19

THE FIRST LAW OF THERMODYNAMICS

19.1. **(a) IDENTIFY and SET UP:** The pressure is constant and the volume increases.
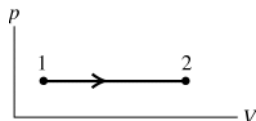

The pV-diagram is sketched in Figure 19.1

Figure 19.1

(b) $W = \int_{V_1}^{V_2} p \, dV$

Since p is constant, $W = p \int_{V_1}^{V_2} dV = p(V_2 - V_1)$

The problem gives T rather than p and V, so use the ideal gas law to rewrite the expression for W.

EXECUTE: $pV = nRT$ so $p_1V_1 = nRT_1$, $p_2V_2 = nRT_2$; subtracting the two equations gives

$p(V_2 - V_1) = nR(T_2 - T_1)$

Thus $W = nR(T_2 - T_1)$ is an alternative expression for the work in a constant pressure process for an ideal gas.

Then $W = nR(T_2 - T_1) = (2.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(107°C - 27°C) = +1330 \text{ J}$

EVALUATE: The gas expands when heated and does positive work.

19.2. **IDENTIFY:** At constant pressure, $W = p\Delta V = nR\Delta T$.

SET UP: $R = 8.3145 \text{ J/mol} \cdot \text{K}$. ΔT has the same numerical value in kelvins and in $C°$.

EXECUTE: $\Delta T = \dfrac{W}{nR} = \dfrac{1.75 \times 10^3 \text{ J}}{(6 \text{ mol}) (8.3145 \text{ J/mol} \cdot \text{K})} = 35.1 \text{ K}$. $\Delta T_K = \Delta T_C$ and $T_2 = 27.0°C + 35.1°C = 62.1°C$.

EVALUATE: When $W > 0$ the gas expands. When p is constant and V increases, T increases.

19.3. **IDENTIFY:** Example 19.1 shows that for an isothermal process $W = nRT \ln(p_1/p_2)$. $pV = nRT$ says V decreases when p increases and T is constant.

SET UP: $T = 358.15 \text{ K}$. $p_2 = 3p_1$.

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.3.

(b) $W = (2.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(358.15 \text{ K})\ln\left(\dfrac{p_1}{3p_1}\right) = -6540 \text{ J}$.
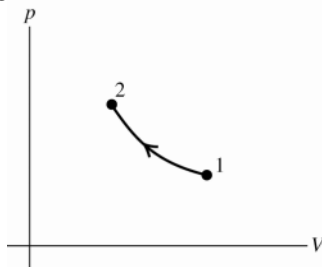
EVALUATE: Since V decreases, W is negative.

Figure 19.3

19.4. **IDENTIFY:** Use the expression for W that is appropriate to this type of process.

SET UP: The volume is constant.

EXECUTE: **(a)** The pV diagram is given in Figure 19.4.

(b) Since $\Delta V = 0$, $W = 0$.

EVALUATE: For any constant volume process the work done is zero.
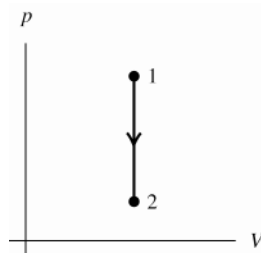

Figure 19.4

19.5.   **IDENTIFY:**   Example 19.1 shows that for an isothermal process $W = nRT \ln(p_1/p_2)$. Solve for p_1.

SET UP: For a compression (V decreases) W is negative, so $W = -518$ J. $T = 295.15$ K.

EXECUTE: **(a)** $\dfrac{W}{nRT} = \ln\left(\dfrac{p_1}{p_2}\right)$. $\dfrac{p_1}{p_2} = e^{W/nRT}$. $\dfrac{W}{nRT} = \dfrac{-518 \text{ J}}{(0.305 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(295.15 \text{ K})} = -0.692$.

$p_1 = p_2 e^{W/nRT} = (1.76 \text{ atm}) e^{-0.692} = 0.881$ atm.

(b) In the process the pressure increases and the volume decreases. The pV-diagram is sketched in Figure 19.5.

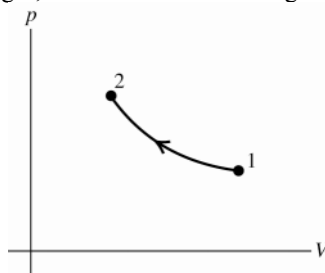**EVALUATE:** W is the work done by the gas, so when the surroundings do work on the gas, W is negative.

Figure 19.5

19.6.   **(a) IDENTIFY** and **SET UP:**   The pV-diagram is sketched in Figure 19.6.
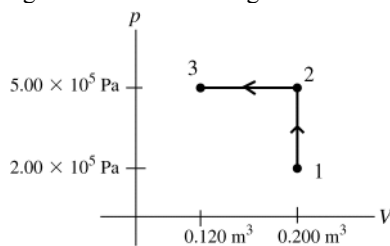

Figure 19.6

(b) Calculate W for each process, using the expression for W that applies to the specific type of process.

EXECUTE: $1 \rightarrow 2$, $\Delta V = 0$, so $W = 0$

$2 \rightarrow 3$

p is constant; so $W = p\,\Delta V = (5.00 \times 10^5 \text{ Pa})(0.120 \text{ m}^3 - 0.200 \text{ m}^3) = -4.00 \times 10^4$ J (W is negative since the volume decreases in the process.)

$W_{\text{tot}} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} = -4.00 \times 10^4$ J

EVALUATE: The volume decreases so the total work done is negative.

19.7.   **IDENTIFY:**   Calculate W for each step using the appropriate expression for each type of process.

SET UP: When p is constant, $W = p\Delta V$. When $\Delta V = 0$, $W = 0$.

EXECUTE: **(a)** $W_{13} = p_1(V_2 - V_1)$, $W_{32} = 0$, $W_{24} = p_2(V_1 - V_2)$ and $W_{41} = 0$. The total work done by the system is

$W_{13} + W_{32} + W_{24} + W_{41} = (p_1 - p_2)(V_2 - V_1)$, which is the area in the pV plane enclosed by the loop.

(b) For the process in reverse, the pressures are the same, but the volume changes are all the negatives of those found in part (a), so the total work is negative of the work found in part (a).

EVALUATE: When $\Delta V > 0$, $W > 0$ and when $\Delta V < 0$, $W < 0$.

19.8.   **IDENTIFY:**   Apply $\Delta U = Q - W$.

SET UP: For an ideal gas, U depends only on T.

EXECUTE: **(a)** V decreases and W is negative.

(b) Since T is constant, $\Delta U = 0$ and $Q = W$. Since W is negative, Q is negative.

(c) $Q = W$, the magnitudes are the same.

EVALUATE: $Q < 0$ means heat flows out of the gas. The plunger does positive work on the gas. The energy added by the positive work done on the gas leaves as heat flow out of the gas and the internal energy of the gas is constant.

19.9.   **IDENTIFY:** $\Delta U = Q - W$. For a constant pressure process, $W = p\Delta V$.

SET UP: $Q = +1.15 \times 10^5$ J, since heat enters the gas.

EXECUTE: **(a)** $W = p\Delta V = (1.80 \times 10^5 \text{ Pa})(0.320 \text{ m}^3 - 0.110 \text{ m}^3) = 3.78 \times 10^4$ J.

(b) $\Delta U = Q - W = 1.15 \times 10^5$ J $- 3.78 \times 10^4$ J $= 7.72 \times 10^4$ J.

EVALUATE: **(c)** $W = p\Delta V$ for a constant pressure process and $\Delta U = Q - W$ both apply to any material. The ideal gas law wasn't used and it doesn't matter if the gas is ideal or not.

19.10.   **IDENTIFY:**   The type of process is not specified. We can use $\Delta U = Q - W$ because this applies to all processes. Calculate ΔU and then from it calculate ΔT.

SET UP: Q is positive since heat goes into the gas; $Q = +1200$ J

W positive since gas expands; $W = +2100$ J

EXECUTE: $\Delta U = 1200$ J $- 2100$ J $= -900$ J

We can also use $\Delta U = n\left(\frac{3}{2}R\right)\Delta T$ since this is true for any process for an ideal gas.

$$\Delta T = \frac{2\,\Delta U}{3nR} = \frac{2(-900 \text{ J})}{3(5.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})} = -14.4\text{C}°$$

$T_2 = T_1 + \Delta T = 127°\text{C} - 14.4\text{C}° = 113°\text{C}$

EVALUATE: More energy leaves the gas in the expansion work than enters as heat. The internal energy therefore decreases, and for an ideal gas this means the temperature decreases. We didn't have to convert ΔT to kelvins since ΔT is the same on the Kelvin and Celsius scales.

19.11.   **IDENTIFY:**   Apply $\Delta U = Q - W$ to the air inside the ball.

SET UP: Since the volume decreases, W is negative. Since the compression is sudden, $Q = 0$.

EXECUTE: $\Delta U = Q - W$ with $Q = 0$ gives $\Delta U = -W$. $W < 0$ so $\Delta U > 0$. $\Delta U = +410$ J.

(b) Since $\Delta U > 0$, the temperature increases.

EVALUATE: When the air is compressed, work is done on the air by the force on the air. The work done on the air increases its energy. No energy leaves the gas as a flow of heat, so the internal energy increases.

19.12.   **IDENTIFY and SET UP:**   Calculate W using the equation for a constant pressure process. Then use $\Delta U = Q - W$ to calculate Q.

(a) EXECUTE: $W = \int_{V_1}^{V_2} p\, dV = p(V_2 - V_1)$ for this constant pressure process.

$W = (2.3 \times 10^5 \text{ Pa})(1.20 \text{ m}^3 - 1.70 \text{ m}^3) = -1.15 \times 10^5$ J   (The volume decreases in the process, so W is negative.)

(b) $\Delta U = Q - W$

$Q = \Delta U + W = -1.40 \times 10^5$ J $+ (-1.15 \times 10^5$ J$) = -2.55 \times 10^5$ J

Q negative means heat flows out of the gas.

(c) EVALUATE: $W = \int_{V_1}^{V_2} p\, dV = p(V_2 - V_1)$ (constant pressure) and $\Delta U = Q - W$ apply to *any* system, not just to an ideal gas. We did not use the ideal gas equation, either directly or indirectly, in any of the calculations, so the results are the same whether the gas is ideal or not.

19.13.   **IDENTIFY:**   Calculate the total food energy value for one doughnut. $K = \frac{1}{2}mv^2$.

SET UP: 1 cal $= 4.186$ J

EXECUTE: **(a)** The energy is $(2.0 \text{ g})(4.0 \text{ kcal/g}) + (17.0 \text{ g})(4.0 \text{ kcal/g}) + (7.0 \text{ g})(9.0 \text{ kcal/g}) = 139$ kcal.

The time required is $(139 \text{ kcal})/(510 \text{ kcal/h}) = 0.273$ h $= 16.4$ min.

(b) $v = \sqrt{2K/m} = \sqrt{2(139 \times 10^3 \text{ cal})(4.186 \text{ J/cal})/(60 \text{ kg})} = 139$ m/s $= 501$ km/h.

EVALUATE: When we set $K = Q$, we must express Q in J, so we can solve for v in m/s.

19.14.   **IDENTIFY:**   Apply $\Delta U = Q - W$.

SET UP: $W > 0$ when the system does work.

EXECUTE: **(a)** The container is said to be well-insulated, so there is no heat transfer.
(b) Stirring requires work. The stirring needs to be irregular so that the stirring mechanism moves against the water, not with the water.
(c) The work mentioned in part (b) is work done *on* the system, so $W < 0$, and since no heat has been transferred, $\Delta U = -W > 0$.
EVALUATE: The stirring adds energy to the liquid and this energy stays in the liquid as an increase in internal energy.

