21

ELECTRIC CHARGE AND ELECTRIC FIELD

21.1. **(a) IDENTIFY and SET UP:** Use the charge of one electron $(-1.602\times10^{-19}\text{ C})$ to find the number of electrons required to produce the net charge.

EXECUTE: The number of excess electrons needed to produce net charge q is

$$\frac{q}{-e}=\frac{-3.20\times10^{-9}\text{ C}}{-1.602\times10^{-19}\text{ C/electron}}=2.00\times10^{10}\text{ electrons.}$$

(b) IDENTIFY and SET UP: Use the atomic mass of lead to find the number of lead atoms in 8.00×10^{-3} kg of lead. From this and the total number of excess electrons, find the number of excess electrons per lead atom.

EXECUTE: The atomic mass of lead is 207×10^{-3} kg/mol, so the number of moles in 8.00×10^{-3} kg is

$n=\dfrac{m_{\text{tot}}}{M}=\dfrac{8.00\times10^{-3}\text{ kg}}{207\times10^{-3}\text{ kg/mol}}=0.03865\text{ mol.}$ N_A (Avogadro's number) is the number of atoms in 1 mole, so the number of lead atoms is $N=nN_\text{A}=(0.03865\text{ mol})(6.022\times10^{23}\text{ atoms/mol})=2.328\times10^{22}$ atoms. The number of excess electrons per lead atom is $\dfrac{2.00\times10^{10}\text{ electrons}}{2.328\times10^{22}\text{ atoms}}=8.59\times10^{-13}.$

EVALUATE: Even this small net charge corresponds to a large number of excess electrons. But the number of atoms in the sphere is much larger still, so the number of excess electrons per lead atom is very small.

21.2. **IDENTIFY:** The charge that flows is the rate of charge flow times the duration of the time interval.

SET UP: The charge of one electron has magnitude $e=1.60\times10^{-19}$ C.

EXECUTE: The rate of charge flow is 20,000 C/s and $t=100\ \mu\text{s}=1.00\times10^{-4}$ s.

$Q=(20{,}000\text{ C/s})(1.00\times10^{-4}\text{ s})=2.00\text{ C}$. The number of electrons is $n_\text{e}=\dfrac{Q}{1.60\times10^{-19}\text{ C}}=1.25\times10^{19}.$

EVALUATE: This is a very large amount of charge and a large number of electrons.

21.3. **IDENTIFY:** From your mass estimate the number of protons in your body. You have an equal number of electrons.

SET UP: Assume a body mass of 70 kg. The charge of one electron is -1.60×10^{-19} C.

EXECUTE: The mass is primarily protons and neutrons of $m=1.67\times10^{-27}$ kg. The total number of protons and neutrons is $n_{\text{p and n}}=\dfrac{70\text{ kg}}{1.67\times10^{-27}\text{ kg}}=4.2\times10^{28}$. About one-half are protons, so $n_\text{p}=2.1\times10^{28}=n_\text{e}$. The number of electrons is about 2.1×10^{28}. The total charge of these electrons is

$Q=(-1.60\times10^{-19}\text{ C/electron})(2.10\times10^{28}\text{ electrons})=-3.35\times10^{9}\text{ C}.$

EVALUATE: This is a huge amount of negative charge. But your body contains an equal number of protons and your net charge is zero. If you carry a net charge, the number of excess or missing electrons is a very small fraction of the total number of electrons in your body.

21.4. **IDENTIFY:** Use the mass m of the ring and the atomic mass M of gold to calculate the number of gold atoms. Each atom has 79 protons and an equal number of electrons.

SET UP: $N_\text{A}=6.02\times10^{23}$ atoms/mol. A proton has charge $+e$.

EXECUTE: The mass of gold is 17.7 g and the atomic weight of gold is 197 g/mol. So the number of atoms is $N_\text{A}n=(6.02\times10^{23}\text{ atoms/mol})\left(\dfrac{17.7\text{ g}}{197\text{ g/mol}}\right)=5.41\times10^{22}$ atoms. The number of protons is

$n_\text{p}=(79\text{ protons/atom})(5.41\times10^{22}\text{ atoms})=4.27\times10^{24}$ protons. $Q=(n_\text{p})(1.60\times10^{-19}\text{ C/proton})=6.83\times10^{5}$ C.

(b) The number of electrons is $n_\text{e}=n_\text{p}=4.27\times10^{24}.$

EVALUATE: The total amount of positive charge in the ring is very large, but there is an equal amount of negative charge.

21.5. **IDENTIFY:** Apply $F = \dfrac{k|q_1q_2|}{r^2}$ and solve for r.

SET UP: $F = 650\text{ N}$.

EXECUTE: $r = \sqrt{\dfrac{k|q_1q_2|}{F}} = \sqrt{\dfrac{(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.0\text{ C})^2}{650\text{ N}}} = 3.7\times 10^3\text{ m} = 3.7\text{ km}$

EVALUATE: Charged objects typically have net charges much less than 1 C.

21.6. **IDENTIFY:** Apply Coulomb's law and calculate the net charge q on each sphere.

SET UP: The magnitude of the charge of an electron is $e = 1.60\times 10^{-19}\text{ C}$.

EXECUTE: $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{r^2}$. This gives $|q| = \sqrt{4\pi\epsilon_0 Fr^2} = \sqrt{4\pi\epsilon_0(4.57\times 10^{-21}\text{ N})(0.200\text{ m})^2} = 1.43\times 10^{-16}\text{ C}$. And therefore, the total number of electrons required is $n = |q|/e = (1.43\times 10^{-16}\text{ C})/(1.60\times 10^{-19}\text{ C/electron}) = 890$ electrons.

EVALUATE: Each sphere has 890 excess electrons and each sphere has a net negative charge. The two like charges repel.

21.7. **IDENTIFY:** Apply Coulomb's law.

SET UP: Consider the force on one of the spheres.

(a) EXECUTE: $q_1 = q_2 = q$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} = \frac{q^2}{4\pi\epsilon_0 r^2} \text{ so } q = r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = 0.150\text{ m}\sqrt{\frac{0.220\text{ N}}{8.988\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = 7.42\times 10^{-7}\text{ C (on each)}$$

(b) $q_2 = 4q_1$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} = \frac{4q_1^2}{4\pi\epsilon_0 r^2} \text{ so } q_1 = r\sqrt{\frac{F}{4(1/4\pi\epsilon_0)}} = \tfrac{1}{2}r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = \tfrac{1}{2}(7.42\times 10^{-7}\text{ C}) = 3.71\times 10^{-7}\text{ C}.$$

And then $q_2 = 4q_1 = 1.48\times 10^{-6}\text{ C}$.

EVALUATE: The force on one sphere is the same magnitude as the force on the other sphere, whether the sphere have equal charges or not.

21.8. **IDENTIFY:** Use the mass of a sphere and the atomic mass of aluminum to find the number of aluminum atoms in one sphere. Each atom has 13 electrons. Apply Coulomb's law and calculate the magnitude of charge $|q|$ on each sphere.

SET UP: $N_A = 6.02\times 10^{23}\text{ atoms/mol}$. $|q| = n'_e e$, where n'_e is the number of electrons removed from one sphere and added to the other.

EXECUTE: **(a)** The total number of electrons on each sphere equals the number of protons.

$$n_e = n_p = (13)(N_A)\left(\frac{0.0250\text{ kg}}{0.026982\text{ kg/mol}}\right) = 7.25\times 10^{24}\text{ electrons}.$$

(b) For a force of 1.00×10^4 N to act between the spheres, $F = 1.00\times 10^4\text{ N} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{r^2}$. This gives $|q| = \sqrt{4\pi\epsilon_0(1.00\times 10^4\text{ N})(0.0800\text{ m})^2} = 8.43\times 10^{-4}\text{ C}$. The number of electrons removed from one sphere and added to the other is $n'_e = |q|/e = 5.27\times 10^{15}$ electrons.

(c) $n'_e/n_e = 7.27\times 10^{-10}$.

EVALUATE: When ordinary objects receive a net charge the fractional change in the total number of electrons in the object is very small.

21.9. **IDENTIFY:** Apply $F = ma$, with $F = k\dfrac{|q_1q_2|}{r^2}$.

SET UP: $a = 25.0g = 245\text{ m/s}^2$. An electron has charge $-e = -1.60\times 10^{-19}\text{ C}$.

EXECUTE: $F = ma = (8.55\times 10^{-3}\text{ kg})(245\text{ m/s}^2) = 2.09\text{ N}$. The spheres have equal charges q, so $F = k\dfrac{q^2}{r^2}$ and $|q| = r\sqrt{\dfrac{F}{k}} = (0.150\text{ m})\sqrt{\dfrac{2.09\text{ N}}{8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = 2.29\times 10^{-6}\text{ C}$. $N = \dfrac{|q|}{e} = \dfrac{2.29\times 10^{-6}\text{ C}}{1.60\times 10^{-19}\text{ C}} = 1.43\times 10^{13}$ electrons. The charges on the spheres have the same sign so the electrical force is repulsive and the spheres accelerate away from each other.

EVALUATE: As the spheres move apart the repulsive force they exert on each other decreases and their acceleration decreases.

21.10. **(a) IDENTIFY:** The electrical attraction of the proton gives the electron an acceleration equal to the acceleration due to gravity on earth.

SET UP: Coulomb's law gives the force and Newton's second law gives the acceleration this force produces. $ma = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$ and $r = \sqrt{\frac{e^2}{4\pi\epsilon_0 ma}}$.

EXECUTE: $r = \sqrt{\frac{(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(9.11\times10^{-31}\ \text{kg})(9.80\ \text{m/s}^2)}} = 5.08\ \text{m}$

EVALUATE: The electron needs to be about 5 m from a single proton to have the same acceleration as it receives from the gravity of the entire earth.

(b) IDENTIFY: The force on the electron comes from the electrical attraction of all the protons in the earth.

SET UP: First find the number n of protons in the earth, and then find the acceleration of the electron using Newton's second law, as in part (a).

$$n = m_E/m_p = (5.97\times10^{24}\ \text{kg})/(1.67\times10^{-27}\ \text{kg}) = 3.57\times10^{51}\ \text{protons.}$$

$$a = F/m = \frac{\frac{1}{4\pi\epsilon_0}\frac{|q_p q_e|}{R_E^2}}{m_e} = \frac{\frac{1}{4\pi\epsilon_0}ne^2}{m_e R_E^2}.$$

EXECUTE: $a = (9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.57\times10^{51})(1.60\times10^{-19}\ \text{C})^2/[(9.11\times10^{-31}\ \text{kg})(6.38\times10^6\ \text{m})^2] = 2.22\times10^{40}\ \text{m/s}^2$. One can ignore the gravitation force since it produces an acceleration of only 9.8 m/s^2 and hence is much much less than the electrical force.

EVALUATE: With the electrical force, the acceleration of the electron would nearly 10^{40} times greater than with gravity, which shows how strong the electrical force is.

21.11. **IDENTIFY:** In a space satellite, the only force accelerating the free proton is the electrical repulsion of the other proton.

SET UP: Coulomb's law gives the force, and Newton's second law gives the acceleration: $a = F/m = (1/4\pi\epsilon_0)(e^2/r^2)/m$.

EXECUTE: **(a)** $a = (9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2/[(0.00250\ \text{m})^2(1.67\times10^{-27}\ \text{kg})] = 2.21\times10^4\ \text{m/s}^2$.

(b) The graphs are sketched in Figure 21.11.

EVALUATE: The electrical force of a single stationary proton gives the moving proton an initial acceleration about 20,000 times as great as the acceleration caused by the gravity of the entire earth. As the protons move farther apart, the electrical force gets weaker, so the acceleration decreases. Since the protons continue to repel, the velocity keeps increasing, but at a decreasing rate.

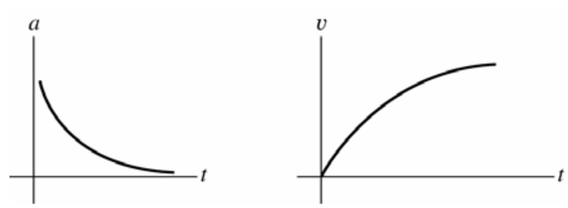


Figure 21.11

21.12. **IDENTIFY:** Apply Coulomb's law.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: **(a)** $F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$. This gives $0.200\ \text{N} = \frac{1}{4\pi\epsilon_0}\frac{(0.550\times10^{-6}\ \text{C})|q_2|}{(0.30\ \text{m})^2}$ and $|q_2| = +3.64\times10^{-6}\ \text{C}$. The force is attractive and $q_1 < 0$, so $q_2 = +3.64\times10^{-6}\ \text{C}$.

(b) $F = 0.200$ N. The force is attractive, so is downward.

EVALUATE: The forces between the two charges obey Newton's third law.

21.13. **IDENTIFY:** Apply Coulomb's law. The two forces on q_3 must have equal magnitudes and opposite directions.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: The force $\vec{F}_2$ that q_2 exerts on q_3 has magnitude $F_2 = k\dfrac{|q_2 q_3|}{r_2^2}$ and is in the +x direction. $\vec{F}_1$ must be in the $-x$ direction, so q_1 must be positive. $F_1 = F_2$ gives $k\dfrac{|q_1||q_3|}{r_1^2} = k\dfrac{|q_2||q_3|}{r_2^2}$.

$$|q_1| = |q_2|\left(\frac{r_1}{r_2}\right)^2 = (3.00\text{ nC})\left(\frac{2.00\text{ cm}}{4.00\text{ cm}}\right)^2 = 0.750\text{ nC}.$$

EVALUATE: The result for the magnitude of q_1 doesn't depend on the magnitude of q_2.

21.14. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on Q.

SET UP: The force that q_1 exerts on Q is repulsive, as in Example 21.4, but now the force that q_2 exerts is attractive.

EXECUTE: The x-components cancel. We only need the y-components, and each charge contributes equally.

$F_{1y} = F_{2y} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{(2.0\times10^{-6}\text{ C})(4.0\times10^{-6}\text{ C})}{(0.500\text{ m})^2}\sin\alpha = -0.173\text{ N}$ (since $\sin\alpha = 0.600$). Therefore, the total force is $2F = 0.35\text{ N}$, in the $-y$-direction.

EVALUATE: If q_1 is $-2.0\ \mu\text{C}$ and q_2 is $+2.0\ \mu\text{C}$, then the net force is in the +y-direction.

21.15. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on q_1.

SET UP: Like charges repel and unlike charges attract, so $\vec{F}_2$ and $\vec{F}_3$ are both in the +x-direction.

EXECUTE: $F_2 = k\dfrac{|q_1 q_2|}{r_{12}^2} = 6.749\times10^{-5}\text{ N}$, $F_3 = k\dfrac{|q_1 q_3|}{r_{13}^2} = 1.124\times10^{-4}\text{ N}$. $F = F_2 + F_3 = 1.8\times10^{-4}\text{ N}$.

$F = 1.8\times10^{-4}$ N and is in the +x-direction.

EVALUATE: Comparing our results to those in Example 21.3, we see that $\vec{F}_{1\text{ on }3} = -\vec{F}_{3\text{ on }1}$, as required by Newton's third law.

21.16. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on q_2.

SET UP: $\vec{F}_{2\text{ on }1}$ is in the +y-direction.

EXECUTE: $F_{2\text{on }1} = \dfrac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.0\times10^{-6}\text{ C})(2.0\times10^{-6}\text{ C})}{(0.60\text{ m})^2} = 0.100\text{ N}$. $(F_{2\text{ on }1})_x = 0$ and $(F_{2\text{ on }1})_y = +0.100\text{ N}$. $F_{Q\text{ on }1}$ is equal and opposite to $F_{1\text{ on }Q}$ (Example 21.4), so $(F_{Q\text{ on }1})_x = -0.23\text{N}$ and $(F_{Q\text{ on }1})_y = 0.17\text{ N}$. $F_x = (F_{2\text{ on }1})_x + (F_{Q\text{ on }1})_x = -0.23\text{ N}$. $F_y = (F_{2\text{ on }1})_y + (F_{Q\text{ on }1})_y = 0.100\text{ N} + 0.17\text{ N} = 0.27\text{ N}$.

The magnitude of the total force is $F = \sqrt{(0.23\text{ N})^2 + (0.27\text{ N})^2} = 0.35\text{ N}$. $\tan^{-1}\dfrac{0.23}{0.27} = 40°$, so $\vec{F}$ is $40°$ counterclockwise from the +y axis, or $130°$ counterclockwise from the +x axis.

EVALUATE: Both forces on q_1 are repulsive and are directed away from the charges that exert them.

21.17. **IDENTIFY** and **SET UP:** Apply Coulomb's law to calculate the force exerted by q_2 and q_3 on q_1. Add these forces as vectors to get the net force. The target variable is the x-coordinate of q_3.

EXECUTE: $\vec{F}_2$ is in the x-direction.

$F_2 = k\dfrac{|q_1 q_2|}{r_{12}^2} = 3.37\text{ N}$, so $F_{2x} = +3.37\text{ N}$

$F_x = F_{2x} + F_{3x}$ and $F_x = -7.00\text{ N}$

$F_{3x} = F_x - F_{2x} = -7.00\text{ N} - 3.37\text{ N} = -10.37\text{ N}$

For F_{3x} to be negative, q_3 must be on the $-x$-axis.

$F_3 = k\dfrac{|q_1 q_3|}{x^2}$, so $|x| = \sqrt{\dfrac{k|q_1 q_3|}{F_3}} = 0.144\text{ m}$, so $x = -0.144\text{ m}$

EVALUATE: q_2 attracts q_1 in the $+x$-direction so q_3 must attract q_1 in the $-x$-direction, and q_3 is at negative x.

21.18. **IDENTIFY:** Apply Coulomb's law.

SET UP: Like charges repel and unlike charges attract. Let $\vec{F}_{21}$ be the force that q_2 exerts on q_1 and let $\vec{F}_{31}$ be the force that q_3 exerts on q_1.

EXECUTE: The charge q_3 must be to the right of the origin; otherwise both q_2 and q_3 would exert forces in the $+x$ direction. Calculating the two forces:

$$F_{21} = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r_{12}^2} = \frac{(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\ \text{C})(5.00\times10^{-6}\ \text{C})}{(0.200\ \text{m})^2} = 3.375\ \text{N}\text{, in the }+x\text{ direction.}$$

$$F_{31} = \frac{(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\,(3.00\times10^{-6}\ \text{C})\,(8.00\times10^{-6}\ \text{C})}{r_{13}^2} = \frac{0.216\ \text{N}\cdot\text{m}^2}{r_{13}^2}\text{, in the }-x\text{ direction.}$$

We need $F_x = F_{21} - F_{31} = -7.00\ \text{N}$, so $3.375\ \text{N} - \dfrac{0.216\ \text{N}\cdot\text{m}^2}{r_{13}^2} = -7.00\ \text{N}$. $r_{13} = \sqrt{\dfrac{0.216\ \text{N}\cdot\text{m}^2}{3.375\ \text{N} + 7.00\ \text{N}}} = 0.144\ \text{m}$. q_3 is at $x = 0.144\ \text{m}$.

EVALUATE: $F_{31} = 10.4\ \text{N}$. F_{31} is larger than F_{21}, because $|q_3|$ is larger than $|q_2|$ and also because r_{13} is less than r_{12}.

21.19. **IDENTIFY:** Apply Coulomb's law to calculate the force each of the two charges exerts on the third charge. Add these forces as vectors.

SET UP: The three charges are placed as shown in Figure 21.19a.

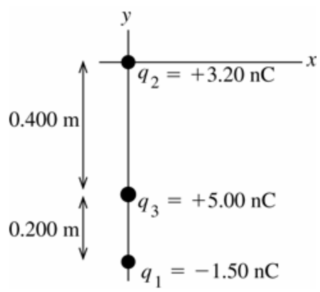
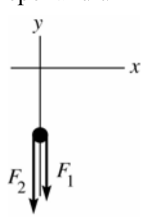


Figure 21.19a

EXECUTE: Like charges repel and unlike attract, so the free-body diagram for q_3 is as shown in Figure 21.19b.



$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_3|}{r_{13}^2}$$

$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2q_3|}{r_{23}^2}$$

Figure 21.19b

$$F_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.50\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 1.685\times10^{-6}\ \text{N}$$

$$F_2 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.20\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(0.400\ \text{m})^2} = 8.988\times10^{-7}\ \text{N}$$

The resultant force is $\vec{R} = \vec{F}_1 + \vec{F}_2$.

$R_x = 0$.

$R_y = F_1 + F_2 = 1.685\times10^{-6}\ \text{N} + 8.988\times10^{-7}\ \text{N} = 2.58\times10^{-6}\ \text{N}$.

The resultant force has magnitude $2.58\times10^{-6}\ \text{N}$ and is in the $-y$-direction.

EVALUATE: The force between q_1 and q_3 is attractive and the force between q_2 and q_3 is replusive.

21.20. **IDENTIFY:** Apply $F = k\dfrac{|qq'|}{r^2}$ to each pair of charges. The net force is the vector sum of the forces due to q_1 and q_2.

SET UP: Like charges repel and unlike charges attract. The charges and their forces on q_3 are shown in Figure 21.20.

EXECUTE: $F_1 = k\dfrac{|q_1 q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(4.00\times10^{-9}\ \text{C})(0.600\times10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 5.394\times10^{-7}\ \text{N}$.

$F_2 = k\dfrac{|q_2 q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(5.00\times10^{-9}\ \text{C})(0.600\times10^{-9}\ \text{C})}{(0.300\ \text{m})^2} = 2.997\times10^{-7}\ \text{N}$.

$F_x = F_{1x} + F_{2x} = +F_1 - F_2 = 2.40\times10^{-7}\ \text{N}$. The net force has magnitude $2.40\times10^{-7}\ \text{N}$ and is in the $+x$ direction.

EVALUATE: Each force is attractive, but the forces are in opposite directions because of the placement of the charges. Since the forces are in opposite directions, the net force is obtained by subtracting their magnitudes.

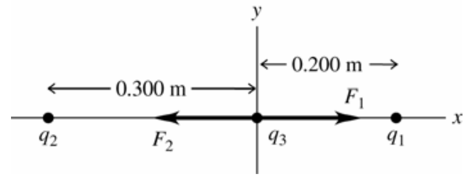


Figure 21.20

21.21. IDENTIFY: Apply Coulomb's law to calculate each force on $-Q$.

SET UP: Let $\vec{F}_1$ be the force exerted by the charge at $y = a$ and let $\vec{F}_2$ be the force exerted by the charge at $y = -a$.

