FLUID MECHANICS

14

14.1. **IDENTIFY:** Use Eq.(14.1) to calculate the mass and then use $w = mg$ to calculate the weight.

SET UP: $\rho = m/V$ so $m = \rho V$ From Table 14.1, $\rho = 7.8\times10^3\ \text{kg/m}^3$.

EXECUTE: For a cylinder of length L and radius R, $V = (\pi R^2)L = \pi(0.01425\ \text{m})^2(0.858\ \text{m}) = 5.474\times10^{-4}\ \text{m}^3$. Then $m = \rho V = (7.8\times10^3\ \text{kg/m}^3)(5.474\times10^{-4}\ \text{m}^3) = 4.27\ \text{kg}$, and $w = mg = (4.27\ \text{kg})(9.80\ \text{m/s}^2) = 41.8\ \text{N}$ (about 9.4 lbs). A cart is not needed.

EVALUATE: The rod is less than 1m long and less than 3 cm in diameter, so a weight of around 10 lbs seems reasonable.

14.2. **IDENTIFY:** Convert gallons to kg. The mass m of a volume V of gasoline is $m = \rho V$.

SET UP: $1\ \text{gal} = 3.788\ \text{L} = 3.788\times10^{-3}\ \text{m}^3$. $1\ \text{m}^3$ of gasoline has a mass of 737 kg.

EXECUTE: $45.0\ \text{mi/gal} = (45.0\ \text{mi/gal})\left(\dfrac{1\ \text{gal}}{3.788\times10^{-3}\ \text{m}^3}\right)\left(\dfrac{1\ \text{m}^3}{737\ \text{kg}}\right) = 16.1\ \text{mi/kg}$

EVALUATE: 1 gallon of gasoline has a mass of 2.79 kg. The car goes fewer miles on 1 kg than on 1 gal, since 1 kg of gasoline is less gasoline than 1 gal of gasoline.

14.3. **IDENTIFY:** $\rho = m/V$

SET UP: The density of gold is $19.3\times10^3\ \text{kg/m}^3$.

EXECUTE: $V = (5.0\times10^{-3}\ \text{m})(15.0\times10^{-3}\ \text{m})(30.0\times10^{-3}\ \text{m}) = 2.25\times10^{-6}\ \text{m}^3$.

$\rho = \dfrac{m}{V} = \dfrac{0.0158\ \text{kg}}{2.25\times10^{-6}\ \text{m}^3} = 7.02\times10^3\ \text{kg/m}^3$. The metal is not pure gold.

EVALUATE: The average density is only 36% that of gold, so at most 36% of the mass is gold.

14.4. **IDENTIFY:** Find the mass of gold that has a value of $\$1.00\times10^6$. Then use the density of gold to find the volume of this mass of gold.

SET UP: For gold, $\rho = 19.3\times10^3\ \text{kg/m}^3$. The volume V of a cube is related to the length L of one side by $V = L^3$.

EXECUTE: $m = (\$1.00\times10^6)\left(\dfrac{1\ \text{troy ounce}}{\$426.60}\right)\left(\dfrac{31.1035\times10^{-3}\ \text{kg}}{1\ \text{troy ounce}}\right) = 72.9\ \text{kg}$. $\rho = \dfrac{m}{V}$ so

$V = \dfrac{m}{\rho} = \dfrac{72.9\ \text{kg}}{19.3\times10^3\ \text{kg/m}^3} = 3.78\times10^{-3}\ \text{m}^3$. $L = V^{1/3} = 0.156\ \text{m} = 15.6\ \text{cm}$.

EVALUATE: The cube of gold would weigh about 160 lbs.

14.5. **IDENTIFY:** Apply $\rho = m/V$ to relate the densities and volumes for the two spheres.

SET UP: For a sphere, $V = \frac{4}{3}\pi r^3$. For lead, $\rho_1 = 11.3\times10^3\ \text{kg/m}^3$ and for aluminum, $\rho_a = 2.7\times10^3\ \text{kg/m}^3$.

EXECUTE: $m = \rho V = \frac{4}{3}\pi r^3\rho$. Same mass means $r_a^3\rho_a = r_1^3\rho_1$. $\dfrac{r_a}{r_1} = \left(\dfrac{\rho_1}{\rho_a}\right)^{1/3} = \left(\dfrac{11.3\times10^3}{2.7\times10^3}\right)^{1/3} = 1.6$.

EVALUATE: The aluminum sphere is larger, since its density is less.

14.6. **IDENTIFY:** Average density is $\rho = m/V$.

SET UP: For a sphere, $V = \frac{4}{3}\pi R^3$. The sun has mass $M_{\text{sun}} = 1.99\times10^{30}\ \text{kg}$ and radius $6.96\times10^8\ \text{m}$.

EXECUTE: **(a)** $\rho = \dfrac{M_{\text{sun}}}{V_{\text{sun}}} = \dfrac{1.99\times10^{30}\ \text{kg}}{\frac{4}{3}\pi(6.96\times10^8\text{m})^3} = \dfrac{1.99\times10^{30}\ \text{kg}}{1.412\times10^{27}\ \text{m}^3} = 1.409\times10^3\ \text{kg/m}^3$

(b) $\rho = \dfrac{1.99\times10^{30}\ \text{kg}}{\frac{4}{3}\pi(2.00\times10^4\ \text{m})^3} = \dfrac{1.99\times10^{30}\ \text{kg}}{3.351\times10^{13}\ \text{m}^3} = 5.94\times10^{16}\ \text{kg/m}^3$

EVALUATE: For comparison, the average density of the earth is $5.5\times10^3\ \text{kg/m}^3$. A neutron star is extremely dense.

14.7. **IDENTIFY:** $w = mg$ and $m = \rho V$. Find the volume V of the pipe.

SET UP: For a hollow cylinder with inner radius R_1 , outer radius R_2 , and length L the volume is $V = \pi(R_2^2 - R_1^2)L$. $R_1 = 1.25\times10^{-2}$ m and $R_2 = 1.75\times10^{-2}$ m

EXECUTE: $V = \pi([0.0175\ \text{m}]^2 - [0.0125\ \text{m}]^2)(1.50\ \text{m}) = 7.07\times10^{-4}\ \text{m}^3$.

$m = \rho V = (8.9\times10^3\ \text{kg/m}^3)(7.07\times10^{-4}\ \text{m}^3) = 6.29\ \text{kg}$. $w = mg = 61.6\ \text{N}$.

EVALUATE: The pipe weights about 14 pounds.

14.8. **IDENTIFY:** The gauge pressure $p - p_0$ at depth h is $p - p_0 = \rho gh$.

SET UP: Ocean water is seawater and has a density of $1.03\times10^3\ \text{kg/m}^3$.

EXECUTE: $p - p_0 = (1.03\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(3200\ \text{m}) = 3.23\times10^7\ \text{Pa}$.

$$p - p_0 = (3.23\times10^7\ \text{Pa})\left(\frac{1\ \text{atm}}{1.013\times10^5\ \text{Pa}}\right) = 319\ \text{atm} .$$

EVALUATE: The gauge pressure is about 320 times the atmospheric pressure at the surface.

14.9. **IDENTIFY:** The gauge pressure $p - p_0$ at depth h is $p - p_0 = \rho gh$.

SET UP: Freshwater has density $1.00\times10^3\ \text{kg/m}^3$ and seawater has density $1.03\times10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** $p - p_0 = (1.00\times10^3\ \text{kg/m}^3)(3.71\ \text{m/s}^2)(500\ \text{m}) = 1.86\times10^6\ \text{Pa}$.

(b) $h = \dfrac{p - p_0}{\rho g} = \dfrac{1.86\times10^6\ \text{Pa}}{(1.03\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 184\ \text{m}$

EVALUATE: The pressure at a given depth is greater on earth because a cylinder of water of that height weighs more on earth than on Mars.

14.10. **IDENTIFY:** The difference in pressure at points with heights y_1 and y_2 is $p - p_0 = \rho g(y_1 - y_2)$. The outward force $F_\perp$ is related to the surface area A by $F_\perp = pA$.

SET UP: For blood, $\rho = 1.06\times10^3\ \text{kg/m}^3$. $y_1 - y_2 = 1.65\ \text{m}$. The surface area of the segment is πDL , where $D = 1.50\times10^{-3}$ m and $L = 2.00\times10^{-2}$ m .

EXECUTE: **(a)** $p_1 - p_2 = (1.06\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.65\ \text{m}) = 1.71\times10^4\ \text{Pa}$.

(b) The additional force due to this pressure difference is $\Delta F_\perp = (p_1 - p_2)A$.

$A = \pi DL = \pi(1.50\times10^{-3}\ \text{m})(2.00\times10^{-2}\ \text{m}) = 9.42\times10^{-5}\ \text{m}^2$. $\Delta F_\perp = (1.71\times10^4\ \text{Pa})(9.42\times10^{-5}\ \text{m}^2) = 1.61\ \text{N}$.

EVALUATE: The pressure difference is about $\frac{1}{6}$ atm .

14.11. **IDENTIFY:** Apply $p = p_0 + \rho gh$.

SET UP: Gauge pressure is $p - p_{\text{air}}$.

EXECUTE: The pressure difference between the top and bottom of the tube must be at least 5980 Pa in order to force fluid into the vein: $\rho gh = 5980$ Pa and

$$h = \frac{5980\ \text{Pa}}{gh} = \frac{5980\ \text{N/m}^2}{(1050\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 0.581\ \text{m} .$$

EVALUATE: The bag of fluid is typically hung from a vertical pole to achieve this height above the patient's arm.

14.12. **IDENTIFY:** $p_0 = p_{\text{surface}} + \rho gh$ where p_{surface} is the pressure at the surface of a liquid and p_0 is the pressure at a depth h below the surface.

SET UP: The density of water is $1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** For the oil layer, $p_{\text{surface}} = p_{\text{atm}}$ and p_0 is the pressure at the oil-water interface.

$p_0 - p_{\text{atm}} = p_{\text{gauge}} = \rho gh = (600\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.120\ \text{m}) = 706\ \text{Pa}$

(b) For the water layer, $p_{\text{surface}} = 706\ \text{Pa} + p_{\text{atm}}$.

$p_0 - p_{\text{atm}} = p_{\text{gauge}} = 706\ \text{Pa} + \rho gh = 706\ \text{Pa} + (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.250\ \text{m}) = 3.16\times10^3\ \text{Pa}$

EVALUATE: The gauge pressure at the bottom of the barrel is due to the combined effects of the oil layer and water layer. The pressure at the bottom of the oil layer is the pressure at the top of the water layer

14.13. **IDENTIFY:** An inflation to 32.0 pounds means a gauge pressure of $32.0\ \text{lb/in.}^2$. The contact area A with the pavement is related to the gauge pressure $p - p_0$ in the tire and the force $F_\perp$ the tire exerts on the pavement by $F_\perp = (p - p_0)A$. By Newton's third law the magnitude of the force the tire exerts on the pavement equals the magnitude of the force the pavement exerts on the car, and this must equal the weight of the car.

SET UP: $14.7\ \text{lb/in.}^2 = 1.013\times10^5\ \text{Pa} = 1\ \text{atm}$. Assume $p_0 = 1\ \text{atm}$.

EXECUTE: **(a)** The gauge pressure is $32.0\ \text{lb/in.}^2 = 2.21\times10^5\ \text{Pa} = 2.18\ \text{atm}$. The absolute pressure is $46.7\ \text{lb/in.}^2 = 3.22\times10^5\ \text{Pa} = 3.18\ \text{atm}$.

(b) No, the tire would touch the pavement at a single point and the contact area would be zero.

(c) $F_\perp = mg = 9.56\times10^3\ \text{N}$. $A = \dfrac{F_\perp}{p - p_0} = \dfrac{9.56\times10^3\ \text{N}}{2.21\times10^5\ \text{Pa}} = 0.0433\ \text{m}^2 = 433\ \text{cm}^2$.

EVALUATE: If the contact area is square, the length of each side for each tire is $\sqrt{\dfrac{433\ \text{cm}^2}{4}} = 10.4\ \text{cm}$. This is a realistic value, based on our observation of the tires of cars.

14.14. **IDENTIFY** and **SET UP:** Use Eq.(14.8) to calculate the gauge pressure at this depth. Use Eq.(14.3) to calculate the force the inside and outside pressures exert on the window, and combine the forces as vectors to find the net force.

