# 23

## ELECTRIC POTENTIAL

**23.1.** **IDENTIFY:** Apply Eq.(23.2) to calculate the work. The electric potential energy of a pair of point charges is given by Eq.(23.9).

**SET UP:** Let the initial position of $q_2$ be point *a* and the final position be point *b*, as shown in Figure 23.1.

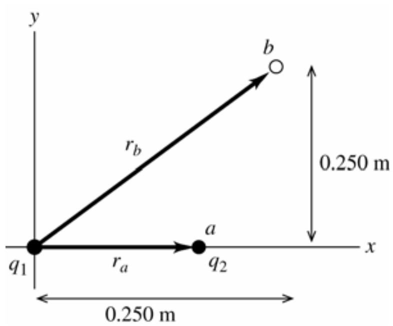



**Figure 23.1**

$$r_a = 0.150 \text{ m}$$

$$r_b = \sqrt{(0.250 \text{ m})^2 + (0.250 \text{ m})^2}$$

$$r_b = 0.3536 \text{ m}$$

**EXECUTE:** $W_{a\to b} = U_a - U_b$

$$U_a = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_a} = (8.988\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(+2.40\times 10^{-6} \text{ C})(-4.30\times 10^{-6} \text{ C})}{0.150 \text{ m}}$$

$$U_a = -0.6184 \text{ J}$$

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_b} = (8.988\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(+2.40\times 10^{-6} \text{ C})(-4.30\times 10^{-6} \text{ C})}{0.3536 \text{ m}}$$

$$U_b = -0.2623 \text{ J}$$

$$W_{a\to b} = U_a - U_b = -0.6184 \text{ J} - (-0.2623 \text{ J}) = -0.356 \text{ J}$$

**EVALUATE:** The attractive force on $q_2$ is toward the origin, so it does negative work on $q_2$ when $q_2$ moves to larger *r*.

**23.2.** **IDENTIFY:** Apply $W_{a\to b} = U_a - U_b$.

**SET UP:** $U_a = +5.4\times 10^{-8}$ J. Solve for $U_b$.

**EXECUTE:** $W_{a\to b} = -1.9\times 10^{-8} \text{ J} = U_a - U_b$. $U_b = U_a - W_{a\to b} = 1.9\times 10^{-8} \text{ J} - (-5.4\times 10^{-8} \text{ J}) = 7.3\times 10^{-8}$ J.

**EVALUATE:** When the electric force does negative work the electrical potential energy increases.

**23.3.** **IDENTIFY:** The work needed to assemble the nucleus is the sum of the electrical potential energies of the protons in the nucleus, relative to infinity.

**SET UP:** The total potential energy is the scalar sum of all the individual potential energies, where each potential energy is $U = (1/4\pi\epsilon_0)(qq_0/r)$. Each charge is *e* and the charges are equidistant from each other, so the total potential energy is $U = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{e^2}{r} + \dfrac{e^2}{r} + \dfrac{e^2}{r}\right) = \dfrac{3e^2}{4\pi\epsilon_0 r}$.

**EXECUTE:** Adding the potential energies gives

$$U = \frac{3e^2}{4\pi\epsilon_0 r} = \frac{3(1.60\times 10^{-19} \text{ C})^2(9.00\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)}{2.00\times 10^{-15} \text{ m}} = 3.46\times 10^{-13} \text{ J} = 2.16 \text{ MeV}$$

**EVALUATE:** This is a small amount of energy on a macroscopic scale, but on the scale of atoms, 2 MeV is quite a lot of energy.

**23.4.** **IDENTIFY:** The work required is the change in electrical potential energy. The protons gain speed after being released because their potential energy is converted into kinetic energy.

**(a) SET UP:** Using the potential energy of a pair of point charges relative to infinity, $U=(1/4\pi\epsilon_0)(qq_0/r)$. we have

$$W=\Delta U=U_2-U_1=\frac{1}{4\pi\epsilon_0}\left(\frac{e^2}{r_2}-\frac{e^2}{r_1}\right).$$

**EXECUTE:** Factoring out the $e^2$ and substituting numbers gives

$$W=\left(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2\left(\frac{1}{3.00\times10^{-15}\ \text{m}}-\frac{1}{2.00\times10^{-15}\ \text{m}}\right)=7.68\times10^{-14}\ \text{J}$$

**(b) SET UP:** The protons have equal momentum, and since they have equal masses, they will have equal speeds and hence equal kinetic energy. $\Delta U=K_1+K_2=2K=2\left(\frac{1}{2}mv^2\right)=mv^2.$

**EXECUTE:** Solving for $v$ gives $v=\sqrt{\dfrac{\Delta U}{m}}=\sqrt{\dfrac{7.68\times10^{-14}\ \text{J}}{1.67\times10^{-27}\ \text{kg}}}=6.78\times10^6$ m/s

**EVALUATE:** The potential energy may seem small (compared to macroscopic energies), but it is enough to give each proton a speed of nearly 7 million m/s.

**23.5.** **(a) IDENTIFY:** Use conservation of energy:

$$K_a+U_a+W_{\text{other}}=K_b+U_b$$

$U$ for the pair of point charges is given by Eq.(23.9).

**SET UP:**

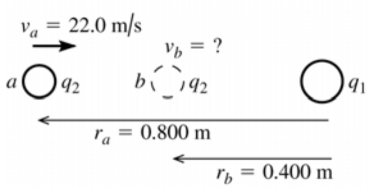



**Figure 23.5a**

Let point $a$ be where $q_2$ is 0.800 m from $q_1$ and point $b$ be where $q_2$ is 0.400 m from $q_1$, as shown in Figure 23.5a.

**EXECUTE:** Only the electric force does work, so $W_{\text{other}}=0$ and $U=\dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r}$.

$$K_a=\tfrac{1}{2}mv_a^2=\tfrac{1}{2}(1.50\times10^{-3}\ \text{kg})(22.0\ \text{m/s})^2=0.3630\ \text{J}$$

$$U_a=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_a}=(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\ \text{C})(-7.80\times10^{-6}\ \text{C})}{0.800\ \text{m}}=+0.2454\ \text{J}$$

$$K_b=\tfrac{1}{2}mv_b^2$$

$$U_b=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_b}=(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\ \text{C})(-7.80\times10^{-6}\ \text{C})}{0.400\ \text{m}}=+0.4907\ \text{J}$$

The conservation of energy equation then gives $K_b=K_a+(U_a-U_b)$

$$\tfrac{1}{2}mv_b^2=+0.3630\ \text{J}+(0.2454\ \text{J}-0.4907\ \text{J})=0.1177\ \text{J}$$

$$v_b=\sqrt{\frac{2(0.1177\ \text{J})}{1.50\times10^{-3}\ \text{kg}}}=12.5\ \text{m/s}$$

**EVALUATE:** The potential energy increases when the two positively charged spheres get closer together, so the kinetic energy and speed decrease.

**(b) IDENTIFY:** Let point $c$ be where $q_2$ has its speed momentarily reduced to zero. Apply conservation of energy to points $a$ and $c$: $K_a+U_a+W_{\text{other}}=K_c+U_c$.

**SET UP:** Points $a$ and $c$ are shown in Figure 23.5b.

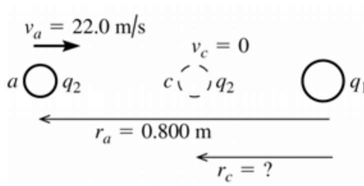



**Figure 23.5b**

**EXECUTE:** $K_a = +0.3630$ J (from part (a))

$U_a = +0.2454$ J (from part (a))

$K_c = 0$ (at distance of closest approach the speed is zero)

$$U_c = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_c}$$

Thus conservation of energy $K_a + U_a = U_c$ gives $\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_c} = +0.3630 \text{ J} + 0.2454 \text{ J} = 0.6084 \text{ J}$

$$r_c = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{0.6084 \text{ J}} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6} \text{ C})(-7.80\times10^{-6} \text{ C})}{+0.6084 \text{ J}} = 0.323 \text{ m.}$$

**EVALUATE:** $U \to \infty$ as $r \to 0$ so $q_2$ will stop no matter what its initial speed is.

**23.6. IDENTIFY:** Apply $U = k\dfrac{q_1q_2}{r}$ and solve for $r$.

**SET UP:** $q_1 = -7.2\times10^{-6}$ C, $q_2 = +2.3\times10^{-6}$ C

**EXECUTE:** $r = \dfrac{kq_1q_2}{U} = \dfrac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(-7.20\times10^{-6} \text{ C})(+2.30\times10^{-6} \text{ C})}{-0.400 \text{ J}} = 0.372 \text{ m}$

**EVALUATE:** The potential energy $U$ is a scalar and can take positive and negative values.

**23.7. (a) IDENTIFY** and **SET UP:** $U$ is given by Eq.(23.9).

**EXECUTE:** $U = \dfrac{1}{4\varepsilon\pi_0}\dfrac{qq'}{r}$

$$U = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(+4.60\times10^{-6} \text{ C})(+1.20\times10^{-6} \text{ C})}{0.250 \text{ m}} = +0.198 \text{ J}$$

**EVALUATE:** The two charges are both of the same sign so their electric potential energy is positive.

**(b) IDENTIFY:** Use conservation of energy: $K_a + U_a + W_{\text{other}} = K_b + U_b$

**SET UP:** Let point $a$ be where $q$ is released and point $b$ be at its final position, as shown in Figure 23.7.

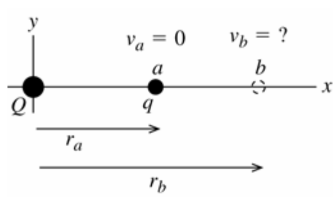



**Figure 23.7**

**EXECUTE:** $K_a = 0$ (released from rest)

$U_a = +0.198$ J (from part (a))

$K_b = \frac{1}{2}mv_b^2$

Only the electric force does work, so $W_{\text{other}} = 0$ and $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{r}$.

(i) $r_b = 0.500$ m

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(+4.60\times10^{-6} \text{ C})(+1.20\times10^{-6} \text{ C})}{0.500 \text{ m}} = +0.0992 \text{ J}$$

Then $K_a + U_a + W_{\text{other}} = K_b + U_b$ gives $K_b = U_a - U_b$ and $\frac{1}{2}mv_b^2 = U_a - U_b$ and

$$v_b = \sqrt{\frac{2(U_a - U_b)}{m}} = \sqrt{\frac{2(+0.198 \text{ J} - 0.0992 \text{ J})}{2.80\times10^{-4} \text{ kg}}} = 26.6 \text{ m/s.}$$

(ii) $r_b = 5.00$ m $r_b$ is now ten times larger than in (i) so $U_b$ is ten times smaller: $U_b = +0.0992 \text{ J}/10 = +0.00992$ J.

$$v_b = \sqrt{\frac{2(U_a - U_b)}{m}} = \sqrt{\frac{2(+0.198 \text{ J} - 0.00992 \text{ J})}{2.80\times10^{-4} \text{ kg}}} = 36.7 \text{ m/s.}$$

(iii) $r_b = 50.0$ m

$r_b$ is now ten times larger than in (ii) so $U_b$ is ten times smaller:

$$U_b = +0.00992 \text{ J}/10 = +0.000992 \text{ J}.$$

$$v_b = \sqrt{\frac{2(U_a - U_b)}{m}} = \sqrt{\frac{2(+0.198 \text{ J} - 0.000992 \text{ J})}{2.80\times 10^{-4} \text{ kg}}} = 37.5 \text{ m/s}.$$

**EVALUATE:** The force between the two charges is repulsive and provides an acceleration to $q$. This causes the speed of $q$ to increase as it moves away from $Q$.

**23.8.** **IDENTIFY:** Call the three charges 1, 2 and 3. $U = U_{12} + U_{13} + U_{23}$

**SET UP:** $U_{12} = U_{23} = U_{13}$ because the charges are equal and each pair of charges has the same separation, 0.500 m.

**EXECUTE:** $U = \dfrac{3kq^2}{0.500 \text{ m}} = \dfrac{3k(1.2\times 10^{-6} \text{ C})^2}{0.500 \text{ m}} = 0.078$ J.

**EVALUATE:** When the three charges are brought in from infinity to the corners of the triangle, the repulsive electrical forces between each pair of charges do negative work and electrical potential energy is stored.

**23.9.** **IDENTIFY:** $U = k\left(\dfrac{q_1q_2}{r_{12}} + \dfrac{q_1q_3}{r_{13}} + \dfrac{q_2q_3}{r_{23}}\right)$

**SET UP:** In part (a), $r_{12} = 0.200$ m, $r_{23} = 0.100$ m and $r_{13} = 0.100$ m. In part (b) let particle 3 have coordinate $x$, so $r_{12} = 0.200$ m, $r_{13} = x$ and $r_{23} = 0.200 - x$.

**EXECUTE:** **(a)** $U = k\left(\dfrac{(4.00 \text{ nC})(-3.00 \text{ nC})}{(0.200 \text{ m})} + \dfrac{(4.00 \text{ nC})(2.00 \text{ nC})}{(0.100 \text{ m})} + \dfrac{(-3.00 \text{ nC})(2.00 \text{ nC})}{(0.100 \text{ m})}\right) = 3.60\times 10^{-7}$ J

**(b)** If $U = 0$, then $0 = k\left(\dfrac{q_1q_2}{r_{12}} + \dfrac{q_1q_3}{x} + \dfrac{q_2q_3}{r_{12} - x}\right)$. Solving for $x$ we find:

$0 = -60 + \dfrac{8}{x} - \dfrac{6}{0.2 - x} \Rightarrow 60x^2 - 26x + 1.6 = 0 \Rightarrow x = 0.074$ m, 0.360 m. Therefore, $x = 0.074$ m since it is the only value between the two charges.

**EVALUATE:** $U_{13}$ is positive and both $U_{23}$ and $U_{12}$ are negative. If $U = 0$, then $|U_{13}| = |U_{23}| + |U_{12}|$. For $x = 0.074$ m, $U_{13} = +9.7\times 10^{-7}$ J, $U_{23} = -4.3\times 10^{-7}$ J and $U_{12} = -5.4\times 10^{-7}$ J. It is true that $U = 0$ at this $x$.

**23.10.** **IDENTIFY:** The work done on the alpha particle is equal to the difference in its potential energy when it is moved from the midpoint of the square to the midpoint of one of the sides.

**SET UP:** We apply the formula $W_{a\to b} = U_a - U_b$. In this case, $a$ is the center of the square and $b$ is the midpoint of one of the sides. Therefore $W_{\text{center}\to\text{side}} = U_{\text{center}} - U_{\text{side}}$.

There are 4 electrons, so the potential energy at the center of the square is 4 times the potential energy of a single alpha-electron pair. At the center of the square, the alpha particle is a distance $r_1 = \sqrt{50}$ nm from each electron. At the midpoint of the side, the alpha is a distance $r_2 = 5.00$ nm from the two nearest electrons and a distance $r_2 = \sqrt{125}$ nm from the two most distant electrons. Using the formula for the potential energy (relative to infinity) of two point charges, $U = (1/4\pi\epsilon_0)(qq_0/r)$, the total work is

$$W_{\text{center}\to\text{side}} = U_{\text{center}} - U_{\text{side}} = 4\frac{1}{4\pi\epsilon_0}\frac{q_\alpha q_e}{r_1} - \left(2\frac{1}{4\pi\epsilon_0}\frac{q_\alpha q_e}{r_2} + 2\frac{1}{4\pi\epsilon_0}\frac{q_\alpha q_e}{r_3}\right)$$

Substituting $q_e = e$ and $q_\alpha = 2e$ and simplifying gives

$$W_{\text{center}\to\text{side}} = -4e^2\frac{1}{4\pi\epsilon_0}\left[\frac{2}{r_1} - \left(\frac{1}{r_2} + \frac{1}{r_3}\right)\right]$$

**EXECUTE:** Substituting the numerical values into the equation for the work gives

$$W = -4\left(1.60\times 10^{-19} \text{ C}\right)^2\left[\frac{2}{\sqrt{50} \text{ m}} - \left(\frac{1}{5.00 \text{ nm}} + \frac{1}{\sqrt{125} \text{ nm}}\right)\right] = 6.08\times 10^{-21} \text{ J ???}$$

**EVALUATE:** Since the work is positive, the system has more potential energy with the alpha particle at the center of the square than it does with it at the midpoint of a side.

**23.11.** **IDENTIFY:** Apply Eq.(23.2). The net work to bring the charges in from infinity is equal to the change in potential energy. The total potential energy is the sum of the potential energies of each pair of charges, calculated from Eq.(23.9).

**SET UP:** Let 1 be where all the charges are infinitely far apart. Let 2 be where the charges are at the corners of the triangle, as shown in Figure 23.11.

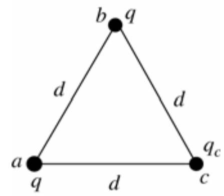



**Figure 23.11**

Let $q_c$ be the third, unknown charge.

**EXECUTE:** $W = -\Delta U = -(U_2 - U_1)$

$$U_1 = 0$$

$$U_2 = U_{ab} + U_{ac} + U_{bc} = \frac{1}{4\pi\epsilon_0 d}(q^2 + 2qq_c)$$

Want $W = 0$, so $W = -(U_2 - U_1)$ gives $0 = -U_2$

$$0 = \frac{1}{4\pi\epsilon_0 d}(q^2 + 2qq_c)$$

$q^2 + 2qq_c = 0$ and $q_c = -q/2$.

**EVALUATE:** The potential energy for the two charges $q$ is positive and for each $q$ with $q_c$ it is negative. There are two of the $q$, $q_c$ terms so must have $q_c < q$.

**23.12.** **IDENTIFY:** Use conservation of energy $U_a + K_a = U_b + K_b$ to find the distance of closest approach $r_b$. The maximum force is at the distance of closest approach, $F = k\dfrac{|q_1 q_2|}{r_b^2}$.

**SET UP:** $K_b = 0$. Initially the two protons are far apart, so $U_a = 0$. A proton has mass $1.67\times10^{-27}$ kg and charge $q = +e = +1.60\times10^{-19}$ C.

