cdot\text{m})(2.75\ \text{A})/[\pi(0.001025\ \text{m})^2] = 1.43\times10^{-2}\ \text{V/m}$

EVALUATE: The field is quite weak, since the potential would drop only a volt in 70 m of wire.

(b) IDENTIFY: Take the ratio of the field in silver to the field in copper.

SET UP: Take the ratio and solve for the field in silver: $E_S = E_C(\rho_S/\rho_C)$

EXECUTE: $E_S = (0.0143\ \text{V/m})[(1.47)/(1.72)] = 1.22 \times 10^{-2}\ \text{V/m}$

EVALUATE: Since silver is a better conductor than copper, the field in silver is smaller than the field in copper.

25.11. **IDENTIFY:** First use Ohm's law to find the resistance at 20.0°C; then calculate the resistivity from the resistance. Finally use the dependence of resistance on temperature to calculate the temperature coefficient of resistance.

SET UP: Ohm's law is $R = V/I$, $R = \rho L/A$, $R = R_0[1 + \alpha(T - T_0)]$, and the radius is one-half the diameter.

EXECUTE: **(a)** At 20.0°C, $R = V/I = (15.0\ \text{V})/(18.5\ \text{A}) = 0.811\ \Omega$. Using $R = \rho L/A$ and solving for ρ gives $\rho = RA/L = R\pi(D/2)^2/L = (0.811\ \Omega)\pi[(0.00500\ \text{m})/2]^2/(1.50\ \text{m}) = 1.06 \times 10^{-6}\ \Omega\cdot\text{m}$.

(b) At 92.0°C, $R = V/I = (15.0\ \text{V})/(17.2\ \text{A}) = 0.872\ \Omega$. Using $R = R_0[1 + \alpha(T - T_0)]$ with T_0 taken as 20.0°C, we have $0.872\ \Omega = (0.811\ \Omega)[1 + \alpha(92.0°\text{C} - 20.0°\text{C})]$. This gives $\alpha = 0.00105\ (\text{C}°)^{-1}$

EVALUATE: The results are typical of ordinary metals.

25.12. **IDENTIFY:** $E = \rho J$, where $J = I/A$. The drift velocity is given by $I = n|q|v_d A$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. $n = 8.5\times10^{28}/\text{m}^3$.

EXECUTE: **(a)** $J = \dfrac{I}{A} = \dfrac{3.6\ \text{A}}{(2.3\times10^{-3}\ \text{m})^2} = 6.81\times10^5\ \text{A/m}^2$.

(b) $E = \rho J = (1.72\times10^{-8}\ \Omega\cdot\text{m})(6.81\times10^5\ \text{A/m}^2) = 0.012\ \text{V/m}$.

(c) The time to travel the wire's length l is

$$t = \frac{l}{v_d} = \frac{ln|q|A}{I} = \frac{(4.0\ \text{m})(8.5\times10^{28}/\text{m}^3)(1.6\times10^{-19}\ \text{C})(2.3\times10^{-3}\ \text{m})^2}{3.6\ \text{A}} = 8.0\times10^4\ \text{s}.$$

$t = 1333\ \text{min} \approx 22\ \text{hrs!}$

EVALUATE: The currents propagate very quickly along the wire but the individual electrons travel very slowly.

25.13. **IDENTIFY:** $E = \rho J$, where $J = I/A$.

SET UP: For tungsten $\rho = 5.25\times10^{-8}\ \Omega\cdot\text{m}$ and for aluminum $\rho = 2.75\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: **(a)** tungsten: $E = \rho J = \dfrac{\rho I}{A} = \dfrac{(5.25\times10^{-8}\ \Omega\cdot\text{m})(0.820\ \text{A})}{(\pi/4)(3.26\times10^{-3}\ \text{m})^2} = 5.16\times10^{-3}\ \text{V/m}$.

(b) aluminum: $E = \rho J = \dfrac{\rho I}{A} = \dfrac{(2.75\times10^{-8}\ \Omega\cdot\text{m})(0.820\ \text{A})}{(\pi/4)(3.26\times10^{-3}\ \text{m})^2} = 2.70\times10^{-3}\ \text{V/m}$.

EVALUATE: A larger electric field is required for tungsten, because it has a larger resistivity.

25.14. **IDENTIFY:** The resistivity of the wire should identify what the material is.

SET UP: $R = \rho L/A$ and the radius of the wire is half its diameter.

EXECUTE: Solve for ρ and substitute the numerical values.

$$\rho = AR/L = \pi(D/2)^2 R/L = \frac{\pi\left([0.00205\ \text{m}]/2\right)^2(0.0290\ \Omega)}{6.50\ \text{m}} = 1.47\times10^{-8}\ \Omega\cdot\text{m}$$

EVALUATE: This result is the same as the resistivity of silver, which implies that the material is silver.

25.15. **(a) IDENTIFY:** Start with the definition of resistivity and use its dependence on temperature to find the electric field.

SET UP: $E = \rho J = \rho_{20}[1 + \alpha(T - T_0)]\dfrac{I}{\pi r^2}$

EXECUTE: $E = (5.25\times10^{-8}\ \Omega\cdot\text{m})[1 + (0.0045/\text{C}°)(120°\text{C} - 20°\text{C})](12.5\ \text{A})/[\pi(0.000500\ \text{m})^2] = 1.21\ \text{V/m}$.

(Note that the resistivity at 120°C turns out to be $7.61\times10^{-8}\ \Omega\cdot\text{m}$.)

EVALUATE: This result is fairly large because tungsten has a larger resisitivity than copper.

(b) IDENTIFY: Relate resistance and resistivity.

SET UP: $R = \rho L/A = \rho L/\pi r^2$

EXECUTE: $R = (7.61\times10^{-8}\ \Omega\cdot\text{m})(0.150\ \text{m})/[\pi(0.000500\ \text{m})^2] = 0.0145\ \Omega$

EVALUATE: Most metals have very low resistance.

(c) IDENTIFY: The potential difference is proportional to the length of wire.

SET UP: $V = EL$

EXECUTE: $V = (1.21\ \text{V/m})(0.150\ \text{m}) = 0.182\ \text{V}$

EVALUATE: We could also calculate $V = IR = (12.5\ \text{A})(0.0145\ \Omega) = 0.181\ \text{V}$, in agreement with part (c).

25.16. **IDENTIFY:** Apply $R=\dfrac{\rho L}{A}$ and solve for L.

SET UP: $A=\pi D^2/4$, where $D=0.462\text{ mm}$.

EXECUTE: $L=\dfrac{RA}{\rho}=\dfrac{(1.00\,\Omega)(\pi/4)(0.462\times10^{-3}\text{ m})^2}{1.72\times10^{-8}\,\Omega\cdot\text{m}}=9.75\text{ m}.$

EVALUATE: The resistance is proportional to the length of the wire.

25.17. **IDENTIFY:** $R=\dfrac{\rho L}{A}$.

SET UP: For copper, $\rho=1.72\times10^{-8}\ \Omega\cdot\text{m}$. $A=\pi r^2$.

EXECUTE: $R=\dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(24.0\text{ m})}{\pi(1.025\times10^{-3}\text{ m})^2}=0.125\ \Omega$

EVALUATE: The resistance is proportional to the length of the piece of wire.

25.18. **IDENTIFY:** $R=\dfrac{\rho L}{A}=\dfrac{\rho L}{\pi d^2/4}$.

SET UP: For aluminum, $\rho_{\text{al}}=2.63\times10^{-8}\ \Omega\cdot\text{m}$. For copper, $\rho_{\text{c}}=1.72\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: $\dfrac{\rho}{d^2}=\dfrac{R\pi}{4L}=$ constant, so $\dfrac{\rho_{\text{al}}}{d_{\text{al}}^2}=\dfrac{\rho_{\text{c}}}{d_{\text{c}}^2}$. $d_{\text{c}}=d_{\text{al}}\sqrt{\dfrac{\rho_{\text{c}}}{\rho_{\text{al}}}}=(3.26\text{ mm})\sqrt{\dfrac{1.72\times10^{-8}\ \Omega\cdot\text{m}}{2.63\times10^{-8}\ \Omega\cdot\text{m}}}=2.64\text{ mm}.$

EVALUATE: Copper has a smaller resistivity, so the copper wire has a smaller diameter in order to have the same resistance as the aluminum wire.

25.19. **IDENTIFY** and **SET UP:** Use Eq. (25.10) to calculate A. Find the volume of the wire and use the density to calculate the mass.

EXECUTE: Find the volume of one of the wires:

$R=\dfrac{\rho L}{A}$ so $A=\dfrac{\rho L}{R}$ and

$$\text{volume}=AL=\frac{\rho L^2}{R}=\frac{\left(1.72\times10^{-8}\Omega\cdot\text{ m}\right)\left(3.50\text{ m}\right)^2}{0.125\ \Omega}=1.686\times10^{-6}\text{ m}^3$$

$$m=\left(\text{density}\right)V=\left(8.9\times10^3\text{ kg/m}^3\right)\left(1.686\times10^{-6}\text{ m}^3\right)=15\text{ g}$$

EVALUATE: The mass we calculated is reasonable for a wire.

25.20. **IDENTIFY:** $R=\dfrac{\rho L}{A}$.

SET UP: The length of the wire in the spring is the circumference πd of each coil times the number of coils.

EXECUTE: $L=(75)\pi d=(75)\pi(3.50\times10^{-2}\text{ m})=8.25\text{ m}.$

$$A=\pi r^2=\pi d^2/4=\pi(3.25\times10^{-3}\text{ m})^2/4=8.30\times10^{-6}\text{ m}^2.$$

$$\rho=\frac{RA}{L}=\frac{(1.74\ \Omega)(8.30\times10^{-6}\text{ m}^2)}{8.25\text{ m}}=1.75\times10^{-6}\ \Omega\cdot\text{m}.$$

EVALUATE: The value of ρ we calculated is about a factor of 100 times larger than ρ for copper. The metal of the spring is not a very good conductor.

25.21. **IDENTIFY:** $R=\dfrac{\rho L}{A}$.

SET UP: $L=1.80\text{ m}$, the length of one side of the cube. $A=L^2$.

EXECUTE: $R=\dfrac{\rho L}{A}=\dfrac{\rho L}{L^2}=\dfrac{\rho}{L}=\dfrac{2.75\times10^{-8}\ \Omega\cdot\text{m}}{1.80\text{ m}}=1.53\times10^{-8}\ \Omega$

EVALUATE: The resistance is very small because A is very much larger than the typical value for a wire.

25.22. **IDENTIFY:** Apply $R_T=R_0(1+\alpha(T-T_0))$.

SET UP: Since $V=IR$ and V is the same, $\dfrac{R_T}{R_{20}}=\dfrac{I_{20}}{I_T}$. For tungsten, $\alpha=4.5\times10^{-3}\ (\text{C}°)^{-1}$.

EXECUTE: The ratio of the current at $20°C$ to that at the higher temperature is $(0.860\text{ A})/(0.220\text{ A}) = 3.909$.

$\frac{R_T}{R_{20}} = 1+\alpha(T-T_0) = 3.909$, where $T_0 = 20°C$.

$$T = T_0 + \frac{R_T/R_{20}-1}{\alpha} = 20°C + \frac{3.909-1}{4.5\times10^{-3}\ (C°)^{-1}} = 666°C.$$

EVALUATE: As the temperature increases, the resistance increases and for constant applied voltage the current decreases. The resistance increases by nearly a factor of four.

25.23. **IDENTIFY:** Relate resistance to resistivity.

SET UP: $R = \rho L/A$

EXECUTE: **(a)** $R = \rho L/A = (0.60\ \Omega\cdot\text{m})(0.25\text{ m})/(0.12\text{ m})^2 = 10.4\ \Omega$

(b) $R = \rho L/A = (0.60\ \Omega\cdot\text{m})(0.12\text{ m})/(0.12\text{ m})(0.25\text{ m}) = 2.4\ \Omega$

EVALUATE: The resistance is greater for the faces that are farther apart.

25.24. **IDENTIFY:** Apply $R = \frac{\rho L}{A}$ and $V = IR$.

SET UP: $A = \pi r^2$

EXECUTE: $\rho = \frac{RA}{L} = \frac{VA}{IL} = \frac{(4.50\text{ V})\pi(6.54\times10^{-4}\text{ m})^2}{(17.6\text{ A})(2.50\text{ m})} = 1.37\times10^{-7}\ \Omega\cdot\text{m}.$

EVALUATE: Our result for ρ shows that the wire is made of a metal with resistivity greater than that of good metallic conductors such as copper and aluminum.

25.25. **IDENTIFY** and **SET UP:** Eq. (25.5) relates the electric field that is given to the current density. $V = EL$ gives the potential difference across a length L of wire and Eq. (25.11) allows us to calculate R.

