MECHANICAL WAVES

15

15.1. IDENTIFY: $v = f\lambda$. $T = 1/f$ is the time for one complete vibration.

SET UP: The frequency of the note one octave higher is 1568 Hz.

EXECUTE: **(a)** $\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{784 \text{ Hz}} = 0.439 \text{ m}$. $T = \frac{1}{f} = 1.28 \text{ ms}$.

(b) $\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{1568 \text{ Hz}} = 0.219 \text{ m}$.

EVALUATE: When f is doubled, λ is halved.

15.2. IDENTIFY: The distance between adjacent dots is λ. $v = f\lambda$. The long-wavelength sound has the lowest frequency, 20.0 Hz, and the short-wavelength sound has the highest frequency, 20.0 kHz.

SET UP: For sound in air, $v = 344 \text{ m/s}$.

EXECUTE: **(a)** Red dots: $\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{20.0 \text{ Hz}} = 17.2 \text{ m}$.

Blue dots: $\lambda = \frac{344 \text{ m/s}}{20.0 \times 10^3 \text{ Hz}} = 0.0172 \text{ m} = 1.72 \text{ cm}$.

(b) In each case the separation easily can be measured with a meterstick.

(c) Red dots: $\lambda = \frac{v}{f} = \frac{1480 \text{ m/s}}{20.0 \text{ Hz}} = 74.0 \text{ m}$.

Blue dots: $\lambda = \frac{1480 \text{ m/s}}{20.0 \times 10^3 \text{ Hz}} = 0.0740 \text{ m} = 7.40 \text{ cm}$. In each case the separation easily can be measured with a meterstick, although for the red dots a long tape measure would be more convenient.

EVALUATE: Larger wavelengths correspond to smaller frequencies. When the wave speed increases, for a given frequency, the wavelength increases.

15.3. IDENTIFY: $v = f\lambda = \lambda / T$.

SET UP: $1.0 \text{ h} = 3600 \text{ s}$. The crest to crest distance is λ.

EXECUTE: $v = \frac{800 \times 10^3 \text{ m}}{3600 \text{ s}} = 220 \text{ m/s}$. $v = \frac{800 \text{ km}}{1.0 \text{ h}} = 800 \text{ km/h}$.

EVALUATE: Since the wave speed is very high, the wave strikes with very little warning.

15.4. IDENTIFY: $f\lambda = v$

SET UP: $1.0 \text{ mm} = 0.0010 \text{ m}$

EXECUTE: $f = \frac{v}{\lambda} = \frac{1500 \text{ m/s}}{0.0010 \text{ m}} = 1.5 \times 10^6 \text{ Hz}$

EVALUATE: The frequency is much higher than the upper range of human hearing.

15.5. IDENTIFY: $v = f\lambda$. $T = 1/f$.

SET UP: $1 \text{ nm} = 10^{-9} \text{ m}$

EXECUTE: **(a)** $\lambda = 400 \text{ nm}$: $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{400 \times 10^{-9} \text{ m}} = 7.50 \times 10^{14} \text{ Hz}$. $T = 1/f = 1.33 \times 10^{-15} \text{ s}$.

$\lambda = 700 \text{ nm}$: $f = \frac{3.00 \times 10^8 \text{ m/s}}{700 \times 10^{-9} \text{ m}} = 4.29 \times 10^{14} \text{ Hz}$. $T = 2.33 \times 10^{-15} \text{ s}$. The frequencies of visible light lie between $4.29 \times 10^{14} \text{ Hz}$ and $7.50 \times 10^{14} \text{ Hz}$. The periods lie between $1.33 \times 10^{-15} \text{ s}$ and $2.33 \times 10^{-15} \text{ s}$.

(b) T is very short and cannot be measured with a stopwatch.

EVALUATE: Longer wavelength corresponds to smaller frequency and larger period.

15.6. **IDENTIFY:** Compare $y(x,t)$ given in the problem to the general form of Eq.(15.4). $f = 1/T$ and $v = f\lambda$

SET UP: The comparison gives $A = 6.50$ mm, $\lambda = 28.0$ cm and $T = 0.0360$ s.

EXECUTE: **(a)** 6.50 mm

(b) 28.0 cm

(c) $f = \dfrac{1}{0.0360 \text{ s}} = 27.8 \text{ Hz}$

(d) $v = (0.280 \text{ m})(27.8 \text{ Hz}) = 7.78 \text{ m/s}$

(e) Since there is a minus sign in front of the t/T term, the wave is traveling in the $+x$-direction.

EVALUATE: The speed of propagation does not depend on the amplitude of the wave.

15.7. **IDENTIFY:** Use Eq.(15.1) to calculate v. $T = 1/f$ and k is defined by Eq.(15.5). The general form of the wave function is given by Eq.(15.8), which is the equation for the transverse displacement.

SET UP: $v = 8.00$ m/s, $A = 0.0700$ m, $\lambda = 0.320$ m

EXECUTE: **(a)** $v = f\lambda$ so $f = v/\lambda = (8.00 \text{ m/s})/(0.320 \text{ m}) = 25.0 \text{ Hz}$

$T = 1/f = 1/25.0 \text{ Hz} = 0.0400 \text{ s}$

$k = 2\pi/\lambda = 2\pi \text{ rad}/0.320 \text{ m} = 19.6 \text{ rad/m}$

(b) For a wave traveling in the $-x$-direction,

$y(x,\ t) = A\cos 2\pi(x/\lambda + t/T)$ (Eq.(15.8).)

At $x = 0$, $y(0,\ t) = A\cos 2\pi(t/T)$, so $y = A$ at $t = 0$. This equation describes the wave specified in the problem.

Substitute in numerical values:

$y(x,\ t) = (0.0700 \text{ m})\cos(2\pi(x/0.320 \text{ m} + t/0.0400 \text{ s}))$.

Or, $y(x,\ t) = (0.0700 \text{ m})\cos((19.6 \text{ m}^{-1})x + (157 \text{ rad/s})t)$.

(c) From part (b), $y = (0.0700 \text{ m})\cos(2\pi(x/0.320 \text{ m} + t/0.0400 \text{ s}))$.

Plug in $x = 0.360$ m and $t = 0.150$ s:

$y = (0.0700 \text{ m})\cos(2\pi(0.360 \text{ m}/0.320 \text{ m} + 0.150 \text{ s}/0.0400 \text{ s}))$

$y = (0.0700 \text{ m})\cos[2\pi(4.875 \text{ rad})] = +0.0495 \text{ m} = +4.95 \text{ cm}$

(d) In part (c) $t = 0.150$ s.

$y = A$ means $\cos(2\pi(x/\lambda + t/T)) = 1$

$\cos\theta = 1$ for $\theta = 0,\ 2\pi,\ 4\pi, \ldots = n(2\pi)$ or $n = 0,\ 1,\ 2, \ldots$

So $y = A$ when $2\pi(x/\lambda + t/T) = n(2\pi)$ or $x/\lambda + x/T = n$

$t = T(n - x/\lambda) = (0.0400 \text{ s})(n - 0.360 \text{ m}/0.320 \text{ m}) = (0.0400 \text{ s})(n - 1.125)$

For $n = 4$, $t = 0.1150$ s (before the instant in part (c))

For $n = 5$, $t = 0.1550$ s (the first occurrence of $y = A$ after the instant in part (c)) Thus the elapsed time is $0.1550 \text{ s} - 0.1500 \text{ s} = 0.0050 \text{ s}$.

EVALUATE: Part (d) says $y = A$ at 0.115 s and next at 0.155 s; the difference between these two times is 0.040 s, which is the period. At $t = 0.150$ s the particle at $x = 0.360$ m is at $y = 4.95$ cm and traveling upward. It takes $T/4 = 0.0100$ s for it to travel from $y = 0$ to $y = A$, so our answer of 0.0050 s is reasonable.

15.8. **IDENTIFY:** The general form of the wave function for a wave traveling in the $-x$-direction is given by Eq.(15.8). The time for one complete cycle to pass a point is the period T and the number that pass per second is the frequency f. The speed of a crest is the wave speed v and the maximum speed of a particle in the medium is $v_{max} = \omega A$.

SET UP: Comparison to Eq.(15.8) gives $A = 3.75$ cm, $k = 0.450$ rad/cm and $\omega = 5.40$ rad/s.

EXECUTE: **(a)** $T = \dfrac{2\pi \text{ rad}}{\omega} = \dfrac{2\pi \text{ rad}}{5.40 \text{ rad/s}} = 1.16 \text{ s}$. In one cycle a wave crest travels a distance

$\lambda = \dfrac{2\pi \text{ rad}}{k} = \dfrac{2\pi \text{ rad}}{0.450 \text{ rad/cm}} = 0.140 \text{ m}$.

(b) $k = 0.450$ rad/cm. $f = 1/T = 0.862 \text{ Hz} = 0.862$ waves/second.

(c) $v = f\lambda = (0.862 \text{ Hz})(0.140 \text{ m}) = 0.121 \text{ m/s}$. $v_{max} = \omega A = (5.40 \text{ rad/s})(3.75 \text{ cm}) = 0.202 \text{ m/s}$.

EVALUATE: The transverse velocity of the particles in the medium (water) is not the same as the velocity of the wave.

15.9. **IDENTIFY:** Evaluate the partial derivatives and see if Eq.(15.12) is satisfied.

SET UP: $\frac{\partial}{\partial x}\cos(kx+\omega t)=-k\sin(kx+\omega t)$. $\frac{\partial}{\partial t}\cos(kx+\omega t)=-\omega\sin(kx+\omega t)$. $\frac{\partial}{\partial x}\sin(kx+\omega t)=k\cos(kx+\omega t)$. $\frac{\partial}{\partial t}\sin(kx+\omega t)=\omega\sin(kx+\omega t)$.

EXECUTE: **(a)** $\frac{\partial^2 y}{\partial x^2}=-Ak^2\cos(kx+\omega t)$. $\frac{\partial^2 y}{\partial t^2}=-A\omega^2\cos(kx+\omega t)$. Eq.(15.12) is satisfied, if $v=\omega/k$.

(b) $\frac{\partial^2 y}{\partial x^2}=-Ak^2\sin(kx+\omega t)$. $\frac{\partial^2 y}{\partial t^2}=-A\omega^2\sin(kx+\omega t)$. Eq.(15.12) is satisfied, if $v=\omega/k$.

(c) $\frac{\partial y}{\partial x}=-kA\sin(kx)$. $\frac{\partial^2 y}{\partial x^2}=-k^2A\cos(kx)$. $\frac{\partial y}{\partial t}=-\omega A\sin(\omega t)$. $\frac{\partial^2 y}{\partial t^2}=-\omega^2A\cos(\omega t)$. Eq.(15.12) is not satisfied.

(d) $v_y=\frac{\partial y}{\partial t}=\omega A\cos(kx+\omega t)$. $a_y=\frac{\partial^2 y}{\partial t^2}=-A\omega^2\sin(kx+\omega t)$

EVALUATE: The functions $\cos(kx+\omega t)$ and $\sin(kx+\omega t)$ differ only in phase.

15.10. **IDENTIFY:** v_y and a_y are given by Eqs.(15.9) and (15.10).

SET UP: The sign of v_y determines the direction of motion of a particle on the string. If $v_y=0$ and $a_y\neq 0$ the speed of the particle is increasing. If $v_y\neq 0$, the particle is speeding up if v_y and a_y have the same sign and slowing down if they have opposite signs.

EXECUTE: **(a)** The graphs are given in Figure 15.10.

(b) (i) $v_y=\omega A\sin(0)=0$ and the particle is instantaneously at rest. $a_y=-\omega^2A\cos(0)=-\omega^2A$ and the particle is speeding up.

(ii) $v_y=\omega A\sin(\pi/4)=\omega A/\sqrt{2}$, and the particle is moving up. $a_y=-\omega^2A\cos(\pi/4)=-\omega^2A/\sqrt{2}$, and the particle is slowing down (v_y and a_y have opposite sign).

(iii) $v_y=\omega A\sin(\pi/2)=\omega A$ and the particle is moving up. $a_y=-\omega^2A\cos(\pi/2)=0$ and the particle is instantaneously not accelerating.

(iv) $v_y=\omega A\sin(3\pi/4)=\omega A/\sqrt{2}$, and the particle is moving up. $a_y=-\omega^2A\cos(3\pi/4)=\omega^2A/\sqrt{2}$, and the particle is speeding up.

(v) $v_y=\omega A\sin(\pi)=0$ and the particle is instantaneously at rest. $a_y=-\omega^2A\cos(\pi)=\omega^2A$ and the particle is speeding up.

(vi) $v_y=\omega A\sin(5\pi/4)=-\omega A/\sqrt{2}$ and the particle is moving down. $a_y=-\omega^2A\cos(5\pi/4)=\omega^2A/\sqrt{2}$ and the particle is slowing down (v_y and a_y have opposite sign).

(vii) $v_y=\omega A\sin(3\pi/2)=-\omega A$ and the particle is moving down. $a_y=-\omega^2A\cos(3\pi/2)=0$ and the particle is instantaneously not accelerating.

(viii) $v_y=\omega A\sin(7\pi/4)=-\omega A/\sqrt{2}$, and the particle is moving down. $a_y=-\omega^2A\cos(7\pi/4)=-\omega^2A/\sqrt{2}$ and the particle is speeding up (v_y and a_y have the same sign).

EVALUATE: At $t=0$ the wave is represented by Figure 15.10a in the textbook: point (i) in the problem corresponds to the origin, and points (ii)-(viii) correspond to the points in the figure labeled 1-7. Our results agree with what is shown in the figure.

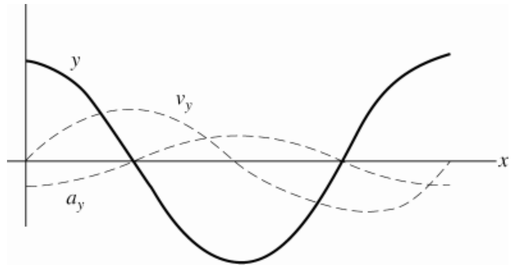

Figure 15.10

15.11. **IDENTIFY** and **SET UP:** Read A and T from the graph. Apply Eq.(15.4) to determine λ and then use Eq.(15.1) to calculate v.

EXECUTE: **(a)** The maximum y is 4 mm (read from graph).

(b) For either x the time for one full cycle is 0.040 s; this is the period.

(c) Since $y = 0$ for $x = 0$ and $t = 0$ and since the wave is traveling in the $+x$-direction then $y(x,\ t) = A\sin[2\pi(t/T - x/\lambda)]$. (The phase is different from the wave described by Eq.(15.4); for that wave $y = A$ for $x = 0,\ t = 0$.) From the graph, if the wave is traveling in the $+x$-direction and if $x = 0$ and $x = 0.090$ m are within one wavelength the peak at $t = 0.01$ s for $x = 0$ moves so that it occurs at $t = 0.035$ s (read from graph so is approximate) for $x = 0.090$ m. The peak for $x = 0$ is the first peak past $t = 0$ so corresponds to the first maximum in $\sin[2\pi(t/T - x/\lambda)]$ and hence occurs at $2\pi(t/T - x/\lambda) = \pi/2$. If this same peak moves to $t_1 = 0.035$ s at $x_1 = 0.090$ m, then

$2\pi(t_1/T - x_1/\lambda) = \pi/2$

Solve for λ: $t_1/T - x_1/\lambda = 1/4$

$x_1/\lambda = t_1/T - 1/4 = 0.035\text{ s}/0.040\text{ s} - 0.25 = 0.625$

$\lambda = x_1/0.625 = 0.090\text{ m}/0.625 = 0.14\text{ m}.$

Then $v = f\lambda = \lambda/T = 0.14\text{ m}/0.040\text{ s} = 3.5\text{ m/s}.$

(d) If the wave is traveling in the $-x$-direction, then $y(x,\ t) = A\sin(2\pi(t/T + x/\lambda))$ and the peak at $t = 0.050$ s for $x = 0$ corresponds to the peak at $t_1 = 0.035$ s for $x_1 = 0.090$ m. This peak at $x = 0$ is the second peak past the origin so corresponds to $2\pi(t/T + x/\lambda) = 5\pi/2$. If this same peak moves to $t_1 = 0.035$ s for $x_1 = 0.090$ m, then

$2\pi(t_1/T + x_1/\lambda) = 5\pi/2.$

$t_1/T + x_1/\lambda = 5/4$

$x_1/\lambda = 5/4 - t_1/T = 5/4 - 0.035\text{ s}/0.040\text{ s} = 0.375$

$\lambda = x_1/0.375 = 0.090\text{ m}/0.375 = 0.24\text{ m}.$

Then $v = f\lambda = \lambda/T = 0.24\text{ m}/0.040\text{ s} = 6.0\text{ m/s}.$

EVALUATE: No. Wouldn't know which point in the wave at $x = 0$ moved to which point at $x = 0.090$ m.

15.12. **IDENTIFY:** $v_y = \dfrac{\partial y}{\partial t}$. $v = f\lambda = \lambda/T$.

SET UP: $\dfrac{\partial}{\partial t} A\cos\left(\dfrac{2\pi}{\lambda}(x - vt)\right) = +A\left(\dfrac{2\pi v}{\lambda}\right)\sin\left(\dfrac{2\pi}{\lambda}(x - vt)\right)$

EXECUTE: **(a)** $A\cos 2\pi\left(\dfrac{x}{\lambda} - \dfrac{t}{T}\right) = +A\cos\dfrac{2\pi}{\lambda}\left(x - \dfrac{\lambda}{T}t\right) = +A\cos\dfrac{2\pi}{\lambda}(x - vt)$ where $\dfrac{\lambda}{T} = \lambda f = v$ has been used.