**EXECUTE:** $K_a = U_b$. $2(\frac{1}{2}mv_a^2) = k\dfrac{q_1 q_2}{r_b}$. $mv_a^2 = k\dfrac{e^2}{r_b}$ and

$$r_b = \frac{ke^2}{mv_a^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(1.67\times10^{-27}\ \text{kg})(1.00\times10^6\ \text{m/s})^2} = 1.38\times10^{-13}\ \text{m}.$$

$$F = k\frac{e^2}{r_b^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{(1.38\times10^{-13}\ \text{C})^2} = 0.012\ \text{N}.$$

**EVALUATE:** The acceleration $a = F/m$ of each proton produced by this force is extremely large.

**23.13.** **IDENTIFY:** $\vec{E}$ points from high potential to low potential. $\dfrac{W_{a\to b}}{q_0} = V_a - V_b$.

**SET UP:** The force on a positive test charge is in the direction of $\vec{E}$.

**EXECUTE:** $V$ decreases in the eastward direction. $A$ is east of $B$, so $V_B > V_A$. $C$ is east of $A$, so $V_C < V_A$. The force on a positive test charge is east, so no work is done on it by the electric force when it moves due south (the force and displacement are perpendicular), and $V_D = V_A$.

**EVALUATE:** The electric potential is constant in a direction perpendicular to the electric field.

**23.14.** **IDENTIFY:** $\dfrac{W_{a\to b}}{q_0} = V_a - V_b$. For a point charge, $V = \dfrac{kq}{r}$.

**SET UP:** Each vacant corner is the same distance, 0.200 m, from each point charge.

**EXECUTE:** Taking the origin at the center of the square, the symmetry means that the potential is the same at the two corners not occupied by the $+5.00\ \mu\text{C}$ charges. This means that no net work is done is moving from one corner to the other.

**EVALUATE:** If the charge $q_0$ moves along a diagonal of the square, the electrical force does positive work for part of the path and negative work for another part of the path, but the net work done is zero.

**23.15.** **IDENTIFY** and **SET UP:** Apply conservation of energy to points $A$ and $B$.

**EXECUTE:** $K_A + U_A = K_B + U_B$

$U = qV$, so $K_A + qV_A = K_B + qV_B$

$K_B = K_A + q(V_A - V_B) = 0.00250\text{ J} + (-5.00\times10^{-6}\text{ C})(200\text{ V} - 800\text{ V}) = 0.00550\text{ J}$

$v_B = \sqrt{2K_B/m} = 7.42\text{ m/s}$

**EVALUATE:** It is faster at $B$; a negative charge gains speed when it moves to higher potential.

**23.16.** **IDENTIFY:** The work-energy theorem says $W_{a\to b} = K_b - K_a$. $\dfrac{W_{a\to b}}{q} = V_a - V_b$.

**SET UP:** Point $a$ is the starting and point $b$ is the ending point. Since the field is uniform, $W_{a\to b} = Fs\cos\phi = E|q|s\cos\phi$. The field is to the left so the force on the positive charge is to the left. The particle moves to the left so $\phi = 0°$ and the work $W_{a\to b}$ is positive.

**EXECUTE:** **(a)** $W_{a\to b} = K_b - K_a = 1.50\times10^{-6}\text{ J} - 0 = 1.50\times10^{-6}\text{ J}$

**(b)** $V_a - V_b = \dfrac{W_{a\to b}}{q} = \dfrac{1.50\times10^{-6}\text{ J}}{4.20\times10^{-9}\text{ C}} = 357\text{ V}$. Point $a$ is at higher potential than point $b$.

**(c)** $E|q|s = W_{a\to b}$, so $E = \dfrac{W_{a\to b}}{|q|s} = \dfrac{V_a - V_b}{s} = \dfrac{357\text{ V}}{6.00\times10^{-2}\text{ m}} = 5.95\times10^{3}\text{ V/m}$.

**EVALUATE:** A positive charge gains kinetic energy when it moves to lower potential; $V_b < V_a$.

**23.17.** **IDENTIFY:** Apply the equation that precedes Eq.(23.17): $W_{a\to b} = q'\int_a^b \vec{E}\cdot d\vec{l}$.

**SET UP:** Use coordinates where $+y$ is upward and $+x$ is to the right. Then $\vec{E} = E\hat{j}$ with $E = 4.00\times10^{4}\text{ N/C}$.

**(a)** The path is sketched in Figure 23.17a.

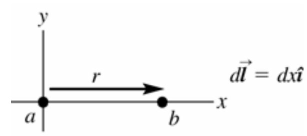



**Figure 23.17a**

**EXECUTE:** $\vec{E}\cdot d\vec{l} = (E\hat{j})\cdot(dx\hat{i}) = 0$ so $W_{a\to b} = q'\int_a^b \vec{E}\cdot d\vec{l} = 0$.

**EVALUATE:** The electric force on the positive charge is upward (in the direction of the electric field) and does no work for a horizontal displacement of the charge.

**(b) SET UP:** The path is sketched in Figure 23.17b.

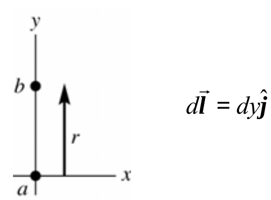



**Figure 23.17b**

**EXECUTE:** $\vec{E}\cdot d\vec{l} = (E\hat{j})\cdot(dy\hat{j}) = E\,dy$

$$W_{a\to b} = q'\int_a^b \vec{E}\cdot d\vec{l} = q'E\int_a^b dy = q'E(y_b - y_a)$$

$y_b - y_a = +0.670\text{ m}$, positive since the displacement is upward and we have taken $+y$ to be upward.

$$W_{a\to b} = q'E(y_b - y_a) = (+28.0\times10^{-9}\text{ C})(4.00\times10^{4}\text{ N/C})(+0.670\text{ m}) = +7.50\times10^{-4}\text{ J}.$$

**EVALUATE:** The electric force on the positive charge is upward so it does positive work for an upward displacement of the charge.

**(c) SET UP:** The path is sketched in Figure 23.17c.

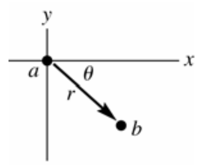



**Figure 23.17c**

$y_a = 0$

$y_b = -r\sin\theta = -(2.60\text{ m})\sin 45° = -1.838\text{ m}$

The vertical component of the 2.60 m displacement is 1.838 m downward.

**EXECUTE:** $d\vec{l} = dx\hat{i} + dy\hat{j}$ (The displacement has both horizontal and vertical components.)

$\vec{E}\cdot d\vec{l} = (E\hat{j})\cdot(dx\hat{i} + dy\hat{j}) = E\,dy$ (Only the vertical component of the displacement contributes to the work.)

$$W_{a\to b} = q'\int_a^b \vec{E}\cdot d\vec{l} = q'E\int_a^b dy = q'E(y_b - y_a)$$

$$W_{a\to b} = q'E(y_b - y_a) = (+28.0\times10^{-9}\text{C})(4.00\times10^4\text{ N/C})(-1.838\text{ m}) = -2.06\times10^{-3}\text{ J}.$$

**EVALUATE:** The electric force on the positive charge is upward so it does negative work for a displacement of the charge that has a downward component.

**23.18.** **IDENTIFY:** Apply $K_a + U_a = K_b + U_b$.

**SET UP:** Let $q_1 = +3.00\text{ nC}$ and $q_2 = +2.00\text{ nC}$. At point $a$, $r_{1a} = r_{2a} = 0.250\text{ m}$. At point $b$, $r_{1b} = 0.100\text{ m}$ and $r_{2b} = 0.400\text{ m}$. The electron has $q = -e$ and $m_e = 9.11\times10^{-31}\text{ kg}$. $K_a = 0$ since the electron is released from rest.

**EXECUTE:** $-\dfrac{keq_1}{r_{1a}} - \dfrac{keq_2}{r_{2a}} = -\dfrac{keq_1}{r_{1b}} - \dfrac{keq_2}{r_{2b}} + \dfrac{1}{2}m_e v_b^2$.

$$E_a = K_a + U_a = k(-1.60\times10^{-19}\text{ C})\left(\frac{(3.00\times10^{-9}\text{ C})}{0.250\text{ m}} + \frac{(2.00\times10^{-9}\text{ C})}{0.250\text{ m}}\right) = -2.88\times10^{-17}\text{ J}.$$

$$E_b = K_b + U_b = k(-1.60\times10^{-19}\text{ C})\left(\frac{(3.00\times10^{-9}\text{ C})}{0.100\text{ m}} + \frac{(2.00\times10^{-9}\text{ C}}{0.400\text{ m}}\right) + \frac{1}{2}m_e v_b^2 = -5.04\times10^{-17}\text{ J} + \frac{1}{2}m_e v_b^2$$

Setting $E_a = E_b$ gives $v_b = \sqrt{\dfrac{2}{9.11\times10^{-31}\text{ kg}}(5.04\times10^{-17}\text{ J} - 2.88\times10^{-17}\text{ J})} = 6.89\times10^6\text{ m/s}.$

**EVALUATE:** $V_a = V_{1a} + V_{2a} = 180\text{ V}$. $V_b = V_{1b} + V_{2b} = 315\text{ V}$. $V_b > V_a$. The negatively charged electron gains kinetic energy when it moves to higher potential.

**23.19.** **IDENTIFY** and **SET UP:** For a point charge $V = \dfrac{kq}{r}$. Solve for $r$.

**EXECUTE:** **(a)** $r = \dfrac{kq}{V} = \dfrac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.50\times10^{-11}\text{ C})}{90.0\text{ V}} = 2.50\times10^{-3}\text{ m} = 2.50\text{ mm}$

**(b)** $Vr = kq = \text{constant}$ so $V_1 r_1 = V_2 r_2$. $r_2 = r_1\left(\dfrac{V_1}{V_2}\right) = (2.50\text{ mm})\left(\dfrac{90.0\text{ V}}{30.0\text{ V}}\right) = 7.50\text{ mm}$.

**EVALUATE:** The potential of a positive charge is positive and decreases as the distance from the point charge increases.

**23.20.** **IDENTIFY:** The total potential is the *scalar* sum of the individual potentials, but the net electric field is the *vector* sum of the two fields.

**SET UP:** The net potential can only be zero if one charge is positive and the other is negative, since it is a scalar. The electric field can only be zero if the two fields point in opposite directions.

**EXECUTE:** **(a)** (i) Since both charges have the same sign, there are no points for which the potential is zero.

(ii) The two electric fields are in opposite directions only between the two charges, and midway between them the fields have equal magnitudes. So $E = 0$ midway between the charges, but $V$ is never zero.

**(b)** (i) The two potentials have equal magnitude but opposite sign midway between the charges, so $V = 0$ midway between the charges, but $E \neq 0$ there since the fields point in the same direction.

(ii) Between the two charges, the fields point in the same direction, so $E$ cannot be zero there. In the other two regions, the field due to the nearer charge is always greater than the field due to the more distant charge, so they cannot cancel. Hence $E$ is not zero anywhere.

**EVALUATE:** It does *not* follow that the electric field is zero where the potential is zero, or that the potential is zero where the electric field is zero.

**23.21.** **IDENTIFY:** $V = \dfrac{1}{4\pi\epsilon_0}\sum_i \dfrac{q_i}{r_i}$

**SET UP:** The locations of the changes and points $A$ and $B$ are sketched in Figure 23.21.

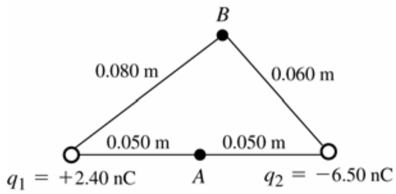



**Figure 23.21**

**EXECUTE:** **(a)** $V_A = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r_{A1}} + \dfrac{q_2}{r_{A2}}\right)$

$$V_A = (8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.40\times 10^{-9}\ \text{C}}{0.050\ \text{m}} + \frac{-6.50\times 10^{-9}\ \text{C}}{0.050\ \text{m}}\right) = -737\ \text{V}$$

**(b)** $V_B = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r_{B1}} + \dfrac{q_2}{r_{B2}}\right)$

$$V_B = (8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.40\times 10^{-9}\ \text{C}}{0.080\ \text{m}} + \frac{-6.50\times 10^{-9}\ \text{C}}{0.060\ \text{m}}\right) = -704\ \text{V}$$

**(c) IDENTIFY** and **SET UP:** Use Eq.(23.13) and the results of parts (a) and (b) to calculate $W$.

**EXECUTE:** $W_{B\to A} = q'(V_B - V_A) = (2.50\times 10^{-9}\ \text{C})(-704\ \text{V} - (-737\ \text{V})) = +8.2\times 10^{-8}\ \text{J}$

**EVALUATE:** The electric force does positive work on the positive charge when it moves from higher potential (point $B$) to lower potential (point $A$).

**23.22.** **IDENTIFY:** For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

**SET UP:** **(a)** The positions of the two charges are shown in Figure 23.22a. $r = \sqrt{a^2 + x^2}$.

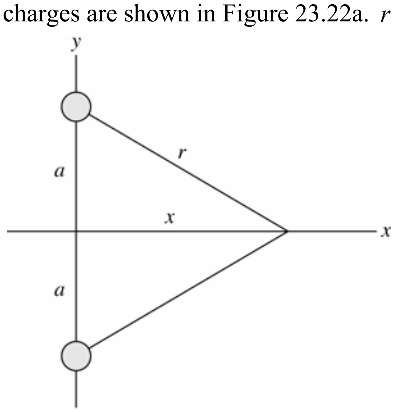



**Figure 23.22a**

**EXECUTE:** **(b)** $V_0 = 2\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{a}$.

**(c)** $V(x) = 2\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r} = 2\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{\sqrt{a^2 + x^2}}$

**(d)** The graph of $V$ versus $x$ is sketched in Figure 23.22b.

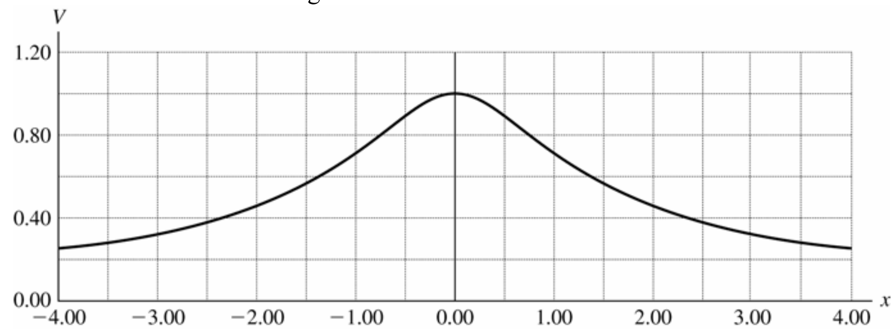



**Figure 23.22b**

**EVALUATE:** **(e)** When $x >> a$, $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{2q}{x}$, just like a point charge of charge $+2q$. At distances from the charges much greater than their separation, the two charges act like a single point charge.

**23.23.** **IDENTIFY:** For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

**SET UP:** **(a)** The positions of the two charges are shown in Figure 23.23.

**EXECUTE:** **(b)** $V = \dfrac{kq}{r} + \dfrac{k(-q)}{r} = 0.$

**(c)** The potential along the $x$-axis is always zero, so a graph would be flat.

**(d)** If the two charges are interchanged, then the results of (b) and (c) still hold. The potential is zero.

**EVALUATE:** The potential is zero at any point on the $x$-axis because any point on the $x$-axis is equidistant from the two charges.

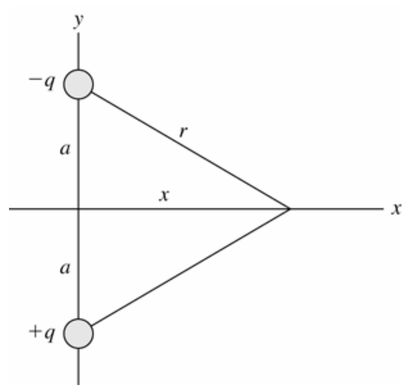



**Figure 23.23**

**23.24.** **IDENTIFY:** For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

**SET UP:** Consider the distances from the point on the $y$-axis to each charge for the three regions $-a \le y \le a$ (between the two charges), $y > a$ (above both charges) and $y < -a$ (below both charges).

**EXECUTE:** **(a)** $|y| < a: V = \dfrac{kq}{(a+y)} - \dfrac{kq}{(a-y)} = \dfrac{2kqy}{y^2-a^2}$. $y > a: V = \dfrac{kq}{(a+y)} - \dfrac{kq}{y-a} = \dfrac{-2kqa}{y^2-a^2}$.

$y < -a: V = \dfrac{-kq}{(a+y)} - \dfrac{kq}{(-y+a)} = \dfrac{2kqa}{y^2-a^2}$.

A general expression valid for any $y$ is $V = k\left(\dfrac{-q}{|y-a|} + \dfrac{q}{|y+a|}\right)$.

**(b)** The graph of $V$ versus $y$ is sketched in Figure 23.24.

**(c)** $y >> a: V = \dfrac{-2kqa}{y^2-a^2} \approx \dfrac{-2kqa}{y^2}$.

**(d)** If the charges are interchanged, then the potential is of the opposite sign.

**EVALUATE:** $V = 0$ at $y = 0$. $V \to +\infty$ as the positive charge is approached and $V \to -\infty$ as the negative charge is approached.

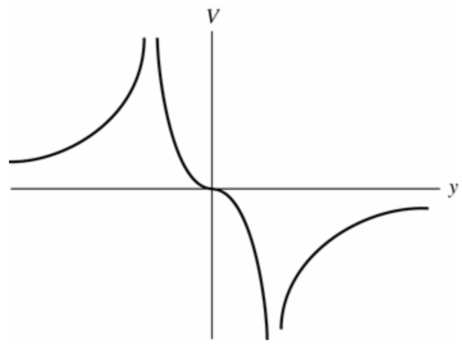



**Figure 23.24**

**23.25. IDENTIFY:** For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

**SET UP:** **(a)** The positions of the two charges are shown in Figure 23.25a.

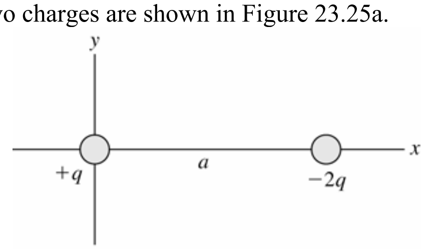



**Figure 23.25a**

**(b)** $x > a: V = \dfrac{kq}{x} - \dfrac{2kq}{x-a} = \dfrac{-kq(x+a)}{x(x-a)}$. $0 < x < a: V = \dfrac{kq}{x} - \dfrac{2kq}{a-x} = \dfrac{kq(3x-a)}{x(x-a)}$.