EXECUTE: **(a)** Eq. (25.5): $\rho = E/J$ so $J = E/\rho$

From Table 25.1 the resistivity for gold is $2.44\times10^{-8}\ \Omega\cdot\text{m}$.

$$J = \frac{E}{\rho} = \frac{0.49\text{ V/m}}{2.44\times10^{-8}\ \Omega\cdot\text{m}} = 2.008\times10^7\text{ A/m}^2$$

$$I = JA = J\pi r^2 = \left(2.008\times10^7\text{ A/m}^2\right)\pi\left(0.41\times10^{-3}\text{ m}\right)^2 = 11\text{ A}$$

(b) $V = EL = (0.49\text{ V/m})(6.4\text{ m}) = 3.1\text{ V}$

(c) We can use Ohm's law (Eq. (25.11)): $V = IR$.

$$R = \frac{V}{I} = \frac{3.1\text{ V}}{11\text{ A}} = 0.28\ \Omega$$

EVALUATE: We can also calculate R from the resistivity and the dimensions of the wire (Eq. 25.10):

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{\left(2.44\times10^{-8}\ \Omega\cdot\text{m}\right)(6.4\text{ m})}{\pi\left(0.42\times10^{-3}\text{ m}\right)^2} = 0.28\ \Omega,\ \text{which checks.}$$

25.26. **IDENTIFY** and **SET UP:** Use $V = EL$ to calculate E and then $\rho = E/J$ to calculate ρ.

EXECUTE: **(a)** $E = \frac{V}{L} = \frac{0.938\text{ V}}{0.750\text{ m}} = 1.25\text{ V/m}$

(b) $E = \rho J$ so $\rho = \frac{E}{J} = \frac{1.25\text{ V/m}}{4.40\times10^7\text{ A/m}^2} = 2.84\times10^{-8}\ \Omega\cdot\text{m}$

EVALUATE: This value of ρ is similar to that for the good metallic conductors in Table 25.1.

25.27. **IDENTIFY:** Apply $R = R_0\left[1+\alpha\left(T-T_0\right)\right]$ to calculate the resistance at the second temperature.

(a) SET UP: $\alpha = 0.0004\ \left(C°\right)^{-1}$ (Table 25.1). Let T_0 be 0.0°C and T be 11.5°C.

EXECUTE: $R_0 = \frac{R}{1+\alpha\left(T-T_0\right)} = \frac{100.0\ \Omega}{1+\left(0.0004\ \left(C°\right)^{-1}\left(11.5\ C°\right)\right)} = 99.54\ \Omega$

(b) SET UP: $\alpha = -0.0005\ \left(C°\right)^{-1}$ (Table 25.2). Let $T_0 = 0.0°C$ and $T = 25.8°C$.

EXECUTE: $R = R_0\left[1+\alpha\left(T-T_0\right)\right] = 0.0160\ \Omega\left[1+\left(-0.0005\ \left(C°\right)^{-1}\right)\left(25.8\ C°\right)\right] = 0.0158\ \Omega$

EVALUATE: Nichrome, like most metallic conductors, has a positive α and its resistance increases with temperature. For carbon, α is negative and its resistance decreases as T increases.

25.28. IDENTIFY: $R_T = R_0[1+\alpha(T-T_0)]$

SET UP: $R_0 = 217.3\ \Omega$. $R_T = 215.8\ \Omega$. For carbon, $\alpha = -0.00050\ (\text{C}°)^{-1}$.

EXECUTE: $T - T_0 = \dfrac{(R_T/R_0)-1}{\alpha} = \dfrac{(215.8\ \Omega/217.3\ \Omega)-1}{-0.00050\ (\text{C}°)^{-1}} = 13.8\ \text{C}°$. $T = 13.8\ \text{C}° + 4.0°\text{C} = 17.8°\text{C}$.

EVALUATE: For carbon, α is negative so R decreases as T increases.

25.29. IDENTIFY and SET UP: Apply $R = \dfrac{\rho L}{A}$ to determine the effect of increasing A and L.

EXECUTE: **(a)** If 120 strands of wire are placed side by side, we are effectively increasing the area of the current carrier by 120. So the resistance is smaller by that factor: $R = (5.60\times10^{-6}\ \Omega)/120 = 4.67\times10^{-8}\ \Omega$.

(b) If 120 strands of wire are placed end to end, we are effectively increasing the length of the wire by 120, and so $R = (5.60\times10^{-6}\ \Omega)120 = 6.72\times10^{-4}\ \Omega$.

EVALUATE: Placing the strands side by side decreases the resistance and placing them end to end increases the resistance.

25.30. IDENTIFY: When the ohmmeter is connected between the opposite faces, the current flows along its length, but when the meter is connected between the inner and outer surfaces, the current flows radially outward.

(a) SET UP: For a hollow cylinder, $R = \rho L/A$, where $A = \pi(b^2 - a^2)$.

EXECUTE: $R = \rho L/A = \dfrac{\rho L}{\pi\left(b^2-a^2\right)} = \dfrac{\left(2.75\times10^{-8}\ \Omega\cdot\text{m}\right)(2.50\ \text{m})}{\pi\left[(0.0460\ \text{m})^2-(0.0320\ \text{m})^2\right]} = 2.00\times10^{-5}\ \Omega$

(b) SET UP: For radial current flow from $r = a$ to $r = b$, $R = (\rho/2\pi L)\ln(b/a)$ (Example 25.4)

EXECUTE: $R = \dfrac{\rho}{2\pi L}\ln(b/a) = \dfrac{2.75\times10^{-8}\ \Omega\cdot\text{m}}{2\pi(2.50\ \text{m})}\ln\left(\dfrac{4.60\ \text{cm}}{3.20\ \text{cm}}\right) = 6.35\times10^{-10}\ \Omega$

EVALUATE: The resistance is much smaller for the radial flow because the current flows through a much smaller distance and the area through which it flows is much larger.

25.31. IDENTIFY: Use $R = \dfrac{\rho L}{A}$ to calculate R and then apply $V = IR$. $P = VI$ and energy $= Pt$

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. $A = \pi r^2$, where $r = 0.050$ m.

EXECUTE: **(a)** $R = \dfrac{\rho L}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(100\times10^3\text{m})}{\pi(0.050\ \text{m})^2} = 0.219\ \Omega$. $V = IR = (125\ \text{A})(0.219\ \Omega) = 27.4\ \text{V}$.

(b) $P = VI = (27.4\ \text{V})(125\ \text{A}) = 3422\ \text{W} = 3422\ \text{J/s}$ and energy $= Pt = (3422\ \text{J/s})(3600\ \text{s}) = 1.23\times10^7\ \text{J}$.

EVALUATE: The rate of electrical energy loss in the cable is large, over 3 kW.

25.32. IDENTIFY: When current passes through a battery in the direction from the − terminal toward the + terminal, the terminal voltage V_{ab} of the battery is $V_{ab} = \mathcal{E} - Ir$. Also, $V_{ab} = IR$, the potential across the circuit resistor.

SET UP: $\mathcal{E} = 24.0\ \text{V}$. $I = 4.00\ \text{A}$.

EXECUTE: **(a)** $V_{ab} = \mathcal{E} - Ir$ gives $r = \dfrac{\mathcal{E} - V_{ab}}{I} = \dfrac{24.0\ \text{V} - 21.2\ \text{V}}{4.00\ \text{A}} = 0.700\ \Omega$.

(b) $V_{ab} - IR = 0$ so $R = \dfrac{V_{ab}}{I} = \dfrac{21.2\ \text{V}}{4.00\ \text{A}} = 5.30\ \Omega$.

EVALUATE: The voltage drop across the internal resistance of the battery causes the terminal voltage of the battery to be less than its emf. The total resistance in the circuit is $R + r = 6.00\ \Omega$. $I = \dfrac{24.0\ \text{V}}{6.00\ \Omega} = 4.00\ \text{A}$, which agrees with the value specified in the problem.

25.33. IDENTIFY: $V = \mathcal{E} - Ir$.

SET UP: The graph gives $V = 9.0$ V when $I = 0$ and $I = 2.0$ A when $V = 0$.

EXECUTE: **(a)** $\mathcal{E}$ is equal to the terminal voltage when the current is zero. From the graph, this is 9.0 V.

(b) When the terminal voltage is zero, the potential drop across the internal resistance is just equal in magnitude to the internal emf, so $rI = \mathcal{E}$, which gives $r = \mathcal{E}/I = (9.0\ \text{V})/(2.0\ \text{A}) = 4.5\ \Omega$.

EVALUATE: The terminal voltage decreases as the current through the battery increases.

25.34. (a) IDENTIFY: The *idealized* ammeter has no resistance so there is no potential drop across it. Therefore it acts like a short circuit across the terminals of the battery and removes the 4.00-Ω resistor from the circuit. Thus the only resistance in the circuit is the 2.00-Ω internal resistance of the battery.

SET UP: Use Ohm's law: $I = \mathcal{E}/r$.

EXECUTE: $I = (10.0\ \text{V})/(2.00\ \Omega) = 5.00\ \text{A}$.

(b) The zero-resistance ammeter is in parallel with the 4.00-Ω resistor, so all the current goes through the ammeter. If no current goes through the 4.00-Ω resistor, the potential drop across it must be zero.

(c) The terminal voltage is zero since there is no potential drop across the ammeter.

EVALUATE: An ammeter should *never* be connected this way because it would seriously alter the circuit!

25.35. **IDENTIFY:** The terminal voltage of the battery is $V_{ab} = \mathcal{E} - Ir$. The voltmeter reads the potential difference between its terminals.

SET UP: An ideal voltmeter has infinite resistance.

EXECUTE: **(a)** Since an ideal voltmeter has infinite resistance, so there would be NO current through the $2.0\ \Omega$ resistor.

(b) $V_{ab} = \mathcal{E} = 5.0\text{ V}$; since there is no current there is no voltage lost over the internal resistance.

(c) The voltmeter reading is therefore 5.0 V since with no current flowing there is no voltage drop across either resistor.

EVALUATE: This not the proper way to connect a voltmeter. If we wish to measure the terminal voltage of the battery in a circuit that does not include the voltmeter, then connect the voltmeter across the terminals of the battery.

25.36. **IDENTIFY:** The sum of the potential changes around the circuit loop is zero. Potential decreases by *IR* when going through a resistor in the direction of the current and increases by $\mathcal{E}$ when passing through an emf in the direction from the $-$ to + terminal.

SET UP: The current is counterclockwise, because the 16 V battery determines the direction of current flow.

EXECUTE: $+16.0\text{ V} - 8.0\text{ V} - I(1.6\ \Omega + 5.0\ \Omega + 1.4\ \Omega + 9.0\ \Omega) = 0$

$$I = \frac{16.0\text{ V} - 8.0\text{ V}}{1.6\ \Omega + 5.0\ \Omega + 1.4\ \Omega + 9.0\ \Omega} = 0.47\text{ A}$$

(b) $V_b + 16.0\text{ V} - I(1.6\ \Omega) = V_a$, so $V_a - V_b = V_{ab} = 16.0\text{ V} - (1.6\ \Omega)(0.47\text{ A}) = 15.2\text{ V}$.

(c) $V_c + 8.0\text{ V} + I(1.4\ \Omega + 5.0\ \Omega) = V_a$ so $V_{ac} = (5.0\ \Omega)(0.47\text{ A}) + (1.4\ \Omega)(0.47\text{ A}) + 8.0\text{ V} = 11.0\text{ V}$.

(d) The graph is sketched in Figure 25.36.

EVALUATE: $V_{cb} = (0.47\text{ A})(9.0\ \Omega) = 4.2\text{ V}$. The potential at point *b* is 15.2 V below the potential at point *a* and the potential at point *c* is 11.0 V below the potential at point *a*, so the potential of point *c* is $15.2\text{ V} - 11.0\text{ V} = 4.2\text{ V}$ above the potential of point *b*.

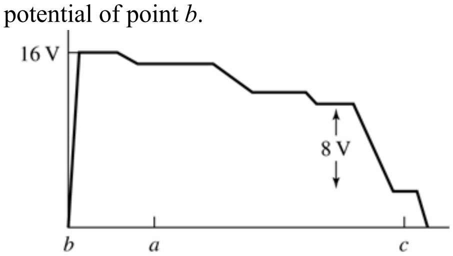


Figure 25.36

25.37. **IDENTIFY:** The voltmeter reads the potential difference V_{ab} between the terminals of the battery.

SET UP: <u>open circuit</u> $I = 0$. The circuit is sketched in Figure 25.37a.

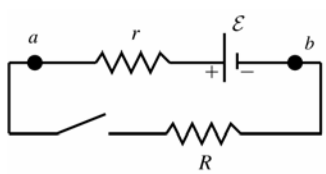


Figure 25.37a

EXECUTE: $V_{ab} = \mathcal{E} = 3.08\text{ V}$

SET UP: <u>switch closed</u> The circuit is sketched in Figure 35.37b.