19.15.   **IDENTIFY:**   Apply $\Delta U = Q - W$ to the gas.

   SET UP: For the process, $\Delta V = 0$. $Q = +400$ J since heat goes into the gas.

   EXECUTE: **(a)** Since $\Delta V = 0$, $W = 0$.

   (b) $pV = nRT$ says $\dfrac{p}{T} = \dfrac{nR}{V} =$ constant. Since p doubles, T doubles. $T_b = 2T_a$.

   (c) Since $W = 0$, $\Delta U = Q = +400$ J. $U_b = U_a + 400$ J.

   EVALUATE: For an ideal gas, when T increases, U increases.

19.16.   **IDENTIFY:**   Apply $\Delta U = Q - W$. $|W|$ is the area under the path in the pV-plane.

   SET UP: $W > 0$ when V increases.
   EXECUTE: **(a)** The greatest work is done along the path that bounds the largest area above the V-axis in the p-V plane, which is path 1. The least work is done along path 3.
   (b) $W > 0$ in all three cases; $Q = \Delta U + W$, so $Q > 0$ for all three, with the greatest Q for the greatest work, that along path 1. When $Q > 0$, heat is absorbed.

   EVALUATE: ΔU is path independent and depends only on the initial and final states. W and Q are path independent and can have different values for different paths between the same initial and final states.

19.17.   **IDENTIFY:** $\Delta U = Q - W$. W is the area under the path in the pV-diagram. When the volume increases, $W > 0$.

   SET UP: For a complete cycle, $\Delta U = 0$.
   EXECUTE: **(a)** and **(b)** The clockwise loop (I) encloses a larger area in the p-V plane than the counterclockwise loop (II). Clockwise loops represent positive work and counterclockwise loops negative work, so $W_{\mathrm{I}} > 0$ and $W_{\mathrm{II}} < 0$. Over one complete cycle, the net work $W_{\mathrm{I}} + W_{\mathrm{II}} > 0$, and the net work done by the system is positive.
   (c) For the complete cycle, $\Delta U = 0$ and so $W = Q$. From part (a), $W > 0$, so $Q > 0$, and heat flows into the system.
   (d) Consider each loop as beginning and ending at the intersection point of the loops. Around each loop, $\Delta U = 0$, so $Q = W$; then, $Q_{\mathrm{I}} = W_{\mathrm{I}} > 0$ and $Q_{\mathrm{II}} = W_{\mathrm{II}} < 0$. Heat flows into the system for loop I and out of the system for loop II.
   EVALUATE: W and Q are path dependent and are in general not zero for a cycle.

19.18.   **IDENTIFY** and **SET UP:**   Deduce information about Q and W from the problem statement and then apply the first law, $\Delta U = Q - W$, to infer whether Q is positive or negative.

   EXECUTE: **(a)** For the water $\Delta T > 0$, so by $Q = mc\,\Delta T$ heat has been added to the water. Thus heat energy comes from the burning fuel-oxygen mixture, and Q for the system (fuel and oxygen) is negative.
   (b) Constant volume implies $W = 0$.
   (c) The 1st law (Eq.19.4) says $\Delta U = Q - W$.
   $Q < 0$, $W = 0$ so by the 1st law $\Delta U < 0$. The internal energy of the fuel-oxygen mixture decreased.
   EVALUATE: In this process internal energy from the fuel-oxygen mixture was transferred to the water, raising its temperature.

19.19.   **IDENTIFY:** $\Delta U = Q - W$. For a constant pressure process, $W = p\Delta V$.

   SET UP: $Q = +2.20 \times 10^6$ J; $Q > 0$ since this amount of heat goes into the water. $p = 2.00$ atm $= 2.03 \times 10^5$ Pa.

   EXECUTE: **(a)** $W = p\Delta V = (2.03 \times 10^5 \text{ Pa})(0.824 \text{ m}^3 - 1.00 \times 10^{-3} \text{ m}^3) = 1.67 \times 10^5$ J

   (b) $\Delta U = Q - W = 2.20 \times 10^6 \text{ J} - 1.67 \times 10^5 \text{ J} = 2.03 \times 10^6$ J.

   EVALUATE: 2.20×10^6 J of energy enters the water. 1.67×10^5 J of energy leaves the materials through expansion work and the remainder stays in the material as an increase in internal energy.

19.20.   **IDENTIFY:** $\Delta U = Q - W$

   SET UP: $Q < 0$ when heat leaves the gas.

   EXECUTE: For an isothermal process, $\Delta U = 0$, so $W = Q = -335$ J.

   EVALUATE: In a compression the volume decreases and $W < 0$.

19.21. **IDENTIFY:** For a constant pressure process, $W = p\Delta V$, $Q = nC_p\Delta T$ and $\Delta U = nC_V\Delta T$. $\Delta U = Q - W$ and $C_p = C_V + R$. For an ideal gas, $p\Delta V = nR\Delta T$.

SET UP: From Table 19.1, $C_V = 28.46$ J/mol·K.

EXECUTE: **(a)** The pV diagram is given in Figure 19.21.

(b) $W = pV_2 - pV_1 = nR(T_2 - T_1) = (0.250 \text{ mol})(8.3145 \text{ J/mol·K})(100.0 \text{ K}) = 208$ J.

(c) The work is done on the piston.

(d) Since Eq. (19.13) holds for any process, $\Delta U = nC_V\Delta T = (0.250 \text{ mol})(28.46 \text{ J/mol·K})(100.0 \text{ K}) = 712$ J.

(e) Either $Q = nC_p\Delta T$ or $Q = \Delta U + W$ gives $Q = 920$ J to three significant figures.

(f) The lower pressure would mean a correspondingly larger volume, and the net result would be that the work done would be the same as that found in part (b).

EVALUATE: $W = nR\Delta T$, so W, Q and ΔU all depend only on ΔT. When T increases at constant pressure, V increases and $W > 0$. ΔU and Q are also positive when T increases.
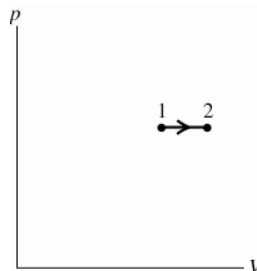

Figure 19.21

19.22. **IDENTIFY:** For constant volume $Q = nC_V\Delta T$. For constant pressure, $Q = nC_p\Delta T$. For any process of an ideal gas, $\Delta U = nC_V\Delta T$.

SET UP: $R = 8.315$ J/mol·K. For helium, $C_V = 12.47$ J/mol·K and $C_p = 20.78$ J/mol·K.

EXECUTE: **(a)** $Q = nC_V\Delta T = (0.0100 \text{ mol})(12.47\text{ J/mol·K})(40.0 \text{ C°}) = 4.99$ J. The pV-diagram is sketched in Figure 19.22a.

(b) $Q = nC_p\Delta T = (0.0100 \text{ mol})(20.78 \text{ J/mol·K})(40.0 \text{ C°}) = 8.31$ J. The pV-diagram is sketched in Figure 19.22b.

(c) More heat is required for the constant pressure process. ΔU is the same in both cases. For constant volume $W = 0$ and for constant pressure $W > 0$. The additional heat energy required for constant pressure goes into expansion work.

(d) $\Delta U = nC_V\Delta T = 4.99$ J for both processes. ΔU is path independent and for an ideal gas depends only on ΔT.
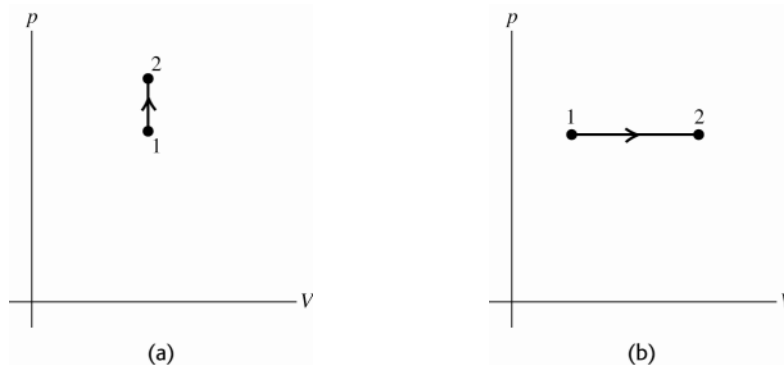
EVALUATE: $C_p = C_V + R$, so $C_p > C_V$.

Figure 19.22

19.23. **IDENTIFY:** For constant volume, $Q = nC_V\Delta T$. For constant pressure, $Q = nC_p\Delta T$.

SET UP: From Table 19.1, $C_V = 20.76$ J/mol·K and $C_p = 29.07$ J/mol·K

EXECUTE: **(a)** Using Equation (19.12), $\Delta T = \dfrac{Q}{nC_V} = \dfrac{645 \text{ J}}{(0.185 \text{ mol})(20.76 \text{ J/mol·K})} = 167.9$ K and $T = 948$ K.

The pV-diagram is sketched in Figure 19.23a.

(b) Using Equation (19.14), $\Delta T = \dfrac{Q}{nC_p} = \dfrac{645\text{ J}}{(0.185\text{ mol})(29.07\text{ J/mol}\cdot\text{K})} = 119.9$ K and $T = 900$ K.

The pV-diagram is sketched in Figure 19.23b.

EVALUATE: At constant pressure some of the heat energy added to the gas leaves the gas as expansion work and the internal energy change is less than if the same amount of heat energy is added at constant volume. ΔT is proportional to ΔU.
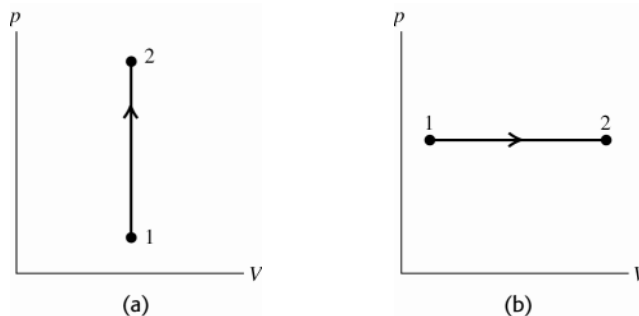

Figure 19.23

19.24.   **IDENTIFY** and **SET UP:**   Use information about the pressure and volume in the ideal gas law to determine the sign of ΔT, and from that the sign of Q.

EXECUTE: For constant p, $Q = nC_p\,\Delta T$

Since the gas is ideal, $pV = nRT$ and for constant p, $p\Delta V = nR\,\Delta T$.

$$Q = nC_p\left(\frac{p\Delta V}{nR}\right) = \left(\frac{C_p}{R}\right)p\Delta V$$

Since the gas expands, $\Delta V > 0$ and therefore $Q > 0$. $Q > 0$ means heat goes into gas.

EVALUATE: Heat flows into the gas, W is positive and the internal energy increases. It must be that $Q > W$.

19.25.   **IDENTIFY:** $\Delta U = Q - W$. For an ideal gas, $\Delta U = C_V\,\Delta T$, and at constant pressure, $W = p\Delta V = nR\,\Delta T$.

SET UP: $C_V = \frac{3}{2}R$ for a monatomic gas.

EXECUTE: $\Delta U = n(\frac{3}{2}R)\Delta T = \frac{3}{2}p\Delta V = \frac{3}{2}W$. Then $Q = \Delta U + W = \frac{5}{2}W$, so $W/Q = \frac{2}{5}$.