EXECUTE: **(a)** gauge pressure $= p - p_0 = \rho gh$ From Table 14.1 the density of seawater is $1.03\times10^3\ \text{kg/m}^3$, so

$$p - p_0 = \rho gh = (1.03\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(250\ \text{m}) = 2.52\times10^6\ \text{Pa}$$

(b) The force on each side of the window is $F = pA$. Inside the pressure is p_0 and outside in the water the pressure is $p = p_0 + \rho gh$. The forces are shown in Figure 14.14.

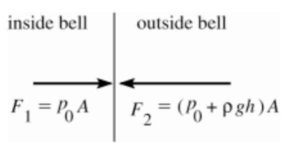

Figure 14.14

The net force is

$$F_2 - F_1 = (p_0 + \rho gh)A - p_0 A = (\rho gh)A$$

$$F_2 - F_1 = (2.52\times10^6\ \text{Pa})\pi(0.150\ \text{m})^2$$

$$F_2 - F_1 = 1.78\times10^5\ \text{N}$$

EVALUATE: The pressure at this depth is very large, over 20 times normal air pressure, and the net force on the window is huge. Diving bells used at such depths must be constructed to withstand these large forces.

14.15. **IDENTIFY:** $p_{\text{gauge}} = p_0 - p_{\text{atm}} = \rho gh$.

SET UP: $1\ \text{atm} = 1.013\times10^5\ \text{Pa}$. The density of water is $1.00\times10^3\ \text{kg/m}^3$. The gauge pressure must equal the pressure difference due to a column of water $1370\ \text{m} - 730\ \text{m} = 640\ \text{m}$ tall.

EXECUTE: $(1.00\times10^3\ \text{m}^3)(9.80\ \text{m/s}^2)(640\ \text{m}) = 6.27\times10^6\ \text{Pa} = 61.9\ \text{atm}$

EVALUATE: The gauge pressure required is directly proportional to the height to which the water is pumped.

14.16. **IDENTIFY** and **SET UP:** Use Eq.(14.6) to calculate the pressure at the specified depths in the open tube. The pressure is the same at all points the same distance from the bottom of the tubes, so the pressure calculated in part (b) is the pressure in the tank. Gauge pressure is the difference between the absolute pressure and air pressure.

EXECUTE: $p_a = 980\ \text{millibar} = 9.80\times10^4\ \text{Pa}$

(a) Apply $p = p_0 + \rho gh$ to the right-hand tube. The top of this tube is open to the air so $p_0 = p_a$. The density of the liquid (mercury) is $13.6\times10^3\ \text{kg/m}^3$.

Thus $p = 9.80\times10^4\ \text{Pa} + (13.6\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0700\ \text{m}) = 1.07\times10^5\ \text{Pa}$.

(b) $p = p_0 + \rho gh = 9.80\times10^4\ \text{Pa} + (13.6\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0400\ \text{m}) = 1.03\times10^5\ \text{Pa}$.

(c) Since $y_2 - y_1 = 4.00\ \text{cm}$ the pressure at the mercury surface in the left-hand end tube equals that calculated in part (b). Thus the absolute pressure of gas in the tank is $1.03\times10^5\ \text{Pa}$.

(d) $p - p_0 = \rho gh = (13.6\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0400\ \text{m}) = 5.33\times10^3\ \text{Pa}$.

EVALUATE: If Eq.(14.8) is evaluated with the density of mercury and $p - p_a = 1\ \text{atm} = 1.01\times10^5\ \text{Pa}$, then $h = 76\ \text{cm}$. The mercury columns here are much shorter than 76 cm, so the gauge pressures are much less than $1.0\times10^5\ \text{Pa}$.

14.17. **IDENTIFY:** Apply $p = p_0 + \rho gh$.

SET UP: For water, $\rho = 1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: $p - p_{\text{air}} = \rho gh = (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(6.1\ \text{m}) = 6.0\times10^4\ \text{Pa}$.

EVALUATE: The pressure difference increases linearly with depth.

14.18. **IDENTIFY** and **SET UP:** Apply Eq.(14.6) to the water and mercury columns. The pressure at the bottom of the water column is the pressure at the top of the mercury column.

EXECUTE: With just the mercury, the gauge pressure at the bottom of the cylinder is $p = p_0 + \rho_{\rm m} g h_{\rm m}$. With the water to a depth $h_{\rm w}$, the gauge pressure at the bottom of the cylinder is $p = p_0 + \rho_{\rm m} g h_{\rm m} + \rho_{\rm w} g h_{\rm w}$. If this is to be double the first value, then $\rho_{\rm w} g h_{\rm w} = \rho_{\rm m} g h_{\rm m}$.

$$h_{\rm w} = h_{\rm m}(\rho_{\rm m}/\rho_{\rm w}) = (0.0500 \text{ m})(13.6\times10^3/1.00\times10^3) = 0.680 \text{ m}$$

The volume of water is $V = hA = (0.680 \text{ m})(12.0\times10^{-4} \text{ m}^2) = 8.16\times10^{-4} \text{ m}^3 = 816 \text{ cm}^3$

EVALUATE: The density of mercury is 13.6 times the density of water and $(13.6)(5 \text{ cm}) = 68 \text{ cm}$, so the pressure increase from the top to the bottom of a 68-cm tall column of water is the same as the pressure increase from top to bottom for a 5-cm tall column of mercury.

14.19. **IDENTIFY:** Assume the pressure at the upper surface of the ice is $p_0 = 1.013\times10^5$ Pa . The pressure at the surface of the water is increased from p_0 by $\rho_{\rm ice} g h_{\rm ice}$ and then increases further with depth in the water.

SET UP: $\rho_{\rm ice} = 0.92\times10^3 \text{ kg/m}^3$ and $\rho_{\rm water} = 1.00\times10^3 \text{ kg/m}^3$.

EXECUTE: $p - p_0 = \rho_{\rm ice} g h_{\rm ice} + \rho_{\rm water} g h_{\rm water}$.

$p - p_0 = (0.92\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1.75 \text{ m}) + (1.00\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(2.50 \text{ m})$.

$p - p_0 = 4.03\times10^4$ Pa .

$p = p_0 + 4.03\times10^4 \text{ Pa} = 1.42\times10^5$ Pa .

EVALUATE: The gauge pressure at the surface of the water must be sufficient to apply an upward force on a section of ice equal to the weight of that section.

14.20. **IDENTIFY:** Apply $p = p_0 + \rho g h$, where p_0 is the pressure at the surface of the fluid. Gauge pressure is $p - p_{\rm air}$.

SET UP: For water, $\rho = 1.00\times10^3 \text{ kg/m}^3$.

EXECUTE: **(a)** The pressure difference between the surface of the water and the bottom is due to the weight of the water and is still 2500 Pa after the pressure increase above the surface. But the surface pressure increase is also transmitted to the fluid, making the total difference from atmospheric pressure $2500 \text{ Pa} + 1500 \text{ Pa} = 4000 \text{ Pa}$.

(b) Initially, the pressure due to the water alone is 2500 Pa $= \rho g h$. Thus $h = \dfrac{2500 \text{ N/m}^2}{(1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 0.255 \text{ m}.$

To keep the bottom gauge pressure at 2500 Pa after the 1500 Pa increase at the surface, the pressure due to the water's weight must be reduced to 1000 Pa: $h = \dfrac{1000 \text{ N/m}^2}{(1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 0.102 \text{ m}.$ Thus the water must be lowered by $0.255 \text{ m} - 0.102 \text{ m} = 0.153 \text{ m}.$.

EVALUATE: Note that $\rho g h$, with $h = 0.153$ m , is 1500 Pa.

14.21. **IDENTIFY:** $p = p_0 + \rho g h$. $F = pA$.

SET UP: For seawater, $\rho = 1.03\times10^3 \text{ kg/m}^3$

EXECUTE: The force *F* that must be applied is the difference between the upward force of the water and the downward forces of the air and the weight of the hatch. The difference between the pressure inside and out is the gauge pressure, so

$$F = (\rho g h)\, A - w = (1.03\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(30 \text{ m})(0.75 \text{ m}^2) - 300 \text{ N} = 2.27\times10^5 \text{ N}.$$

EVALUATE: The force due to the gauge pressure of the water is much larger than the weight of the hatch and would be impossible for the crew to apply it just by pushing.

14.22. **IDENTIFY:** The force on an area *A* due to pressure *p* is $F_\perp = pA$. Use $p - p_0 = \rho g h$ to find the pressure inside the tank, at the bottom.

SET UP: $1 \text{ atm} = 1.013\times10^5$ Pa . For benzene, $\rho = 0.90\times10^3 \text{ kg/m}^3$. The area of the bottom of the tank is $\pi D^2/4$, where $D = 1.72$ m . The area of the vertical walls of the tank is $\pi D L$, where $L = 11.50$ m .

EXECUTE: **(a)** At the bottom of the tank,

$p = p_0 + \rho g h = 92(1.013\times10^5 \text{ Pa}) + (0.90\times10^3 \text{ kg/m}^3)(0.894)(9.80 \text{ m/s}^2)(11.50 \text{ m}).$

$p = 9.32\times10^6 \text{ Pa} + 9.07\times10^4 \text{ Pa} = 9.41\times10^6$ Pa . $F_\perp = pA = (9.41\times10^6 \text{ Pa})\pi(1.72 \text{ m})^2/4 = 2.19\times10^7$ N .

(b) At the outside surface of the bottom of the tank, the air pressure is $p = (92)(1.013\times10^5 \text{ Pa}) = 9.32\times10^6$ Pa .

$F_\perp = pA = (9.32\times10^6 \text{ Pa})\pi(1.72 \text{ m})^2/4 = 2.17\times10^7$ N .

(c) $F_{\perp} = pA = 92(1.013\times 10^5\ \text{Pa})\pi(1.72\ \text{m})(11.5\ \text{m}) = 5.79\times 10^8\ \text{N}$

EVALUATE: Most of the force in part (a) is due to the 92 atm of air pressure above the surface of the benzene and the net force on the bottom of the tank is much less than the inward and outward forces.

14.23. **IDENTIFY:** The gauge pressure at the top of the oil column must produce a force on the disk that is equal to its weight.

SET UP: The area of the bottom of the disk is $A = \pi r^2 = \pi(0.150\ \text{m})^2 = 0.0707\ \text{m}^2$.

EXECUTE: **(a)** $p - p_0 = \dfrac{w}{A} = \dfrac{45.0\ \text{N}}{0.0707\ \text{m}^2} = 636\ \text{Pa}$.

(b) The increase in pressure produces a force on the disk equal to the increase in weight. By Pascal's law the increase in pressure is transmitted to all points in the oil.

(i) $\Delta p = \dfrac{83.0\ \text{N}}{0.0707\ \text{m}^2} = 1170\ \text{Pa}$. (ii) 1170 Pa

EVALUATE: The absolute pressure at the top of the oil produces an upward force on the disk but this force is partially balanced by the force due to the air pressure at the top of the disk.

14.24. **IDENTIFY:** $F_2 = \dfrac{A_2}{A_1}F_1$. F_2 must equal the weight $w = mg$ of the car.

SET UP: $A = \pi D^2/4$. D_1 is the diameter of the vessel at the piston where F_1 is applied and D_2 of the diameter at the car.

EXECUTE: $mg = \dfrac{\pi D_2^2/4}{\pi D_1^2/4}F_1$. $\dfrac{D_2}{D_1} = \sqrt{\dfrac{mg}{F_1}} = \sqrt{\dfrac{(1520\ \text{kg})(9.80\ \text{m/s}^2)}{125\ \text{N}}} = 10.9$

EVALUATE: The diameter is smaller where the force is smaller, so the pressure will be the same at both pistons.

14.25. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the piston, with $+y$ upward. $F = pA$.

SET UP: $1\ \text{atm} = 1.013\times 10^5\ \text{Pa}$. The force diagram for the piston is given in Figure 14.25. p is the absolute pressure of the hydraulic fluid.

EXECUTE: $pA - w - p_{\text{atm}}A = 0$ and $p - p_{\text{atm}} = p_{\text{gauge}} = \dfrac{w}{A} = \dfrac{mg}{\pi r^2} = \dfrac{(1200\ \text{kg})(9.80\ \text{m/s}^2)}{\pi(0.15\ \text{m})^2} = 1.7\times 10^5\ \text{Pa} = 1.7\ \text{atm}$

EVALUATE: The larger the diameter of the piston, the smaller the gauge pressure required to lift the car.