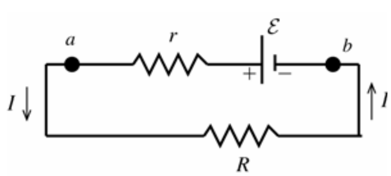


Figure 25.37b

EXECUTE:

$V_{ab} = \mathcal{E} - Ir = 2.97\text{ V}$

$$r = \frac{\mathcal{E} - 2.97\text{ V}}{I}$$

$$r = \frac{3.08\text{ V} - 2.97\text{ V}}{1.65\text{ A}} = 0.067\ \Omega$$

And $V_{ab} = IR$ so $R = \dfrac{V_{ab}}{I} = \dfrac{2.97\text{ V}}{1.65\text{ A}} = 1.80\ \Omega.$

EVALUATE: When current flows through the battery there is a voltage drop across its internal resistance and its terminal voltage *V* is less than its emf.

25.38. **IDENTIFY:** The sum of the potential changes around the loop is zero.

SET UP: The voltmeter reads the IR voltage across the $9.0\ \Omega$ resistor. The current in the circuit is counterclockwise because the 16 V battery determines the direction of the current flow.

EXECUTE: **(a)** $V_{bc} = 1.9\text{ V}$ gives $I = V_{bc}/R_{bc} = 1.9\text{ V}/9.0\ \Omega = 0.21\text{ A}$.

(b) $16.0\text{ V} - 8.0\text{ V} = (1.6\ \Omega + 9.0\ \Omega + 1.4\ \Omega + R)(0.21\text{ A})$ and $R = \dfrac{5.48\text{ V}}{0.21\text{ A}} = 26.1\ \Omega$.

(c) The graph is sketched in Figure 25.38.

EVALUATE: In Exercise 25.36 the current is 0.47 A. When the $5.0\ \Omega$ resistor is replaced by the $26.1\ \Omega$ resistor the current decreases to 0.21 A.

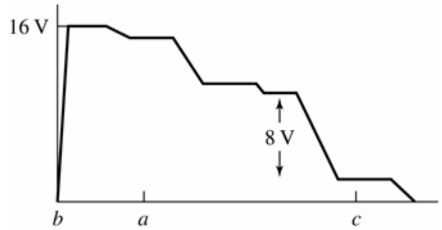


Figure 25.38

25.39. **(a) IDENTIFY** and **SET UP:** Assume that the current is clockwise. The circuit is sketched in Figure 25.39a.

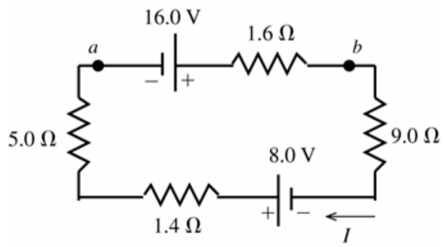


Figure 25.39a

Add up the potential rises and drops as travel clockwise around the circuit.

EXECUTE: $16.0\text{ V} - I(1.6\ \Omega) - I(9.0\ \Omega) + 8.0\text{ V} - I(1.4\ \Omega) - I(5.0\ \Omega) = 0$

$$I = \frac{16.0\text{ V} + 8.0\text{ V}}{9.0\ \Omega + 1.4\ \Omega + 5.0\ \Omega + 1.6\ \Omega} = \frac{24.0\text{ V}}{17.0\ \Omega} = 1.41\text{ A, clockwise}$$

EVALUATE: The 16.0 V battery drives the current clockwise more strongly than the 8.0 V battery does in the opposite direction.

(b) IDENTIFY and **SET UP:** Start at point a and travel through the battery to point b, keeping track of the potential changes. At point b the potential is V_b.

EXECUTE: $V_a + 16.0\text{ V} - I(1.6\ \Omega) = V_b$

$V_a - V_b = -16.0\text{ V} + (1.41\text{ A})(1.6\ \Omega)$

$V_{ab} = -16.0\text{ V} + 2.3\text{ V} = -13.7\text{ V}$ (point a is at lower potential; it is the negative terminal)

EVALUATE: Could also go counterclockwise from a to b:

$V_a + (1.41\text{ A})(5.0\ \Omega) + (1.41\text{ A})(1.4\ \Omega) - 8.0\text{ V} + (1.41\text{ A})(9.0\ \Omega) = V_b$

$V_{ab} = -13.7\text{ V}$, which checks.

(c) IDENTIFY and **SET UP:** State at point a and travel through the battery to point c, keeping track of the potential changes.

EXECUTE: $V_a + 16.0\text{ V} - I(1.6\ \Omega) - I(9.0\ \Omega) = V_c$

$V_a - V_c = -16.0\text{ V} + (1.41\text{ A})(1.6\ \Omega + 9.0\ \Omega)$

$V_{ac} = -16.0\text{ V} + 15.0\text{ V} = -1.0\text{ V}$ (point a is at lower potential than point c)

EVALUATE: Could also go counterclockwise from a to c:

$V_a + (1.41\text{ A})(5.0\ \Omega) + (1.41\text{ A})(1.4\ \Omega) - 8.0\text{ V} = V_c$

$V_{ac} = -1.0\text{ V}$, which checks.

(d) Call the potential zero at point *a*. Travel clockwise around the circuit. The graph is sketched in Figure 25.39b.

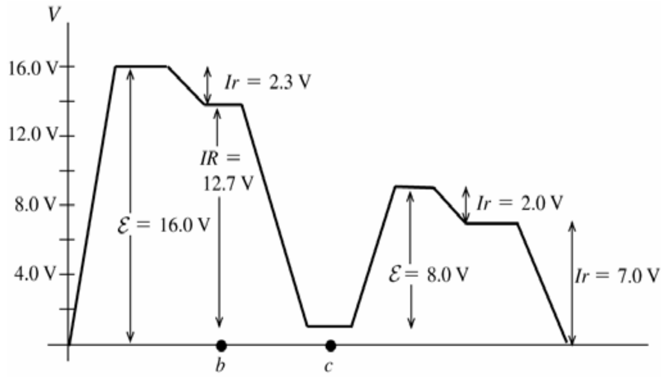


Figure 25.39b

25.40. **IDENTIFY:** Ohm's law says $R = \dfrac{V_{ab}}{I}$ is a constant.

SET UP: **(a)** The graph is given in Figure 25.40a.

EXECUTE: **(b)** No. The graph of V_{ab} versus I is not a straight line so Thyrite does not obey Ohm's law.

(c) The graph of R versus I is given in Figure 25.40b. R is not constant; it decreases as I increases.

EVALUATE: Not all materials obey Ohm's law.

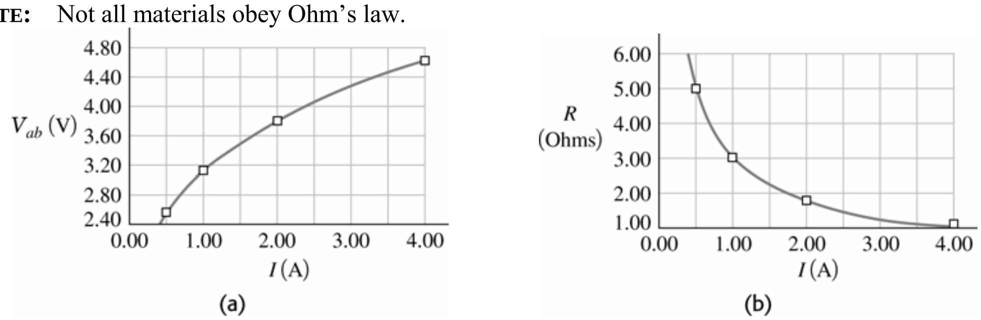


Figure 25.40

25.41. **IDENTIFY:** Ohm's law says $R = \dfrac{V_{ab}}{I}$ is a constant.

SET UP: **(a)** The graph is given in Figure 25.41.

EXECUTE: **(b)** The graph of V_{ab} versus I is a straight line so Nichrome obeys Ohm's law.

(c) R is the slope of the graph in part (a). $R = \dfrac{15.52\ \text{V} - 1.94\ \text{V}}{4.00\ \text{A} - 0.50\ \text{A}} = 3.88\ \Omega.$

EVALUATE: V_{ab}/I for every I gives the same result for R, $R = 3.88\ \Omega.$

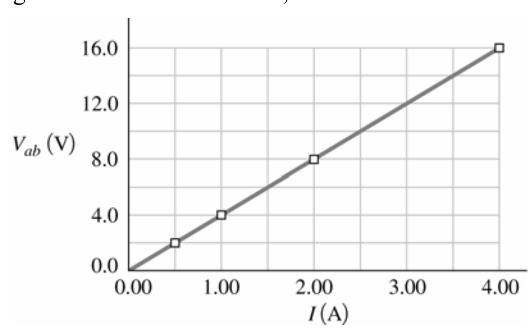


Figure 25.41

25.42. **IDENTIFY** and **SET UP:** For a resistor, $P = VI = V^2/R$ and $V = IR$.

EXECUTE: **(a)** $R = \dfrac{V^2}{P} = \dfrac{(15.0\ \text{V})^2}{327\ \text{W}} = 0.688\ \Omega$

(b) $I = \dfrac{V}{R} = \dfrac{15.0\ \text{V}}{0.688\ \Omega} = 21.8\ \text{A}$

EVALUATE: We could also write $P = VI$ to calculate $I = \dfrac{P}{V} = \dfrac{327\ \text{W}}{15.0\ \text{V}} = 21.8\ \text{A}.$

25.43. **IDENTIFY:** The bulbs are each connected across a 120-V potential difference.

SET UP: Use $P = V^2/R$ to solve for R and Ohm's law ($I = V/R$) to find the current.

EXECUTE: **(a)** $R = V^2/P = (120\text{ V})^2/(100\text{ W}) = 144\ \Omega$.

(b) $R = V^2/P = (120\text{ V})^2/(60\text{ W}) = 240\ \Omega$

(c) For the 100-W bulb: $I = V/R = (120\text{ V})/(144\ \Omega) = 0.833\text{ A}$

For the 60-W bulb: $I = (120\text{ V})/(240\ \Omega) = 0.500\text{ A}$

EVALUATE: The 60-W bulb has *more* resistance than the 100-W bulb, so it draws less current.

25.44. **IDENTIFY:** Across 120 V, a 75-W bulb dissipates 75 W. Use this fact to find its resistance, and then find the power the bulb dissipates across 220 V.

SET UP: $P = V^2/R$, so $R = V^2/P$

EXECUTE: Across 120 V: $R = (120\text{ V})^2/(75\text{ W}) = 192\ \Omega$. Across a 220-V line, its power will be $P = V^2/R = (220\text{ V})^2/(192\ \Omega) = 252\text{ W}$.

EVALUATE: The bulb dissipates much more power across 220 V, so it would likely blow out at the higher voltage. An alternative solution to the problem is to take the ratio of the powers.

$\dfrac{P_{220}}{P_{120}} = \dfrac{V_{220}^2/R}{V_{120}^2/R} = \left(\dfrac{V_{220}}{V_{120}}\right)^2 = \left(\dfrac{220}{120}\right)^2$. This gives $P_{220} = (75\text{ W})\left(\dfrac{220}{120}\right)^2 = 252\text{ W}$.

25.45. **IDENTIFY:** A "100-W" European bulb dissipates 100 W when used across 220 V.

(a) SET UP: Take the ratio of the power in the US to the power in Europe, as in the alternative method for problem 25.44, using $P = V^2/R$.

EXECUTE: $\dfrac{P_{\text{US}}}{P_{\text{E}}} = \dfrac{V_{\text{US}}^2/R}{V_{\text{E}}^2/R} = \left(\dfrac{V_{\text{US}}}{V_{\text{E}}}\right)^2 = \left(\dfrac{120\text{ V}}{220\text{ V}}\right)^2$. This gives $P_{\text{US}} = (100\text{ W})\left(\dfrac{120\text{ V}}{220\text{ V}}\right)^2 = 29.8\text{ W}$.

(b) SET UP: Use $P = IV$ to find the current.

EXECUTE: $I = P/V = (29.8\text{ W})/(120\text{ V}) = 0.248\text{ A}$

EVALUATE: The bulb draws considerably less power in the U.S., so it would be much dimmer than in Europe.

25.46. **IDENTIFY:** $P = VI$. Energy $= Pt$.

SET UP: $P = (9.0\text{ V})(0.13\text{ A}) = 1.17\text{ W}$

EXECUTE: $\text{Energy} = (1.17\text{ W})(1.5\text{ h})(3600\text{ s/h}) = 6320\text{ J}$

EVALUATE: The energy consumed is proportional to the voltage, to the current and to the time.

25.47. **IDENTIFY** and **SET UP:** By definition $p = \dfrac{P}{LA}$. Use $P = VI$, $E = VL$ and $I = JA$ to rewrite this expression in terms of the specified variables.