EXECUTE: **(a)** The two forces on $-Q$ are shown in Figure 21.21a. $\sin\theta = \dfrac{a}{(a^2+x^2)^{1/2}}$ and $r = (a^2+x^2)^{1/2}$ is the distance between q and $-Q$ and between $-q$ and $-Q$.

(b) $F_x = F_{1x} + F_{2x} = 0$. $F_y = F_{1y} + F_{2y} = 2\dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{(a^2+x^2)}\sin\theta = \dfrac{1}{4\pi\epsilon_0}\dfrac{2qQa}{(a^2+x^2)^{3/2}}$.

(c) At $x = 0$, $F_y = \dfrac{1}{4\pi\epsilon_0}\dfrac{2qQ}{a^2}$, in the $+y$ direction.

(d) The graph of F_y versus x is given in Figure 21.21b.

EVALUATE: $F_x = 0$ for all values of x and $F_y > 0$ for all x.

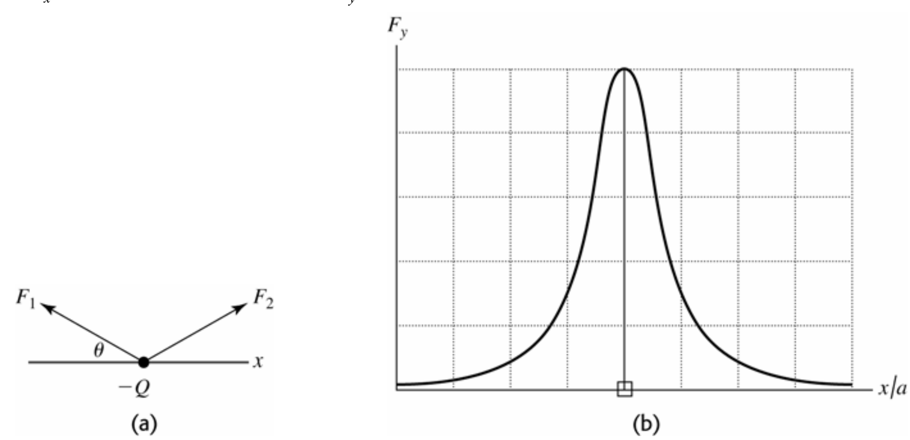


Figure 21.21

21.22. IDENTIFY: Apply Coulomb's law to calculate each force on $-Q$.

SET UP: Let $\vec{F}_1$ be the force exerted by the charge at $y = a$ and let $\vec{F}_2$ be the force exerted by the charge at $y = -a$. The distance between each charge q and Q is $r = \left(a^2+x^2\right)^{1/2}$. $\cos\theta = \dfrac{|x|}{\left(a^2+x^2\right)^{1/2}}$.

EXECUTE: **(a)** The two forces on $-Q$ are shown in Figure 21.22a.

(b) When $x > 0$, F_{1x} and F_{2x} are negative. $F_x = F_{1x} + F_{2x} = -2\dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{(a^2+x^2)}\cos\theta = \dfrac{1}{4\pi\epsilon_0}\dfrac{-2qQx}{(a^2+x^2)^{3/2}}$. When $x < 0$, F_{1x} and F_{2x} are positive and the same expression for F_x applies. $F_y = F_{1y} + F_{2y} = 0$.

(c) At $x = 0$, $F_x = 0$.

(d) The graph of F_x versus x is sketched in Figure 21.22b.

EVALUATE: The direction of the net force on $-Q$ is always toward the origin.

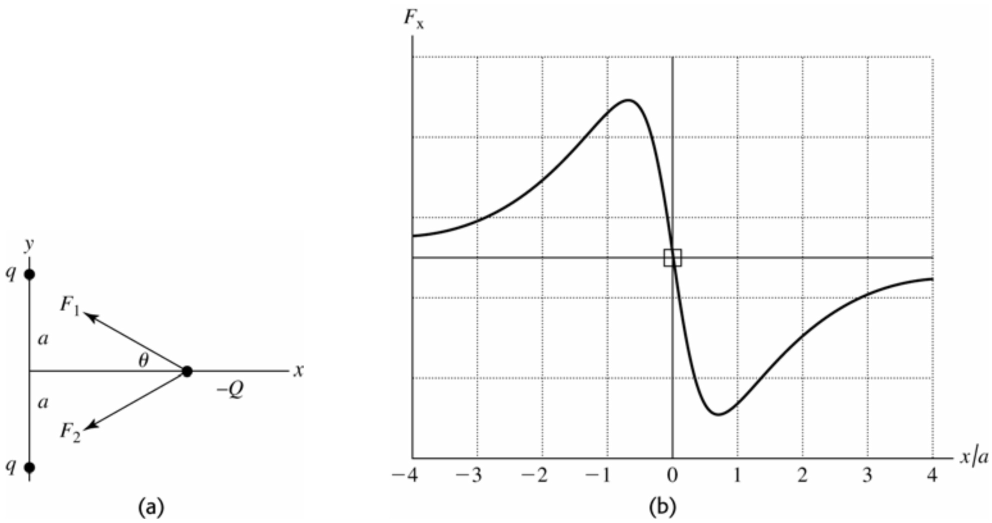
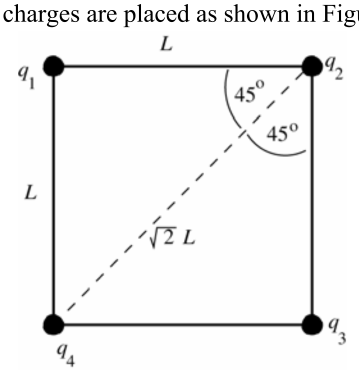


Figure 21.22

21.23. IDENTIFY: Apply Coulomb's law to calculate the force exerted on one of the charges by each of the other three and then add these forces as vectors.

(a) SET UP: The charges are placed as shown in Figure 21.23a.



$q_1 = q_2 = q_3 = q_4 = q$

Figure 21.23a

Consider forces on q_4. The free-body diagram is given in Figure 21.23b. Take the y-axis to be parallel to the diagonal between q_2 and q_4 and let $+y$ be in the direction away from q_2. Then $\vec{F}_2$ is in the $+y$-direction.

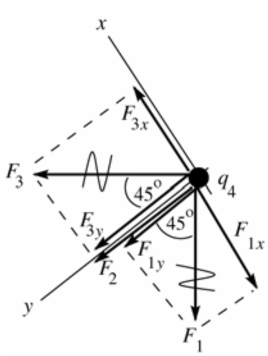


Figure 21.23b

EXECUTE: $F_3 = F_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{L^2}$

$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{q^2}{2L^2}$$

$$F_{1x} = -F_1 \sin 45° = -F_1/\sqrt{2}$$

$$F_{1y} = +F_1 \cos 45° = +F_1/\sqrt{2}$$

$$F_{3x} = +F_3 \sin 45° = +F_3/\sqrt{2}$$

$$F_{3y} = +F_3 \cos 45° = +F_3/\sqrt{2}$$

$$F_{2x} = 0,\; F_{2y} = F_2$$

(b) $R_x = F_{1x} + F_{2x} + F_{3x} = 0$

$$R_y = F_{1y} + F_{2y} + F_{3y} = (2/\sqrt{2})\frac{1}{4\pi\epsilon_0}\frac{q^2}{L^2} + \frac{1}{4\pi\epsilon_0}\frac{q^2}{2L^2} = \frac{q^2}{8\pi\epsilon_0 L^2}(1+2\sqrt{2})$$

$R = \dfrac{q^2}{8\pi\epsilon_0 L^2}(1+2\sqrt{2})$. Same for all four charges.

EVALUATE: In general the resultant force on one of the charges is directed away from the opposite corner. The forces are all repulsive since the charges are all the same. By symmetry the net force on one charge can have no component perpendicular to the diagonal of the square.

21.24. IDENTIFY: Apply $F = \frac{k|qq'|}{r^2}$ to find the force of each charge on $+q$. The net force is the vector sum of the individual forces.

SET UP: Let $q_1 = +2.50\ \mu\text{C}$ and $q_2 = -3.50\ \mu\text{C}$. The charge $+q$ must be to the left of q_1 or to the right of q_2 in order for the two forces to be in opposite directions. But for the two forces to have equal magnitudes, $+q$ must be closer to the charge q_1, since this charge has the smaller magnitude. Therefore, the two forces can combine to give zero net force only in the region to the left of q_1. Let $+q$ be a distance *d* to the left of q_1, so it is a distance $d + 0.600\ \text{m}$ from q_2.

EXECUTE: $F_1 = F_2$ gives $\frac{kq|q_1|}{d^2} = \frac{kq|q_2|}{(d+0.600\ \text{m})^2}$. $d = \pm\sqrt{\frac{|q_1|}{|q_2|}}(d + 0.600\ \text{m}) = \pm(0.8452)(d + 0.600\ \text{m})$. *d* must be positive, so $d = \frac{(0.8452)(0.600\ \text{m})}{1 - 0.8452} = 3.27\ \text{m}$. The net force would be zero when $+q$ is at $x = -3.27\ \text{m}$.

EVALUATE: When $+q$ is at $x = -3.27\ \text{m}$, $\vec{F}_1$ is in the $-x$ direction and $\vec{F}_2$ is in the $+x$ direction.

21.25. IDENTIFY: $F = |q|E$. Since the field is uniform, the force and acceleration are constant and we can use a constant acceleration equation to find the final speed.

SET UP: A proton has charge $+e$ and mass $1.67\times10^{-27}\ \text{kg}$.

EXECUTE: **(a)** $F = (1.60\times10^{-19}\ \text{C})(2.75\times10^{3}\ \text{N/C}) = 4.40\times10^{-16}\ \text{N}$

(b) $a = \frac{F}{m} = \frac{4.40\times10^{-16}\ \text{N}}{1.67\times10^{-27}\ \text{kg}} = 2.63\times10^{11}\ \text{m/s}^2$

(c) $v_x = v_{0x} + a_x t$ gives $v = (2.63\times10^{11}\ \text{m/s}^2)(1.00\times10^{-6}\ \text{s}) = 2.63\times10^{5}\ \text{m/s}$

EVALUATE: The acceleration is very large and the gravity force on the proton can be ignored.

21.26. IDENTIFY: For a point charge, $E = k\frac{|q|}{r^2}$.

SET UP: $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: **(a)** The field is toward the negative charge so is downward.

$E = (8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{3.00\times10^{-9}\ \text{C}}{(0.250\ \text{m})^2} = 432\ \text{N/C}$.

(b) $r = \sqrt{\frac{k|q|}{E}} = \sqrt{\frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-9}\ \text{C})}{12.0\ \text{N/C}}} = 1.50\ \text{m}$

EVALUATE: At different points the electric field has different directions, but it is always directed toward the negative point charge.

21.27. IDENTIFY: The acceleration that stops the charge is produced by the force that the electric field exerts on it. Since the field and the acceleration are constant, we can use the standard kinematics formulas to find acceleration and time.

(a) SET UP: First use kinematics to find the proton's acceleration. $v_x = 0$ when it stops. Then find the electric field needed to cause this acceleration using the fact that $F = qE$.

EXECUTE: $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$. $0 = (4.50\times10^6\ \text{m/s})^2 + 2a(0.0320\ \text{m})$ and $a = 3.16\times10^{14}\ \text{m/s}^2$. Now find the electric field, with $q = e$. $eE = ma$ and $E = ma/e = (1.67\times10^{-27}\ \text{kg})(3.16\times10^{14}\ \text{m/s}^2)/(1.60\times10^{-19}\ \text{C}) = 3.30\times10^6\ \text{N/C}$, to the left.

(b) SET UP: Kinematics gives $v = v_0 + at$, and $v = 0$ when the electron stops, so $t = v_0/a$.

EXECUTE: $t = v_0/a = (4.50\times10^6\ \text{m/s})/(3.16\times10^{14}\ \text{m/s}^2) = 1.42\times10^{-8}\ \text{s} = 14.2\ \text{ns}$

(c) SET UP: In part (a) we saw that the electric field is proportional to *m*, so we can use the ratio of the electric fields. $E_\text{e}/E_\text{p} = m_\text{e}/m_\text{p}$ and $E_\text{e} = (m_\text{e}/m_\text{p})E_\text{p}$.

EXECUTE: $E_\text{e} = [(9.11\times10^{-31}\ \text{kg})/(1.67\times10^{-27}\ \text{kg})](3.30\times10^6\ \text{N/C}) = 1.80\times10^3\ \text{N/C}$, to the right

EVALUATE: Even a modest electric field, such as the ones in this situation, can produce enormous accelerations for electrons and protons.

21.28. **IDENTIFY:** Use constant acceleration equations to calculate the upward acceleration a and then apply $\vec{F} = q\vec{E}$ to calculate the electric field.

SET UP: Let $+y$ be upward. An electron has charge $q = -e$.

EXECUTE: **(a)** $v_{0y} = 0$ and $a_y = a$, so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = \frac{1}{2}at^2$. Then

$$a = \frac{2(y - y_0)}{t^2} = \frac{2(4.50\text{ m})}{(3.00\times10^{-6}\text{ s})^2} = 1.00\times10^{12}\text{ m/s}^2.\quad E = \frac{F}{q} = \frac{ma}{q} = \frac{(9.11\times10^{-31}\text{ kg})(1.00\times10^{12}\text{ m/s}^2)}{1.60\times10^{-19}\text{ C}} = 5.69\text{ N/C}$$

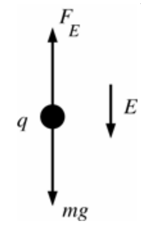
The force is up, so the electric field must be *downward* since the electron has negative charge.

(b) The electron's acceleration is $\sim10^{11}g$, so gravity must be negligibly small compared to the electrical force.

EVALUATE: Since the electric field is uniform, the force it exerts is constant and the electron moves with constant acceleration.

21.29. **(a) IDENTIFY:** Eq. (21.4) relates the electric field, charge of the particle, and the force on the particle. If the particle is to remain stationary the net force on it must be zero.

SET UP: The free-body diagram for the particle is sketched in Figure 21.29. The weight is *mg,* downward. For the net force to be zero the force exerted by the electric field must be upward. The electric field is downward. Since the electric field and the electric force are in opposite directions the charge of the particle is negative.



$mg = |q|E$

Figure 21.29

EXECUTE: $|q| = \frac{mg}{E} = \frac{(1.45\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{650\text{ N/C}} = 2.19\times10^{-5}\text{ C}$ and $q = -21.9\ \mu\text{C}$

(b) SET UP: The electrical force has magnitude $F_E = |q|E = eE.$ The weight of a proton is $w = mg.$ $F_E = w$ so $eE = mg$

EXECUTE: $E = \frac{mg}{e} = \frac{(1.673\times10^{-27}\text{ kg})(9.80\text{ m/s}^2)}{1.602\times10^{-19}\text{ C}} = 1.02\times10^{-7}\text{ N/C}.$

This is a very small electric field.

EVALUATE: In both cases $|q|E = mg$ and $E = (m/|q|)g.$ In part (b) the $m/|q|$ ratio is much smaller $(\sim10^{-8})$ than in part (a) $(\sim10^{-2})$ so E is much smaller in (b). For subatomic particles gravity can usually be ignored compared to electric forces.

21.30. **IDENTIFY:** Apply $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$.

SET UP: The iron nucleus has charge $+26e.$ A proton has charge $+e$.

EXECUTE: **(a)** $E = \frac{1}{4\pi\epsilon_0}\frac{(26)(1.60\times10^{-19}\text{ C})}{(6.00\times10^{-10}\text{ m})^2} = 1.04\times10^{11}\text{ N/C}.$

(b) $E_{\text{proton}} = \frac{1}{4\pi\epsilon_0}\frac{(1.60\times10^{-19}\text{ C})}{(5.29\times10^{-11}\text{ m})^2} = 5.15\times10^{11}\text{ N/C}.$

EVALUATE: These electric fields are very large. In each case the charge is positive and the electric fields are directed away from the nucleus or proton.

21.31. **IDENTIFY:** For a point charge, $E = k\frac{|q|}{r^2}$. The net field is the vector sum of the fields produced by each charge. A charge q in an electric field $\vec{E}$ experiences a force $\vec{F} = q\vec{E}$.

SET UP: The electric field of a negative charge is directed toward the charge. Point A is 0.100 m from q_2 and 0.150 m from q_1. Point B is 0.100 m from q_1 and 0.350 m from q_2.

EXECUTE: **(a)** The electric fields due to the charges at point A are shown in Figure 21.31a.

$$E_1 = k\frac{|q_1|}{r_{A1}^2} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{6.25\times10^{-9}\text{ C}}{(0.150\text{ m})^2} = 2.50\times10^3\text{ N/C}$$

$$E_2 = k\frac{|q_2|}{r_{A2}^2} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{12.5\times10^{-9}\text{ C}}{(0.100\text{ m})^2} = 1.124\times10^4\text{ N/C}$$

Since the two fields are in opposite directions, we subtract their magnitudes to find the net field.
$E = E_2 - E_1 = 8.74\times10^3$ N/C, to the right.

(b) The electric fields at points B are shown in Figure 21.31b.

$$E_1 = k\frac{|q_1|}{r_{B1}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.25\times10^{-9}\ \text{C}}{(0.100\ \text{m})^2} = 5.619\times10^3\ \text{N/C}$$

$$E_2 = k\frac{|q_2|}{r_{B2}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12.5\times10^{-9}\ \text{C}}{(0.350\ \text{m})^2} = 9.17\times10^2\ \text{N/C}$$

Since the fields are in the same direction, we add their magnitudes to find the net field. $E = E_1 + E_2 = 6.54\times10^3$ N/C, to the right.

(c) At A, $E = 8.74\times10^3$ N/C, to the right. The force on a proton placed at this point would be
$F = qE = (1.60\times10^{-19}\ \text{C})(8.74\times10^3\ \text{N/C}) = 1.40\times10^{-15}$ N, to the right.

EVALUATE: A proton has positive charge so the force that an electric field exerts on it is in the same direction as the field.

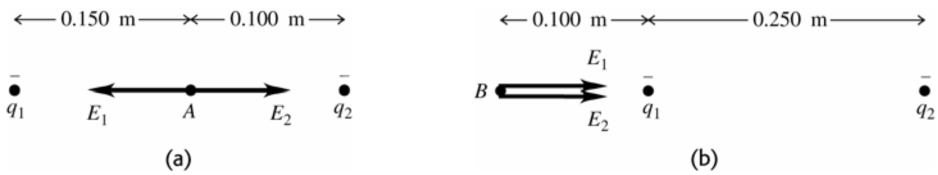


Figure 21.31

21.32. **IDENTIFY:** The electric force is $\vec{F} = q\vec{E}$.

SET UP: The gravity force (weight) has magnitude $w = mg$ and is downward.

EXECUTE: **(a)** To balance the weight the electric force must be upward. The electric field is downward, so for an upward force the charge q of the person must be negative. $w = F$ gives $mg = |q|E$ and

$$|q| = \frac{mg}{E} = \frac{(60\ \text{kg})(9.80\ \text{m/s}^2)}{150\ \text{N/C}} = 3.9\ \text{C}.$$

(b) $F = k\frac{|qq'|}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.9\ \text{C})^2}{(100\ \text{m})^2} = 1.4\times10^7\ \text{N}$. The repulsive force is immense and this is not a feasible means of flight.

EVALUATE: The net charge of charged objects is typically much less than 1 C.

21.33. **IDENTIFY:** Eq. (21.3) gives the force on the particle in terms of its charge and the electric field between the plates. The force is constant and produces a constant acceleration. The motion is similar to projectile motion; use constant acceleration equations for the horizontal and vertical components of the motion.

(a) SET UP: The motion is sketched in Figure 21.33a.

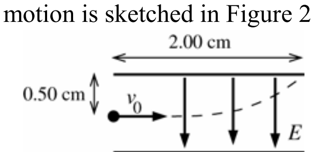


Figure 21.33a

For an electron $q = -e$.

$\vec{F} = q\vec{E}$ and q negative gives that $\vec{F}$ and $\vec{E}$ are in opposite directions, so $\vec{F}$ is upward. The free-body diagram for the electron is given in Figure 21.33b.

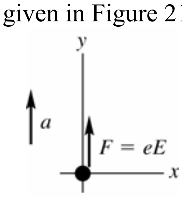


Figure 21.33b

EXECUTE: $\sum F_y = ma_y$

$eE = ma$

Solve the kinematics to find the acceleration of the electron: Just misses upper plate says that $x - x_0 = 2.00$ cm when $y - y_0 = +0.500$ cm.

<u>x-component</u>

$v_{0x} = v_0 = 1.60\times10^6$ m/s, $a_x = 0$, $x - x_0 = 0.0200$ m, $t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$

$t = \dfrac{x - x_0}{v_{0x}} = \dfrac{0.0200\text{ m}}{1.60\times 10^6\text{ m/s}} = 1.25\times 10^{-8}\text{ s}$

In this same time t the electron travels 0.0050 m vertically:

<u>y-component</u>

$t = 1.25\times 10^{-8}\text{s},\ v_{0y} = 0,\ y - y_0 = +0.0050\text{ m},\ a_y = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

$a_y = \dfrac{2(y - y_0)}{t^2} = \dfrac{2(0.0050\text{ m})}{(1.25\times 10^{-8}\text{ s})^2} = 6.40\times 10^{13}\text{ m/s}^2$

(This analysis is very similar to that used in Chapter 3 for projectile motion, except that here the acceleration is upward rather than downward.) This acceleration must be produced by the electric-field force: $eE = ma$

$$E = \frac{ma}{e} = \frac{(9.109\times 10^{-31}\text{ kg})(6.40\times 10^{13}\text{ m/s}^2)}{1.602\times 10^{-19}\text{ C}} = 364\text{ N/C}$$

Note that the acceleration produced by the electric field is <u>much</u> larger than g, the acceleration produced by gravity, so it is perfectly ok to neglect the gravity force on the elctron in this problem.