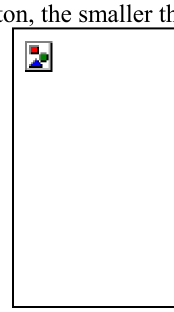

Figure 14.25

14.26. **IDENTIFY:** Apply Newton's 2nd law to the woman plus slab. The buoyancy force exerted by the water is upward and given by $B = \rho_{\text{water}}V_{\text{displ}}g$, where V_{displ} is the volume of water displaced.

SET UP: The floating object is the slab of ice plus the woman; the buoyant force must support both. The volume of water displaced equals the volume V_{ice} of the ice. The free-body diagram is given in Figure 14.26.

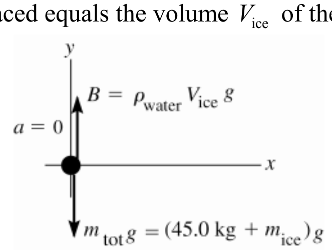

Figure 14.26

EXECUTE: $\sum F_y = ma_y$

$B - m_{\text{tot}}g = 0$

$\rho_{\text{water}}V_{\text{ice}}g = (45.0\ \text{kg} + m_{\text{ice}})g$

But $\rho = m/V$ so $m_{\text{ice}} = p_{\text{ice}}V_{\text{ice}}$

$V_{\text{ice}} = \dfrac{45.0\ \text{kg}}{\rho_{\text{water}} - \rho_{\text{ice}}} = \dfrac{45.0\ \text{kg}}{1000\ \text{kg/m}^3 - 920\ \text{kg/m}^3} = 0.562\ \text{m}^3.$

EVALUATE: The mass of ice is $m_{\text{ice}} = \rho_{\text{ice}}V_{\text{ice}} = 517\ \text{kg}.$

14.27. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the sample, with $+y$ upward. $B = \rho_{\text{water}} V_{\text{obj}} g$.

SET UP: $w = mg = 17.50\text{ N}$ and $m = 1.79\text{ kg}$.

EXECUTE: $T + B - mg = 0$. $B = mg - T = 17.50\text{ N} - 11.20\text{ N} = 6.30\text{ N}$.

$$V_{\text{obj}} = \frac{B}{\rho_{\text{water}} g} = \frac{6.30\text{ N}}{(1.00\times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 6.43\times 10^{-4}\text{ m}^3.$$

$$\rho = \frac{m}{V} = \frac{1.79\text{ kg}}{6.43\times 10^{-4}\text{ m}^3} = 2.78\times 10^3\text{ kg/m}^3.$$

EVALUATE: The density of the sample is greater than that of water and it doesn't float.

14.28. **IDENTIFY:** The upward buoyant force *B* exerted by the liquid equals the weight of the fluid displaced by the object. Since the object floats the buoyant force equals its weight.

SET UP: Glycerin has density $\rho_{\text{gly}} = 1.26\times 10^3\text{ kg/m}^3$ and seawater has density $\rho_{\text{sw}} = 1.03\times 10^3\text{ kg/m}^3$. Let V_{obj} be the volume of the apparatus. $g_{\text{E}} = 9.80\text{ m/s}^2$; $g_{\text{C}} = 4.15\text{ m/s}^2$. Let V_{sub} be the volume submerged on Caasi.

EXECUTE: On earth $B = \rho_{\text{sw}}(0.250 V_{\text{obj}}) g_{\text{E}} = m g_{\text{E}}$. $m = (0.250)\rho_{\text{obj}} V_{\text{sw}}$. On Caasi, $B = \rho_{\text{gly}} V_{\text{sub}} g_{\text{C}} = m g_{\text{C}}$. $m = \rho_{\text{gyl}} V_{\text{sub}}$. The two expressions for *m* must be equal, so $(0.250) V_{\text{obj}} \rho_{\text{sw}} = \rho_{\text{gly}} V_{\text{sub}}$ and

$$V_{\text{sub}} = \left(\frac{0.250\rho_{\text{sw}}}{\rho_{\text{gly}}}\right) V_{\text{obj}} = \left(\frac{[0.250][1.03\times 10^3\text{ kg/m}^3]}{1.26\times 10^3\text{ kg/m}^3}\right) V_{\text{obj}} = 0.204 V_{\text{obj}}.$$

20.4% of the volume will be submerged on Caasi.

EVALUATE: Less volume is submerged in glycerin since the density of glycerin is greater than the density of seawater. The value of *g* on each planet cancels out and has no effect on the answer. The value of *g* changes the weight of the apparatus and the buoyant force by the same factor.

14.29. **IDENTIFY:** For a floating object, the weight of the object equals the upward buoyancy force, *B*, exerted by the fluid.

SET UP: $B = \rho_{\text{fluid}} V_{\text{submerged}} g$. The weight of the object can be written as $w = \rho_{\text{object}} V_{\text{object}} g$. For seawater, $\rho = 1.03\times 10^3\text{ kg/m}^3$.

EXECUTE: **(a)** The displaced fluid must weigh more than the object, so $\rho < \rho_{\text{fluid}}$.

(b) If the ship does not leak, much of the water will be displaced by air or cargo, and the average density of the floating ship is less than that of water.

(c) Let the portion submerged have volume *V*, and the total volume be V_0. Then $\rho V_0 = \rho_{\text{fluid}} V$, so $\dfrac{V}{V_0} = \dfrac{\rho}{\rho_{\text{fluid}}}$ The fraction above the fluid surface is then $1 - \dfrac{\rho}{\rho_{\text{fluid}}}$. If $\rho \to 0$, the entire object floats, and if $\rho \to \rho_{\text{fluid}}$, none of the object is above the surface.

(d) Using the result of part (c), $1 - \dfrac{\rho}{\rho_{\text{fluid}}} = 1 - \dfrac{(0.042\text{ kg})/([5.0][4.0][3.0]\times 10^{-6}\text{m}^3)}{1030\text{kg/m}^3} = 0.32 = 32\%.$

EVALUATE: For a given object, the fraction of the object above the surface increases when the density of the fluid in which it floats increases.

14.30. **IDENTIFY:** $B = \rho_{\text{water}} V_{\text{obj}} g$. The net force on the sphere is zero.

SET UP: The density of water is $1.00\times 10^3\text{ kg/m}^3$.

EXECUTE: **(a)** $B = (1000\text{ kg/m}^3)(0.650\text{ m}^3)(9.80\text{ m/s}^2) = 6.37\times 10^3\text{ N}$

(b) $B = T + mg$ and $m = \dfrac{B - T}{g} = \dfrac{6.37\times 10^3\text{ N} - 900\text{ N}}{9.80\text{ m/s}^2} = 558\text{ kg}$.

(c) Now $B = \rho_{\text{water}} V_{\text{sub}} g$, where V_{sub} is the volume of the sphere that is submerged. $B = mg$. $\rho_{\text{water}} V_{\text{sub}} = mg$ and

$$V_{\text{sub}} = \frac{m}{\rho_{\text{water}}} = \frac{558\text{ kg}}{1000\text{ kg/m}^3} = 0.558\text{ m}^3.\quad \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{0.558\text{ m}^3}{0.650\text{ m}^3} = 0.858 = 85.8\%.$$

EVALUATE: The average density of the sphere is $\rho_{\text{sph}} = \dfrac{m}{V} = \dfrac{558\text{ kg}}{0.650\text{ m}^3} = 858\text{ kg/m}^3$. $\rho_{\text{sph}} < \rho_{\text{water}}$, and that is why it floats with 85.8% of its volume submerged.

14.31. **IDENTIFY** and **SET UP:** Use Eq.(14.8) to calculate the gauge pressure at the two depths.

(a) The distances are shown in Figure 14.31a.

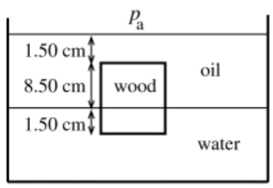

Figure 14.31a

EXECUTE: $p - p_0 = \rho g h$

The upper face is 1.50 cm below the top of the oil, so

$p - p_0 = (7.90\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0150\ \text{m})$

$p - p_0 = 116\ \text{Pa}$

(b) The pressure at the interface is $p_{\text{interface}} = p_a + \rho_{\text{oil}} g(0.100\ \text{m})$. The lower face of the block is 1.50 cm below the interface, so the pressure there is $p = p_{\text{interface}} + \rho_{\text{water}} g(0.0150\ \text{m})$. Combining these two equations gives

$p - p_a = \rho_{\text{oil}} g(0.100\ \text{m}) + \rho_{\text{water}} g(0.0150\ \text{m})$

$p - p_a = [(790\ \text{kg/m}^3)(0.100\ \text{m}) + (1000\ \text{kg/m}^3)(0.0150\ \text{m})](9.80\ \text{m/s}^2)$

$p - p_a = 921\ \text{Pa}$

(c) IDENTIFY and **SET UP:** Consider the forces on the block. The area of each face of the block is $A = (0.100\ \text{m})^2 = 0.0100\ \text{m}^2$. Let the absolute pressure at the top face be p_t and the pressure at the bottom face be p_b. In Eq.(14.3) use these pressures to calculate the force exerted by the fluids at the top and bottom of the block. The free-body diagram for the block is given in Figure 14.31b.

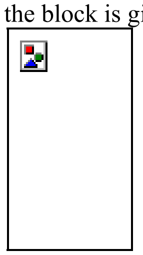

Figure 14.31b

EXECUTE: $\sum F_y = m a_y$

$p_b A - p_t A - mg = 0$

$(p_b - p_t)A = mg$

Note that $(p_b - p_t) = (p_b - p_a) - (p_t - p_a) = 921\ \text{Pa} - 116\ \text{Pa} = 805\ \text{Pa}$; the difference in absolute pressures equals the difference in gauge pressures.

$$m = \frac{(p_b - p_t)A}{g} = \frac{(805\ \text{Pa})(0.0100\ \text{m}^2)}{9.80\ \text{m/s}^2} = 0.821\ \text{kg}.$$

And then $\rho = m/V = 0.821\ \text{kg}/(0.100\ \text{m})^3 = 821\ \text{kg/m}^3$.

EVALUATE: We can calculate the buoyant force as $B = (\rho_{\text{oil}} V_{\text{oil}} + \rho_{\text{water}} V_{\text{water}})g$ where $V_{\text{oil}} = (0.0100\ \text{m}^2)(0.850\ \text{m}) = 8.50\times10^{-4}\ \text{m}^3$ is the volume of oil displaced by the block and $V_{\text{water}} = (0.0100\ \text{m}^2)(0.0150\ \text{m}) = 1.50\times10^{-4}\ \text{m}^3$ is the volume of water displaced by the block. This gives $B = (0.821\ \text{kg})g$. The mass of water displaced equals the mass of the block.

14.32. **IDENTIFY:** The sum of the vertical forces on the ingot is zero. $\rho = m/V$. The buoyant force is $B = \rho_{\text{water}} V_{\text{obj}} g$.

SET UP: The density of aluminum is $2.7\times10^3\ \text{kg/m}^3$. The density of water is $1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** $T = mg = 89\ \text{N}$ so $m = 9.08\ \text{kg}$. $V = \dfrac{m}{\rho} = \dfrac{9.08\ \text{kg}}{2.7\times10^3\ \text{kg/m}^3} = 3.36\times10^{-3}\ \text{m}^3 = 3.4\ \text{L}$.

(b) When the ingot is totally immersed in the water while suspended, $T + B - mg = 0$.

$B = \rho_{\text{water}} V_{\text{obj}} g = (1.00\times10^3\ \text{kg/m}^3)(3.36\times10^{-3}\ \text{m}^3)(9.80\ \text{m/s}^2) = 32.9\ \text{N}$. $T = mg - B = 89\ \text{N} - 32.9\ \text{N} = 56\ \text{N}$.

EVALUATE: The buoyant force is equal to the difference between the apparent weight when the object is submerged in the fluid and the actual gravity force on the object.

14.33. **IDENTIFY:** The vertical forces on the rock sum to zero. The buoyant force equals the weight of liquid displaced by the rock. $V = \frac{4}{3}\pi R^3$.

SET UP: The density of water is $1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: The rock displaces a volume of water whose weight is $39.2\ \text{N} - 28.4\ \text{N} = 10.8\ \text{N}$. The mass of this much water is thus $10.8\ \text{N}/(9.80\ \text{m/s}^2) = 1.102\ \text{kg}$ and its volume, equal to the rock's volume, is

$\dfrac{1.102\ \text{kg}}{1.00\times10^3\ \text{kg/m}^3} = 1.102\times10^{-3}\ \text{m}^3$. The weight of unknown liquid displaced is $39.2\ \text{N} - 18.6\ \text{N} = 20.6\ \text{N}$, and its mass is $20.6\ \text{N}/(9.80\ \text{m/s}^2) = 2.102\ \text{kg}$. The liquid's density is thus $2.102\ \text{kg}/(1.102\times10^{-3}\ \text{m}^3) = 1.91\times10^3\ \text{kg/m}^3$.