EXECUTE: **(a)** E is related to V and J is related to I, so use $P = VI$. This gives $p = \dfrac{VI}{LA}$

$\dfrac{V}{L} = E$ and $\dfrac{I}{A} = J$ so $p = EJ$

(b) J is related to I and ρ is related to R, so use $P = IR^2$. This gives $p = \dfrac{I^2R}{LA}$.

$I = JA$ and $R = \dfrac{\rho L}{A}$ so $p = \dfrac{J^2A^2\rho L}{LA^2}\rho J^2$

(c) E is related to V and ρ is related to R, so use $P = V^2/R$. This gives $p = \dfrac{V^2}{RLA}$.

$V = EL$ and $R = \dfrac{\rho L}{A}$ so $p = \dfrac{E^2L^2}{LA}\left(\dfrac{A}{\rho L}\right) = \dfrac{E^2}{\rho}$.

EVALUATE: For a given material (ρ constant), p is proportional to J^2 or to E^2.

25.48. **IDENTIFY:** Calculate the current in the circuit. The power output of a battery is its terminal voltage times the current through it. The power dissipated in a resistor is I^2R.

SET UP: The sum of the potential changes around the circuit is zero.

EXECUTE: **(a)** $I = \dfrac{8.0\text{ V}}{17\ \Omega} = 0.47\text{ A}$. Then $P_{9\Omega} = I^2R = (0.47\text{ A})^2(5.0\ \Omega) = 1.1\text{ W}$ and

$P_{9\Omega} = I^2R = (0.47\text{ A})^2(9.0\ \Omega) = 2.0\text{ W}$.

(b) $P_{16\text{V}} = \mathcal{E}I - I^2r = (16\text{ V})(0.47\text{ A}) - (0.47\text{ A})^2(1.6\ \Omega) = 7.2\text{ W}$.

(c) $P_{8\text{V}} = \mathcal{E}I + Ir^2 = (8.0\text{ V})(0.47\text{ A}) + (0.47\text{ A})^2(1.4\ \Omega) = 4.1\text{ W}$.

EVALUATE: **(d)** $(\text{b}) = (\text{a}) + (\text{c})$. The rate at which the 16.0 V battery delivers electrical energy to the circuit equals the rate at which it is consumed in the 8.0 V battery and the $5.0\ \Omega$ and $9.0\ \Omega$ resistors.

25.49. **(a) IDENTIFY** and **SET UP:** $P = VI$ and energy = (power) × (time).

EXECUTE: $P = VI = (12\text{ V})(60\text{ A}) = 720\text{ W}$

The battery can provide this for 1.0 h, so the energy the battery has stored is

$U = Pt = (720\text{ W})(3600\text{ s}) = 2.6\times10^{6}\text{ J}$

(b) IDENTIFY and **SET UP:** For gasoline the heat of combustion is $L_c = 46\times10^{6}\text{ J/kg}$. Solve for the mass m required to supply the energy calculated in part (a) and use density $\rho = m/V$ to calculate V.

EXECUTE: The mass of gasoline that supplies 2.6×10^{6} J is $m = \dfrac{2.6\times10^{6}\text{ J}}{46\times10^{6}\text{ J/kg}} = 0.0565\text{ kg}.$

The volume of this mass of gasoline is

$$V = \frac{m}{\rho} = \frac{0.0565\text{ kg}}{900\text{ kg/m}^3} = 6.3\times10^{-5}\text{ m}^3\left(\frac{1000\text{ L}}{1\text{ m}^3}\right) = 0.063\text{ L}$$

(c) IDENTIFY and **SET UP:** Energy = (power) × (time); the energy is that calculated in part (a).

EXECUTE: $U = Pt,\ t = \dfrac{U}{P} = \dfrac{2.6\times10^{6}\text{ J}}{450\text{ W}} = 5800\text{ s} = 97\text{ min} = 1.6\text{ h}.$

EVALUATE: The battery discharges at a rate of 720 W (for 60 A) and is charged at a rate of 450 W, so it takes longer to charge than to discharge.

25.50. **IDENTIFY:** The rate of conversion of chemical to electrical energy in an emf is $\mathcal{E}I$. The rate of dissipation of electrical energy in a resistor R is I^2R.

SET UP: Example 25.10 finds that $I = 1.2$ A for this circuit. In Example 25.9, $\mathcal{E}I = 24$ W and $I^2r = 8$ W. In Example 25.10, $I^2R = 12$ W, or 11.5 W if expressed to three significant figures.

EXECUTE: **(a)** $P = \mathcal{E}I = (12\text{ V})(1.2\text{ A}) = 14.4\text{ W}$. This is less than the previous value of 24 W.

(b) The energy dissipated in the battery is $P = I^2r = (1.2\text{ A})^2(2.0\ \Omega) = 2.9$ W. This is less than 8 W, the amount found in Example (25.9).

(c) The net power output of the battery is $14.4\text{ W} - 2.9\text{ W} = 11.5\text{ W}$. This is the same as the power dissipated in the $8.0\ \Omega$ resistor.

EVALUATE: With the larger circuit resistance the current is less and the power input and power consumption are less.

25.51. **IDENTIFY:** Some of the power generated by the internal emf of the battery is dissipated across the battery's internal resistance, so it is not available to the bulb.

SET UP: Use $P = I^2R$ and take the ratio of the power dissipated in the internal resistance r to the total power.

EXECUTE: $\dfrac{P_r}{P_{\text{Total}}} = \dfrac{I^2r}{I^2(r+R)} = \dfrac{r}{r+R} = \dfrac{3.5\ \Omega}{28.5\ \Omega} = 0.123 = 12.3\%$

EVALUATE: About 88% of the power of the battery goes to the bulb. The rest appears as heat in the internal resistance.

25.52. **IDENTIFY:** The voltmeter reads the terminal voltage of the battery, which is the potential difference across the appliance. The terminal voltage is less than 15.0 V because some potential is lost across the internal resistance of the battery.

(a) SET UP: $P = V^2/R$ gives the power dissipated by the appliance.

EXECUTE: $P = (11.3\text{ V})^2/(75.0\ \Omega) = 1.70$ W

(b) SET UP: The drop in terminal voltage ($\mathcal{E} - V_{ab}$) is due to the potential drop across the internal resistance r. Use $Ir = \mathcal{E} - V_{ab}$ to find the internal resistance r, but first find the current using $P = IV$.

EXECUTE: $I = P/V = (1.70\text{ W})/(11.3\text{ V}) = 0.151$ A. Then $Ir = \mathcal{E} - V_{ab}$ gives $(0.151\text{ A})r = 15.0\text{ V} - 11.3\text{ V}$ and $r = 24.6\ \Omega$.

EVALUATE: The full 15.0 V of the battery would be available only when no current (or a very small current) is flowing in the circuit. This would be the base if the appliance had a resistance much greater than 24.6 Ω.

25.53. **IDENTIFY:** Solve for the current I in the circuit. Apply Eq. (25.17) to the specified circuit elements to find the rates of energy conversion.

SET UP: The circuit is sketched in Figure 25.53.

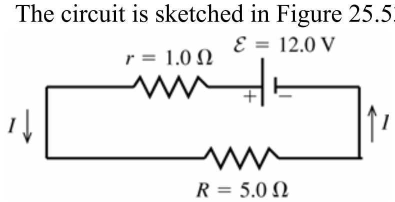


Figure 25.53

EXECUTE: Compute I:

$\mathcal{E} - Ir - IR = 0$

$$I = \frac{\mathcal{E}}{r+R} = \frac{12.0\text{ V}}{1.0\ \Omega + 5.0\ \Omega} = 2.00\text{ A}$$

(a) The rate of conversion of chemical energy to electrical energy in the emf of the battery is $P = \mathcal{E}I = (12.0 \text{ V})(2.00 \text{ A}) = 24.0 \text{ W}$.

(b) The rate of dissipation of electrical energy in the internal resistance of the battery is $P = I^2 r = (2.00 \text{ A})^2 (1.0 \ \Omega) = 4.0 \text{ W}$.

(c) The rate of dissipation of electrical energy in the external resistor R is $P = I^2 R = (2.00 \text{ A})^2 (5.0 \ \Omega) = 20.0 \text{ W}$.

EVALUATE: The rate of production of electrical energy in the circuit is 24.0 W. The total rate of consumption of electrical energy in the circuit is 4.00 W + 20.0 W = 24.0 W. Equal rate of production and consumption of electrical energy are required by energy conservation.

25.54. **IDENTIFY:** The power delivered to the bulb is $I^2 R$. Energy $= Pt$.

SET UP: The circuit is sketched in Figure 25.54. r_{total} is the combined internal resistance of both batteries.

EXECUTE: **(a)** $r_{\text{total}} = 0$. The sum of the potential changes around the circuit is zero, so $1.5 \text{ V} + 1.5 \text{ V} - I(17 \ \Omega) = 0$. $I = 0.1765 \text{ A}$. $P = I^2 R = (0.1765 \text{ A})^2 (17 \ \Omega) = 0.530 \text{ W}$. This is also $(3.0 \text{ V})(0.1765 \text{ A})$.

(b) Energy $= (0.530 \text{ W})(5.0 \text{ h})(3600 \text{ s/h}) = 9540 \text{ J}$

(c) $P = \dfrac{0.530 \text{ W}}{2} = 0.265 \text{ W}$. $P = I^2 R$ so $I = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{0.265 \text{ W}}{17 \ \Omega}} = 0.125 \text{ A}$.

The sum of the potential changes around the circuit is zero, so $1.5 \text{ V} + 1.5 \text{ V} - IR - Ir_{\text{total}} = 0$.

$$r_{\text{total}} = \frac{3.0 \text{ V} - (0.125 \text{ A})(17 \ \Omega)}{0.125 \text{ A}} = 7.0 \ \Omega.$$

EVALUATE: When the power to the bulb has decreased to half its initial value, the total internal resistance of the two batteries is nearly half the resistance of the bulb. Compared to a single battery, using two identical batteries in series doubles the emf but also doubles the total internal resistance.

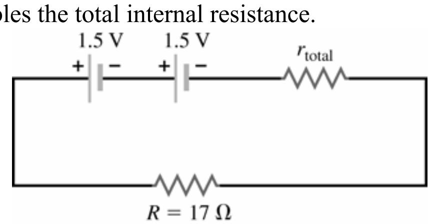


Figure 25.54

25.55. **IDENTIFY:** $P = I^2 R = \dfrac{V^2}{R} = VI$. $V = IR$.

SET UP: The heater consumes 540 W when $V = 120 \text{ V}$. Energy $= Pt$.

EXECUTE: **(a)** $P = \dfrac{V^2}{R}$ so $R = \dfrac{V^2}{P} = \dfrac{(120 \text{ V})^2}{540 \text{ W}} = 26.7 \ \Omega$

(b) $P = VI$ so $I = \dfrac{P}{V} = \dfrac{540 \text{ W}}{120 \text{ V}} = 4.50 \text{ A}$

(c) Assuming that R remains $26.7 \ \Omega$, $P = \dfrac{V^2}{R} = \dfrac{(110 \text{ V})^2}{26.7 \ \Omega} = 453 \text{ W}$. P is smaller by a factor of $(110/120)^2$.

EVALUATE: **(d)** With the lower line voltage the current will decrease and the operating temperature will decrease. R will be less than $26.7 \ \Omega$ and the power consumed will be greater than the value calculated in part (c).

25.56. **IDENTIFY:** From Eq. (25.24), $\rho = \dfrac{m}{ne^2 \tau}$.

SET UP: For silicon, $\rho = 2300 \ \Omega \cdot \text{m}$.

EXECUTE: **(a)** $\tau = \dfrac{m}{ne^2 \rho} = \dfrac{9.11 \times 10^{-31} \text{ kg}}{(1.0 \times 10^{16} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})^2 (2300 \ \Omega \cdot \text{m})} = 1.55 \times 10^{-12} \text{ s}$.

EVALUATE: **(b)** The number of free electrons in copper $(8.5 \times 10^{28} \text{ m}^{-3})$ is much larger than in pure silicon $(1.0 \times 10^{16} \text{ m}^{-3})$. A smaller density of current carriers means a higher resistivity.

25.57. **(a) IDENTIFY** and **SET UP:** Use $R = \dfrac{\rho L}{A}$.