(b) $v_y = \dfrac{\partial y}{\partial t} = \dfrac{2\pi v}{\lambda} A\sin\dfrac{2\pi}{\lambda}(x - vt).$

(c) The speed is the greatest when the sine is 1, and that speed is $2\pi vA/\lambda$. This will be equal to v if $A = \lambda/2\pi$, less than v if $A < \lambda/2\pi$ and greater than v if $A > \lambda/2\pi$.

EVALUATE: The propagation speed applies to all points on the string. The transverse speed of a particle of the string depends on both x and t.

15.13. **IDENTIFY:** Follow the procedure specified in the problem.

SET UP: For λ and x in cm, v in cm/s and t in s, the argument of the cosine is in radians.

EXECUTE: **(a)** $t = 0$:

***x* (cm)**	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
***y* (cm)**	0.300	0.212	0	–0.212	–0.300	–0.212	0	0.212	0.300

The graph is shown in Figure 15.13a.

(b) (i) $t = 0.400$ s:

***x* (cm)**	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
***y* (cm)**	–0.221	–0.0131	0.203	0.300	0.221	0.0131	–0.203	–0.300	–0.221

The graph is shown in Figure 15.13b.

(ii) $t = 0.800$ s:

***x* (cm)**	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
***y* (cm)**	0.0262	–0.193	–0.300	–0.230	–0.0262	0.193	0.300	0.230	0.0262

The graph is shown in Figure 15.13c.

(iii) The graphs show that the wave is traveling in $+x$-direction.

EVALUATE: We know that Eq.(15.3) is for a wave traveling in the $+x$-direction, and $y(x,t)$ is derived from this. This is consistent with the direction of propagation we deduced from our graph.

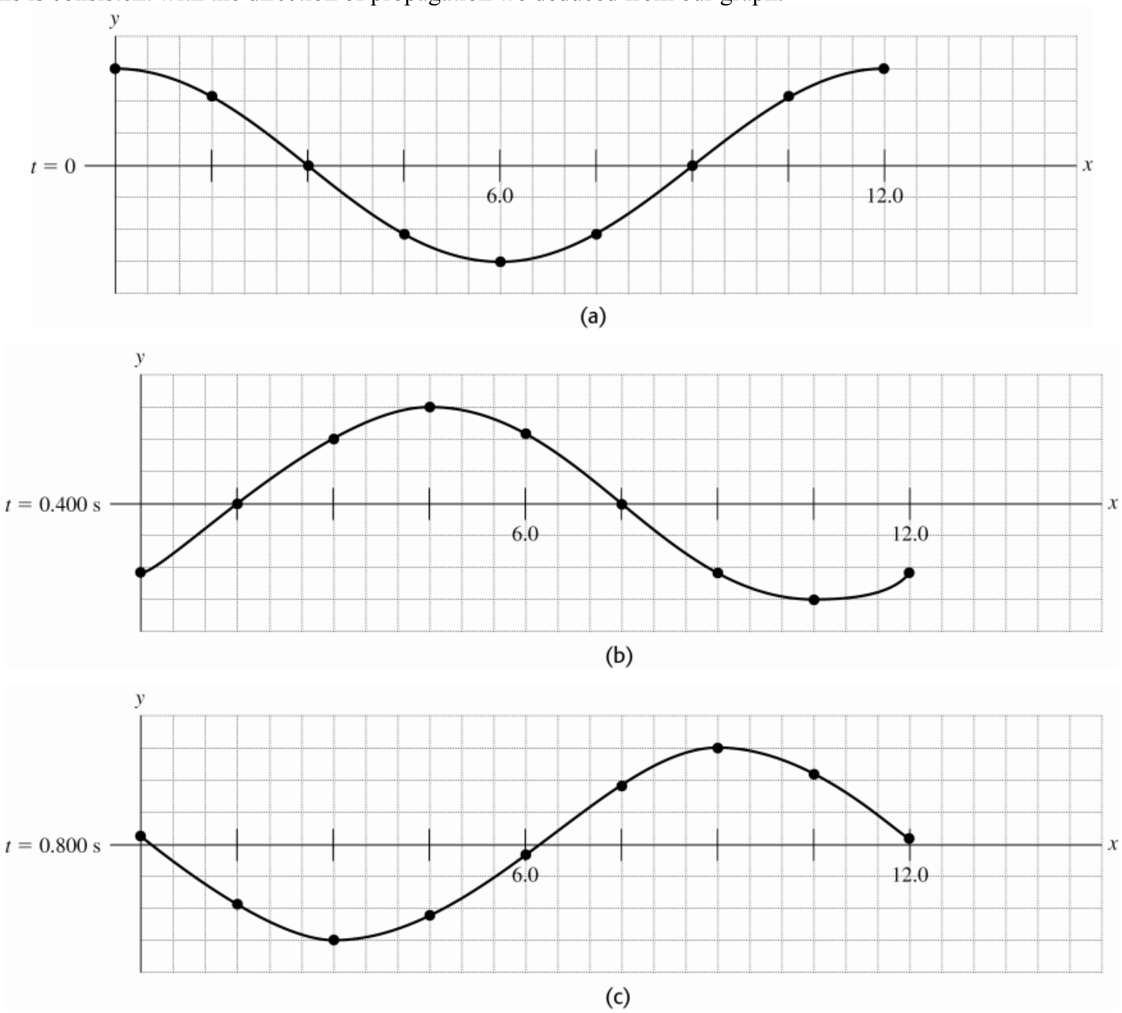

Figure 15.13

15.14. **IDENTIFY:** The frequency and wavelength determine the wave speed and the wave speed depends on the tension.

SET UP: $v = \sqrt{\frac{F}{\mu}}$. $\mu = m/L$. $v = f\lambda$.

EXECUTE: $F = \mu v^2 = \mu(f\lambda)^2 = \frac{0.120 \text{ kg}}{2.50 \text{ m}}([40.0 \text{ Hz}][0.750 \text{ m}])^2 = 43.2 \text{ N}$

EVALUATE: If the frequency is held fixed, increasing the tension will increase the wavelength.

15.15. **IDENTIFY** and **SET UP:** Use Eq.(15.13) to calculate the wave speed. Then use Eq.(15.1) to calculate the wavelength.

EXECUTE: **(a)** The tension F in the rope is the weight of the hanging mass:

$F = mg = (1.50 \text{ kg})(9.80 \text{ m/s}^2) = 14.7 \text{ N}$

$v = \sqrt{F/\mu} = \sqrt{14.7 \text{ N}/(0.0550 \text{ kg/m})} = 16.3 \text{ m/s}$

(b) $v = f\lambda$ so $\lambda = v/f = (16.3 \text{ m/s})/120 \text{ Hz} = 0.136 \text{ m}$.

(c) EVALUATE: $v = \sqrt{F/\mu}$, where $F = mg$. Doubling m increases v by a factor of $\sqrt{2}$. $\lambda = v/f$. f remains 120 Hz and v increases by a factor of $\sqrt{2}$, so λ increases by a factor of $\sqrt{2}$.

15.16. **IDENTIFY:** For transverse waves on a string, $v = \sqrt{F/\mu}$. The general form of the equation for waves traveling in the $+x$-direction is $y(x,t) = A\cos(kx - \omega t)$. For waves traveling in the $-x$-direction it is $y(x,t) = A\cos(kx + \omega t)$. $v = \omega/k$.

SET UP: Comparison to the general equation gives $A = 8.50$ mm, $k = 172$ rad/m and $\omega = 2730$ rad/s. The string has mass 0.128 kg and $\mu = m/L = 0.0850$ kg/m.

EXECUTE: **(a)** $v = \dfrac{\omega}{k} = \dfrac{2730 \text{ rad/s}}{172 \text{ rad/m}} = 15.9 \text{ m/s}$. $t = \dfrac{d}{v} = \dfrac{1.50 \text{ m}}{15.9 \text{ m/s}} = 0.0943 \text{ s}$.

(b) $W = F = \mu v^2 = (0.0850 \text{ kg/m})(15.9 \text{ m/s})^2 = 21.5 \text{ N}$.

(c) $\lambda = \dfrac{2\pi \text{ rad}}{k} = \dfrac{2\pi \text{ rad}}{172 \text{ rad/m}} = 0.0365 \text{ m}$. The number of wavelengths along the length of the string is $\dfrac{1.50 \text{ m}}{0.0365 \text{ m}} = 41.1$.

(d) For a wave traveling in the opposite direction, $y(x,t) = (8.50 \text{ mm})\cos([172 \text{ rad/m}]x + [2730 \text{ rad/s}]t)$

EVALUATE: We have assumed that the tension in the string is constant and equal to *W*. In reality the tension will vary along the length of the string because of the weight of the string and the wave speed will therefore vary along the string. The tension at the lower end of the string will be $W = 21.5$ N and at the upper end it is $W + 1.25 \text{ N} = 22.8 \text{ N}$, an increase of 6%.

15.17. **IDENTIFY:** For transverse waves on a string, $v = \sqrt{F/\mu}$. $v = f\lambda$.

SET UP: The wire has $\mu = m/L = (0.0165 \text{ kg})/(0.750 \text{ m}) = 0.0220 \text{ kg/m}$.

EXECUTE: **(a)** $v = f\lambda = (875 \text{ Hz})(3.33\times10^{-2} \text{ m}) = 29.1 \text{ m/s}$. The tension is $F = \mu v^2 = (0.0220 \text{ kg/m})(29.1 \text{ m/s})^2 = 18.6 \text{ N}$.

(b) $v = 29.1$ m/s

EVALUATE: If λ is kept fixed, the wave speed and the frequency increase when the tension is increased.

15.18. **IDENTIFY:** Apply $\sum F_y = 0$ to determine the tension at different points of the rope. $v = \sqrt{F/\mu}$.

SET UP: From Example 15.3, $m_{\text{samples}} = 20.0$ kg, $m_{\text{rope}} = 2.00$ kg and $\mu = 0.0250$ kg/m

EXECUTE: **(a)** The tension at the bottom of the rope is due to the weight of the load, and the speed is the same 88.5 m/s as found in Example 15.3.

(b) The tension at the middle of the rope is $(21.0 \text{ kg})(9.80 \text{ m/s}^2) = 205.8$ N and the wave speed is 90.7 m/s.

(c) The tension at the top of the rope is $(22.0 \text{ kg})(9.80 \text{ m/s}^2) = 215.6$ m/s and the speed is 92.9 m/s. (See Challenge Problem (15.82) for the effects of varying tension on the time it takes to send signals.)

EVALUATE: The tension increases toward the top of the rope, so the wave speed increases from the bottom of the rope to the top of the rope.

15.19. **IDENTIFY:** $v = \sqrt{F/\mu}$. $v = f\lambda$. The general form for $y(x,t)$ is given in Eq.(15.4), where $T = 1/f$. Eq.(15.10) says that the maximum transverse acceleration is $a_{\text{max}} = \omega^2 A = (2\pi f)^2 A$.

SET UP: $\mu = 0.0500$ kg/m

EXECUTE: **(a)** $v = \sqrt{F/\mu} = \sqrt{(5.00 \text{ N})/(0.0500 \text{ kg/m})} = 10.0 \text{ m/s}$

(b) $\lambda = v/f = (10.0 \text{ m/s})/(40.0 \text{ Hz}) = 0.250 \text{ m}$

(c) $y(x,t) = A\cos(kx - \omega t)$. $k = 2\pi/\lambda = 8.00\pi$ rad/m; $\omega = 2\pi f = 80.0\pi$ rad/s.

$y(x,t) = (3.00 \text{ cm})\cos[\pi(8.00 \text{ rad/m})x - (80.0\pi \text{ rad/s})t]$

(d) $v_y = +A\omega\sin(kx - \omega t)$ and $a_y = -A\omega^2\cos(kx - \omega t)$. $a_{y,\text{max}} = A\omega^2 = A(2\pi f)^2 = 1890 \text{ m/s}^2$.

(e) $a_{y,\text{max}}$ is much larger than *g*, so it is a reasonable approximation to ignore gravity.

EVALUATE: $y(x,t)$ in part (c) gives $y(0,0) = A$, which does correspond to the oscillator having maximum upward displacement at $t = 0$.

15.20. **IDENTIFY:** Apply Eq.(15.25).

SET UP: $\omega = 2\pi f$. $\mu = m/L$.

EXECUTE: **(a)** $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$. $P_{\text{av}} = \frac{1}{2}\sqrt{\left(\dfrac{3.00\times10^{-3} \text{ kg}}{0.80 \text{ m}}\right)(25.0 \text{ N})}(2\pi(120.0 \text{ Hz}))^2(1.6\times10^{-3} \text{ m})^2 = 0.223 \text{ W}$ or 0.22 W to two figures.

(b) P_{av} is proportional to A^2, so halving the amplitude quarters the average power, to 0.056 W.

EVALUATE: The average power is also proportional to the square of the frequency.

15.21. **IDENTIFY:** For a point source, $I = \dfrac{P}{4\pi r^2}$ and $\dfrac{I_1}{I_2} = \dfrac{r_2^2}{r_1^2}$.

SET UP: $1\ \mu\text{W} = 10^{-6}\ \text{W}$

EXECUTE: **(a)** $r_2 = r_1\sqrt{\dfrac{I_1}{I_2}} = (30.0\ \text{m})\sqrt{\dfrac{10.0\ \text{W/m}^2}{1\times 10^{-6}\ \text{W/m}^2}} = 95\ \text{km}$

(b) $\dfrac{I_2}{I_3} = \dfrac{r_3^2}{r_2^2}$, with $I_2 = 1.0\ \mu\text{W/m}^2$ and $r_3 = 2r_2$. $I_3 = I_2\left(\dfrac{r_2}{r_3}\right)^2 = I_2/4 = 0.25\ \mu\text{W/m}^2$.

(c) $P = I(4\pi r^2) = (10.0\ \text{W/m}^2)(4\pi)(30.0\ \text{m})^2 = 1.1\times 10^5\ \text{W}$

EVALUATE: These are approximate calculations, that assume the sound is emitted uniformly in all directions and that ignore the effects of reflection, for example reflections from the ground.

15.22. **IDENTIFY:** Apply Eq.(15.26).

SET UP: $I_1 = 0.11\ \text{W/m}^2$. $r_1 = 7.5\ \text{m}$. Set $I_2 = 1.0\ \text{W/m}^2$ and solve for r_2.

EXECUTE: $r_2 = r_1\sqrt{\dfrac{I_1}{I_2}} = (7.5\ \text{m})\sqrt{\dfrac{0.11\ \text{W/m}^2}{1.0\ \text{W/m}^2}} = 2.5\ \text{m}$, so it is possible to move

$r_1 - r_2 = 7.5\ \text{m} - 2.5\ \text{m} = 5.0\ \text{m}$ closer to the source.

EVALUATE: I increases as the distance r of the observer from the source decreases.

15.23. **IDENTIFY** and **SET UP:** Apply Eq.(15.26) to relate I and r.

Power is related to intensity at a distance r by $P = I(4\pi r^2)$. Energy is power times time.

EXECUTE: **(a)** $I_1 r_1^2 = I_2 r_2^2$

$I_2 = I_1(r_1/r_2)^2 = (0.026\ \text{W/m}^2)(4.3\ \text{m/3.1 m})^2 = 0.050\ \text{W/m}^2$

(b) $P = 4\pi r^2 I = 4\pi(4.3\ \text{m})^2(0.026\ \text{W/m}^2) = 6.04\ \text{W}$

$\text{Energy} = Pt = (6.04\ \text{W})(3600\ \text{s}) = 2.2\times 10^4\ \text{J}$

EVALUATE: We could have used $r = 3.1\ \text{m}$ and $I = 0.050\ \text{W/m}^2$ in $P = 4\pi r^2 I$ and would have obtained the same P. Intensity becomes less as r increases because the radiated power spreads over a sphere of larger area.

15.24. **IDENTIFY:** The tension and mass per unit length of the rope determine the wave speed. Compare $y(x,t)$ given in the problem to the general form given in Eq.(15.8). $v = \omega/k$. The average power is given by Eq. (15.25).

SET UP: Comparison with Eq.(15.8) gives $A = 2.33\ \text{mm}$, $k = 6.98\ \text{rad/m}$ and $\omega = 742\ \text{rad/s}$.

EXECUTE: **(a)** $A = 2.30\ \text{mm}$

(b) $f = \dfrac{\omega}{2\pi} = \dfrac{742\ \text{rad/s}}{2\pi} = 118\ \text{Hz}$.

(c) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{6.98\ \text{rad/m}} = 0.90\ \text{m}$

(d) $v = \dfrac{\omega}{k} = \dfrac{742\ \text{rad/s}}{6.98\ \text{rad/m}} = 106\ \text{m/s}$

(e) The wave is traveling in the $-x$ direction because the phase of $y(x,\ t)$ has the form $kx + \omega t$.

(f) The linear mass density is $\mu = (3.38\times 10^{-3}\ \text{kg})/(1.35\ \text{m}) = 2.504\times 10^{-3}\ \text{kg/m}$, so the tension is $F = \mu v^2 = (2.504\times 10^{-3}\ \text{kg/m})(106.3\ \text{m/s})^2 = 28.3\ \text{N}$.

(g) $P_{\text{av}} = \tfrac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 = \tfrac{1}{2}\sqrt{(2.50\times 10^{-3}\ \text{kg/m})(28.3\ \text{N})}(742\ \text{rad/s})^2(2.30\times 10^{-3}\ \text{m})^2 = 0.39\ \text{W}$

EVALUATE: In part (d) we could also calculate the wave speed as $v = f\lambda$ and we would obtain the same result.