EVALUATE: The density of the unknown liquid is roughly twice the density of water.

14.34. IDENTIFY: The volume flow rate is Av.

SET UP: $Av = 0.750\ \text{m/s}^3$. $A = \pi D^2/4$.

EXECUTE: **(a)** $v\pi D^2/4 = 0.750\ \text{m/s}^3$. $v = \dfrac{4(0.750\ \text{m/s}^3)}{\pi(4.50\times10^{-2}\ \text{m})^2} = 472\ \text{m/s}$.

(b) vD^2 must be constant, so $v_1D_1^2 = v_2D_2^2$. $v_2 = v_1\left(\dfrac{D_1}{D_2}\right)^2 = (472\ \text{m/s})\left(\dfrac{D_1}{3D_1}\right)^2 = 52.4\ \text{m/s}$.

EVALUATE: The larger the hole, the smaller the speed of the fluid as it exits.

14.35. IDENTIFY: Apply the equation of continuity, $v_1A_1 = v_2A_2$.

SET UP: $A = \pi r^2$

EXECUTE: $v_2 = v_1(A_1/A_2)$. $A_1 = \pi(0.80\ \text{cm})^2$, $A_2 = 20\pi(0.10\ \text{cm})^2$. $v_2 = (3.0\ \text{m/s})\dfrac{\pi(0.80)^2}{20\pi(0.10)^2} = 9.6\ \text{m/s}$.

EVALUATE: The total area of the shower head openings is less than the cross section area of the pipe, and the speed of the water in the shower head opening is greater than its speed in the pipe.

14.36. IDENTIFY: $v_1A_1 = v_2A_2$. The volume flow rate is vA.

SET UP: $1.00\ \text{h} = 3600\ \text{s}$.

EXECUTE: **(a)** $v_2 = v_1\left(\dfrac{A_1}{A_2}\right) = (3.50\ \text{m/s})\left(\dfrac{0.070\ \text{m}^2}{0.105\ \text{m}^2}\right) = 2.33\ \text{m/s}$

(b) $v_2 = v_1\left(\dfrac{A_1}{A_2}\right) = (3.50\ \text{m/s})\left(\dfrac{0.070\ \text{m}^2}{0.047\ \text{m}^2}\right) = 5.21\ \text{m/s}$

(c) $V = v_1A_1t = (3.50\ \text{m/s})(0.070\ \text{m}^2)(3600\ \text{s}) = 882\ \text{m}^3$.

EVALUATE: The equation of continuity says the volume flow rate is the same at all points in the pipe.

14.37. IDENTIFY and SET UP: Apply Eq.(14.10). In part (a) the target variable is V. In part (b) solve for A and then from that get the radius of the pipe.

EXECUTE: **(a)** $vA = 1.20\ \text{m}^3/\text{s}$

$$v = \frac{1.20\ \text{m}^3/\text{s}}{A} = \frac{1.20\ \text{m}^3/\text{s}}{\pi r^2} = \frac{1.20\ \text{m}^3/\text{s}}{\pi(0.150\ \text{m})^2} = 17.0\ \text{m/s}$$

(b) $vA = 1.20\ \text{m}^3/\text{s}$

$v\pi r^2 = 1.20\ \text{m}^3/\text{s}$

$$r = \sqrt{\frac{1.20\ \text{m}^3/\text{s}}{v\pi}} = \sqrt{\frac{1.20\ \text{m}^3/\text{s}}{(3.80\ \text{m/s})\pi}} = 0.317\ \text{m}$$

EVALUATE: The speed is greater where the area and radius are smaller.

14.38. IDENTIFY: The volume flow rate is equal to Av.

SET UP: In the equation preceding Eq.(14.10), label the densities of the two points ρ_1 and ρ_2.

EXECUTE: **(a)** From the equation preceding Eq.(14.10), dividing by the time interval dt gives Eq.(14.12).

(b) The volume flow rate decreases by 1.50%.

EVALUATE: When the density increases, the volume flow rate decreases; it is the mass flow rate that remains constant.

14.39. IDENTIFY and SET UP:

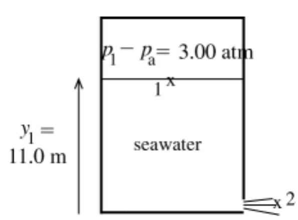

Figure 14.39

Apply Bernoulli's equation with points 1 and 2 chosen as shown in Figure 14.39. Let $y = 0$ at the bottom of the tank so $y_1 = 11.0\ \text{m}$ and $y_2 = 0$. The target variable is v_2.

$p_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$

$A_1v_1 = A_2v_2$, so $v_1 = (A_2/A_1)v_2$. But the cross-section area of the tank (A_1) is much larger than the cross-section area of the hole (A_2), so $v_1 << v_2$ and the $\tfrac{1}{2}\rho v_1^2$ term can be neglected.

EXECUTE: This gives $\frac{1}{2}\rho v_2^2 = (p_1 - p_2) + \rho g y_1$.

Use $p_2 = p_a$ and solve for v_2:

$$v_2 = \sqrt{2(p_1 - p_a)/\rho + 2gy_1} = \sqrt{\frac{2(3.039\times10^5\ \text{Pa})}{1030\ \text{kg/m}^3} + 2(9.80\ \text{m/s}^2)(11.0\ \text{m})}$$

$v_2 = 28.4$ m/s

EVALUATE: If the pressure at the top surface of the water were air pressure, then Toricelli's theorem (Example 14.8) gives $v_2 = \sqrt{2g(y_1 - y_2)} = 14.7$ m/s. The actual afflux speed is much larger than this due to the excess pressure at the top of the tank.

14.40. **IDENTIFY:** Toricelli's theorem says the speed of efflux is $v = \sqrt{2gh}$, where h is the distance of the small hole below the surface of the water in the tank. The volume flow rate is vA.

SET UP: $A = \pi D^2/4$, with $D = 6.00\times10^{-3}$ m.

EXECUTE: **(a)** $v = \sqrt{2(9.80\ \text{m/s}^2)(14.0\ \text{m})} = 16.6$ m/s

(b) $vA = (16.6\ \text{m/s})\pi(6.00\times10^{-3}\ \text{m})^2/4 = 4.69\times10^{-4}\ \text{m}^3/\text{s}$. A volume of $4.69\times10^{-4}\ \text{m}^3 = 0.469$ L is discharged each second.

EVALUATE: We have assumed that the diameter of the hole is much less than the diameter of the tank.

14.41. **IDENTIFY** and **SET UP:**

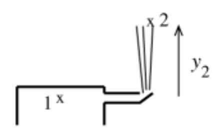

Figure 14.41

Apply Bernoulli's equation to points 1 and 2 as shown in Figure 14.41. Point 1 is in the mains and point 2 is at the maximum height reached by the stream, so $v_2 = 0$.

Solve for p_1 and then convert this absolute pressure to gauge pressure.

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

Let $y_1 = 0$, $y_2 = 15.0$ m. The mains have large diameter, so $v_1 \approx 0$.

Thus $p_1 = p_2 + \rho g y_2$.

But $p_2 = p_a$, so $p_1 - p_a = \rho g y_2 = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(15.0\ \text{m}) = 1.47\times10^5$ Pa.

EVALUATE: This is the gauge pressure at the bottom of a column of water 15.0 m high.

14.42. **IDENTIFY:** Apply Bernoulli's equation to the two points.

SET UP: The continuity equation says $v_1 A_1 = v_2 A_2$. In Eq.(14.17) either absolute or gauge pressures can be used at both points.

EXECUTE: Using $v_2 = \frac{1}{4}v_1$,

$$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2) = p_1 + \rho\left[\left(\frac{15}{32}\right)v_1^2 + g(y_1 - y_2)\right]$$

$$p_2 = 5.00\times10^4\ \text{Pa} + (1.00\times10^3\ \text{kg/m}^3)\left(\frac{15}{32}(3.00\ \text{m/s})^2 + (9.80\ \text{m/s}^2)(11.0\ \text{m})\right) = 1.62\times10^5\ \text{Pa}.$$

EVALUATE: The decrease in speed and the decrease in height at point 2 both cause the pressure at point 2 to be greater than the pressure at point 1.

14.43. **IDENTIFY:** Apply Bernoulli's equation to the air flowing past the wing. $F = pA$.

SET UP: Let point 1 be at the top surface and point 2 be at the bottom surface. Neglect the $\rho g(y_1 - y_2)$ term in Bernoulli's equation. In calculating the net force take $+y$ to be upward.

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$.

$p_2 - p_1 = \frac{1}{2}\rho(v_1^2 - v_2^2) = \frac{1}{2}(1.20\ \text{kg/m}^3)([70.0\ \text{m/s}]^2 - [60.0\ \text{m/s}]^2) = 780$ Pa.

The net force exerted by the air is $p_2 A - p_1 A = (780\ \text{Pa})(16.2\ \text{m}^2) = 12{,}600$ N. The net force is upward.

EVALUATE: The pressure is lower where the fluid speed is higher.

14.44. **IDENTIFY:** $\rho = m/V$. Apply the equation of continuity and Bernoulli's equation to points 1 and 2.

SET UP: The density of water is 1 kg/L.

EXECUTE: **(a)** $\dfrac{(220)(0.355\ \text{kg})}{60.0\ \text{s}} = 1.30$ kg/s.

(b) The density of the liquid is $\frac{0.355 \text{ kg}}{0.355\times 10^{-3} \text{ m}^3} = 1000 \text{ kg/m}^3$, and so the volume flow rate is

$\frac{1.30 \text{ kg/s}}{1000 \text{ kg/m}^3} = 1.30\times 10^{-3} \text{ m}^3\text{/s} = 1.30 \text{ L/s}$. This result may also be obtained from $\frac{(220)(0.355 \text{ L})}{60.0 \text{ s}} = 1.30 \text{ L/s}$.

(c) $v_1 = \frac{1.30\times 10^{-3} \text{ m}^3\text{/s}}{2.00\times 10^{-4} \text{m}^2} = 6.50 \text{ m/s}$. $v_2 = v_1/4 = 1.63 \text{ m/s}$.

(d) $p_1 = p_2 + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho g\left(y_2 - y_1\right)$.

$p_1 = 152 \text{ kPa} + (1000 \text{ kg/m}^3)\left(\tfrac{1}{2}[(1.63 \text{ m/s})^2 - (6.50 \text{ m/s})^2] + (9.80 \text{ m/s}^2)(-1.35 \text{ m})\right)$. $p_1 = 119 \text{ kPa}$.

EVALUATE: The increase in height and the increase in fluid speed at point 1 both cause the pressure at point 1 to be less than the pressure at point 2.

14.45. **IDENTIFY:** Apply Bernoulli's equation to the two points.

SET UP: $y_1 = y_2$. $v_1A_1 = v_2A_2$. $A_2 = 2A_1$.

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$. $v_2 = v_1\left(\frac{A_1}{A_2}\right) = (2.50 \text{ m/s})\left(\frac{A_1}{2A_1}\right) = 1.25 \text{ m/s}$.

$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 1.80\times 10^4 \text{ Pa} + \frac{1}{2}(1000 \text{ kg/m}^3)([2.50 \text{ m/s}]^2 - [1.25 \text{ m/s}]^2) = 2.03\times 10^4 \text{ Pa}$

EVALUATE: The gauge pressure is higher at the second point because the water speed is less there.

14.46. **IDENTIFY** and **SET UP:** Let point 1 be where $r_1 = 4.00$ cm and point 2 be where $r_2 = 2.00$ cm. The volume flow rate vA has the value $7200 \text{ cm}^3\text{/s}$ at all points in the pipe. Apply Eq.(14.10) to find the fluid speed at points 1 and 2 and then use Bernoulli's equation for these two points to find p_2.

EXECUTE: $v_1A_1 = v_1\pi r_1^2 = 7200 \text{ cm}^3$, so $v_1 = 1.43 \text{ m/s}$

$v_2A_2 = v_2\pi r_2^2 = 7200 \text{ cm}^3$, so $v_2 = 5.73 \text{ m/s}$

$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$y_1 = y_2$ and $p_1 = 2.40\times 10^5$ Pa, so $p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 2.25\times 10^5 \text{ Pa}$

EVALUATE: Where the area decreases the speed increases and the pressure decreases.