EXECUTE: $\rho = \dfrac{RA}{L} = \dfrac{(0.104 \ \Omega)\pi (1.25 \times 10^{-3} \text{ m})^2}{14.0 \text{ m}} = 3.65 \times 10^{-8} \ \Omega \cdot \text{m}$

EVALUATE: This value is similar to that for good metallic conductors in Table 25.1.
(b) IDENTIFY and SET UP: Use $V = EL$ to calculate E and then Ohm's law gives I.
EXECUTE: $V = EL = (1.28\text{ V/m})(14.0\text{ m}) = 17.9\text{ V}$

$$I = \frac{V}{R} = \frac{17.9\text{ V}}{0.104\ \Omega} = 172\text{ A}$$

EVALUATE: We could do the calculation another way:

$$E = \rho J \text{ so } J = \frac{E}{\rho} = \frac{1.28\text{ V/m}}{3.65\times 10^{-8}\ \Omega\cdot\text{m}} = 3.51\times 10^{7}\text{ A/m}^2$$

$$I = JA = \left(3.51\times 10^{7}\text{ A/m}^2\right)\pi\left(1.25\times 10^{-3}\text{ m}\right)^2 = 172\text{ A, which checks}$$

(c) IDENTIFY and SET UP: Calculate $J = I/A$ or $J = E/\rho$ and then use Eq. (25.3) for the target variable v_{d}.
EXECUTE: $J = n|q|v_{\text{d}} = nev_{\text{d}}$

$$v_{\text{d}} = \frac{J}{ne} = \frac{3.51\times 10^{7}\text{ A/m}^2}{\left(8.5\times 10^{28}\text{ m}^{-3}\right)\left(1.602\times 10^{-19}\text{ C}\right)} = 2.58\times 10^{-3}\text{ m/s} = 2.58\text{ mm/s}$$

EVALUATE: Even for this very large current the drift speed is small.

25.58. IDENTIFY: Use $R = \dfrac{\rho L}{A}$ to calculate the resistance of the silver tube. Then $I = V/R$.

SET UP: For silver, $\rho = 1.47\times 10^{-8}\ \Omega\cdot\text{m}$. The silver tube is sketched in Figure 25.58. Since the thickness $T = 0.100$ mm is much smaller than the radius, $r = 2.00$ cm, the cross section area of the silver is $2\pi rT$. The length of the tube is $l = 25.0$ m.

EXECUTE: $I = \dfrac{V}{R} = \dfrac{V}{\rho l/A} = \dfrac{VA}{\rho l} = \dfrac{V(2\pi rT)}{\rho l} = \dfrac{(12\text{ V})(2\pi)(2.00\times 10^{-2}\text{ m})(0.100\times 10^{-3}\text{ m})}{(1.47\times 10^{-8}\ \Omega\cdot\text{m})(25.0\text{ m})} = 410\text{ A}$

EVALUATE: The resistance is small, $R = 0.0292\ \Omega$, so 12.0 V produces a large current.

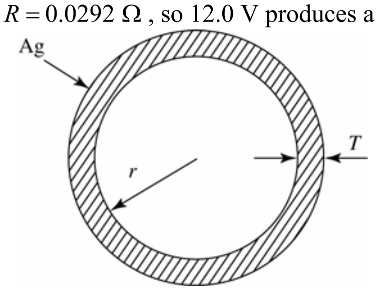


Figure 25.58

25.59. IDENTIFY and SET UP: With the voltmeter connected across the terminals of the battery there is no current through the battery and the voltmeter reading is the battery emf; $\mathcal{E} = 12.6$ V.
With a wire of resistance R connected to the battery current I flows and $\mathcal{E} - Ir - IR = 0$, where r is the internal resistance of the battery. Apply this equation to each piece of wire to get two equations in the two unknowns.
EXECUTE: Call the resistance of the 20.0-m piece R_1; then the resistance of the 40.0-m piece is $R_2 = 2R_1$.

$$\mathcal{E} - I_1 r - I_1 R_1 = 0;\qquad 12.6\text{ V} - (7.00\text{ A})r - (7.00\text{ A})R_1 = 0$$

$$\mathcal{E} - I_2 r - I_2(2R_1) = 0;\qquad 12.6\text{ V} - (4.20\text{ A})r - (4.20\text{ A})(2R_1) = 0$$

Solving these two equations in two unknowns gives $R_1 = 1.20\ \Omega$. This is the resistance of 20.0 m, so the resistance of one meter is $\left[1.20\ \Omega/(20.0\text{ m})\right](1.00\text{ m}) = 0.060\ \Omega$

EVALUATE: We can also solve for r and we get $r = 0.600\ \Omega$. When measuring small resistances, the internal resistance of the battery has a large effect.

25.60. IDENTIFY: Conservation of charge requires that the current is the same in both sections. The voltage drops across each section add, so $R = R_{\text{Cu}} + R_{\text{Ag}}$. The total resistance is the sum of the resistances of each section.

$E = \rho J = \dfrac{\rho I}{A}$, so $E = \dfrac{IR}{L}$, where R is the resistance of a section and L is its length.

SET UP: For copper, $\rho_{\text{Cu}} = 1.72\times 10^{-8}\ \Omega\cdot\text{m}$. For silver, $\rho_{\text{Ag}} = 1.47\times 10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: **(a)** $I = \dfrac{V}{R} = \dfrac{V}{R_{Cu} + R_{Ag}}$. $R_{Cu} = \dfrac{\rho_{Cu} L_{Cu}}{A_{Cu}} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(0.8\ \text{m})}{(\pi/4)(6.0\times10^{-4}\text{m})^2} = 0.049\ \Omega$ and

$R_{Ag} = \dfrac{\rho_{Ag} L_{Ag}}{A_{Ag}} = \dfrac{(1.47\times10^{-8}\ \Omega\cdot\text{m})(1.2\ \text{m})}{(\pi/4)(6.0\times10^{-4}\ \text{m})^2} = 0.062\ \Omega$. This gives $I = \dfrac{5.0\ \text{V}}{0.049\ \Omega + 0.062\ \Omega} = 45\ \text{A}$.

The current in the copper wire is 45 A.

(b) The current in the silver wire is 45 A, the same as that in the copper wire or else charge would build up at their interface.

(c) $E_{Cu} = J\rho_{Cu} = \dfrac{IR_{Cu}}{L_{Cu}} = \dfrac{(45\ \text{A})(0.049\ \Omega)}{0.8\ \text{m}} = 2.76\ \text{V/m}$.

(d) $E_{Ag} = J\rho_{Ag} = \dfrac{IR_{Ag}}{L_{Ag}} = \dfrac{(45\ \text{A})(0.062\ \Omega)}{1.2\ \text{m}} = 2.33\ \text{V/m}$.

(e) $V_{Ag} = IR_{Ag} = (45\ \text{A})(0.062\ \Omega) = 2.79\ \text{V}$.

EVALUATE: For the copper section, $V_{Cu} = IR_{Cu} = 2.21\ \text{V}$. Note that $V_{Cu} + V_{Ag} = 5.0\ \text{V}$, the voltage applied across the ends of the composite wire.

25.61. **IDENTIFY:** Conservation of charge requires that the current be the same in both sections of the wire.

$E = \rho J = \dfrac{\rho I}{A}$. For each section, $V = IR = JAR = \left(\dfrac{EA}{\rho}\right)\left(\dfrac{\rho L}{A}\right) = EL$. The voltages across each section add.

SET UP: $A = (\pi/4)D^2$, where D is the diameter.

EXECUTE: **(a)** The current must be the same in both sections of the wire, so the current in the thin end is 2.5 mA.

(b) $E_{1.6\text{mm}} = \rho J = \dfrac{\rho I}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(2.5\times10^{-3}\ \text{A})}{(\pi/4)(1.6\times10^{-3}\ \text{m})^2} = 2.14\times10^{-5}\ \text{V/m}$.

(c) $E_{0.8\text{mm}} = \rho J = \dfrac{\rho I}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(2.5\times10^{-3}\ \text{A})}{(\pi/4)(0.80\times10^{-3}\ \text{m})^2} = 8.55\times10^{-5}\ \text{V/m}$. This is $4E_{1.6\text{mm}}$.

(d) $V = E_{1.6\text{mm}} L_{1.6\ \text{mm}} + E_{0.8\ \text{mm}} L_{0.8\ \text{mm}}$. $V = (2.14\times10^{-5}\ \text{V/m})(1.20\ \text{m}) + (8.55\times10^{-5}\ \text{V/m})(1.80\ \text{m}) = 1.80\times10^{-4}\ \text{V}$.

EVALUATE: The currents are the same but the current density is larger in the thinner section and the electric field is larger there.

25.62. **IDENTIFY:** $I = JA$.

SET UP: From Example 25.1, an 18-gauge wire has $A = 8.17\times10^{-3}\ \text{cm}^2$.

EXECUTE: **(a)** $I = JA = (1.0\times10^5\ \text{A/cm}^2)(8.17\times10^{-3}\ \text{cm}^2) = 820\ \text{A}$

(b) $A = I/J = (1000\ \text{A})/(1.0\times10^6\ \text{A/cm}^2) = 1.0\times10^{-3}\ \text{cm}^2$. $A = \pi r^2$ so

$r = \sqrt{A/\pi} = \sqrt{(1.0\times10^{-3}\ \text{cm}^2)/\pi} = 0.0178\ \text{cm}$ and $d = 2r = 0.36\ \text{mm}$.

EVALUATE: These wires can carry very large currents.

25.63. **(a) IDENTIFY:** Apply Eq. (25.10) to calculate the resistance of each thin disk and then integrate over the truncated cone to find the total resistance.

SET UP:

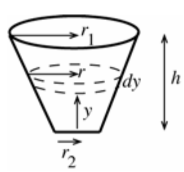


Figure 25.63

EXECUTE: The radius of a truncated cone a distance y above the bottom is given by $r = r_2 + (y/h)(r_1 - r_2) = r_2 + y\beta$ with $\beta = (r_1 - r_2)/h$

Consider a thin slice a distance y above the bottom. The slice has thickness dy and radius r. The resistance of the slice is

$$dR = \frac{\rho dy}{A} = \frac{\rho dy}{\pi r^2} = \frac{\rho dy}{\pi (r_2 + \beta y)^2}$$

The total resistance of the cone if obtained by integrating over these thin slices:

$$R = \int dR = \frac{\rho}{\pi}\int_0^h \frac{dy}{(r_2 + \beta y)^2} = \frac{\rho}{\pi}\left[-\frac{1}{\beta}(r_2 + y\beta)^{-1}\right]_0^h = -\frac{\rho}{\pi\beta}\left[\frac{1}{r_2 + h\beta} - \frac{1}{r_2}\right]$$

But $r_2 + h\beta = r_1$

$$R = \frac{\rho}{\pi\beta}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = \frac{\rho}{\pi}\left(\frac{h}{r_1 - r_2}\right)\left(\frac{r_1 - r_2}{r_1 r_2}\right) = \frac{\rho h}{\pi r_1 r_2}$$

(b) EVALUATE: Let $r_1 = r_2 = r$. Then $R = \rho h/\pi r^2 = \rho L/A$ where $A = \pi r^2$ and $L = h$. This agrees with Eq. (25.10).

25.64. IDENTIFY: Divide the region into thin spherical shells of radius r and thickness dr. The total resistance is the sum of the resistances of the thin shells and can be obtained by integration.

SET UP: $I = V/R$ and $J = I/4\pi r^2$, where $4\pi r^2$ is the surface area of a shell of radius r.

EXECUTE: **(a)** $dR = \dfrac{\rho dr}{4\pi r^2} \Rightarrow R = \dfrac{\rho}{4\pi}\displaystyle\int_a^b \frac{dr}{r^2} = -\frac{\rho}{4\pi}\frac{1}{r}\bigg|_a^b = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\rho}{4\pi}\left(\frac{b-a}{ab}\right).$

(b) $I = \dfrac{V_{ab}}{R} = \dfrac{V_{ab}4\pi ab}{\rho(b-a)}$ and $J = \dfrac{I}{A} = \dfrac{V_{ab}4\pi ab}{\rho(b-a)4\pi r^2} = \dfrac{V_{ab}ab}{\rho(b-a)r^2}.$

(c) If the thickness of the shells is small, then $4\pi ab \approx 4\pi a^2$ is the surface area of the conducting material.

$R = \dfrac{\rho}{4\pi}\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = \dfrac{\rho(b-a)}{4\pi ab} \approx \dfrac{\rho L}{4\pi a^2} = \dfrac{\rho L}{A}$, where $L = b - a$.

EVALUATE: The current density in the material is proportional to $1/r^2$.

25.65. IDENTIFY and SET UP: Use $E = \rho J$ to calculate the current density between the plates. Let A be the area of each plate; then $I = JA$.

EXECUTE: $J = \dfrac{E}{\rho}$ and $E = \dfrac{\sigma}{K\epsilon_0} = \dfrac{Q}{KA\epsilon_0}$

Thus $J = \dfrac{Q}{KA\epsilon_0\rho}$ and $I = JA = \dfrac{Q}{K\epsilon_0\rho}$, as was to be shown.

EVALUATE: $C = K\epsilon_0 A/d$ and $V = Q/C = Qd/K\epsilon_0 A$ so the result can also be written as $I = VA/d\rho$. The resistance of the dielectric is $R = V/I = d\rho/A$, which agrees with Eq. (25.10).