(b) $a = \dfrac{eE}{m_p} = \dfrac{(1.602\times 10^{-19}\text{ C})(364\text{ N/C})}{1.673\times 10^{-27}\text{ kg}} = 3.49\times 10^{10}\text{ m/s}^2$

This is much less than the acceleration of the electron in part (a) so the vertical deflection is less and the proton won't hit the plates. The proton has the same initial speed, so the proton takes the same time $t = 1.25\times 10^{-8}\text{ s}$ to travel horizontally the length of the plates. The force on the proton is downward (in the same direction as $\vec{E}$, since q is positive), so the acceleration is downward and $a_y = -3.49\times 10^{10}\text{ m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(-3.49\times 10^{10}\text{ m/s}^2)(1.25\times 10^{-8}\text{ s})^2 = -2.73\times 10^{-6}\text{ m}$. The displacement is 2.73×10^{-6} m, downward.

(c) EVALUATE: The displacements are in opposite directions because the electron has negative charge and the proton has positive charge. The electron and proton have the same magnitude of charge, so the force the electric field exerts has the same magnitude for each charge. But the proton has a mass larger by a factor of 1836 so its acceleration and its vertical displacement are smaller by this factor.

21.34. IDENTIFY: Apply Eq.(21.7) to calculate the electric field due to each charge and add the two field vectors to find the resultant field.

SET UP: For q_1, $\hat{\boldsymbol{r}} = \hat{\boldsymbol{j}}$. For q_2, $\hat{\boldsymbol{r}} = \cos\theta\hat{\boldsymbol{i}} + \sin\theta\hat{\boldsymbol{j}}$, where θ is the angle between $\vec{E}_2$ and the $+x$-axis.

EXECUTE: **(a)** $\vec{E}_1 = \dfrac{q_1}{4\pi\epsilon_0 r_1^2}\hat{\boldsymbol{j}} = \dfrac{(9.0\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(-5.00\times 10^{-9}\text{ C})}{(0.0400\text{ m})^2} = (-2.813\times 10^4\text{ N/C})\,\hat{\boldsymbol{j}}$.

$\left|\vec{E}_2\right| = \dfrac{q_2}{4\pi\epsilon_0 r_2^2} = \dfrac{(9.0\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(3.00\times 10^{-9}\text{ C})}{(0.0300\text{ m})^2 + (0.0400\text{ m})^2} = 1.080\times 10^4\text{ N/C}$. The angle of $\vec{E}_2$, measured from the x-axis, is $180° - \tan^{-1}\left(\dfrac{4.00\text{ cm}}{3.00\text{ cm}}\right) = 126.9°$ Thus

$\vec{E}_2 = (1.080\times 10^4\text{ N/C})(\hat{\boldsymbol{i}}\cos 126.9° + \hat{\boldsymbol{j}}\sin 126.9°) = (-6.485\times 10^3\text{ N/C})\hat{\boldsymbol{i}} + (8.64\times 10^3\text{ N/C})\hat{\boldsymbol{j}}$

(b) The resultant field is $\vec{E}_1 + \vec{E}_2 = (-6.485\times 10^3\text{ N/C})\hat{\boldsymbol{i}} + (-2.813\times 10^4\text{ N/C} + 8.64\times 10^3\text{ N/C})\hat{\boldsymbol{j}}$.

$$\vec{E}_1 + \vec{E}_2 = (-6.485\times 10^3\text{ N/C})\hat{\boldsymbol{i}} - (1.95\times 10^4\text{ N/C})\hat{\boldsymbol{j}}.$$

EVALUATE: $\vec{E}_1$ is toward q_1 since q_1 is negative. $\vec{E}_2$ is directed away from q_2, since q_2 is positive.

21.35. IDENTIFY: Apply constant acceleration equations to the motion of the electron.

SET UP: Let $+x$ be to the right and let $+y$ be downward. The electron moves 2.00 cm to the right and 0.50 cm downward.

EXECUTE: Use the horizontal motion to find the time when the electron emerges from the field.

$x - x_0 = 0.0200\text{ m},\ a_x = 0,\ v_{0x} = 1.60\times 10^6\text{ m/s}$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = 1.25\times 10^{-8}$ s. Since $a_x = 0$,

$v_x = 1.60\times 10^6\text{ m/s}$. $y - y_0 = 0.0050\text{ m},\ v_{0y} = 0,\ t = 1.25\times 10^{-8}$ s. $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $v_y = 8.00\times 10^5\text{ m/s}$.

Then $v = \sqrt{v_x^2 + v_y^2} = 1.79\times 10^6\text{ m/s}$.

EVALUATE: $v_y = v_{0y} + a_y t$ gives $a_y = 6.4\times 10^{13}\text{ m/s}^2$. The electric field between the plates is

$E = \dfrac{ma_y}{e} = \dfrac{(9.11\times 10^{-31}\text{ kg})(6.4\times 10^{13}\text{ m/s}^2)}{1.60\times 10^{-19}\text{ C}} = 364\text{ V/m}$. This is not a very large field.

21.36. **IDENTIFY:** Use the components of $\vec{E}$ from Example 21.6 to calculate the magnitude and direction of $\vec{E}$. Use $\vec{F} = q\vec{E}$ to calculate the force on the -2.5 nC charge and use Newton's third law for the force on the -8.0 nC charge.

SET UP: From Example 21.6, $\vec{E} = (-11\text{ N/C})\hat{i} + (14\text{ N/C})\hat{j}$.

EXECUTE: **(a)** $E = \sqrt{E_x^2 + E_y^2} = \sqrt{(-11\text{ N/C})^2 + (14\text{ N/C})^2} = 17.8\text{ N/C}$. $\tan^{-1}\left(\frac{|E_y|}{|E_x|}\right) = \tan^{-1}(14/11) = 51.8°$, so

$\theta = 128°$ counterclockwise from the +x-axis.

(b) (i) $\vec{F} = \vec{E}q$ so $F = (17.8\text{ N/C})(2.5\times10^{-9}\text{ C}) = 4.45\times10^{-8}\text{ N}$, at $52°$ below the +x-axis.

(ii) 4.45×10^{-8} N at $128°$ counterclockwise from the +x-axis.

EVALUATE: The forces in part (b) are repulsive so they are along the line connecting the two charges and in each case the force is directed away from the charge that exerts it.

21.37. **IDENTIFY** and **SET UP:** The electric force is given by Eq. (21.3). The gravitational force is $w_e = m_e g$. Compare these forces.

(a) EXECUTE: $w_e = (9.109\times10^{-31}\text{ kg})(9.80\text{ m/s}^2) = 8.93\times10^{-30}\text{ N}$

In Examples 21.7 and 21.8, $E = 1.00\times10^4$ N/C, so the electric force on the electron has magnitude

$F_E = |q|E = eE = (1.602\times10^{-19}\text{ C})(1.00\times10^4\text{ N/C}) = 1.602\times10^{-15}\text{ N}$.

$$\frac{w_e}{F_E} = \frac{8.93\times10^{-30}\text{ N}}{1.602\times10^{-15}\text{ N}} = 5.57\times10^{-15}$$

The gravitational force is much smaller than the electric force and can be neglected.

(b) $mg = |q|E$

$m = |q|E/g = (1.602\times10^{-19}\text{ C})(1.00\times10^4\text{ N/C})/(9.80\text{ m/s}^2) = 1.63\times10^{-16}\text{ kg}$

$$\frac{m}{m_e} = \frac{1.63\times10^{-16}\text{ kg}}{9.109\times10^{-31}\text{ kg}} = 1.79\times10^{14};\ m = 1.79\times10^{14} m_e.$$

EVALUATE: m is much larger than m_e. We found in part (a) that if $m = m_e$ the gravitational force is much smaller than the electric force. $|q|$ is the same so the electric force remains the same. To get w large enough to equal F_E, the mass must be made much larger.

(c) The electric field in the region between the plates is uniform so the force it exerts on the charged object is independent of where between the plates the object is placed.

21.38. **IDENTIFY:** Apply constant acceleration equations to the motion of the proton. $E = F/|q|$.

SET UP: A proton has mass $m_p = 1.67\times10^{-27}$ kg and charge $+e$. Let +x be in the direction of motion of the proton.

EXECUTE: **(a)** $v_{0x} = 0$. $a = \frac{eE}{m_p}$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x - x_0 = \frac{1}{2}a_x t^2 = \frac{1}{2}\frac{eE}{m_p}t^2$. Solving for E gives

$$E = \frac{2(0.0160\text{ m})(1.67\times10^{-27}\text{ kg})}{(1.60\times10^{-19}\text{ C})(1.50\times10^{-6}\text{ s})^2} = 148\text{ N/C}.$$

(b) $v_x = v_{0x} + a_x t = \frac{eE}{m_p}t = 2.13\times10^4\text{ m/s}$.

EVALUATE: The electric field is directed from the positively charged plate toward the negatively charged plate and the force on the proton is also in this direction.

21.39. **IDENTIFY:** Find the angle θ that $\hat{r}$ makes with the +x-axis. Then $\hat{r} = (\cos\theta)\hat{i} + (\sin\theta)\hat{j}$.

SET UP: $\tan\theta = y/x$

EXECUTE: **(a)** $\tan^{-1}\left(\frac{-1.35}{0}\right) = -\frac{\pi}{2}$ rad. $\hat{r} = -\hat{j}$.

(b) $\tan^{-1}\left(\frac{12}{12}\right) = \frac{\pi}{4}$ rad. $\hat{r} = \frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j}$.

(c) $\tan^{-1}\left(\frac{2.6}{+1.10}\right) = 1.97\text{ rad} = 112.9°$. $\hat{r} = -0.39\hat{i} + 0.92\hat{j}$ (Second quadrant).

EVALUATE: In each case we can verify that $\hat{r}$ is a unit vector, because $\hat{r}\cdot\hat{r} = 1$.

21.40. **IDENTIFY:** The net force on each charge must be zero.

SET UP: The force diagram for the $-6.50\ \mu\text{C}$ charge is given in Figure 21.40. F_E is the force exerted on the charge by the uniform electric field. The charge is negative and the field is to the right, so the force exerted by the field is to the left. F_q is the force exerted by the other point charge. The two charges have opposite signs, so the force is attractive. Take the $+x$ axis to be to the right, as shown in the figure.

EXECUTE: **(a)** $F = |q|E = (6.50\times10^{-6}\text{ C})(1.85\times10^{8}\text{ N/C}) = 1.20\times10^{3}\text{ N}$

$$F_q = k\frac{|q_1q_2|}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(6.50\times10^{-6}\text{ C})(8.75\times10^{-6}\text{ C})}{(0.0250\text{ m})^2} = 8.18\times10^2\text{ N}$$

$\sum F_x = 0$ gives $T + F_q - F_E = 0$ and $T = F_E - F_q = 382\text{ N}$.

(b) Now F_q is to the left, since like charges repel.

$\sum F_x = 0$ gives $T - F_q - F_E = 0$ and $T = F_E + F_q = 2.02\times10^3\text{ N}$.

EVALUATE: The tension is much larger when both charges have the same sign, so the force one charge exerts on the other is repulsive.

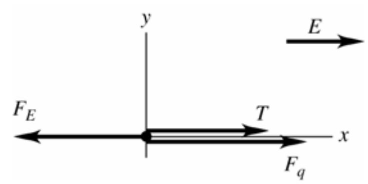


Figure 21.40

21.41. **IDENTIFY** and **SET UP:** Use $\vec{E}$ in Eq. (21.3) to calculate $\vec{F}$, $\vec{F} = m\vec{a}$ to calculate $\vec{a}$, and a constant acceleration equation to calculate the final velocity. Let $+x$ be east.

(a) EXECUTE: $F_x = |q|E = (1.602\times10^{-19}\text{ C})(1.50\text{ N/C}) = 2.403\times10^{-19}\text{ N}$

$a_x = F_x/m = (2.403\times10^{-19}\text{ N})/(9.109\times10^{-31}\text{ kg}) = +2.638\times10^{11}\text{ m/s}^2$

$v_{0x} = +4.50\times10^5\text{ m/s},\ a_x = +2.638\times10^{11}\text{ m/s}^2,\ x - x_0 = 0.375\text{ m},\ v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 6.33\times10^5\text{ m/s}$

EVALUATE: $\vec{E}$ is west and q is negative, so $\vec{F}$ is east and the electron speeds up.

(b) EXECUTE: $F_x = -|q|E = -(1.602\times10^{-19}\text{ C})(1.50\text{ N/C}) = -2.403\times10^{-19}\text{ N}$

$a_x = F_x/m = (-2.403\times10^{-19}\text{ N})/(1.673\times10^{-27}\text{ kg}) = -1.436\times10^8\text{ m/s}^2$

$v_{0x} = +1.90\times10^4\text{m/s},\ a_x = -1.436\times10^8\text{ m/s}^2,\ x - x_0 = 0.375\text{ m},\ v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 1.59\times10^4\text{ m/s}$

EVALUATE: $q > 0$ so $\vec{F}$ is west and the proton slows down.

21.42. **IDENTIFY:** Coulomb's law for a single point-charge gives the electric field.

(a) SET UP: Coulomb's law for a point-charge is $E = (1/4\pi\epsilon_0)q/r^2$.

EXECUTE: $E = (9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})/(1.50\times10^{-15}\ \text{m})^2 = 6.40\times10^{20}$ N/C

(b) Taking the ratio of the electric fields gives

$$E/E_{\text{plates}} = (6.40\times10^{20}\text{ N/C})/(1.00\times10^4\text{ N/C}) = 6.40\times10^{16}\text{ times as strong}$$

EVALUATE: The electric field within the nucleus is huge compared to typical laboratory fields!

21.43. **IDENTIFY:** Calculate the electric field due to each charge and find the vector sum of these two fields.

SET UP: At points on the x-axis only the x component of each field is nonzero. The electric field of a point charge points away from the charge if it is positive and toward it if it is negative.

EXECUTE: **(a)** Halfway between the two charges, $E = 0$.

(b) For $|x| < a$, $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right) = -\dfrac{4q}{4\pi\epsilon_0}\dfrac{ax}{(x^2-a^2)^2}$.

For $x > a$, $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right) = \dfrac{2q}{4\pi\epsilon_0}\dfrac{x^2+a^2}{(x^2-a^2)^2}$.

For $x < -a$, $E_x = \dfrac{-1}{4\pi\epsilon_0}\left(\dfrac{q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right) = -\dfrac{2q}{4\pi\epsilon_0}\dfrac{x^2+a^2}{(x^2-a^2)^2}$.

The graph of E_x versus x is sketched in Figure 21.43.

EVALUATE: The magnitude of the field approaches infinity at the location of one of the point charges.

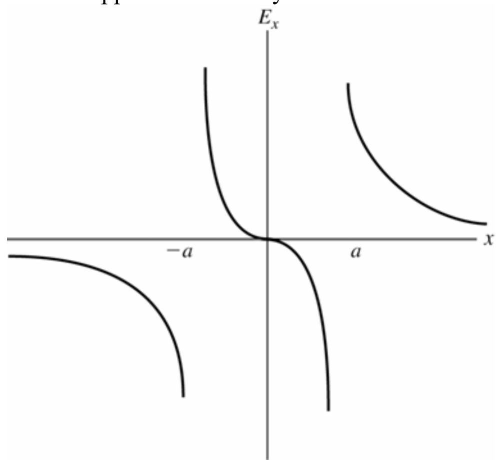


Figure 21.43

21.44. **IDENTIFY:** For a point charge, $E = k\frac{|q|}{r^2}$. For the net electric field to be zero, $\vec{E}_1$ and $\vec{E}_2$ must have equal magnitudes and opposite directions.

SET UP: Let $q_1 = +0.500$ nC and $q_2 = +8.00$ nC. $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: The two charges and the directions of their electric fields in three regions are shown in Figure 21.44. Only in region II are the two electric fields in opposite directions. Consider a point a distance x from q_1 so a distance $1.20\text{ m} - x$ from q_2. $E_1 = E_2$ gives $k\frac{0.500\text{ nC}}{x^2} = k\frac{8.00\text{ nC}}{(1.20 - x)^2}$. $16x^2 = (1.20\text{ m} - x)^2$. $4x = \pm(1.20\text{ m} - x)$ and $x = 0.24$ m is the positive solution. The electric field is zero at a point between the two charges, 0.24 m from the 0.500 nC charge and 0.96 m from the 8.00 nC charge.

EVALUATE: There is only one point along the line connecting the two charges where the net electric field is zero. This point is closer to the charge that has the smaller magnitude.

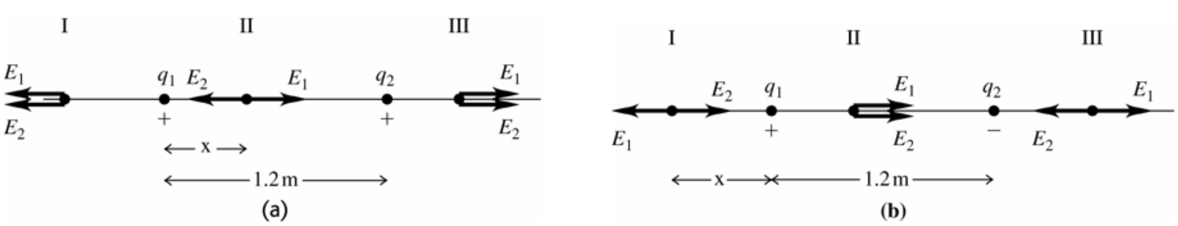


Figure 21.44

21.45. **IDENTIFY:** Eq.(21.7) gives the electric field of each point charge. Use the principle of superposition and add the electric field vectors. In part (b) use Eq.(21.3) to calculate the force, using the electric field calculated in part (a).

(a) SET UP: The placement of charges is sketched in Figure 21.45a.

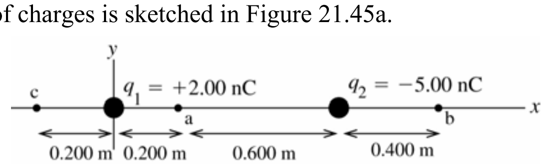


Figure 21.45a

The electric field of a point charge is directed away from the point charge if the charge is positive and toward the point charge if the charge is negative. The magnitude of the electric field is $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$, where r is the distance between the point where the field is calculated and the point charge.

(i) At point a the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.45b.

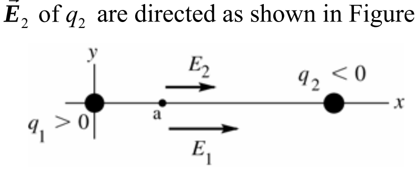


Figure 21.45b

EXECUTE: $E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 449.4\ \text{N/C}$

$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(0.600\ \text{m})^2} = 124.8\ \text{N/C}$

$E_{1x} = 449.4\ \text{N/C}, E_{1y} = 0$

$E_{2x} = 124.8\ \text{N/C}, E_{2y} = 0$

$E_x = E_{1x} + E_{2x} = +449.4\ \text{N/C} + 124.8\ \text{N/C} = +574.2\ \text{N/C}$

$E_y = E_{1y} + E_{2y} = 0$

The resultant field at point a has magnitude 574 N/C and is in the $+x$-direction.

(ii) **SET UP:** At point b the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.45c.

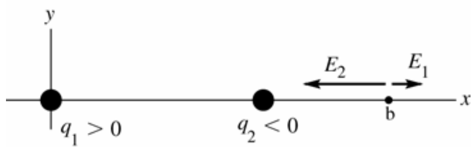


Figure 21.45c

EXECUTE: $E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(1.20\ \text{m})^2} = 12.5\ \text{N/C}$

$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{5.00\times10^{-9}\ \text{C}}{(0.400\ \text{m})^2} = 280.9\ \text{N/C}$

$E_{1x} = 12.5\ \text{N/C}, E_{1y} = 0$

$E_{2x} = -280.9\ \text{N/C}, E_{2y} = 0$

$E_x = E_{1x} + E_{2x} = +12.5\ \text{N/C} - 280.9\ \text{N/C} = -268.4\ \text{N/C}$

$E_y = E_{1y} + E_{2y} = 0$

The resultant field at point b has magnitude 268 N/C and is in the $-x$-direction.

(iii) **SET UP:** At point c the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.45d.

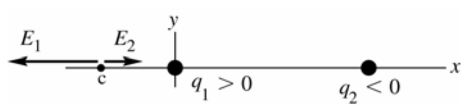


Figure 21.45d

EXECUTE: $E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 449.4\ \text{N/C}$

$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{5.00\times10^{-9}\ \text{C}}{(1.00\ \text{m})^2} = 44.9\ \text{N/C}$

$E_{1x} = -449.4\ \text{N/C}, E_{1y} = 0$

$E_{2x} = +44.9\ \text{N/C}, E_{2y} = 0$

$E_x = E_{1x} + E_{2x} = -449.4\ \text{N/C} + 44.9\ \text{N/C} = -404.5\ \text{N/C}$

$E_y = E_{1y} + E_{2y} = 0$

The resultant field at point b has magnitude 404 N/C and is in the $-x$-direction.

(b) SET UP: Since we have calculated $\vec{E}$ at each point the simplest way to get the force is to use $\vec{F} = -e\vec{E}$.

EXECUTE: (i) $F = (1.602\times10^{-19}\ \text{C})(574.2\ \text{N/C}) = 9.20\times10^{-17}\ \text{N}$, $-x$-direction

(ii) $F = (1.602\times10^{-19}\ \text{C})(268.4\ \text{N/C}) = 4.30\times10^{-17}\ \text{N}$, $+x$-direction

(iii) $F = (1.602\times10^{-19}\ \text{C})(404.5\ \text{N/C}) = 6.48\times10^{-17}\ \text{N}$, $+x$-direction

EVALUATE: The general rule for electric field direction is away from positive charge and toward negative charge. Whether the field is in the $+x$- or $-x$-direction depends on where the field point is relative to the charge that produces the field. In part (a) the field magnitudes were added because the fields were in the same direction and in (b) and (c) the field magnitudes were subtracted because the two fields were in opposite directions. In part (b) we could have used Coulomb's law to find the forces on the electron due to the two charges and then added these force vectors, but using the resultant electric field is much easier.

21.46. **IDENTIFY:** Apply Eq.(21.7) to calculate the field due to each charge and then require that the vector sum of the two fields to be zero.

SET UP: The field of each charge is directed toward the charge if it is negative and away from the charge if it is positive .

EXECUTE: The point where the two fields cancel each other will have to be closer to the negative charge, because it is smaller. Also, it can't be between the two charges, since the two fields would then act in the same direction. We could use Coulomb's law to calculate the actual values, but a simpler way is to note that the 8.00 nC charge is twice as large as the -4.00 nC charge. The zero point will therefore have to be a factor of $\sqrt{2}$ farther from the 8.00 nC charge for the two fields to have equal magnitude. Calling x the distance from the –4.00 nC charge: $1.20 + x = \sqrt{2}x$ and $x = 2.90\text{ m}$.