14.47. **IDENTIFY:** $F = pA$, where A is the cross-sectional area presented by a hemisphere. The force F_{bb} that the body builder must apply must equal in magnitude the net force on each hemisphere due to the air inside and outside the sphere.

SET UP: $A = \pi\frac{D^2}{4}$.

EXECUTE: **(a)** $F_{bb} = \left(p_0 - p\right)\pi\frac{D^2}{4}$.

(b) The force on each hemisphere due to the atmosphere is

$\pi(5.00\times 10^{-2} \text{ m})^2(1.013\times 10^5 \text{ Pa/atm})(0.975 \text{ atm}) = 776 \text{ N}$. The bodybuilder must exert this force on each hemisphere to pull them apart.

EVALUATE: The force is about 170 lbs, feasible only for a very strong person. The force required is proportional to the square of the diameter of the hemispheres.

14.48. **IDENTIFY:** Apply $p = p_0 + \rho g h$ and $\Delta V = -\frac{(\Delta p)V_0}{B}$, where B is the bulk modulus.

SET UP: Seawater has density $\rho = 1.03\times 10^3 \text{ kg/m}^3$. The bulk modulus of water is $B = 2.2\times 10^9 \text{ Pa}$.

$p_{air} = 1.01\times 10^5 \text{ Pa}$.

EXECUTE: **(a)** $p_0 = p_{air} + \rho g h = 1.01\times 10^5 \text{ Pa} + (1.03\times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(10.92\times 10^3 \text{ m}) = 1.10\times 10^8 \text{ Pa}$

(b) At the surface 1.00 m^3 of seawater has mass 1.03×10^3 kg. At a depth of 10.92 km the change in volume is

$\Delta V = -\frac{(\Delta p)V_0}{B} = -\frac{(1.10\times 10^8 \text{ Pa})(1.00 \text{ m}^3)}{2.2\times 10^9 \text{ Pa}} = -0.050 \text{ m}^3$. The volume of this mass of water at this depth therefore is $V = V_0 + \Delta V = 0.950 \text{ m}^3$. $\rho = \frac{m}{V} = \frac{1.03\times 10^3 \text{ kg}}{0.950 \text{ m}^3} = 1.08\times 10^3 \text{ kg/m}^3$. The density is 5% larger than at the surface.

EVALUATE: For water B is small and a very large increase in pressure corresponds to a small fractional change in volume.

14.49. **IDENTIFY:** In part (a), the force is the weight of the water. In part (b), the pressure due to the water at a depth *h* is ρgh. $F = pA$ and $m = \rho V$.

SET UP: The density of water is $1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** The weight of the water is

$\rho gV = (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)\big((5.00\ \text{m})(4.0\ \text{m})(3.0\ \text{m})\big) = 5.9\times10^5\ \text{N}$,

(b) Integration gives the expected result that the force is what it would be if the pressure were uniform and equal to the pressure at the midpoint. If *d* is the depth of the pool and *A* is the area of one end of the pool, then

$$F = \rho gA\frac{d}{2} = (1.00\times10^3\text{kg/m}^3)(9.80\ \text{m/s}^2)\big((4.0\ \text{m})(3.0\ \text{m})\big)(1.50\ \text{m}) = 1.76\times10^5\ \text{N}.$$

EVALUATE: The answer to part (a) can be obtained as $F = pA$, where $p = \rho gd$ is the gauge pressure at the bottom of the pool and $A = (5.0\ \text{m})(4.0\ \text{m})$ is the area of the bottom of the pool.

14.50. **IDENTIFY:** Use Eq.(14.8) to find the gauge pressure versus depth, use Eq.(14.3) to relate the pressure to the force on a strip of the gate, calculate the torque as force times moment arm, and follow the procedure outlined in the hint to calculate the total torque.

SET UP: The gate is sketched in Figure 14.50a

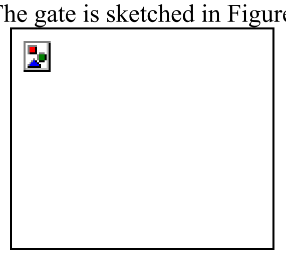

Figure 14.50a

Let τ_u be the torque due to the net force of the water on the upper half of the gate, and τ_1 be the torque due to the force on the lower half.

With the indicated sign convention, τ_1 is positive and τ_u is negative, so the net torque about the hinge is $\tau = \tau_1 - \tau_u$. Let *H* be the height of the gate.

Upper-half of gate:

Calculate the torque due to the force on a narrow strip of height *dy* located a distance *y* below the top of the gate, as shown in Figure 14.50b. Then integrate to get the total torque.

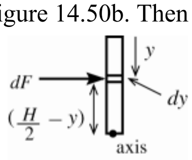

Figure 14.50b

The net force on the strip is $dF = p(y)\,dA$, where $p(y) = \rho gy$ is the pressure at this depth and $dA = W\,dy$ with $W = 4.00$ m

$dF = \rho gyW\,dy$

The moment arm is $(H/2 - y)$, so $d\tau = pgW(H/2 - y)y\,dy$.

$$\tau_u = \int_0^{H/2} d\tau = \rho gW\int_0^{H/2}(H/2 - y)y\,dy = \rho gW((H/4)y^2 - y^3/3)\Big|_0^{H/2}$$

$$\tau_u = \rho gW(H^3/16 - H^3/24) = \rho gW(H^3/48)$$

$$\tau_u = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(4.00\ \text{m})(2.00\ \text{m})^3/48 = 6.533\times10^3\ \text{N}\cdot\text{m}$$

Lower-half of gate:

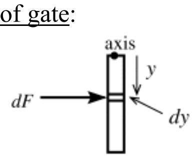

Figure 14.50c

Consider the narrow strip shown in Figure 14.50c
The depth of the strip is $(H/2 + y)$
so the force *dF* is
$dF = p(y)\,dA = \rho g(H/2 + y)W\,dy$

The moment arm is *y*, so $d\tau = \rho gW(H/2 + y)y\,dy$.

$$\tau_1 = \int_0^{H/2} d\tau = \rho gW\int_0^{H/2}(H/2 + y)y\,dy = \rho gW((H/4)y^2 + y^3/3)\Big|_0^{H/2}$$

$$\tau_1 = \rho gW(H^3/16 + H^3/24) = \rho gW(5H^3/48)$$

$$\tau_1 = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(4.00\ \text{m})5(2.00\ \text{m})^3/48 = 3.267\times10^4\ \text{N}\cdot\text{m}$$

Then $\tau = \tau_1 - \tau_u = 3.267\times10^4\ \text{N}\cdot\text{m} - 6.533\times10^3\ \text{N}\cdot\text{m} = 2.61\times10^4\ \text{N}\cdot\text{m}$.

EVALUATE: The forces and torques on the upper and lower halves of the gate are in opposite directions so find the net value by subtracting the magnitudes. The torque on the lower half is larger than the torque on the upper half since pressure increases with depth.

14.51. **IDENTIFY:** Compute the force and the torque on a thin, horizontal strip at a depth h and integrate to find the total force and torque.

SET UP: The strip has an area $dA = (dh)L$,where dh is the height of the strip and L is its length. $A = HL$. The height of the strip about the bottom of the dam is $H - h$.

EXECUTE: **(a)** $dF = pdA = \rho ghLdh$. $F = \int_0^H dF = \rho gL\int_0^H hdh = \rho gLH^2/2 = \rho gAH/2$.

(b) The torque about the bottom on a strip of vertical thickness dh is $d\tau = dF\left(H-h\right) = \rho gLh\left(H-h\right)dh,$ and integrating from $h = 0$ to $h = H$ gives $\tau = \rho gLH^3/6 = \rho gAH^2/6.$

(c) The force depends on the width and on the square of the depth, and the torque about the bottom depends on the width and the cube of the depth; the surface area of the lake does not affect either result (for a given width).

EVALUATE: The force is equal to the average pressure, at depth $H/2$, times the area A of the vertical side of the dam that faces the lake. But the torque is not equal to $F(H/2)$,where $H/2$ is the moment arm for a force acting at the center of the dam.

14.52. **IDENTIFY:** The information about Europa allows us to evaluate g at the surface of Europa. Since there is no atmosphere, $p_0 = 0$ at the surface. The pressure at depth h is $p = \rho gh$. The inward force on the window is $F_\perp = pA$.

SET UP: $g = \dfrac{Gm}{R^2}$, where $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$. $R = 1.565\times10^6$ m . Assume the ocean water has density $\rho = 1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: $g = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(4.78\times10^{22}\ \text{kg})}{(1.565\times10^6\ \text{m})^2} = 1.30\ \text{m/s}^2$. The maximum pressure at the window is

$$p = \frac{9750\ \text{N}}{(0.250\ \text{m})^2} = 1.56\times10^5\ \text{Pa}\ .\ \ p = \rho gh \text{ so } h = \frac{1.56\times10^5\ \text{Pa}}{(1.00\times10^3\ \text{kg/m}^3)(1.30\ \text{m/s}^2)} = 120\ \text{m}\ .$$

EVALUATE: 9750 N is the inward force exerted by the surrounding water. This will also be the net force on the window if the pressure inside the submarine is essentially zero.

14.53. **IDENTIFY** and **SET UP:** Apply Eq.(14.6) and solve for g.
Then use Eq.(12.4) to relate g to the mass of the planet.

EXECUTE: $p - p_0 = \rho gd.$

This expression gives that $g = (p - p_0)/\rho d = (p - p_0)V/md.$

But also $g = Gm_\text{p}/R^2$ (Eq.(12.4) applied to the planet rather than to earth.)

Setting these two expressions for g equal gives $Gm_\text{p}/R^2 = (p - p_0)V/md$ and $m_\text{p} = (p - p_0)VR^2/Gmd.$

EVALUATE: The greater p is at a given depth, the greater g is for the planet and greater g means greater m_p.

14.54. **IDENTIFY:** The buoyant force B equals the weight of the air displaced by the balloon.

SET UP: $B = \rho_\text{air}Vg$. Let g_M be the value of g for Mars. For a sphere $V = \frac{4}{3}\pi R^3$. The surface area of a sphere is given by $A = 4\pi R^2$. The mass of the balloon is $(5.00\times10^{-3}\ \text{kg/m}^2)(4\pi R^2)$.

EXECUTE: **(a)** $B = mg_\text{M}$. $\rho_\text{air}Vg_\text{M} = mg_\text{M}$. $\rho_\text{air}\frac{4}{3}\pi R^3 = (5.00\times10^{-3}\ \text{kg/m}^2)(4\pi R^2)$.

$$R = \frac{3(5.00\times10^{-3}\ \text{kg/m}^2)}{\rho_\text{air}} = 0.974\ \text{m}\ .\ \ m = (5.00\times10^{-3}\ \text{kg/m}^2)(4\pi R^2) = 0.0596\ \text{kg}\ .$$

(b) $F_\text{net} = B - mg = ma$. $B = \rho_\text{air}Vg = \rho_\text{air}\frac{4}{3}\pi R^3 g = (1.20\ \text{kg/m}^3)\left(\dfrac{4\pi}{3}\right)(0.974\ \text{m})^3(9.80\ \text{m/s}^2) = 45.5\ \text{N}$.

$$a = \frac{B - mg}{m} = \frac{45.5\ \text{N} - (0.0596\ \text{kg})(9.80\ \text{m/s}^2)}{0.0596\ \text{m}} = 754\ \text{m/s}^2\text{, upward.}$$

(c) $B = m_\text{tot}g$. $\rho_\text{air}Vg = (m_\text{balloon} + m_\text{load})g$. $m_\text{load} = \rho_\text{air}\frac{4}{3}\pi R^3 - (5.00\times10^{-3}\ \text{kg/m}^2)4\pi R^2$.

$$m_\text{load} = (0.0154\ \text{kg/m}^3)\left(\frac{4\pi}{3}\right)(5[0.974\ \text{m}])^3 - (5.00\times10^{-3}\ \text{kg/m}^2)(4\pi)(5[0.974\ \text{m}])^2$$

$$m_\text{load} = 7.45\ \text{kg} - 1.49\ \text{kg} = 5.96\ \text{kg}$$

EVALUATE: The buoyant force is proportional to R^3 and the mass of the balloon is proportional to R^2, so the load that can be carried increases when the radius of the balloon increases. We calculated the mass of the load. To find the weight of the load we would need to know the value of g for Mars.