15.25. **IDENTIFY:** $P = 4\pi r^2 I$

SET UP: From Example 15.5, $I = 0.250\ \text{W/m}^2$ at $r = 15.0\ \text{m}$

EXECUTE: $P = 4\pi r^2 I = 4\pi(15.0\ \text{m})^2(0.250\ \text{W/m}^2) = 707\ \text{W}$

EVALUATE: $I = 0.010\ \text{W/m}^2$ at $75.0\ \text{m}$ and $4\pi(75.0\ \text{m})^2(0.010\ \text{W/m}^2) = 707\ \text{W}$. P is the average power of the sinusoidal waves emitted by the source.

15.26. **IDENTIFY:** The distance the wave shape travels in time *t* is *vt*. The wave pulse reflects at the end of the string, at point *O*.
SET UP: The reflected pulse is inverted when *O* is a fixed end and is not inverted when *O* is a free end.
EXECUTE: **(a)** The wave form for the given times, respectively, is shown in Figure 15.26a.
(b) The wave form for the given times, respectively, is shown in Figure 15.26b.
EVALUATE: For the fixed end the result of the reflection is an inverted pulse traveling to the left and for the free end the result is an upright pulse traveling to the left.

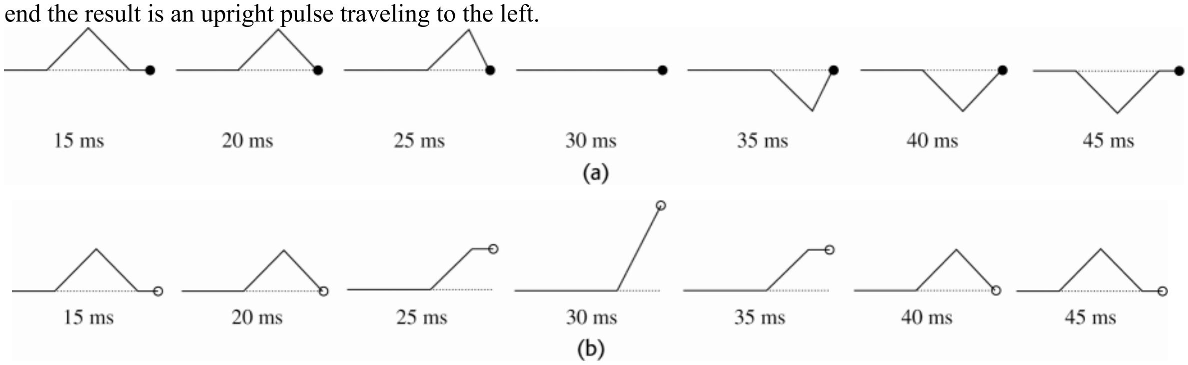

Figure 15.26

15.27. **IDENTIFY:** The distance the wave shape travels in time *t* is *vt*. The wave pulse reflects at the end of the string, at point *O*.
SET UP: The reflected pulse is inverted when *O* is a fixed end and is not inverted when *O* is a free end.
EXECUTE: **(a)** The wave form for the given times, respectively, is shown in Figure 15.27a.
(b) The wave form for the given times, respectively, is shown in Figure 15.27b.
EVALUATE: For the fixed end the result of the reflection is an inverted pulse traveling to the left and for the free end the result is an upright pulse traveling to the left.

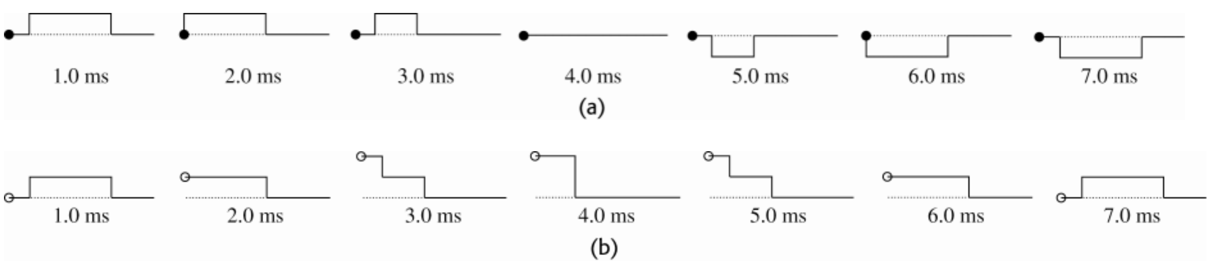

Figure 15.27

15.28. **IDENTIFY:** Apply the principle of superposition.
SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.
EXECUTE: The shape of the string at each specified time is shown in Figure 15.28.
EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

Figure 15.28

15.29. **IDENTIFY:** Apply the principle of superposition.
SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.
EXECUTE: The shape of the string at each specified time is shown in Figure 15.29.

EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

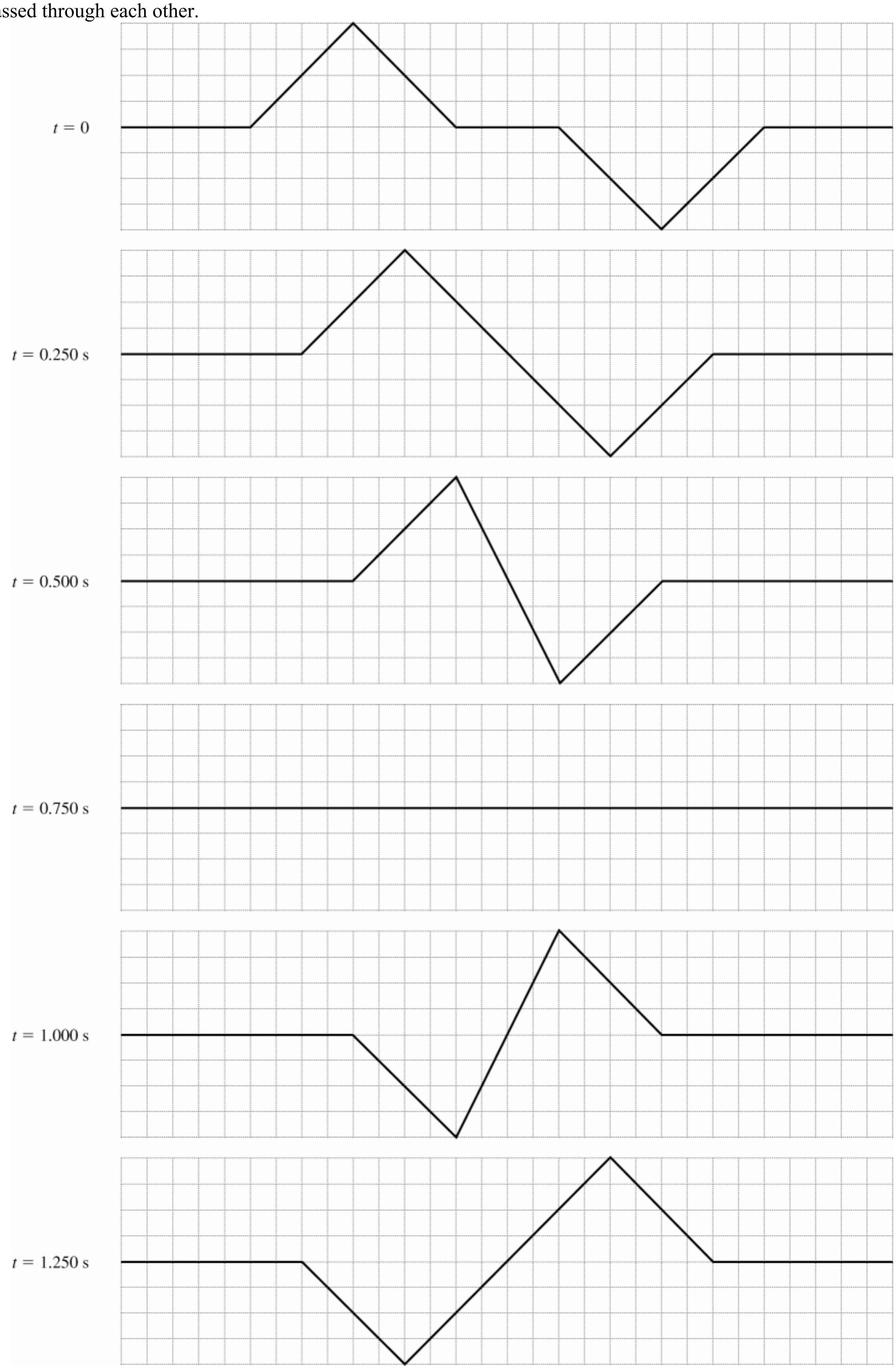

Figure 15.29

15.30. **IDENTIFY:** Apply the principle of superposition.
SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.
EXECUTE: The shape of the string at each specified time is shown in Figure 15.30.
EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

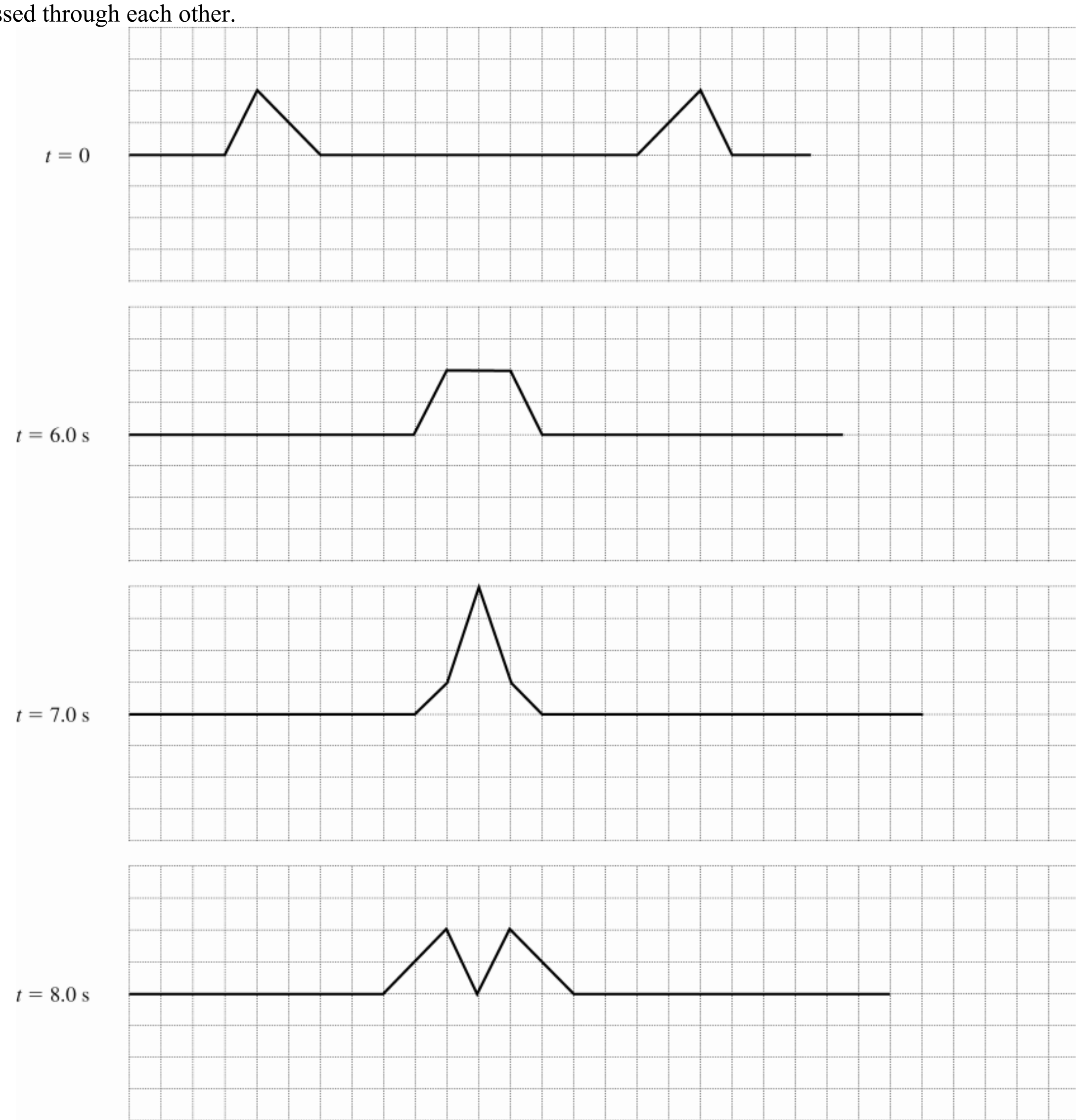

Figure 15.30

15.31. **IDENTIFY:** Apply the principle of superposition.
SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.
EXECUTE: The shape of the string at each specified time is shown in Figure 15.31.
EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

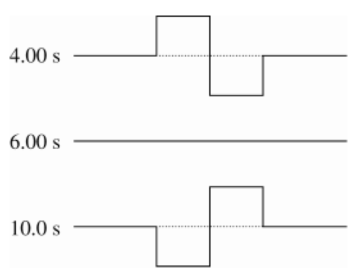

Figure 15.31

15.32. **IDENTIFY:** $y_{net} = y_1 + y_2$. The string never moves at values of x for which $\sin kx = 0$.

SET UP: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

EXECUTE: **(a)** $y_{net} = A\sin(kx + \omega t) + A\sin(kx - \omega t)$.

$y_{net} = A[\sin(kx)\cos(\omega t) + \cos(kx)\sin(\omega t) + \sin(kx)\cos(\omega t) - \cos(kx)\sin(\omega t)] = 2A\sin(kx)\cos(\omega t)$

(b) $\sin kx = 0$ for $kx = n\pi,\ n = 0, 1, 2, \ldots$. $x = \dfrac{n\pi}{k} = \dfrac{n\pi}{2\pi/\lambda} = \dfrac{n\lambda}{2}$.

EVALUATE: Using $y = A\sin(kx \pm \omega t)$ instead of $y = A\cos(kx \pm \omega t)$ corresponds to a particular choice of phase and corresponds to $y = 0$ at $x = 0$, for all t.

15.33. **IDENTIFY** and **SET UP:** Nodes occur where $\sin kx = 0$ and antinodes are where $\sin kx = \pm 1$.

EXECUTE: Eq.(15.28): $y = (A_{SW}\sin kx)\sin \omega t$

(a) At a node $y = 0$ for all t. This requires that $\sin kx = 0$ and this occurs for $kx = n\pi$, $n = 0, 1, 2, \ldots$

$$x = n\pi / k = \frac{n\pi}{0.750\pi \text{ rad/m}} = (1.33 \text{ m})n,\ n = 0, 1, 2, \ldots$$

(b) At an antinode $\sin kx = \pm 1$ so y will have maximum amplitude. This occurs when $kx = \left(n + \tfrac{1}{2}\right)\pi$, $n = 0, 1, 2, \ldots$

$$x = \left(n + \tfrac{1}{2}\right)\pi / k = \left(n + \tfrac{1}{2}\right)\frac{\pi}{0.750\pi \text{ rad/m}} = (1.33 \text{ m})\left(n + \tfrac{1}{2}\right),\ n = 0, 1, 2, \ldots$$

EVALUATE: $\lambda = 2\pi / k = 2.66$ m. Adjacent nodes are separated by $\lambda/2$, adjacent antinodes are separated by $\lambda/2$, and the node to antinode distance is $\lambda/4$.

15.34. **IDENTIFY:** Apply Eqs.(15.28) and (15.1). At an antinode, $y(t) = A_{SW}\sin \omega t$. k and ω for the standing wave have the same values as for the two traveling waves.

SET UP: $A_{SW} = 0.850$ cm. The antinode to antinode distance is $\lambda/2$, so $\lambda = 30.0$ cm. $v_y = \partial y / \partial t$.

EXECUTE: **(a)** The node to node distance is $\lambda/2 = 15.0$ cm.

(b) λ is the same as for the standing wave, so $\lambda = 30.0$ cm. $A = \tfrac{1}{2}A_{SW} = 0.425$ cm.

$$v = f\lambda = \frac{\lambda}{T} = \frac{0.300 \text{ m}}{0.0750 \text{ s}} = 13.3 \text{ m/s}.$$

(c) $v_y = \dfrac{\partial y}{\partial t} = A_{SW}\omega \sin kx \cos \omega t$. At an antinode $\sin kx = 1$, so $v_y = A_{SW}\omega \cos \omega t$. $v_{max} = A_{SW}\omega$.

$$\omega = \frac{2\pi \text{ rad}}{T} = \frac{2\pi \text{ rad}}{0.0750 \text{ s}} = 83.8 \text{ rad/s}.\ v_{max} = (0.850 \times 10^{-2} \text{ m})(83.8 \text{ rad/s}) = 0.0712 \text{ m/s}.\ v_{min} = 0.$$

(d) The distance from a node to an adjacent antinode is $\lambda/4 = 7.50$ cm.

EVALUATE: The maximum transverse speed for a point at an antinode of the standing wave is twice the maximum transverse speed for each traveling wave, since $A_{SW} = 2A$.

15.35. **IDENTIFY:** Evaluate $\partial^2 y / \partial x^2$ and $\partial^2 y / \partial t^2$ and see if Eq.(15.12) is satisfied for $v = \omega / k$.