EVALUATE: This point is 4.10 m from the 8.00 nC charge. The two fields at this point are in opposite directions and have equal magnitudes.

21.47. **IDENTIFY:** $E = k\dfrac{|q|}{r^2}$. The net field is the vector sum of the fields due to each charge.

SET UP: The electric field of a negative charge is directed toward the charge. Label the charges q_1, q_2 and q_3, as shown in Figure 21.47a. This figure also shows additional distances and angles. The electric fields at point *P* are shown in Figure 21.47b. This figure also shows the *xy* coordinates we will use and the *x* and *y* components of the fields $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$.

EXECUTE: $E_1 = E_3 = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{5.00\times10^{-6}\text{ C}}{(0.100\text{ m})^2} = 4.49\times10^6\text{ N/C}$

$E_2 = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{2.00\times10^{-6}\text{ C}}{(0.0600\text{ m})^2} = 4.99\times10^6\text{ N/C}$

$E_y = E_{1y} + E_{2y} + E_{3y} = 0$ and $E_x = E_{1x} + E_{2x} + E_{3x} = E_2 + 2E_1\cos 53.1° = 1.04\times10^7\text{ N/C}$

$E = 1.04\times10^7\text{ N/C}$, toward the $-2.00\ \mu\text{C}$ charge.

EVALUATE: The *x*-components of the fields of all three charges are in the same direction.

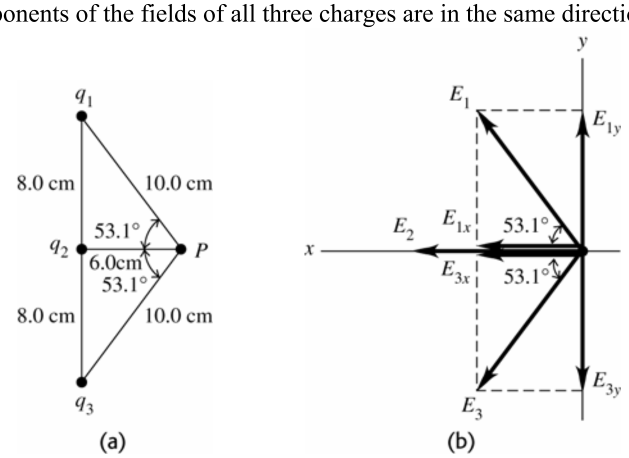


Figure 21.47

21.48. **IDENTIFY:** A positive and negative charge, of equal magnitude *q*, are on the *x*-axis, a distance *a* from the origin. Apply Eq.(21.7) to calculate the field due to each charge and then calculate the vector sum of these fields.

SET UP: $\vec{E}$ due to a point charge is directed away from the charge if it is positive and directed toward the charge if it is negative.

EXECUTE: **(a)** Halfway between the charges, both fields are in the $-x$-direction and $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{2q}{a^2}$, in the $-x$-direction .

(b) $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{-q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right)$ for $|x| < a$. $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{-q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right)$ for $x > a$.

$E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{-q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right)$ for $x < -a$. E_x is graphed in Figure 21.48.

EVALUATE: At points on the x axis and between the charges, E_x is in the $-x$-direction because the fields from both charges are in this direction. For $x < -a$ and $x > +a$, the fields from the two charges are in opposite directions and the field from the closer charge is larger in magnitude.

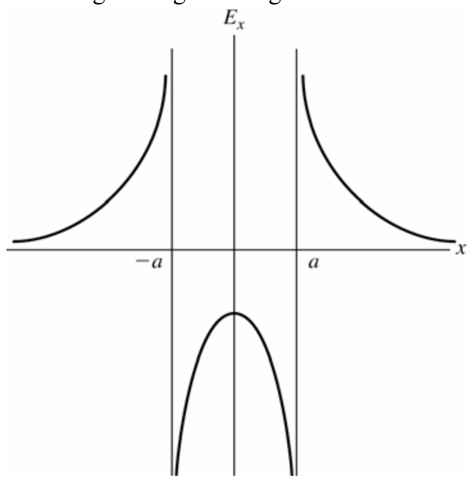


Figure 21.48

21.49. IDENTIFY: The electric field of a positive charge is directed radially outward from the charge and has magnitude $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$. The resultant electric field is the vector sum of the fields of the individual charges.

SET UP: The placement of the charges is shown in Figure 21.49a.

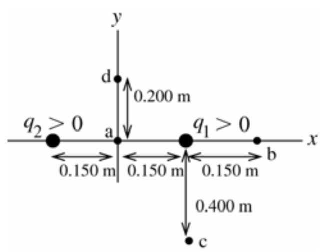
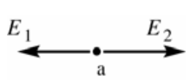


Figure 21.49a

EXECUTE: **(a)** The directions of the two fields are shown in Figure 21.49b.

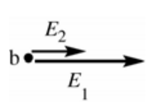


$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$ with $r = 0.150$ m.

$E = E_2 - E_1 = 0;\ E_x = 0,\ E_y = 0$

Figure 21.49b

(b) The two fields have the directions shown in Figure 21.49c.



$E = E_1 + E_2$, in the $+x$-direction

Figure 21.49c

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times 10^{-9}\ \text{C}}{(0.150\ \text{m})^2} = 2396.8\ \text{N/C}$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times 10^{-9}\ \text{C}}{(0.450\ \text{m})^2} = 266.3\ \text{N/C}$$

$$E = E_1 + E_2 = 2396.8\ \text{N/C} + 266.3\ \text{N/C} = 2660\ \text{N/C};\ E_x = +2260\ \text{N/C},\ E_y = 0$$

(c) The two fields have the directions shown in Figure 21.49d.

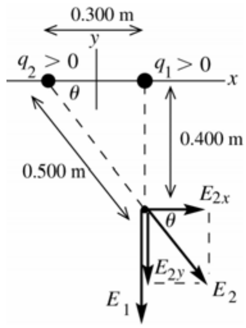


Figure 21.49d

$$\sin\theta = \frac{0.400\text{ m}}{0.500\text{ m}} = 0.800$$

$$\cos\theta = \frac{0.300\text{ m}}{0.500\text{ m}} = 0.600$$

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2}$$

$$E_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.400\text{ m})^2} = 337.1\text{ N/C}$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2}$$

$$E_2 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.500\text{ m})^2} = 215.7\text{ N/C}$$

$$E_{1x} = 0,\ E_{1y} = -E_1 = -337.1\text{ N/C}$$

$$E_{2x} = +E_2\cos\theta = +(215.7\text{ N/C})(0.600) = +129.4\text{ N/C}$$

$$E_{2y} = -E_2\sin\theta = -(215.7\text{ N/C})(0.800) = -172.6\text{ N/C}$$

$$E_x = E_{1x} + E_{2x} = +129\text{ N/C}$$

$$E_y = E_{1y} + E_{2y} = -337.1\text{ N/C } -172.6\text{ N/C } = -510\text{ N/C}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(129\text{ N/C})^2 + (-510\text{ N/C})^2} = 526\text{ N/C}$$

$\vec{E}$ and its components are shown in Figure 21.49e.

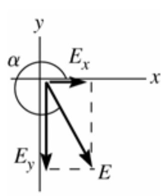


Figure 21.49e

$$\tan\alpha = \frac{E_y}{E_x}$$

$$\tan\alpha = \frac{-510\text{ N/C}}{+129\text{ N/C}} = -3.953$$

$\alpha = 284°\text{C}$, counterclockwise from $+x$-axis

(d) The two fields have the directions shown in Figure 21.49f.

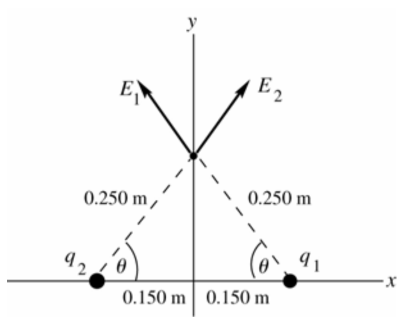


Figure 21.49f

$$\sin\theta = \frac{0.200\text{ m}}{0.250\text{ m}} = 0.800$$

The components of the two fields are shown in Figure 21.49g.

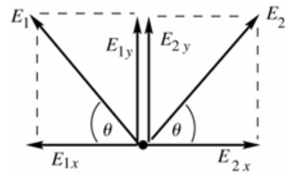


Figure 21.49g

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$$

$$E_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.250\ \text{m})^2}$$

$$E_1 = E_2 = 862.8\ \text{N/C}$$

$E_{1x} = -E_1\cos\theta,\ E_{2x} = +E_2\cos\theta$

$E_x = E_{1x} + E_{2x} = 0$

$E_{1y} = +E_1\sin\theta,\ E_{2y} = +E_2\sin\theta$

$E_y = E_{1y} + E_{2y} = 2E_{1y} = 2E_1\sin\theta = 2(862.8\ \text{N/C})(0.800) = 1380\ \text{N/C}$

$E = 1380$ N/C, in the $+y$-direction.

EVALUATE: Point a is symmetrically placed between identical charges, so symmetry tells us the electric field must be zero. Point b is to the right of both charges and both electric fields are in the $+x$-direction and the resultant field is in this direction. At point c both fields have a downward component and the field of q_2 has a component to the right, so the net $\vec{E}$ is in the 4th quadrant. At point d both fields have an upward component but by symmetry they have equal and opposite x-components so the net field is in the $+y$-direction. We can use this sort of reasoning to deduce the general direction of the net field before doing any calculations.

21.50. **IDENTIFY:** Apply Eq.(21.7) to calculate the field due to each charge and then calculate the vector sum of those fields.

SET UP: The fields due to q_1 and to q_2 are sketched in Figure 21.50.

EXECUTE: $\vec{E}_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{(6.00\times10^{-9}\ \text{C})}{(0.6\ \text{m})^2}(-\hat{i}) = -150\hat{i}$ N/C .

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}(4.00\times10^{-9}\ \text{C})\left(\frac{1}{(1.00\ \text{m})^2}(0.600)\hat{i} + \frac{1}{(1.00\ \text{m})^2}(0.800)\hat{j}\right) = (21.6\hat{i} + 28.8\hat{j})\,\text{N/C}\,.$$

$\vec{E} = \vec{E}_1 + \vec{E}_2 = (-128.4\ \text{N/C})\hat{i} + (28.8\ \text{N/C})\hat{j}$. $E = \sqrt{(128.4\ \text{N/C})^2 + (28.8\ \text{N/C})^2} = 131.6$ N/C at

$\theta = \tan^{-1}\left(\dfrac{28.8}{128.4}\right) = 12.6°$ above the $-x$ axis and therefore $196.2°$ counterclockwise from the $+x$ axis.

EVALUATE: $\vec{E}_1$ is directed toward q_1 because q_1 is negative and $\vec{E}_2$ is directed away from q_2 because q_2 is positive.

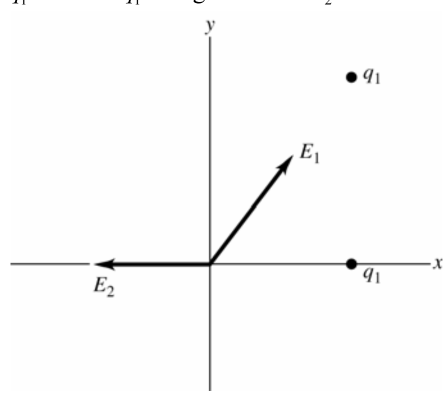


Figure 21.50

21.51. **IDENTIFY:** The resultant electric field is the vector sum of the field $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2.

SET UP: The placement of the charges is shown in Figure 21.51a.

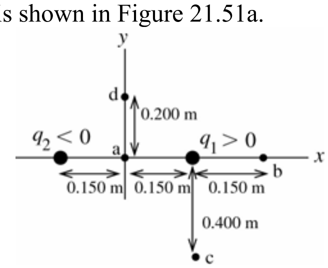


Figure 21.51a

EXECUTE: **(a)** The directions of the two fields are shown in Figure 21.51b.

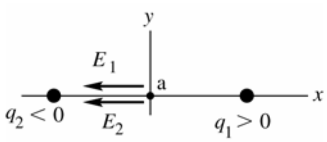


Figure 21.51b

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2}$$

$$E_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.150\ \text{m})^2}$$

$$E_1 = E_2 = 2397\ \text{N/C}$$

$E_{1x} = -2397$ N/C, $E_{1y} = 0$ $E_{2x} = -2397$ N/C, $E_{2y} = 0$

$E_x = E_{1x} + E_{2x} = 2(-2397\ \text{N/C}) = -4790\ \text{N/C}$

$E_y = E_{1y} + E_{2y} = 0$

The resultant electric field at point a in the sketch has magnitude 4790 N/C and is in the $-x$-direction.

(b) The directions of the two fields are shown in Figure 21.51c.

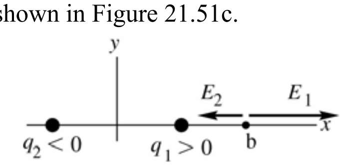


Figure 21.51c

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.150\ \text{m})^2} = 2397\ \text{N/C}$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.450\ \text{m})^2} = 266\ \text{N/C}$$

$E_{1x} = +2397$ N/C, $E_{1y} = 0$ $E_{2x} = -266$ N/C, $E_{2y} = 0$

$E_x = E_{1x} + E_{2x} = +2397\ \text{N/C} - 266\ \text{N/C} = +2130\ \text{N/C}$

$E_y = E_{1y} + E_{2y} = 0$

The resultant electric field at point b in the sketch has magnitude 2130 N/C and is in the $+x$-direction.

(c) The placement of the charges is shown in Figure 21.51d.

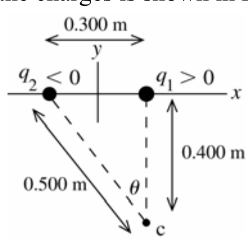


Figure 21.51d

$$\sin\theta = \frac{0.300\ \text{m}}{0.500\ \text{m}} = 0.600$$

$$\cos\theta = \frac{0.400\ \text{m}}{0.500\ \text{m}} = 0.800$$

The directions of the two fields are shown in Figure 21.51e.

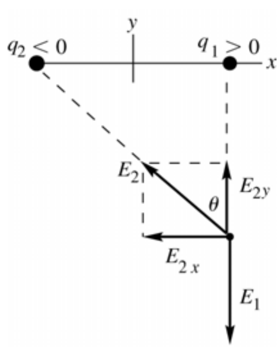


Figure 21.51e

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2}$$

$$E_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.400\ \text{m})^2}$$

$$E_1 = 337.0\ \text{N/C}$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2}$$

$$E_2 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.500\ \text{m})^2}$$

$$E_2 = 215.7\ \text{N/C}$$

$E_{1x} = 0$, $E_{1y} = -E_1 = -337.0\ \text{N/C}$

$E_{2x} = -E_2\sin\theta = -(215.7\ \text{N/C})(0.600) = -129.4\ \text{N/C}$

$E_{2y} = +E_2\cos\theta = +(215.7\ \text{N/C})(0.800) = +172.6\ \text{N/C}$

$E_x = E_{1x} + E_{2x} = -129 \text{ N/C}$

$E_y = E_{1y} + E_{2y} = -337.0 \text{ N/C } +172.6 \text{ N/C} = -164 \text{ N/C}$

$E = \sqrt{E_x^2 + E_y^2} = 209 \text{ N/C}$

The field $\vec{E}$ and its components are shown in Figure 21.51f.

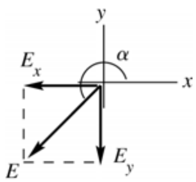


Figure 21.51f

$\tan\alpha = \dfrac{E_y}{E_x}$

$\tan\alpha = \dfrac{-164 \text{ N/C}}{-129 \text{ N/C}} = +1.271$

$\alpha = 232°$, counterclockwise from $+x$-axis

(d) The placement of the charges is shown in Figure 21.51g.

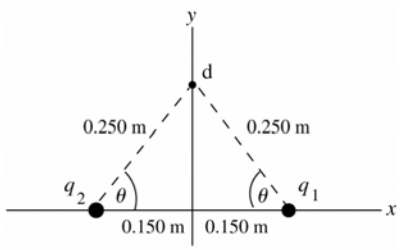


Figure 21.51g

$\sin\theta = \dfrac{0.200 \text{ m}}{0.250 \text{ m}} = 0.800$

$\cos\theta = \dfrac{0.150 \text{ m}}{0.250 \text{ m}} = 0.600$

The directions of the two fields are shown in Figure 21.51h.

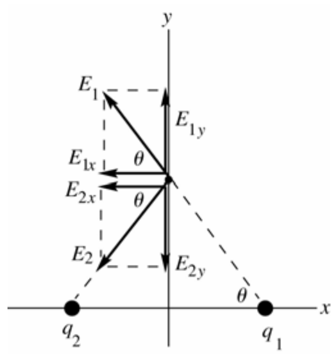


Figure 21.51h

$E_1 = E_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$

$E_1 = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)\dfrac{6.00 \times 10^{-9} \text{ C}}{(0.250 \text{ m})^2}$

$E_1 = 862.8 \text{ N/C}$

$\text{E}_2 = E_1 = 862.8 \text{ N/C}$

$E_{1x} = -E_1\cos\theta,\ E_{2x} = -E_2\cos\theta$

$E_x = E_{1x} + E_{2x} = -2(862.8 \text{ N/C})(0.600) = -1040 \text{ N/C}$

$E_{1y} = +E_1\sin\theta,\ E_{2y} = -E_2\sin\theta$

$E_y = E_{1y} + E_{2y} = 0$

$E = 1040$ N/C, in the $-x$-direction.

EVALUATE: The electric field produced by a charge is toward a negative charge and away from a positive charge. As in Exercise 21.45, we can use this rule to deduce the direction of the resultant field at each point before doing any calculations.

21.52. **IDENTIFY:** For a long straight wire, $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$.

SET UP: $\dfrac{1}{2\pi\epsilon_0} = 4.49 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.

EXECUTE: $r = \dfrac{1.5 \times 10^{-10} \text{ C/m}}{2\pi\epsilon_0 (2.50 \text{ N/C})} = 1.08 \text{ m}$

EVALUATE: For a point charge, E is proportional to $1/r^2$. For a long straight line of charge, E is proportional to $1/r$.

21.53. **IDENTIFY:** Apply Eq.(21.10) for the finite line of charge and $E = \dfrac{\lambda}{2\pi\epsilon_0}$ for the infinite line of charge.

SET UP: For the infinite line of positive charge, $\vec{E}$ is in the $+x$ direction.

EXECUTE: **(a)** For a line of charge of length $2a$ centered at the origin and lying along the y-axis, the electric field is given by Eq.(21.10): $\vec{E} = \dfrac{1}{2\pi\epsilon_0}\dfrac{\lambda}{x\sqrt{x^2/a^2+1}}\hat{i}$.

(b) For an infinite line of charge: $\vec{E} = \dfrac{\lambda}{2\pi\epsilon_0 x}\hat{i}$. Graphs of electric field versus position for both distributions of charge are shown in Figure 21.53.

EVALUATE: For small x, close to the line of charge, the field due to the finite line approaches that of the infinite line of charge. As x increases, the field due to the infinite line falls off more slowly and is larger than the field of the finite line.

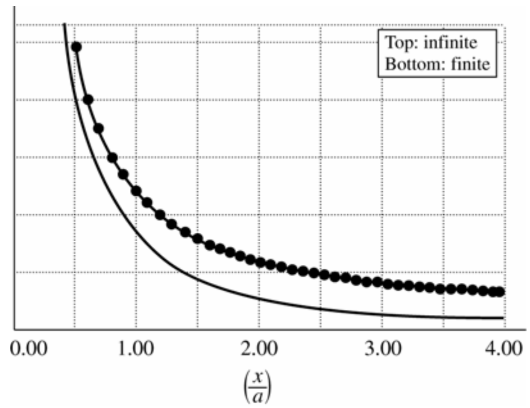


Figure 21.53

21.54. **(a) IDENTIFY:** The field is caused by a finite uniformly charged wire.

SET UP: The field for such a wire a distance x from its midpoint is

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x\sqrt{(x/a)^2+1}} = 2\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\lambda}{x\sqrt{(x/a)^2+1}} .$$

EXECUTE: $E = \dfrac{(18.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(175\times10^{-9}\ \text{C/m})}{(0.0600\ \text{m})\sqrt{\left(\dfrac{6.00\ \text{cm}}{4.25\ \text{cm}}\right)^2+1}}$ = 3.03 × 10^4 N/C, directed upward.

(b) IDENTIFY: The field is caused by a uniformly charged circular wire.

SET UP: The field for such a wire a distance x from its midpoint is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$. We first find the radius of the circle using $2\pi r = l$.

EXECUTE: Solving for r gives $r = l/2\pi = (8.50\ \text{cm})/2\pi = 1.353$ cm

The charge on this circle is $Q = \lambda l = (175\ \text{nC/m})(0.0850\ \text{m}) = 14.88$ nC

The electric field is

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{3/2}} = \frac{(9.00\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(14.88\times10^{-9}\,\text{C/m})(0.0600\,\text{m})}{\left[(0.0600\,\text{m})^2+(0.01353\,\text{m})^2\right]^{3/2}}$$

$$E = 3.45 \times 10^4\ \text{N/C, upward.}$$

EVALUATE: In both cases, the fields are of the same order of magnitude, but the values are different because the charge has been bent into different shapes.

21.55. **IDENTIFY:** For a ring of charge, the electric field is given by Eq. (21.8). $\vec{F} = q\vec{E}$. In part (b) use Newton's third law to relate the force on the ring to the force exerted by the ring.

SET UP: $Q = 0.125\times10^{-9}$ C, $a = 0.025$ m and $x = 0.400$ m .

EXECUTE: **(a)** $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}\hat{i} = (7.0\ \text{N/C})\hat{i}$.

(b) $\vec{F}_{\text{on ring}} = -\vec{F}_{\text{on q}} = -q\vec{E} = -(-2.50\times10^{-6}\text{C})\ (7.0\ \text{N/C})\hat{i} = (1.75\times10^{-5}\ \text{N})\hat{i}$

EVALUATE: Charges q and Q have opposite sign, so the force that q exerts on the ring is attractive.