14.55. **IDENTIFY:** Follow the procedure outlined in part (b). For a spherically symmetric object, with total mass m and radius r, at points on the surface of the object, $g(r) = Gm/r^2$.

SET UP: The earth has mass $m_E = 5.97\times10^{24}$ kg. If $g(r)$ is a maximum at $r = r_{max}$, then $\frac{dg}{dr} = 0$ for $r = r_{max}$.

EXECUTE: **(a)** At $r = 0$, the model predicts $\rho = A = 12{,}700\ \text{kg/m}^3$ and at $r = R$, the model predicts $\rho = A - BR = 12{,}700\ \text{kg/m}^3 - (1.50\times10^{-3}\ \text{kg/m}^4)(6.37\times10^6\ \text{m}) = 3.15\times10^3\ \text{kg/m}^3$.

(b) and **(c)** $M = \int dm = 4\pi\int_0^R [A - Br]r^2 dr = 4\pi\left[\frac{AR^3}{3} - \frac{BR^4}{4}\right] = \left(\frac{4\pi R^3}{3}\right)\left[A - \frac{3BR}{4}\right]$.

$$M = \left(\frac{4\pi(6.37\times10^6\ \text{m})^3}{3}\right)\left[12{,}700\ \text{kg/m}^3 - \frac{3(1.50\times10^{-3}\ \text{kg/m}^4)(6.37\times10^6\ \text{m})}{4}\right] = 5.99\times10^{24}\ \text{kg}$$

which is within 0.36% of the earth's mass.

(d) If $m(r)$ is used to denote the mass contained in a sphere of radius r, then $g = Gm(r)/r^2$. Using the same integration as that in part (b), with an upper limit of r instead of R gives the result.

(e) $g = 0$ at $r = 0$, and g at $r = R$,

$$g = Gm(R)/R^2 = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2\text{kg}^2)(5.99\times10^{24}\ \text{kg})/(6.37\times10^6\ \text{m})^2 = 9.85\ \text{m/s}^2.$$

(f) $\frac{dg}{dr} = \left(\frac{4\pi G}{3}\right)\frac{d}{dr}\left[Ar - \frac{3Br^2}{4}\right] = \left(\frac{4\pi G}{3}\right)\left[A - \frac{3Br}{2}\right]$. Setting this equal to zero gives $r = 2A/3B = 5.64\times10^6$ m, and at this radius $g = \left(\frac{4\pi G}{3}\right)\left(\frac{2A}{3B}\right)\left[A - \left(\frac{3}{4}\right)B\left(\frac{2A}{3B}\right)\right] = \frac{4\pi GA^2}{9B}$.

$$g = \frac{4\pi(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(12{,}700\ \text{kg/m}^3)^2}{9(1.50\times10^{-3}\ \text{kg/m}^4)} = 10.02\ \text{m/s}^2.$$

EVALUATE: If the earth were a uniform sphere of density ρ, then $g(r) = \frac{\rho V(r)}{r^2} = \left(\frac{4\pi\rho G}{3}\right)r$, the same as setting $B = 0$ and $A = \rho$ in $g(r)$ in part (d). If r_{max} is the value of r in part (f) where $g(r)$ is a maximum, then $r_{max}/R = 0.885$. For a uniform sphere, $g(r)$ is maximum at the surface.

14.56. **IDENTIFY:** Follow the procedure outlined in part (a).

SET UP: The earth has mass $M = 5.97\times10^{24}$ kg and radius $R = 6.38\times10^6$ m. Let $g_S = 9.80\ \text{m/s}^2$

EXECUTE: **(a)** Equation (14.4), with the radius r instead of height y, becomes $dp = -\rho g(r)\,dr = -\rho g_s(r/R)dr$. This form shows that the pressure decreases with increasing radius. Integrating, with $p = 0$ at $r = R$,

$$p = -\frac{\rho g_s}{R}\int_R^r r\,dr = \frac{\rho g_s}{R}\int_r^R r\,dr = \frac{\rho g_s}{2R}(R^2 - r^2).$$

(b) Using the above expression with $r = 0$ and $\rho = \frac{M}{V} = \frac{3M}{4\pi R^3}$,

$$p(0) = \frac{3(5.97\times10^{24}\ \text{kg})(9.80\ \text{m/s}^2)}{8\pi(6.38\times10^6\ \text{m})^2} = 1.71\times10^{11}\ \text{Pa}.$$

(c) While the same order of magnitude, this is not in very good agreement with the estimated value. In more realistic density models (see Problem 14.55), the concentration of mass at lower radii leads to a higher pressure.

EVALUATE: In this model, the pressure at the center of the earth is about 10^6 times what it is at the surface.

14.57. **(a) IDENTIFY** and **SET UP:**

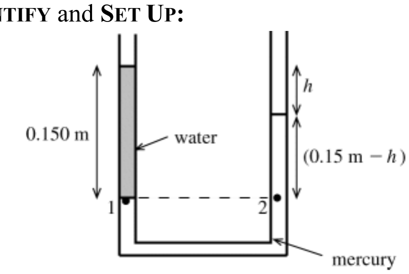

Figure 14.57

Apply $p = p_0 + \rho gh$ to the water in the left-hand arm of the tube. See Figure 14.57.

EXECUTE: $p_0 = p_a$, so the gauge pressure at the interface (point 1) is

$p - p_a = \rho g h = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.150\ \text{m}) = 1470\ \text{Pa}$

(b) IDENTIFY and **SET UP:** The pressure at point 1 equals the pressure at point 2. Apply Eq.(14.6) to the right-hand arm of the tube and solve for h.

EXECUTE: $p_1 = p_a + p_w g(0.150\ \text{m})$ and $p_2 = p_a + \rho_{Hg} g(0.150\ \text{m} - h)$

$p_1 = p_2$ implies $\rho_w g(0.150\ \text{m}) = \rho_{Hg} g(0.150\ \text{m} - h)$

$$0.150\ \text{m} - h = \frac{\rho_w(0.150\ \text{m})}{\rho_{Hg}} = \frac{(1000\ \text{kg/m}^3)(0.150\ \text{m})}{13.6\times 10^3\ \text{kg/m}^3} = 0.011\ \text{m}$$

$h = 0.150\ \text{m} - 0.011\ \text{m} = 0.139\ \text{m} = 13.9\ \text{cm}$

EVALUATE: The height of mercury above the bottom level of the water is 1.1 cm. This height of mercury produces the same gauge pressure as a height of 15.0 cm of water.

14.58. IDENTIFY: Follow the procedure outlined in the hint. $F = pA$.

SET UP: The circular ring has area $dA = (2\pi R)dy$. The pressure due to the molasses at depth y is $\rho g y$.

EXECUTE: $F = \int_0^h (\rho g y)(2\pi R)dy = \rho g \pi R h^2$ where R and h are the radius and height of the tank. Using the given numerical values gives $F = 5.07\times 10^8\ \text{N}$.

EVALUATE: The net outward force is the area of the wall of the tank, $A = 2\pi R h$, times the average pressure, the pressure $\rho g h/2$ at depth $h/2$.

14.59. IDENTIFY: Apply Newton's 2nd law to the barge plus its contents. Apply Archimedes' principle to express the buoyancy force B in terms of the volume of the barge.

SET UP: The free-body diagram for the barge plus coal is given in Figure 14.59.

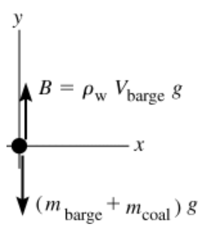

Figure 14.59

EXECUTE: $\sum F_y = ma_y$

$B - (m_{barge} + m_{coal})g = 0$

$\rho_w V_{barge} g = (m_{marge} + m_{coal})g$

$m_{coal} = \rho_w V_{barge} - m_{barge}$

$V_{barge} = (22\ \text{m})(12\ \text{m})(40\ \text{m}) = 1.056\times 10^4\ \text{m}^3$

The mass of the barge is $m_{barge} = \rho_s V_s$, where s refers to steel.

From Table 14.1, $\rho_s = 7800\ \text{kg/m}^3$. The volume V_s is 0.040 m times the total area of the five pieces of steel that make up the barge:

$$V_s = (0.040\ \text{m})\left[2(22\ \text{m})(12\ \text{m}) + 2(40\ \text{m})(12\ \text{m}) + (22\ \text{m})(40\ \text{m})\right] = 94.7\ \text{m}^3.$$

Therefore, $m_{barge} = \rho_s V_s = (7800\ \text{kg/m}^3)(94.7\ \text{m}^3) = 7.39\times 10^5\ \text{kg}$.

Then $m_{coal} = \rho_w V_{barge} - m_{barge} = (1000\ \text{kg/m}^3)(1.056\times 10^4\ \text{m}^3) - 7.39\times 10^5\ \text{kg} = 9.8\times 10^6\ \text{kg}$.

The volume of this mass of coal is $V_{coal} = m_{coal}/\rho_{coal} = 9.8\times 10^6\ \text{kg}/1500\ \text{kg/m}^3 = 6500\ \text{m}^3$; this is less that V_{barge} so it will fit into the barge.

EVALUATE: The buoyancy force B must support both the weight of the coal and also the weight of the barge. The weight of the coal is about 13 times the weight of the barge. The buoyancy force increases when more of the barge is submerged, so when it holds the maximum mass of coal the barge is fully submerged.

14.60. IDENTIFY: The buoyant force on the balloon must equal the total weight of the balloon fabric, the basket and its contents and the gas inside the balloon. $m_{gas} = \rho_{gas} V$. $B = \rho_{air} V g$.

SET UP: The total weight, exclusive of the gas inside the balloon, is $900\ \text{N} + 1700\ \text{N} + 3200\ \text{N} = 5800\ \text{N}$

EXECUTE: $5800\ \text{N} + \rho_{gas} V_g = \rho_{air} V_g$ and $\rho_{gas} = 1.23\ \text{kg/m}^3 - \dfrac{(5800\ \text{N})}{(9.80\ \text{m/s}^2)(2200\ \text{m}^3)} = 0.96\ \text{kg/m}^3$.

EVALUATE: The volume of a given mass of gas increases when the gas is heated, and the density of the gas therefore decreases.

14.61. IDENTIFY: Apply Newton's 2nd law to the car. The buoyancy force is given by Archimedes' principle.

(a) SET UP: The free-body diagram for the floating car is given in Figure 14.61. (V_{sub} is the volume that is submerged.)

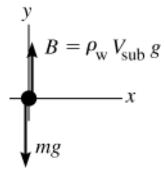

Figure 14.61

EXECUTE: $\sum F_y = ma_y$

$B - mg = 0$

$\rho_w V_{sub} g - mg = 0$

$V_{sub} = m/\rho_w = (900 \text{ kg})/(1000 \text{ kg/m}^3) = 0.900 \text{ m}^3$

$V_{sub}/V_{obj} = (0.900 \text{ m}^3)/(3.0 \text{ m}^3) = 0.30 = 30\%$

EVALUATE: The average density of the car is $(900 \text{ kg})/(3.0 \text{ m}^3) = 300 \text{ kg/m}^3$. $\rho_{car}/\rho_{water} = 0.30$; this equals V_{sub}/V_{obj}.

(b) SET UP: When the car starts to sink it is fully submerged and the buoyant force is equal to the weight of the car plus the water that is inside it.

EXECUTE: When the car is full submerged $V_{sub} = V$, the volume of the car and

$B = \rho_{water} V g = (1000 \text{ kg/m}^3)(3.0 \text{ m}^3)(9.80 \text{ m/s}^2) = 2.94 \times 10^4 \text{ N}$

The weight of the car is $mg = (900 \text{ kg})(9.80 \text{ m/s}^2) = 8820 \text{ N}$.

Thus the weight of the water in the car when it sinks is the buoyant force minus the weight of the car itself:

$m_{water} = (2.94 \times 10^4 \text{ N} - 8820 \text{ N})/(9.80 \text{ m/s}^2) = 2.10 \times 10^3 \text{ kg}$

And $V_{water} = m_{water}/\rho_{water} = (2.10 \times 10^3 \text{ kg})/(1000 \text{ kg/m}^3) = 2.10 \text{ m}^3$

The fraction this is of the total interior volume is $(2.10 \text{ m}^3)/(3.00 \text{ m}^3) = 0.70 = 70\%$

EVALUATE: The average density of the car plus the water inside it is $(900 \text{ kg} + 2100 \text{ kg})/(3.0 \text{ m}^3) = 1000 \text{ kg/m}^3$, so $\rho_{car} = \rho_{water}$ when the car starts to sink.