SET UP: $\dfrac{\partial}{\partial x}\sin kx = k\cos kx$. $\dfrac{\partial}{\partial x}\cos kx = -k\sin kx$. $\dfrac{\partial}{\partial t}\sin \omega t = \omega \cos \omega t$. $\dfrac{\partial}{\partial t}\cos \omega t = -\omega \sin \omega t$

EXECUTE: **(a)** $\dfrac{\partial^2 y}{\partial x^2} = -k^2\left[A_{sw}\sin \omega t\right]\sin kx$, $\dfrac{\partial^2 y}{\partial t^2} = -\omega^2\left[A_{sw}\sin \omega t\right]\sin kx$, so for $y(x,t)$ to be a solution of Eq.(15.12), $-k^2 = \dfrac{-\omega^2}{v^2}$, and $v = \dfrac{\omega}{k}$.

(b) A standing wave is built up by the superposition of traveling waves, to which the relationship $v = \lambda / k$ applies.

EVALUATE: $y(x,t) = (A_{SW}\sin kx)\sin \omega t$ is a solution of the wave equation because it is a sum of solutions to the wave equation.

15.36. **IDENTIFY** and **SET UP:** $\cos(kx \pm \omega t) = \cos kx \cos \omega t \mp \sin kx \sin \omega t$

EXECUTE: $y_1 + y_2 = A\,[-\cos(kx + \omega t) + \cos(kx - \omega t)]$.

$y_1 + y_2 = A[-\cos kx \cos \omega t + \sin kx \sin \omega t + \cos kx \cos \omega t + \sin kx \sin \omega t] = 2A\sin kx \sin \omega t$.

EVALUATE: The derivation shows that the standing wave of Eq.(15.28) results from the combination of two waves with the same A, k, and ω that are traveling in opposite directions.

15.37. IDENTIFY: Evaluate $\partial^2 y/\partial x^2$ and $\partial^2 y/\partial t^2$ and show that Eq.(15.12) is satisfied.

SET UP: $\dfrac{\partial}{\partial x}(y_1+y_2)=\dfrac{\partial y_1}{\partial x}+\dfrac{\partial y_2}{\partial x}$ and $\dfrac{\partial}{\partial t}(y_1+y_2)=\dfrac{\partial y_1}{\partial t}+\dfrac{\partial y_2}{\partial t}$

EXECUTE: $\dfrac{\partial^2 y}{\partial x^2}=\dfrac{\partial^2 y_1}{\partial x^2}+\dfrac{\partial^2 y_2}{\partial x^2}$ and $\dfrac{\partial^2 y}{\partial t^2}=\dfrac{\partial^2 y_1}{\partial t^2}+\dfrac{\partial^2 y_2}{\partial t^2}$. The functions y_1 and y_2 are given as being solutions to the wave equation, so

$\dfrac{\partial^2 y}{\partial x^2}=\dfrac{\partial^2 y_1}{\partial x^2}+\dfrac{\partial^2 y_2}{\partial x^2}=\left(\dfrac{1}{v^2}\right)\dfrac{\partial^2 y_1}{\partial t^2}+\left(\dfrac{1}{v^2}\right)\dfrac{\partial^2 y_2}{\partial t^2}=\left(\dfrac{1}{v^2}\right)\left[\dfrac{\partial^2 y_1}{\partial t^2}+\dfrac{\partial^2 y_2}{\partial t^2}\right]=\left(\dfrac{1}{v^2}\right)\dfrac{\partial^2 y}{\partial t^2}$ and so $y=y_1+y_2$ is a solution of Eq. (15.12).

EVALUATE: The wave equation is a linear equation, as it is linear in the derivatives, and differentiation is a linear operation.

15.38. IDENTIFY: For a string fixed at both ends, $\lambda_n=\dfrac{2L}{n}$ and $f_n=n\left(\dfrac{v}{2L}\right)$.

SET UP: For the fundamental, $n=1$. For the second overtone, $n=3$. For the fourth harmonic, $n=4$.

EXECUTE: **(a)** $\lambda_1=2L=3.00\text{ m}$. $f_1=\dfrac{v}{2L}=\dfrac{(48.0\text{ m/s})}{2(1.50\text{ m})}=16.0\text{ Hz}$.

(b) $\lambda_3=\lambda_1/3=1.00\text{ m}$. $f_2=3f_1=48.0\text{ Hz}$.

(c) $\lambda_4=\lambda_1/4=0.75\text{ m}$. $f_3=4f_1=64.0\text{ Hz}$.

EVALUATE: As n increases, λ decreases and f increases.

15.39. IDENTIFY: Use Eq.(15.1) for v and Eq.(15.13) for the tension F. $v_y=\partial y/\partial t$ and $a_y=\partial v_y/\partial t$.

(a) SET UP: The fundamental standing wave is sketched in Figure 15.39.

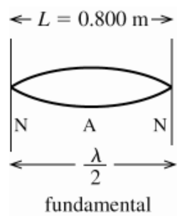

Figure 15.39

$f=60.0\text{ Hz}$

From the sketch,

$\lambda/2=L$ so

$\lambda=2L=1.60\text{ m}$

EXECUTE: $v=f\lambda=(60.0\text{ Hz})(1.60\text{ m})=96.0\text{ m/s}$

(b) The tension is related to the wave speed by Eq.(15.13):

$v=\sqrt{F/\mu}$ so $F=\mu v^2$.

$\mu=m/L=0.0400\text{ kg}/0.800\text{ m}=0.0500\text{ kg/m}$

$F=\mu v^2=(0.0500\text{ kg/m})(96.0\text{ m/s})^2=461\text{ N}$.

(c) $\omega=2\pi f=377\text{ rad/s}$ and $y(x,\ t)=A_{\text{SW}}\sin kx\sin\omega t$

$v_y=\omega A_{\text{SW}}\sin kx\cos\omega t$; $a_y=-\omega^2 A_{\text{SW}}\sin kx\sin\omega t$

$(v_y)_{\max}=\omega A_{\text{SW}}=(377\text{ rad/s})(0.300\text{ cm})=1.13\text{ m/s}$.

$(a_y)_{\max}=\omega^2 A_{\text{SW}}=(377\text{ rad/s})^2(0.300\text{ cm})=426\text{ m/s}^2$.

EVALUATE: The transverse velocity is different from the wave velocity. The wave velocity and tension are similar in magnitude to the values in the Examples in the text. Note that the transverse acceleration is quite large.

15.40. IDENTIFY: The fundamental frequency depends on the wave speed, and that in turn depends on the tension.

SET UP: $v=\sqrt{\dfrac{F}{\mu}}$ where $\mu=m/L$. $f_1=\dfrac{v}{2L}$. The nth harmonic has frequency $f_n=nf_1$.

EXECUTE: **(a)** $v=\sqrt{\dfrac{F}{m/L}}=\sqrt{\dfrac{FL}{m}}=\sqrt{\dfrac{(800\text{ N})(0.400\text{ m})}{3.00\times10^{-3}\text{ kg}}}=327\text{ m/s}$. $f_1=\dfrac{v}{2L}=\dfrac{327\text{ m/s}}{2(0.400\text{ m})}=409\text{ Hz}$.

(b) $n=\dfrac{10{,}000\text{ Hz}}{f_1}=24.4$. The 24^{th} harmonic is the highest that could be heard.

EVALUATE: In part (b) we use the fact that a standing wave on the wire produces a sound wave in air of the same frequency.

15.41. **IDENTIFY:** Compare $y(x,\ t)$ given in the problem to Eq.(15.28). From the frequency and wavelength for the third harmonic find these values for the eighth harmonic.

(a) SET UP: The third harmonic standing wave pattern is sketched in Figure 15.41.

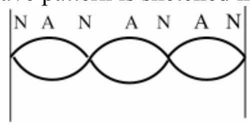

Figure 15.41

EXECUTE: **(b)** Eq. (15.28) gives the general equation for a standing wave on a string:

$y(x,\ t) = (A_{\text{SW}} \sin kx)\sin \omega t$

$A_{\text{SW}} = 2A$, so $A = A_{\text{SW}}/2 = (5.60\text{ cm})/2 = 2.80\text{ cm}$

(c) The sketch in part (a) shows that $L = 3(\lambda/2)$. $k = 2\pi/\lambda$, $\lambda = 2\pi/k$

Comparison of $y(x,\ t)$ given in the problem to Eq. (15.28) gives $k = 0.0340$ rad/cm. So,

$\lambda = 2\pi/(0.0340\text{ rad/cm}) = 184.8\text{ cm}$

$L = 3(\lambda/2) = 277\text{ cm}$

(d) $\lambda = 185$ cm, from part (c)

$\omega = 50.0$ rad/s so $f = \omega/2\pi = 7.96$ Hz

period $T = 1/f = 0.126$ s

$v = f\lambda = 1470$ cm/s

(e) $v_y = dy/dt = \omega A_{\text{SW}} \sin kx \cos \omega t$

$v_{y,\ \max} = \omega A_{\text{SW}} = (50.0\text{ rad/s})(5.60\text{ cm}) = 280\text{ cm/s}$

(f) $f_3 = 7.96\text{ Hz} = 3f_1$, so $f_1 = 2.65$ Hz is the fundamental

$f_8 = 8f_1 = 21.2$ Hz; $\omega_8 = 2\pi f_8 = 133$ rad/s

$\lambda = v/f = (1470\text{ cm/s})/(21.2\text{ Hz}) = 69.3$ cm and $k = 2\pi/\lambda = 0.0906$ rad/cm

$y(x,\ t) = (5.60\text{ cm})\sin([0.0906\text{ rad/cm}]x)\sin([133\text{ rad/s}]t)$

EVALUATE: The wavelength and frequency of the standing wave equals the wavelength and frequency of the two traveling waves that combine to form the standing wave. In the 8th harmonic the frequency and wave number are larger than in the 3rd harmonic.

15.42. **IDENTIFY:** Compare the $y(x,t)$ specified in the problem to the general form of Eq.(15.28).

SET UP: The comparison gives $A_{\text{SW}} = 4.44$ mm, $k = 32.5$ rad/m and $\omega = 754$ rad/s.

EXECUTE: **(a)** $A = \frac{1}{2}A_{\text{SW}} = \frac{1}{2}(4.44\text{ mm}) = 2.22\text{ mm}$.

(b) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{32.5\ \text{rad/m}} = 0.193\text{ m}.$

(c) $f = \dfrac{\omega}{2\pi} = \dfrac{754\ \text{rad/s}}{2\pi} = 120\text{ Hz}.$

(d) $v = \dfrac{\omega}{k} = \dfrac{754\ \text{rad/s}}{32.5\ \text{rad/m}} = 23.2\ \text{m/s}.$

(e) If the wave traveling in the $+x$ direction is written as $y_1(x,t) = A\cos(kx - \omega t)$, then the wave traveling in the $-x$-direction is $y_2(x,t) = -A\cos(kx + \omega t)$, where $A = 2.22$ mm from part (a), $k = 32.5$ rad/m and $\omega = 754$ rad/s.

(f) The harmonic cannot be determined because the length of the string is not specified.

EVALUATE: The two traveling waves that produce the standing wave are identical except for their direction of propagation.

15.43. **(a) IDENTIFY** and **SET UP:** Use the angular frequency and wave number for the traveling waves in Eq.(15.28) for the standing wave.

EXECUTE: The traveling wave is $y(x,\ t) = (2.30\text{ mm})\cos([6.98\text{ rad/m}]x) + [742\text{ rad/s}]t)$

$A = 2.30$ mm so $A_{\text{SW}} = 4.60$ mm; $k = 6.98$ rad/m and $\omega = 742$ rad/s

The general equation for a standing wave is $y(x,\ t) = (A_{\text{SW}} \sin kx)\sin \omega t$, so

$y(x,\ t) = (4.60\text{ mm})\sin([6.98\text{ rad/m}]x)\sin([742\text{ rad/s}]t)$

(b) IDENTIFY and **SET UP:** Compare the wavelength to the length of the rope in order to identify the harmonic.

EXECUTE: $L = 1.35$ m (from Exercise 15.24)

$\lambda = 2\pi/k = 0.900$ m

$L = 3(\lambda/2)$, so this is the 3rd harmonic

(c) For this 3rd harmonic, $f = \omega/2\pi = 118$ Hz

$f_3 = 3f_1$ so $f_1 = (118 \text{ Hz})/3 = 39.3$ Hz

EVALUATE: The wavelength and frequency of the standing wave equals the wavelength and frequency of the two traveling waves that combine to form the standing wave. The *n*th harmonic has *n* node-to-node segments and the node-to-node distance is $\lambda/2$, so the relation between L and λ for the *n*th harmonic is $L = n(\lambda/2)$.

15.44. IDENTIFY: $v = \sqrt{F/\mu}$. $v = f\lambda$. The standing waves have wavelengths $\lambda_n = \dfrac{2L}{n}$ and frequencies $f_n = nf_1$. The standing wave on the string and the sound wave it produces have the same frequency.

SET UP: For the fundamental $n = 1$ and for the second overtone $n = 3$. The string has $\mu = m/L = (8.75\times10^{-3} \text{ kg})/(0.750 \text{ m}) = 1.17\times10^{-2} \text{ kg/m}$.

EXECUTE: **(a)** $\lambda = 2L/3 = 2(0.750 \text{ m})/3 = 0.500 \text{ m}$. The sound wave has frequency $f = \dfrac{v}{\lambda} = \dfrac{344 \text{ m/s}}{3.35\times10^{-2} \text{ m}} = 1.03\times10^4 \text{ Hz}$. For waves on the string,

$v = f\lambda = (1.03\times10^4 \text{ Hz})(0.500 \text{ m}) = 5.15\times10^3 \text{ m/s}$. The tension in the string is

$F = \mu v^2 = (1.17\times10^{-2} \text{ kg/m})(5.15\times10^3 \text{ m/s})^2 = 3.10\times10^5 \text{ N}$.

(b) $f_1 = f_3/3 = (1.03\times10^4 \text{ Hz})/3 = 3.43\times10^3 \text{ Hz}$.

EVALUATE: The waves on the string have a much longer wavelength than the sound waves in the air because the speed of the waves on the string is much greater than the speed of sound in air.

15.45. IDENTIFY and SET UP: Use the information given about the A_4 note to find the wave speed, that depends on the linear mass density of the string and the tension. The wave speed isn't affected by the placement of the fingers on the bridge. Then find the wavelength for the D_5 note and relate this to the length of the vibrating portion of the string.

EXECUTE: **(a)** $f = 440$ Hz when a length $L = 0.600$ m vibrates; use this information to calculate the speed v of waves on the string. For the fundamental $\lambda/2 = L$ so $\lambda = 2L = 2(0.600 \text{ m}) = 1.20$ m. Then

$v = f\lambda = (440 \text{ Hz})(1.20 \text{ m}) = 528$ m/s. Now find the length $L = x$ of the string that makes $f = 587$ Hz.

$$\lambda = \frac{v}{f} = \frac{528 \text{ m/s}}{587 \text{ Hz}} = 0.900 \text{ m}$$

$L = \lambda/2 = 0.450$ m, so $x = 0.450 \text{ m} = 45.0$ cm.

(b) No retuning means same wave speed as in part (a). Find the length of vibrating string needed to produce $f = 392$ Hz.

$$\lambda = \frac{v}{f} = \frac{528 \text{ m/s}}{392 \text{ Hz}} = 1.35 \text{ m}$$

$L = \lambda/2 = 0.675$ m; string is shorter than this. No, not possible.

EVALUATE: Shortening the length of this vibrating string increases the frequency of the fundamental.

15.46. IDENTIFY: $y(x,t) = (A_{SW}\sin kx)\sin\omega t$. $v_y = \partial y/\partial t$. $a_y = \partial^2 y/\partial t^2$.

SET UP: $v_{max} = (A_{SW}\sin kx)\omega$. $a_{max} = (A_{SW}\sin kx)\omega^2$.

EXECUTE: **(a)** (i) $x = \dfrac{\lambda}{2}$ is a node, and there is no motion. (ii) $x = \dfrac{\lambda}{4}$ is an antinode, and $v_{max} = A(2\pi f) = 2\pi fA$, $a_{max} = (2\pi f)v_{max} = 4\pi^2 f^2 A$. (iii) $\cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$ and this factor multiplies the results of (ii), so $v_{max} = \sqrt{2}\pi fA$, $a_{max} = 2\sqrt{2}\pi^2 f^2 A$.

(b) The amplitude is $2A\sin kx$, or (i) 0, (ii) $2A$, (iii) $2A/\sqrt{2}$.

(c) The time between the extremes of the motion is the same for any point on the string (although the period of the zero motion at a node might be considered indeterminate) and is $1/2f$.

EVALUATE: Any point in a standing wave moves in SHM. All points move with the same frequency but have different amplitude.

15.47. IDENTIFY: For the fundamental, $f_1 = \dfrac{v}{2L}$. $v = \sqrt{F/\mu}$. A standing wave on a string with frequency *f* produces a sound wave that also has frequency *f*.

SET UP: $f_1 = 245$ Hz. $L = 0.635$ m.

EXECUTE: **(a)** $v = 2f_1L = 2(245 \text{ Hz})(0.635 \text{ m}) = 311$ m/s.

(b) The frequency of the fundamental mode is proportional to the speed and hence to the square root of the tension; $(245\ \text{Hz})\sqrt{1.01} = 246\ \text{Hz}$.

(c) The frequency will be the same, 245 Hz. The wavelength will be $\lambda_{\text{air}} = v_{\text{air}}/f = (344\ \text{m/s})/(245\ \text{Hz}) = 1.40\ \text{m}$, which is larger than the wavelength of standing wave on the string by a factor of the ratio of the speeds.

EVALUATE: Increasing the tension increases the wave speed and this in turn increases the frequencies of the standing waves. The wavelength of each normal mode depends only on the length of the string and doesn't change when the tension changes.

15.48. **IDENTIFY:** The ends of the stick are free, so they must be displacement antinodes. The first harmonic has one node, at the center of the stick, and each successive harmonic adds one node.