21.56. IDENTIFY: We must use the appropriate electric field formula: a uniform disk in (a), a ring in (b) because all the charge is along the rim of the disk, and a point-charge in (c).

(a) SET UP: First find the surface charge density (Q/A), then use the formula for the field due to a disk of charge, $E_x = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{(R/x)^2 + 1}}\right]$.

EXECUTE: The surface charge density is $\sigma = \frac{Q}{A} = \frac{Q}{\pi r^2} = \frac{6.50 \times 10^{-9}\text{ C}}{\pi(0.0125\text{ m})^2} = 1.324 \times 10^{-5}\text{ C/m}^2$.

The electric field is

$$E_x = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{(R/x)^2 + 1}}\right] = \frac{1.324 \times 10^{-5}\text{ C/m}^2}{2(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\left[1 - \frac{1}{\sqrt{\left(\frac{1.25\text{ cm}}{2.00\text{ cm}}\right)^2 + 1}}\right]$$

$E_x = 1.14 \times 10^5$ N/C, toward the center of the disk.

(b) SET UP: For a ring of charge, the field is $E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{\left(x^2 + a^2\right)^{3/2}}$.

EXECUTE: Substituting into the electric field formula gives

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{\left(x^2 + a^2\right)^{3/2}} = \frac{(9.00 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(6.50 \times 10^{-9}\text{ C})(0.0200\text{ m})}{\left[(0.0200\text{ m})^2 + (0.0125\text{ m})^2\right]^{3/2}}$$

$E = 8.92 \times 10^4$ N/C, toward the center of the disk.

(c) SET UP: For a point charge, $E = \left(1/4\pi\epsilon_0\right)q/r^2$.

EXECUTE: $E = (9.00 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(6.50 \times 10^{-9}\text{ C})/(0.0200\text{ m})^2 = 1.46 \times 10^5$ N/C

(d) EVALUATE: With the ring, more of the charge is farther from P than with the disk. Also with the ring the component of the electric field parallel to the plane of the ring is greater than with the disk, and this component cancels. With the point charge in (c), all the field vectors add with no cancellation, and all the charge is closer to point P than in the other two cases.

21.57. IDENTIFY: By superposition we can add the electric fields from two parallel sheets of charge.

SET UP: The field due to each sheet of charge has magnitude $\sigma/2\epsilon_0$ and is directed toward a sheet of negative charge and away from a sheet of positive charge.

(a) The two fields are in opposite directions and $E = 0$.

(b) The two fields are in opposite directions and $E = 0$.

(c) The fields of both sheets are downward and $E = 2\frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$, directed downward.

EVALUATE: The field produced by an infinite sheet of charge is uniform, independent of distance from the sheet.

21.58. IDENTIFY and SET UP: The electric field produced by an infinite sheet of charge with charge density σ has magnitude $E = \frac{|\sigma|}{2\epsilon_0}$. The field is directed toward the sheet if it has negative charge and is away from the sheet if it has positive charge.

EXECUTE: **(a)** The field lines are sketched in Figure 21.58a.

(b) The field lines are sketched in Figure 21.58b.

EVALUATE: The spacing of the field lines indicates the strength of the field. In part (a) the two fields add between the sheets and subtract in the regions to the left of A and to the right of B. In part (b) the opposite is true.

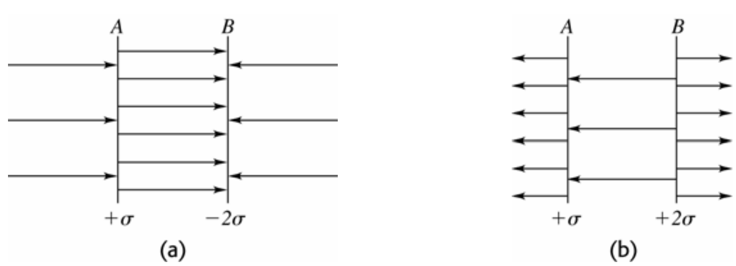


Figure 21.58

21.59. **IDENTIFY:** The force on the particle at any point is always tangent to the electric field line at that point.
SET UP: The instantaneous velocity determines the path of the particle.
EXECUTE: In Fig.21.29a the field lines are straight lines so the force is always in a straight line and velocity and acceleration are always in the same direction. The particle moves in a straight line along a field line, with increasing speed. In Fig.21.29b the field lines are curved. As the particle moves its velocity and acceleration are not in the same direction and the trajectory does not follow a field line.
EVALUATE: In two-dimensional motion the velocity is always tangent to the trajectory but the velocity is not always in the direction of the net force on the particle.

21.60. **IDENTIFY:** The field appears like that of a point charge a long way from the disk and an infinite sheet close to the disk's center. The field is symmetrical on the right and left.
SET UP: For a positive point charge, E is proportional to $1/r^2$ and is directed radially outward. For an infinite sheet of positive charge, the field is uniform and is directed away from the sheet.
EXECUTE: The field is sketched in Figure 21.60.
EVALUATE: Near the disk the field lines are parallel and equally spaced, which corresponds to a uniform field. Far from the disk the field lines are getting farther apart, corresponding to the $1/r^2$ dependence for a point charge.

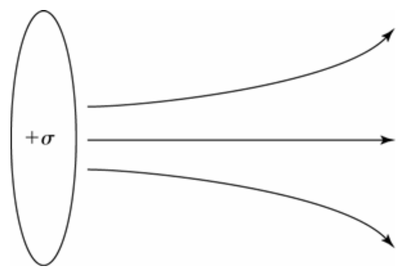


Figure 21.60

21.61. **IDENTIFY:** Use symmetry to deduce the nature of the field lines.
(a) SET UP: The only distinguishable direction is toward the line or away from the line, so the electric field lines are perpendicular to the line of charge, as shown in Figure 21.61a.

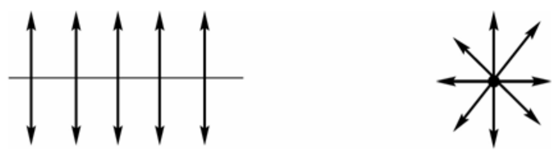


Figure 21.61a

(b) EXECUTE and EVALUATE: The magnitude of the electric field is inversely proportional to the spacing of the field lines. Consider a circle of radius r with the line of charge passing through the center, as shown in Figure 21.61b.

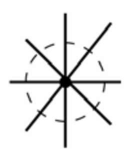


Figure 21.61b

The spacing of field lines is the same all around the circle, and in the direction perpendicular to the plane of the circle the lines are equally spaced, so E depends only on the distance r. The number of field lines passing out through the circle is independent of the radius of the circle, so the spacing of the field lines is proportional to the reciprocal of the circumference $2\pi r$ of the circle. Hence E is proportional to $1/r$.

21.62. **IDENTIFY:** Field lines are directed away from a positive charge and toward a negative charge. The density of field lines is proportional to the magnitude of the electric field.
SET UP: The field lines represent the resultant field at each point, the net field that is the vector sum of the fields due to each of the three charges.
EXECUTE: **(a)** Since field lines pass from positive charges and toward negative charges, we can deduce that the top charge is positive, middle is negative, and bottom is positive.
(b) The electric field is the smallest on the horizontal line through the middle charge, at two positions on either side where the field lines are least dense. Here the y-components of the field are cancelled between the positive charges and the negative charge cancels the x-component of the field from the two positive charges.
EVALUATE: Far from all three charges the field is the same as the field of a point charge equal to the algebraic sum of the three charges.

21.63. **(a) IDENTIFY** and **SET UP:** Use Eq.(21.14) to relate the dipole moment to the charge magnitude and the separation d of the two charges. The direction is from the negative charge toward the positive charge.
EXECUTE: $p = qd = (4.5\times10^{-9}\ \text{C})(3.1\times10^{-3}\ \text{m}) = 1.4\times10^{-11}\ \text{C}\cdot\text{m}$; The direction of $\vec{p}$ is from q_1 toward q_2.
(b) IDENTIFY and **SET UP:** Use Eq. (21.15) to relate the magnitudes of the torque and field.
EXECUTE: $\tau = pE\sin\phi$, with ϕ as defined in Figure 21.63, so

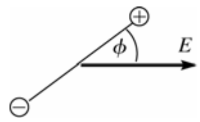


Figure 21.63

$$E = \frac{\tau}{p\sin\phi}$$

$$E = \frac{7.2\times10^{-9}\ \text{N}\cdot\text{m}}{(1.4\times10^{-11}\ \text{C}\cdot\text{m})\sin 36.9^\circ} = 860\ \text{N/C}$$

EVALUATE: Eq.(21.15) gives the torque about an axis through the center of the dipole. But the forces on the two charges form a couple (Problem 11.53) and the torque is the same for any axis parallel to this one. The force on each charge is $|q|E$ and the maximum moment arm for an axis at the center is $d/2$, so the maximum torque is $2(|q|E)(d/2) = 1.2\times10^{-8}\ \text{N}\cdot\text{m}$. The torque for the orientation of the dipole in the problem is less than this maximum.

21.64. **(a) IDENTIFY:** The potential energy is given by Eq.(21.17).
SET UP: $U(\phi) = -\vec{p}\cdot\vec{E} = -pE\cos\phi$, where ϕ is the angle between $\vec{p}$ and $\vec{E}$.
EXECUTE: parallel: $\phi = 0$ and $U(0^\circ) = -pE$

perpendicular: $\phi = 90^\circ$ and $U(90^\circ) = 0$

$\Delta U = U(90^\circ) - U(0^\circ) = pE = (5.0\times10^{-30}\ \text{C}\cdot\text{m})(1.6\times10^{6}\ \text{N/C}) = 8.0\times10^{-24}\ \text{J}.$

(b) $\frac{3}{2}kT = \Delta U$ so $T = \dfrac{2\Delta U}{3k} = \dfrac{2(8.0\times10^{-24}\ \text{J})}{3(1.381\times10^{-23}\ \text{J/K})} = 0.39\ \text{K}$

EVALUATE: Only at very low temperatures are the dipoles of the molecules aligned by a field of this strength. A much larger field would be required for alignment at room temperature.

21.65. **IDENTIFY:** Follow the procedure specified in part (a) of the problem.
SET UP: Use that $y >> d$.

EXECUTE: **(a)** $\dfrac{1}{(y-d/2)^2} - \dfrac{1}{(y+d/2)^2} = \dfrac{(y+d/2)^2-(y-d/2)^2}{(y^2-d^2/4)^2} = \dfrac{2yd}{(y^2-d^2/4)^2}$. This gives

$E_y = \dfrac{q}{4\pi\epsilon_0}\dfrac{2yd}{(y^2-d^2/4)^2} = \dfrac{qd}{2\pi\epsilon_0}\dfrac{y}{(y^2-d^2/4)^2}$. Since $y^2 >> d^2/4$, $E_y \approx \dfrac{p}{2\pi\epsilon_0 y^3}$.

(b) For points on the $-y$-axis, $\vec{E}_-$ is in the $+y$ direction and $\vec{E}_+$ is in the $-y$ direction. The field point is closer to $-q$, so the net field is upward. A similar derivation gives $E_y \approx \dfrac{p}{2\pi\epsilon_0 y^3}$. E_y has the same magnitude and direction at points where $y >> d$ as where $y << -d$.

EVALUATE: E falls off like $1/r^3$ for a dipole, which is faster than the $1/r^2$ for a point charge. The total charge of the dipole is zero.

21.66. **IDENTIFY:** Calculate the electric field due to the dipole and then apply $\vec{F} = q\vec{E}$.

SET UP: From Example 21.15, $E_{\text{dipole}}(x) = \dfrac{p}{2\pi\epsilon_0 x^3}$.

EXECUTE: $E_{\text{dipole}} = \dfrac{6.17\times10^{-30}\ \text{C}\cdot\text{m}}{2\pi\epsilon_0(3.0\times10^{-9}\ \text{m})^3} = 4.11\times10^{6}\ \text{N/C}$. The electric force is $F = qE =$ $(1.60\times10^{-19}\ \text{C})(4.11\times10^{6}\ \text{N/C}) = 6.58\times10^{-13}\ \text{N}$ and is toward the water molecule (negative x-direction).

EVALUATE: $\vec{E}_{\text{dipole}}$ is in the direction of $\vec{p}$, so is in the $+x$ direction. The charge q of the ion is negative, so $\vec{F}$ is directed opposite to $\vec{E}$ and is therefore in the $-x$ direction.

21.67. **IDENTIFY:** Like charges repel and unlike charges attract. The force increases as the distance between the charges decreases.
SET UP: The forces on the dipole that is between the slanted dipoles are sketched in Figure 21.67a.

EXECUTE: The forces are attractive because the + and − charges of the two dipoles are closest. The forces are toward the slanted dipoles so have a net upward component. In Figure 21.67b, adjacent dipoles charges of opposite sign are closer than charges of the same sign so the attractive forces are larger than the repulsive forces and the dipoles attract.

EVALUATE: Each dipole has zero net charge, but because of the charge separation there is a non-zero force between dipoles.

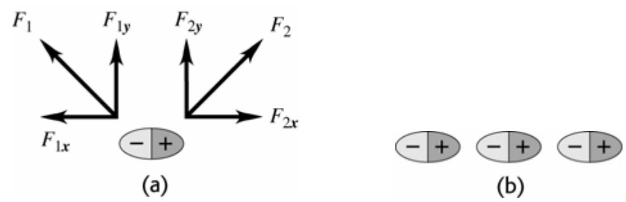


Figure 21.67

21.68. IDENTIFY: Find the vector sum of the fields due to each charge in the dipole.

SET UP: A point on the x-axis with coordinate x is a distance $r = \sqrt{(d/2)^2 + x^2}$ from each charge.

EXECUTE: **(a)** The magnitude of the field the due to each charge is $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(d/2)^2 + x^2}\right)$,

where d is the distance between the two charges. The x-components of the forces due to the two charges are equal and oppositely directed and so cancel each other. The two fields have equal y-components,

so $E = 2E_y = \frac{2q}{4\pi\epsilon_0}\left(\frac{1}{(d/2)^2 + x^2}\right)\sin\theta$, where θ is the angle below the x-axis for both fields. $\sin\theta = \frac{d/2}{\sqrt{(d/2)^2 + x^2}}$

and $E_{\text{dipole}} = \left(\frac{2q}{4\pi\epsilon_0}\right)\left(\frac{1}{(d/2)^2 + x^2}\right)\left(\frac{d/2}{\sqrt{(d/2)^2 + x^2}}\right) = \frac{qd}{4\pi\epsilon_0((d/2)^2 + x^2)^{3/2}}$. The field is the $-y$ direction.

(b) At large x, $x^2 >> (d/2)^2$, so the expression in part (a) reduces to the approximation $E_{\text{dipole}} \approx \frac{qd}{4\pi\epsilon_0 x^3}$.

EVALUATE: Example 21.15 shows that at points on the $+y$ axis far from the dipole, $E_{\text{dipole}} \approx \frac{qd}{2\pi\epsilon_0 y^3}$. The expression in part (b) for points on the x axis has a similar form.

21.69. IDENTIFY: The torque on a dipole in an electric field is given by $\vec{\tau} = \vec{p} \times \vec{E}$.

SET UP: $\tau = pE\sin\phi$, where ϕ is the angle between the direction of $\vec{p}$ and the direction of $\vec{E}$.

EXECUTE: **(a)** The torque is zero when $\vec{p}$ is aligned either in the *same* direction as $\vec{E}$ or in the *opposite* direction, as shown in Figure 21.69a.

(b) The stable orientation is when $\vec{p}$ is aligned in the *same* direction as $\vec{E}$. In this case a small rotation of the dipole results in a torque directed so as to bring $\vec{p}$ back into alignment with $\vec{E}$. When $\vec{p}$ is directed opposite to $\vec{E}$, a small displacement results in a torque that takes $\vec{p}$ farther from alignment with $\vec{E}$.

(c) Field lines for E_{dipole} in the stable orientation are sketched in Figure 21.69b.

EVALUATE: The field of the dipole is directed from the + charge toward the − charge.

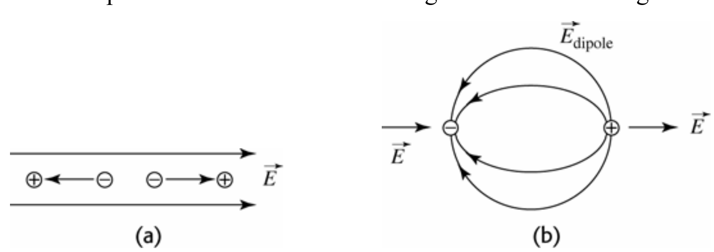


Figure 21.69

21.70. IDENTIFY: The plates produce a uniform electric field in the space between them. This field exerts torque on a dipole and gives it potential energy.

SET UP: The electric field between the plates is given by $E = \sigma/\epsilon_0$, and the dipole moment is $p = ed$. The potential energy of the dipole due to the field is $U = -\vec{p}\cdot\vec{E} = -pE\cos\phi$, and the torque the field exerts on it is $\tau = pE\sin\phi$.

EXECUTE: **(a)** The potential energy, $U = -\vec{p}\cdot\vec{E} = -pE\cos\phi$, is a maximum when $\phi = 180°$. The field between the plates is $E = \sigma/\epsilon_0$, giving

$$U_{max} = (1.60\times10^{-19}\text{ C})(220\times10^{-9}\text{ m})(125\times10^{-6}\text{ C/m}^2)/(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = 4.97\times10^{-19}\text{ J}$$

The orientation is parallel to the electric field (perpendicular to the plates) with the positive charge of the dipole toward the positive plate.
(b) The torque, $\tau = pE\sin\phi$, is a maximum when $\phi = 90°$ or $270°$. In this case

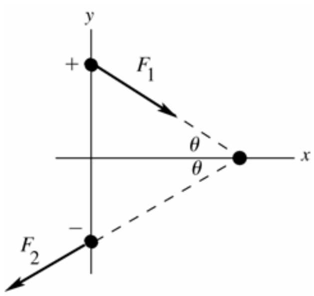
$$\tau_{max} = pE = p\sigma/\epsilon_0 = ed\sigma/\epsilon_0$$

$$\tau_{max} = (1.60\times10^{-19}\text{ C})(220\times10^{-9}\text{ m})(125\times10^{-6}\text{ C/m}^2)/(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)$$

$$\tau_{max} = 4.97\times10^{-19}\text{ N}\cdot\text{m}$$

The dipole is oriented perpendicular to the electric field (parallel to the plates).
(c) $F = 0$.
EVALUATE: When the potential energy is a maximum, the torque is zero. In both cases, the net force on the dipole is zero because the forces on the charges are equal but opposite (which would not be true in a nonuniform electric field).

21.71. **(a) IDENTIFY:** Use Coulomb's law to calculate each force and then add them as vectors to obtain the net force. Torque is force times moment arm.
SET UP: The two forces on each charge in the dipole are shown in Figure 21.71a.



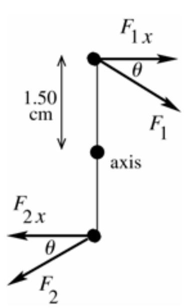
$\sin\theta = 1.50/2.00$ so $\theta = 48.6°$
Opposite charges attract and like charges repel.
$F_x = F_{1x} + F_{2x} = 0$

Figure 21.71a

EXECUTE: $F_1 = k\dfrac{|qq'|}{r^2} = k\dfrac{(5.00\times10^{-6}\text{ C})(10.0\times10^{-6}\text{ C})}{(0.0200\text{ m})^2} = 1.124\times10^3\text{ N}$

$F_{1y} = -F_1\sin\theta = -842.6\text{ N}$

$F_{2y} = -842.6$ N so $F_y = F_{1y} + F_{2y} = -1680$ N (in the direction from the $+5.00\text{-}\mu\text{C}$ charge toward the $-5.00\text{-}\mu\text{C}$ charge).
EVALUATE: The x-components cancel and the y-components add.
(b) SET UP: Refer to Figure 21.71b.



The y-components have zero moment arm and therefore zero torque.
F_{1x} and F_{2x} both produce clockwise torques.

Figure 21.71b

EXECUTE: $F_{1X} = F_1\cos\theta = 743.1\text{ N}$

$\tau = 2(F_{1x})(0.0150\text{ m}) = 22.3\text{ N}\cdot\text{m}$, clockwise
EVALUATE: The electric field produced by the $-10.00\mu\text{C}$ charge is not uniform so Eq. (21.15) does not apply.

21.72. **IDENTIFY:** Apply $F = k\dfrac{|qq'|}{r^2}$ for each pair of charges and find the vector sum of the forces that q_1 and q_2 exert on q_3.

SET UP: Like charges repel and unlike charges attract. The three charges and the forces on q_3 are shown in Figure 21.72.

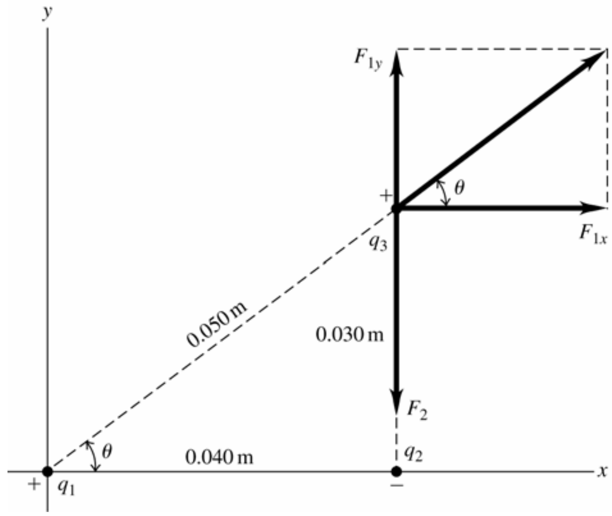


Figure 21.72

EXECUTE: **(a)** $F_1 = k\dfrac{|q_1 q_3|}{r_1^2} = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(5.00\times 10^{-9}\ \text{C})(6.00\times 10^{-9}\ \text{C})}{(0.0500\ \text{m})^2} = 1.079\times 10^{-4}\ \text{C}$.

$\theta = 36.9°$. $F_{1x} = +F_1\cos\theta = 8.63\times 10^{-5}\ \text{N}$. $F_{1y} = +F_1\sin\theta = 6.48\times 10^{-5}\ \text{N}$.