14.62. IDENTIFY: For a floating object, the buoyant force equals the weight of the object. $B = \rho_{fluid} V_{submerged} g$.

SET UP: Water has density $\rho = 1.00 \text{ g/cm}^3$.

EXECUTE: **(a)** The volume displaced must be that which has the same weight and mass as the ice,

$\frac{9.70 \text{ gm}}{1.00 \text{ gm/cm}^3} = 9.70 \text{ cm}^3$.

(b) No; when melted, the cube produces the same volume of water as was displaced by the floating cube, and the water level does not change.

(c) $\frac{9.70 \text{ gm}}{1.05 \text{ gm/cm}^3} = 9.24 \text{ cm}^3$

(d) The melted water takes up more volume than the salt water displaced, and so 0.46 cm^3 flows over.

EVALUATE: The volume of water from the melted cube is less than the volume of the ice cube, but the cube floats with only part of its volume submerged.

14.63. IDENTIFY: For a floating object the buoyant force equals the weight of the object. The buoyant force when the wood sinks is $B = \rho_{water} V_{tot} g$, where V_{tot} is the volume of the wood plus the volume of the lead. $\rho = m/V$.

SET UP: The density of lead is $11.3 \times 10^3 \text{ kg/m}^3$.

EXECUTE: $V_{wood} = (0.600 \text{ m})(0.250 \text{ m})(0.080 \text{ m}) = 0.0120 \text{ m}^3$.

$m_{wood} = \rho_{wood} V_{wood} = (600 \text{ kg/m}^3)(0.0120 \text{ m}^3) = 7.20 \text{ kg}$.

$B = (m_{wood} + m_{lead})g$. Using $B = \rho_{water} V_{tot} g$ and $V_{tot} = V_{wood} + V_{lead}$ gives $\rho_{water}(V_{wood} + V_{lead})g = (m_{wood} + m_{lead})g$.

$m_{lead} = \rho_{lead} V_{lead}$ then gives $\rho_{water} V_{wood} + \rho_{water} V_{lead} = m_{wood} + \rho_{lead} V_{lead}$.

$V_{lead} = \frac{\rho_{water} V_{wood} - m_{wood}}{\rho_{lead} - \rho_{water}} = \frac{(1000 \text{ kg/m}^3)(0.0120 \text{ m}^3) - 7.20 \text{ kg}}{11.3 \times 10^3 \text{ kg/m}^3 - 1000 \text{ kg/m}^3} = 4.66 \times 10^{-4} \text{ m}^3$. $m_{lead} = \rho_{lead} V_{lead} = 5.27 \text{ kg}$.

EVALUATE: The volume of the lead is only 3.9% of the volume of the wood. If the contribution of the volume of the lead to F_B is neglected, the calculation is simplified: $\rho_{water} V_{wood} g = (m_{wood} + m_{lead})g$ and $m_{lead} = 4.8 \text{ kg}$. The result of this calculation is in error by about 9%.

14.64. **IDENTIFY:** The fraction f of the volume that floats above the fluid is $f = 1 - \frac{\rho}{\rho_{\text{fluid}}}$, where ρ is the average density of the hydrometer (see Problem 14.29). This gives $\rho_{\text{fluid}} = \rho\frac{1}{1-f}$.

SET UP: The volume above the surface is hA, where h is the height of the stem above the surface and $A = 0.400\ \text{cm}^2$.

EXECUTE: If two fluids are observed to have floating fraction f_1 and f_2, $\rho_2 = \rho_1\frac{1-f_1}{1-f_2}$. Using

$f_1 = \frac{(8.00\ \text{cm})(0.400\ \text{cm}^2)}{(13.2\ \text{cm}^3)} = 0.242,\ f_2 = \frac{(3.20\ \text{cm})(0.400\ \text{cm}^2)}{(13.2\ \text{cm}^3)} = 0.097$ gives $\rho_{\text{alcohol}} = (0.839)\rho_{\text{water}} = 839\ \text{kg/m}^3$.

EVALUATE: $\rho_{\text{alcohol}} < \rho_{\text{water}}$. When ρ_{fluid} increases, the fraction f of the object's volume that is above the surface increases.

14.65. **(a) IDENTIFY:** Apply Newton's 2nd law to the airship. The buoyancy force is given by Archimedes' principle; the fluid that exerts this force is the air.

SET UP: The free-body diagram for the dirigible is given in Figure 14.65. The lift corresponds to a mass $m_{\text{lift}} = (120\times10^3\ \text{N})/(9.80\ \text{m/s}^2) = 1.224\times10^4\ \text{kg}$. The mass m_{tot} is $1.224\times10^4\ \text{kg}$ plus the mass m_{gas} of the gas that fills the dirigible. B is the buoyant force exerted by the air.

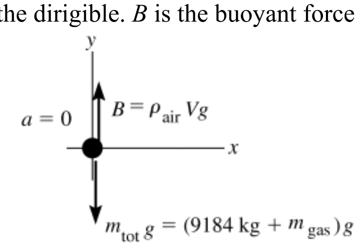

Figure 14.65

EXECUTE: $\sum F_y = ma_y$

$B - m_{\text{tot}}g = 0$

$\rho_{\text{air}}Vg = (1.224\times10^4\ \text{kg} + m_{\text{gas}})g$

Write m_{gas} in terms of V: $m_{\text{gas}} = \rho_{\text{gas}}V$

And let g divide out; the equation becomes $\rho_{\text{air}}V = 1.224\times10^4\ \text{kg} + \rho_{\text{gas}}V$

$$V = \frac{1.224\times10^4\ \text{kg}}{1.20\ \text{kg/m}^3 - 0.0899\ \text{kg/m}^3} = 1.10\times10^4\ \text{m}^3$$

EVALUATE: The density of the airship is less than the density of air and the airship is totally submerged in the air, so the buoyancy force exceeds the weight of the airship.

(b) SET UP: Let m_{lift} be the mass that could be lifted.

EXECUTE: From part (a), $m_{\text{lift}} = (\rho_{\text{air}} - \rho_{\text{gas}})V = (1.20\ \text{kg/m}^3 - 0.166\ \text{kg/m}^3)(1.10\times10^4\ \text{m}^3) = 1.14\times10^4\ \text{kg}$.

The lift force is $m_{\text{lift}} = (1.14\times10^4\ \text{kg})(9.80\ \text{m/s}^2) = 112\ \text{kN}$.

EVALUATE: The density of helium is less than that of air but greater than that of hydrogen. Helium provides lift, but less lift than hydrogen. Hydrogen is not used because it is highly explosive in air.

14.66. **IDENTIFY:** The vertical forces on the floating object must sum to zero. The buoyant force B applied to the object by the liquid is given by Archimedes's principle. The motion is SHM if the net force on the object is of the form $F_y = -ky$ and then $T = 2\pi\sqrt{m/k}$.

SET UP: Take $+y$ to be downward.

EXECUTE: **(a)** $V_{\text{submerged}} = LA$, where L is the vertical distance from the surface of the liquid to the bottom of the object. Archimedes' principle states $\rho gLA = Mg$, so $L = \frac{M}{\rho A}$.

(b) The buoyant force is $\rho gA(L+y) = Mg + F$, where y is the additional distance the object moves downward. Using the result of part (a) and solving for y gives $y = \frac{F}{\rho gA}$.

(c) The net force is $F_{\text{net}} = Mg - \rho gA(L+y) = -\rho gAy$. $k = \rho gA$, and the period of oscillation is

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M}{\rho gA}}.$$

EVALUATE: The force F determines the amplitude of the motion but the period does not depend on how much force was applied.

14.67. **IDENTIFY:** Apply the results of problem 14.66.

SET UP: The additional force F applied to the buoy is the weight $w = mg$ of the man.

EXECUTE: **(a)** $x = \dfrac{w}{\rho g A} = \dfrac{mg}{\rho g A} = \dfrac{m}{\rho A} = \dfrac{(70.0\text{ kg})}{(1.03\times 10^3\text{ kg/m}^3)\pi(0.450\text{ m})^2} = 0.107\text{ m}.$

(b) Note that in part (c) of Problem 14.66, M is the mass of the buoy, not the mass of the man, and A is the cross-section area of the buoy, not the amplitude. The period is then

$$T = 2\pi\sqrt{\frac{(950\text{ kg})}{(1.03\times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)\pi(0.450\text{ m})^2}} = 2.42\text{ s}$$

EVALUATE: The period is independent of the mass of the man.

14.68. **IDENTIFY:** After the water leaves the hose the only force on it is gravity. Use conservation of energy to relate the initial speed to the height the water reaches. The volume flow rate is Av.

SET UP: $A = \pi D^2/4$

EXECUTE: **(a)** $\tfrac{1}{2}mv^2 = mgh$. $v = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(35.0\text{ m})} = 26.2\text{ m/s}$. $(\pi D^2/4)v = 0.500\text{ m/s}^3$.

$D = \sqrt{\dfrac{4(0.500\text{ m/s}^3)}{\pi v}} = \sqrt{\dfrac{4(0.500\text{ m/s}^3)}{\pi(26.3\text{ m/s})}} = 0.156\text{ m} = 15.6\text{ cm}$.

(b) $D^2 v$ is constant so if D is twice as great then v is decreased by a factor of 4. h is proportional to v^2, so h is decreased by a factor of 16. $h = \dfrac{35.0\text{ m}}{16} = 2.19\text{ m}$.

EVALUATE: The larger the diameter of the nozzle the smaller the speed with which the water leaves the hose and the smaller the maximum height.

14.69. **IDENTIFY:** Find the horizontal range x as a function of the height y of the hole above the base of the cylinder. Then find the value of y for which x is a maximum. Once the water leaves the hole it moves in projectile motion.

SET UP: Apply Bernoulli's equation to points 1 and 2, where point 1 is at the surface of the water and point 2 is in the stream as the water leaves the hole. Since the hole is small the volume flow rate out the hole is small and $v_1 \approx 0$. $y_1 - y_2 = H - y$ and $p_1 = p_2 = \rho_{\text{air}}$. For the projectile motion, take $+y$ to be upward; $a_x = 0$ and $a_y = -9.80\text{ m/s}^2$.

EXECUTE: **(a)** $p_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$ gives $v_2 = \sqrt{2g(H-y)}$. In the projectile motion, $v_{0y} = 0$ and $y - y_0 = -y$, so $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2y}{g}}$. The horizontal range is $x = v_{0x}t = v_2 t = 2\sqrt{y(H-y)}$. The y that gives maximum x satisfies $\dfrac{dx}{dy} = 0$. $(Hy - y^2)^{-1/2}(H - 2y) = 0$ and $y = H/2$.

(b) $x = 2\sqrt{y(H-y)} = 2\sqrt{(H/2)(H-H/2)} = H$.

EVALUATE: A smaller y gives a larger v_2, but a smaller time in the air after the water leaves the hole.

14.70. **IDENTIFY:** Bernoulli's equation gives the speed at which water exits the hole, and from this we can calculate the volume flow rate. This will depend on the height h of the water remaining in the tank. Integrate to find h versus t. The time for the tank to empty is t for which $h = 0$.

SET UP: Apply Bernoulli's equation to point 1 at the top of the tank and point 2 at the hole. Assume the cross sectional area A_1 of the tank is much larger than the area A_2 of the hole. $v_1 = -\dfrac{dh}{dt}$, where the minus sign is because h is decreasing and dh/dt is negative, whereas v_1 is positive.

EXECUTE: $p_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$ gives $v_2^2 = 2gh + v_1^2$. $A_1 v_1 = A_2 v_2$ gives $v_1^2 = \left(\dfrac{A_2}{A_1}\right)^2 v_2^2$ and $v_2^2\left(1 - \left[\dfrac{A_2}{A_1}\right]^2\right) = 2gh$. $A_2 << A_1$ so $v_2 = \sqrt{2gh}$. $v_1 = -\dfrac{dh}{dt} = \dfrac{A_2}{A_1}v_2$ and $v_2 = -\dfrac{A_1}{A_2}\dfrac{dh}{dt}$. Combining these two equations for v_2 gives $\dfrac{dh}{dt} = -\dfrac{A_2}{A_1}\sqrt{2g}h^{1/2}$. $\dfrac{dh}{h^{1/2}} = -\left(\dfrac{A_2}{A_1}\sqrt{2g}\right)dt$. $\int_{h_0}^{h}\dfrac{dh}{h^{1/2}} = -\left(\dfrac{A_2}{A_1}\sqrt{2g}\right)\int_0^t dt$ gives $2\left(\sqrt{h} - \sqrt{h_0}\right) = -\dfrac{A_2}{A_1}\sqrt{2g}t$.