EVALUATE: For diatomic or polyatomic gases, C_V is a different multiple of R and the fraction of Q that is used for expansion work is different.

19.26.   **IDENTIFY:**   For an ideal gas, $\Delta U = C_V\,\Delta T$, and at constant pressure, $p\Delta V = nR\Delta T$.

SET UP: $C_V = \frac{3}{2}R$ for a monatomic gas.

EXECUTE: $\Delta U = n\left(\frac{3}{2}R\right)\Delta T = \frac{3}{2}p\Delta V = \frac{3}{2}(4.00\times10^4\text{ Pa})(8.00\times10^{-3}\text{ m}^3 - 2.00\times10^{-3}\text{ m}^3) = 360$ J.

EVALUATE: $W = nR\Delta T = \frac{2}{3}\Delta U = 240$ J. $Q = nC_p\Delta T = n(\frac{5}{2}R)\Delta T = \frac{5}{3}\Delta U = 600$ J. 600 J of heat energy flows into the gas. 240 J leaves as expansion work and 360 J remains in the gas as an increase in internal energy.

19.27.   **IDENTIFY:**   For a constant volume process, $Q = nC_V\Delta T$. For a constant pressure process, $Q = nC_p\Delta T$. For any process of an ideal gas, $\Delta U = nC_V\Delta T$.

SET UP: From Table 19.1, for N_2, $C_V = 20.76$ J/mol $\cdot$ K and $C_p = 29.07$ J/mol $\cdot$ K. Heat is added, so Q is positive and $Q = +1557$ J.

EXECUTE: **(a)** $\Delta T = \dfrac{Q}{nC_V} = \dfrac{1557\text{ J}}{(3.00\text{ mol})(20.76\text{ J/mol}\cdot\text{K})} = +25.0$ K

(b) $\Delta T = \dfrac{Q}{nC_p} = \dfrac{1557\text{ J}}{(3.00\text{ mol})(29.07\text{ J/mol}\cdot\text{K})} = +17.9$ K

(c) $\Delta U = nC_V\Delta T$ for either process, so ΔU is larger when ΔT is larger. The final internal energy is larger for the constant volume process in (a).

EVALUATE: For constant volume $W = 0$ and all the energy added as heat stays in the gas as internal energy. For the constant pressure process the gas expands and $W > 0$. Part of the energy added as heat leaves the gas as expansion work done by the gas.

19.28.   **IDENTIFY:**   Apply $pV = nRT$ to calculate T. For this constant pressure process, $W = p\Delta V$. $Q = nC_p\Delta T$. Use $\Delta U = Q - W$ to relate Q, W and ΔU.

SET UP: 2.50 atm = 2.53×10^5 Pa. For a monatomic ideal gas, $C_V = 12.47$ J/mol·K and $C_p = 20.78$ J/mol·K.

EXECUTE: **(a)** $T_1 = \dfrac{pV_1}{nR} = \dfrac{(2.53 \times 10^5 \text{ Pa})(3.20 \times 10^{-2} \text{ m}^2)}{(3.00 \text{ mol})(8.314 \text{ J/mol·K})} = 325$ K.

$T_2 = \dfrac{pV_2}{nR} = \dfrac{(2.53 \times 10^5 \text{ Pa})(4.50 \times 10^{-2} \text{ m}^2)}{(3.00 \text{ mol})(8.314 \text{ J/mol·K})} = 456$ K.

(b) $W = p\Delta V = (2.53 \times 10^5 \text{ Pa})(4.50 \times 10^{-2} \text{ m}^3 - 3.20 \times 10^{-2} \text{ m}^3) = 3.29 \times 10^3$ J

(c) $Q = nC_p\Delta T = (3.00 \text{ mol})(20.78 \text{ J/mol·K})(456 \text{ K} - 325 \text{ K}) = 8.17 \times 10^3$ J

(d) $\Delta U = Q - W = 4.88 \times 10^3$ J

EVALUATE: We could also calculate ΔU as $\Delta U = nC_V\Delta T = (3.00 \text{ mol})(12.47 \text{ J/mol·K})(456 \text{ K} - 325 \text{ K}) = 4.90 \times 10^3$ J, which agrees with the value we calculated in part (d).

19.29.   **IDENTIFY:**   Calculate W and ΔU and then use the first law to calculate Q.

(a) SET UP: $W = \displaystyle\int_{V_1}^{V_2} p \, dV$

$pV = nRT$ so $p = nRT/V$

$W = \displaystyle\int_{V_1}^{V_2} (nRT/V) \, dV = nRT \int_{V_1}^{V_2} dV/V = nRT \ln(V_2/V_1)$ (work done during an isothermal process).

EXECUTE: $W = (0.150 \text{ mol})(8.3145 \text{ J/mol·K})(350 \text{ K}) \ln(0.25V_1/V_1) = (436.5 \text{ J}) \ln(0.25) = -605$ J.

EVALUATE: W for the gas is negative, since the volume decreases.

(b) EXECUTE: $\Delta U = nC_V\Delta T$ for any ideal gas process.

$\Delta T = 0$ (isothermal) so $\Delta U = 0$.

EVALUATE: $\Delta U = 0$ for any ideal gas process in which T doesn't change.

(c) EXECUTE: $\Delta U = Q - W$

$\Delta U = 0$ so $Q = W = -605$ J. (Q is negative; the gas liberates 605 J of heat to the surroundings.)

EVALUATE: $Q = nC_V\Delta T$ is only for a constant volume process so doesn't apply here.

$Q = nC_p\Delta T$ is only for a constant pressure process so doesn't apply here.

19.30.   **IDENTIFY:** $C_p = C_V + R$ and $\gamma = \dfrac{C_p}{C_V}$.

SET UP: $R = 8.315$ J/mol·K

EXECUTE: $C_p = C_V + R$. $\gamma = \dfrac{C_p}{C_V} = 1 + \dfrac{R}{C_V}$. $C_V = \dfrac{R}{\gamma - 1} = \dfrac{8.315 \text{ J/mol·K}}{0.127} = 65.5$ J/mol·K. Then

$C_p = C_V + R = 73.8$ J/mol·K.

EVALUATE: The value of C_V is about twice the values for the polyatomic gases in Table 19.1. A propane molecule has more atoms and hence more internal degrees of freedom than the polyatomic gases in the table.

19.31.   **IDENTIFY:** $\Delta U = Q - W$. Apply $Q = nC_p\Delta T$ to calculate C_p. Apply $\Delta U = nC_V\Delta T$ to calculate C_V. $\gamma = C_p/C_V$.

SET UP: $\Delta T = 15.0 \text{ C}° = 15.0$ K. Since heat is added, $Q = +970$ J.

EXECUTE: **(a)** $\Delta U = Q - W = +970 \text{ J} - 223 \text{ J} = 747$ J

(b) $C_p = \dfrac{Q}{n\Delta T} = \dfrac{970 \text{ J}}{(1.75 \text{ mol})(15.0 \text{ K})} = 37.0$ J/mol·K. $C_V = \dfrac{\Delta U}{n\Delta T} = \dfrac{747 \text{ J}}{(1.75 \text{ mol})(15.0 \text{ K})} = 28.5$ J/mol·K.

$\gamma = \dfrac{C_p}{C_V} = \dfrac{37.0 \text{ J/mol·K}}{28.5 \text{ J/mol·K}} = 1.30$

EVALUATE: The value of γ we calculated is similar to the values given in Tables 19.1 for polyatomic gases.

19.32.   **IDENTIFY and SET UP:**   For an ideal gas $\Delta U = nC_V\Delta T$. The sign of ΔU is the same as the sign of ΔT. Combine Eq.(19.22) and the ideal gas law to obtain an equation relating T and p, and use it to determine the sign of ΔT.

EXECUTE: $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ and $V = nRT/p$ so, $T_1^\gamma p_1^{1-\gamma} = T_2^\gamma p_2^{1-\gamma}$ and $T_2^\gamma = T_1^\gamma (p_2/p_1)^{\gamma-1}$

$p_2 < p_1$ and $\gamma - 1$ is positive so $T_2 < T_1$. ΔT is negative so ΔU is negative; the energy of the gas decreases.

EVALUATE: Eq.(19.24) shows that the volume increases for this process, so it is an adiabatic expansion. In an adiabatic expansion the temperature decreases.

19.33. **IDENTIFY:** For an adiabatic process of an ideal gas, $p_1V_1^\gamma = p_2V_2^\gamma$, $W = \dfrac{1}{\gamma - 1}(p_1V_1 - p_2V_2)$ and $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$.

SET UP: For a monatomic ideal gas $\gamma = 5/3$.

EXECUTE: **(a)** $p_2 = p_1\left(\dfrac{V_1}{V_2}\right)^\gamma = (1.50\times10^5 \text{ Pa})\left(\dfrac{0.0800 \text{ m}^3}{0.0400 \text{ m}^3}\right)^{5/3} = 4.76\times10^5 \text{ Pa}.$

(b) This result may be substituted into Eq.(19.26), or, substituting the above form for p_2,

$W = \dfrac{1}{\gamma - 1}p_1V_1\left(1 - (V_1/V_2)^{\gamma-1}\right) = \dfrac{3}{2}(1.50\times10^5 \text{ Pa})(0.0800 \text{ m}^3)\left(1 - \left(\dfrac{0.0800}{0.0400}\right)^{2/3}\right) = -1.06\times10^4 \text{ J}.$

(c) From Eq.(19.22), $(T_2/T_1) = (V_2/V_1)^{\gamma-1} = (0.0800/0.0400)^{2/3} = 1.59$, and since the final temperature is higher than the initial temperature, the gas is heated.

EVALUATE: In an adiabatic compression $W < 0$ since $\Delta V < 0$. $Q = 0$ so $\Delta U = -W$. $\Delta U > 0$ and the temperature increases.

19.34. **IDENTIFY and SET UP:** **(a)** In the process the pressure increases and the volume decreases. The pV-diagram is sketched in Figure 19.34.
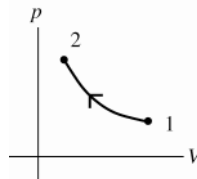

Figure 19.34

(b) For an adiabatic process for an ideal gas

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$, $p_1V_1^\gamma = p_2V_2^\gamma$, and $pV = nRT$

EXECUTE: From the first equation, $T_2 = T_1(V_1/V_2)^{\gamma-1} = (293 \text{ K})(V_1/0.0900V_1)^{1.4-1}$

$T_2 = (293 \text{ K})(11.11)^{0.4} = 768 \text{ K} = 495°C$

(Note: In the equation $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ the temperature *must* be in kelvins.)

$p_1V_1^\gamma = p_2V_2^\gamma$ implies $p_2 = p_1(V_1/V_2)^\gamma = (1.00 \text{ atm})(V_1/0.0900V_1)^{1.4}$

$p_2 = (1.00 \text{ atm})(11.11)^{1.4} = 29.1 \text{ atm}$

EVALUATE: Alternatively, we can use $pV = nRT$ to calculate p_2: n, R constant implies $pV/T = nR = $ constant so $p_1V_1/T_1 = p_2V_2/T_2$

$p_2 = p_1(V_1/V_2)(T_2/T_1) = (1.00 \text{ atm})(V_1/0.0900V_1)(768 \text{ K}/293 \text{ K}) = 29.1 \text{ atm}$, which checks.

19.35. **IDENTIFY:** For an adiabatic process of an ideal gas, $W = \dfrac{1}{\gamma - 1}(p_1V_1 - p_2V_2)$ and $p_1V_1^\gamma = p_2V_2^\gamma$.