$F_2 = k\dfrac{|q_2 q_3|}{r_2^2} = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2.00\times 10^{-9}\ \text{C})(6.00\times 10^{-9}\ \text{C})}{(0.0300\ \text{m})^2} = 1.20\times 10^{-4}\ \text{C}$.

$F_{2x} = 0$, $F_{2y} = -F_2 = -1.20\times 10^{-4}\ \text{N}$. $F_x = F_{1x} + F_{2x} = 8.63\times 10^{-5}\ \text{N}$.

$F_y = F_{1y} + F_{2y} = 6.48\times 10^{-5}\ \text{N} + (-1.20\times 10^{-4}\ \text{N}) = -5.52\times 10^{-5}\ \text{N}$.

(b) $F = \sqrt{F_x^2 + F_y^2} = 1.02\times 10^{-4}\ \text{N}$. $\tan\phi = \left|\dfrac{F_y}{F_x}\right| = 0.640$. $\phi = 32.6°$, below the $+x$ axis.

EVALUATE: The individual forces on q_3 are computed from Coulomb's law and then added as vectors, using components.

21.73. **(a) IDENTIFY:** Use Coulomb's law to calculate the force exerted by each Q on q and add these forces as vectors to find the resultant force. Make the approximation $x >> a$ and compare the net force to $F = -kx$ to deduce k and then $f = (1/2\pi)\sqrt{k/m}$.

SET UP: The placement of the charges is shown in Figure 21.73a.

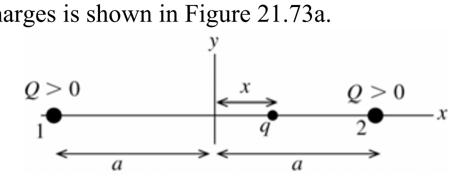


Figure 21.73a

EXECUTE: Find the net force on q.

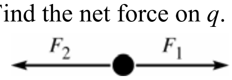


Figure 21.73b

$F_x = F_{1x} + F_{2x}$ and $F_{1x} = +F_1,\ F_{2x} = -F_2$

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{qQ}{(a+x)^2},\quad F_2 = \frac{1}{4\pi\epsilon_0}\frac{qQ}{(a-x)^2}$$

$$F_x = F_1 - F_2 = \frac{qQ}{4\pi\epsilon_0}\left[\frac{1}{(a+x)^2} - \frac{1}{(a-x)^2}\right]$$

$$F_x = \frac{qQ}{4\pi\epsilon_0 a^2}\left[+\left(1+\frac{x}{a}\right)^{-2} - \left(1-\frac{x}{a}\right)^{-2}\right]$$

Since $x << a$ we can use the binomial expansion for $(1-x/a)^{-2}$ and $(1+x/a)^{-2}$ and keep only the first two terms: $(1+z)^n \approx 1+nz$. For $(1-x/a)^{-2}$, $z=-x/a$ and $n=-2$ so $(1-x/a)^{-2} \approx 1+2x/a$. For $(1+x/a)^{-2}$, $z=+x/a$ and $n=-2$ so $(1+x/a)^{-2} \approx 1-2x/a$. Then $F \approx \frac{qQ}{4\pi\epsilon_0 a^2}\left[\left(1-\frac{2x}{a}\right)-\left(1+\frac{2x}{a}\right)\right] = -\left(\frac{qQ}{\pi\epsilon_0 a^3}\right)x$. For simple harmonic motion $F=-kx$ and the frequency of oscillation is $f=(1/2\pi)\sqrt{k/m}$. The net force here is of this form, with $k = qQ/\pi\epsilon_0 a^3$. Thus $f = \frac{1}{2\pi}\sqrt{\frac{qQ}{\pi\epsilon_0 m a^3}}$.

(b) The forces and their components are shown in Figure 21.73c.

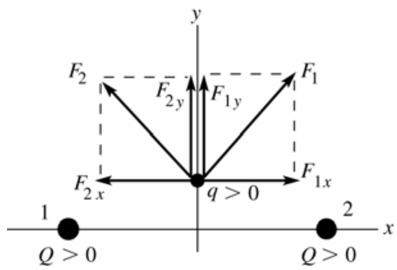


Figure 21.73c

The x-components of the forces exerted by the two charges cancel, the y-components add, and the net force is in the $+y$-direction when $y > 0$ and in the $-y$-direction when $y < 0$. The charge moves away from the origin on the y-axis and never returns.

EVALUATE: The directions of the forces and of the net force depend on where q is located relative to the other two charges. In part (a), $F = 0$ at $x = 0$ and when the charge q is displaced in the $+x$- or $-x$-direction the net force is a restoring force, directed to return q to $x = 0$. The charge oscillates back and forth, similar to a mass on a spring.

21.74. **IDENTIFY:** Apply $\sum F_x = 0$ and $\sum F_y = 0$ to one of the spheres.

SET UP: The free-body diagram is sketched in Figure 21.74. F_e is the repulsive Coulomb force between the spheres. For small θ, $\sin\theta \approx \tan\theta$.

EXECUTE: $\sum F_x = T\sin\theta - F_e = 0$ and $\sum F_y = T\cos\theta - mg = 0$. So $\frac{mg\sin\theta}{\cos\theta} = F_e = \frac{kq^2}{d^2}$. But $\tan\theta \approx \sin\theta = \frac{d}{2L}$, so $d^3 = \frac{2kq^2L}{mg}$ and $d = \left(\frac{q^2L}{2\pi\epsilon_0 mg}\right)^{1/3}$.

EVALUATE: d increases when q increases.

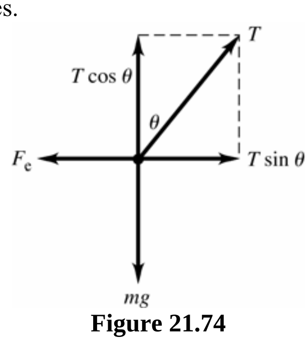


Figure 21.74

21.75. **IDENTIFY:** Use Coulomb's law for the force that one sphere exerts on the other and apply the 1st condition of equilibrium to one of the spheres.

(a) SET UP: The placement of the spheres is sketched in Figure 21.75a.

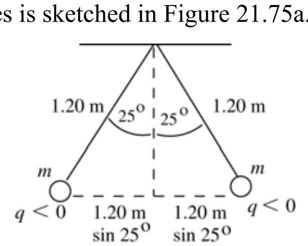


Figure 21.75a

The free-body diagrams for each sphere are given in Figure 21.75b.

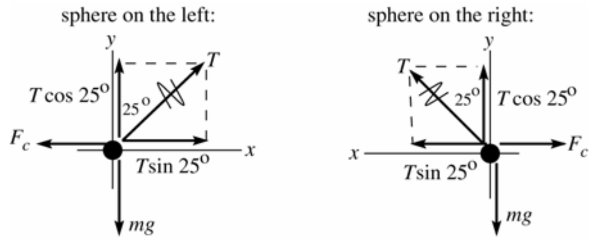


Figure 21.75b

F_c is the repulsive Coulomb force exerted by one sphere on the other.

(b) EXECUTE: From either force diagram in part (a): $\sum F_y = ma_y$

$T\cos 25.0° - mg = 0$ and $T = \dfrac{mg}{\cos 25.0°}$

$\sum F_x = ma_x$

$T\sin 25.0° - F_c = 0$ and $F_c = T\sin 25.0°$

Use the first equation to eliminate T in the second: $F_c = (mg/\cos 25.0°)(\sin 25.0°) = mg\tan 25.0°$

$$F_c = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{[2(1.20\text{ m})\sin 25.0°]^2}$$

Combine this with $F_c = mg\tan 25.0°$ and get $mg\tan 25.0° = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{[2(1.20\text{ m})\sin 25.0°]^2}$

$$q = (2.40\text{ m})\sin 25.0°\sqrt{\frac{mg\tan 25.0°}{(1/4\pi\epsilon_0)}}$$

$$q = (2.40\text{ m})\sin 25.0°\sqrt{\frac{(15.0\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)\tan 25.0°}{8.988\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = 2.80\times 10^{-6}\text{ C}$$

(c) The separation between the two spheres is given by $2L\sin\theta$. $q = 2.80\,\mu\text{C}$ as found in part (b).

$F_c = (1/4\pi\epsilon_0)q^2/(2L\sin\theta)^2$ and $F_c = mg\tan\theta$. Thus $(1/4\pi\epsilon_0)q^2/(2L\sin\theta)^2 = mg\tan\theta$.

$$(\sin\theta)^2\tan\theta = \frac{1}{4\pi\epsilon_0}\frac{q^2}{4L^2mg} = (8.988\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(2.80\times 10^{-6}\text{ C})^2}{4(0.600\text{ m})^2(15.0\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)} = 0.3328.$$

Solve this equation by trial and error. This will go quicker if we can make a good estimate of the value of θ that solves the equation. For θ small, $\tan\theta \approx \sin\theta$. With this approximation the equation becomes $\sin^3\theta = 0.3328$ and $\sin\theta = 0.6930$, so $\theta = 43.9°$. Now refine this guess:

θ	$\sin^2\theta\tan\theta$	
45.0°	0.5000	
40.0°	0.3467	
39.6°	0.3361	
39.5°	0.3335	
39.4°	0.3309	so $\theta = 39.5°$

EVALUATE: The expression in part (c) says $\theta \to 0$ as $L \to \infty$ and $\theta \to 90°$ as $L \to 0$. When L is decreased from the value in part (a), θ increases.

21.76. IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to each sphere.

SET UP: **(a)** Free body diagrams are given in Figure 21.76. F_e is the repulsive electric force that one sphere exerts on the other.

EXECUTE: **(b)** $T = mg/\cos 20° = 0.0834$ N, so $F_e = T\sin 20° = 0.0285\text{ N} = \dfrac{kq_1q_2}{r_1^2}$. (Note:

$r_1 = 2(0.500\text{ m})\sin 20° = 0.342$ m.)

(c) From part (b), $q_1q_2 = 3.71\times 10^{-13}\text{ C}^2$.

(d) The charges on the spheres are made equal by connecting them with a wire, but we still have $F_e = mg\tan\theta = 0.0453\ \text{N} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{r_2^2}$, where $Q = \dfrac{q_1+q_2}{2}$. But the separation r_2 is known:

$r_2 = 2(0.500\ \text{m})\sin 30° = 0.500\ \text{m}$. Hence: $Q = \dfrac{q_1+q_2}{2} = \sqrt{4\pi\epsilon_0 F_e r_2^2} = 1.12\times10^{-6}\ \text{C}$. This equation, along with that from part (c), gives us two equations in q_1 and q_2: $q_1 + q_2 = 2.24\times10^{-6}\ \text{C}$ and $q_1 q_2 = 3.71\times10^{-13}\ \text{C}^2$. By elimination, substitution and after solving the resulting quadratic equation, we find: $q_1 = 2.06\times10^{-6}\ \text{C}$ and $q_2 = 1.80\times10^{-7}\ \text{C}$.

EVALUATE: After the spheres are connected by the wire, the charge on sphere 1 decreases and the charge on sphere 2 increases. The product of the charges on the sphere increases and the thread makes a larger angle with the vertical.

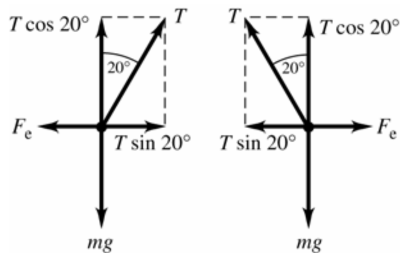


Figure 21.76

21.77. **IDENTIFY** and **SET UP:** Use Avogadro's number to find the number of Na^+ and Cl^- ions and the total positive and negative charge. Use Coulomb's law to calculate the electric force and $\vec{F} = m\vec{a}$ to calculate the acceleration.

(a) EXECUTE: The number of Na^+ ions in 0.100 mol of NaCl is $N = nN_A$. The charge of one ion is $+e$, so the total charge is $q_1 = nN_A e =$ $(0.100\ \text{mol})(6.022\times10^{23}\ \text{ions/mol})(1.602\times10^{-19}\ \text{C/ion}) = 9.647\times10^3\ \text{C}$

There are the same number of Cl^- ions and each has charge $-e$, so $q_2 = -9.647\times10^3\ \text{C}$.

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(9.647\times10^3\ \text{C})^2}{(0.0200\ \text{m})^2} = 2.09\times10^{21}\ \text{N}$$

(b) $a = F/m$. Need the mass of 0.100 mol of Cl^- ions. For Cl, $M = 35.453\times10^{-3}$ kg/mol, so

$m = (0.100\ \text{mol})(35.453\times10^{-3}\ \text{kg/mol}) = 35.45\times10^{-4}\ \text{kg}$. Then $a = \dfrac{F}{m} = \dfrac{2.09\times10^{21}\ \text{N}}{35.45\times10^{-4}\ \text{kg}} = 5.90\times10^{23}\ \text{m/s}^2$.

(c) EVALUATE: Is is not reasonable to have such a huge force. The net charges of objects are rarely larger than 1 μC; a charge of 10^4 C is immense. A small amount of material contains huge amounts of positive and negative charges.

21.78. **IDENTIFY:** For the acceleration (and hence the force) on Q to be upward, as indicated, the forces due to q_1 and q_2 must have equal strengths, so q_1 and q_2 must have equal magnitudes. Furthermore, for the force to be upward, q_1 must be positive and q_2 must be negative.

SET UP: Since we know the acceleration of Q, Newton's second law gives us the magnitude of the force on it. We can then add the force components using $F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$. The electrical force on Q is given by Coulomb's law, $F_{Qq_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qq_1}{r^2}$ (for q_1) and likewise for q_2.

EXECUTE: First find the net force: $F = ma = (0.00500\ \text{kg})(324\ \text{m/s}^2) = 1.62$ N. Now add the force components, calling θ the angle between the line connecting q_1 and q_2 and the line connecting q_1 and Q.

$F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$ and $F_{Qq_1} = \dfrac{F}{2\cos\theta} = \dfrac{1.62\ \text{N}}{2\left(\dfrac{2.25\ \text{cm}}{3.00\ \text{cm}}\right)} = 1.08$ N. Now find the charges by solving for q_1 in Coulomb's law and use the fact that q_1 and q_2 have equal magnitudes but opposite signs.

$$F_{Qq_1} = \frac{1}{4\pi\epsilon_0}\frac{Qq_1}{r^2} \text{ and } q_1 = \frac{r^2F_{Qq_1}}{\dfrac{1}{4\pi\epsilon_0}Q} = \frac{(0.0300\ \text{m})^2(1.08\ \text{N})}{(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.75\times10^{-6}\ \text{C})} = 6.17\times10^{-8}\ \text{C}.$$

$q_2 = -q_1 = -6.17\times10^{-8}\ \text{C}$.

EVALUATE: Simple reasoning allows us first to conclude that q_1 and q_2 must have equal magnitudes but opposite signs, which makes the equations much easier to set up than if we had tried to solve the problem in the general case. As Q accelerates and hence moves upward, the magnitude of the acceleration vector will change in a complicated way.

21.79. IDENTIFY: Use Coulomb's law to calculate the forces between pairs of charges and sum these forces as vectors to find the net charge.

(a) SET UP: The forces are sketched in Figure 21.79a.

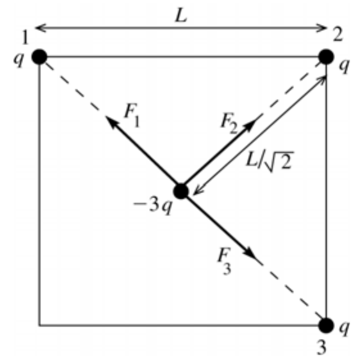


Figure 21.79a

EXECUTE: $\vec{F}_1 + \vec{F}_3 = \mathbf{0}$, so the net force is $\vec{F} = \vec{F}_2$.

$F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q(3q)}{(L/\sqrt{2})^2} = \dfrac{6q^2}{4\pi\epsilon_0 L^2}$, away from the vacant corner.

(b) SET UP: The forces are sketched in Figure 21.79b.

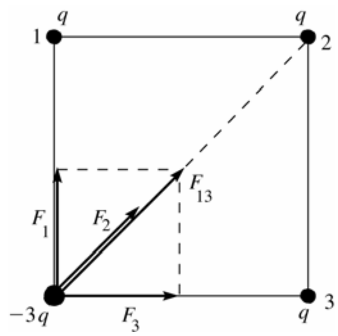


Figure 21.79b

EXECUTE: $F_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q(3q)}{\left(\sqrt{2}L\right)^2} = \dfrac{3q^2}{4\pi\epsilon_0\left(2L^2\right)}$

$F_1 = F_3 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q(3q)}{L^2} = \dfrac{3q^2}{4\pi\epsilon_0 L^2}$

The vector sum of F_1 and F_3 is $F_{13} = \sqrt{F_1^2 + F_3^2}$.

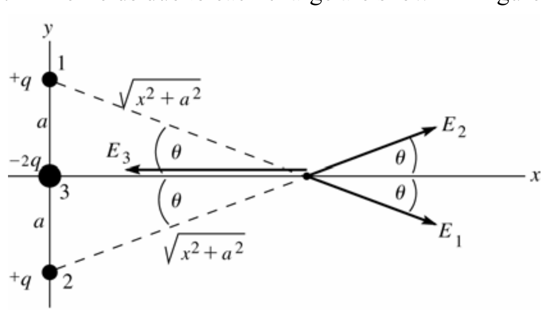
$F_{13} = \sqrt{2}F_1 = \dfrac{3\sqrt{2}q^2}{4\pi\epsilon_0 L^2}$; $\vec{F}_{13}$ and $\vec{F}_2$ are in the same direction.

$F = F_{13} + F_2 = \dfrac{3q^2}{4\pi\epsilon_0 L^2}\left(\sqrt{2} + \dfrac{1}{2}\right)$, and is directed toward the center of the square.

EVALUATE: By symmetry the net force is along the diagonal of the square. The net force is only slightly larger when the $-3q$ charge is at the center. Here it is closer to the charge at point 2 but the other two forces cancel.

21.80. IDENTIFY: Use Eq.(21.7) for the electric field produced by each point charge. Apply the principle of superposition and add the fields as vectors to find the net field.

(a) SET UP: The fields due to each charge are shown in Figure 21.80a.



$\cos\theta = \dfrac{x}{\sqrt{x^2 + a^2}}$

Figure 21.80a

EXECUTE: The components of the fields are given in Figure 21.80b.

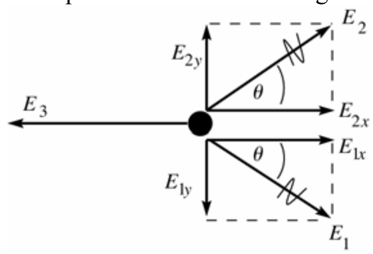


Figure 21.80b

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{a^2+x^2}\right)$$

$$E_3 = \frac{1}{4\pi\epsilon_0}\left(\frac{2q}{x^2}\right)$$

$E_{1y} = -E_1\sin\theta,\ E_{2y} = +E_2\sin\theta$ so $E_y = E_{1y} + E_{2y} = 0.$

$$E_{1x} = E_{2x} = +E_1\cos\theta = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{a^2+x^2}\right)\left(\frac{x}{\sqrt{x^2+a^2}}\right),\ E_{3x} = -E_3$$

$$E_x = E_{1x} + E_{2x} + E_{3x} = 2\left(\frac{1}{4\pi\epsilon_0}\left(\frac{q}{a^2+x^2}\right)\left(\frac{x}{\sqrt{x^2+a^2}}\right)\right) - \frac{2q}{4\pi\epsilon_0 x^2}$$

$$E_x = -\frac{2q}{4\pi\epsilon_0}\left(\frac{1}{x^2} - \frac{x}{\left(a^2+x^2\right)^{3/2}}\right) = -\frac{2q}{4\pi\epsilon_0 x^2}\left(1 - \frac{1}{\left(1+a^2/x^2\right)^{3/2}}\right)$$

Thus $E = \dfrac{2q}{4\pi\epsilon_0 x^2}\left(1 - \dfrac{1}{\left(1+a^2/x^2\right)^{3/2}}\right)$, in the $-x$-direction.

(b) $x \gg a$ implies $a^2/x^2 \ll 1$ and $\left(1+a^2/x^2\right)^{-3/2} \approx 1 - 3a^2/2x^2$.

Thus $E \approx \dfrac{2q}{4\pi\epsilon_0 x^2}\left(1 - \left(1 - \dfrac{3a^2}{2x^2}\right)\right) = \dfrac{3qa^2}{4\pi\epsilon_0 x^4}.$

EVALUATE: $E \sim 1/x^4$. For a point charge $E \sim 1/x^2$ and for a dipole $E \sim 1/x^3$. The total charge is zero so at large distances the electric field should decrease faster with distance than for a point charge. By symmetry $\vec{E}$ must lie along the x-axis, which is the result we found in part (a).

21.81. **IDENTIFY:** The small bags of protons behave like point-masses and point-charges since they are extremely far apart.
SET UP: For point-particles, we use Newton's formula for universal gravitation ($F = Gm_1m_2/r^2$) and Coulomb's law. The number of protons is the mass of protons in the bag divided by the mass of a single proton.

EXECUTE: **(a)** $(0.0010\text{ kg})/(1.67\times10^{-27}\text{ kg}) = 6.0\times10^{23}$ protons

(b) Using Coulomb's law, where the separation is twice is the radius of the earth, we have
$F_{\text{electrical}} = (9.00 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.0 \times 10^{23} \times 1.60 \times 10^{-19}\ \text{C})^2/(2 \times 6.38 \times 10^6\ \text{m})^2 = 5.1 \times 10^5\ \text{N}$
$F_{\text{grav}} = (6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.0010\ \text{kg})^2/(2 \times 6.38 \times 10^6\ \text{m})^2 = 4.1 \times 10^{-31}\ \text{N}$
(c) EVALUATE: The electrical force (≈200,000 lb!) is certainly large enough to feel, but the gravitational force clearly is not since it is about 10^{36} times weaker.

21.82. **IDENTIFY:** We can treat the protons as point-charges and use Coulomb's law.
SET UP: (a) Coulomb's law is $F = (1/4\pi\epsilon_0)|q_1q_2|/r^2$.

EXECUTE: $F = (9.00 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\ \text{C})^2/(2.0 \times 10^{-15}\ \text{m}) = 58\ \text{N} = 13$ lb, which is certainly large enough to feel.
(b) EVALUATE: Something must be holding the nucleus together by opposing this enormous repulsion. This is the strong nuclear force.