25.66. IDENTIFY: As the resistance R varies, the current in the circuit also varies, which causes the potential drop across the internal resistance of the battery to vary.

SET UP: The largest current will occur when $R = 0$, and the smallest current will occur when $R \to \infty$. The largest terminal voltage will occur when the current is zero ($R \to \infty$) and the smallest terminal voltage will be when the current is a maximum ($R = 0$).

EXECUTE: **(a)** As $R \to \infty$, $I \to 0$, so $V_{ab} \to \mathcal{E} = 15.0$ V, which is the largest reading of the voltmeter. When $R = 0$, the current is largest at (15.0 V)/(4.00 Ω) = 3.75 A, so the smallest terminal voltage is $V_{ab} = \mathcal{E} - rI = 15.0$ V – (4.00 Ω)(3.75 A) = 0.

(b) From part (a), the maximum current is 3.75 A when $R = 0$, and the minimum current is 0.00 A when $R \to \infty$.

(c) The graphs are sketched in Figure 25.66.

EVALUATE: Increasing the resistance R increases the terminal voltage, but at the same time it decreases the current in the circuit.

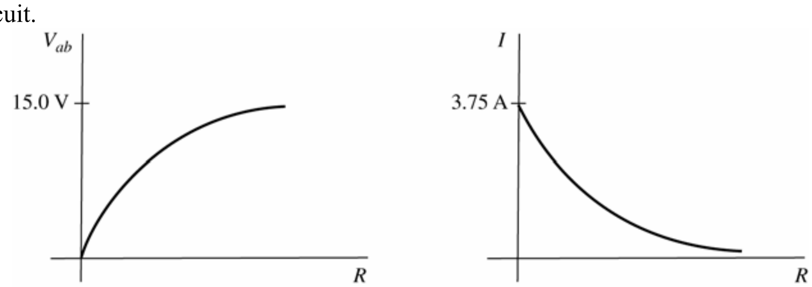


Figure 25.66

25.67. IDENTIFY: Apply $R = \dfrac{\rho L}{A}$.

SET UP: For mercury at 20°C, $\rho = 9.5\times10^{-7}\ \Omega\cdot\text{m}$, $\alpha = 0.00088\ (\text{C}°)^{-1}$ and $\beta = 18\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: **(a)** $R = \dfrac{\rho L}{A} = \dfrac{(9.5\times10^{-7}\ \Omega\cdot\text{m})(0.12\ \text{m})}{(\pi/4)(0.0016\ \text{m})^2} = 0.057\ \Omega.$

(b) $\rho(T) = \rho_0(1+\alpha\Delta T)$ gives $\rho(60°\text{ C}) = (9.5\times10^{-7}\ \Omega\cdot\text{m})(1+(0.00088\ (\text{C}°)^{-1})(40\ \text{C}°) = 9.83\times10^{-7}\ \Omega\cdot\text{m}$, so $\Delta\rho = 3.34\times10^{-8}\ \Omega\cdot\text{m}$.

(c) $\Delta V = \beta V_0\Delta T$ gives $A\Delta L = A\ (\beta L_0\Delta T)$. Therefore

$\Delta L = \beta L_0\Delta T = (18\times10^{-5}\ (\text{C}°)^{-1})(0.12\ \text{m})(40\ \text{C}°) = 8.64\times10^{-4}\ \text{m} = 0.86\ \text{mm}$. The cross sectional area of the mercury remains constant because the diameter of the glass tube doesn't change. All of the change in volume of the mercury must be accommodated by a change in length of the mercury column.

(d) $R = \dfrac{\rho L}{A}$ gives $\Delta R = \dfrac{L\Delta\rho}{A} + \dfrac{\rho\Delta L}{A}$.

$$\Delta R = \frac{(3.34\times10^{-8}\ \Omega\cdot\text{m})(0.12\ \text{m})}{(\pi/4)(0.0016\ \text{m})^2} + \frac{(95\times10^{-8}\ \Omega\cdot\text{m})(0.86\times10^{-3}\ \text{m})}{(\pi/4)(0.0016\ \text{m})^2} = 2.40\times10^{-3}\ \Omega.$$

EVALUATE: **(e)** From Equation (25.12),

$$\alpha = \frac{1}{\Delta T}\left(\frac{R}{R_0}-1\right) = \frac{1}{40\ \text{C}°}\left(\frac{(0.057\ \Omega + 2.40\times10^{-3}\ \Omega)}{0.057\ \Omega}-1\right) = 1.1\times10^{-3}\ (\text{C}°)^{-1}.$$

This value is 25% greater than the temperature coefficient of resistivity and the length increase is important.

25.68. **IDENTIFY:** Consider the potential changes around the circuit. For a complete loop the sum of the potential changes is zero.

SET UP: There is a potential drop of IR when you pass through a resistor in the direction of the current.

EXECUTE: **(a)** $I = \dfrac{8.0\ \text{V} - 4.0\ \text{V}}{24.0\ \Omega} = 0.167\ \text{A}$. $V_d + 8.00\ \text{V} - I(0.50\ \Omega + 8.00\ \Omega) = V_a$, so

$V_{ad} = 8.00\ \text{V} - (0.167\ \text{A})\ (8.50\ \Omega) = 6.58\ \text{V}$.

(b) The terminal voltage is $V_{bc} = V_b - V_c$. $V_c + 4.00\ \text{V} + I(0.50\ \Omega) = V_b$ and

$V_{bc} = +\ 4.00\ \text{V} + (0.167\ \text{A})\ (0.50\ \Omega) = +\ 4.08\ \text{V}$.

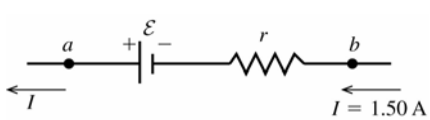
(c) Adding another battery at point d in the opposite sense to the 8.0 V battery produces a counterclockwise current with magnitude $I = \dfrac{10.3\ \text{V} - 8.0\ \text{V} + 4.0\ \text{V}}{24.5\ \Omega} = 0.257\ \text{A}$. Then $V_c + 4.00\ \text{V} - I(0.50\ \Omega) = V_b$ and

$V_{bc} = 4.00\ \text{V} - (0.257\ \text{A})\ (0.50\ \Omega) = 3.87\ \text{V}$.

EVALUATE: When current enters the battery at its negative terminal, as in part (c), the terminal voltage is less than its emf. When current enters the battery at the positive terminal, as in part (b), the terminal voltage is greater than its emf.

25.69. **IDENTIFY:** In each case write the terminal voltage in terms of $\mathcal{E}$, I, and r. Since I is known, this gives two equations in the two unknowns $\mathcal{E}$ and r.

SET UP: The battery with the 1.50 A current is sketched in Figure 25.69a.



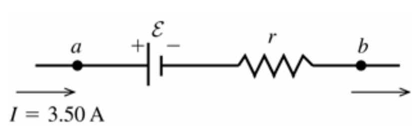
$V_{ab} = 8.4\ \text{V}$

$V_{ab} = \mathcal{E} - Ir$

$\mathcal{E} - (1.50\ \text{A})r = 8.4\ \text{V}$

Figure 25.69a

The battery with the 3.50 A current is sketched in Figure 25.69b.



$V_{ab} = 9.4\ \text{V}$

$V_{ab} = \mathcal{E} + Ir$

$\mathcal{E} + (3.5\ \text{A})r = 9.4\ \text{V}$

Figure 25.69b

EXECUTE: **(a)** Solve the first equation for $\mathcal{E}$ and use that result in the second equation:

$\mathcal{E} = 8.4\ \text{V} + (1.50\ \text{A})r$

$8.4\ \text{V} + (1.50\ \text{A})r + (3.50\ \text{A})r = 9.4\ \text{V}$

$(5.00\ \text{A})r = 1.0\ \text{V}$ so $r = \dfrac{1.0\ \text{V}}{5.00\ \text{A}} = 0.20\ \Omega$

(b) Then $\mathcal{E} = 8.4\text{ V} + (1.50\text{ A})r = 8.4\text{ V} + (1.50\text{ A})(0.20\ \Omega) = 8.7\text{ V}$

EVALUATE: When the current passes through the emf in the direction from $-$ to $+$, the terminal voltage is less than the emf and when it passes through from $+$ to $-$, the terminal voltage is greater than the emf.

25.70. IDENTIFY: $V = IR$. $P = I^2R$.

SET UP: The total resistance is the resistance of the person plus the internal resistance of the power supply.

EXECUTE: **(a)** $I = \dfrac{V}{R_{\text{tot}}} = \dfrac{14\times10^3\text{ V}}{10\times10^3\ \Omega + 2000\ \Omega} = 1.17\text{ A}$

(b) $P = I^2R = (1.17\text{ A})^2(10\times10^3\ \Omega) = 1.37\times10^4\text{ J} = 13.7\text{ kJ}$

(c) $R_{\text{tot}} = \dfrac{V}{I} = \dfrac{14\times10^3\text{ V}}{1.00\times10^{-3}\text{ A}} = 14\times10^6\ \Omega$. The resistance of the power supply would need to be $14\times10^6\ \Omega - 10\times10^3\ \Omega = 14\times10^6\ \Omega = 14\text{ M}\Omega$.

EVALUATE: The current through the body in part (a) is large enough to be fatal.

25.71. IDENTIFY: $R = \dfrac{\rho L}{A}$. $V = IR$. $P = I^2R$.

SET UP: The area of the end of a cylinder of radius r is πr^2.

EXECUTE: **(a)** $R = \dfrac{(5.0\ \Omega\cdot\text{m})(1.6\text{ m})}{\pi(0.050\text{ m})^2} = 1.0\times10^3\ \Omega$

(b) $V = IR = (100\times10^{-3}\text{ A})(1.0\times10^3\ \Omega) = 100\text{ V}$

(c) $P = I^2R = (100\times10^{-3}\text{ A})^2(1.0\times10^3\ \Omega) = 10\text{ W}$

EVALUATE: The resistance between the hands when the skin is wet is about a factor of ten less than when the skin is dry (Problem 25.70).

25.72. IDENTIFY: The cost of operating an appliance is proportional to the amount of energy consumed. The energy depends on the power the item consumes and the length of time for which it is operated.

SET UP: At a constant power, the energy is equal to Pt, and the total cost is the cost per kilowatt-hour (kWh) times the time the energy (in kWh).

EXECUTE: **(a)** Use the fact that 1.00 kWh = (1000 J/s)(3600 s) = 3.60×10^6 J, and one year contains 3.156×10^7 s.

$$(75\text{ J/s})\left(\frac{3.156\times10^7\text{ s}}{1\text{ yr}}\right)\left(\frac{\$0.120}{3.60\times10^6\text{ J}}\right) = \$78.90$$

(b) At 8 h/day, the refrigerator runs for 1/3 of a year. Using the same procedure as above gives

$$(400\text{ J/s})\left(\frac{1}{3}\right)\left(\frac{3.156\times10^7\text{ s}}{1\text{ yr}}\right)\left(\frac{\$0.120}{3.60\times10^6\text{ J}}\right) = \$140.27$$

EVALUATE: Electric lights can be a substantial part of the cost of electricity in the home if they are left on for a long time!

25.73. IDENTIFY: Set the sum of the potential rises and drops around the circuit equal to zero and solve for I.

SET UP: The circuit is sketched in Figure 25.73.

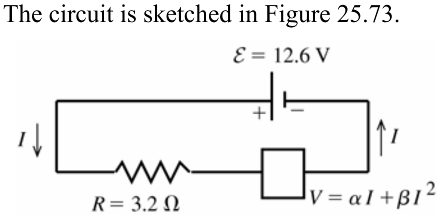


Figure 25.73

EXECUTE:

$\mathcal{E} - IR - V = 0$

$\mathcal{E} - IR - \alpha I - \beta I^2 = 0$

$\beta I^2 + (R+\alpha)I - \mathcal{E} = 0$

The quadratic formula gives $I = (1/2\beta)\left[-(R+\alpha) \pm \sqrt{(R+\alpha)^2 + 4\beta\mathcal{E}}\right]$

I must be positive, so take the + sign

$I = (1/2\beta)\left[-(R+\alpha) + \sqrt{(R+\alpha)^2 + 4\beta\mathcal{E}}\right]$

$I = -2.692\text{ A} + 4.116\text{ A} = 1.42\text{ A}$

EVALUATE: For this I the voltage across the thermistor is 8.0 V. The voltage across the resistor must then be $12.6\text{ V} - 8.0\text{ V} = 4.6\text{ V}$, and this agrees with Ohm's law for the resistor.

25.74. **(a) IDENTIFY:** The rate of heating (power) in the cable depends on the potential difference across the cable and the resistance of the cable.