SET UP: $\gamma = 1.40$ for an ideal diatomic gas. $1 \text{ atm} = 1.013\times10^5 \text{ Pa}$ and $1 \text{ L} = 10^{-3} \text{ m}^3$.

EXECUTE: $Q = \Delta U + W = 0$ for an adiabatic process, so $\Delta U = -W = \dfrac{1}{\gamma - 1}(p_2V_2 - p_1V_1)$. $p_1 = 1.22\times10^5 \text{ Pa}$.

$p_2 = p_1(V_1/V_2)^\gamma = (1.22\times10^5 \text{ Pa})(3)^{1.4} = 5.68\times10^5 \text{ Pa}.$

$W = \dfrac{1}{0.40}\left([5.68\times10^5 \text{ Pa}][10\times10^{-3} \text{ m}^3] - [1.22\times10^5 \text{ Pa}][30\times10^{-3} \text{ m}^3]\right) = 5.05\times10^3 \text{ J}$. The internal energy increases because work is done *on* the gas $(\Delta U > 0)$ and $Q = 0$. The temperature increases because the internal energy has increased.

EVALUATE: In an adiabatic compression $W < 0$ since $\Delta V < 0$. $Q = 0$ so $\Delta U = -W$. $\Delta U > 0$ and the temperature increases.

19.36. **IDENTIFY:** Assume the expansion is adiabatic. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ relates V and T. Assume the air behaves as an ideal gas, so $\Delta U = nC_V\Delta T$. Use $pV = nRT$ to calculate n.

SET UP: For air, $C_V = 29.76 \text{ J/mol·K}$ and $\gamma = 1.40$. $V_2 = 0.800V_1$. $T_1 = 293.15 \text{ K}$. $p_1 = 2.026\times10^5 \text{ Pa}$. For a sphere, $V = \dfrac{4}{3}\pi r^3$.

EXECUTE: **(a)** $T_2 = T_1 \left(\dfrac{V_1}{V_2} \right)^{\gamma-1} = (293.15 \text{ K}) \left(\dfrac{V_1}{0.800V_1} \right)^{0.40} = 320.5 \text{ K} = 47.4°\text{C}.$

(b) $V_1 = \frac{4}{3}\pi r^3 = \dfrac{4\pi}{3}(0.1195 \text{ m})^3 = 7.15\times10^{-3} \text{ m}^3. \quad n = \dfrac{p_1 V_1}{RT_1} = \dfrac{(2.026\times10^5 \text{ Pa})(7.15\times10^{-3} \text{ m}^3)}{(8.314 \text{ J/mol}\cdot\text{K})(293.15 \text{ K})} = 0.594 \text{ mol}.$

$\Delta U = nC_V \Delta T = (0.594 \text{ mol})(20.76 \text{ J/mol}\cdot\text{K})(321 \text{ K} - 293 \text{ K}) = 345 \text{ J}.$

EVALUATE: We could also use $\Delta U = W = \dfrac{1}{\gamma-1}(p_1 V_1 - p_2 V_2)$ to calculate $\Delta U,$ if we first found p_2 from $pV = nRT.$

19.37.   **(a) IDENTIFY** and **SET UP:**   In the expansion the pressure decreases and the volume increases. The pV-diagram is sketched in Figure 19.37.
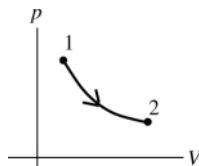
Figure 19.37

(b) Adiabatic means $Q = 0.$

Then $\Delta U = Q - W$ gives $W = -\Delta U = -nC_V \, \Delta T = nC_V(T_1 - T_2)$ (Eq.19.25).

$C_V = 12.47 \text{ J/mol}\cdot\text{K}$ (Table 19.1)

EXECUTE: $W = (0.450 \text{ mol})(12.47 \text{ J/mol}\cdot\text{K})(50.0°\text{C} - 10.0°\text{C}) = +224 \text{ J}$

W positive for $\Delta V > 0$ (expansion)

(c) $\Delta U = -W = -224 \text{ J}.$

EVALUATE: There is no heat energy input. The energy for doing the expansion work comes from the internal energy of the gas, which therefore decreases. For an ideal gas, when T decreases, U decreases.

19.38.   **IDENTIFY:** $pV = nRT.$ For an adiabatic process, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}.$

SET UP: For an ideal monatomic gas, $\gamma = 5/3.$

EXECUTE: **(a)** $T = \dfrac{pV}{nR} = \dfrac{(1.00\times10^5 \text{ Pa})\,(2.50\times10^{-3} \text{ m}^3)}{(0.1 \text{ mol})\,(8.3145 \text{ J/mol}\cdot\text{K})} = 301 \text{ K}.$

(b) (i) Isothermal: If the expansion is *isothermal,* the process occurs at constant temperature and the final temperature is the same as the initial temperature, namely 301 K. $p_2 = p_1(V_1/V_2) = \frac{1}{2}p_1 = 5.00\times10^4 \text{ Pa}.$

(ii) Isobaric: $\Delta p = 0$ so $p_2 = 1.00\times10^5 \text{ Pa}.$ $T_2 = T_1(V_2/V_1) = 2T_1 = 602 \text{ K}.$

(iii) Adiabatic: Using Equation (19.22), $T_2 = \dfrac{T_1 V_1^{\gamma-1}}{V_2^{\gamma-1}} = \dfrac{(301 \text{ K})(V_1)^{0.67}}{(2V_1)^{0.67}} = (301 \text{ K})\left(\tfrac{1}{2}\right)^{0.67} = 189 \text{ K}.$

EVALUATE: In an isobaric expansion, T increases. In an adiabatic expansion, T decreases.

19.39.   **IDENTIFY:**   Combine $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ with $pV = nRT$ to obtain an expression relating T and p for an adiabatic process of an ideal gas.

SET UP: $T_1 = 299.15 \text{ K}$

EXECUTE: $V = \dfrac{nRT}{p}$ so $T_1 \left(\dfrac{nRT_1}{p_1} \right)^{\gamma-1} = T_2 \left(\dfrac{nRT_2}{p_2} \right)^{\gamma-1}$ and $\dfrac{T_1^\gamma}{p_1^{\gamma-1}} = \dfrac{T_2^\gamma}{p_2^{\gamma-1}}.$

$T_2 = T_1 \left(\dfrac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} = (299.15 \text{ K}) \left(\dfrac{0.850\times10^5 \text{ Pa}}{1.01\times10^5 \text{ Pa}} \right)^{0.4/1.4} = 284.8 \text{ K} = 11.6 \text{ °C}$

EVALUATE: For an adiabatic process of an ideal gas, when the pressure decreases the temperature decreases.

19.40.   **IDENTIFY:**   Apply $\Delta U = Q - W.$ For any process of an ideal gas, $\Delta U = nC_V \Delta T.$ For an isothermal expansion,

$W = nRT \ln \left(\dfrac{V_2}{V_1} \right) = nRT \ln \left(\dfrac{p_1}{p_2} \right).$

SET UP: $T = 288.15 \text{ K}.$ $\dfrac{p_1}{p_2} = \dfrac{V_2}{V_1} = 2.00.$

EXECUTE: **(a)** $\Delta U = 0$ since $\Delta T = 0.$

(b) $W = (1.50 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(288.15 \text{ K})\ln(2.00) = 2.49\times10^3 \text{ J}.$ $W > 0$ and work is done by the gas. Since $\Delta U = 0,$ $Q = W = +2.49\times10^3 \text{ J}.$ $Q > 0$ so heat flows into the gas.

EVALUATE: When the volume increases, W is positive.

19.41.   IDENTIFY and SET UP: For an ideal gas, $pV = nRT$. The work done is the area under the path in the pV-diagram.

EXECUTE: (a) The product pV increases and this indicates a temperature increase.

(b) The work is the area in the pV plane bounded by the blue line representing the process and the verticals at V_a and V_b. The area of this trapezoid is $\frac{1}{2}(p_b + p_a)(V_b - V_a) = \frac{1}{2}(2.40 \times 10^5 \text{ Pa})(0.0400 \text{ m}^3) = 4800 \text{ J}$.

EVALUATE: The work done is the average pressure, $\frac{1}{2}(p_1 + p_2)$, times the volume increase.

19.42.   IDENTIFY: Use $pV = nRT$ to calculate T. W is the area under the process in the pV-diagram. Use $\Delta U = nC_V \Delta T$ and $\Delta U = Q - W$ to calculate Q.

SET UP: In state c, $p_c = 2.0 \times 10^5$ Pa and $V_c = 0.0040$ m^3. In state a, $p_a = 4.0 \times 10^5$ Pa and $V_a = 0.0020$ m^3.

EXECUTE: (a) $T_c = \dfrac{p_c V_c}{nR} = \dfrac{(2.0 \times 10^5 \text{ Pa})(0.0040 \text{ m}^3)}{(0.500 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = 192 \text{ K}$

(b) $W = \frac{1}{2}(4.0 \times 10^5 \text{ Pa} + 2.0 \times 10^5 \text{ Pa})(0.0030 \text{ m}^3 - 0.0020 \text{ m}^3) + (2.0 \times 10^5 \text{ Pa})(0.0040 \text{ m}^3 - 0.0030 \text{ m}^3)$

$W = +500$ J. 500 J of work is done by the gas.

(c) $T_a = \dfrac{p_a V_a}{nR} = \dfrac{(4.0 \times 10^5 \text{ Pa})(0.0020 \text{ m}^3)}{(0.500 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = 192$ K. For the process, $\Delta T = 0$, so $\Delta U = 0$ and $Q = W = +500$ J.

500 J of heat enters the system.

EVALUATE: The work done by the gas is positive since the volume increases.

19.43.   IDENTIFY: Use $\Delta U = Q - W$ and the fact that ΔU is path independent.

$W > 0$ when the volume increases, $W < 0$ when the volume decreases, and $W = 0$ when the volume is constant. $Q > 0$ if heat flows into the system.
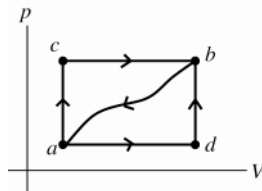
SET UP: The paths are sketched in Figure 19.43.

$Q_{acb} = +90.0$ J (positive since heat flows in)

$W_{acb} = +60.0$ J (positive since $\Delta V > 0$)

Figure 19.43

EXECUTE: (a) $\Delta U = Q - W$

ΔU is path independent; Q and W depend on the path.

$\Delta U = U_b - U_a$

This can be calculated for any path from a to b, in particular for path acb: $\Delta U_{a \to b} = Q_{acb} - W_{acb} = 90.0 \text{ J} - 60.0 \text{ J} = 30.0 \text{ J}$.

Now apply $\Delta U = Q - W$ to path adb; $\Delta U = 30.0$ J for this path also.

$W_{adb} = +15.0$ J (positive since $\Delta V > 0$)

$\Delta U_{a \to b} = Q_{adb} - W_{adb}$ so $Q_{acb} = \Delta U_{a \to b} + W_{adb} = 30.0 \text{ J} + 15.0 \text{ J} = +45.0 \text{ J}$

(b) Apply $\Delta U = Q - W$ to path ba: $\Delta U_{b \to a} = Q_{ba} - W_{ba}$

$W_{ba} = -35.0$ J (negative since $\Delta V < 0$)

$\Delta U_{b \to a} = U_a - U_b = -(U_b - U_a) = -\Delta U_{a \to b} = -30.0$ J

Then $Q_{ba} = \Delta U_{b \to a} + W_{ba} = -30.0 \text{ J} - 35.0 \text{ J} = -65.0 \text{ J}$.