$x < 0: V = \dfrac{-kq}{x} + \dfrac{2kq}{x-a} = \dfrac{kq(x+a)}{x(x-a)}$. A general expression valid for any $y$ is $V = k\left(\dfrac{q}{|x|} - \dfrac{2q}{|x-a|}\right)$.

**(c)** The potential is zero at $x = -a$ and $a/3$.

**(d)** The graph of $V$ versus $x$ is sketched in Figure 23.25b.

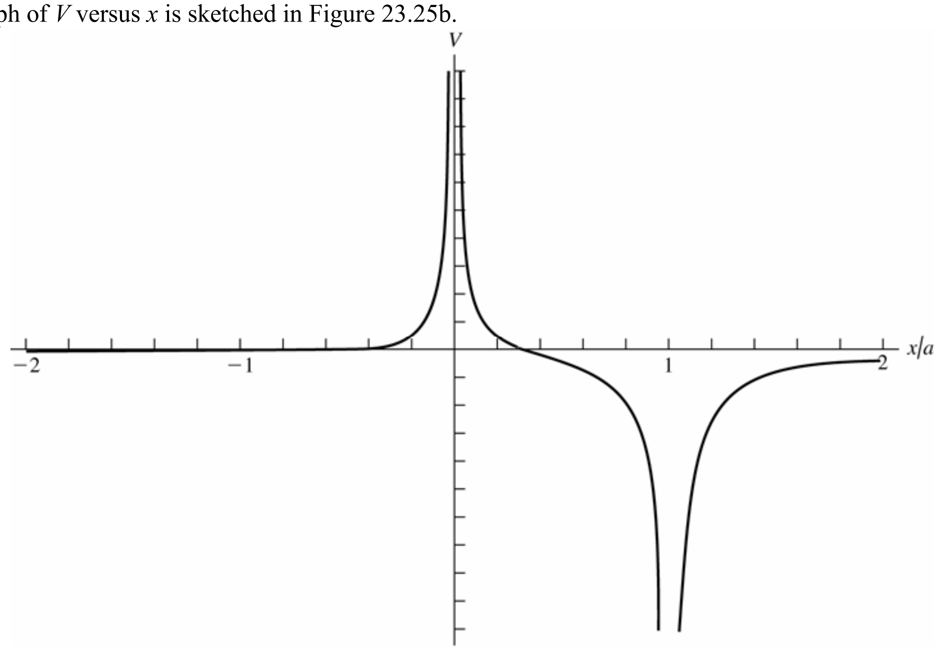



**Figure 23.25b**

**EVALUATE:** **(e)** For $x >> a: V \approx \dfrac{-kqx}{x^2} = \dfrac{-kq}{x}$, which is the same as the potential of a point charge $-q$. Far from the two charges they appear to be a point charge with a charge that is the algebraic sum of their two charges.

**23.26.** **IDENTIFY:** For a point charge, $V = \frac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

**SET UP:** The distance of a point with coordinate $y$ from the positive charge is $|y|$ and the distance from the negative charge is $r = \sqrt{a^2 + y^2}$.

**EXECUTE:** **(a)** $V = \frac{kq}{|y|} - \frac{2kq}{r} = kq\left(\frac{1}{|y|} - \frac{2}{\sqrt{a^2 + y^2}}\right)$.

**(b)** $V = 0$, when $y^2 = \frac{a^2 + y^2}{4} \Rightarrow 3y^2 = a^2 \Rightarrow y = \pm\frac{a}{\sqrt{3}}$.

**(c)** The graph of $V$ versus $y$ is sketched in Figure 23.26. $V \to \infty$ as the positive charge at the origin is approached.

**EVALUATE:** **(d)** $y >> a : V \approx kq\left(\frac{1}{y} - \frac{2}{y}\right) = -\frac{kq}{y}$, which is the potential of a point charge $-q$. Far from the two charges they appear to be a point charge with a charge that is the algebraic sum of their two charges.

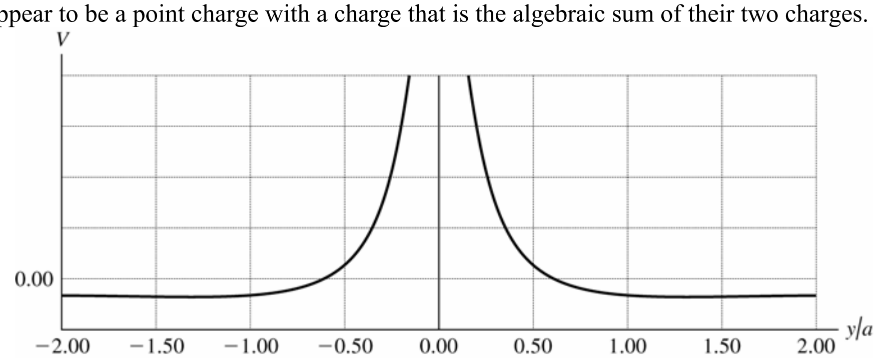



**Figure 23.26**

**23.27.** **IDENTIFY:** $K_a + qV_a = K_b + qV_b$.

**SET UP:** Let point $a$ be at the cathode and let point $b$ be at the anode. $K_a = 0$. $V_b - V_a = 295\text{ V}$. An electron has $q = -e$ and $m = 9.11\times10^{-31}\text{ kg}$.

**EXECUTE:** $K_b = q(V_a - V_b) = -(1.60\times10^{-19}\text{ C})(-295\text{ V}) = 4.72\times10^{-17}\text{ J}$. $K_b = \frac{1}{2}mv_b^2$, so

$v_b = \sqrt{\frac{2(4.72\times10^{-17}\text{ J})}{9.11\times10^{-31}\text{ kg}}} = 1.01\times10^7\text{ m/s}.$

**EVALUATE:** The negatively charged electron gains kinetic energy when it moves to higher potential.

**23.28.** **IDENTIFY:** For a point charge, $E = \frac{k|q|}{r^2}$ and $V = \frac{kq}{r}$.

**SET UP:** The electric field is directed toward a negative charge and away from a positive charge.

**EXECUTE:** **(a)** $V > 0$ so $q > 0$. $\frac{V}{E} = \frac{kq/r}{k|q|/r^2} = \left(\frac{kq}{r}\right)\left(\frac{r^2}{kq}\right) = r$. $r = \frac{4.98\text{ V}}{12.0\text{ V/m}} = 0.415\text{ m}$.

**(b)** $q = \frac{rV}{k} = \frac{(0.415\text{ m})(4.98\text{ V})}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 2.30\times10^{-10}\text{ C}$

**(c)** $q > 0$, so the electric field is directed away from the charge.

**EVALUATE:** The ratio of $V$ to $E$ due to a point charge increases as the distance $r$ from the charge increases, because $E$ falls off as $1/r^2$ and $V$ falls off as $1/r$.

**23.29.** **(a) IDENTIFY** and **SET UP:** The direction of $\vec{E}$ is always from high potential to low potential so point $b$ is at higher potential.

**(b)** Apply Eq.(23.17) to relate $V_b - V_a$ to $E$.

**EXECUTE:** $V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\,dx = E(x_b - x_a)$.

$$E = \frac{V_b - V_a}{x_b - x_a} = \frac{+240\text{ V}}{0.90\text{ m} - 0.60\text{ m}} = 800\text{ V/m}$$

**(c)** $W_{b\to a} = q(V_b - V_a) = (-0.200\times10^{-6}\ \text{C})(+240\ \text{V}) = -4.80\times10^{-5}\ \text{J}$.

**EVALUATE:** The electric force does negative work on a negative charge when the negative charge moves from high potential (point *b*) to low potential (point *a*).

**23.30. IDENTIFY:** For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges. For a point charge, $E = \dfrac{k|q|}{r^2}$. The net electric field is the vector sum of the electric fields of the two charges.

**SET UP:** $\vec{E}$ produced by a point charge is directed away from the point charge if it is positive and toward the charge if it is negative.

**EXECUTE:** **(a)** $V = V_Q + V_{2Q} > 0$, so *V* is zero nowhere except for infinitely far from the charges. The fields can cancel only between the charges, because only there are the fields of the two charges in opposite directions. Consider a point a distance *x* from *Q* and $d - x$ from 2*Q*, as shown in Figure 23.30a. $E_Q = E_{2Q} \to \dfrac{kQ}{x^2} = \dfrac{k(2Q)}{(d-x)^2} \to (d-x)^2 = 2x^2$.

$x = \dfrac{d}{1+\sqrt{2}}$. The other root, $x = \dfrac{d}{1-\sqrt{2}}$, does not lie between the charges.

**(b)** *V* can be zero in 2 places, *A* and *B*, as shown in Figure 23.30b. Point *A* is a distance *x* from $-Q$ and $d - x$ from 2*Q*. *B* is a distance *y* from $-Q$ and $d + y$ from 2*Q*. At $A$: $\dfrac{k(-Q)}{x} + \dfrac{k(2Q)}{d-x} = 0 \to x = d/3$.

At $B$: $\dfrac{k(-Q)}{y} + \dfrac{k(2Q)}{d+y} = 0 \to y = d$.

The two electric fields are in opposite directions to the left of $-Q$ or to the right of 2*Q* in Figure 23.30c. But for the magnitudes to be equal, the point must be closer to the charge with smaller magnitude of charge. This can be the case only in the region to the left of $-Q$. $E_Q = E_{2Q}$ gives $\dfrac{kQ}{x^2} = \dfrac{k(2Q)}{(d+x)^2}$ and $x = \dfrac{d}{\sqrt{2}-1}$.

**EVALUATE:** **(d)** *E* and *V* are not zero at the same places. $\vec{E}$ is a vector and *V* is a scalar. *E* is proportional to $1/r^2$ and *V* is proportional to $1/r$. $\vec{E}$ is related to the force on a test charge and $\Delta V$ is related to the work done on a test charge when it moves from one point to another.


**Figure 23.30**

**23.31. IDENTIFY** and **SET UP:** Apply conservation of energy, Eq.(23.3). Use Eq.(23.12) to express *U* in terms of *V*.

**(a) EXECUTE:** $K_1 + qV_1 = K_2 + qV_2$

$q(V_1 - V_2) = K_2 - K_1$; $\quad q = -1.602\times10^{-19}\ \text{C}$

$K_1 = \frac{1}{2}m_e v_1^2 = 4.099\times10^{-18}\ \text{J}$; $\quad K_2 = \frac{1}{2}m_e v_2^2 = 2.915\times10^{-17}\ \text{J}$

$V_1 - V_2 = \dfrac{K_2 - K_1}{q} = -156\ \text{V}$

**EVALUATE:** The electron gains kinetic energy when it moves to higher potential.

**(b) EXECUTE:** Now $K_1 = 2.915\times10^{-17}\ \text{J}, K_2 = 0$

$$V_1 - V_2 = \frac{K_2 - K_1}{q} = +182\ \text{V}$$

**EVALUATE:** The electron loses kinetic energy when it moves to lower potential.

**23.32. IDENTIFY** and **SET UP:** Expressions for the electric potential inside and outside a solid conducting sphere are derived in Example 23.8.

**EXECUTE:** **(a)** This is outside the sphere, so $V = \dfrac{kq}{r} = \dfrac{k(3.50\times10^{-9}\ \text{C})}{0.480\ \text{m}} = 65.6\ \text{V}$.

**(b)** This is at the surface of the sphere, so $V = \dfrac{k(3.50\times10^{-9}\ \text{C})}{0.240\ \text{m}} = 131\ \text{V}$.

**(c)** This is inside the sphere. The potential has the same value as at the surface, 131 V.

**EVALUATE:** All points of a conductor are at the same potential.

**23.33.** **(a) IDENTIFY and SET UP:** The electric field on the ring's axis is calculated in Example 21.10. The force on the electron exerted by this field is given by Eq.(21.3).

**EXECUTE:** When the electron is on either side of the center of the ring, the ring exerts an attractive force directed toward the center of the ring. This restoring force produces oscillatory motion of the electron along the axis of the ring, with amplitude 30.0 cm. The force on the electron is *not* of the form $F = -kx$ so the oscillatory motion is not simple harmonic motion.

**(b) IDENTIFY:** Apply conservation of energy to the motion of the electron.

**SET UP:** $K_a + U_a = K_b + U_b$ with *a* at the initial position of the electron and *b* at the center of the ring. From Example 23.11, $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{\sqrt{x^2+R^2}}$, where $R$ is the radius of the ring.

**EXECUTE:** $x_a = 30.0 \text{ cm}, x_b = 0.$

$K_a = 0$ (released from rest), $K_b = \frac{1}{2}mv^2$

Thus $\frac{1}{2}mv^2 = U_a - U_b$

And $U = qV = -eV$ so $v = \sqrt{\dfrac{2e(V_b - V_a)}{m}}$.

$$V_a = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x_a^2+R^2}} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{24.0\times10^{-9}\text{ C}}{\sqrt{(0.300\text{ m})^2+(0.150\text{ m})^2}}$$

$$V_a = 643 \text{ V}$$

$$V_b = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x_b^2+R^2}} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{24.0\times10^{-9}\text{ C}}{0.150\text{ m}} = 1438 \text{ V}$$

$$v = \sqrt{\frac{2e(V_b - V_a)}{m}} = \sqrt{\frac{2(1.602\times10^{-19}\text{ C})(1438\text{ V} - 643\text{ V})}{9.109\times10^{-31}\text{ kg}}} = 1.67\times10^7 \text{ m/s}$$

**EVALUATE:** The positively charged ring attracts the negatively charged electron and accelerates it. The electron has its maximum speed at this point. When the electron moves past the center of the ring the force on it is opposite to its motion and it slows down.

**23.34.** **IDENTIFY:** Example 23.10 shows that for a line of charge, $V_a - V_b = \dfrac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$. Apply conservation of energy to the motion of the proton.

**SET UP:** Let point *a* be 18.0 cm from the line and let point *b* be at the distance of closest approach, where $K_b = 0$.

**EXECUTE:** **(a)** $K_a = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\text{ kg})(1.50\times10^3\text{ m/s})^2 = 1.88\times10^{-21}\text{ J}$.

**(b)** $K_a + qV_a = K_b + qV_b$. $V_a - V_b = \dfrac{K_b - K_a}{q} = \dfrac{-1.88\times10^{-21}\text{ J}}{1.60\times10^{-19}\text{ C}} = -0.01175\text{ V}$. $\ln(r_b/r_a) = \left(\dfrac{2\pi\epsilon_0}{\lambda}\right)(-0.01175\text{ V})$.

$r_b = r_a\exp\left(\dfrac{2\pi\epsilon_0(-0.01175\text{ V})}{\lambda}\right) = (0.180\text{ m})\exp\left(-\dfrac{2\pi\epsilon_0(0.01175\text{ V})}{5.00\times10^{-12}\text{ C/m}}\right) = 0.158\text{ m}$.

**EVALUATE:** The potential increases with decreasing distance from the line of charge. As the positively charged proton approaches the line of charge it gains electrical potential energy and loses kinetic energy.

**23.35.** **IDENTIFY:** The voltmeter measures the potential difference between the two points. We must relate this quantity to the linear charge density on the wire.

**SET UP:** For a very long (infinite) wire, the potential difference between two points is $\Delta V = \dfrac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$.

**EXECUTE:** **(a)** Solving for $\lambda$ gives

$$\lambda = \frac{(\Delta V)2\pi\epsilon_0}{\ln(r_b/r_a)} = \frac{575\text{ V}}{(18\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\ln\left(\dfrac{3.50\text{ cm}}{2.50\text{ cm}}\right)} = 9.49\times10^{-8}\text{ C/m}$$

**(b)** The meter will read less than 575 V because the electric field is weaker over this 1.00-cm distance than it was over the 1.00-cm distance in part (a).

**(c)** The potential difference is zero because both probes are at the same distance from the wire, and hence at the same potential.

**EVALUATE:** Since a voltmeter measures potential difference, we are actually given $\Delta V$, even though that is not stated explicitly in the problem. We must also be careful when using the formula for the potential difference because each $r$ is the distance from the *center* of the cylinder, not from the surface.

**23.36.** **IDENTIFY:** The voltmeter reads the potential difference between the two points where the probes are placed. Therefore we must relate the potential difference to the distances of these points from the center of the cylinder. For points outside the cylinder, its electric field behaves like that of a line of charge.

**SET UP:** Using $\Delta V = \dfrac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$ and solving for $r_b$, we have $r_b = r_a e^{2\pi\epsilon_0 \Delta V/\lambda}$.

**EXECUTE:** The exponent is $\dfrac{\left(\dfrac{1}{2\times 9.00\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}\right)(175\ \text{V})}{15.0\times 10^{-9}\ \text{C/m}} = 0.648$, which gives

$$r_b = (2.50\ \text{cm})\, e^{0.648} = 4.78\ \text{cm}.$$

The distance above the *surface* is 4.78 cm – 2.50 cm = 2.28 cm.

**EVALUATE:** Since a voltmeter measures potential difference, we are actually given $\Delta V$, even though that is not stated explicitly in the problem. We must also be careful when using the formula for the potential difference because each $r$ is the distance from the *center* of the cylinder, not from the surface.

**23.37.** **IDENTIFY:** For points outside the cylinder, its electric field behaves like that of a line of charge. Since a voltmeter reads potential difference, that is what we need to calculate.

**SET UP:** The potential difference is $\Delta V = \dfrac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$.

**EXECUTE:** **(a)** Substituting numbers gives

$$\Delta V = \frac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a) = (8.50\times 10^{-6}\ \text{C/m})(2\times 9.00\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\ln\left(\frac{10.0\ \text{cm}}{6.00\ \text{cm}}\right)$$

$$\Delta V = 7.82\times 10^4\ \text{V} = 78{,}200\ \text{V} = 78.2\ \text{kV}$$

**(b)** $E = 0$ inside the cylinder, so the potential is constant there, meaning that the voltmeter reads zero.

**EVALUATE:** Caution! The fact that the voltmeter reads zero in part (b) does not mean that $V = 0$ inside the cylinder. The electric field is zero, but the potential is constant and equal to the potential at the surface.