SET UP: The node to node and antinode to antinode distance is $\lambda/2$.

EXECUTE: The standing wave patterns for the first three harmonics are shown in Figure 15.48.

1st harmonic: $L = \frac{1}{2}\lambda_1 \to \lambda_1 = 2L = 4.0\ \text{m}$. 2nd harmonic: $L = 1\lambda_2 \to \lambda_2 = L = 2.0\ \text{m}$.

3rd harmonic: $L = \frac{3}{2}\lambda_3 \to \lambda_3 = \frac{2L}{3} = 1.33\ \text{m}$.

EVALUATE: The higher the harmonic the shorter the wavelength.

Figure 15.48

15.49. **IDENTIFY** and **SET UP:** Calculate v, ω, and k from Eqs.(15.1), (15.5), and (15.6). Then apply Eq.(15.7) to obtain $y(x, t)$.

$A = 2.50 \times 10^{-3}\ \text{m}$, $\lambda = 1.80\ \text{m}$, $v = 36.0\ \text{m/s}$

EXECUTE: **(a)** $v = f\lambda$ so $f = v/\lambda = (36.0\ \text{m/s})/1.80\ \text{m} = 20.0\ \text{Hz}$

$\omega = 2\pi f = 2\pi(20.0\ \text{Hz}) = 126\ \text{rad/s}$

$k = 2\pi/\lambda = 2\pi\ \text{rad}/1.80\ \text{m} = 3.49\ \text{rad/m}$

(b) For a wave traveling to the right, $y(x, t) = A\cos(kx - \omega t)$. This equation gives that the $x = 0$ end of the string has maximum upward displacement at $t = 0$.

Put in the numbers: $y(x, t) = (2.50 \times 10^{-3}\ \text{m})\cos((3.49\ \text{rad/m})x - (126\ \text{rad/s})t)$.

(c) The left hand end is located at $x = 0$. Put this value into the equation of part (b):

$y(0, t) = +(2.50 \times 10^{-3}\ \text{m})\cos((126\ \text{rad/s})t)$.

(d) Put $x = 1.35\ \text{m}$ into the equation of part (b):

$y(1.35\ \text{m}, t) = (2.50 \times 10^{-3}\ \text{m})\cos((3.49\ \text{rad/m})(1.35\ \text{m}) - (126\ \text{rad/s})t)$.

$y(1.35\ \text{m}, t) = (2.50 \times 10^{-3}\ \text{m})\cos(4.71\ \text{rad}) - (126\ \text{rad/s})t$

$4.71\ \text{rad} = 3\pi/2$ and $\cos(\theta) = \cos(-\theta)$, so $y(1.35\ \text{m}, t) = (2.50 \times 10^{-3}\ \text{m})\cos((126\ \text{rad/s})t - 3\pi/2\ \text{rad})$

(e) $y = A\cos(kx - \omega t)$ ((part b))

The transverse velocity is given by $v_y = \frac{\partial y}{\partial t} = A\frac{\partial}{\partial t}\cos(kx - \omega t) = +A\omega\sin(kx - \omega t)$.

The maximum v_y is $A\omega = (2.50 \times 10^{-3}\ \text{m})(126\ \text{rad/s}) = 0.315\ \text{m/s}$.

(f) $y(x, t) = (2.50 \times 10^{-3}\ \text{m})\cos((3.49\ \text{rad/m})x - (126\ \text{rad/s})t)$

$t = 0.0625\ \text{s}$ and $x = 1.35\ \text{m}$ gives

$y = (2.50 \times 10^{-3}\ \text{m})\cos((3.49\ \text{rad/m})(1.35\ \text{m}) - (126\ \text{rad/s})(0.0625\ \text{s})) = -2.50 \times 10^{-3}\ \text{m}$.

$v_y = +A\omega\sin(kx - \omega t) = +(0.315\ \text{m/s})\sin((3.49\ \text{rad/m})x - (126\ \text{rad/s})t)$

$t = 0.0625\ \text{s}$ and $x = 1.35\ \text{m}$ gives

$v_y = (0.315\ \text{m/s})\sin((3.49\ \text{rad/m})(1.35\ \text{m}) - (126\ \text{rad/s})(0.0625\ \text{s})) = 0.0$

EVALUATE: The results of part (f) illustrate that $v_y = 0$ when $y = \pm A$, as we saw from SHM in Chapter 13.

15.50. **IDENTIFY:** Compare $y(x,t)$ given in the problem to the general form given in Eq.(15.8).

SET UP: The comparison gives $A = 0.750\ \text{cm}$, $k = 0.400\pi\ \text{rad/cm}$ and $\omega = 250\pi\ \text{rad/s}$.

EXECUTE: **(a)** $A = 0.750$ cm, $\lambda = \dfrac{2}{0.400 \text{ rad/cm}} = 5.00$ cm, $f = 125$ Hz, $T = \dfrac{1}{f} = 0.00800$ s and $v = \lambda f = 6.25$ m/s.

(b) The sketches of the shape of the rope at each time are given in Figure 15.50.

(c) To stay with a wavefront as t increases, x decreases and so the wave is moving in the $-x$-direction.

(d) From Eq. (15.13), the tension is $F = \mu v^2 = (0.50 \text{ kg/m})\ (6.25 \text{ m/s})^2 = 19.5 \text{ N}$.

(e) $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2 = 54.2$ W.

EVALUATE: The argument of the cosine is $(kx + \omega t)$ for a wave traveling in the $-x$-direction, and that is the case here.

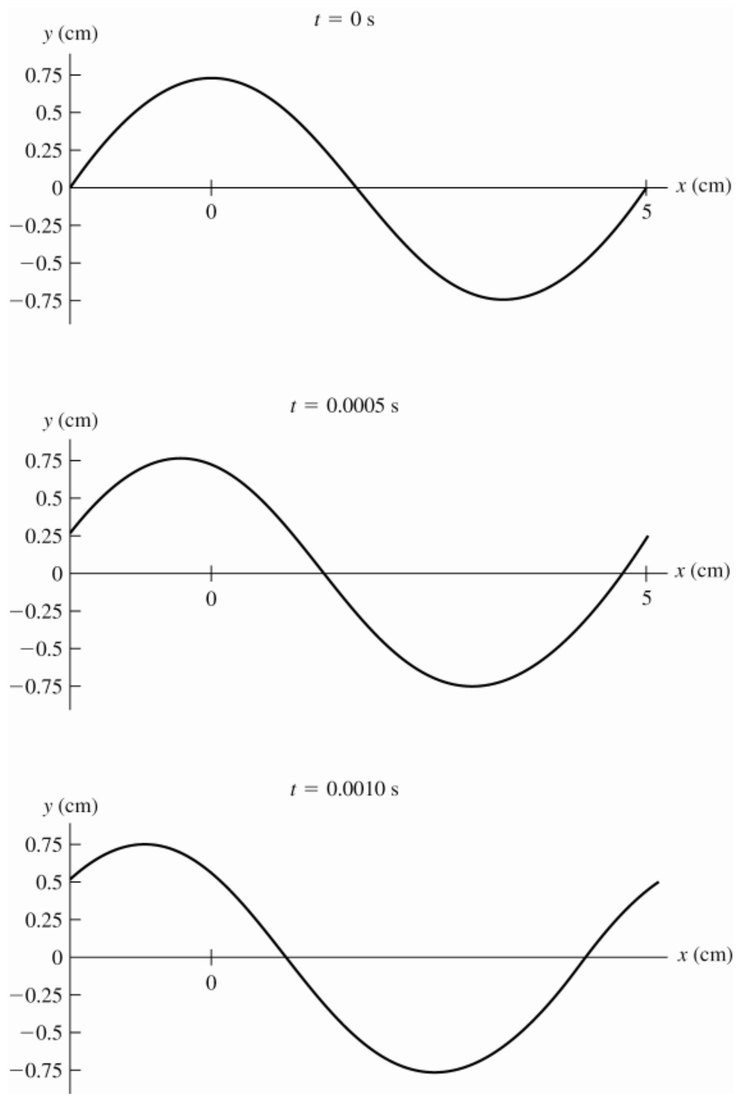

Figure 15.50

15.51. **IDENTIFY:** The speed in each segment is $v = \sqrt{F/\mu}$. The time to travel through a segment is $t = L/v$.

SET UP: The travel times for each segment are $t_1 = L\sqrt{\dfrac{\mu_1}{F}}$, $t_2 = L\sqrt{\dfrac{4\mu_1}{F}}$, and $t_3 = L\sqrt{\dfrac{\mu_1}{4F}}$.

EXECUTE: Adding the travel times gives $t_{total} = L\sqrt{\dfrac{\mu_1}{F}} + 2L\sqrt{\dfrac{\mu_1}{F}} + \frac{1}{2}L\sqrt{\dfrac{\mu_1}{F}} = \frac{7}{2}L\sqrt{\dfrac{\mu_1}{F}}$.

(b) No. The speed in a segment depends only on F and μ for that segment.

EVALUATE: The wave speed is greater and its travel time smaller when the mass per unit length of the segment decreases.

15.52. **IDENTIFY:** Apply $\sum \tau_z = 0$ to find the tension in each wire. Use $v = \sqrt{F/\mu}$ to calculate the wave speed for each wire and then $t = L/v$ is the time for each pulse to reach the ceiling, where $L = 1.25$ m.

SET UP: The wires have $\mu = \frac{m}{L} = \frac{2.50 \text{ N}}{(9.80 \text{ m/s}^2)(1.25 \text{ m})} = 0.204$ kg/m. The free-body diagram for the beam is given in Figure 15.52. Take the axis to be at the end of the beam where wire A is attached.

EXECUTE: $\sum \tau_z = 0$ gives $T_B L = w(L/3)$ and $T_B = w/3 = 583$ N. $T_A + T_B = 1750$ N, so $T_A = 1167$ N.

$v_A = \sqrt{\frac{T_A}{\mu}} = \sqrt{\frac{1167 \text{ N}}{0.204 \text{ kg/m}}} = 75.6$ m/s. $t_A = \frac{1.25 \text{ m}}{75.6 \text{ m/s}} = 0.0165$ s. $v_B = \sqrt{\frac{583 \text{ N}}{0.204 \text{ kg/m}}} = 53.5$ m/s.

$t_B = \frac{1.25 \text{ m}}{53.5 \text{ m/s}} = 0.0234$ s. $\Delta t = t_B - t_A = 6.9$ ms.

EVALUATE: The wave pulse travels faster in wire A, since that wire has the greater tension.

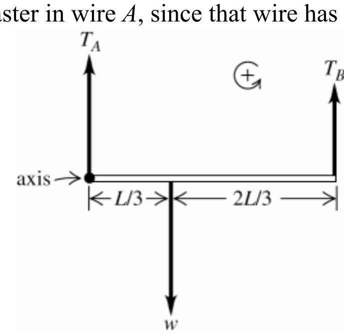

Figure 15.52

15.53. **IDENTIFY** and **SET UP:** The transverse speed of a point of the rope is $v_y = \partial y/\partial t$ where $y(x, t)$ is given by Eq.(15.7).

EXECUTE: **(a)** $y(x, t) = A\cos(kx - \omega t)$

$v_y = dy/dt = +A\omega \sin(kx - \omega t)$

$v_{y,\text{max}} = A\omega = 2\pi f A$

$f = \frac{v}{\lambda}$ and $v = \sqrt{\frac{F}{(m/L)}}$, so $f = \left(\frac{1}{\lambda}\right)\sqrt{\frac{FL}{M}}$

$v_{y,\text{max}} = \left(\frac{2\pi A}{\lambda}\right)\sqrt{\frac{FL}{M}}$

(b) To double $v_{y,\text{max}}$ increase F by a factor of 4.

EVALUATE: Increasing the tension increases the wave speed v which in turn increases the oscillation frequency. With the amplitude held fixed, increasing the number of oscillations per second increases the transverse velocity.

15.54. **IDENTIFY:** The maximum vertical acceleration must be at least g.

SET UP: $a_{\text{max}} = \omega^2 A$

EXECUTE: $g = \omega^2 A_{\text{min}}$ and thus $A_{\text{min}} = g/\omega^2$. Using $\omega = 2\pi f = 2\pi v/\lambda$ and $v = \sqrt{F/\mu}$, this becomes $A_{\text{min}} = \frac{g\lambda^2\mu}{4\pi^2 F}$.

EVALUATE: When the amplitude of the motion increases, the maximum acceleration of a point on the rope increases.

15.55. **IDENTIFY** and **SET UP:** Use Eq.(15.1) and $\omega = 2\pi f$ to replace v by ω in Eq.(15.13). Compare this equation to $\omega = \sqrt{k'/m}$ from Chapter 13 to deduce k'.

EXECUTE: **(a)** $\omega = 2\pi f$, $f = v/\lambda$, and $v = \sqrt{F/\mu}$ These equations combine to give

$\omega = 2\pi f = 2\pi(v/\lambda) = (2\pi/\lambda)\sqrt{F/\mu}$.

But also $\omega = \sqrt{k'/m}$. Equating these expressions for ω gives $k' = m(2\pi/\lambda)^2(F/\mu)$

But $m = \mu\,\Delta x$ so $k' = \Delta x(2\pi/\lambda)^2 F$

(b) EVALUATE: The "force constant" k' is independent of the amplitude A and mass per unit length μ, just as is the case for a simple harmonic oscillator. The force constant is proportional to the tension in the string F and inversely proportional to the wavelength λ. The tension supplies the restoring force and the $1/\lambda^2$ factor represents the dependence of the restoring force on the curvature of the string.

15.56. **IDENTIFY:** Apply $\sum \tau_z = 0$ to one post and calculate the tension in the wire. $v = \sqrt{F/\mu}$ for waves on the wire. $v = f\lambda$. The standing wave on the wire and the sound it produces have the same frequency. For standing waves on the wire, $\lambda_n = \dfrac{2L}{n}$.

SET UP: For the 7th overtone, $n = 8$. The wire has $\mu = m/L = (0.732\text{ kg})/(5.00\text{ m}) = 0.146\text{ kg/m}$. The free-body diagram for one of the posts is given in Figure 15.56. Forces at the pivot aren't shown. We take the rotation axis to be at the pivot, so forces at the pivot produce no torque.

EXECUTE: $\sum \tau_z = 0$ gives $w\left(\dfrac{L}{2}\cos 57.0°\right) - T(L\sin 57.0°) = 0$. $T = \dfrac{w}{2\tan 57.0°} = \dfrac{235\text{ N}}{2\tan 57.0°} = 76.3\text{ N}$. For waves on the wire, $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{76.3\text{ N}}{0.146\text{ kg/m}}} = 22.9\text{ m/s}$. For the 7th overtone standing wave on the wire, $\lambda = \dfrac{2L}{8} = \dfrac{2(5.00\text{ m})}{8} = 1.25\text{ m}$. $f = \dfrac{v}{\lambda} = \dfrac{22.9\text{ m/s}}{1.25\text{ m}} = 18.3\text{ Hz}$. The sound waves have frequency 18.3 Hz and wavelength $\lambda = \dfrac{344\text{ m/s}}{18.3\text{ Hz}} = 18.8\text{ m}$

EVALUATE: The frequency of the sound wave is at the lower limit of audible frequencies. The wavelength of the standing wave on the wire is much less than the wavelength of the sound waves, because the speed of the waves on the wire is much less than the speed of sound in air.

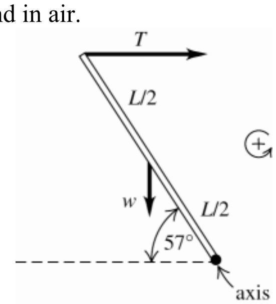

Figure 15.56

15.57. **IDENTIFY:** The magnitude of the transverse velocity is related to the slope of the *t* versus *x* curve. The transverse acceleration is related to the curvature of the graph, to the rate at which the slope is changing.

SET UP: If *y* increases as *t* increases, v_y is positive. a_y has the same sign as v_y if the transverse speed is increasing.

EXECUTE: **(a)** and **(b)** (1): The curve appears to be horizontal, and $v_y = 0$. As the wave moves, the point will begin to move downward, and $a_y < 0$. (2): As the wave moves in the $+x$-direction, the particle will move upward so $v_y > 0$. The portion of the curve to the left of the point is steeper, so $a_y > 0$. (3) The point is moving down, and will increase its speed as the wave moves; $v_y < 0$, $a_y < 0$. (4) The curve appears to be horizontal, and $v_y = 0$. As the wave moves, the point will move away from the *x*-axis, and $a_y > 0$. (5) The point is moving downward, and will increase its speed as the wave moves; $v_y < 0$, $a_y < 0$. (6) The particle is moving upward, but the curve that represents the wave appears to have no curvature, so $v_y > 0$ and $a_y = 0$.

(c) The accelerations, which are related to the curvatures, will not change. The transverse velocities will all change sign.

EVALUATE: At points 1, 3, and 5 the graph has negative curvature and $a_y < 0$. At points 2 and 4 the graph has positive curvature and $a_y > 0$.

15.58. **IDENTIFY:** The time it takes the wave to travel a given distance is determined by the wave speed *v*. A point on the string travels a distance 4*A* in time *T*.

SET UP: $v = f\lambda$. $T = 1/f$.

EXECUTE: **(a)** The wave travels a horizontal distance *d* in a time $t = \dfrac{d}{v} = \dfrac{d}{\lambda f} = \dfrac{8.00\text{ m}}{(0.600\text{ m})(40.0\text{ Hz})} = 0.333\text{ s}$.