($Q_{ba} < 0$; the system liberates heat.)

(c) $U_a = 0$, $U_d = 8.0$ J

$\Delta U_{a \to b} = U_b - U_a = +30.0$ J, so $U_b = +30.0$ J.

process $a \to d$

$\Delta U_{a \to d} = Q_{ad} - W_{ad}$

$\Delta U_{a \to d} = U_d - U_a = +8.0$ J

$W_{adb} = +15.0$ J and $W_{adb} = W_{ad} + W_{db}$. But the work W_{db} for the process $d \to b$ is zero since $\Delta V = 0$ for that process.

Therefore $W_{ad} = W_{adb} = +15.0$ J.

Then $Q_{ad} = \Delta U_{a \to d} + W_{ad} = +8.0 \text{ J} + 15.0 \text{ J} = +23.0 \text{ J}$ (positive implies heat absorbed).

process $d \rightarrow b$

$\Delta U_{d \rightarrow b} = Q_{db} - W_{db}$

$W_{db} = 0$, as already noted.

$\Delta U_{d \rightarrow b} = U_b - U_d = 30.0 \text{ J} - 8.0 \text{ J} = +22.0 \text{ J}.$

Then $Q_{db} = \Delta U_{d \rightarrow b} + W_{db} = +22.0 \text{ J}$ (positive; heat absorbed).

EVALUATE: The signs of our calculated Q_{ad} and Q_{db} agree with the problem statement that heat is absorbed in these processes.

19.44.   **IDENTIFY:** $\Delta U = Q - W.$

SET UP: $W = 0$ when $\Delta V = 0.$

EXECUTE: For each process, $Q = \Delta U + W.$ No work is done in the processes ab and dc, and so $W_{bc} = W_{abc} = 450 \text{ J}$ and $W_{ad} = W_{adc} = 120 \text{ J}.$ The heat flow for each process is: for ab, $Q = 90 \text{ J}.$ For bc, $Q = 440 \text{ J} + 450 \text{ J} = 890 \text{ J}.$ For ad, $Q = 180 \text{ J} + 120 \text{ J} = 300 \text{ J}.$ For dc, $Q = 350 \text{ J}.$ Heat is absorbed in each process. Note that the arrows representing the processes all point in the direction of increasing temperature (increasing U).

EVALUATE: ΔU is path independent so is the same for paths adc and abc. $Q_{adc} = 300 \text{ J} + 350 \text{ J} = 650 \text{ J}.$

$Q_{abc} = 90 \text{ J} + 890 \text{ J} = 980 \text{ J}.$ Q and W are path dependent and are different for these two paths.

19.45.   **IDENTIFY:**   Use $pV = nRT$ to calculate T_c / T_a. Calculate ΔU and W and use $\Delta U = Q - W$ to obtain Q.

SET UP: For path ac, the work done is the area under the line representing the process in the pV-diagram.

EXECUTE: **(a)** $\dfrac{T_c}{T_a} = \dfrac{p_c V_c}{p_a V_a} = \dfrac{(1.0 \times 10^5 \text{ J})(0.060 \text{ m}^3)}{(3.0 \times 10^5 \text{ J})(0.020 \text{ m}^3)} = 1.00.$ $T_c = T_a.$

(b) Since $T_c = T_a$, $\Delta U = 0$ for process abc. For ab, $\Delta V = 0$ and $W_{ab} = 0.$ For bc, p is constant and $W_{bc} = p\Delta V = (1.0 \times 10^5 \text{ Pa})(0.040 \text{ m}^3) = 4.0 \times 10^3 \text{ J}.$ Therefore, $W_{abc} = +4.0 \times 10^3 \text{ J}.$ Since $\Delta U = 0,$

$Q = W = +4.0 \times 10^3 \text{ J}.$ $4.0 \times 10^3 \text{ J}$ of heat flows into the gas during process abc.

(c) $W = \frac{1}{2}(3.0 \times 10^5 \text{ Pa} + 1.0 \times 10^5 \text{ Pa})(0.040 \text{ m}^3) = +8.0 \times 10^3 \text{ J}.$ $Q_{ac} = W_{ac} = +8.0 \times 10^3 \text{ J}.$

EVALUATE: The work done is path dependent and is greater for process ac than for process abc, even though the initial and final states are the same.

19.46.   **IDENTIFY:**   For a cycle, $\Delta U = 0$ and $Q = W.$ Calculate $W.$

SET UP: The magnitude of the work done by the gas during the cycle equals the area enclosed by the cycle in the pV-diagram.

EXECUTE: **(a)** The cycle is sketched in Figure 19.46.

(b) $|W| = (3.50 \times 10^4 \text{ Pa} - 1.50 \times 10^4 \text{ Pa})(0.0435 \text{ m}^3 - 0.0280 \text{ m}^3) = +310 \text{ J}.$ More negative work is done for cd than positive work for ab and the net work is negative. $W = -310 \text{ J}.$

(c) $Q = W = -310 \text{ J}.$ Since $Q < 0$, the net heat flow is out of the gas.

EVALUATE: During each constant pressure process $W = p\Delta V$ and during the constant volume process $W = 0.$
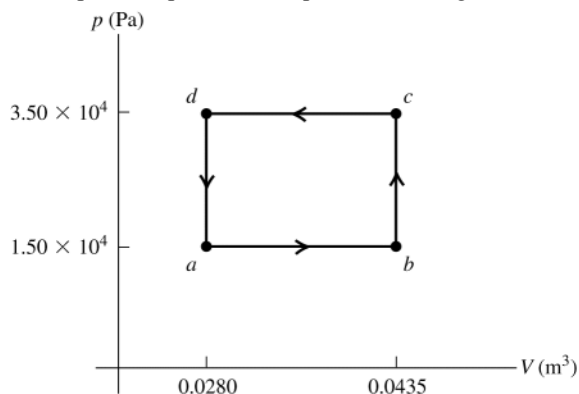

Figure 19.46

19.47.   **IDENTIFY:**   Use the 1st law to relate Q_{tot} to W_{tot} for the cycle.

Calculate W_{ab} and W_{bc} and use what we know about W_{tot} to deduce W_{ca}

(a) SET UP: We aren't told whether the pressure increases or decreases in process bc. The two possibilities for the cycle are sketched in Figure 19.47.
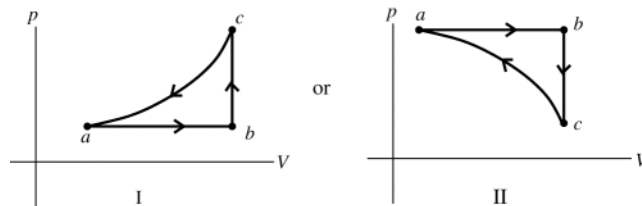

Figure 19.47

In cycle I, the total work is negative and in cycle II the total work is positive. For a cycle, $\Delta U = 0$, so $Q_{tot} = W_{tot}$

The net heat flow for the cycle is out of the gas, so heat $Q_{tot} < 0$ and $W_{tot} < 0$. Sketch I is correct.

(b) EXECUTE: $W_{tot} = Q_{tot} = -800$ J

$W_{tot} = W_{ab} + W_{bc} + W_{ca}$

$W_{bc} = 0$ since $\Delta V = 0$.

$W_{ab} = p\Delta V$ since p is constant. But since it is an ideal gas, $p\Delta V = nR\Delta T$

$W_{ab} = nR(T_b - T_a) = 1660$ J

$W_{ca} = W_{tot} - W_{ab} = -800$ J $- 1660$ J $= -2460$ J

EVALUATE: In process ca the volume decreases and the work W is negative.

19.48. **IDENTIFY:** Apply the appropriate expression for W for each type of process. $pV = nRT$ and $C_p = C_V + R$.

SET UP: $R = 8.315$ J/mol $\cdot$ K

EXECUTE: Path ac has constant pressure, so $W_{ac} = p\Delta V = nR\Delta T$, and

$W_{ac} = nR(T_c - T_a) = (3 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(492 \text{ K} - 300 \text{ K}) = 4.789 \times 10^3$ J.

Path cb is adiabatic $(Q = 0)$, so $W_{cb} = Q - \Delta U = -\Delta U = -nC_V\Delta T$, and using $C_V = C_p - R$,

$W_{cb} = -n(C_p - R)(T_b - T_c) = -(3 \text{ mol})(29.1 \text{ J/mol} \cdot \text{K} - 8.3145 \text{ J/mol} \cdot \text{K})(600 \text{ K} - 492 \text{ K}) = -6.735 \times 10^3$ J.

Path ba has constant volume, so $W_{ba} = 0$. So the total work done is

$$W = W_{ac} + W_{cb} + W_{ba} = 4.789 \times 10^3 \text{ J} - 6.735 \times 10^3 \text{ J} + 0 = -1.95 \times 10^3 \text{ J}.$$

EVALUATE: $W > 0$ when $\Delta V > 0$, $W < 0$ when $\Delta V < 0$ and $W = 0$ when $\Delta V = 0$.

19.49. **IDENTIFY:** Use $Q = nC_V\Delta T$ to calculate the temperature change in the constant volume process and use $pV = nRT$ to calculate the temperature change in the constant pressure process. The work done in the constant volume process is zero and the work done in the constant pressure process is $W = p\Delta V$. Use $Q = nC_p\Delta T$ to calculate the heat flow in the constant pressure process. $\Delta U = nC_V\Delta T$, or $\Delta U = Q - W$.

SET UP: For N_2, $C_V = 20.76$ J/mol $\cdot$ K and $C_p = 29.07$ J/mol $\cdot$ K.

EXECUTE: **(a)** For process ab, $\Delta T = \dfrac{Q}{nC_V} = \dfrac{1.52 \times 10^4 \text{ J}}{(2.50 \text{ mol})(20.76 \text{ J/mol} \cdot \text{K})} = 293$ K. $T_a = 293$ K, so $T_b = 586$ K.

$pV = nRT$ says T doubles when V doubles and p is constant, so $T_c = 2(586 \text{ K}) = 1172$ K $= 899°$C.

(b) For process ab, $W_{ab} = 0$. For process bc,

$W_{bc} = p\Delta V = nR\Delta T = (2.50 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(1172 \text{ K} - 586 \text{ K}) = 1.22 \times 10^4$ J. $W = W_{ab} + W_{bc} = 1.22 \times 10^4$ J.

(c) For process bc, $Q = nC_p\Delta T = (2.50 \text{ mol})(29.07 \text{ J/mol} \cdot \text{K})(1172 \text{ K} - 586 \text{ K}) = 4.26 \times 10^4$ J.

(d) $\Delta U = nC_V\Delta T = (2.50 \text{ mol})(20.76 \text{ J/mol} \cdot \text{K})(1172 \text{ K} - 293 \text{ K}) = 4.56 \times 10^4$ J.

EVALUATE: The total Q is 1.52×10^4 J $+ 4.26 \times 10^4$ J $= 5.78 \times 10^4$ J.

$\Delta U = Q - W = 5.78 \times 10^4$ J $- 1.22 \times 10^4$ J $= 4.56 \times 10^4$ J, which agrees with our results in part (d).

19.50. **IDENTIFY:** For a constant pressure process, $Q = nC_p\Delta T$. $\Delta U = Q - W$. $\Delta U = nC_V\Delta T$ for any ideal gas process.

SET UP: For N_2, $C_V = 20.76$ J/mol $\cdot$ K and $C_p = 29.07$ J/mol $\cdot$ K. $Q < 0$ if heat comes out of the gas.