**23.38.** **IDENTIFY:** The work required is equal to the change in the electrical potential energy of the charge-ring system. We need only look at the beginning and ending points, since the potential difference is independent of path for a conservative field.

**SET UP:** **(a)** $W = \Delta U = q\Delta V = q(V_{\text{center}} - V_\infty) = q\left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{a} - 0\right)$

**EXECUTE:** Substituting numbers gives

$$\Delta U = (3.00\times 10^{-6}\ \text{C})(9.00\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times 10^{-6}\ \text{C})/(0.0400\ \text{m}) = 3.38\ \text{J}$$

**(b)** We can take any path since the potential is independent of path.

**(c) SET UP:** The net force is away from the ring, so the ball will accelerate away. Energy conservation gives $U_0 = K_{\max} = \frac{1}{2}mv^2$.

**EXECUTE:** Solving for $v$ gives

$$v = \sqrt{\frac{2U_0}{m}} = \sqrt{\frac{2(3.38\ \text{J})}{0.00150\ \text{kg}}} = 67.1\ \text{m/s}$$

**EVALUATE:** Direct calculation of the work from the electric field would be extremely difficult, and we would need to know the path followed by the charge. But, since the electric field is conservative, we can bypass all this calculation just by looking at the end points (infinity and the center of the ring) using the potential.

**23.39.** **IDENTIFY:** The electric field is zero everywhere except between the plates, and in this region it is uniform and points from the positive to the negative plate (to the left in Figure 23.32).

**SET UP:** Since the field is uniform between the plates, the potential increases linearly as we go from left to right starting at $b$.

**EXECUTE:** Since the potential is taken to be zero at the left surface of the negative plate ($a$ in Figure 23.32), it is zero everywhere to the left of $b$. It increases linearly from $b$ to $c$, and remains constant (since $E = 0$) past $c$. The graph is sketched in Figure 23.39.

**EVALUATE:** When the electric field is zero, the potential remains constant but not necessarily zero (as to the right of $c$). When the electric field is constant, the potential is linear.

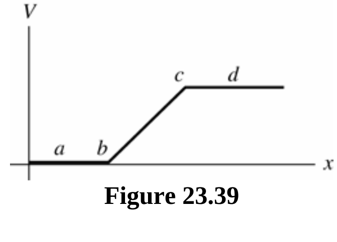



**Figure 23.39**

**23.40. IDENTIFY and SET UP:** For oppositely charged parallel plates, $E = \sigma/\epsilon_0$ between the plates and the potential difference between the plates is $V = Ed$.

**EXECUTE:** **(a)** $E = \dfrac{\sigma}{\epsilon_0} = \dfrac{47.0\times10^{-9}\ \text{C/m}^2}{\epsilon_0} = 5310\ \text{N/C}.$

**(b)** $V = Ed = (5310\ \text{N/C})(0.0220\ \text{m}) = 117\ \text{V}.$

**(c)** The electric field stays the same if the separation of the plates doubles. The potential difference between the plates doubles.

**EVALUATE:** The electric field of an infinite sheet of charge is uniform, independent of distance from the sheet. The force on a test charge between the two plates is constant because the electric field is constant. The potential difference is the work per unit charge on a test charge when it moves from one plate to the other. When the distance doubles the work, which is force times distance, doubles and the potential difference doubles.

**23.41. IDENTIFY and SET UP:** Use the result of Example 23.9 to relate the electric field between the plates to the potential difference between them and their separation. The force this field exerts on the particle is given by Eq.(21.3). Use the equation that precedes Eq.(23.17) to calculate the work.

**EXECUTE:** **(a)** From Example 23.9, $E = \dfrac{V_{ab}}{d} = \dfrac{360\ \text{V}}{0.0450\ \text{m}} = 8000\ \text{V/m}$

**(b)** $F = |q|E = (2.40\times10^{-9}\ \text{C})(8000\ \text{V/m}) = +1.92\times10^{-5}\ \text{N}$

**(c)** The electric field between the plates is shown in Figure 23.41.

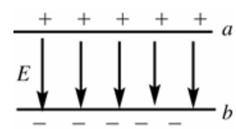



**Figure 23.41**

The plate with positive charge (plate *a*) is at higher potential. The electric field is directed from high potential toward low potential (or, $\vec{E}$ is from + charge toward − charge), so $\vec{E}$ points from *a* to *b*. Hence the force that $\vec{E}$ exerts on the positive charge is from *a* to *b*, so it does positive work.

$W = \int_a^b \vec{F}\cdot d\vec{l} = Fd$, where *d* is the separation between the plates.

$W = Fd = (1.92\times10^{-5}\ \text{N})(0.0450\ \text{m}) = +8.64\times10^{-7}\ \text{J}$

**(d)** $V_a - V_b = +360\ \text{V}$ (plate *a* is at higher potential)

$\Delta U = U_b - U_a = q(V_b - V_a) = (2.40\times10^{-9}\ \text{C})(-360\ \text{V}) = -8.64\times10^{-7}\ \text{J}.$

**EVALUATE:** We see that $W_{a\to b} = -(U_b - U_a) = U_a - U_b$.

**23.42. IDENTIFY:** The electric field is zero inside the sphere, so the potential is constant there. Thus the potential at the center must be the same as at the surface, where it is equivalent to that of a point-charge.

**SET UP:** At the surface, and hence also at the center of the sphere, the field is that of a point-charge, $E = Q/(4\pi\epsilon_0 R)$.

**EXECUTE:** **(a)** Solving for $Q$ and substituting the numbers gives

$$Q = 4\pi\epsilon_0 RV = (0.125\ \text{m})(1500\ \text{V})/(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) = 2.08\times10^{-8}\ \text{C} = 20.8\ \text{nC}$$

**(b)** Since the potential is constant inside the sphere, its value at the surface must be the same as at the center, 1.50 kV.

**EVALUATE:** The electric field inside the sphere is zero, so the potential is constant but is not zero.

**23.43. IDENTIFY and SET UP:** Consider the electric field outside and inside the shell and use that to deduce the potential.

**EXECUTE:** **(a)** The electric field outside the shell is the same as for a point charge at the center of the shell, so the potential outside the shell is the same as for a point charge:

$$V = \frac{q}{4\pi\epsilon_0 r} \text{ for } r > R.$$

The electric field is zero inside the shell, so no work is done on a test charge as it moves inside the shell and all points inside the shell are at the same potential as the surface of the shell: $V = \dfrac{q}{4\pi\epsilon_0 R}$ for $r \le R$.

**(b)** $V = \dfrac{kq}{R}$ so $q = \dfrac{RV}{k} = \dfrac{(0.15\ \text{m})(-1200\ \text{V})}{k} = -20\ \text{nC}$

**(c) EVALUATE:** No, the amount of charge on the sphere is very small. Since $U = qV$ the total amount of electric energy stored on the balloon is only $(20\ \text{nC})(1200\ \text{V}) = 2.4\times10^{-5}\ \text{J}$.

**23.44.** **IDENTIFY:** Example 23.8 shows that the potential of a solid conducting sphere is the same at every point inside the sphere and is equal to its value $V = q/2\pi\epsilon_0 R$ at the surface. Use the given value of $E$ to find $q$.

**SET UP:** For negative charge the electric field is directed toward the charge.
For points outside this spherical charge distribution the field is the same as if all the charge were concentrated at the center.

**EXECUTE:** $E = \dfrac{q}{4\pi\epsilon_0 r^2}$ and $q = 4\pi\epsilon_0 E r^2 = \dfrac{(3800\text{ N/C})(0.200\text{ m})^2}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 1.69\times10^{-8}\text{ C}$.

Since the field is directed inward, the charge must be negative. The potential of a point charge, taking $\infty$ as zero, is $V = \dfrac{q}{4\pi\epsilon_0 r} = \dfrac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(-1.69\times10^{-8}\text{ C})}{0.200\text{ m}} = -760\text{ V}$ at the surface of the sphere. Since the charge all resides on the surface of a conductor, the field inside the sphere due to this symmetrical distribution is zero. No work is therefore done in moving a test charge from just inside the surface to the center, and the potential at the center must also be $-760$ V.

**EVALUATE:** Inside the sphere the electric field is zero and the potential is constant.

**23.45.** **IDENTIFY:** Example 23.9 shows that $V(y) = Ey$, where $y$ is the distance from the negatively charged plate, whose potential is zero. The electric field between the plates is uniform and perpendicular to the plates.

**SET UP:** $V$ increases toward the positively charged plate. $\vec{E}$ is directed from the positively charged plated toward the negatively charged plate.

**EXECUTE:** **(a)** $E = \dfrac{V}{d} = \dfrac{480\text{ V}}{0.0170\text{ m}} = 2.82\times10^4\text{ V/m}$ and $y = \dfrac{V}{E}$. $V = 0$ at $y = 0$, $V = 120$ V at $y = 0.43$ cm, $V = 240$ V at $y = 0.85$ cm, $V = 360$ V at $y = 1.28$ cm and $V = 480$ V at $y = 1.70$ cm. The equipotential surfaces are sketched in Figure 23.45. The surfaces are planes parallel to the plates.
**(b)** The electric field lines are also shown in Figure 23.45. The field lines are perpendicular to the plates and the equipotential lines are parallel to the plates, so the electric field lines and the equipotential lines are mutually perpendicular.
**EVALUATE:** Only differences in potential have physical significance. Letting $V = 0$ at the negative plate is a choice we are free to make.

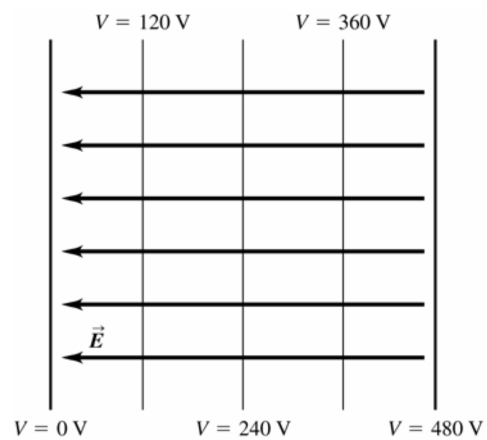



**Figure 23.45**

**23.46.** **IDENTIFY:** By the definition of electric potential, if a positive charge gains potential along a path, then the potential along that path must have increased. The electric field produced by a very large sheet of charge is uniform and is independent of the distance from the sheet.
**(a) SET UP:** No matter what the reference point, we must do work on a positive charge to move it away from the negative sheet.
**EXECUTE:** Since we must do work on the positive charge, it gains potential energy, so the potential increases.

**(b) SET UP:** Since the electric field is uniform and is equal to $\sigma/2\varepsilon_0$, we have $\Delta V = Ed = \dfrac{\sigma}{2\epsilon_0}d$.

**EXECUTE:** Solving for $d$ gives

$$d = \frac{2\epsilon_0 \Delta V}{\sigma} = \frac{2\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)(1.00\text{ V})}{6.00\times10^{-9}\text{ C/m}^2} = 0.00295\text{ m} = 2.95\text{ mm}$$

**EVALUATE:** Since the spacing of the equipotential surfaces ($d$ = 2.95 mm) is independent of the distance from the sheet, the equipotential surfaces are planes parallel to the sheet and spaced 2.95 mm apart.

**23.47** **IDENTIFY** and **SET UP:** Use Eq.(23.19) to calculate the components of $\vec{E}$.

**EXECUTE:** $V = Axy - Bx^2 + Cy$

**(a)** $E_x = -\dfrac{\partial V}{\partial x} = -Ay + 2Bx$

$Ey = -\dfrac{\partial V}{\partial y} = -Ax - C$

$E_z = -\dfrac{\partial V}{\partial z} = 0$

**(b)** $E = 0$ requires that $E_x = E_y = E_z = 0$.

$E_z = 0$ everywhere.

$E_y = 0$ at $x = -C/A$.

And $E_x$ is also equal zero for this *x,* any value of *z,* and $y = 2Bx/A = (2B/A)(-C/A) = -2BC/A^2$.

**EVALUATE:** *V* doesn't depend on *z* so $E_z = 0$ everywhere.

**23.48.** **IDENTIFY:** Apply Eq.(21.19).

**SET UP:** Eq.(21.7) says $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}\hat{r}$ is the electric field due to a point charge *q*.

**EXECUTE:** **(a)** $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{\partial}{\partial x}\left(\dfrac{kQ}{\sqrt{x^2+y^2+z^2}}\right) = \dfrac{kQx}{(x^2+y^2+z^2)^{3/2}} = \dfrac{kQx}{r^3}$.

Similarly, $E_y = \dfrac{kQy}{r^3}$ and $E_z = \dfrac{kQz}{r^3}$.

**(b)** From part (a), $E = \dfrac{kQ}{r^2}\left(\dfrac{x\hat{i}}{r} + \dfrac{y\hat{j}}{r} + \dfrac{z\hat{k}}{r}\right) = \dfrac{kQ}{r^2}\hat{r}$, which agrees with Equation (21.7).

**EVALUATE:** *V* is a scalar. $\vec{E}$ is a vector and has components.

**23.49.** **IDENTIFY** and **SET UP:** For a solid metal sphere or for a spherical shell, $V = \dfrac{kq}{r}$ outside the sphere and $V = \dfrac{kq}{R}$ at all points inside the sphere, where *R* is the radius of the sphere. When the electric field is radial, $E = -\dfrac{\partial V}{\partial r}$.

**EXECUTE:** **(a)** (i) $r < r_a$: This region is inside both spheres. $V = \dfrac{kq}{r_a} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)$.

(ii) $r_a < r < r_b$: This region is outside the inner shell and inside the outer shell. $V = \dfrac{kq}{r} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right)$.

(iii) $r > r_b$: This region is outside both spheres and $V = 0$ since outside a sphere the potential is the same as for point charge. Therefore the potential is the same as for two oppositely charged point charges at the same location. These potentials cancel.

**(b)** $V_a = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q}{r_a} - \dfrac{q}{r_b}\right)$ and $V_b = 0$, so $V_{ab} = \dfrac{1}{4\pi\epsilon_0}q\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)$.

**(c)** Between the spheres $r_a < r < r_b$ and $V = kq\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right)$. $E = -\dfrac{\partial V}{\partial r} = -\dfrac{q}{4\pi\epsilon_0}\dfrac{\partial}{\partial r}\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right) = +\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2} = \dfrac{V_{ab}}{\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)}\dfrac{1}{r^2}$.

**(d)** From Equation (23.23): $E = 0$, since *V* is constant (zero) outside the spheres.

**(e)** If the outer charge is different, then outside the outer sphere the potential is no longer zero but is

$V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r} - \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(q-Q)}{r}$. All potentials inside the outer shell are just shifted by an amount

$V = -\dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r_b}$. Therefore relative potentials within the shells are not affected. Thus (b) and (c) do not change.

However, now that the potential does vary outside the spheres, there is an electric field there:

$E = -\dfrac{\partial V}{\partial r} = -\dfrac{\partial}{\partial r}\left(\dfrac{kq}{r} + \dfrac{-kQ}{r}\right) = \dfrac{kq}{r^2}\left(1 - \dfrac{Q}{q}\right) = \dfrac{k}{r^2}(q-Q)$.

**EVALUATE:** In part (a) the potential is greater than zero for all $r < r_b$.

**23.50.** **IDENTIFY:** Exercise 23.49 shows that $V = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$ for $r < r_a$, $V = kq\left(\frac{1}{r} - \frac{1}{r_b}\right)$ for $r_a < r < r_b$ and

$V_{ab} = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$.

**SET UP:** $E = \frac{kq}{r^2}$, radially outward, for $r_a \le r \le r_b$

**EXECUTE:** **(a)** $V_{ab} = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = 500\text{ V}$ gives $q = \dfrac{500\text{ V}}{k\left(\dfrac{1}{0.012\text{ m}} - \dfrac{1}{0.096\text{ m}}\right)} = 7.62 \times 10^{-10}\text{ C}$.

**(b)** $V_b = 0$ so $V_a = 500\text{ V}$. The inner metal sphere is an equipotential with $V = 500\text{ V}$. $\frac{1}{r} = \frac{1}{r_a} + \frac{V}{kq}$. $V = 400\text{ V}$ at $r = 1.45\text{ cm}$, $V = 300\text{ V}$ at $r = 1.85\text{ cm}$, $V = 200\text{ V}$ at $r = 2.53\text{ cm}$, $V = 100\text{ V}$ at $r = 4.00\text{ cm}$, $V = 0$ at $r = 9.60\text{ cm}$. The equipotential surfaces are sketched in Figure 23.50.

**EVALUATE:** **(c)** The equipotential surfaces are concentric spheres and the electric field lines are radial, so the field lines and equipotential surfaces are mutually perpendicular. The equipotentials are closest at smaller *r*, where the electric field is largest.

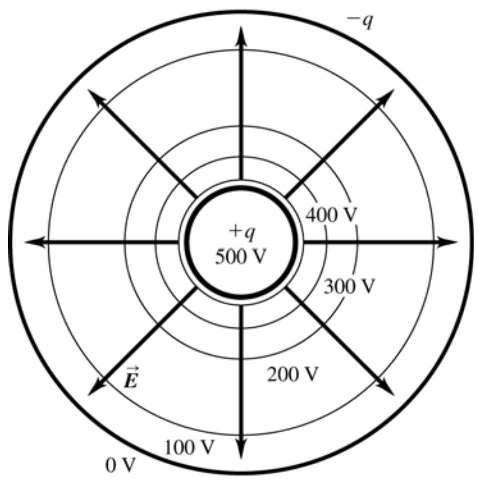



**Figure 23.50**

**23.51.** **IDENTIFY:** Outside the cylinder it is equivalent to a line of charge at its center.

**SET UP:** The difference in potential between the surface of the cylinder (a distance *R* from the central axis) and a general point a distance *r* from the central axis is given by $\Delta V = \frac{\lambda}{2\pi\epsilon_0}\ln(r/R)$.

**EXECUTE:** **(a)** The potential difference depends only on *r*, and not direction. Therefore all points at the same value of *r* will be at the same potential. Thus the equipotential surfaces are cylinders coaxial with the given cylinder.

**(b)** Solving $\Delta V = \frac{\lambda}{2\pi\epsilon_0}\ln(r/R)$ for *r*, gives $r = Re^{2\pi\epsilon_0 \Delta V/\lambda}$.