(b) A point on the string will travel a vertical distance of $4A$ each cycle. Although the transverse velocity $v_y(x,t)$ is not constant, a distance of $h = 8.00\text{ m}$ corresponds to a whole number of cycles, $n = h/(4A) = (8.00\text{ m})/[4(5.00\times10^{-3}\text{ m})] = 400$, so the amount of time is $t = nT = n/f = (400)/(40.0\text{ Hz}) = 10.0\text{ s}$.

EVALUATE: **(c)** The time in part (a) is independent of amplitude but the time in part (b) depends on the amplitude of the wave. For (b), the time is halved if the amplitude is doubled.

15.59. **IDENTIFY:** $y^2(x,y) + z^2(x,y) = A^2$. The trajectory is a circle of radius A.

SET UP: $v_y = \partial y/\partial t$, $v_z = \partial z/\partial t$. $a_y = \partial v_y/\partial t$, $a_z = \partial v_z/\partial t$

EXECUTE: At $t = 0$, $y(0,0) = A$, $z(0,0) = 0$. At $t = \pi/2\omega$, $y(0,\pi/2\omega) = 0$, $z(0,\pi/2\omega) = -A$.
At $t = \pi/\omega$, $y(0,\pi/\omega) = -A$, $z(0,\pi/\omega) = 0$. At $t = 3\pi/2\omega$, $y(0,3\pi/2\omega) = 0$, $z(0,3\pi/2\omega) = A$. The trajectory and these points are sketched in Figure 15.59.

(b) $v_y = \partial y/\partial t = +A\omega\sin(kx-\omega t)$, $v_z = \partial z/\partial t = -A\omega\cos(kx-\omega t)$.

$\vec{v} = v_y\hat{j} + v_z\hat{k} = A\omega[\sin(kx-\omega)\hat{j} - \cos(kx-\omega t)\hat{k}]$. $v = \sqrt{v_y^2 + v_z^2} = A\omega$ so the speed is constant.

$\vec{r} = y\hat{j} + z\hat{k}$. $\vec{r}\cdot\vec{v} = yv_y + zv_z = A^2\omega\sin(kx-\omega t)\cos(kx-\omega t) - A^2\omega\cos(kx-\omega t)\sin(kx-\omega t) = 0$.

$\vec{r}\cdot\vec{v} = 0$, so $\vec{v}$ is tangent to the circular path.

(c) $a_y = \partial v_y/\partial t = -A\omega^2\cos(kx-\omega t)$, $a_z = \partial v_z/\partial t = -A\omega^2\sin(kx-\omega t)$

$\vec{r}\cdot\vec{a} = ya_y + za_z = -A^2\omega^2[\cos^2(kx-\omega t) + \sin^2(kx-\omega t)] = -A^2\omega^2$. $r = A$, $\vec{r}\cdot\vec{a} = -ra$.

$\vec{r}\cdot\vec{a} = ra\cos\phi$ so $\phi = 180°$ and $\vec{a}$ is opposite in direction to $\vec{r}$; $\vec{a}$ is radially inward. For these $y(x,t)$ and $z(x,t)$, $y^2 + z^2 = A^2$, so the path is again circular, but the particle rotates in the opposite sense compared to part (a).

EVALUATE: The wave propagates in the $+x$-direction. The displacement is transverse, so $\vec{v}$ and $\vec{a}$ lie in the yz-plane.

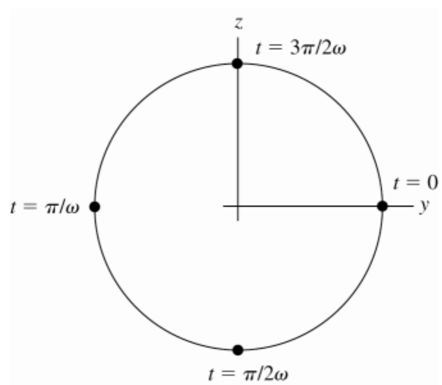

Figure 15.59

15.60. **IDENTIFY:** The wavelengths of the standing waves on the wire are given by $\lambda_n = \frac{2L}{n}$. When the ball is changed the wavelength changes because the length of the wire changes; $\Delta l = \frac{Fl_0}{AY}$.

SET UP: For the third harmonic, $n = 3$. For copper, $Y = 11\times10^{10}\text{ Pa}$. The wire has cross-sectional area $A = \pi r^2 = \pi(0.512\times10^{-3}\text{ m})^2 = 8.24\times10^{-7}\text{ m}^2$

EXECUTE: **(a)** $\lambda_3 = \frac{2(1.20\text{ m})}{3} = 0.800\text{ m}$

(b) The increase in length when the 100.0 N ball is replaced by the 500.0 N ball is given by $\Delta l = \frac{(\Delta F)l_0}{AY}$, where $\Delta F = 400.0\text{ N}$ is the increase in the force applied to the end of the wire.

$\Delta l = \frac{(400.0\text{ N})(1.20\text{ m})}{(8.24\times10^{-7}\text{ m}^2)(11\times10^{10}\text{ Pa})} = 5.30\times10^{-3}\text{ m}$. The change in wavelength is $\Delta\lambda = \frac{2}{3}\Delta l = 3.5\text{ mm}$.

EVALUATE: The change in tension changes the wave speed and that in turn changes the frequency of the standing wave, but the problem asks only about the wavelength.

15.61. **IDENTIFY:** Follow the procedure specified in part (b).

SET UP: If $u = x - vt$, then $\frac{\partial u}{\partial t} = -v$ and $\frac{\partial u}{\partial x} = 1$.

EXECUTE: **(a)** As time goes on, someone moving with the wave would need to move in such a way that the wave appears to have the same shape. If this motion can be described by $x = vt + b$, with b a constant, then $y(x,t) = f(b)$, and the waveform is the same to such an observer.

(b) $\frac{\partial^2 y}{\partial x^2} = \frac{d^2 f}{du^2}$ and $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{d^2 f}{du^2}$, so $y(x,t) = f(x - vt)$ is a solution to the wave equation with wave speed v.

(c) This is of the form $y(x,t) = f(u)$, with $u = x - vt$ and $f(u) = De^{-B^2(x - Ct/B)^2}$. The result of part (b) may be used to determine the speed $v = C/B$.

EVALUATE: The wave in part (c) moves in the $+x$-direction. The speed of the wave is independent of the constant D.

15.62. **IDENTIFY:** The phase angle determines the value of y for $x = 0$, $t = 0$ but does not affect the shape of the $y(x,t)$ versus x or t graph.

SET UP: $\frac{\partial \cos(kx - \omega t + \phi)}{\partial t} = -\omega \sin(kx - \omega t + \phi)$.

EXECUTE: **(a)** The graphs for each ϕ are sketched in Figure 15.62.

(b) $\frac{\partial y}{\partial t} = -\omega A \sin(kx - \omega t + \phi)$

(c) No. $\phi = \pi/4$ or $\phi = 3\pi/4$ would both give $A/\sqrt{2}$. If the particle is known to be moving downward, the result of part (b) shows that $\cos\phi < 0$, and so $\phi = 3\pi/4$.

(d) To identify ϕ uniquely, the quadrant in which ϕ lies must be known. In physical terms, the signs of both the position and velocity, and the magnitude of either, are necessary to determine ϕ (within additive multiples of 2π).

EVALUATE: The phase $\phi = 0$ corresponds to $y = A$ at $x = 0$, $t = 0$.

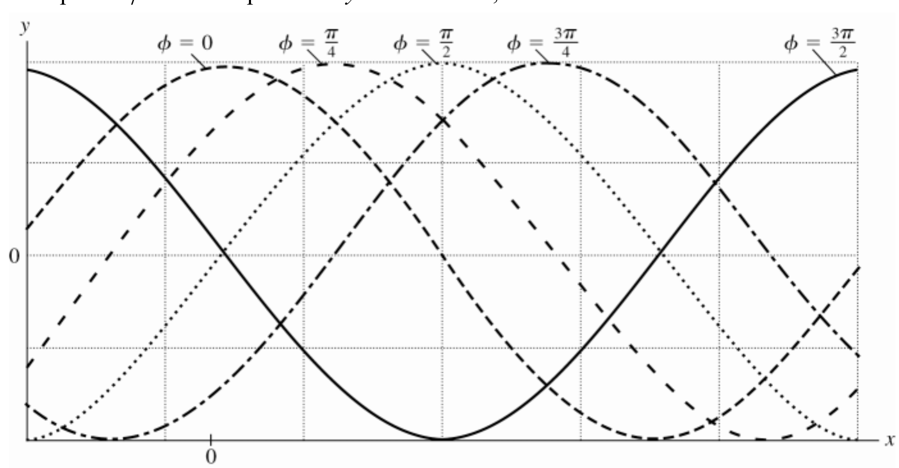

Figure 15.62

15.63. **IDENTIFY** and **SET UP:** Use Eq.(15.13) to replace μ, and then Eq.(15.6) to replace v.

EXECUTE: **(a)** Eq.(15.25): $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$

$v = \sqrt{F/\mu}$ says $\sqrt{\mu} = \sqrt{F}/v$ so $P_{av} = \frac{1}{2}\left(\sqrt{F}/v\right)\sqrt{F}\omega^2 A^2 = \frac{1}{2}F\omega^2 A^2/v$

$\omega = 2\pi f$ so $\omega/v = 2\pi f/v = 2\pi/\lambda = k$ and $P_{av} = \frac{1}{2}Fk\omega A^2$, as was to be shown.

(b) IDENTIFY: For the ω dependence, use Eq.(15.25) since it involves just ω, not k: $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$.

SET UP: P_{av}, μ, A all constant so $\sqrt{F}\omega^2$ is constant, and $\sqrt{F_1}\omega_1^2 = \sqrt{F_2}\omega_2^2$.

EXECUTE: $\omega_2 = \omega_1(F_1/F_2)^{1/4} = \omega_1(F_1/4F_1)^{1/4} = \omega_1(4)^{-1/4} = \omega_1/\sqrt{2}$

ω must be changed by a factor of $1/\sqrt{2}$ (decreased)

IDENTIFY: For the k dependence, use the equation derived in part (a), $P_{av} = \frac{1}{2}Fk\omega A^2$.

SET UP: If P_{av} and A are constant then $Fk\omega$ must be constant, and $F_1k_1\omega_1 = F_2k_2\omega_2$.

EXECUTE: $k_2 = k_1\left(\frac{F_1}{F_2}\right)\left(\frac{\omega_1}{\omega_2}\right) = k_1\left(\frac{F_1}{4F_1}\right)\left(\frac{\omega_1}{\omega_1/\sqrt{2}}\right) = k_1\frac{\sqrt{2}}{4} = k_1\sqrt{\frac{2}{16}} = k_1/\sqrt{8}$

k must be changed by a factor of $1/\sqrt{8}$ (decreased).

EVALUATE: Power is the transverse force times the transverse velocity. To keep P_{av} constant the transverse velocity must be decreased when F is increased, and this is done by decreasing ω.

15.64. **IDENTIFY:** The wave moves in the $+x$ direction with speed v, so to obtain $y(x,t)$ replace x with $x - vt$ in the expression for $y(x,0)$.

SET UP: $P(x,t)$ is given by Eq.(15.21).

EXECUTE: **(a)** The wave pulse is sketched in Figure 15.64.

(b)

$$y(x,t) = \begin{cases} 0 & \text{for} \quad (x-vt) < -L \\ h(L+x-vt)/L & \text{for} \quad -L < (x-vt) < 0 \\ h(L-x+vt)/L & \text{for} \quad 0 < (x-vt) < L \\ 0 & \text{for} \quad (x-vt) > L \end{cases}$$

(c) From Eq.(15.21):

$$P(x,t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t} = \begin{cases} -F(0)(0) = 0 & \text{for} \quad (x-vt) < -L \\ -F(h/L)(-hv/L) = Fv(h/L)^2 & \text{for} \quad -L < (x-vt) < 0 \\ -F(-h/L)(hv/L) = Fv(h/L)^2 & \text{for} \quad 0 < (x-vt) < L \\ -F(0)(0) = 0 & \text{for} \quad (x-vt) > L \end{cases}$$

Thus the instantaneous power is zero except for $-L < (x-vt) < L$, where it has the constant value $Fv(h/L)^2$.

EVALUATE: For this pulse the transverse velocity v_y is constant in magnitude and has opposite sign on either side of the peak of the pulse.

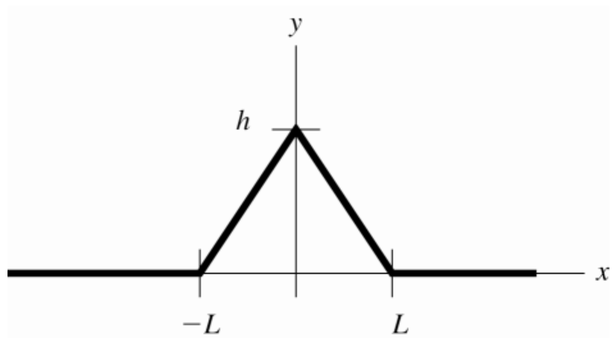

Figure 15.64

15.65. **IDENTIFY** and **SET UP:** The average power is given by Eq.(15.25). Rewrite this expression in terms of v and λ in place of F and ω.

EXECUTE: **(a)** $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$

$v = \sqrt{F/\mu}$ so $\sqrt{F} = v\sqrt{\mu}$

$\omega = 2\pi f = 2\pi(v/\lambda)$

Using these two expressions to replace $\sqrt{F}$ and ω gives $P_{av} = 2\mu\pi^2 v^3 A^2/\lambda^2$; $\mu = (6.00\times10^{-3}\text{ kg})/(8.00\text{ m})$

$$A = \left(\frac{2\lambda^2 P_{av}}{4\pi^2 v^3 \mu}\right) = 7.07\text{ cm}$$

(b) EVALUATE: $P_{av} \sim v^3$ so doubling v increases P_{av} by a factor of 8.

$P_{av} = 8(50.0\text{ W}) = 400.0\text{ W}$

15.66. **IDENTIFY:** Draw the graphs specified in part (a).

SET UP: When $y(x,t)$ is a maximum, the slope $\partial y/\partial x$ is zero. The slope has maximum magnitude when $y(x,t) = 0$.

EXECUTE: **(a)** The graph is sketched in Figure 15.66a.

(b) The power is a maximum where the displacement is zero, and the power is a minimum of zero when the magnitude of the displacement is a maximum.

(c) The energy flow is always in the same direction.

(d) In this case, $\frac{\partial y}{\partial x} = -kA\sin(kx + \omega t)$ and Eq.(15.22) becomes $P(x,t) = -Fk\omega A^2 \sin^2(kx + \omega t)$. The power is now negative (energy flows in the $-x$-direction), but the qualitative relations of part (b) are unchanged. The graph is sketched in Figure 15.66b.

EVALUATE: $\cos\theta$ and $\sin\theta$ are $180°$ out of phase, so for fixed *t*, maximum *y* corresponds to zero *P* and $y = 0$ corresponds to maximum *P*.

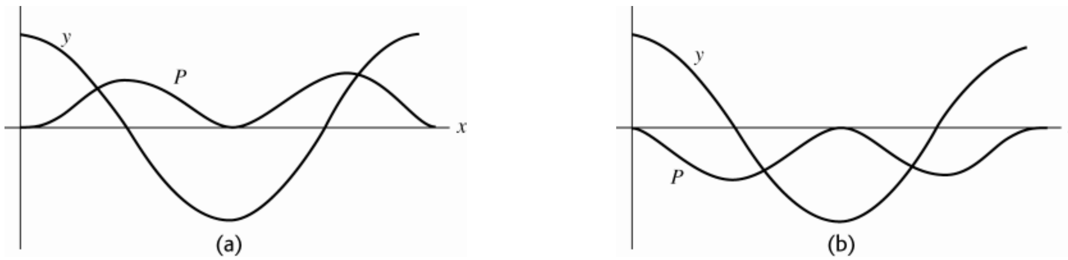

Figure 15.66

15.67. **IDENTIFY** and **SET UP:** $v = \sqrt{F/\mu}$. The coefficient of linear expansion α is defined by $\Delta L = L_0 \alpha\, \Delta T$. This can be combined with $Y = \frac{F/A}{\Delta L/L_0}$ to give $\Delta F = -Y\alpha A \Delta T$ for the change in tension when the temperature changes by ΔT. Combine the two equations and solve for α.

EXECUTE: $v_1 = \sqrt{F/\mu}$, $v_1^2 = F/\mu$ and $F = \mu v_1^2$

The length and hence μ stay the same but the tension decreases by $\Delta F = -Y\alpha A\, \Delta T$.

$v_2 = \sqrt{(F + \Delta F)/\mu} = \sqrt{(F - Y\alpha A\, \Delta T)/\mu}$

$v_2^2 = F/\mu - Y\alpha A\, \Delta T/\mu = v_1^2 - Y\alpha A\, \Delta T/\mu$

And $\mu = m/L$ so $A/\mu = AL/m = V/m = 1/\rho$. (*A* is the cross-sectional area of the wire, *V* is the volume of a length *L*.) Thus $v_1^2 - v_2^2 = \alpha(Y\, \Delta T/\rho)$ and $\alpha = \frac{v_1^2 - v_2^2}{(Y/\rho)\, \Delta T}$

EVALUATE: When *T* increases the tension decreases and *v* decreases.

15.68. **IDENTIFY:** The time between positions 1 and 5 is equal to $T/2$. $v = f\lambda$. The velocity of points on the string is given by Eq.(15.9).

SET UP: Four flashes occur from position 1 to position 5, so the elapsed time is $4\left(\frac{60\text{ s}}{5000}\right) = 0.048\text{ s}$. The figure in the problem shows that $\lambda = L = 0.500\text{ m}$. At point *P* the amplitude of the standing wave is 1.5 cm.