EXECUTE: **(a)** $n = \dfrac{Q}{C_p\Delta T} = \dfrac{+2.5 \times 10^4 \text{ J}}{(29.07 \text{ J/mol} \cdot \text{K})(40.0 \text{ K})} = 21.5$ mol.

(b) $\Delta U = nC_V\Delta T = Q(C_V / C_p) = (-2.5 \times 10^4 \text{ J})(20.76/29.07) = -1.79 \times 10^4$ J.

(c) $W = Q - \Delta U = -7.15 \times 10^3$ J.

(d) ΔU is the same for both processes, and if $\Delta V = 0$, $W = 0$ and $Q = \Delta U = -1.79 \times 10^4$ J.

EVALUATE: For a given ΔT, Q is larger when the pressure is constant than when the volume is constant.

19.51.   **IDENTIFY** and **SET UP:**   Use the first law to calculate W and then use $W = p\Delta V$ for the constant pressure process to calculate ΔV.

EXECUTE: $\Delta U = Q - W$

$Q = -2.15 \times 10^5$ J (negative since heat energy goes out of the system)

$\Delta U = 0$ so $W = Q = -2.15 \times 10^5$ J

Constant pressure, so $W = \int_{V_1}^{V_2} p\, dV = p(V_2 - V_1) = p\Delta V$.

Then $\Delta V = \dfrac{W}{p} = \dfrac{-2.15 \times 10^5 \text{ J}}{9.50 \times 10^5 \text{ Pa}} = -0.226$ m^3.

EVALUATE: Positive work is done on the system by its surroundings; this inputs to the system the energy that then leaves the system as heat. Both Eq.(19.4) and (19.2) apply to all processes for any system, not just to an ideal gas.

19.52.   **IDENTIFY:** $pV = nRT$. For an isothermal process $W = nRT \ln(V_2 / V_1)$. For a constant pressure process, $W = p\Delta V$.

SET UP: $1 \text{ L} = 10^{-3}$ m^3.

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.52.

(b) At constant temperature, the product pV is constant, so $V_2 = V_1(p_1 / p_2) = (1.5 \text{ L})\left(\dfrac{1.00 \times 10^5 \text{ Pa}}{2.50 \times 10^4 \text{ Pa}}\right) = 6.00$ L. The final pressure is given as being the same as $p_3 = p_2 = 2.5 \times 10^4$ Pa. The final volume is the same as the initial volume, so $T_3 = T_1(p_3 / p_1) = 75.0$ K.

(c) Treating the gas as ideal, the work done in the first process is $W = nRT \ln(V_2 / V_1) = p_1 V_1 \ln(p_1 / p_2)$.

$W = (1.00 \times 10^5 \text{ Pa})(1.5 \times 10^{-3} \text{ m}^3) \ln\left(\dfrac{1.00 \times 10^5 \text{ Pa}}{2.50 \times 10^4 \text{ Pa}}\right) = 208$ J.
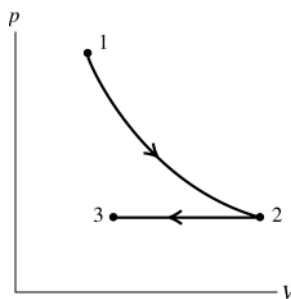
For the second process, $W = p_2(V_3 - V_2) = p_2(V_1 - V_2) = p_2 V_1(1 - (p_1 / p_2))$.

$W = (2.50 \times 10^4 \text{ Pa})(1.5 \times 10^{-3} \text{ m}^3)\left(1 - \dfrac{1.00 \times 10^5 \text{ Pa}}{2.50 \times 10^4 \text{ Pa}}\right) = -113$ J.

The total work done is $208 \text{ J} - 113 \text{ J} = 95$ J.

(d) Heat at constant volume. No work would be done by the gas or on the gas during this process.

EVALUATE: When the volume increases, $W > 0$. When the volume decreases, $W < 0$.

Figure 19.52

19.53.   **IDENTIFY:** $\Delta V = V_0 \beta \Delta T$. $W = p\Delta V$ since the force applied to the piston is constant. $Q = mc_p \Delta T$. $\Delta U = Q - W$.

SET UP: $m = \rho V$

EXECUTE: **(a)** The fractional change in volume is

$\Delta V = V_0 \beta \Delta T = (1.20 \times 10^{-2} \text{ m}^3)(1.20 \times 10^{-3} \text{ K}^{-1})(30.0 \text{ K}) = 4.32 \times 10^{-4}$ m^3.

(b) $W = p\Delta V = (F/A)\Delta V = ((3.00 \times 10^4 \text{ N})/(0.0200 \text{ m}^2))(4.32 \times 10^{-4} \text{ m}^3) = 648$ J.

(c) $Q = mc_p \Delta T = V_0 \rho c_p \Delta T = (1.20 \times 10^{-2} \text{ m}^3)(791 \text{ kg/m}^3)(2.51 \times 10^3 \text{ J/kg} \cdot \text{K})(30.0 \text{ K})$.

$Q = 7.15 \times 10^5$ J.

(d) $\Delta U = Q - W = 7.15 \times 10^5$ J to three figures.

(e) Under these conditions W is much less than Q and there is no substantial difference between c_V and c_p.

EVALUATE: $\Delta U = Q - W$ is valid for any material. For liquids the expansion work is much less than Q.

19.54. **IDENTIFY:** $\Delta V = \beta V_0 \Delta T$. $W = p\Delta V$ since the applied pressure (air pressure) is constant. $Q = mc_p \Delta T$. $\Delta U = Q - W$.

SET UP: For copper, $\beta = 5.1 \times 10^{-3}$ (C°)$^{-1}$, $c_p = 390$ J/kg·K and $\rho = 8.90 \times 10^3$ kg/m^3.

EXECUTE: **(a)** $\Delta V = \beta \Delta T V_0 = (5.1 \times 10^{-5} (C°)^{-1})(70.0 \ C°)(2.00 \times 10^{-2} \ m)^3 = 2.86 \times 10^{-8} \ m^3$.

(b) $W = p\Delta V = 2.88 \times 10^{-3}$ J.

(c) $Q = mc_p \Delta T = \rho V_0 c_p \Delta T = (8.9 \times 10^3 \ kg/m^3)(8.00 \times 10^{-6} \ m^3)(390 \ J/kg \cdot K)(70.0 \ C°) = 1944$ J.

(d) To three figures, $\Delta U = Q = 1940$ J.

(e) Under these conditions, the difference is not substantial, since W is much less than Q.

EVALUATE: $\Delta U = Q - W$ applies to any material. For solids the expansion work is much less than Q.

19.55. **IDENTIFY** and **SET UP:** The heat produced from the reaction is $Q_{reaction} = mL_{reaction}$, where $L_{reaction}$ is the heat of reaction of the chemicals.

$Q_{reaction} = W + \Delta U_{spray}$

EXECUTE: For a mass m of spray, $W = \frac{1}{2}mv^2 = \frac{1}{2}m(19 \ m/s)^2 = (180.5 \ J/kg)m$ and

$\Delta U_{spray} = Q_{spray} = mc\Delta T = m(4190 \ J/kg \cdot K)(100°C - 20°C) = (335,200 \ J/kg)m$.

Then $Q_{reaction} = (180 \ J/kg + 335,200 \ J/kg)m = (335,380 \ J/kg)m$ and $Q_{reaction} = mL_{reaction}$ implies

$mL_{reaction} = (335,380 \ J/kg)m$.

The mass m divides out and $L_{reaction} = 3.4 \times 10^5$ J/kg

EVALUATE: The amount of energy converted to work is negligible for the two significant figures to which the answer should be expressed. Almost all of the energy produced in the reaction goes into heating the compound.

19.56. **IDENTIFY:** The process is adiabatic. Apply $p_1 V_1^\gamma = p_2 V_2^\gamma$ and $pV = nRT$. $Q = 0$ so

$\Delta U = -W = -\frac{1}{\gamma - 1}(p_1 V_1 - p_2 V_2)$.

SET UP: For helium, $\gamma = 1.67$. $p_1 = 1.00$ atm $= 1.013 \times 10^5$ Pa. $V_1 = 2.00 \times 10^3 \ m^3$.

$p_2 = 0.900$ atm $= 9.117 \times 10^4$ Pa. $T_1 = 288.15$ K.

EXECUTE: **(a)** $V_2^\gamma = V_1^\gamma \left(\frac{p_1}{p_2}\right)$. $V_2 = V_1 \left(\frac{p_1}{p_2}\right)^{1/\gamma} = (2.00 \times 10^3 \ m^3)\left(\frac{1.00 \ atm}{0.900 \ atm}\right)^{1/1.67} = 2.13 \times 10^3 \ m^3$.

(b) $pV = nRT$ gives $\frac{T_1}{p_1 V_1} = \frac{T_2}{p_2 V_2}$.

$T_2 = T_1 \left(\frac{p_2}{p_1}\right)\left(\frac{V_2}{V_1}\right) = (288.15 \ K)\left(\frac{0.900 \ atm}{1.00 \ atm}\right)\left(\frac{2.13 \times 10^3 \ m^3}{2.00 \times 10^3 \ m^3}\right) = 276.2$ K $= 3.0°C$.

(c) $\Delta U = -\frac{1}{0.67}([1.013 \times 10^5 \ Pa)(2.00 \times 10^3 \ m^3)] - [9.117 \times 10^4 \ Pa)(2.13 \times 10^3 \ m^3)] = -1.25 \times 10^7$ J.

EVALUATE: The internal energy decreases when the temperature decreases.

19.57. **IDENTIFY:** For an adiabatic process of an ideal gas, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$. $pV = nRT$.

SET UP: For air, $\gamma = 1.40 = \frac{7}{5}$.

EXECUTE: **(a)** As the air moves to lower altitude its density increases; under an adiabatic compression, the temperature rises. If the wind is fast-moving, Q is not as likely to be significant, and modeling the process as adiabatic (no heat loss to the surroundings) is more accurate.

(b) $V = \frac{nRT}{p}$, so $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ gives $T_1^\gamma p_1^{1-\gamma} = T_2^\gamma p_2^{1-\gamma}$. The temperature at the higher pressure is

$T_2 = T_1 (p_1 / p_2)^{(\gamma-1)/\gamma} = (258.15 \ K)([8.12 \times 10^4 \ Pa]/[5.60 \times 10^4 \ Pa])^{2/7} = 287.1$ K $= 13.0°C$ so the temperature would rise by 11.9 C°.

EVALUATE: In an adiabatic compression, $Q = 0$ but the temperature rises because of the work done on the gas.

19.58.   **IDENTIFY:**   For constant pressure , $W = p\Delta V$. For an adiabatic process of an ideal gas, $W = \dfrac{C_V}{R}(p_1V_1 - p_2V_2)$ and $p_1V_1^\gamma = p_2V_2^\gamma$.

SET UP: $\gamma = \dfrac{C_p}{C_V} = \dfrac{C_p + C_V}{C_V} = 1 + \dfrac{R}{C_V}$

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.58.

(b) The work done is $W = p_0(2V_0 - V_0) + \dfrac{C_V}{R}(p_0(2V_0) - p_3(4V_0))$. $p_3 = p_0(2V_0/4V_0)^\gamma$ and so

$W = p_0V_0\left[1 + \dfrac{C_V}{R}(2 - 2^{2-\gamma})\right]$.   Note that p_0 is the absolute pressure.