For 10 V, the exponent is (10 V)/[(2 × 9.00 × $10^9$ N · $m^2/C^2$)(1.50 × $10^{-9}$ C/m)] = 0.370, which gives *r* = (2.00 cm) $e^{0.370}$ = 2.90 cm. Likewise, the other radii are 4.20 cm (for 20 V) and 6.08 cm (for 30 V).

**(c)** $\Delta r_1$ = 2.90 cm – 2.00 cm = 0.90 cm; $\Delta r_2$ = 4.20 cm – 2.90 cm = 1.30 cm; $\Delta r_3$ = 6.08 cm – 4.20 cm = 1.88 cm

**EVALUATE:** As we can see, $\Delta r$ increases, so the surfaces get farther apart. This is very different from a sheet of charge, where the surfaces are equally spaced planes.

**23.52.** **IDENTIFY:** The electric field is the negative gradient of the potential.

**SET UP:** $E_x = -\frac{\partial V}{\partial x}$, so $E_x$ is the negative slope of the graph of *V* as a function of *x*.

**EXECUTE:** The graph is sketched in Figure 23.52. Up to *a*, *V* is constant, so $E_x = 0$. From *a* to *b*, *V* is linear with a positive slope, so $E_x$ is a negative constant. Past *b*, the *V*-*x* graph has a decreasing positive slope which approaches zero, so $E_x$ is negative and approaches zero.

**EVALUATE:** Notice that an increasing potential does not necessarily mean that the electric field is increasing.

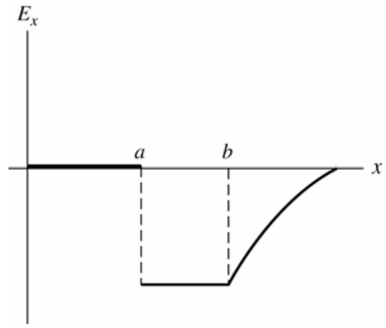



**Figure 23.52**

**23.53.** **(a) IDENTIFY:** Apply the work-energy theorem, Eq.(6.6).

**SET UP:** Points *a* and *b* are shown in Figure 23.53a.

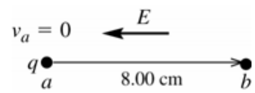



**Figure 23.53a**

**EXECUTE:** $W_{\text{tot}} = \Delta K = K_b - K_a = K_b = 4.35 \times 10^{-5}$ J

The electric force $F_E$ and the additional force $F$ both do work, so that $W_{\text{tot}} = W_{F_E} + W_F$.

$$W_{F_E} = W_{\text{tot}} - W_F = 4.35 \times 10^{-5}\text{ J} - 6.50 \times 10^{-5}\text{ J} = -2.15 \times 10^{-5}\text{ J}$$

**EVALUATE:** The forces on the charged particle are shown in Figure 23.53b.

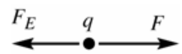



**Figure 23.53b**

The electric force is to the left (in the direction of the electric field since the particle has positive charge). The displacement is to the right, so the electric force does negative work. The additional force $F$ is in the direction of the displacement, so it does positive work.

**(b) IDENTIFY** and **SET UP:** For the work done by the electric force, $W_{a\to b} = q(V_a - V_b)$

**EXECUTE:** $V_a - V_b = \dfrac{W_{a\to b}}{q} = \dfrac{-2.15 \times 10^{-5}\text{ J}}{7.60 \times 10^{-9}\text{ C}} = -2.83 \times 10^3$ V.

**EVALUATE:** The starting point (point *a*) is at $2.83 \times 10^3$ V lower potential than the ending point (point *b*). We know that $V_b > V_a$ because the electric field always points from high potential toward low potential.

**(c) IDENTIFY:** Calculate *E* from $V_a - V_b$ and the separation *d* between the two points.

**SET UP:** Since the electric field is uniform and directed opposite to the displacement $W_{a\to b} = -F_E d = -qEd$, where $d = 8.00$ cm is the displacement of the particle.

**EXECUTE:** $E = -\dfrac{W_{a\to b}}{qd} = -\dfrac{V_a - V_b}{d} = \dfrac{-2.83 \times 10^3\text{ V}}{0.0800\text{ m}} = 3.54 \times 10^4$ V/m.

**EVALUATE:** In part (a), $W_{\text{tot}}$ is the total work done by both forces. In parts (b) and (c) $W_{a\to b}$ is the work done just by the electric force.

**23.54.** **IDENTIFY:** The electric force between the electron and proton is attractive and has magnitude $F = \dfrac{ke^2}{r^2}$. For circular motion the acceleration is $a_{\text{rad}} = v^2/r$. $U = -k\dfrac{e^2}{r}$.

**SET UP:** $e = 1.60 \times 10^{-19}$ C. 1 eV $= 1.60 \times 10^{-19}$ J.

**EXECUTE:** **(a)** $\dfrac{mv^2}{r} = \dfrac{ke^2}{r^2}$ and $v = \sqrt{\dfrac{ke^2}{mr}}$.

**(b)** $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\dfrac{ke^2}{r} = -\dfrac{1}{2}U$

**(c)** $E = K + U = \frac{1}{2}U = -\frac{1}{2}\frac{ke^2}{r} = -\frac{1}{2}\frac{k(1.60\times10^{-19}\ \text{C})^2}{5.29\times10^{-11}\ \text{m}} = -2.17\times10^{-18}\ \text{J} = -13.6\ \text{eV}$ .

**EVALUATE:** The total energy is negative, so the electron is bound to the proton. Work must be done on the electron to take it far from the proton.

**23.55.** **IDENTIFY** and **SET UP:** Calculate the components of $\vec{E}$ from Eq.(23.19). Eq.(21.3) gives $\vec{F}$ from $\vec{E}$**.**

**EXECUTE:** **(a)** $V = Cx^{4/3}$

$$C = V/x^{4/3} = 240\ \text{V}/(13.0\times10^{-3}\ \text{m})^{4/3} = 7.85\times10^4\ \text{V/m}^{4/3}$$

**(b)** $E_x = -\frac{\partial V}{\partial x} = -\frac{4}{3}Cx^{1/3} = -(1.05\times10^5\ \text{V/m}^{4/3})x^{1/3}$

The minus sign means that $E_x$ is in the $-x$-direction, which says that $\vec{E}$ points from the positive anode toward the negative cathode.

**(c)** $\vec{F} = q\vec{E}$ so $F_x = -eE_x = \frac{4}{3}eCx^{1/3}$

Halfway between the electrodes means $x = 6.50\times10^{-3}$ m.

$$F_x = \tfrac{4}{3}(1.602\times10^{-19}\ \text{C})(7.85\times10^4\ \text{V/m}^{4/3})(6.50\times10^{-3}\ \text{m})^{1/3} = 3.13\times10^{-15}\ \text{N}$$

$F_x$ is positive, so the force is directed toward the positive anode.

**EVALUATE:** $V$ depends only on $x$, so $E_y = E_z = 0$. $\vec{E}$ is directed from high potential (anode) to low potential (cathode). The electron has negative charge, so the force on it is directed opposite to the electric field.

**23.56.** **IDENTIFY:** At each point ($a$ and $b$), the potential is the sum of the potentials due to *both* spheres. The voltmeter reads the difference between these two potentials. The spheres behave like a point-charges since the meter is connected to the surface of each one.

**SET UP:** **(a)** Call $a$ the point on the surface of one sphere and $b$ the point on the surface of the other sphere, call $r$ the radius of each sphere, and call $d$ the center-to-center distance between the spheres. The potential difference $V_{ab}$ between points $a$ and $b$ is then

$$V_b - V_a = V_{ab} = \frac{1}{4\pi\epsilon_0}\left[\frac{-q}{r} + \frac{q}{d-r} - \left(\frac{q}{r} + \frac{-q}{d-r}\right)\right] = \frac{2q}{4\pi\varepsilon_0}\left(\frac{1}{d-r} - \frac{1}{r}\right)$$

**EXECUTE:** Substituting the numbers gives

$$V_b - V_a = 2(175\ \mu\text{C})\left(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{0.750\ \text{m}} - \frac{1}{0.250\ \text{m}}\right) = -8.40\times10^6\ \text{V}$$

The meter reads 8.40 MV.

**(b)** Since $V_b - V_a$ is negative, $V_a > V_b$, so point $a$ is at the higher potential.

**EVALUATE:** An easy way to see that the potential at $a$ is higher than the potential at $b$ is that it would take positive work to move a positive test charge from $b$ to $a$ since this charge would be attracted by the negative sphere and repelled by the positive sphere.

**23.57.** **IDENTIFY:** $U = \frac{kq_1q_2}{r}$

**SET UP:** Eight charges means there are $8(8-1)/2 = 28$ pairs. There are 12 pairs of $q$ and $-q$ separated by $d$, 12 pairs of equal charges separated by $\sqrt{2}d$ and 4 pairs of $q$ and $-q$ separated by $\sqrt{3}d$ .

**EXECUTE:** **(a)** $U = kq^2\left(-\frac{12}{d} + \frac{12}{\sqrt{2}d} - \frac{4}{\sqrt{3}d}\right) = -\frac{12kq^2}{d}\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}}\right) = -1.46q^2/\pi\epsilon_0 d$

**EVALUATE:** **(b)** The fact that the electric potential energy is less than zero means that it is energetically favorable for the crystal ions to be together.

**23.58.** **IDENTIFY:** For two small spheres, $U = \frac{kq_1q_2}{r}$. For part (b) apply conservation of energy.

**SET UP:** Let $q_1 = 2.00\ \mu\text{C}$ and $q_2 = -3.50\ \mu\text{C}$ . Let $r_a = 0.250$ m and $r_b \to \infty$ .

**EXECUTE:** **(a)** $U = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\ \text{C})(-3.50\times10^{-6}\ \text{C})}{0.250\ \text{m}} = -0.252\ \text{J}$

**(b)** $K_b = 0$ . $U_b = 0$ . $U_a = -0.252$ J . $K_a + U_a = K_b + U_b$ gives $K_a = 0.252$ J . $K_a = \frac{1}{2}mv_a^2$ , so

$v_a = \sqrt{\frac{2K_a}{m}} = \sqrt{\frac{2(0.252\ \text{J})}{1.50\times10^{-3}\ \text{kg}}} = 18.3\ \text{m/s}$

**EVALUATE:** As the sphere moves away, the attractive electrical force exerted by the other sphere does negative work and removes all the kinetic energy it initially had. Note that it doesn't matter which sphere is held fixed and which is shot away; the answer to part (b) is unaffected.

**23.59.** **(a) IDENTIFY:** Use Eq.(23.10) for the electron and each proton.

**SET UP:** The positions of the particles are shown in Figure 23.59a.


$r = (1.07\times10^{-10}\ \text{m})/2 = 0.535\times10^{-10}\ \text{m}$

**Figure 23.59a**

**EXECUTE:** The potential energy of interaction of the electron with each proton is

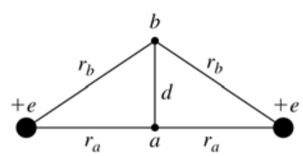
$U = \dfrac{1}{4\pi\epsilon_0}\dfrac{(-e^2)}{r}$, so the total potential energy is

$$U = -\frac{2e^2}{4\pi\epsilon_0 r} = -\frac{2(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{0.535\times10^{-10}\ \text{m}} = -8.60\times10^{-18}\ \text{J}$$

$$U = -8.60\times10^{-18}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = -53.7\ \text{eV}$$

**EVALUATE:** The electron and proton have charges of opposite signs, so the potential energy of the system is negative.

**(b) IDENTIFY** and **SET UP:** The positions of the protons and points $a$ and $b$ are shown in Figure 23.59b.




$r_b = \sqrt{r_a^2 + d^2}$

$r_a = r = 0.535\times10^{-10}\ \text{m}$

**Figure 23.59b**

Apply $K_a + U_a + W_{\text{other}} = K_b + U_b$ with point $a$ midway between the protons and point $b$ where the electron instantaneously has $v = 0$ (at its maximum displacement $d$ from point $a$).

**EXECUTE:** Only the Coulomb force does work, so $W_{\text{other}} = 0$.

$U_a = -8.60\times10^{-18}\ \text{J}$ (from part (a))

$K_a = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(9.109\times10^{-31}\ \text{kg})(1.50\times10^6\ \text{m/s})^2 = 1.025\times10^{-18}\ \text{J}$

$K_b = 0$

$U_b = -2ke^2/r_b$

Then $U_b = K_a + U_a - K_b = 1.025\times10^{-18}\ \text{J} - 8.60\times10^{-18}\ \text{J} = -7.575\times10^{-18}\ \text{J}$.

$$r_b = -\frac{2ke^2}{U_b} = -\frac{2(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{-7.575\times10^{-18}\ \text{J}} = 6.075\times10^{-11}\ \text{m}$$

Then $d = \sqrt{r_b^2 - r_a^2} = \sqrt{(6.075\times10^{-11}\ \text{m})^2 - (5.35\times10^{-11}\ \text{m})^2} = 2.88\times10^{-11}\ \text{m}$.

**EVALUATE:** The force on the electron pulls it back toward the midpoint. The transverse distance the electron moves is about 0.27 times the separation of the protons.

**23.60.** **IDENTIFY:** Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere. The electric force on the sphere is $F_e = qE$. The potential difference between the plates is $V = Ed$.

**SET UP:** The free-body diagram for the sphere is given in Figure 23.56.

**EXECUTE:** $T\cos\theta = mg$ and $T\sin\theta = F_e$ gives $F_e = mg\tan\theta = (1.50\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)\tan(30°) = 0.0085\ \text{N}$.

$F_e = Eq = \dfrac{Vq}{d}$ and $V = \dfrac{Fd}{q} = \dfrac{(0.0085\ \text{N})(0.0500\ \text{m})}{8.90\times10^{-6}\ \text{C}} = 47.8\ \text{V}$.

**EVALUATE:** $E = V/d = 956$ V/m . $E = \sigma/\epsilon_0$ and $\sigma = E\epsilon_0 = 8.46\times10^{-9}$ C/m$^2$ .

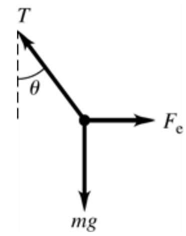



**Figure 23.60**

**23.61.** **(a) IDENTIFY:** The potential at any point is the sum of the potentials due to each of the two charged conductors.
**SET UP:** From Example 23.10, for a conducting cylinder with charge per unit length $\lambda$ the potential outside the cylinder is given by $V = (\lambda/2\pi\epsilon_0)\ln(r_0/r)$ where $r$ is the distance from the cylinder axis and $r_0$ is the distance from the axis for which we take $V = 0.$ Inside the cylinder the potential has the same value as on the cylinder surface. The electric field is the same for a solid conducting cylinder or for a hollow conducting tube so this expression for $V$ applies to both. This problem says to take $r_0 = b.$

**EXECUTE:** For the hollow tube of radius $b$ and charge per unit length $-\lambda$: outside $V = -(\lambda/2\pi\epsilon_0)\ln(b/r);$ inside $V = 0$ since $V = 0$ at $r = b.$
For the metal cylinder of radius $a$ and charge per unit length $\lambda$:
outside $V = (\lambda/2\pi\epsilon_0)\ln(b/r),$ inside $V = (\lambda/2\pi\epsilon_0)\ln(b/a),$ the value at $r = a.$
(i) $r < a$; inside both $V = (\lambda/2\pi\epsilon_0)\ln(b/a)$
(ii) $a < r < b$; outside cylinder, inside tube $V = (\lambda/2\pi\epsilon_0)\ln(b/r)$
(iii) $r > b$; outside both the potentials are equal in magnitude and opposite in sign so $V = 0.$
**(b)** For $r = a,$ $V_a = (\lambda/2\pi\epsilon_0)\ln(b/a).$
For $r = b,$ $V_b = 0.$
Thus $V_{ab} = V_a - V_b = (\lambda/2\pi\epsilon_0)\ln(b/a).$
**(c) IDENTIFY** and **SET UP:** Use Eq.(23.23) to calculate $E$.
**EXECUTE:** $E = -\dfrac{\partial V}{\partial r} = -\dfrac{\lambda}{2\pi\epsilon_0}\dfrac{\partial}{\partial r}\ln\left(\dfrac{b}{r}\right) = -\dfrac{\lambda}{2\pi\epsilon_0}\left(\dfrac{r}{b}\right)\left(-\dfrac{b}{r^2}\right) = \dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}.$
**(d)** The electric field between the cylinders is due only to the inner cylinder, so $V_{ab}$ is not changed, $V_{ab} = (\lambda/2\pi\epsilon_0)\ln(b/a).$
**EVALUATE:** The electric field is not uniform between the cylinders, so $V_{ab} \neq E(b-a).$

**23.62.** **IDENTIFY:** The wire and hollow cylinder form coaxial cylinders. Problem 23.61 gives $E(r) = \dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$ .

**SET UP:** $a = 145\times10^{-6}$ m , $b = 0.0180$ m .

**EXECUTE:** $E = \dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$ and $V_{ab} = E\ln(b/a)r = (2.00\times10^4\ \text{N/C})(\ln(0.018\ \text{m}/145\times10^{-6}\ \text{m}))0.012\ \text{m} = 1157$ V.

**EVALUATE:** The electric field at any $r$ is directly proportional to the potential difference between the wire and the cylinder.

**23.63.** **IDENTIFY** and **SET UP:** Use Eq.(21.3) to calculate $\vec{F}$ and then $\vec{F} = m\vec{a}$ gives $\vec{a}.$
**EXECUTE:** **(a)** $\vec{F}_E = q\vec{E}.$ Since $q = -e$ is negative $\vec{F}_E$ and $\vec{E}$ are in opposite directions; $\vec{E}$ is upward so $\vec{F}_E$ is downward. The magnitude of $F_E$ is

$$F_E = |q|E = eE = (1.602\times10^{-19}\ \text{C})(1.10\times10^3\ \text{N/C}) = 1.76\times10^{-16}\ \text{N}.$$

**(b)** Calculate the acceleration of the electron produced by the electric force:

$$a = \frac{F}{m} = \frac{1.76\times10^{-16}\ \text{N}}{9.109\times10^{-31}\ \text{kg}} = 1.93\times10^{14}\ \text{m/s}^2$$

**EVALUATE:** This is much larger than $g = 9.80$ m/s$^2$, so the gravity force on the electron can be neglected. $\vec{F}_E$ is downward, so $\vec{a}$ is downward.
**(c) IDENTIFY** and **SET UP:** The acceleration is constant and downward, so the motion is like that of a projectile. Use the horizontal motion to find the time and then use the time to find the vertical displacement.