21.83. **IDENTIFY:** Estimate the number of protons in the textbook and from this find the net charge of the textbook. Apply Coulomb's law to find the force and use $F_{\text{net}} = ma$ to find the acceleration.

SET UP: With the mass of the book about 1.0 kg, most of which is protons and neutrons, we find that the number of protons is $\frac{1}{2}(1.0\text{ kg})/(1.67\times10^{-27}\text{ kg}) = 3.0\times10^{26}$.

EXECUTE: **(a)** The charge difference present if the electron's charge was 99.999% of the proton's is
$\Delta q = (3.0\times10^{26})(0.00001)(1.6\times10^{-19}\ \text{C}) = 480\ \text{C}$.

(b) $F = k(\Delta q)^2/r^2 = k(480\ \text{C})^2/(5.0\ \text{m})^2 = 8.3\times10^{13}\ \text{N}$, and is repulsive.

$a = F/m = (8.3\times10^{13}\ \text{N})/(1\ \text{kg}) = 8.3\times10^{13}\ \text{m/s}^2$.

EXECUTE: **(c)** Even the slightest charge imbalance in matter would lead to explosive repulsion!

21.84. **IDENTIFY:** The electric field exerts equal and opposite forces on the two balls, causing them to swing away from each other. When the balls hang stationary, they are in equilibrium so the forces on them (electrical, gravitational, and tension in the strings) must balance.
SET UP: (a) The force on the left ball is in the direction of the electric field, so it must be positive, while the force on the right ball is opposite to the electric field, so it must be negative.
(b) Balancing horizontal and vertical forces gives $qE = T \sin \theta/2$ and $mg = T \cos \theta/2$.
EXECUTE: Solving for the angle θ gives: $\theta = 2 \arctan(qE/mg)$.
(c) As $E \to \infty$, $\theta \to 2 \arctan(\infty) = 2 (\pi/2) = \pi = 180°$
EVALUATE: If the field were large enough, the gravitational force would not be important, so the strings would be horizontal.

21.85. **IDENTIFY** and **SET UP:** Use the density of copper to calculate the number of moles and then the number of atoms. Calculate the net charge and then use Coulomb's law to calculate the force.

EXECUTE: **(a)** $m = \rho V = \rho\left(\frac{4}{3}\pi r^3\right) = \left(8.9\times10^3\ \text{kg/m}^3\right)\left(\frac{4}{3}\pi\right)\left(1.00\times10^{-3}\ \text{m}\right)^3 = 3.728\times10^{-5}\ \text{kg}$

$n = m/M = \left(3.728\times10^{-5}\ \text{kg}\right)/\left(63.546\times10^{-3}\ \text{kg/mol}\right) = 5.867\times10^{-4}\ \text{mol}$

$N = nN_A = 3.5\times10^{20}$ atoms

(b) $N_e = (29)\left(3.5\times10^{20}\right) = 1.015\times10^{22}$ electrons and protons

$q_{\text{net}} = eN_e - (0.99900)eN_e = \left(0.100\times10^{-2}\right)\left(1.602\times10^{-19}\ \text{C}\right)\left(1.015\times10^{22}\right) = 1.6\ \text{C}$

$$F = k\frac{q^2}{r^2} = k\frac{(1.6\ \text{C})^2}{(1.00\ \text{m})^2} = 2.3\times10^{10}\ \text{N}$$

EVALUATE: The amount of positive and negative charge in even small objects is immense. If the charge of an electron and a proton weren't exactly equal, objects would have large net charges.

21.86. **IDENTIFY:** Apply constant acceleration equations to a drop to find the acceleration. Then use $F = ma$ to find the force and $F = |q|E$ to find $|q|$.

SET UP: Let $D = 2.0$ cm be the horizontal distance the drop travels and $d = 0.30$ mm be its vertical displacement. Let $+x$ be horizontal and in the direction from the nozzle toward the paper and let $+y$ be vertical, in the direction of the deflection of the drop. $a_x = 0$ and $a_y = a$.

EXECUTE: First, the mass of the drop: $m = \rho V = (1000\ \text{kg/m}^3)\left(\frac{4\pi(15.0\times10^{-6}\ \text{m})^3}{3}\right) = 1.41\times10^{-11}\ \text{kg}$. Next, the time of flight: $t = D/v = (0.020\ \text{m})/(20\ \text{m/s}) = 0.00100\ \text{s}$. $d = \frac{1}{2}at^2$. $a = \frac{2d}{t^2} = \frac{2(3.00\times10^{-4}\ \text{m})}{(0.001\ \text{s})^2} = 600\ \text{m/s}^2$.

Then $a = F/m = qE/m$ gives $q = ma/E = \frac{(1.41\times10^{-11}\ \text{kg})(600\ \text{m/s}^2)}{8.00\times10^4\ \text{N/C}} = 1.06\times10^{-13}\ \text{C}$.

EVALUATE: Since q is positive the vertical deflection is in the direction of the electric field.

21.87. **IDENTIFY:** Eq. (21.3) gives the force exerted by the electric field. This force is constant since the electric field is uniform and gives the proton a constant acceleration. Apply the constant acceleration equations for the x- and y-components of the motion, just as for projectile motion.
(a) SET UP: The electric field is upward so the electric force on the positively charged proton is upward and has magnitude $F = eE$. Use coordinates where positive y is downward. Then applying $\sum \vec{F} = m\vec{a}$ to the proton gives that $a_x = 0$ and $a_y = -eE/m$. In these coordinates the initial velocity has components $v_x = +v_0 \cos\alpha$ and $v_y = +v_0 \sin\alpha$, as shown in Figure 21.87a.

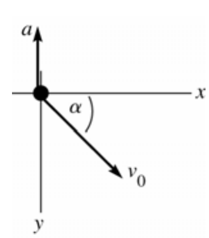


Figure 21.87a

EXECUTE: Finding h_{max}: At $y = h_{max}$ the y-component of the velocity is zero.

$v_y = 0,\ v_{0y} = v_0 \sin\alpha,\ a_y = -eE/m,\ y - y_0 = h_{max} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y}$$

$$h_{max} = \frac{-v_0^2 \sin^2\alpha}{2(-eE/m)} = \frac{mv_0^2 \sin^2\alpha}{2eE}$$

(b) Use the vertical motion to find the time t: $y - y_0 = 0,\ v_{0y} = v_0 \sin\alpha,\ a_y = -eE/m,\ t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

With $y - y_0 = 0$ this gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(v_0 \sin\alpha)}{-eE/m} = \dfrac{2mv_0 \sin\alpha}{eE}$

Then use the x-component motion to find d: $a_x = 0,\ v_{0x} = v_0 \cos\alpha,\ t = 2mv_0 \sin\alpha / eE,\ x - x_0 = d = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $d = v_0 \cos\alpha\left(\dfrac{2mv_0 \sin\alpha}{eE}\right) = \dfrac{mv_0^2 2\sin\alpha\cos\alpha}{eE} = \dfrac{mv_0^2 \sin 2\alpha}{eE}$

(c) The trajectory of the proton is sketched in Figure 21.87b.

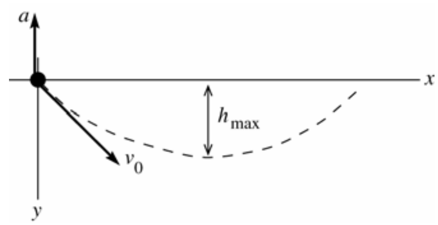


Figure 21.87b

(d) Use the expression in part (a): $h_{max} = \dfrac{[(4.00\times10^5\ \text{m/s})(\sin 30.0°)]^2(1.673\times10^{-27}\ \text{kg})}{2(1.602\times10^{-19}\ \text{C})(500\ \text{N/C})} = 0.418\ \text{m}$

Use the expression in part (b): $d = \dfrac{(1.673\times10^{-27}\ \text{kg})(4.00\times10^5\ \text{m/s})^2 \sin 60.0°}{(1.602\times10^{-19}\ \text{C})(500\ \text{N/C})} = 2.89\ \text{m}$

EVALUATE: In part (a), $a_y = -eE/m = -4.8\times10^{10}\ \text{m/s}^2$. This is much larger in magnitude than g, the acceleration due to gravity, so it is reasonable to ignore gravity. The motion is just like projectile motion, except that the acceleration is upward rather than downward and has a much different magnitude. h_{max} and d increase when α or v_0 increase and decrease when E increases.

21.88. **IDENTIFY:** $E_x = E_{1x} + E_{2x}$. Use Eq.(21.7) for the electric field due to each point charge.

SET UP: $\vec{E}$ is directed away from positive charges and toward negative charges.

EXECUTE: **(a)** $E_x = +50.0\ \text{N/C}$. $E_{1x} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{4.00\times10^{-9}\ \text{C}}{(0.60\ \text{m})^2} = +99.9\ \text{N/C}$.

$E_x = E_{1x} + E_{2x}$, so $E_{2x} = E_x - E_{1x} = +50.0\ \text{N/C} - 99.9\ \text{N/C} = -49.9\ \text{N/C}$. Since E_{2x} is negative, q_2 must be negative. $|q_2| = \dfrac{|E_{2x}|r_2^2}{(1/4\pi\epsilon_0)} = \dfrac{(49.9\ \text{N/C})(1.20\ \text{m})^2}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 7.99\times10^{-9}\ \text{C}$. $q_2 = -7.99\times10^{-9}\ \text{C}$

(b) $E_x = -50.0\ \text{N/C}$. $E_{1x} = +99.9\ \text{N/C}$, as in part (a). $E_{2x} = E_x - E_{1x} = -149.9\ \text{N/C}$. q_2 is negative.

$|q_2| = \dfrac{|E_{2x}|r_2^2}{(1/4\pi\epsilon_0)} = \dfrac{(149.9\ \text{N/C})(1.20\ \text{m})^2}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 2.40\times10^{-8}\ \text{C}$. $q_2 = -2.40\times10^{-8}\ \text{C}$.

EVALUATE: q_2 would be positive if E_{2x} were positive.

21.89. **IDENTIFY:** Divide the charge distribution into infinitesimal segments of length dx. Calculate E_x and E_u due to a segment and integrate to find the total field.

SET UP: The charge dQ of a segment of length dx is $dQ = (Q/a)dx$. The distance between a segment at x and the charge q is $a + r - x$. $(1 - y)^{-1} \approx 1 + y$ when $|y| << 1$.

EXECUTE: **(a)** $dE_x = \frac{1}{4\pi\epsilon_0}\frac{dQ}{(a+r-x)^2}$ so $E_x = \frac{1}{4\pi\epsilon_0}\int_0^a \frac{Qdx}{a(a+r-x)^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{1}{r} - \frac{1}{a+r}\right)$.

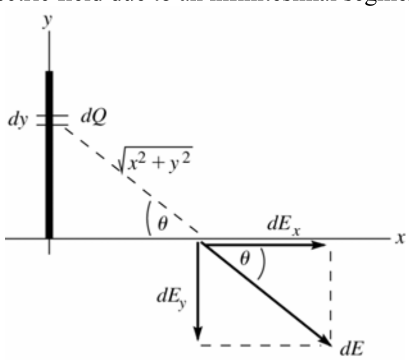
$a + r = x$, so $E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{1}{x-a} - \frac{1}{x}\right)$. $E_y = 0$.

(b) $\vec{F} = q\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{a}\left(\frac{1}{x-a} - \frac{1}{x}\right)\hat{i}$.

EVALUATE: **(c)** For $x >> a$, $F = \frac{kqQ}{ax}((1 - a/x)^{-1} - 1) = \frac{kqQ}{ax}(1 + a/x + \cdots - 1) \approx \frac{kqQ}{x^2} \approx \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}$. (Note that for $x >> a$, $r = x - a \approx x$.) The charge distribution looks like a point charge from far away, so the force takes the form of the force between a pair of point charges.

21.90. **IDENTIFY:** Use Eq. (21.7) to calculate the electric field due to a small slice of the line of charge and integrate as in Example 21.11. Use Eq. (21.3) to calculate $\vec{F}$.

SET UP: The electric field due to an infinitesimal segment of the line of charge is sketched in Figure 21.90.



$$\sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$$

Figure 21.90

Slice the charge distribution up into small pieces of length dy. The charge dQ in each slice is $dQ = Q(dy/a)$. The electric field this produces at a distance x along the x-axis is dE. Calculate the components of $d\vec{E}$ and then integrate over the charge distribution to find the components of the total field.

EXECUTE: $dE = \frac{1}{4\pi\epsilon_0}\left(\frac{dQ}{x^2 + y^2}\right) = \frac{Q}{4\pi\epsilon_0 a}\left(\frac{dy}{x^2 + y^2}\right)$

$$dE_x = dE\cos\theta = \frac{Qx}{4\pi\epsilon_0 a}\left(\frac{dy}{\left(x^2 + y^2\right)^{3/2}}\right)$$

$$dE_y = -dE\sin\theta = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{ydy}{\left(x^2 + y^2\right)^{3/2}}\right)$$

$$E_x = \int dE_x = -\frac{Qx}{4\pi\epsilon_0 a}\int_0^a \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{Qx}{4\pi\epsilon_0 a}\left[\frac{1}{x^2}\frac{y}{\sqrt{x^2 + y^2}}\right]_0^a = \frac{Q}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2 + a^2}}$$

$$E_y = \int dEy = -\frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{ydy}{(x^2 + y^2)^{3/2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[-\frac{1}{\sqrt{x^2 + y^2}}\right]_0^a = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2 + a^2}}\right)$$

(b) $\vec{F} = q_0\vec{E}$

$$F_x = -qE_x = \frac{-qQ}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2 + a^2}};\ F_y = -qE_y = \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2 + a^2}}\right)$$

(c) For $x >> a$, $\frac{1}{\sqrt{x^2 + a^2}} = \frac{1}{x}\left(1 + \frac{a^2}{x^2}\right)^{-1/2} = \frac{1}{x}\left(1 - \frac{a^2}{2x^2}\right) = \frac{1}{x} - \frac{a^2}{2x^3}$

$$F_x \approx -\frac{qQ}{4\pi\epsilon_0 x^2},\ F_y \approx \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x} - \frac{1}{x} + \frac{a^2}{2x^3}\right) = \frac{qQa}{8\pi\epsilon_0 x^3}$$

EVALUATE: For $x >> a$, $F_y << F_x$ and $F \approx |F_x| = \dfrac{qQ}{4\pi\epsilon_0 x^2}$ and $\vec{F}$ is in the $-x$-direction. For $x >> a$ the charge distribution Q acts like a point charge.

21.91. IDENTIFY: Apply Eq.(21.9) from Example 21.11.

SET UP: $a = 2.50$ cm. Replace Q by $|Q|$. Since Q is negative, $\vec{E}$ is toward the line of charge and $\vec{E} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|Q|}{x\sqrt{x^2+a^2}}\hat{i}$.

EXECUTE: $\vec{E} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|Q|}{x\sqrt{x^2+a^2}}\hat{i} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{9.00\times 10^{-9}\ \text{C}}{(0.100\ \text{m})\sqrt{(0.100\ \text{m})^2+(0.025\ \text{m})^2}}\hat{i} = (-7850\ \text{N/C})\hat{i}$.

(b) The electric field is less than that at the same distance from a point charge (8100 N/C). For large x, $(x+a)^{-1/2} = \dfrac{1}{x}(1+a^2/x^2)^{-1/2} \approx \dfrac{1}{x}\left(1-\dfrac{a^2}{2x^2}\right)$. $E_{x\to\infty} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{x^2}\left(1-\dfrac{a^2}{2x^2}+\cdots\right)$. The first correction term to the point charge result is negative.

(c) For a 1% difference, we need the first term in the expansion beyond the point charge result to be less than 0.010: $\dfrac{a^2}{2x^2} \approx 0.010 \Rightarrow x \approx a\sqrt{1/(2(0.010))} = 0.025\sqrt{1/0.020} \Rightarrow x \approx 0.177$ m.

EVALUATE: At $x = 10.0$ cm (part b), the exact result for the line of charge is 3.1% smaller than for a point charge. It is sensible, therefore, that the difference is 1.0% at a somewhat larger distance, 17.7 cm.

21.92. IDENTIFY: The electrical force has magnitude $F = \dfrac{kQ^2}{r^2}$ and is attractive. Apply $\sum \vec{F} = m\vec{a}$ to the earth.

SET UP: For a circular orbit, $a = \dfrac{v^2}{r}$. The period T is $\dfrac{2\pi r}{v}$. The mass of the earth is $m_E = 5.97\times 10^{24}$ kg, the orbit radius of the earth is 1.50×10^{11} m and its orbital period is 3.146×10^{7} s.

EXECUTE: $F = ma$ gives $\dfrac{kQ}{r^2} = m_E\dfrac{v^2}{r}$. $v^2 = \dfrac{4\pi^2 r^2}{T^2}$, so

$$Q = \sqrt{\frac{m_E 4\pi^2 r^3}{kT^2}} = \sqrt{\frac{(5.97\times 10^{24}\ \text{kg})(4)(\pi^2)(1.50\times 10^{11}\ \text{m})^3}{(8.99\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.146\times 10^{7}\ \text{s})^2}} = 2.99\times 10^{17}\ \text{C}.$$

EVALUATE: A very large net charge would be required.

21.93. IDENTIFY: Apply Eq.(21.11).

SET UP: $\sigma = Q/A = Q/\pi R^2$. $(1+y^2)^{-1/2} \approx 1 - y^2/2$, when $y^2 << 1$.

EXECUTE: **(a)** $E = \dfrac{\sigma}{2\epsilon_0}\left[1-\left(R^2/x^2+1\right)^{-1/2}\right]$.

$$E = \frac{4.00\ \text{pC}/\pi(0.025\ \text{m})^2}{2\epsilon_0}\left[1-\left(\frac{(0.025\ \text{m})^2}{(0.200\ \text{m})^2}+1\right)^{-1/2}\right] = 0.89\ \text{N/C}$$, in the $+x$ direction.

(b) For $x >> R$ $E = \dfrac{\sigma}{2\epsilon_0}[1-(1-R^2/2x^2+\cdots)] \approx \dfrac{\sigma}{2\epsilon_0}\dfrac{R^2}{2x^2} = \dfrac{\sigma\pi R^2}{4\pi\epsilon_0 x^2} = \dfrac{Q}{4\pi\epsilon_0 x^2}$.

(c) The electric field of (a) is less than that of the point charge (0.90 N/C) since the first correction term to the point charge result is negative.

(d) For $x = 0.200$ m, the percent difference is $\dfrac{(0.90-0.89)}{0.89} = 0.01 = 1\%$. For $x = 0.100$ m, $E_{\text{disk}} = 3.43$ N/C and $E_{\text{point}} = 3.60$ N/C, so the percent difference is $\dfrac{(3.60-3.43)}{3.60} = 0.047 \approx 5\%$.

EVALUATE: The field of a disk becomes closer to the field of a point charge as the distance from the disk increases. At $x = 10.0$ cm, $R/x = 25\%$ and the percent difference between the field of the disk and the field of a point charge is 5%.

21.94. IDENTIFY: Apply the procedure specified in the problem.

SET UP: $\int_{x_1}^{x_2} f(x)dx = -\int_{x_2}^{x_1} f(x)dx$.

EXECUTE: **(a)** For $f(x) = f(-x)$, $\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx = \int_{0}^{-a} f(-x)d(-x) + \int_{0}^{a} f(x)dx$. Now replace $-x$ with y. This gives $\int_{-a}^{a} f(x)dx = \int_{0}^{a} f(y)dy + \int_{0}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$.

(b) For $g(x) = -g(-x)$, $\int_{-a}^{a} g(x)dx = \int_{-a}^{0} g(x)dx + \int_{0}^{a} g(x)dx = -\int_{0}^{-a} -g(-x)(-d(-x)) + \int_{0}^{a} g(x)dx$. Now replace $-x$ with y. This gives $\int_{-a}^{a} g(x)dx = -\int_{0}^{a} g(y)dy + \int_{0}^{a} g(x)dx = 0$.

(c) The integrand in E_y for Example 21.11 is odd, so $E_y = 0$.

EVALUATE: In Example 21.11, $E_y = 0$ because for each infinitesimal segment in the upper half of the line of charge, there is a corresponding infinitesimal segment in the bottom half of the line that has E_y in the opposite direction.

21.95. **IDENTIFY:** Find the resultant electric field due to the two point charges. Then use $\vec{F} = q\vec{E}$ to calculate the force on the point charge.

SET UP: Use the results of Problems 21.90 and 21.89.

EXECUTE: **(a)** The y-components of the electric field cancel, and the x-component from both charges, as given in Problem 21.90, is $E_x = \frac{1}{4\pi\epsilon_0}\frac{-2Q}{a}\left(\frac{1}{y} - \frac{1}{(y^2+a^2)^{1/2}}\right)$. Therefore, $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{-2Qq}{a}\left(\frac{1}{y} - \frac{1}{(y^2+a^2)^{1/2}}\right)\hat{i}$. If $y >> a$

$\vec{F} \approx \frac{1}{4\pi\epsilon_0}\frac{-2Qq}{ay}(1-(1-a^2/2y^2+\cdots))\hat{i} = -\frac{1}{4\pi\epsilon_0}\frac{Qqa}{y^3}\hat{i}$.

(b) If the point charge is now on the x-axis the two halves of the charge distribution provide different forces, though still along the x-axis, as given in Problem 21.89: $\vec{F}_+ = q\vec{E}_+ = \frac{1}{4\pi\epsilon_0}\frac{Qq}{a}\left(\frac{1}{x-a} - \frac{1}{x}\right)\hat{i}$

and $\vec{F}_- = q\,\vec{E}_- = -\frac{1}{4\pi\epsilon_0}\frac{Qq}{a}\left(\frac{1}{x} - \frac{1}{x+a}\right)\hat{i}$. Therefore, $\vec{F} = \vec{F}_+ + \vec{F}_- = \frac{1}{4\pi\epsilon_0}\frac{Qq}{a}\left(\frac{1}{x-a} - \frac{2}{x} + \frac{1}{x+a}\right)\hat{i}$. For $x >> a$,

$\vec{F} \approx \frac{1}{4\pi\epsilon_0}\frac{Qq}{ax}\left(\left(1+\frac{a}{x}+\frac{a^2}{x^2}+\ldots\right)-2+\left(1-\frac{a}{x}+\frac{a^2}{x^2}-\ldots\right)\right)\hat{i} = \frac{1}{4\pi\epsilon_0}\frac{2Qqa}{x^3}\hat{i}$.