(c) The most direct way to find the temperature is to find the ratio of the final pressure and volume to the original

and treat the air as an ideal gas. $p_3 = p_2\left(\dfrac{V_2}{V_3}\right)^\gamma = p_1\left(\dfrac{V_2}{V_3}\right)^\gamma$, since $p_1 = p_2$. Then

$$T_3 = T_0\dfrac{p_3V_3}{p_1V_1} = T_0\left(\dfrac{V_2}{V_3}\right)^\gamma\left(\dfrac{V_3}{V_1}\right) = T_0\left(\dfrac{1}{2}\right)^\gamma 4 = T_0(2)^{2-\gamma}.$$

(d) Since $n = \dfrac{p_0V_0}{RT_0}$, $Q = \dfrac{p_0V_0}{RT_0}(C_V + R)(2T_0 - T_0) = p_0V_0\left(\dfrac{C_V}{R} + 1\right)$. This amount of heat flows into the gas, since

$Q > 0$.

EVALUATE: In the isobaric expansion the temperature doubles and in the adiabatic expansion the temperature decreases. If the gas is diatomic, with $\gamma = \tfrac{7}{5}$, $2 - \gamma = \tfrac{3}{5}$ and $T_3 = 3.03T_0$, $W = 2.21p_0V_0$ and $Q = 3.50p_0V_0$.

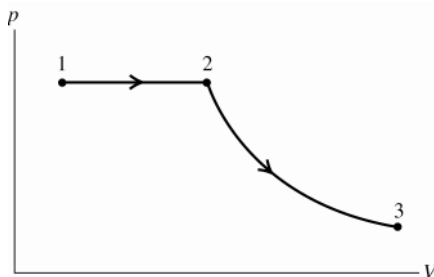$\Delta U = 1.29p_0V_0$. $\Delta U > 0$ and this is consistent with an increase in temperature.

Figure 19.58

19.59.   **IDENTIFY:**   Assume that the gas is ideal and that the process is adiabatic. Apply Eqs.(19.22) and (19.24) to relate pressure and volume and temperature and volume. The distance the piston moves is related to the volume of the gas. Use Eq.(19.25) to calculate W.

(a) SET UP: $\gamma = C_p/C_V = (C_V + R)/C_V = 1 + R/C_V = 1.40$. The two positions of the piston are shown in Figure 19.59.
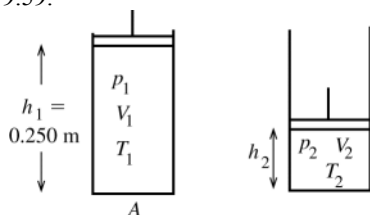

$p_1 = 1.01\times10^5$ Pa
$p_2 = 4.20\times10^5$ Pa $+ p_{air} = 5.21\times10^6$ Pa
$V_1 = h_1A$
$V_2 = h_2A$

Figure 19.59

EXECUTE: adiabatic process: $p_1V_1^\gamma = p_2V_2^\gamma$

$p_1h_1^\gamma A^\gamma = p_2h_2^\gamma A^\gamma$

$h_2 = h_1\left(\dfrac{p_1}{p_2}\right)^{1/\gamma} = (0.250\text{ m})\left(\dfrac{1.01\times10^5\text{ Pa}}{5.21\times10^5\text{ Pa}}\right)^{1/1.40} = 0.0774$ m

The piston has moved a distance $h_1 - h_2 = 0.250\text{ m} - 0.0774\text{ m} = 0.173$ m.

(b) $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$T_1 h_1^{\gamma-1} A^{\gamma-1} = T_2 h_2^{\gamma-1} A^{\gamma-1}$

$T_2 = T_1 \left(\dfrac{h_1}{h_2} \right)^{\gamma-1} = 300.1 \text{ K} \left(\dfrac{0.250 \text{ m}}{0.0774 \text{ m}} \right)^{0.40} = 479.7 \text{ K} = 207°\text{C}$

(c) $W = nC_V (T_1 - T_2)$ (Eq.19.25)

$W = (20.0 \text{ mol})(20.8 \text{ J/mol} \cdot \text{K})(300.1 \text{ K} - 479.7 \text{ K}) = -7.47 \times 10^4 \text{ J}$

EVALUATE: In an adiabatic compression of an ideal gas the temperature increases. In any compression the work W is negative.

19.60.   **IDENTIFY:** $m = \rho V$. The density of air is given by $\rho = \dfrac{pM}{RT}$. For an adiabatic process, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$.

$pV = nRT$

SET UP: Using $V = \dfrac{nRT}{p}$ in $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ gives $T_1 p_1^{1-\gamma} = T_2 p_2^{1-\gamma}$.

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.60.
(b) The final temperature is the same as the initial temperature, and the density is proportional to the absolute pressure. The mass needed to fill the cylinder is then

$$m = p_0 V \frac{p}{p_{\text{air}}} = (1.23 \text{ kg/m}^3)(575 \times 10^{-6} \text{ m}^3) \frac{1.45 \times 10^5 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} = 1.02 \times 10^{-3} \text{ kg}.$$

Without the turbocharger or intercooler the mass of air at $T = 15.0°\text{C}$ and $p = 1.01 \times 10^5$ Pa in a cylinder is

$m = \rho_0 V = 7.07 \times 10^{-4}$ kg. The increase in power is proportional to the increase in mass of air in the cylinder; the

percentage increase is $\dfrac{1.02 \times 10^{-3} \text{ kg}}{7.07 \times 10^{-4} \text{ kg}} - 1 = 0.44 = 44\%$.
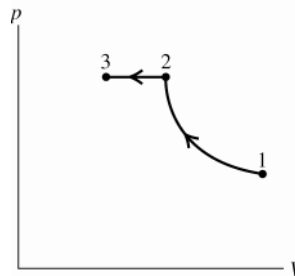
(c) The temperature after the adiabatic process is $T_2 = T_1 \left(\dfrac{p_2}{p_1} \right)^{(\gamma-1)/\gamma}$. The density becomes

$\rho = \rho_0 \left(\dfrac{T_1}{T_2} \right) \left(\dfrac{p_2}{p_1} \right) = \rho_0 \left(\dfrac{p_2}{p_1} \right)^{(1-\gamma)/\gamma} \left(\dfrac{p_2}{p_1} \right) = \rho_0 \left(\dfrac{p_2}{p_1} \right)^{1/\gamma}$. The mass of air in the cylinder is

$$m = (1.23 \text{ kg/m}^3)(575 \times 10^{-6} \text{ m}^3) \left(\frac{1.45 \times 10^5 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} \right)^{1/1.40} = 9.16 \times 10^{-4} \text{ kg},$$

The percentage increase in power is $\dfrac{9.16 \times 10^{-4} \text{ kg}}{7.07 \times 10^{-4} \text{ kg}} - 1 = 0.30 = 30\%$.

EVALUATE: The turbocharger and intercooler each have an appreciable effect on the engine power.

Figure 19.60

19.61.   **IDENTIFY:**   In each case calculate either ΔU or Q for the specific type of process and then apply the first law.
(a) SET UP: <u>isothermal</u> $(\Delta T = 0)$ $\Delta U = Q - W$; $W = +300$ J

For any process of an ideal gas, $\Delta U = nC_V \Delta T$.

EXECUTE: Therefore, for an ideal gas, if $\Delta T = 0$ then $\Delta U = 0$ and $Q = W = +300$ J.

(b) SET UP: <u>adiabatic</u> $(Q = 0)$

$\Delta U = Q - W$; $W = +300$ J

EXECUTE: $Q = 0$ says $\Delta U = -W = -300$ J

(c) SET UP: isobaric $\Delta p = 0$

Use W to calculate ΔT and then calculate Q.

EXECUTE: $W = p\Delta T = nR\Delta T$; $\Delta T = W/nR$

$Q = nC_p\Delta T$ and for a monatomic ideal gas $C_p = \frac{5}{2}R$

Thus $Q = n\frac{5}{2}R\Delta T = (5Rn/2)(W/nR) = 5W/2 = +750$ J.

$\Delta U = nC_V\Delta T$ for any ideal gas process and $C_V = C_p - R = \frac{3}{2}R$.

Thus $\Delta U = 3W/2 = +450$ J

EVALUATE: 300 J of energy leaves the gas when it performs expansion work. In the isothermal process this energy is replaced by heat flow into the gas and the internal energy remains the same. In the adiabatic process the energy used in doing the work decreases the internal energy. In the isobaric process 750 J of heat energy enters the gas, 300 J leaves as the work done and 450 J remains in the gas as increased internal energy.

19.62.   **IDENTIFY:** $pV = nRT$. For the isobaric process, $W = p\Delta V = nR\Delta T$. For the isothermal process,

$$W = nRT\ln\left(\frac{V_f}{V_i}\right).$$

SET UP: $R = 8.315$ J/mol $\cdot$ K

EXECUTE: **(a)** The pV diagram for these processes is sketched in Figure 19.62.

(b) Find T_2. For process $1 \rightarrow 2$, n, R, and p are constant so $\dfrac{T}{V} = \dfrac{p}{nR} =$ constant. $\dfrac{T_1}{V_1} = \dfrac{T_2}{V_2}$ and

$$T_2 = T_1\left(\frac{V_2}{V_1}\right) = (355 \text{ K})(2) = 710 \text{ K}.$$

(c) The maximum pressure is for state 3. For process $2 \rightarrow 3$, n, R, and T are constant. $p_2V_2 = p_3V_3$ and

$$p_3 = p_2\left(\frac{V_2}{V_3}\right) = (2.40 \times 10^5 \text{ Pa})(2) = 4.80 \times 10^5 \text{ Pa}.$$

(d) process $1 \rightarrow 2$: $W = p\Delta V = nR\Delta T = (0.250 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(710 \text{ K} - 355 \text{ K}) = 738$ K.

process $2 \rightarrow 3$: $W = nRT\ln\left(\dfrac{V_3}{V_2}\right) = (0.250 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(710 \text{ K})\ln\left(\dfrac{1}{2}\right) = -1023$ J.

process $3 \rightarrow 1$: $\Delta V = 0$ and $W = 0$.

The total work done is $738 \text{ J} + (-1023 \text{ J}) = -285$ J. This is the work done by the gas. The work done on the gas is 285 J.

EVALUATE: The final pressure and volume are the same as the initial pressure and volume, so the final state is the same as the initial state. For the cycle, $\Delta U = 0$ and $Q = W = -285$ J. During the cycle, 285 J of heat energy must leave the gas.

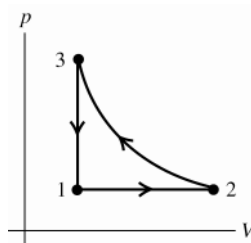**Figure 19.62**

19.63.   **IDENTIFY** and **SET UP:**   Use the ideal gas law, the first law and expressions for Q and W for specific types of processes.

EXECUTE: **(a)** initial expansion (state $1 \rightarrow$ state 2)

$p_1 = 2.40 \times 10^5$ Pa, $T_1 = 355$ K, $p_2 = 2.40 \times 10^5$ Pa, $V_2 = 2V_1$

$pV = nRT$; $T/V = p/nR =$ constant, so $T_1/V_1 = T_2/V_2$ and $T_2 = T_1(V_2/V_1) = 355 \text{ K}(2V_1/V_1) = 710$ K

$\Delta p = 0$ so $W = p\Delta V = nR\Delta T = (0.250 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(710 \text{ K} - 355 \text{ K}) = +738$ J

$Q = nC_p\Delta T = (0.250 \text{ mol})(29.17 \text{ J/mol} \cdot \text{K})(710 \text{ K} - 355 \text{ K}) = +2590$ J

$\Delta U = Q - W = 2590 \text{ J} - 738 \text{ J} = 1850$ J

(b) At the beginning of the final cooling process (cooling at constant volume), $T = 710$ K. The gas returns to its original volume and pressure, so also to its original temperature of 355 K.