**EXECUTE:** x-component

$v_{0x} = 6.50\times10^6$ m/s; $a_x = 0$; $x - x_0 = 0.060$ m; $t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ and the $a_x$ term is zero, so

$$t = \frac{x - x_0}{v_{0x}} = \frac{0.060 \text{ m}}{6.50\times10^6 \text{ m/s}} = 9.231\times10^{-9} \text{ s}$$

y-component

$v_{0y} = 0$; $a_y = 1.93\times10^{14}$ m/s$^2$; $t = 9.231\times10^{-9}$ m/s; $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

$y - y_0 = \frac{1}{2}(1.93\times10^{14} \text{ m/s}^2)(9.231\times10^{-9} \text{ s})^2 = 0.00822 \text{ m} = 0.822 \text{ cm}$

**(d)** The velocity and its components as the electron leaves the plates are sketched in Figure 23.63.

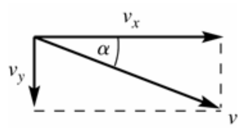



**Figure 23.63**

$v_x = v_{0x} = 6.50\times10^6$ m/s (since $a_x = 0$)

$v_y = v_{0y} + a_y t$

$v_y = 0 + (1.93\times10^{14} \text{ m/s}^2)(9.231\times10^{-9} \text{ s})$

$v_y = 1.782\times10^6$ m/s

$$\tan\alpha = \frac{v_y}{v_x} = \frac{1.782\times10^6 \text{ m/s}}{6.50\times10^6 \text{ m/s}} = 0.2742 \text{ so } \alpha = 15.3°.$$

**EVALUATE:** The greater the electric field or the smaller the initial speed the greater the downward deflection.

**(e) IDENTIFY** and **SET UP:** Consider the motion of the electron after it leaves the region between the plates. Outside the plates there is no electric field, so $a = 0$. (Gravity can still be neglected since the electron is traveling at such high speed and the times are small.) Use the horizontal motion to find the time it takes the electron to travel 0.120 m horizontally to the screen. From this time find the distance downward that the electron travels.

**EXECUTE:** x-component

$v_{0x} = 6.50\times10^6$ m/s; $a_x = 0$; $x - x_0 = 0.120$ m; $t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ and the $a_x$ term is term is zero, so

$$t = \frac{x - x_0}{v_{0x}} = \frac{0.120 \text{ m}}{6.50\times10^6 \text{ m/s}} = 1.846\times10^{-8} \text{ s}$$

y-component

$v_{0y} = 1.782\times10^6$ m/s (from part (b)); $a_y = 0$; $t = 1.846\times10^{-8}$ m/s; $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (1.782\times10^6 \text{ m/s})(1.846\times10^{-8} \text{ s}) = 0.0329 \text{ m} = 3.29 \text{ cm}$

**EVALUATE:** The electron travels downward a distance 0.822 cm while it is between the plates and a distance 3.29 cm while traveling from the edge of the plates to the screen. The total downward deflection is $0.822 \text{ cm} + 3.29 \text{ cm} = 4.11 \text{ cm}$.

The horizontal distance between the plates is half the horizontal distance the electron travels after it leaves the plates. And the vertical velocity of the electron increases as it travels between the plates, so it makes sense for it to have greater downward displacement during the motion after it leaves the plates.

**23.64.** **IDENTIFY:** The charge on the plates and the electric field between them depend on the potential difference across the plates. Since we do not know the numerical potential, we shall call this potential $V$ and find the answers in terms of $V$.

**(a) SET UP:** For two parallel plates, the potential difference between them is $V = Ed = \frac{\sigma}{\epsilon_0}d = \frac{Qd}{\epsilon_0 A}$.

**EXECUTE:** Solving for $Q$ gives

$$Q = \epsilon_0 AV/d = (8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(0.030 \text{ m})^2 V/(0.0050 \text{ m})$$

$$Q = 1.59V\times10^{-12} \text{ C} = 1.59V \text{ pC, when } V \text{ is in volts.}$$

**(b)** $E = V/d = V/(0.0050 \text{ m}) = 200V$ V/m, with $V$ in volts.

**(c) SET UP:** Energy conservation gives $\frac{1}{2}mv^2 = eV$.

**EXECUTE:** Solving for $v$ gives

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60\times10^{-19} \text{ C})V}{9.11\times10^{-31} \text{ kg}}} = 5.93\times10^5 V^{1/2} \text{ m/s, with } V \text{ in volts}$$

**EVALUATE:** Typical voltages in student laboratory work run up to around 25 V, so the charge on the plates is typically about around 40 pC, the electric field is about 5000 V/m, and the electron speed would be about 3 million m/s.

**23.65.** **(a) IDENTIFY** and **SET UP:** Problem 23.61 derived that $E = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r}$, where $a$ is the radius of the inner cylinder (wire) and $b$ is the radius of the outer hollow cylinder. The potential difference between the two cylinders is $V_{ab}$. Use this expression to calculate $E$ at the specified $r$.

**EXECUTE:** Midway between the wire and the cylinder wall is at a radius of $r = (a+b)/2 = (90.0\times10^{-6}\ \text{m} + 0.140\ \text{m})/2 = 0.07004\ \text{m}.$

$$E = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r} = \frac{50.0\times10^{3}\ \text{V}}{\ln(0.140\ \text{m}/90.0\times10^{-6}\ \text{m})(0.07004\ \text{m})} = 9.71\times10^{4}\ \text{V/m}$$

**(b) IDENTIFY** and **SET UP:** The electric force is given by Eq.(21.3). Set this equal to ten times the weight of the particle and solve for $|q|$, the magnitude of the charge on the particle.

**EXECUTE:** $F_E = 10mg$

$$|q|E = 10mg \ \text{ and } \ |q| = \frac{10mg}{E} = \frac{10(30.0\times10^{-9}\ \text{kg})(9.80\ \text{m/s}^2)}{9.71\times10^{4}\ \text{V/m}} = 3.03\times10^{-11}\ \text{C}$$

**EVALUATE:** It requires only this modest net charge for the electric force to be much larger than the weight.

**23.66.** **(a) IDENTIFY:** Calculate the potential due to each thin ring and integrate over the disk to find the potential. $V$ is a scalar so no components are involved.

**SET UP:** Consider a thin ring of radius $y$ and width $dy$. The ring has area $2\pi y\,dy$ so the charge on the ring is $dq = \sigma(2\pi y\,dy)$.

**EXECUTE:** The result of Example 23.11 then says that the potential due to this thin ring at the point on the axis at a distance $x$ from the ring is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{\sqrt{x^2+y^2}} = \frac{2\pi\sigma}{4\pi\epsilon_0}\frac{y\,dy}{\sqrt{x^2+y^2}}$$

$$V = \int dV = \frac{\sigma}{2\epsilon_0}\int_0^R \frac{y\,dy}{\sqrt{x^2+y^2}} = \frac{\sigma}{2\epsilon_0}\left[\sqrt{x^2+y^2}\right]_0^R = \frac{\sigma}{2\epsilon_0}(\sqrt{x^2+R^2} - x)$$

**EVALUATE:** For $x \gg R$ this result should reduce to the potential of a point charge with $Q = \sigma\pi R^2$.

$$\sqrt{x^2+R^2} = x(1+R^2/x^2)^{1/2} \approx x(1+R^2/2x^2) \ \text{ so } \ \sqrt{x^2+R^2} - x \approx R^2/2x$$

Then $V \approx \frac{\sigma}{2\epsilon_0}\frac{R^2}{2x} = \frac{\sigma\pi R^2}{4\pi\epsilon_0 x} = \frac{Q}{4\pi\epsilon_0 x}$, as expected.

**(b) IDENTIFY** and **SET UP:** Use Eq.(23.19) to calculate $E_x$.

**EXECUTE:** $E_x = -\frac{\partial V}{\partial x} = -\frac{\sigma}{2\epsilon_0}\left(\frac{x}{\sqrt{x^2+R^2}} - 1\right) = \frac{\sigma x}{2\epsilon_0}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right).$

**EVALUATE:** Our result agrees with Eq.(21.11) in Example 21.12.

**23.67.** **(a) IDENTIFY:** Use $V_a - V_b = \int_a^b \vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{l}}.$

**SET UP:** From Problem 22.48, $E(r) = \frac{\lambda r}{2\pi\epsilon_0 R^2}$ for $r \le R$ (inside the cylindrical charge distribution) and $E(r) = \frac{\lambda r}{2\pi\epsilon_0 r}$ for $r \ge R$. Let $V = 0$ at $r = R$ (at the surface of the cylinder).

**EXECUTE:** $r > R$

Take point $a$ to be at $R$ and point $b$ to be at $r$, where $r > R$. Let $d\vec{\boldsymbol{l}} = d\vec{\boldsymbol{r}}$. $\vec{\boldsymbol{E}}$ and $d\vec{\boldsymbol{r}}$ are both radially outward, so $\vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{r}} = E\,dr$. Thus $V_R - V_r = \int_R^r E\,dr$. Then $V_R = 0$ gives $V_r = -\int_R^r E\,dr$. In this interval $(r > R)$, $E(r) = \lambda/2\pi\epsilon_0 r$, so

$$V_r = -\int_R^r \frac{\lambda}{2\pi\epsilon_0 r}dr = -\frac{\lambda}{2\pi\epsilon_0}\int_R^r \frac{dr}{r} = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r}{R}\right).$$

**EVALUATE:** This expression gives $V_r = 0$ when $r = R$ and the potential decreases (becomes a negative number of larger magnitude) with increasing distance from the cylinder.

**EXECUTE:** $r < R$

Take point $a$ at $r$, where $r < R$, and point $b$ at $R$. $\vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{r}} = E\,dr$ as before. Thus $V_r - V_R = \int_r^R E\,dr$. Then $V_R = 0$ gives $V_r = \int_r^R E\,dr$. In this interval $(r < R)$, $E(r) = \lambda r / 2\pi\epsilon_0 R^2$, so

$$V_r = \int_r^R \frac{\lambda}{2\pi\epsilon_0 R^2} dr = \frac{\lambda}{2\pi\epsilon_0 R^2}\int_r^R r\,dr = \frac{\lambda}{2\pi\epsilon_0 R^2}\left(\frac{R^2}{2} - \frac{r^2}{2}\right).$$

$$V_r = \frac{\lambda}{4\pi\epsilon_0}\left(1 - \left(\frac{r}{R}\right)^2\right).$$

**EVALUATE:** This expression also gives $V_r = 0$ when $r = R$. The potential is $\lambda / 4\pi\epsilon_0$ at $r = 0$ and decreases with increasing $r$.

**(b) EXECUTE:** Graphs of $V$ and $E$ as functions of $r$ are sketched in Figure 23.67.

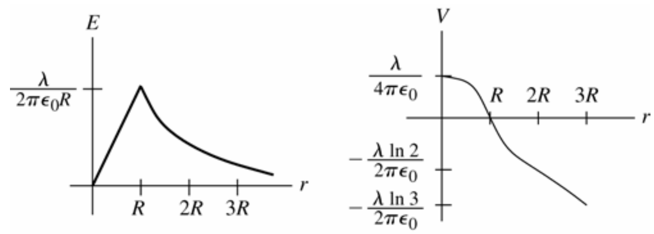



**Figure 23.67**

**EVALUATE:** $E$ at any $r$ is the negative of the slope of $V(r)$ at that $r$ (Eq.23.23).

**23.68.** **IDENTIFY:** The alpha particles start out with kinetic energy and wind up with electrical potential energy at closest approach to the nucleus.

**SET UP:** **(a)** The energy of the system is conserved, with $U = (1/4\pi\epsilon_0)(qq_0 / r)$ being the electric potential energy. With the charge of the alpha particle being $2e$ and that of the gold nucleus being $Ze$, we have

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0}\frac{2Ze^2}{R}$$

**EXECUTE:** Solving for $v$ and using $Z = 79$ for gold gives

$$v = \sqrt{\left(\frac{1}{4\pi\epsilon_0}\right)\frac{4Ze^2}{mR}} = \sqrt{\frac{\left(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4)(79)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(6.7\times10^{-27}\ \text{kg}\right)\left(5.6\times10^{-15}\ \text{m}\right)}} = 4.4\times10^7\ \text{m/s}$$

We have neglected any relativistic effects.

**(b)** Outside the atom, it is neutral. Inside the atom, we can model the 79 electrons as a uniform spherical shell, which produces no electric field inside of itself, so the only electric field is that of the nucleus.

**EVALUATE:** Neglecting relativistic effects was not such a good idea since the speed in part (a) is over 10% the speed of light. Modeling 79 electrons as a uniform spherical shell is reasonable, but we would not want to do this with small atoms.

**23.69.** **IDENTIFY:** $V_a - V_b = \int_a^b \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}$ .

**SET UP:** From Example 21.10, we have: $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2 + a^2)^{3/2}}$. $\vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = E_x dx$ . Let $a = \infty$ so $V_a = 0$ .

**EXECUTE:** $V = -\dfrac{Q}{4\pi\epsilon_0}\displaystyle\int_\infty^x \frac{x'}{(x'^2 + a^2)^{3/2}}dx' = \frac{Q}{4\pi\epsilon_0}u^{-1/2}\Bigg|_{u=\infty}^{u=x^2+a^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2 + a^2}}$ .

**EVALUATE:** Our result agrees with Eq.(23.16) in Example 23.11.

**23.70.** **IDENTIFY:** Divide the rod into infinitesimal segments with charge $dq$. The potential $dV$ due to the segment is $dV = \dfrac{1}{4\pi\epsilon_0}\dfrac{dq}{r}$ . Integrate over the rod to find the total potential.

**SET UP:** $dq = \lambda dl$ , with $\lambda = Q / \pi a$ and $dl = a\ d\theta$ .

**EXECUTE:** $dV = \dfrac{1}{4\pi\epsilon_0}\dfrac{dq}{r} = \dfrac{1}{4\pi\epsilon_0}\dfrac{\lambda\,dl}{a} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{\pi a}\dfrac{dl}{a} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q\,d\theta}{\pi a}$ . $V = \dfrac{1}{4\pi\epsilon_0}\displaystyle\int_0^\pi \frac{Q\,d\theta}{\pi a} = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}$ .

**EVALUATE:** All the charge of the ring is the same distance $a$ from the center of curvature.

**23.71.** **IDENTIFY:** We must integrate to find the total energy because the energy to bring in more charge depends on the charge already present.

**SET UP:** If $\rho$ is the uniform volume charge density, the charge of a spherical shell or radius $r$ and thickness $dr$ is $dq = \rho\, 4\pi r^2\, dr$, and $\rho = Q/(4/3\ \pi R^3)$. The charge already present in a sphere of radius $r$ is $q = \rho(4/3\ \pi r^3)$. The energy to bring the charge $dq$ to the surface of the charge $q$ is $Vdq$, where $V$ is the potential due to $q$, which is $q/4\pi\epsilon_0 r$.

**EXECUTE:** The total energy to assemble the entire sphere of radius $R$ and charge $Q$ is sum (integral) of the tiny increments of energy.

$$U = \int V dq = \int \frac{q}{4\pi\epsilon_0 r} dq = \int_0^R \frac{\rho \frac{4}{3}\pi r^3}{4\pi\epsilon_0 r}\left(\rho 4\pi r^2 dr\right) = \frac{3}{5}\left(\frac{1}{4\pi\epsilon_0}\frac{Q^2}{R}\right)$$

where we have substituted $\rho = Q/(4/3\ \pi R^3)$ and simplified the result.

**EVALUATE:** For a point-charge, $R \to 0$ so $U \to \infty$, which means that a point-charge should have infinite self-energy. This suggests that either point-charges are impossible, or that our present treatment of physics is not adequate at the extremely small scale, or both.

**23.72.** **IDENTIFY:** $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$ . The electric field is radially outward, so $\vec{E} \cdot d\vec{l} = E\ dr$ .

**SET UP:** Let $a = \infty$ , so $V_a = 0$ .

**EXECUTE:** From Example 22.9, we have the following. For $r > R: E = \dfrac{kQ}{r^2}$ and $V = -kQ\displaystyle\int_\infty^r \frac{dr'}{r'^2} = \frac{kQ}{r}$ .

For $r < R: E = \dfrac{kQr}{R^3}$ and $V = -\displaystyle\int_\infty^R \vec{E} \cdot d\vec{r}' - \int_R^r \vec{E} \cdot d\vec{r}' = \frac{kQ}{R} - \frac{kQ}{R^3}\int_R^r r'dr' = \frac{kQ}{R} - \frac{kQ}{R^3}\frac{1}{2}r'^2\Big|_R^r = \frac{kQ}{R} + \frac{kQ}{2R} - \frac{kQr^2}{2R^3} = \frac{kQ}{2R}\left[3 - \frac{r^2}{R^2}\right]$ .

**(b)** The graphs of $V$ and $E$ versus $r$ are sketched in Figure 23.72.

**EVALUATE:** For $r < R$ the potential depends on the electric field in the region $r$ to $\infty$ .

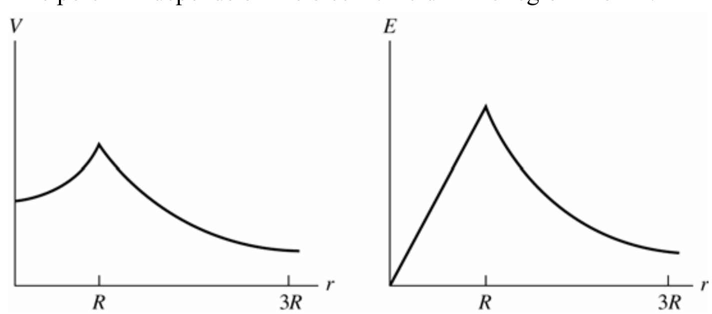



**Figure 23.72**

**23.73.** **IDENTIFY:** Problem 23.70 shows that $V_r = \dfrac{Q}{8\pi\epsilon_0 R}(3 - r^2/R^2)$ for $r \le R$ and $V_r = \dfrac{Q}{4\pi\epsilon_0 r}$ for $r \ge R$ .