SET UP: The power is $P = V^2/R$ and the resistance is $R = \rho L/A$. The diameter D of the cable is twice its radius. $P = \dfrac{V^2}{R} = \dfrac{V^2}{(\rho L/A)} = \dfrac{AV^2}{\rho L} = \dfrac{\pi r^2 V^2}{\rho L}$. The electric field in the cable is equal to the potential difference across its ends divided by the length of the cable: $E = V/L$.

EXECUTE: Solving for r and using the resistivity of copper gives

$$r = \sqrt{\frac{P\rho L}{\pi V^2}} = \sqrt{\frac{(50.0\text{ W})(1.72\times10^{-8}\ \Omega\cdot\text{m})(1500\text{ m})}{\pi(220.0\text{ V})^2}} = 9.21\times10^{-5}\text{ m.}\ \ D = 2r = 0.184\text{ mm}$$

(b) SET UP: $E = V/L$

EXECUTE: $E = (220\text{ V})/(1500\text{ m}) = 0.147\text{ V/m}$

EVALUATE: This would be an extremely thin (and hence fragile) cable.

25.75. **IDENTIFY:** The ammeter acts as a resistance in the circuit loop. Set the sum of the potential rises and drops around the circuit equal to zero.

(a) SET UP: The circuit with the ammeter is sketched in Figure 25.75a.

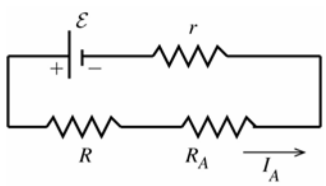


Figure 25.75a

EXECUTE:

$$I_A = \frac{\mathcal{E}}{r + R + R_A}$$

$$\mathcal{E} = I_A(r + R + R_A)$$

SET UP: The circuit with the ammeter removed is sketched in Figure 25.75b.

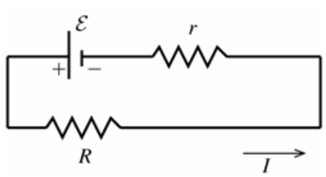


Figure 25.75b

EXECUTE:

$$I = \frac{\mathcal{E}}{R + r}$$

Combining the two equations gives

$$I = \left(\frac{1}{R+r}\right) I_A(r + R + R_A) = I_A\left(1 + \frac{R_A}{r+R}\right)$$

(b) Want $I_A = 0.990I$. Use this in the result for part (a).

$$I = 0.990I\left(1 + \frac{R_A}{r+R}\right)$$

$$0.010 = 0.990\left(\frac{R_A}{r+R}\right)$$

$$R_A = (r+R)(0.010/0.990) = (0.45\ \Omega + 3.80\ \Omega)(0.010/0.990) = 0.0429\ \Omega$$

(c) $I - I_A = \dfrac{\mathcal{E}}{r+R} - \dfrac{\mathcal{E}}{r+R+R_A}$

$$I - I_A = \mathcal{E}\left(\frac{r + R + R_A - r - R}{(r+R)(r+R+R_A)}\right) = \frac{\mathcal{E}R_A}{(r+R)(r+R+R_A)}.$$

EVALUATE: The difference between I and I_A increases as R_A increases. If R_A is larger than the value calculated in part (b) then I_A differs from I by more than 1.0%.

25.76. **IDENTIFY:** Since the resistivity is a function of the position along the length of the cylinder, we must integrate to find the resistance.

(a) SET UP: The resistance of a cross-section of thickness dx is $dR = \rho dx/A$.

EXECUTE: Using the given function for the resistivity and integrating gives

$$R = \int \frac{\rho dx}{A} = \int_0^L \frac{(a + bx^2)dx}{\pi r^2} = \frac{aL + bL^3/3}{\pi r^2}.$$

Now get the constants a and b: $\rho(0) = a = 2.25 \times 10^{-8}\ \Omega\cdot\text{m}$ and

$\rho(L) = a + bL^2$ gives $8.50 \times 10^{-8}\ \Omega\cdot\text{m} = 2.25 \times 10^{-8}\ \Omega\cdot\text{m} + b(1.50\text{ m})^2$

which gives $b = 2.78 \times 10^{-8}\ \Omega/\text{m}$. Now use the above result to find R.

$$R = \frac{\left(2.25\times10^{-8}\ \Omega\cdot\text{m}\right)(1.50\text{ m}) + \left(2.78\times10^{-8}\ \Omega/\text{m}\right)(1.50\text{ m})^3/3}{\pi(0.0110\text{ m})^2} = 1.71 \times 10^{-4}\ \Omega = 171\ \mu\Omega$$

(b) IDENTIFY: Use the definition of resistivity to find the electric field at the midpoint of the cylinder, where $x = L/2$.

SET UP: $E = \rho J$. Evaluate the resistivity, using the given formula, for $x = L/2$.

EXECUTE: At the midpoint, $x = L/2$, giving $E = \dfrac{\rho I}{\pi r^2} = \dfrac{\left[a + b\left(L/2\right)^2\right]I}{\pi r^2}$.

$$E = \frac{\left[2.25\times10^{-8}\ \Omega\cdot\text{m} + \left(2.78\times10^{-8}\ \Omega/\text{m}\right)(0.750\text{ m})^2\right](1.75\text{ A})}{\pi(0.0110\text{ m})^2} = 1.76\times10^{-4}\ \text{V/m}$$

(c) IDENTIFY: For the first segment, the result is the same as in part (a) except that the upper limit of the integral is $L/2$ instead of L.

SET UP: Integrating using the upper limit of $L/2$ gives $R_1 = \dfrac{a\left(L/2\right) + (b/3)\left(L^3/8\right)}{\pi r^2}$.

EXECUTE: Substituting the numbers gives

$$R_1 = \frac{\left(2.25\times10^{-8}\ \Omega\cdot\text{m}\right)(0.750\text{ m}) + (2.78\times10^{-8}\ \Omega/\text{m})/3\left((1.50\text{ m})^3/8\right)}{\pi(0.0110\text{ m})^2} = 5.47\times10^{-5}\ \Omega$$

The resistance R_2 of the second half is equal to the total resistance minus the resistance of the first half.

$R_2 = R - R_1 = 1.71 \times 10^{-4}\ \Omega - 5.47 \times10^{-5}\ \Omega = 1.16 \times 10^{-4}\ \Omega$

EVALUATE: The second half has a greater resistance than the first half because the resistance increases with distance along the cylinder.

25.77. **IDENTIFY:** The power supplied to the house is $P = VI$. The rate at which electrical energy is dissipated in the wires is I^2R, where $R = \dfrac{\rho L}{A}$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: **(a)** The line voltage, current to be drawn, and wire diameter are what must be considered in household wiring.

(b) $P = VI$ gives $I = \dfrac{P}{V} = \dfrac{4200\text{ W}}{120\text{ V}} = 35\text{ A}$, so the 8-gauge wire is necessary, since it can carry up to 40 A.

(c) $P = I^2R = \dfrac{I^2\rho L}{A} = \dfrac{(35\text{ A})^2\,(1.72\times10^{-8}\ \Omega\cdot\text{m})\,(42.0\text{ m})}{(\pi/4)\,(0.00326\text{ m})^2} = 106\text{ W}$.

(d) If 6-gauge wire is used, $P = \dfrac{I^2\rho L}{A} = \dfrac{(35\text{ A})^2\,(1.72\times10^{-8}\ \Omega\cdot\text{m})\,(42\text{ m})}{(\pi/4)\,)\,(0.00412\text{ m})^2} = 66\text{ W}$. The decrease in energy consumption is $\Delta E = \Delta Pt = (40\text{ W})\,(365\text{ days/yr})\,(12\text{ h/day}) = 175\text{ kWh/yr}$ and the savings is $(175\text{ kWh/yr})\,(\$0.11/\text{kWh}) = \$19.25$ per year.

EVALUATE: The cost of the 4200 W used by the appliances is \$2020. The savings is about 1%.

25.78. **IDENTIFY:** $R_T = R_0(1 + \alpha[T - T_0])$. $R = \dfrac{V}{I}$. $P = VI$.

SET UP: When the temperature increases the resistance increases and the current decreases.

EXECUTE: **(a)** $\dfrac{V}{I_T} = \dfrac{V}{I_0}(1 + \alpha[T - T_0])$. $I_0 = I_T(1 + \alpha[T - T_0])$.

$$T - T_0 = \frac{I_0 - I_T}{\alpha I_T} = \frac{1.35\text{ A} - 1.23\text{ A}}{(1.23\text{ A})(4.5\times10^{-4}\ (\text{C}^\circ)^{-1})} = 217\ \text{C}^\circ.\quad T = 20^\circ\text{C} + 217^\circ\text{C} = 237^\circ\text{C}$$

(b) (i) $P = VI = (120\text{ V})(1.35\text{ A}) = 162\text{ W}$ (ii) $P = (120\text{ V})(1.23\text{ A}) = 148\text{ W}$

EVALUATE: $P = V^2/R$ shows that the power dissipated decreases when the resistance increases.

25.79. **(a) IDENTIFY:** Set the sum of the potential rises and drops around the circuit equal to zero and solve for the resulting equation for the current I. Apply Eq. (25.17) to each circuit element to find the power associated with it.

SET UP: The circuit is sketched in Figure 25.79.

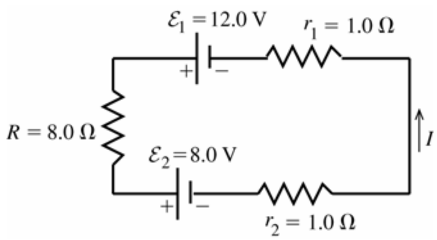


Figure 25.79

EXECUTE:

$\mathcal{E}_1 - \mathcal{E}_2 - I(r_1 + r_2 + R) = 0$

$I = \dfrac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + r_2 + R}$

$I = \dfrac{12.0\text{ V} - 8.0\text{ V}}{1.0\ \Omega + 1.0\ \Omega + 8.0\ \Omega}$

$I = 0.40$ A

(b) $P = I^2R + I^2r_1 + I^2r_2 = I^2(R + r_1 + r_2) = (0.40\text{ A})^2(8.0\ \Omega + 1.0\ \Omega + 1.0\ \Omega)$

$P = 1.6$ W

(c) Chemical energy is converted to electrical energy in a battery when the current goes through the battery from the negative to the positive terminal, so the electrical energy of the charges increases as the current passes through. This happens in the 12.0 V battery, and the rate of production of electrical energy is

$P = \mathcal{E}_1 I = (12.0\text{ V})(0.40\text{ A}) = 4.8$ W.

(d) Electrical energy is converted to chemical energy in a battery when the current goes through the battery from the positive to the negative terminal, so the electrical energy of the charges decreases as the current passes through. This happens in the 8.0 V battery, and the rate of consumption of electrical energy is

$P = \mathcal{E}_2 I = (8.0\text{ V})(0.40\text{ V}) = 3.2$ W.

(e) EVALUATE: Total rate of production of electrical energy = 4.8 W. Total rate of consumption of electrical energy = 1.6 W + 3.2 W = 4.8 W, which equals the rate of production, as it must.

25.80. **IDENTIFY:** Apply $R = \dfrac{\rho L}{A}$ for each material. The total resistance is the sum of the resistances of the rod and the wire. The rate at which energy is dissipated is I^2R.

SET UP: For steel, $\rho = 2.0 \times 10^{-7}\ \Omega\cdot\text{m}$. For copper, $\rho = 1.72 \times 10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: **(a)** $R_{\text{steel}} = \dfrac{\rho L}{A} = \dfrac{(2.0 \times 10^{-7}\Omega\cdot\text{m})\,(2.0\text{ m})}{(\pi/4)\,(0.018\text{ m})^2} = 1.57 \times 10^{-3}\ \Omega$ and

$R_{\text{Cu}} = \dfrac{\rho L}{A} = \dfrac{(1.72 \times 10^{-8}\ \Omega\cdot\text{m})\,(35\text{ m})}{(\pi/4)\,(0.008\text{ m})^2} = 0.012\ \Omega$. This gives

$V = IR = I\,(R_{\text{steel}} + R_{\text{Cu}}) = (15000\text{ A})\,(1.57 \times 10^{-3}\ \Omega + 0.012\ \Omega) = 204$ V.

(b) $E = Pt = I^2Rt = (15000\text{ A})^2(0.0136\ \Omega)\,(65 \times 10^{-6}\text{ s}) = 199$ J.

EVALUATE: I^2R is large but t is very small, so the energy deposited is small. The wire and rod each have a mass of about 1 kg, so their temperature rise due to the deposited energy will be small.

25.81. **IDENTIFY** and **SET UP:** The terminal voltage is $V_{ab} = \mathcal{E} - Ir = IR$, where R is the resistance connected to the battery. During the charging the terminal voltage is $V_{ab} = \mathcal{E} + Ir$. $P = VI$ and energy is $E = Pt$. I^2r is the rate at which energy is dissipated in the internal resistance of the battery.