EXECUTE: **(a)** $T/2 = 0.048\text{ s}$ and $T = 0.096\text{ s}$. $f = 1/T = 10.4\text{ Hz}$. $\lambda = 0.500\text{ m}$.

(b) The fundamental standing wave has nodes at each end and no nodes in between. This standing wave has one additional node. This is the 1st overtone and 2nd harmonic.

(c) $v = f\lambda = (10.4\text{ Hz})(0.500\text{ m}) = 5.20\text{ m/s}$.

(d) In position 1, point *P* is at its maximum displacement and its speed is zero. In position 3, point *P* is passing through its equilibrium position and its speed is $v_{max} = \omega A = 2\pi fA = 2\pi(10.4\text{ Hz})(0.015\text{ m}) = 0.980\text{ m/s}$.

(e) $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{FL}{m}}$ and $m = \frac{FL}{v^2} = \frac{(1.00\text{ N})(0.500\text{ m})}{(5.20\text{ m/s})^2} = 18.5\text{ g}$.

EVALUATE: The standing wave is produced by traveling waves moving in opposite directions. Each point on the string moves in SHM, and the amplitude of this motion varies with position along the string.

15.69. **IDENTIFY** and **SET UP:** There is a node at the post and there must be a node at the clothespin. There could be additional nodes in between. The distance between adjacent nodes is $\lambda/2$, so the distance between *any* two nodes is $n(\lambda/2)$ for $n = 1, 2, 3, \ldots$ This must equal 45.0 cm, since there are nodes at the post and clothespin. Use this in Eq.(15.1) to get an expression for the possible frequencies *f*.

EXECUTE: $45.0\text{ cm} = n(\lambda/2)$, $\lambda = v/f$, so $f = n[v/(90.0\text{ cm})] = (0.800\text{ Hz})n$, $n = 1, 2, 3, \ldots$

EVALUATE: Higher frequencies have smaller wavelengths, so more node-to-node segments fit between the post and clothespin.

15.70. IDENTIFY: The displacement of the string at any point is $y(x,t) = (A_{SW}\sin kx)\sin\omega t$. For the fundamental mode $\lambda = 2L$, so at the midpoint of the string $\sin kx = \sin(2\pi/\lambda)(L/2) = 1$, and $y = A_{SW}\sin\omega t$. The transverse velocity is $v_y = \partial y/\partial t$ and the transverse acceleration is $a_y = \partial v_y/\partial t$.

SET UP: Taking derivatives gives $v_y = \dfrac{\partial y}{\partial t} = \omega A_{SW}\cos\omega t$, with maximum value $v_{y,\max} = \omega A_{SW}$, and

$a_y = \dfrac{\partial v_y}{\partial t} = -\omega^2 A_{SW}\sin\omega t$, with maximum value $a_{y,\max} = \omega^2 A_{SW}$.

EXECUTE: $\omega = a_{y,\max}/v_{y,\max} = (8.40\times10^3\ \text{m/s}^2)/(3.80\ \text{m/s}) = 2.21\times10^3\ \text{rad/s}$, and then

$A_{SW} = v_{y,\max}/\omega = (3.80\ \text{m/s})/(2.21\times10^3\ \text{rad/s}) = 1.72\times10^{-3}\ \text{m}$.

(b) $v = \lambda f = (2L)(\omega/2\pi) = L\omega/\pi = (0.386\ \text{m})(2.21\times10^3\ \text{rad/s})/\pi = 272\ \text{m/s}$.

EVALUATE: The maximum transverse velocity and acceleration will have different (smaller) values at other points on the string.

15.71. IDENTIFY: To show this relationship is valid, take the second time derivative.

SET UP: $\dfrac{\partial}{\partial t}\sin\omega t = \cos\omega t$. $\dfrac{\partial}{\partial t}\cos\omega t = -\omega\sin\omega t$.

EXECUTE: **(a)** $\dfrac{\partial^2 y(x,t)}{\partial t^2} = \dfrac{\partial^2}{\partial t^2}[(A_{SW}\sin kx)\sin\omega t] = \omega\dfrac{\partial}{\partial t}[(A_{SW}\sin kx)\cos\omega t]$

$\dfrac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2\left[(A_{sw}\sin kx)\sin\omega t\right] = -\omega^2 y(x,t)$. This equation shows that $a_y = -\omega^2 y$. This is characteristic of simple harmonic motion; each particle of the string moves in simple harmonic motion.

(b) Yes, the traveling wave is also a solution of this equation. When a string carries a traveling wave each point on the string moves in simple harmonic motion.

EVALUATE: A standing wave is the superposition of two traveling waves, so it is not surprising that for both types of waves the particles on the string move in SHM.

15.72. IDENTIFY and SET UP: Carry out the analysis specified in the problem.

EXECUTE: **(a)** The wave moving to the left is inverted and reflected; the reflection means that the wave moving to the left is the same function of $-x$, and the inversion means that the function is $-f(-x)$.

(b) The wave that is the sum is $f(x) - f(-x)$ (an inherently odd function), and for any f, $f(0) - f(-0) = 0$.

(c) The wave is reflected but not inverted (see the discussion in part (a) above), so the wave moving to the left in Figure 15.21 in the textbook is $+f(-x)$.

(d) $\dfrac{dy}{dx} = \dfrac{d}{dx}(f(x) + f(-x)) = \dfrac{df(x)}{dx} + \dfrac{df(-x)}{dx} = \dfrac{df(x)}{dx} + \dfrac{df(-x)}{d(-x)}\dfrac{d(-x)}{dx} = \dfrac{df}{dx} - \left.\dfrac{df}{dx}\right|_{x=-x}$.

At $x = 0$, the terms are the same and the derivative is zero.

EVALUATE: Our results verify the behavior shown in Figures 15.20 and 15.21 in the textbook.

15.73. IDENTIFY: Carry out the derivation as done in the text for Eq.(15.28). The transverse velocity is $v_y = \partial y/\partial t$ and the transverse acceleration is $a_y = \partial v_y/\partial t$.

(a) SET UP: For reflection from a free end of a string the reflected wave is *not* inverted, so

$y(x,\ t) = y_1(x,\ t) + y_2(x,\ t)$, where

$y_1(x,\ t) = A\cos(kx + \omega t)$ (traveling to the left)

$y_2(x,\ t) = A\cos(kx - \omega t)$ (traveling to the right)

Thus $y(x,\ t) = A[\cos(kx + \omega t) + \cos(kx - \omega t)]$.

EXECUTE: Apply the trig identity $\cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$ with $a = kx$ and $b = \omega t$:

$\cos(kx + \omega t) = \cos kx\cos\omega t - \sin kx\sin\omega t$ and

$\cos(kx - \omega t) = \cos kx\cos\omega t + \sin kx\sin\omega t$.

Then $y(x,\ t) = (2A\cos kx)\cos\omega t$ (the other two terms cancel)

(b) For $x = 0$, $\cos kx = 1$ and $y(x,\ t) = 2A\cos\omega t$. The amplitude of the simple harmonic motion at $x = 0$ is $2A$, which is the maximum for this standing wave, so $x = 0$ is an antinode.

(c) $y_{\max} = 2A$ from part (b).

$v_y = \dfrac{\partial y}{\partial t} = \dfrac{\partial}{\partial t}[(2A\cos kx)\cos\omega t] = 2A\cos kx\dfrac{\partial\cos\omega t}{\partial t} = -2A\omega\cos kx\sin\omega t.$

At $x=0$, $v_y=-2A\omega\sin\omega t$ and $(v_y)_{\max}=2A\omega$

$$a_y=\frac{\partial^2 y}{\partial t^2}=\frac{\partial v_y}{\partial t}=-2A\omega\cos kx\frac{\partial\sin\omega t}{\partial t}=-2A\omega^2\cos kx\cos\omega t$$

At $x=0$, $a_y=-2A\omega^2\cos\omega t$ and $(a_y)_{\max}=2A\omega^2$.

EVALUATE: The expressions for $(v_y)_{\max}$ and $(a_y)_{\max}$ are the same as at the antinodes for the standing wave of a string fixed at both ends.

15.74. **IDENTIFY:** The standing wave is given by Eq.(15.28).

SET UP: At an antinode, $\sin kx=1$. $v_{y,\max}=\omega A$. $a_{y,\max}=\omega^2 A$.

EXECUTE: **(a)** $\lambda=v/f=(192.0\text{ m/s})/(240.0\text{ Hz})=0.800\text{ m}$, and the wave amplitude is $A_{\text{SW}}=0.400\text{ cm}$. The amplitude of the motion at the given points is

(i) $(0.400\text{ cm})\sin(\pi)=0$ (a node) (ii) $(0.400\text{ cm})\sin(\pi/2)=0.400\text{ cm}$ (an antinode)

(iii) $(0.400\text{ cm})\sin(\pi/4)=0.283\text{ cm}$

(b) The time is half of the period, or $1/(2f)=2.08\times10^{-3}\text{s}$.

(c) In each case, the maximum velocity is the amplitude multiplied by $\omega=2\pi f$ and the maximum acceleration is the amplitude multiplied by $\omega^2=4\pi^2 f^2$:

(i) 0, 0; (ii) 6.03 m/s, $9.10\times10^3\text{ m/s}^2$; (iii) 4.27 m/s, $6.43\times10^3\text{ m/s}^2$.

EVALUATE: The amplitude, maximum transverse velocity, and maximum transverse acceleration vary along the length of the string. But the period of the simple harmonic motion of particles of the string is the same at all points on the string.

15.75. **IDENTIFY:** The standing wave frequencies are given by $f_n=n\left(\frac{v}{2L}\right)$. $v=\sqrt{F/\mu}$. Use the density of steel to calculate μ for the wire.

SET UP: For steel, $\rho=7.8\times10^3\text{ kg/m}^3$. For the first overtone standing wave, $n=2$.

EXECUTE: $v=\frac{2Lf_2}{2}=(0.550\text{ m})(311\text{ Hz})=171\text{ m/s}$. The volume of the wire is $V=(\pi r^2)L$. $m=\rho V$ so

$$\mu=\frac{m}{L}=\frac{\rho V}{L}=\rho\pi r^2=(7.8\times10^3\text{ kg/m}^3)\pi(0.57\times10^{-3}\text{ m})^2=7.96\times10^{-3}\text{ kg/m}.$$ The tension is

$$F=\mu v^2=(7.96\times10^{-3}\text{ kg/m})(171\text{ m/s})^2=233\text{ N}.$$

EVALUATE: The tension is not large enough to cause much change in length of the wire.

15.76. **IDENTIFY:** The mass and breaking stress determine the length and radius of the string. $f_1=\frac{v}{2L}$, with $v=\sqrt{\frac{F}{\mu}}$.

SET UP: The tensile stress is $F/\pi r^2$.

EXECUTE: **(a)** The breaking stress is $\frac{F}{\pi r^2}=7.0\times10^8\text{ N/m}^2$ and the maximum tension is $F=900\text{ N}$, so solving for r gives the minimum radius $r=\sqrt{\frac{900\text{ N}}{\pi(7.0\times10^8\text{ N/m}^2)}}=6.4\times10^{-4}\text{ m}$. The mass and density are fixed,

$\rho=\frac{M}{\pi r^2 L}$ so the minimum radius gives the maximum length

$$L=\frac{M}{\pi r^2\rho}=\frac{4.0\times10^{-3}\text{ kg}}{\pi(6.4\times10^{-4}\text{ m})^2(7800\text{ kg/m}^3)}=0.40\text{ m}.$$

(b) The fundamental frequency is $f_1=\frac{1}{2L}\sqrt{\frac{F}{\mu}}=\frac{1}{2L}\sqrt{\frac{F}{M/L}}=\frac{1}{2}\sqrt{\frac{F}{ML}}$. Assuming the maximum length of the string is free to vibrate, the highest fundamental frequency occurs when $F=900\text{ N}$ and

$$f_1=\tfrac{1}{2}\sqrt{\frac{900\text{ N}}{(4.0\times10^{-3}\text{ kg})(0.40\text{ m})}}=376\text{ Hz}.$$

EVALUATE: If the radius was any smaller the breaking stress would be exceeded. If the radius were greater, so the stress was less than the maximum value, then the length would be less to achieve the same total mass.

15.77. **IDENTIFY:** At a node, $y(x,t)=0$ for all *t*. y_1+y_2 is a standing wave if the locations of the nodes don't depend on *t*.

SET UP: The string is fixed at each end so for all harmonics the ends are nodes. The second harmonic is the first overtone and has one additional node.

EXECUTE: **(a)** The fundamental has nodes only at the ends, $x = 0$ and $x = L$.
(b) For the second harmonic, the wavelength is the length of the string, and the nodes are at $x = 0, x = L/2$ and $x = L$.
(c) The graphs are sketched in Figure 15.77.
(d) The graphs in part (c) show that the locations of the nodes and antinodes between the ends vary in time.
EVALUATE: The sum of two standing waves of different frequencies is not a standing wave.

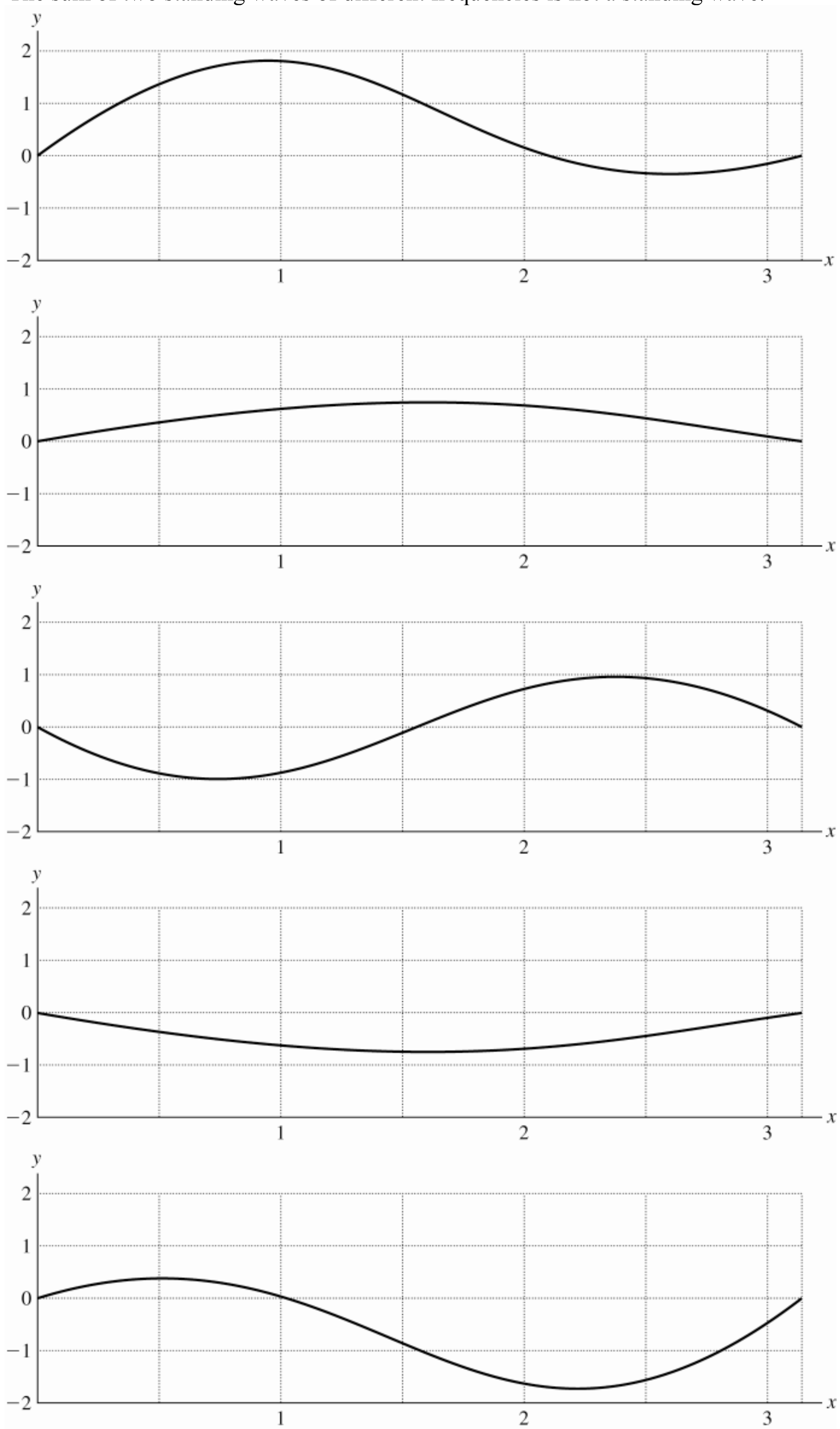

Figure 15.77

15.78. **IDENTIFY:** $f_1 = \dfrac{v}{2L}$. The buoyancy force B that the water exerts on the object reduces the tension in the wire. $B = \rho_{\text{fluid}} V_{\text{submerged}} g$.

SET UP: For aluminum, $\rho_a = 2700\ \text{kg/m}^3$. For water, $\rho_w = 1000\ \text{kg/m}^3$. Since the sculpture is completely submerged, $V_{\text{submerged}} = V_{\text{object}} = V$.