**SET UP:** $V_0 = \dfrac{3Q}{8\pi\epsilon_0 R}, V_R = \dfrac{Q}{4\pi\epsilon_0 R}$

**EXECUTE:** **(a)** $V_0 - V_R = \dfrac{Q}{8\pi\epsilon_0 R}$

**(b)** If $Q > 0$ , $V$ is higher at the center. If $Q < 0$ , $V$ is higher at the surface.

**EVALUATE:** For $Q > 0$ the electric field is radially outward, $\vec{E}$ is directed toward lower potential, so $V$ is higher at the center. If $Q < 0$ , the electric field is directed radially inward and $V$ is higher at the surface.

**23.74.** **IDENTIFY:** For $r < c$ , $E = 0$ and the potential is constant. For $r > c$ , $E$ is the same as for a point charge and $V = \dfrac{kq}{r}$ .

**SET UP:** $V_\infty = 0$

**EXECUTE:** **(a)** Points $a, b$, and $c$ are all at the same potential, so $V_a - V_b = V_b - V_c = V_a - V_c = 0$ .

$$V_c - V_\infty = \frac{kq}{R} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(150\times10^{-6}\ \text{C})}{0.60\ \text{m}} = 2.25\times10^6\ \text{V}$$

**(b)** They are all at the same potential.

**(c)** Only $V_c - V_\infty$ would change; it would be $-2.25\times10^6$ V.

**EVALUATE:** The voltmeter reads the potential difference between the two points to which it is connected.

**23.75. IDENTIFY and SET UP:** Apply $F_r = -dU/dr$ and Newton's third law.

**EXECUTE:** **(a)** The electrical potential energy for a spherical shell with uniform surface charge density and a point charge $q$ outside the shell is the same as if the shell is replaced by a point charge at its center. Since $F_r = -dU/dr$, this means the force the shell exerts on the point charge is the same as if the shell were replaced by a point charge at its center. But by Newton's 3rd law, the force $q$ exerts on the shell is the same as if the shell were a point charge. But $q$ can be replaced by a spherical shell with uniform surface charge and the force is the same, so the force between the shells is the same as if they were both replaced by point charges at their centers. And since the force is the same as for point charges, the electrical potential energy for the pair of spheres is the same as for a pair of point charges.

**(b)** The potential for solid insulating spheres with uniform charge density is the same outside of the sphere as for a spherical shell, so the same result holds.

**(c)** The result doesn't hold for conducting spheres or shells because when two charged conductors are brought close together, the forces between them causes the charges to redistribute and the charges are no longer distributed uniformly over the surfaces.

**EVALUATE:** For the insulating shells or spheres, $F = k\dfrac{|q_1q_2|}{r^2}$ and $U = \dfrac{kq_1q_2}{r}$, where $q_1$ and $q_2$ are the charges of the objects and $r$ is the distance between their centers.

**23.76. IDENTIFY:** Apply Newton's second law to calculate the acceleration. Apply conservation of energy and conservation of momentum to the motions of the spheres.

**SET UP:** Problem 23.75 shows that $F = k\dfrac{|q_1q_2|}{r^2}$ and $U = \dfrac{kq_1q_2}{r}$, where $q_1$ and $q_2$ are the charges of the objects and $r$ is the distance between their centers.

**EXECUTE:** Maximum speed occurs when the spheres are very far apart. Energy conservation gives $\dfrac{kq_1q_2}{r} = \dfrac{1}{2}m_{50}v_{50}^2 + \dfrac{1}{2}m_{150}v_{150}^2$. Momentum conservation gives $m_{50}v_{50} = m_{150}v_{150}$ and $v_{50} = 3v_{150}$. $r = 0.50$ m. Solve for $v_{50}$ and $v_{150}$: $v_{50} = 12.7\ \text{m/s}$, $v_{150} = 4.24\ \text{m/s}$. Maximum acceleration occurs just after spheres are released. $\sum F = ma$ gives $\dfrac{kq_1q_2}{r^2} = m_{150}a_{150}$. $\dfrac{(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10^{-5}\ \text{C})(3\times10^{-5}\ \text{C})}{(0.50\ \text{m})^2} = (0.15\ \text{kg})a_{150}$. $a_{150} = 72.0\ \text{m/s}^2$ and $a_{50} = 3a_{150} = 216\ \text{m/s}^2$.

**EVALUATE:** The more massive sphere has a smaller acceleration and a smaller final speed.

**23.77. IDENTIFY:** Use Eq.(23.17) to calculate $V_{ab}$.

**SET UP:** From Problem 22.43, for $R \le r \le 2R$ (between the sphere and the shell) $E = Q/4\pi\epsilon_0 r^2$

Take $a$ at $R$ and $b$ at $2R$.

**EXECUTE:** $V_{ab} = V_a - V_b = \int_R^{2R} E\,dr = \dfrac{Q}{4\pi\epsilon_0}\int_R^{2R}\dfrac{dr}{r^2} = \dfrac{Q}{4\pi\epsilon_0}\left[-\dfrac{1}{r}\right]_R^{2R} = \dfrac{Q}{4\pi\epsilon_0}\left(\dfrac{1}{R} - \dfrac{1}{2R}\right)$

$$V_{ab} = \frac{Q}{8\pi\epsilon_0 R}$$

**EVALUATE:** The electric field is radially outward and points in the direction of decreasing potential, so the sphere is at higher potential than the shell.

**23.78. IDENTIFY:** $V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$

**SET UP:** $\vec{E}$ is radially outward, so $\vec{E}\cdot d\vec{l} = E\ dr$. Problem 22.42 shows that $E(r) = 0$ for $r \le a$, $E(r) = kq/r^2$ for $a < r < b$, $E(r) = 0$ for $b < r < c$ and $E(r) = kq/r^2$ for $r > c$.

**EXECUTE:** **(a)** At $r = c$: $V_c = -\int_\infty^c \dfrac{kq}{r^2}dr = \dfrac{kq}{c}$.

**(b)** At $r = b$: $V_b = -\int_\infty^c \vec{E}\cdot d\vec{r} - \int_c^b \vec{E}\cdot d\vec{r} = \dfrac{kq}{c} - 0 = \dfrac{kq}{c}$.

**(c)** At $r = a$: $V_a = -\int_\infty^c \vec{E}\cdot d\vec{r} - \int_c^b \vec{E}\cdot d\vec{r} - \int_b^a \vec{E}\cdot d\vec{r} = \dfrac{kq}{c} - kq\int_b^a \dfrac{dr}{r^2} = kq\left[\dfrac{1}{c} - \dfrac{1}{b} + \dfrac{1}{a}\right]$

**(d)** At $r = 0$: $V_0 = kq\left[\dfrac{1}{c} - \dfrac{1}{b} + \dfrac{1}{a}\right]$ since it is inside a metal sphere, and thus at the same potential as its surface.

**EVALUATE:** The potential difference between the two conductors is $V_a - V_b = kq\left[\dfrac{1}{a} - \dfrac{1}{b}\right]$.

**23.79. IDENTIFY:** Slice the rod into thin slices and use Eq.(23.14) to calculate the potential due to each slice. Integrate over the length of the rod to find the total potential at each point.

**(a) SET UP:** An infinitesimal slice of the rod and its distance from point $P$ are shown in Figure 23.79a.

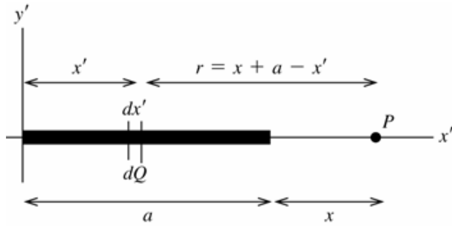



**Figure 23.79a**

Use coordinates with the origin at the left-hand end of the rod and one axis along the rod. Call the axes $x'$ and $y'$ so as not to confuse them with the distance $x$ given in the problem.

**EXECUTE:** Slice the charged rod up into thin slices of width $dx'$. Each slice has charge $dQ = Q(dx'/a)$ and a distance $r = x + a - x'$ from point $P$. The potential at $P$ due to the small slice $dQ$ is

$$dV = \frac{1}{4\pi\epsilon_0}\left(\frac{dQ}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{dx'}{x+a-x'}\right).$$

Compute the total $V$ at $P$ due to the entire rod by integrating $dV$ over the length of the rod $(x' = 0$ to $x' = a)$:

$$V = \int dV = \frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{dx'}{(x+a-x')} = \frac{Q}{4\pi\epsilon_0 a}[-\ln(x+a-x')]_0^a = \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{x+a}{x}\right).$$

**EVALUATE:** As $x \to \infty$, $V \to \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{x}{x}\right) = 0.$

**(b) SET UP:** An infinitesimal slice of the rod and its distance from point $R$ are shown in Figure 23.79b.

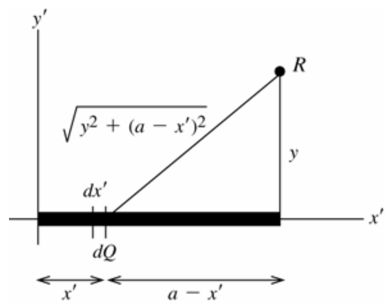



**Figure 23.79b**

$dQ = (Q/a)dx'$ as in part (a)

Each slice $dQ$ is a distance $r = \sqrt{y^2 + (a-x')^2}$ from point $R$.

**EXECUTE:** The potential $dV$ at $R$ due to the small slice $dQ$ is

$$dV = \frac{1}{4\pi\epsilon_0}\left(\frac{dQ}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\frac{dx'}{\sqrt{y^2+(a-x')^2}}.$$

$$V = \int dV = \frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{dx'}{\sqrt{y^2+(a-x')^2}}.$$

In the integral make the change of variable $u = a - x'$; $du = -dx'$

$$V = -\frac{Q}{4\pi\epsilon_0 a}\int_a^0 \frac{du}{\sqrt{y^2+u^2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[\ln(u+\sqrt{y^2+u^2})\right]_a^0$$

$$V = -\frac{Q}{4\pi\epsilon_0 a}[\ln y - \ln(a+\sqrt{y^2+a^2})] = \frac{Q}{4\pi\epsilon_0 a}\left[\ln\left(\frac{a+\sqrt{a^2+y^2}}{y}\right)\right].$$

(The expression for the integral was found in appendix B.)

**EVALUATE:** As $y \to \infty$, $V \to \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{y}{y}\right) = 0.$

**(c) SET UP:** *part (a):* $V = \dfrac{Q}{4\pi\epsilon_0 a}\ln\left(\dfrac{x+a}{x}\right) = \dfrac{Q}{4\pi\epsilon_0 a}\ln\left(1+\dfrac{a}{x}\right).$

From Appendix B, $\ln(1+u) = u - u^2/2\ldots$, so $\ln(1+a/x) = a/x - a^2/2x^2$ and this becomes $a/x$ when $x$ is large.

**EXECUTE:** Thus $V \to \dfrac{Q}{4\pi\epsilon_0 a}\left(\dfrac{a}{x}\right) = \dfrac{Q}{4\pi\epsilon_0 a}$. For large *x, V* becomes the potential of a point charge.

*part (b):* $V = \dfrac{Q}{4\pi\epsilon_0 a}\left[\ln\left(\dfrac{a+\sqrt{a^2+y^2}}{y}\right)\right] = \dfrac{Q}{4\pi\epsilon_0 a}\ln\left(\dfrac{a}{y}+\sqrt{1+\dfrac{a^2}{y^2}}\right).$

From Appendix B, $\sqrt{1+a^2/y^2} = (1+a^2/y^2)^{1/2} = 1+a^2/2y^2+\ldots$

Thus $a/y+\sqrt{1+a^2/y^2} \to 1+a/y+a^2/2y^2+\ldots \to 1+a/y$. And then using $\ln(1+u)\approx u$ gives

$$V \to \frac{Q}{4\pi\epsilon_0 a}\ln(1+a/y) \to \frac{Q}{4\pi\epsilon_0 a}\left(\frac{a}{y}\right) = \frac{Q}{4\pi\epsilon_0 y}.$$

**EVALUATE:** For large *y, V* becomes the potential of a point charge.

**23.80. IDENTIFY:** The potential at the surface of a uniformly charged sphere is $V = \dfrac{kQ}{R}$.

**SET UP:** For a sphere, $V = \dfrac{4}{3}\pi R^3$. When the raindrops merge, the total charge and volume is conserved.

**EXECUTE:** **(a)** $V = \dfrac{kQ}{R} = \dfrac{k(-1.20\times10^{-12}\text{ C})}{6.50\times10^{-4}\text{ m}} = -16.6\text{ V}$.

**(b)** The volume doubles, so the radius increases by the cube root of two: $R_{\text{new}} = \sqrt[3]{2}\,R = 8.19\times10^{-4}\text{ m}$ and the new charge is $Q_{\text{new}} = 2Q = -2.40\times10^{-12}$ C. The new potential is $V_{\text{new}} = \dfrac{kQ_{\text{new}}}{R_{\text{new}}} = \dfrac{k(-2.40\times10^{-12}\text{ C})}{8.19\times10^{-4}\text{ m}} = -26.4\text{ V}$.

**EVALUATE:** The charge doubles but the radius also increases and the potential at the surface increases by only a factor of $\dfrac{2}{2^{1/3}} = 2^{2/3}$.

**23.81. (a) IDENTIFY and SET UP:** The potential at the surface of a charged conducting sphere is given by Example 23.8: $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{R}$. For spheres *A* and *B* this gives

$$V_A = \frac{Q_A}{4\pi\epsilon_0 R_A} \text{ and } V_B = \frac{Q_B}{4\pi\epsilon_0 R_B}.$$

**EXECUTE:** $V_A = V_B$ gives $Q_A/4\pi\epsilon_0 R_A = Q_B/4\pi\epsilon_0 R_B$ and $Q_B/Q_A = R_B/R_A$. And then $R_A = 3R_B$ implies $Q_B/Q_A = 1/3$.

**(b) IDENTIFY and SET UP:** The electric field at the surface of a charged conducting sphere is given in Example 22.5:

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{R^2}.$$

**EXECUTE:** For spheres *A* and *B* this gives

$$E_A = \frac{|Q_A|}{4\pi\epsilon_0 R_A^2} \text{ and } E_B = \frac{|Q_B|}{4\pi\epsilon_0 R_B^2}.$$

$$\frac{E_B}{E_A} = \left(\frac{|Q_B|}{4\pi\epsilon_0 R^2{}_B}\right)\left(\frac{4\pi\epsilon_0 R_A^2}{|Q_A|}\right) = |Q_B/Q_A|(R_A/R_B)^2 = (1/3)(3)^2 = 3.$$

**EVALUATE:** The sphere with the larger radius needs more net charge to produce the same potential. We can write $E = V/R$ for a sphere, so with equal potentials the sphere with the smaller *R* has the larger *V*.

**23.82. IDENTIFY:** Apply conservation of energy, $K_a + U_a = K_b + U_b$.

**SET UP:** Assume the particles initially are far apart, so $U_a = 0$, The alpha particle has zero speed at the distance of closest approach, so $K_b = 0$. $1\text{ eV} = 1.60\times10^{-19}$ J. The alpha particle has charge $+2e$ and the lead nucleus has charge $+82e$.

**EXECUTE:** Set the alpha particle's kinetic energy equal to its potential energy: $K_a = U_b$ gives

$11.0\text{ MeV} = \dfrac{k(2e)(82e)}{r}$ and $r = \dfrac{k(164)(1.60\times10^{-19}\text{ C})^2}{(11.0\times10^6\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 2.15\times10^{-14}\text{ m}$.

**EVALUATE:** The calculation assumes that at the distance of closest approach the alpha particle is outside the radius of the lead nucleus.

**23.83.** **IDENTIFY** and **SET UP:** The potential at the surface is given by Example 23.8 and the electric field at the surface is given by Example 22.5. The charge initially on sphere 1 spreads between the two spheres such as to bring them to the same potential.

**EXECUTE:** **(a)** $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_1}{R_1^2}$, $V_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_1}{R_1} = R_1E_1$

**(b)** Two conditions must be met:

1) Let $q_1$and $q_2$ be the final potentials of each sphere. Then $q_1 + q_2 = Q_1$ (charge conservation)

2) Let $V_1$ and $V_2$ be the final potentials of each sphere. All points of a conductor are at the same potential, so $V_1 = V_2$.

$V_1 = V_2$ requires that $\dfrac{1}{4\pi\epsilon_0}\dfrac{q_1}{R_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_2}{R_2}$ and then $q_1/R_1 = q_2/R_2$

$$q_1R_2 = q_2R_1 = (Q_1 - q_1)R_1$$

This gives $q_1 = (R_1/[R_1+R_2])Q_1$ and $q_2 = Q_1 - q_1 = Q_1(1 - R_1/[R_1+R_2]) = Q_1(R_2/[R_1+R_2])$

**(c)** $V_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1}{R_1} = \dfrac{Q_1}{4\pi\epsilon_0(R_1+R_2)}$ and $V_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_2}{R_2} = \dfrac{Q_1}{4\pi\epsilon_0(R_1+R_2)}$, which equals $V_1$ as it should.

**(d)** $E_1 = \dfrac{V_1}{R_1} = \dfrac{Q_1}{4\pi\epsilon_0 R_1(R_1+R_2)}$. $E_2 = \dfrac{V_2}{R_2} = \dfrac{Q_1}{4\pi\epsilon_0 R_2(R_1+R_2)}$.

**EVALUATE:** Part (a) says $q_2 = q_1(R_2/R_1)$. The sphere with the larger radius needs more charge to produce the same potential at its surface. When $R_1 = R_2$, $q_1 = q_2 = Q_1/2$. The sphere with the larger radius has the smaller electric field at its surface.

**23.84.** **IDENTIFY:** Apply $V_a - V_b = \int_a^b \vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{l}}$

**SET UP:** From Problem 22.57, for $r \ge R$, $E = \dfrac{kQ}{r^2}$. For $r \le R$, $E = \dfrac{kQ}{r^2}\left[4\dfrac{r^3}{R^3} - 3\dfrac{r^4}{R^4}\right]$.