EXECUTE: **(a)** $V_{ab} = \mathcal{E} + Ir = 12.0\text{ V} + (10.0\text{ A})\,(0.24\ \Omega) = 14.4$ V.

(b) $E = Pt = IVt = (10\text{ A})\,(14.4\text{ V})\,(5)\,(3600\text{ s}) = 2.59 \times 10^6$ J.

(c) $E_{\text{diss}} = P_{\text{diss}}t = I^2rt = (10\text{ A})^2\ (0.24\ \Omega)\,(5)\,(3600\text{ s}) = 4.32 \times 10^5$ J.

(d) Discharged at 10 A: $I = \dfrac{\mathcal{E}}{r + R} \Rightarrow R = \dfrac{\mathcal{E} - Ir}{I} = \dfrac{12.0\text{ V} - (10\text{ A})\,(0.24\ \Omega)}{10\text{ A}} = 0.96\ \Omega.$

(e) $E = Pt = IVt = (10\text{ A})\,(9.6\text{ V})\,(5)\,(3600\text{ s}) = 1.73 \times 10^6$ J.

(f) Since the current through the internal resistance is the same as before, there is the same energy dissipated as in (c): $E_{\text{diss}} = 4.32 \times 10^5$ J.

(g) Part of the energy originally supplied was stored in the battery and part was lost in the internal resistance. So the stored energy was less than what was supplied during charging. Then when discharging, even more energy is lost in the internal resistance, and only what is left is dissipated by the external resistor.

25.82. **IDENTIFY** and **SET UP:** The terminal voltage is $V_{ab} = \mathcal{E} - Ir = IR$, where R is the resistance connected to the battery. During the charging the terminal voltage is $V_{ab} = \mathcal{E} + Ir$. $P = VI$ and energy is $E = Pt$. $I^2 r$ is the rate at which energy is dissipated in the internal resistance of the battery.

EXECUTE: **(a)** $V_{ab} = \mathcal{E} + Ir = 12.0\text{ V} + (30\text{ A})(0.24\,\Omega) = 19.2\text{ V}$.

(b) $E = Pt = IVt = (30\text{ A})(19.2\text{ V})(1.7)(3600\text{ s}) = 3.53\times10^6\text{ J}$.

(c) $E_{\text{diss}} = P_{\text{diss}}t = I^2Rt = (30\text{ A})^2(0.24\,\Omega)(1.7)(3600\text{ s}) = 1.32\times10^6\text{ J}$.

(d) Discharged at 30 A: $I = \dfrac{\mathcal{E}}{r+R}$ gives $R = \dfrac{\mathcal{E} - Ir}{I} = \dfrac{12.0\text{ V} - (30\text{ A})(0.24\,\Omega)}{30\text{ A}} = 0.16\,\Omega$.

(e) $E = Pt = I^2Rt = (30\text{ A})^2(0.16\,\Omega)(1.7)(3600\text{ s}) = 8.81\times10^5\text{ J}$.

(f) Since the current through the internal resistance is the same as before, there is the same energy dissipated as in (c): $E_{\text{diss}} = 1.32\times10^6\text{ J}$.

(g) Again, part of the energy originally supplied was stored in the battery and part was lost in the internal resistance. So the stored energy was less than what was supplied during charging. Then when discharging, even more energy is lost in the internal resistance, and what is left is dissipated over the external resistor. This time, at a higher current, much more energy is lost in the internal resistance. Slow charging and discharging is more energy efficient.

25.83. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** $\Sigma F = ma = |q|E$ gives $\dfrac{|q|}{m} = \dfrac{a}{E}$.

(b) If the electric field is constant, $V_{bc} = EL$ and $\dfrac{|q|}{m} = \dfrac{aL}{V_{bc}}$

(c) The free charges are "left behind" so the left end of the rod is negatively charged, while the right end is positively charged. Thus the right end, point *c*, is at the higher potential.

(d) $a = \dfrac{V_{bc}|q|}{mL} = \dfrac{(1.0\times10^{-3}\text{ V})(1.6\times10^{-19}\text{ C})}{(9.11\times10^{-31}\text{ kg})(0.50\text{ m})} = 3.5\times10^8\text{ m/s}^2$.

EVALUATE: **(e)** Performing the experiment in a rotational way enables one to keep the experimental apparatus in a localized area—whereas an acceleration like that obtained in (d), if linear, would quickly have the apparatus moving at high speeds and large distances. Also, the rotating spool of thin wire can have many turns of wire and the total potential is the sum of the potentials in each turn, the potential in each turn times the number of turns.

25.84. **IDENTIFY:** $\mathcal{E} - IR - V = 0$

SET UP: With $T = 293\text{ K}$, $\dfrac{e}{kT} = 39.6\text{ V}^{-1}$.

EXECUTE: **(a)** $\mathcal{E} = IR + V$ gives $2.00\text{ V} = I(1.0\,\Omega) + V$. Dropping units and using the expression given in the problem for *I*, this becomes $2.00 = I_S[\exp(eV/kT) - 1] + V$.

(b) For $I_S = 1.50\times10^{-3}$ A and $T = 293$ K, $1333 = \exp[39.6\,V] - 1 + 667V$. Trial and error shows that the right-hand side (rhs) above, for specific *V* values, equals 1333 V when $V = 0.179$ V. The current then is just $I = I_S[\exp(39.6\,V) - 1] = (1.5\times10^{-3}\text{ A})\left(\exp([39.6][0.179]) - 1]\right) = 1.80\text{ A}$.

EVALUATE: The voltage across the resistor *R* is 1.80 V. The diode does not obey Ohm's law.

25.85. **IDENTIFY:** Apply $R = \dfrac{\rho L}{A}$ to find the resistance of a thin slice of the rod and integrate to find the total *R*. $V = IR$. Also find $R(x)$, the resistance of a length *x* of the rod.

SET UP: $E(x) = \rho(x)J$

EXECUTE: **(a)** $dR = \dfrac{\rho\,dx}{A} = \dfrac{\rho_0\exp[-x/L]\,dx}{A}$ so

$R = \dfrac{\rho_0}{A}\displaystyle\int_0^L \exp[-x/L]\,dx = \dfrac{\rho_0}{A}[-L\exp[-x/L]]_0^L = \dfrac{\rho_0 L}{A}(1 - e^{-1})$ and $I = \dfrac{V_0}{R} = \dfrac{V_0 A}{\rho_0 L(1 - e^{-1})}$. With an upper limit of *x* rather than *L* in the integration, $R(x) = \dfrac{\rho_0 L}{A}\left(1 - e^{-x/L}\right)$.

(b) $E(x) = \rho(x)J = \dfrac{I\,\rho_0 e^{-x/L}}{A} = \dfrac{V_0 e^{-x/L}}{L\left(1 - e^{-1}\right)}$.

(c) $V = V_0 - IR(x)$. $V = V_0 - \left(\frac{V_0 A}{\rho_0 L[1-e^{-1}]}\right)\left(\frac{\rho_0 L}{A}\right)\left(1-e^{-x/L}\right) = V_0 \frac{(e^{-x/L} - e^{-1})}{(1-e^{-1})}$

(d) Graphs of resistivity, electric field and potential from $x = 0$ to L are given in Figure 25.85. Each quantity is given in terms of the indicated unit.

EVALUATE: The current is the same at all points in the rod. Where the resistivity is larger the electric field must be larger, in order to produce the same current density.

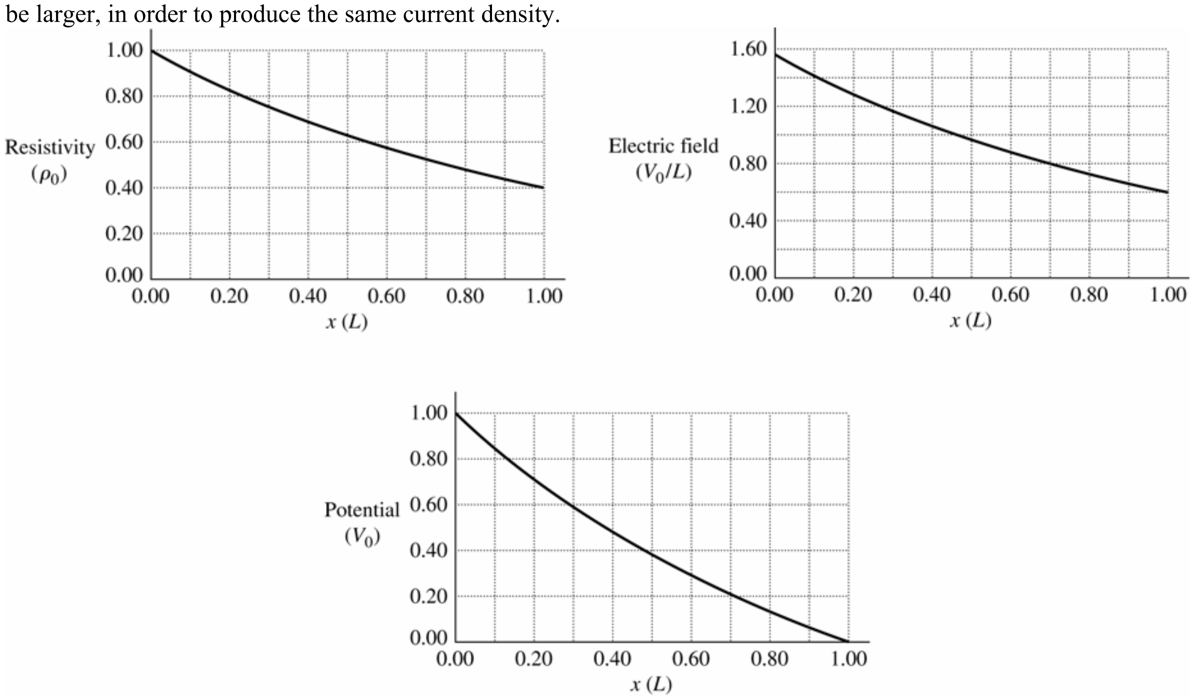


Figure 25.85

25.86. **IDENTIFY:** The power output of the source is $VI = (\mathcal{E} - Ir)I$.

SET UP: The short-circuit current is $I_{\text{short circuit}} = \mathcal{E}/r$.

EXECUTE: **(a)** $P = \mathcal{E}I - I^2 r$, so $\frac{dP}{dI} = \mathcal{E} - 2Ir = 0$ for maximum power output and $I_{P\text{ max}} = \frac{1}{2}\frac{\mathcal{E}}{r} = \frac{1}{2} I_{\text{short circuit}}$.

(b) For the maximum power output of part (a), $I = \frac{\mathcal{E}}{r+R} = \frac{1}{2}\frac{\mathcal{E}}{r}$. $r + R = 2r$ and $R = r$.

Then, $P = I^2 R = \left(\frac{\mathcal{E}}{2r}\right)^2 r = \frac{\mathcal{E}^2}{4r}$.

EVALUATE: When R is smaller than r, I is large and the $I^2 r$ losses in the battery are large. When R is larger than r, I is small and the power output $\mathcal{E}I$ of the battery emf is small.

25.87. **IDENTIFY:** Use $\alpha = -n/T$ in $\alpha = \frac{1}{\rho}\frac{d\rho}{dT}$ to get a separable differential equation that can be integrated.

SET UP: For carbon, $\rho = 3.5\times10^{-5}\ \Omega\cdot\text{m}$ and $\alpha = -5\times10^{-4}\ (\text{K})^{-1}$.

EXECUTE: **(a)** $\alpha = \frac{1}{\rho}\left(\frac{d\rho}{dT}\right) = -\frac{n}{T} \Rightarrow \frac{ndT}{T} = \frac{d\rho}{\rho} \Rightarrow \ln(T^{-n}) = \ln(\rho) \Rightarrow \rho = \frac{a}{T^n}$.

(b) $n = -\alpha T = -(-5\times10^{-4}\ (\text{K})^{-1})(293\ \text{K}) = 0.15$.

$\rho = \frac{a}{T^n} \Rightarrow a = \rho T^n = (3.5\times10^{-5}\ \Omega\cdot\text{m})(293\ \text{K})^{0.15} = 8.0\times10^{-5}\ \Omega\cdot\text{m}\cdot\text{K}^{0.15}$.

(c) $T = -196°\text{C} = 77\ \text{K}$: $\rho = \frac{8.0\times10^{-5}}{(77\ \text{K})^{0.15}} = 4.3\times10^{-5}\ \Omega\cdot\text{m}$.

$T = -300°\text{C} = 573\ \text{K}$: $\rho = \frac{8.0\times10^{-5}}{(573\ \text{K})^{0.15}} = 3.2\times10^{-5}\ \Omega\cdot\text{m}$.

EVALUATE: α is negative and decreases as T decreases, so ρ changes more rapidly with temperature at lower temperatures.