EVALUATE: If the charge distributed along the x-axis were all positive or all negative, the force would be proportional to $1/y^2$ in part (a) and to $1/x^2$ in part (b), when y or x is very large.

21.96. **IDENTIFY:** Divide the semicircle into infinitesimal segments. Find the electric field $d\vec{E}$ due to each segment and integrate over the semicircle to find the total electric field.

SET UP: The electric fields along the x-direction from the left and right halves of the semicircle cancel. The remaining y-component points in the negative y-direction. The charge per unit length of the semicircle is $\lambda = \frac{Q}{\pi a}$ and $dE = \frac{k\lambda\, dl}{a^2} = \frac{k\lambda\, d\theta}{a}$.

EXECUTE: $dE_y = dE\sin\theta = \frac{k\lambda\sin\theta\, d\theta}{a}$. Therefore, $E_y = \frac{2k\lambda}{a}\int_0^{\pi/2}\sin\theta\, d\theta = \frac{2k\lambda}{a}[-\cos\theta]_0^{\pi/2} = \frac{2k\lambda}{a} = \frac{2kQ}{\pi a^2}$, in the $-y$-direction.

EVALUATE: For a full circle of charge the electric field at the center would be zero. For a quarter-circle of charge, in the first quadrant, the electric field at the center of curvature would have nonzero x and y components. The calculation for the semicircle is particularly simple, because all the charge is the same distance from point P.

21.97. **IDENTIFY:** Divide the charge distribution into small segments, use the point charge formula for the electric field due to each small segment and integrate over the charge distribution to find the x and y components of the total field.

SET UP: Consider the small segment shown in Figure 21.97a.

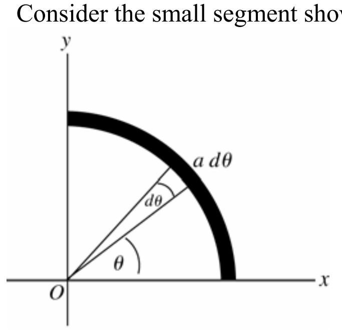


Figure 21.97a

EXECUTE: A small segment that subtends angle $d\theta$ has length $a\,d\theta$ and contains charge $dQ = \left(\frac{a d\theta}{\frac{1}{2}\pi a}\right)Q = \frac{2Q}{\pi}d\theta$. ($\frac{1}{2}\pi a$ is the total length of the charge distribution.)

The charge is negative, so the field at the origin is directed toward the small segment. The small segment is located at angle θ as shown in the sketch. The electric field due to dQ is shown in Figure 21.97b, along with its components.

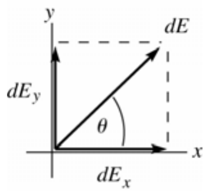


Figure 21.97b

$$dE = \frac{1}{4\pi\epsilon_0}\frac{|dQ|}{a^2}$$

$$dE = \frac{Q}{2\pi^2\epsilon_0 a^2}d\theta$$

$$dE_x = dE\cos\theta = \left(Q/2\pi^2\epsilon_0 a^2\right)\cos\theta d\theta$$

$$E_x = \int dE_x = \frac{Q}{2\pi^2\epsilon_0 a^2}\int_0^{\pi/2}\cos\theta d\theta = \frac{Q}{2\pi^2\epsilon_0 a^2}\left(\sin\theta\Big|_0^{\pi/2}\right) = \frac{Q}{2\pi^2\epsilon_0 a^2}$$

$$dE_y = dE\sin\theta = \left(Q/2\pi^2\epsilon_0 a^2\right)\sin\theta d\theta$$

$$E_y = \int dE_y = \frac{Q}{2\pi^2\epsilon_0 a^2}\int_0^{\pi/2}\sin\theta d\theta = \frac{Q}{2\pi^2\epsilon_0 a^2}\left(-\cos\theta\Big|_0^{\pi/2}\right) = \frac{Q}{2\pi^2\epsilon_0 a^2}$$

EVALUATE: Note that $E_x = E_y$, as expected from symmetry.

21.98. IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere, with x horizontal and y vertical.

SET UP: The free-body diagram for the sphere is given in Figure 21.98. The electric field $\vec{E}$ of the sheet is directed away from the sheet and has magnitude $E = \frac{\sigma}{2\epsilon_0}$ (Eq.21.12).

EXECUTE: $\sum F_y = 0$ gives $T\cos\alpha = mg$ and $T = \frac{mg}{\cos\alpha}$. $\sum F_x = 0$ gives $T\sin\alpha = \frac{q\sigma}{2\epsilon_0}$ and $T = \frac{q\sigma}{2\epsilon_0\sin\alpha}$.

Combining these two equations we have $\frac{mg}{\cos\alpha} = \frac{q\sigma}{2\epsilon_0\sin\alpha}$ and $\tan\alpha = \frac{q\sigma}{2\epsilon_0 mg}$. Therefore, $\alpha = \arctan\left(\frac{q\sigma}{2\epsilon_0 mg}\right)$.

EVALUATE: The electric field of the sheet, and hence the force it exerts on the sphere, is independent of the distance of the sphere from the sheet.

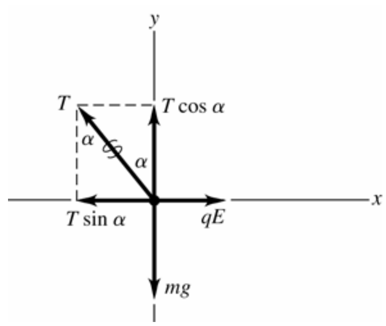


Figure 21.98

21.99. IDENTIFY: Each wire produces an electric field at P due to a finite wire. These fields add by vector addition.

SET UP: Each field has magnitude $\frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}$. The field due to the negative wire points to the left, while the field due to the positive wire points downward, making the two fields perpendicular to each other and of equal magnitude. The net field is the vector sum of these two, which is $E_{net} = 2E_1\cos 45° = 2\frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\cos 45°$. In part (b), the electrical force on an electron at P is eE.

EXECUTE: **(a)** The net field is $E_{net} = 2\frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\cos 45°$.

$$E_{net} = \frac{2\left(9.00\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.50\times10^{-6}\,\text{C}\right)\cos 45°}{(0.600\text{ m})\sqrt{(0.600\text{ m})^2+(0.600\text{ m})^2}} = 6.25\times10^4\text{ N/C}.$$

The direction is 225° counterclockwise from an axis pointing to the right through the positive wire.

(b) $F = eE = (1.60 \times 10^{-19}\text{ C})(6.25 \times 10^4\text{ N/C}) = 1.00 \times 10^{-14}$ N, opposite to the direction of the electric field, since the electron has negative charge.

EVALUATE: Since the electric fields due to the two wires have equal magnitudes and are perpendicular to each other, we only have to calculate one of them in the solution.

21.100. **IDENTIFY:** Each sheet produces an electric field that is independent of the distance from the sheet. The net field is the vector sum of the two fields.

SET UP: The formula for each field is $E = \sigma/2\epsilon_0$, and the net field is the vector sum of these,

$E_{\text{net}} = \dfrac{\sigma_B}{2\epsilon_0} \pm \dfrac{\sigma_A}{2\epsilon_0} = \dfrac{\sigma_B \pm \sigma_A}{2\epsilon_0}$, where we use the + or – sign depending on whether the fields are in the same or opposite directions and σ_B and σ_A are the magnitudes of the surface charges.

EXECUTE: **(a)** The two fields oppose and the field of B is stronger than that of A, so

$$E_{\text{net}} = \frac{\sigma_B}{2\epsilon_0} - \frac{\sigma_A}{2\epsilon_0} = \frac{\sigma_B - \sigma_A}{2\epsilon_0} = \frac{11.6\,\mu\text{C/m}^2 - 9.50\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = 1.19 \times 10^5 \text{ N/C, to the right.}$$

(b) The fields are now in the same direction, so their magnitudes add.

$$E_{\text{net}} = (11.6\ \mu\text{C/m}^2 + 9.50\ \mu\text{C/m}^2)/2\,\epsilon_0 = 1.19 \times 10^6 \text{ N/C, to the right}$$

(c) The fields add but now point to the left, so $E_{\text{net}} = 1.19 \times 10^6$ N/C, to the left.

EVALUATE: We can simplify the calculations by sketching the fields and doing an algebraic solution first.

21.101. **IDENTIFY:** Each sheet produces an electric field that is independent of the distance from the sheet. The net field is the vector sum of the two fields.

SET UP: The formula for each field is $E = \sigma / 2\epsilon_0$, and the net field is the vector sum of these,

$E_{\text{net}} = \dfrac{\sigma_B}{2\epsilon_0} \pm \dfrac{\sigma_A}{2\epsilon_0} = \dfrac{\sigma_B \pm \sigma_A}{2\epsilon_0}$, where we use the + or – sign depending on whether the fields are in the same or opposite directions and σ_B and σ_A are the magnitudes of the surface charges.

EXECUTE: **(a)** The fields add and point to the left, giving $E_{\text{net}} = 1.19 \times 10^6$ N/C.

(b) The fields oppose and point to the left, so $E_{\text{net}} = 1.19 \times 10^5$ N/C.

(c) The fields oppose but now point to the right, giving $E_{\text{net}} = 1.19 \times 10^5$ N/C.

EVALUATE: We can simplify the calculations by sketching the fields and doing an algebraic solution first.

21.102. **IDENTIFY:** The sheets produce an electric field in the region between them which is the vector sum of the fields from the two sheets.

SET UP: The force on the negative oil droplet must be upward to balance gravity. The net electric field between the sheets is $E = \sigma / \epsilon_0$, and the electrical force on the droplet must balance gravity, so $qE = mg$.

EXECUTE: **(a)** The electrical force on the drop must be upward, so the field should point downward since the drop is negative.

(b) The charge of the drop is $5e$, so $qE = mg$. $(5e)(\sigma / \epsilon_0) = mg$ and

$$\sigma = \frac{mg\epsilon_0}{5e} = \frac{\left(324\times10^{-9}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}{5\left(1.60\times10^{-19}\ \text{C}\right)} = 35.1 \text{ C/m}^2$$

EVALUATE: Balancing oil droplets between plates was the basis of the Milliken Oil-Drop Experiment which produced the first measurement of the mass of an electron.

21.103. **IDENTIFY** and **SET UP:** Example 21.12 gives the electric field due to one infinite sheet. Add the two fields as vectors.

EXECUTE: The electric field due to the first sheet, which is in the xy-plane, is $\vec{E}_1 = (\sigma / 2\epsilon_0)\hat{k}$ for $z > 0$ and $\vec{E}_1 = -(\sigma/2\epsilon_0)\hat{k}$ for $z < 0$. We can write this as $\vec{E}_1 = (\sigma/2\epsilon_0)(z/|z|)\hat{k}$, since $z/|z| = +1$ for $z > 0$ and $z/|z| = -z/z = -1$ for $z < 0$. Similarly, we can write the electric field due to the second sheet as $\vec{E}_2 = -(\sigma/2\epsilon_0)(x/|x|)\hat{i}$, since its charge density is $-\sigma$. The net field is $\vec{E} = \vec{E}_1 + \vec{E}_2 = (\sigma/2\epsilon_0)\left(-(x/|x|)\hat{i} + (z/|z|)\hat{k}\right)$.

EVALUATE: The electric field is independent of the y-component of the field point since displacement in the $\pm y$- direction is parallel to both planes. The field depends on which side of each plane the field is located.

21.104. **IDENTIFY:** Apply Eq.(21.11) for the electric field of a disk. The hole can be described by adding a disk of charge density $-\sigma$ and radius R_1 to a solid disk of charge density $+\sigma$ and radius R_2.

SET UP: The area of the annulus is $\pi(R_2^2 - R_1^2)\sigma$. The electric field of a disk, Eq.(21.11) is
$E = \frac{\sigma}{2\epsilon_0}\left[1 - 1\big/\sqrt{(R/x)^2+1}\right]$.

EXECUTE: **(a)** $Q = A\sigma = \pi(R_2^2 - R_1^2)\sigma$

(b) $\vec{E}(x) = \frac{\sigma}{2\epsilon_0}\left(\left[1-1/\sqrt{(R_2/x)^2+1}\right]-\left[1-1/\sqrt{(R_1/x)^2+1}\right]\right)\frac{|x|}{x}\hat{i}$. $\vec{E}(x) = \frac{-\sigma}{2\epsilon_0}\left(1/\sqrt{(R_1/x)^2+1}-1/\sqrt{(R_2/x)^2+1}\right)\frac{|x|}{x}\hat{i}$.
The electric field is in the $+x$ direction at points above the disk and in the $-x$ direction at points below the disk, and the factor $\frac{|x|}{x}\hat{i}$ specifies these directions.

(c) Note that $1\big/\sqrt{(R_1/x)^2+1} = \frac{|x|}{R_1}(1+(x/R_1)^2)^{-1/2} \approx \frac{|x|}{R_1}$. This gives $\vec{E}(x) = \frac{\sigma}{2\epsilon_0}\left(\frac{x}{R_1}-\frac{x}{R_2}\right)\frac{|x|^2}{x}\hat{i} = \frac{\sigma}{2\epsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)x\hat{i}$.
Sufficiently close means that $(x/R_1)^2 << 1$.

(d) $F_x = qE_x = -\frac{q\sigma}{2\epsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)x$. The force is in the form of Hooke's law: $F_x = -kx$, with $k = \frac{q\sigma}{2\epsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$.
$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{q\sigma}{2\epsilon_0 m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$.

EVALUATE: The frequency is independent of the initial position of the particle, so long as this position is sufficiently close to the center of the annulus for $(x/R_1)^2$ to be small.

21.105. **IDENTIFY:** Apply Coulomb's law to calculate the forces that q_1 and q_2 exert on q_3, and add these force vectors to get the net force.
SET UP: Like charges repel and unlike charges attract. Let $+x$ be to the right and $+y$ be toward the top of the page.
EXECUTE: **(a)** The four possible force diagrams are sketched in Figure 21.105a.
Only the last picture can result in a net force in the $-x$-direction.
(b) $q_1 = -2.00\ \mu$C, $q_3 = +4.00\ \mu$C, and $q_2 > 0$.

(c) The forces $\vec{F}_1$ and $\vec{F}_2$ and their components are sketched in Figure 21.105b.
$F_y = 0 = -\frac{1}{4\pi\epsilon_0}\frac{|q_1||q_3|}{(0.0400\text{ m})^2}\sin\theta_1 + \frac{1}{4\pi\epsilon_0}\frac{|q_2||q_3|}{(0.0300\text{ m})^2}\sin\theta_2$. This gives
$q_2 = \frac{9}{16}|q_1|\frac{\sin\theta_1}{\sin\theta_2} = \frac{9}{16}|q_1|\frac{3/5}{4/5} = \frac{27}{64}|q_1| = 0.843\ \mu\text{C}$.

(d) $F_x = F_{1x} + F_{2x}$ and $F_y = 0$, so $F = |q_3|\frac{1}{4\pi\epsilon_0}\left(\frac{|q_1|}{(0.0400\text{ m})^2}\frac{4}{5}+\frac{|q_2|}{(0.0300\text{ m})^2}\frac{3}{5}\right) = 56.2\text{ N}$.

EVALUATE: The net force $\vec{F}$ on q_3 is in the same direction as the resultant electric field at the location of q_3 due to q_1 and q_2.

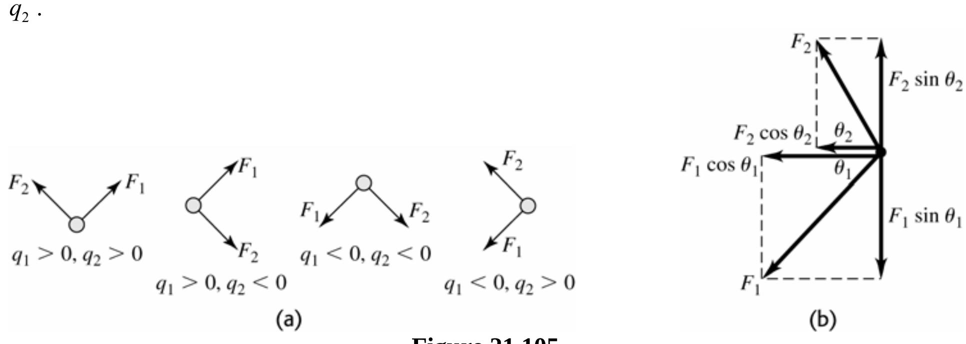


Figure 21.105

21.106. **IDENTIFY:** Calculate the electric field at P due to each charge and add these field vectors to get the net field.
SET UP: The electric field of a point charge is directed away from a positive charge and toward a negative charge. Let $+x$ be to the right and let $+y$ be toward the top of the page.
EXECUTE: **(a)** The four possible diagrams are sketched in Figure 21.106a.
The first diagram is the only one in which the electric field must point in the negative y-direction.
(b) $q_1 = -3.00\ \mu$C, and $q_2 < 0$.

(c) The electric fields $\vec{E}_1$ and $\vec{E}_2$ and their components are sketched in Figure 24.106b. $\cos\theta_1 = \frac{5}{13}$, $\sin\theta_1 = \frac{12}{13}$, $\cos\theta_2 = \frac{12}{13}$ and $\sin\theta_2 = \frac{5}{13}$. $E_x = 0 = -\frac{k|q_1|}{(0.050\text{ m})^2}\frac{5}{13} + \frac{k|q_2|}{(0.120\text{ m})^2}\frac{12}{13}$. This gives $\frac{k|q_2|}{(0.120\text{ m})^2} = \frac{k|q_1|}{(0.050\text{ m})^2}\frac{5}{12}$.
Solving for $|q_2|$ gives $|q_2| = 7.2\ \mu\text{C}$, so $q_2 = -7.2\ \mu\text{C}$. Then

$$E_y = -\frac{k|q_1|}{(0.050\text{ m})^2}\frac{12}{13} - \frac{kq_2}{(0.120\text{ m})^2}\frac{5}{13} = -1.17\times10^7\ \text{N/C}.\quad E = 1.17\times10^7\ \text{N/C}.$$

EVALUATE: With q_1 known, specifying the direction of $\vec{E}$ determines both q_2 and E.

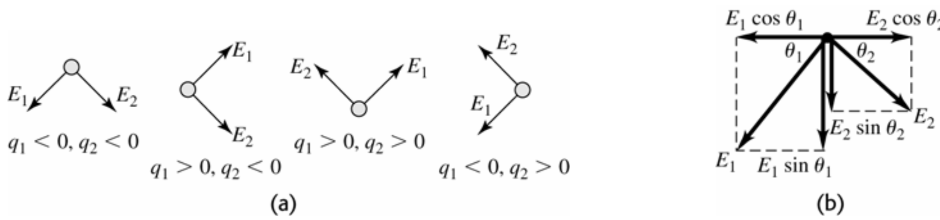


Figure 21.106

21.107. IDENTIFY: To find the electric field due to the second rod, divide that rod into infinitesimal segments of length dx, calculate the field dE due to each segment and integrate over the length of the rod to find the total field due to the rod. Use $d\vec{F} = dq\,\vec{E}$ to find the force the electric field of the second rod exerts on each infinitesimal segment of the first rod.

SET UP: An infinitesimal segment of the second rod is sketched in Figure 21.107. $dQ = (Q/L)dx'$.

EXECUTE: **(a)** $dE = \frac{k\,dQ}{(x+a/2+L-x')^2} = \frac{kQ}{L}\frac{dx'}{(x+a/2+L-x')^2}$.

$$E_x = \int_0^L dE_x = \frac{kQ}{L}\int_0^L \frac{dx'}{(x+a/2+L-x')^2} = \frac{kQ}{L}\left[\frac{1}{x+a/2+L-x'}\right]_0^L = \frac{kQ}{L}\left(\frac{1}{x+a/2} - \frac{1}{x+a/2+L}\right).$$

$$E_x = \frac{2kQ}{L}\left(\frac{1}{2x+a} - \frac{1}{2L+2x+a}\right).$$

(b) Now consider the force that the field of the second rod exerts on an infinitesimal segment dq of the first rod. This force is in the $+x$-direction. $dF = dq\,E$.

$$F = \int E\,dq = \int_{a/2}^{L+a/2}\frac{EQ}{L}\,dx = \frac{2kQ^2}{L^2}\int_{a/2}^{L+a/2}\left(\frac{1}{2x+a} - \frac{1}{2L+2x+a}\right)dx.$$

$$F = \frac{2kQ^2}{L^2}\frac{1}{2}\left([\ln(a+2x)]_{a/2}^{L+a/2} - [\ln(2L+2x+a)]_{a/2}^{L+a/2}\right) = \frac{kQ^2}{L^2}\ln\left(\left(\frac{a+2L+a}{2a}\right)\left(\frac{2L+2a}{4L+2a}\right)\right).$$

$$F = \frac{kQ^2}{L^2}\ln\left(\frac{(a+L)^2}{a(a+2L)}\right).$$

(c) For $a >> L$, $F = \frac{kQ^2}{L^2}\ln\left(\frac{a^2(1+L/a)^2}{a^2(1+2L/a)}\right) = \frac{kQ^2}{L^2}(2\ln(1+L/a) - \ln(1+2L/a))$.

For small z, $\ln(1+z) \approx z - \frac{z^2}{2}$. Therefore, for $a >> L$, $F \approx \frac{kQ^2}{L^2}\left(2\left(\frac{L}{a} - \frac{L^2}{2a^2} + \cdots\right) - \left(\frac{2L}{a} - \frac{2L^2}{a^2} + \cdots\right)\right) \approx \frac{kQ^2}{a^2}$.

EVALUATE: The distance between adjacent ends of the rods is a. When $a >> L$ the distance between the rods is much greater than their lengths and they interact as point charges.

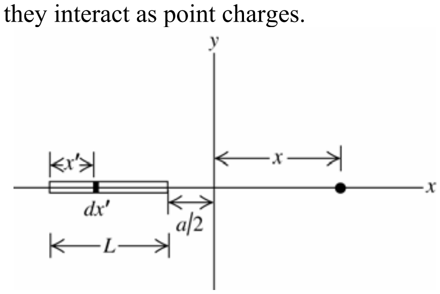


Figure 21.107