$\Delta V = 0$ so $W = 0$

$Q = nC_V \Delta T = (0.250 \text{ mol})(20.85 \text{ J/mol} \cdot \text{K})(355 \text{ K} - 710 \text{ K}) = -1850$ J

$\Delta U = Q - W = -1850$ J.

(c) For any ideal gas process $\Delta U = nC_V \Delta T$. For an isothermal process $\Delta T = 0$, so $\Delta U = 0$.

EVALUATE: The three processes return the gas to its initial state, so $\Delta U_{\text{total}} = 0$; our results agree with this.

19.64.   **IDENTIFY:** $pV = nRT$. For an adiabatic process of an ideal gas, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$.
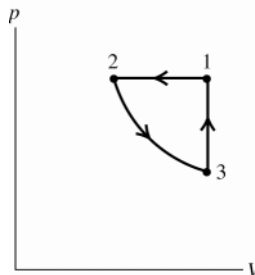
SET UP: For N_2, $\gamma = 1.40$.

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.64.

(b) At constant pressure, halving the volume halves the Kelvin temperature, and the temperature at the beginning of the adiabatic expansion is 150 K. The volume doubles during the adiabatic expansion, and from Eq. (19.22), the temperature at the end of the expansion is $(150 \text{ K})(1/2)^{0.40} = 114$ K.

(c) The minimum pressure occurs at the end of the adiabatic expansion (state 3). During the final heating the volume is held constant, so the minimum pressure is proportional to the Kelvin temperature,

$p_{\min} = (1.80 \times 10^5 \text{ Pa})(114 \text{ K}/300 \text{ K}) = 6.82 \times 10^4$ Pa.

EVALUATE: In the adiabatic expansion the temperature decreases.

Figure 19.64

19.65.   **IDENTIFY:**   Use the appropriate expressions for Q, W and ΔU for each type of process. $\Delta U = Q - W$ can also be used.

SET UP: For N_2, $C_V = 20.76$ J/mol $\cdot$ K and $C_p = 29.07$ J/mol $\cdot$ K.

EXECUTE: **(a)** $W = p\Delta V = nR\Delta T = (0.150 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(-150 \text{ K}) = -187$ J,

$Q = nC_p \Delta T = (0.150 \text{ mol})(29.07 \text{ mol} \cdot \text{K})(-150 \text{ K}) = -654$ J, $\Delta U = Q - W = -467$ J.

(b) From Eq. (19.24), using the expression for the temperature found in Problem 19.64,

$W = \dfrac{1}{0.40}(0.150 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(150 \text{ K})(1 - (1/2^{0.40}) = 113$ J. $Q = 0$ for an adiabatic process, and

$\Delta U = Q - W = -W = -113$ J.

(c) $\Delta V = 0$, so $W = 0$. Using the temperature change as found in Problem 19.64 and part (b),

$Q = nC_V \Delta T = (0.150 \text{ mol})(20.76 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 113.7 \text{ K}) = 580$ J and $\Delta U = Q - W = Q = 580$ J.

EVALUATE: For each process we could also use $\Delta U = nC_V \Delta T$ to calculate ΔU.

19.66.   **IDENTIFY:**   Use the appropriate expression for W for each type of process.

SET UP: For a monatomic ideal gas, $\gamma = 5/3$ and $C_V = 3R/2$.

EXECUTE: **(a)** $W = nRT \ln(V_2/V_1) = nRT \ln(3) = 3.29 \times 10^3$ J.

(b) $Q = 0$ so $W = -\Delta U = -nC_V \Delta T$. $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ gives $T_2 = T_1(1/3)^{2/3}$. Then

$W = nC_V T_1(1 - (1/3)^{2/3}) = 2.33 \times 10^3$ J.

(c) $V_2 = 3V_1$, so $W = p\Delta V = 2pV_1 = 2nRT_1 = 6.00 \times 10^3$ J.

(d) Each process is shown in Figure 19.66. The most work done is in the isobaric process, as the pressure is maintained at its original value. The least work is done in the adiabatic process.

(e) The isobaric process involves the most work and the largest temperature increase, and so requires the most heat. Adiabatic processes involve no heat transfer, and so the magnitude is zero.

(f) The isobaric process doubles the Kelvin temperature, and so has the largest change in internal energy. The isothermal process necessarily involves no change in internal energy.

EVALUATE: The work done is the area under the path for the process in the pV-diagram. Figure 19.66 shows that the work done is greatest in the isobaric process and least in the adiabatic process.
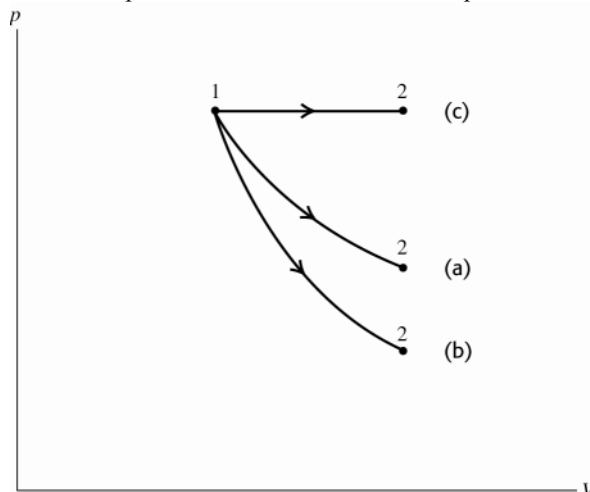

Figure 19.66

19.67.   **IDENTIFY:**   Assume the compression is adiabatic. Apply $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ and $pV = nRT$.

SET UP: For N_2, $\gamma = 1.40$. $V_1 = 3.00$ L, $p = 1.00$ atm $= 1.013 \times 10^5$ Pa, $T = 273.15$ K. $V_2 = V_1/2 = 1.50$ L.

EXECUTE: **(a)** $T_2 = T_1 \left(\dfrac{V_1}{V_2} \right)^{\gamma-1} = (273.15 \text{ K}) \left(\dfrac{V_1}{V_1/2} \right)^{0.40} = (273.15 \text{ K})(2)^{0.4} = 360.4 \text{ K} = 87.3°C$. $\dfrac{p_1 V_1}{T_1} = \dfrac{p_2 V_2}{T_2}$.

$p_2 = p_1 \left(\dfrac{V_1}{V_2} \right) \left(\dfrac{T_2}{T_1} \right) = (1.00 \text{ atm}) \left(\dfrac{V_1}{V_1/2} \right) \left(\dfrac{360.4 \text{ K}}{273.15 \text{ K}} \right) = 2.64$ atm.

(b) p is constant, so $\dfrac{V}{T} = \dfrac{nR}{T} = $ constant and $\dfrac{V_2}{T_2} = \dfrac{V_3}{T_3}$. $V_3 = V_2 \left(\dfrac{T_3}{T_2} \right) = (1.50 \text{ L}) \left(\dfrac{273.15 \text{ K}}{360.4 \text{ K}} \right) = 1.14$ L.

EVALUATE: In an adiabatic compression the temperature increases.

19.68.   **IDENTIFY:**   At equilibrium the net upward force of the gas on the piston equals the weight of the piston. When the piston moves upward the gas expands, the pressure of the gas drops and there is a net downward force on the piston. For simple harmonic motion the net force has the form $F_y = -ky$, for a displacement y from equilibrium,

and $f = \dfrac{1}{2\pi} \sqrt{\dfrac{k}{m}}$.

SET UP: $pV = nRT$. T is constant.

(a) The difference between the pressure, inside and outside the cylinder, multiplied by the area of the piston, must be the weight of the piston. The pressure in the trapped gas is $p_0 + \dfrac{mg}{A} = p_0 + \dfrac{mg}{\pi r^2}$.

(b) When the piston is a distance $h + y$ above the cylinder, the pressure in the trapped gas is $\left(p_0 + \dfrac{mg}{\pi r^2} \right) \left(\dfrac{h}{h + y} \right)$

and for values of y small compared to h, $\dfrac{h}{h + y} = \left(1 + \dfrac{y}{h} \right)^{-1} \sim 1 - \dfrac{y}{h}$. The net force, taking the positive direction to

be upward, is the then $F_y = \left[\left(p_0 + \dfrac{mg}{\pi r^2} \right) \left(1 - \dfrac{y}{h} \right) - p_0 \right] (\pi r^2) - mg = -\left(\dfrac{y}{h} \right) (p_0 \pi r^2 + mg)$.

This form shows that for positive h, the net force is down; the trapped gas is at a lower pressure than the equilibrium pressure, and so the net force tends to restore the piston to equilibrium.

(c) The angular frequency of small oscillations would be given by $\omega^2 = \dfrac{(p_0 \pi r^2 + mg)/h}{m} = \dfrac{g}{h} \left(1 + \dfrac{p_0 \pi r^2}{mg} \right)$.

$f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi} \sqrt{\dfrac{g}{h} \left(1 + \dfrac{p_0 \pi r^2}{mg} \right)^{1/2}}$.

If the displacements are not small, the motion is not simple harmonic. This can be seen be considering what happens if $y \sim -h$; the gas is compressed to a very small volume, and the force due to the pressure of the gas would become unboundedly large for a finite displacement, which is not characteristic of simple harmonic motion. If $y \gg h$ (but not so large that the piston leaves the cylinder), the force due to the pressure of the gas becomes small, and the restoring force due to the atmosphere and the weight would tend toward a constant, and this is not characteristic of simple harmonic motion.

EVALUATE: The assumption of small oscillations was made when $\dfrac{h}{h+y}$ was replaced by $1-y/h$; this is accurate only when y/h is small.

19.69.   **IDENTIFY:** $W = \int_{V_1}^{V_2} p\,dV$.

SET UP: For an isothermal process of an ideal gas, $W = nRT \ln\left(V_2/V_1\right)$.

EXECUTE: **(a)** Solving for p as a function of V and T and integrating with respect to V,

$p = \dfrac{nRT}{V-nb} - \dfrac{an^2}{V^2}$ and $W = \int_{V_1}^{V_2} p\,dV = nRT\ln\left[\dfrac{V_2-nb}{V_1-nb}\right] + an^2\left[\dfrac{1}{V_2} - \dfrac{1}{V_1}\right]$.

When $a = b = 0$, $W = nRT \ln\left(V_2/V_1\right)$, as expected.

(b) (i) Using the expression found in part (a),
$W = (1.80 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})$

$\times \ \ln\left[\dfrac{\left(4.00\times10^{-3} \text{ m}^3\right) - (1.80 \text{ mol})\left(6.38\times10^{-5} \text{ m}^2/\text{mol}\right)}{\left(2.00\times10^{-3} \text{ m}^3\right) - (1.80 \text{ mol})\left(6.38\times10^{-5} \text{ m}^2/\text{mol}\right)}\right]$

$+ \left(0.554 \text{ J} \cdot \text{m}^3/\text{mol}^2\right)(1.80 \text{ mol})^2\left[\dfrac{1}{4.00\times10^{-3} \text{ m}^3} - \dfrac{1}{2.00\times10^{-3} \text{ m}^3}\right]$

$W = 2.80\times10^3 \text{ J}$.

(ii) $W = nRT \ln(2) = 3.11\times10^3 \text{ J}$.

(c) The work for the ideal gas is larger by about 300 J. For this case, the difference due to nonzero a is more than that due to nonzero b. The presence of a nonzero a indicates that the molecules are attracted to each other and so do not do as much work in the expansion.

EVALUATE: The difference in the two results for W is about 10%, which can be considered to be important.