**EXECUTE:** **(a)** $r \ge R$: $E = \dfrac{kQ}{r^2} \Rightarrow V = -\int_\infty^r \dfrac{kQ}{r'^2}dr' = \dfrac{kQ}{r}$, which is the potential of a point charge.

**(b)** $r \le R$: $E = \dfrac{kQ}{r^2}\left[4\dfrac{r^3}{R^3} - 3\dfrac{r^4}{R^4}\right]$ and $V = -\int_\infty^R Edr' - \int_R^r Edr' = \dfrac{kQ}{R}\left[1 - 2\dfrac{r^2}{R^2} + 2\dfrac{R^2}{R^2} + \dfrac{r^3}{R^3} - \dfrac{R^3}{R^3}\right] = \dfrac{kQ}{R}\left[\dfrac{r^3}{R^3} - 2\dfrac{r^2}{R^2} + 2\right]$.

**EVALUATE:** At $r = R$, $V = \dfrac{kQ}{R}$. At $r = 0$, $V = \dfrac{2kQ}{R}$. The electric field is radially outward and $V$ increases as $r$ decreases.

**23.85.** **IDENTIFY:** Apply conservation of energy: $E_\text{i} = E_\text{f}$.

**SET UP:** In the collision the initial kinetic energy of the two particles is converted into potential energy at the distance of closest approach.

**EXECUTE:** **(a)** The two protons must approach to a distance of $2r_\text{p}$, where $r_\text{p}$ is the radius of a proton.

$E_\text{i} = E_\text{f}$ gives $2\left[\dfrac{1}{2}m_\text{p}v^2\right] = \dfrac{ke^2}{2r_\text{p}}$ and $v = \sqrt{\dfrac{k(1.60\times10^{-19}\text{ C})^2}{2(1.2\times10^{-15}\text{ m})(1.67\times10^{-27}\text{ kg})}} = 7.58\times10^6\text{ m/s}$.

**(b)** For a helium-helium collision, the charges and masses change from (a) and

$v = \sqrt{\dfrac{k(2(1.60\times10^{-19}\text{ C}))^2}{(3.5\times10^{-15}\text{ m})(2.99)(1.67\times10^{-27}\text{ kg})}} = 7.26\times10^6\text{ m/s}$.

**(c)** $K = \dfrac{3kT}{2} = \dfrac{mv^2}{2}$. $T_\text{p} = \dfrac{m_\text{p}v^2}{3k} = \dfrac{(1.67\times10^{-27}\text{ kg})(7.58\times10^6\text{ m/s})^2}{3(1.38\times10^{-23}\text{ J/K})} = 2.3\times10^9\text{ K}$.

$$T_\text{He} = \frac{m_\text{He}v^2}{3k} = \frac{(2.99)(1.67\times10^{-27}\text{ kg})(7.26\times10^6\text{ m/s})^2}{3(1.38\times10^{-23}\text{ J/K})} = 6.4\times10^9\text{ K}.$$

**(d)** These calculations were based on the particles' average speed. The distribution of speeds ensures that there are always a certain percentage with a speed greater than the average speed, and these particles can undergo the necessary reactions in the sun's core.

**EVALUATE:** The kinetic energies required for fusion correspond to very high temperatures.

**23.86. IDENTIFY** and **SET UP:** Apply Eq.(23.20). $\frac{W_{a\to b}}{q_0} = V_a - V_b$ and $V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$.

**EXECUTE:** **(a)** $\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} = -2Ax\hat{i} + 6Ay\hat{j} - 2Az\hat{k}$

**(b)** A charge is moved in along the $z$-axis. The work done is given by $W = q\int_{z_0}^{0} \vec{E}\cdot\hat{k}dz = q\int_{z_0}^{0}(-2Az)dz = +(Aq)z_0^2$.

Therefore, $A = \frac{W_{a\to b}}{qz_0^2} = \frac{6.00\times10^{-5}\text{ J}}{(1.5\times10^{-6}\text{ C})(0.250\text{ m})^2} = 640\text{ V/m}^2$.

**(c)** $\vec{E}(0,0,0.250) = -2(640\text{ V/m}^2)(0.250\text{ m})\hat{k} = -(320\text{ V/m})\hat{k}$.

**(d)** In every plane parallel to the $xz$-plane, $y$ is constant, so $V(x,y,z) = Ax^2 + Az^2 - C$, where $C = 3Ay^2$.

$x^2 + z^2 = \frac{V+C}{A} = R^2$, which is the equation for a circle since $R$ is constant as long as we have constant potential on those planes.

**(e)** $V = 1280\text{ V}$ and $y = 2.00\text{ m}$, so $x^2 + z^2 = \frac{1280\text{ V} + 3(640\text{ V/m}^2)(2.00\text{ m})^2}{640\text{ V/m}^2} = 14.0\text{ m}^2$ and the radius of the circle is $3.74\text{ m}$.

**EVALUATE:** In any plane parallel to the $xz$-plane, $\vec{E}$ projected onto the plane is radial and hence perpendicular to the equipotential circles.

**23.87. IDENTIFY:** Apply conservation of energy to the motion of the daughter nuclei.

**SET UP:** Problem 23.73 shows that the electrical potential energy of the two nuclei is the same as if all their charge was concentrated at their centers.

**EXECUTE:** **(a)** The two daughter nuclei have half the volume of the original uranium nucleus, so their radii are smaller by a factor of the cube root of 2: $r = \frac{7.4\times10^{-15}\text{ m}}{\sqrt[3]{2}} = 5.9\times10^{-15}\text{ m}$.

**(b)** $U = \frac{k(46e)^2}{2r} = \frac{k(46)^2(1.60\times10^{-19}\text{ C})^2}{1.18\times10^{-14}\text{ m}} = 4.14\times10^{-11}\text{ J}$. $U = 2K$, where $K$ is the final kinetic energy of each nucleus. $K = U/2 = (4.14\times10^{-11}\text{ J})/2 = 2.07\times10^{-11}\text{ J}$.

**(c)** If we have 10.0 kg of uranium, then the number of nuclei is $n = \frac{10.0\text{ kg}}{(236\text{ u})(1.66\times10^{-27}\text{ kg/u})} = 2.55\times10^{25}$ nuclei.

And each releases energy $U$, so $E = nU = (2.55\times10^{25})(4.14\times10^{-11}\text{ J}) = 1.06\times10^{15}\text{ J} = 253\text{ kilotons of TNT}$.

**(d)** We could call an atomic bomb an "electric" bomb since the electric potential energy provides the kinetic energy of the particles.

**EVALUATE:** This simple model considers only the electrical force between the daughter nuclei and neglects the nuclear force.

**23.88. IDENTIFY** and **SET UP:** In part (a) apply $E = -\frac{\partial V}{\partial r}$. In part (b) apply Gauss's law.

**EXECUTE:** **(a)** For $r \le a$, $E = -\frac{\partial V}{\partial r} = -\frac{\rho_0 a^2}{18\epsilon_0}\left[-6\frac{r}{a^2} + 6\frac{r^2}{a^3}\right] = \frac{\rho_0 a}{3\epsilon_0}\left[\frac{r}{a} - \frac{r^2}{a^2}\right]$. For $r \ge a$, $E = -\frac{\partial V}{\partial r} = 0$. $\vec{E}$ has only a radial component because $V$ depends only on $r$.

**(b)** For $r \le a$, Gauss's law gives $E_r 4\pi r^2 = \frac{Q_r}{\epsilon_0} = \frac{\rho_0 a}{3\epsilon_0}\left[\frac{r}{a} - \frac{r^2}{a^2}\right]4\pi r^2$ and

$E_{r+dr}4\pi(r^2 + 2rdr) = \frac{Q_{r+dr}}{\epsilon_0} = \frac{\rho_0 a}{3\epsilon_0}\left[\frac{r+dr}{a} - \frac{(r^2+2rdr)}{a^2}\right]4\pi(r^2+2rdr)$. Therefore,

$\frac{Q_{r+dr} - Q_r}{\epsilon_0} = \frac{\rho(r)4\pi r^2\, dr}{\epsilon_0} \approx \frac{\rho_0 a 4\pi r^2 dr}{3\epsilon_0}\left[-\frac{2r}{a^2} + \frac{2}{a} - \frac{2r}{a^2} + \frac{1}{a}\right]$ and $\rho(r) = \frac{\rho_0}{3}\left[3 - \frac{4r}{a}\right] = \rho_0\left[1 - \frac{4r}{3a}\right]$.

**(c)** For $r \ge a$, $\rho(r) = 0$, so the total charge enclosed will be given by

$$Q = 4\pi \int_0^a \rho(r) r^2 dr = 4\pi\rho_0 \int_0^a \left[ r^2 - \frac{4r^3}{3a} \right] dr = 4\pi\rho_0 \left[ \frac{1}{3}r^3 - \frac{r^4}{3a} \right]_0^a = 0.$$

**EVALUATE:** Apply Gauss's law to a sphere of radius $r > R$. The result of part (c) says that $Q_{\text{encl}} = 0$, so $E = 0$. This agrees with the result we calculated in part (a).

**23.89. IDENTIFY:** Angular momentum and energy must be conserved.

**SET UP:** At the distance of closest approach the speed is not zero. $E = K + U$. $q_1 = 2e$, $q_2 = 82e$.

**EXECUTE:** $mv_1 b = mv_2 r_2$. $E_1 = E_2$ gives $E_1 = \frac{1}{2}mv_2^2 + \frac{kq_1q_2}{r_2}$. $E_1 = 11\text{ MeV} = 1.76 \times 10^{-12}$ J. $r_2$ is the distance of closest approach. Substituting in for $v_2 = v_1\left(\frac{b}{r_2}\right)$ we find $E_1 = E_1\frac{b^2}{r_2^2} + \frac{kq_1q_2}{r_2}$.

$(E_1)r_2^2 - (kq_1q_2)r_2 - E_1b^2 = 0$. For $b = 10^{-12}$ m, $r_2 = 1.01 \times 10^{-12}$ m. For $b = 10^{-13}$ m, $r_2 = 1.11 \times 10^{-13}$ m. And for $b = 10^{-14}$ m, $r_2 = 2.54 \times 10^{-14}$ m.

**EVALUATE:** As $b$ decreases the collision is closer to being head-on and the distance of closest approach decreases. Problem 23.82 shows that the distance of closest approach is $2.15 \times 10^{-14}$ m when $b = 0$.

**23.90. IDENTIFY:** Consider the potential due to an infinitesimal slice of the cylinder and integrate over the length of the cylinder to find the total potential. The electric field is along the axis of the tube and is given by $E = -\frac{\partial V}{\partial x}$.

**SET UP:** Use the expression from Example 23.11 for the potential due to each infinitesimal slice. Let the slice be at coordinate *z* along the *x*-axis, relative to the center of the tube.

**EXECUTE:** **(a)** For an infinitesimal slice of the finite cylinder, we have the potential

$$dV = \frac{k\,dQ}{\sqrt{(x-z)^2 + R^2}} = \frac{kQ}{L}\frac{dz}{\sqrt{(x-z)^2 + R^2}}.$$ Integrating gives

$$V = \frac{kQ}{L}\int_{-L/2}^{L/2} \frac{dz}{\sqrt{(x-z)^2 + R^2}} = \frac{kQ}{L}\int_{-L/2-x}^{L/2-x} \frac{du}{\sqrt{u^2 + R^2}}$$ where $u = x - z$. Therefore,

$$V = \frac{kQ}{L}\ln\left[\frac{\sqrt{(L/2 - x)^2 + R^2} + (L/2 - x)}{\sqrt{(L/2 + x)^2 + R^2} - L/2 - x}\right]$$ on the cylinder axis.

**(b)** For $L << R$, $V \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{(L/2 - x)^2 + R^2} + L/2 - x}{\sqrt{(L/2 + x)^2 + R^2} - L/2 - x}\right] \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{x^2 - xL + R^2} + L/2 - x}{\sqrt{x^2 + xL + R^2} - L/2 - x}\right]$.

$$V \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{1 - xL/(R^2 + x^2)} + (L/2 - x)/\sqrt{R^2 + x^2}}{\sqrt{1 + xL/(R^2 + x^2)} + (-L/2 - x)/\sqrt{R^2 + x^2}}\right] = \frac{kQ}{L}\ln\left[\frac{1 - xL/2(R^2 + x^2) + (L/2 - x)/\sqrt{R^2 + x^2}}{1 + xL/2(R^2 + x^2) + (-L/2 - x)/\sqrt{R^2 + x^2}}\right].$$

$$V \approx \frac{kQ}{L}\ln\left[\frac{1 + L/2\sqrt{R^2 + x^2}}{1 - L/2\sqrt{R^2 + x^2}}\right] = \frac{kQ}{L}\left(\ln\left[1 + \frac{L}{2\sqrt{R^2 + x^2}}\right] - \ln\left[1 - \frac{L}{2\sqrt{R^2 + x^2}}\right]\right).$$

$$V \approx \frac{kQ}{L}\frac{2L}{2\sqrt{x^2 + R^2}} = \frac{kQ}{\sqrt{x^2 + R^2}},$$ which is the same as for a ring.

**(c)** $$E_x = -\frac{\partial V}{\partial x} = \frac{2kQ\left(\sqrt{(L - 2x)^2 + 4R^2} - \sqrt{(L + 2x)^2 + 4R^2}\right)}{\sqrt{(L - 2x)^2 + 4R^2}\sqrt{(L + 2x)^2 + 4R^2}}$$

**EVALUATE:** For $L << R$ the expression for $E_x$ reduces to that for a ring of charge, as given in Example 23.14.

**23.91. IDENTIFY:** When the oil drop is at rest, the upward force $|q|E$ from the electric field equals the downward weight of the drop. When the drop is falling at its terminal speed, the upward viscous force equals the downward weight of the drop.

**SET UP:** The volume of the drop is related to its radius *r* by $V = \frac{4}{3}\pi r^3$.

**EXECUTE:** **(a)** $F_{\text{g}} = mg = \frac{4\pi r^3}{3}\rho g$. $F_{\text{e}} = |q|E = |q|V_{AB}/d$. $F_{\text{e}} = F_{\text{g}}$ gives $|q| = \frac{4\pi}{3}\frac{\rho r^3 gd}{V_{AB}}$.

**(b)** $\dfrac{4\pi r^3}{3}\rho g = 6\pi\eta r v_t$ gives $r = \sqrt{\dfrac{9\eta v_t}{2\rho g}}$. Using this result to replace *r* in the expression in part (a) gives

$$|q| = \frac{4\pi}{3}\frac{\rho g d}{V_{AB}}\left[\sqrt{\frac{9\eta v_t}{2\rho g}}\right]^3 = 18\pi\frac{d}{V_{AB}}\sqrt{\frac{\eta^3 v_t^3}{2\rho g}}.$$

**(c)** $|q| = 18\pi\dfrac{10^{-3}\text{ m}}{9.16\text{ V}}\sqrt{\dfrac{(1.81\times10^{-5}\text{ N}\cdot\text{s/m}^2)^3(1.00\times10^{-3}\text{ m/39.3 s})^3}{2(824\text{ kg/m}^3)(9.80\text{ m/s}^2)}} = 4.80\times10^{-19}\text{ C} = 3e$. The drop has acquired three excess electrons.

$$r = \sqrt{\frac{9(1.81\times10^{-5}\text{ N}\cdot\text{s/m}^2)(1.00\times10^{-3}\text{ m/39.3 s})}{2(824\text{ kg/m}^3)(9.80\text{ m/s}^2)}} = 5.07\times10^{-7}\text{ m} = 0.507\ \mu\text{m}.$$

**EVALUATE:** The weight of the drop is $\left(\dfrac{4\pi r^3}{3}\right)\rho g = 4.4\times10^{-15}\text{ N}$. The density of air at room temperature is $1.2\text{ kg/m}^3$, so the buoyancy force is $\rho_{\text{air}}Vg = 6.4\times10^{-18}\text{ N}$ and can be neglected.

**23.92. IDENTIFY:** $v_{\text{cm}} = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2}$

**SET UP:** $E = K_1 + K_2 + U$, where $U = \dfrac{kq_1q_2}{r}$.

**EXECUTE:** **(a)** $v_{\text{cm}} = \dfrac{(6\times10^{-5}\text{ kg})(400\text{ m/s}) + (3\times10^{-5}\text{ kg})(1300\text{ m/s})}{6.0\times10^{-5}\text{ kg} + 3.0\times10^{-5}\text{ kg}} = 700\text{ m/s}$

**(b)** $E_{\text{rel}} = \dfrac{1}{2}m_1{v_1}^2 + \dfrac{1}{2}m_2{v_2}^2 + \dfrac{kq_1q_2}{r} - \dfrac{1}{2}(m_1 + m_2)v_{\text{cm}}^2$. After expanding the center of mass velocity and collecting like terms $E_{\text{rel}} = \dfrac{1}{2}\dfrac{m_1m_2}{m_1 + m_2}[{v_1}^2 + {v_2}^2 - 2v_1v_2] + \dfrac{kq_1q_2}{r} = \dfrac{1}{2}\mu(v_1 - v_2)^2 + \dfrac{kq_1q_2}{r}$.

**(c)** $E_{\text{rel}} = \dfrac{1}{2}(2.0\times10^{-5}\text{ kg})(900\text{ m/s})^2 + \dfrac{k(2.0\times10^{-6}\text{ C})(-5.0\times10^{-6}\text{ C})}{0.0090\text{ m}} = -1.9\text{ J}$

**(d)** Since the energy is less than zero, the system is "bound."

**(e)** The maximum separation is when the velocity is zero: $-1.9\text{ J} = \dfrac{kq_1q_2}{r}$ gives

$$r = \frac{k(2.0\times10^{-6}\text{ C})(-5.0\times10^{-6}\text{ C})}{-1.9\text{ J}} = 0.047\text{ m}.$$

**(f)** Now using $v_1 = 400\text{ m/s}$ and $v_2 = 1800\text{ m/s}$, we find $E_{\text{rel}} = +9.6\text{ J}$. The particles do escape, and the final relative velocity is $|v_1 - v_2| = \sqrt{\dfrac{2E_{\text{rel}}}{\mu}} = \sqrt{\dfrac{2(9.6\text{ J})}{2.0\times10^{-5}\text{ kg}}} = 980\text{ m/s}$.

**EVALUATE:** For an isolated system the velocity of the center of mass is constant and the system must retain the kinetic energy associated with the motion of the center of mass.