EXECUTE: L is constant, so $\dfrac{f_{\text{air}}}{v_{\text{air}}} = \dfrac{f_w}{v_w}$ and the fundamental frequency when the sculpture is submerged is $f_w = f_{\text{air}}\left(\dfrac{v_w}{v_{\text{air}}}\right)$, with $f_{\text{air}} = 250.0\ \text{Hz}$. $v = \sqrt{\dfrac{F}{\mu}}$ so $\dfrac{v_w}{v_{\text{air}}} = \sqrt{\dfrac{F_w}{F_{\text{air}}}}$. When the sculpture is in air, $F_{\text{air}} = w = mg = \rho_a V g$.

When the sculpture is submerged in water, $F_w = w - B = (\rho_a - \rho_w)Vg$. $\dfrac{v_w}{v_{\text{air}}} = \sqrt{\dfrac{\rho_a - \rho_w}{\rho_a}}$ and

$$f_w = (250.0\ \text{Hz})\sqrt{1 - \frac{1000\ \text{kg/m}^3}{2700\ \text{kg/m}^3}} = 198\ \text{Hz}.$$

EVALUATE: We have neglected the buoyant force on the wire itself.

15.79. **IDENTIFY:** Compute the wavelength from the length of the string. Use Eq.(15.1) to calculate the wave speed and then apply Eq.(15.13) to relate this to the tension.

(a) SET UP: The tension F is related to the wave speed by $v = \sqrt{F/\mu}$ (Eq.(15.13)), so use the information given to calculate v.

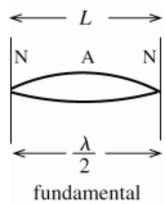

Figure 15.79

EXECUTE: $\lambda/2 = L$

$\lambda = 2L = 2(0.600\ \text{m}) = 1.20\ \text{m}$

$v = f\lambda = (65.4\ \text{Hz})(1.20\ \text{m}) = 78.5\ \text{m/s}$

$\mu = m/L = 14.4\times10^{-3}\ \text{kg}/0.600\ \text{m} = 0.024\ \text{kg/m}$

Then $F = \mu v^2 = (0.024\ \text{kg/m})(78.5\ \text{m/s})^2 = 148\ \text{N}$.

(b) SET UP: $F = \mu v^2$ and $v = f\lambda$ give $F = \mu f^2\lambda^2$.

μ is a property of the string so is constant.

λ is determined by the length of the string so stays constant.

μ, λ constant implies $F/f^2 = \mu\lambda^2 = \text{constant}$, so $F_1/f_1^2 = F_2/f_2^2$

EXECUTE: $F_2 = F_1\left(\dfrac{f_2}{f_1}\right)^2 = (148\ \text{N})\left(\dfrac{73.4\ \text{Hz}}{65.4\ \text{Hz}}\right)^2 = 186\ \text{N}$.

The percent change in F is $\dfrac{F_2 - F_1}{F_1} = \dfrac{186\ \text{N} - 148\ \text{N}}{148\ \text{N}} = 0.26 = 26\%$.

EVALUATE: The wave speed and tension we calculated are similar in magnitude to values in the Examples. Since the frequency is proportional to $\sqrt{F}$, a 26% increase in tension is required to produce a 13% increase in the frequency.

15.80. **IDENTIFY** and **SET UP:** Consider the derivation of the speed of a longitudinal wave in Section 15.4.

EXECUTE: **(a)** The quantity of interest is the change in force per fractional length change. The force constant k' is the change in force per length change, so the force change per fractional length change is $k'L$, the applied force at one end is $F = (k'L)(v_y/v)$ and the longitudinal impulse when this force is applied for a time t is $k'Lt\,v_y/v$. The change in longitudinal momentum is $((vt)m/L)v_y$ and equating the expressions, canceling a factor of t and solving for v gives $v^2 = L^2 k'/m$.

(b) $v = (2.00\ \text{m})\sqrt{(1.50\ \text{N/m})/(0.250\ \text{kg})} = 4.90\ \text{m/s}$

EVALUATE: A larger k' corresponds to a stiffer spring and for a stiffer spring the wave speed is greater.

15.81. **IDENTIFY:** Carry out the analysis specified in the problem.

SET UP: The kinetic energy of a very short segment Δx is $\Delta K = \frac{1}{2}(\Delta m)v_y^2$. $v_y = \partial y/\partial t$. The work done by the tension is F times the increase in length of the segment. Let the potential energy be zero when the segment is unstretched.

EXECUTE: **(a)** $u_\text{k} = \dfrac{\Delta K}{\Delta x} = \dfrac{(1/2)\Delta m v_y^2}{\Delta m/\mu} = \dfrac{1}{2}\mu\left(\dfrac{\partial y}{\partial t}\right)^2.$

(b) $\dfrac{\partial y}{\partial t} = \omega A \sin(kx - \omega t)$ and so $u_\text{k} = \dfrac{1}{2}\mu\omega^2 A^2 \sin^2(kx - \omega t).$

(c) The piece has width Δx and height $\Delta x \dfrac{\partial y}{\partial x}$, and so the length of the piece is

$\left((\Delta x)^2 + \left(\Delta x \dfrac{\partial y}{\partial x}\right)^2\right)^{1/2} = \Delta x\left(1 + \left(\dfrac{\partial y}{\partial x}\right)^2\right)^{1/2} \approx \Delta x\left[1 + \dfrac{1}{2}\left(\dfrac{\partial y}{\partial x}\right)^2\right]$, where the relation given in the hint has been used.

(d) $u_\text{p} = F\dfrac{\Delta x\left[1 + \frac{1}{2}(\partial y/\partial x)^2\right] - \Delta x}{\Delta x} = \dfrac{1}{2}F\left(\dfrac{\partial y}{\partial x}\right)^2.$

(e) $\dfrac{\partial y}{\partial x} = -kA \sin(kx - \omega t)$, and so $u_\text{p} = \dfrac{1}{2}Fk^2 A^2 \sin^2(kx - \omega t)$.

(f) Comparison with the result of part (c) with $k^2 = \omega^2/v^2 = \omega^2 \mu/F$ shows that for a sinusoidal wave $u_\text{k} = u_\text{p}$.

(g) The graph is given in Figure 15.81. In this graph, u_k and u_p coincide, as shown in part (f). At $y = 0$, the string is stretched the most, and is moving the fastest, so u_k and u_p are maximized. At the extremes of y, the string is unstretched and is not moving, so u_k and u_p are both at their minimum of zero.

(h) $u_\text{k} + u_\text{p} = Fk^2A^2\sin^2(kx - \omega t) = Fk(\omega/v)A^2\sin^2(kx - \omega t) = \dfrac{P}{v}.$

EVALUATE: The energy density travels with the wave, and the rate at which the energy is transported is the product of the density per unit length and the speed.

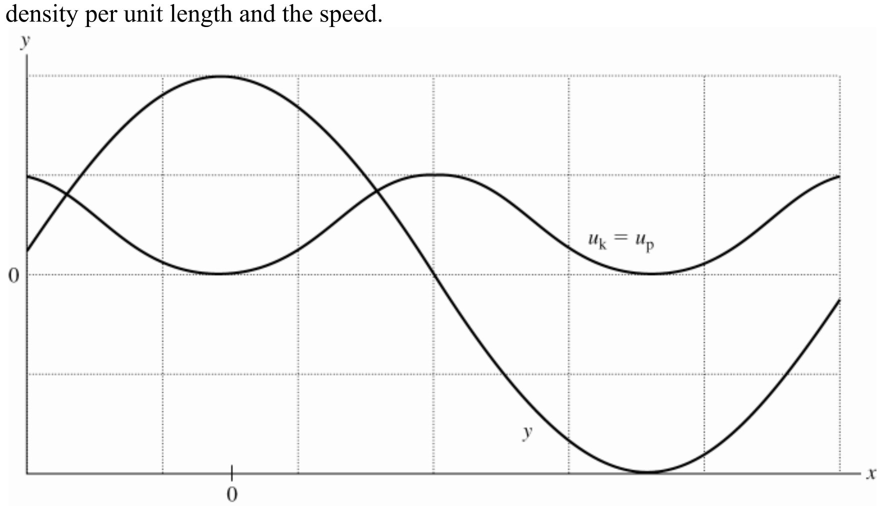

Figure 15.81

15.82. **IDENTIFY:** Apply $\sum F_y = 0$ to segments of the cable. The forces are the weight of the diver, the weight of the segment of the cable, the tension in the cable and the buoyant force on the segment of the cable and on the diver.

SET UP: The buoyant force on an object of volume V that is completely submerged in water is $B = \rho_\text{water}Vg$.

EXECUTE: **(a)** The tension is the difference between the diver's weight and the buoyant force,

$F = (m - \rho_\text{water}V)g = (120\text{ kg} - (1000\text{ kg/m}^3)(0.0800\text{ m}^3)(9.80\text{ m/s}^2)) = 392\text{ N}.$

(b) The increase in tension will be the weight of the cable between the diver and the point at x, minus the buoyant force. This increase in tension is then

$(\mu x - \rho(Ax))g = (1.10\text{ kg/m} - (1000\text{ kg/m}^3)\pi(1.00\times10^{-2}\text{ m})^2)(9.80\text{ m/s}^2)x = (7.70\text{ N/m})x$. The tension as a function of x is then $F(x) = (392\text{ N}) + (7.70\text{ N/m})x.$

(c) Denote the tension as $F(x) = F_0 + ax$, where $F_0 = 392\ \text{N}$ and $a = 7.70\ \text{N/m}$. Then the speed of transverse waves as a function of x is $v = \frac{dx}{dt} = \sqrt{(F_0 + ax)/\mu}$ and the time t needed for a wave to reach the surface is found from $t = \int dt = \int \frac{dx}{dx/dt} = \int \frac{\sqrt{\mu}}{\sqrt{F_0 + ax}} dx.$

Let the length of the cable be L, so $t = \sqrt{\mu}\int_0^L \frac{dx}{\sqrt{F_0 + ax}} = \sqrt{\mu}\frac{2}{a}\sqrt{F_0 + ax}\Big|_0^L = \frac{2\sqrt{\mu}}{a}\left(\sqrt{F_0 + aL} - \sqrt{F_0}\right)$.

$$t = \frac{2\sqrt{1.10\ \text{kg/m}}}{7.70\ \text{N/m}}\left(\sqrt{392\ \text{N} + (7.70\ \text{N/m})(100\ \text{m})} - \sqrt{392\ \text{N}}\right) = 3.98\ \text{s}.$$

EVALUATE: If the weight of the cable and the buoyant force on the cable are neglected, then the tension would have the constant value calculated in part (a). Then $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{392\ \text{N}}{1.10\ \text{kg/m}}} = 18.9\ \text{m/s}$ and $t = \frac{L}{v} = 5.92\ \text{s}$. The weight of the cable increases the tension along the cable and the time is reduced from this value.

15.83. IDENTIFY: The tension in the rope will vary with radius r.

SET UP: The tension at a distance r from the center must supply the force to keep the mass of the rope that is further out than r accelerating inward. The mass of this piece in $m\frac{L-r}{L}$, and its center of mass moves in a circle of radius $\frac{L+r}{2}$.

EXECUTE: $T(r) = \left[m\frac{L-r}{L}\right]\omega^2\left[\frac{L+r}{2}\right] = \frac{m\omega^2}{2L}(L^2 - r^2)$. The speed of propagation as a function of distance is $v(r) = \frac{dr}{dt} = \sqrt{\frac{T(r)}{\mu}} = \sqrt{\frac{TL}{m}} = \frac{\omega}{\sqrt{2}}\sqrt{L^2 - r^2}$, where $\frac{dr}{dt} > 0$ has been chosen for a wave traveling from the center to the edge. Separating variables and integrating, the time t is

$$t = \int dt = \frac{\sqrt{2}}{\omega}\int_0^L \frac{dr}{\sqrt{L^2 - r^2}}.$$

The integral may be found in a table, or in Appendix B. The integral is done explicitly by letting $r = L\sin\theta$, $dr = L\cos\theta\, d\theta$, $\sqrt{L^2 - r^2} = L\cos\theta$, so that $\int \frac{dr}{\sqrt{L^2 - r^2}} = \theta = \arcsin\frac{r}{L}$, and $t = \frac{\sqrt{2}}{\omega}\arcsin(1) = \frac{\pi}{\omega\sqrt{2}}$.

EVALUATE: An equivalent method for obtaining $T(r)$ is to consider the net force on a piece of the rope with length dr and mass $dm = dr\, m/L$. The tension must vary in such a way that $T(r) - T(r + dr) = -\omega^2 r\, dm$, or $\frac{dT}{dr} = -(m\omega^2/L)r\, dr$. This is integrated to obtained $T(r) = -(m\omega^2/2L)r^2 + C$, where C is a constant of integration. The tension must vanish at $r = L$, from which $C = (m\omega^2 L/2)$ and the previous result is obtained.

15.84. IDENTIFY: Carry out the calculation specified in part (a).

SET UP: $\frac{\partial y}{\partial x} = kA_{\text{SW}}\cos kx \sin\omega t$, $\frac{\partial y}{\partial t} = -\omega A_{\text{SW}}\sin kx \cos\omega t$. $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$.

EXECUTE: The instantaneous power is

$$P = FA_{\text{SW}}^2\omega k(\sin kx \cos kx)(\sin\omega t \cos\omega t) = \frac{1}{4}FA_{\text{SW}}^2\omega k \sin(2kx)\sin(2\omega t).$$

(b) The average value of P is proportional to the average value of $\sin(2\omega t)$, and the average of the sine function is zero; $P_{\text{av}} = 0$.

(c) The graphs are given in Figure 15.84. The waveform is the solid line, and the power is the dashed line. At time $t = 0$, $y = 0$ and $P = 0$ and the graphs coincide.

(d) When the standing wave is at its maximum displacement at all points, all of the energy is potential, and is concentrated at the places where the slope is steepest (the nodes). When the standing wave has zero displacement, all of the energy is kinetic, concentrated where the particles are moving the fastest (the antinodes). Thus, the energy must be transferred from the nodes to the antinodes, and back again, twice in each cycle. Note that $|P|$ is greatest midway between adjacent nodes and antinodes, and that P vanishes at the nodes and antinodes.

EVALUATE: There is energy flow back and forth between the nodes, but there is no net flow of energy along the string.

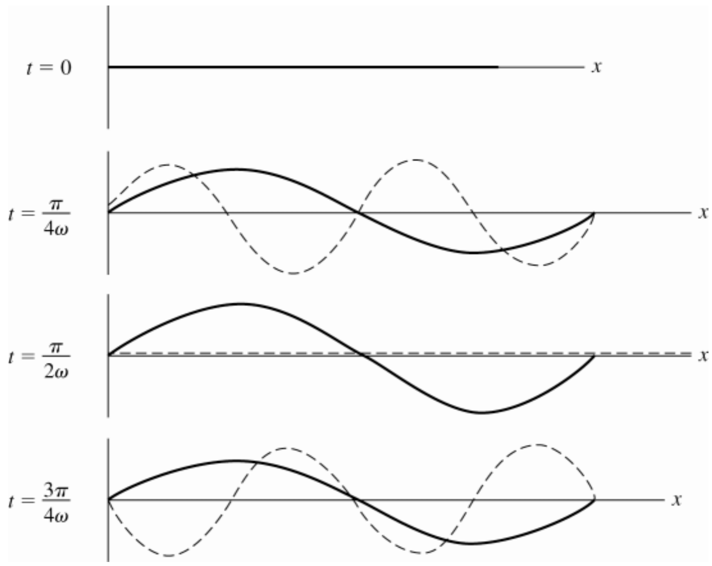

Figure 15.84

15.85. **IDENTIFY:** For a string, $f_n = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$.

SET UP: For the fundamental, $n = 1$. Solving for F gives $F = \mu 4L^2 f^2$. Note that $\mu = \pi r^2 \rho$, so $\mu = \pi(0.203\times10^{-3}\text{ m})^2(7800\text{ kg/m}^3) = 1.01\times10^{-3}\text{ kg/m}$.

EXECUTE: **(a)** $F = (1.01\times10^{-3}\text{ kg/m})4(0.635\text{ m})^2(247.0\text{ Hz})^2 = 99.4\text{ N}$

(b) To find the fractional change in the frequency we must take the ratio of Δf to f: $f = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$ and

$$\Delta f = \Delta\left(\frac{1}{2L}\sqrt{\frac{F}{\mu}}\right) = \Delta\left(\frac{1}{2L\sqrt{\mu}}F^{\frac{1}{2}}\right) = \frac{1}{2L\sqrt{\mu}}\Delta\left(F^{\frac{1}{2}}\right) = \frac{1}{2L\sqrt{\mu}}\frac{1}{2}\frac{\Delta F}{\sqrt{F}}$$

Now divide both sides by the original equation for f and cancel terms: $\frac{\Delta f}{f} = \frac{\frac{1}{2L\sqrt{\mu}}\frac{1}{2}\frac{\Delta F}{\sqrt{F}}}{\frac{1}{2L}\sqrt{\frac{F}{\mu}}} = \frac{1}{2}\frac{\Delta F}{F}$.

(c) The coefficient of thermal expansion α is defined by $\Delta l = l_0 \alpha \Delta T$. Combining this with $Y = \frac{F/A}{\Delta l/l_0}$ gives $\Delta F = -Y\alpha A\Delta T$. $\Delta F = -\left(2.00\times10^{11}\text{ Pa}\right)\left(1.20\times10^{-5}/\text{C}°\right)\pi(0.203\times10^{-3}\text{ m})^2(11°\text{C}) = 3.4\text{ N}$. Then $\Delta F/F - 0.034$, $\Delta f/f = -0.017$ and $\Delta f = -4.2\text{ Hz}$. The pitch falls. This also explains the constant tuning in the string sections of symphonic orchestras.

EVALUATE: An increase in temperature causes a decrease in tension of the string, and this lowers the frequency of each standing wave.