$h(t) = \left(\sqrt{h_0} - \dfrac{A_2}{A_1}\left(\sqrt{\dfrac{g}{2}}\right)t\right)^2$.

(b) $h = 0$ when $t = \frac{A_1}{A_2}\sqrt{\frac{2h_0}{g}}$.

EVALUATE: The time t for the tank to empty decreases when the area of the hole is larger. t increases when A_1 increases because for fixed h_0 an increase in A_1 corresponds to a greater volume of water initially in the tank.

14.71. IDENTIFY: Apply the 2nd condition of equilibrium to the balance arm and apply the first condition of equilibrium to the block and to the brass mass. The buoyancy force on the wood is given by Archimedes' principle and the buoyancy force on the brass mass is ignored.

SET UP: The objects and forces are sketched in Figure 14.71a.

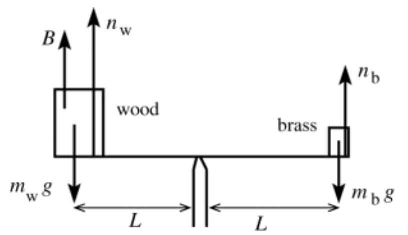

Figure 14.71a

The buoyant force on the brass is neglected, but we include the buoyant force B on the block of wood. n_w and n_b are the normal forces exerted by the balance arm on which the objects sit.

The free-body diagram for the balance arm is given in Figure 14.71b.

Figure 14.71b

EXECUTE: $\tau_P = 0$

$n_w L - n_b L = 0$

$n_w = n_b$

SET UP: The free-body diagram for the brass mass is given in Figure 14.71c.

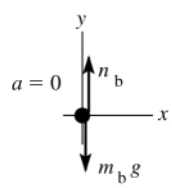

Figure 14.71c

EXECUTE: $\sum F_y = ma_y$

$n_b - m_b g = 0$

$n_b = m_b g$

The free-body diagram for the block of wood is given in Figure 14.71d.

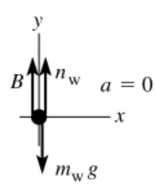

Figure 14.71d

$\sum F_y = ma_y$

$n_w + B - m_w g = 0$

$n_w = m_w g - B$

But $n_b = n_w$ implies $m_b g = m_w g - B$.

And $B = \rho_{air} V_w g = \rho_{air}(m_w/\rho_w)g$, so $m_b g = m_w g - \rho_{air}(m_w/\rho_w)g$.

$$m_w = \frac{m_b}{1-\rho_{air}/\rho_w} = \frac{0.0950 \text{ kg}}{1-((1.20 \text{ kg/m}^3)/(150 \text{ kg/m}^3))} = 0.0958 \text{ kg}$$

EVALUATE: The mass of the wood is greater than the mass of the brass; the wood is partially supported by the buoyancy force exerted by the air. The buoyancy in air of the brass can be neglected because the density of brass is much more than the density of air; the buoyancy force exerted on the brass by the air is much less than the weight of the brass. The density of the balsa wood is much less than the density of the brass, so the buoyancy force on the balsa wood is not such a small fraction of its weight.

14.72. IDENTIFY: $B = \rho V_A g$. Apply Newton's second law to the beaker, liquid and block as a combined object and also to the block as a single object.

SET UP: Take $+y$ upward. Let F_D and F_E be the forces corresponding to the scale reading.

EXECUTE: Forces on the combined object: $F_D + F_E - (w_A + w_B + w_C) = 0$. $w_A = F_D + F_E - w_B - w_C$.
D and E read mass rather than weight, so write the equation as $m_A = m_D + m_E - m_B - m_C$. $m_D = F_D/g$ is the reading in kg of scale D; a similar statement applies to m_E.

$m_A = 3.50\text{ kg} + 7.50\text{ kg} - 1.00\text{ kg} - 1.80\text{ kg} = 8.20\text{ kg}$.

Forces on A: $B + F_D - w_A = 0$. $\rho V_A g + F_D - m_A g = 0$. $\rho V_A + m_D = m_A$.

$$\rho = \frac{m_A - m_D}{V_A} = \frac{8.20\text{ kg} - 3.50\text{ kg}}{3.80\times10^{-3}\text{ m}^3} = 1.24\times10^3\text{ kg/m}^3$$

(b) D reads the mass of A: 8.20 kg. E reads the total mass of B and C: 2.80 kg.
EVALUATE: The sum of the readings of the two scales remains the same.

14.73. IDENTIFY: Apply Newton's 2nd law to the ingot. Use the expression for the buoyancy force given by Archimedes' principle to solve for the volume of the ingot. Then use the facts that the total mass is the mass of the gold plus the mass of the aluminum and that the volume of the ingot is the volume of the gold plus the volume of the aluminum.

SET UP: The free-body diagram for the piece of alloy is given in Figure 14.73.

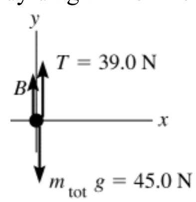

Figure 14.73

EXECUTE: $\sum F_y = ma_y$

$B + T - m_{\text{tot}} g = 0$

$B = m_{\text{tot}} g - T$

$B = 45.0\text{ N} - 39.0\text{ N} = 6.0\text{ N}$

Also, $m_{\text{tot}} g = 45.0\text{ N}$ so $m_{\text{tot}} = 45.0\text{ N}/(9.80\text{ m/s}^2) = 4.59$ kg.

We can use the known value of the buoyant force to calculate the volume of the object: $B = \rho_{\text{w}} V_{\text{obj}} g = 6.0\text{ N}$

$$V_{\text{obj}} = \frac{6.0\text{ N}}{\rho_{\text{w}} g} = \frac{6.0\text{ N}}{(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 6.122\times10^{-4}\text{ m}^3$$

We know two things:

(1) The mass m_{g} of the gold plus the mass m_{a} of the aluminum must add to m_{tot}: $m_{\text{g}} + m_{\text{a}} = m_{\text{tot}}$

We write this in terms of the volumes V_{g} and V_{a} of the gold and aluminum: $\rho_{\text{g}} V_{\text{g}} + \rho_{\text{a}} V_{\text{a}} = m_{\text{tot}}$

(2) The volumes V_{a} and V_{g} must add to give V_{obj}: $V_{\text{a}} + V_{\text{g}} = V_{\text{obj}}$ so that $V_{\text{a}} = V_{\text{obj}} - V_{\text{g}}$

Use this in the equation in (1) to eliminate V_{a}: $\rho_{\text{g}} V_{\text{g}} + \rho_{\text{a}}(V_{\text{obj}} - V_{\text{g}}) = m_{\text{tot}}$

$$V_{\text{g}} = \frac{m_{\text{tot}} - \rho_{\text{a}} V_{\text{obj}}}{\rho_{\text{g}} - \rho_{\text{a}}} = \frac{4.59\text{ kg} - (2.7\times10^3\text{ kg/m}^3)(6.122\times10^{-4}\text{ m}^3)}{19.3\times10^3\text{ kg/m}^3 - 2.7\times10^3\text{ kg/m}^3} = 1.769\times10^{-4}\text{ m}^3.$$

Then $m_{\text{g}} = \rho_{\text{g}} V_{\text{g}} = (19.3\times10^3\text{ kg/m}^3)(1.769\times10^{-4}\text{ m}^3) = 3.41\text{ kg}$ and the weight of gold is $w_{\text{g}} = m_{\text{g}} g = 33.4$ N.

EVALUATE: The gold is 29% of the volume but 74% of the mass, since the density of gold is much greater than the density of aluminum.

14.74. IDENTIFY: Apply $\sum F_y = ma_y$ to the ball, with $+y$ upward. The buoyant force is given by Archimedes's principle.

SET UP: The ball's volume is $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12.0\text{ cm})^3 = 7238\text{ cm}^3$. As it floats, it displaces a weight of water equal to its weight.

EXECUTE: **(a)** By pushing the ball under water, you displace an additional amount of water equal to 84% of the ball's volume or $(0.84)(7238\text{ cm}^3) = 6080\text{ cm}^3$. This much water has a mass of $6080\text{ g} = 6.080\text{ kg}$ and weighs $(6.080\text{ kg})(9.80\text{ m/s}^2) = 59.6\text{ N}$, which is how hard you'll have to push to submerge the ball.

(b) The upward force on the ball in excess of its own weight was found in part (a): 59.6 N. The ball's mass is equal to the mass of water displaced when the ball is floating:

$$(0.16)(7238\text{ cm}^3)(1.00\text{ g/cm}^3) = 1158\text{ g} = 1.158\text{ kg},$$

and its acceleration upon release is thus $a = \frac{F_{\text{net}}}{m} = \frac{59.6\text{ N}}{1.158\text{ kg}} = 51.5\text{ m/s}^2$.

EVALUATE: When the ball is totally immersed the upward buoyant force on it is much larger than its weight.

14.75. **(a) IDENTIFY:** Apply Newton's 2nd law to the crown. The buoyancy force is given by Archimedes' principle. The target variable is the ratio ρ_c/ρ_w (c = crown, w = water).

SET UP: The free-body diagram for the crown is given in Figure 14.75.

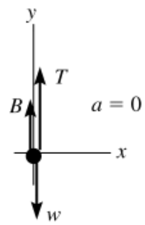

Figure 14.75

EXECUTE: $\sum F_y = ma_y$

$T + B - w = 0$

$T = fw$

$B = \rho_w V_c g$, where

ρ_w = density of water,

V_c = volume of crown

Then $fw + \rho_w V_c g - w = 0.$

$(1-f)w = \rho_w V_c g$

Use $w = \rho_c V_c g$, where ρ_c = density of crown.

$(1-f)\rho_c V_c g = \rho_w V_c g$

$\dfrac{\rho_c}{\rho_w} = \dfrac{1}{1-f}$, as was to be shown.

$f \to 0$ gives $\rho_c/\rho_w = 1$ and $T = 0$. These values are consistent. If the density of the crown equals the density of the water, the crown just floats, fully submerged, and the tension should be zero.

When $f \to 1$, $\rho_c >> \rho_w$ and $T = w$. If $\rho_c >> \rho_w$ then B is negligible relative to the weight w of the crown and T should equal w.

(b) "apparent weight" equals T in rope when the crown is immersed in water. $T = fw$, so need to compute f.

$\rho_c = 19.3\times10^3\ \text{kg/m}^3$; $\rho_w = 1.00\times10^3\ \text{kg/m}^3$

$\dfrac{\rho_c}{\rho_w} = \dfrac{1}{1-f}$ gives $\dfrac{19.3\times10^3\ \text{kg/m}^3}{1.00\times10^3\ \text{kg/m}^3} = \dfrac{1}{1-f}$

$19.3 = 1/(1-f)$ and $f = 0.9482$

Then $T = fw = (0.9482)(12.9\ \text{N}) = 12.2\ \text{N}.$

(c) Now the density of the crown is very nearly the density of lead;

$\rho_c = 11.3\times10^3\ \text{kg/m}^3.$

$\dfrac{\rho_c}{\rho_w} = \dfrac{1}{1-f}$ gives $\dfrac{11.3\times10^3\ \text{kg/m}^3}{1.00\times10^3\ \text{kg/m}^3} = \dfrac{1}{1-f}$

$11.3 = 1/(1-f)$ and $f = 0.9115$

Then $T = fw = (0.9115)(12.9\ \text{N}) = 11.8\ \text{N}.$

EVALUATE: In part (c) the average density of the crown is less than in part (b), so the volume is greater. B is greater and T is less. These measurements can be used to determine if the crown is solid gold, without damaging the crown.

14.76. **IDENTIFY:** Problem 14.75 says $\dfrac{\rho_{\text{object}}}{\rho_{\text{fluid}}} = \dfrac{1}{1-f}$, where the apparent weight of the object when it is totally immersed in the fluid is fw.

SET UP: For the object in water, $f_{\text{water}} = w_{\text{water}}/w$ and for the object in the unknown fluid, $f_{\text{fluid}} = w_{\text{fluid}}/w$.

EXECUTE: **(a)** $\dfrac{\rho_{\text{steel}}}{\rho_{\text{fluid}}} = \dfrac{w}{w - w_{\text{fluid}}}$, $\dfrac{\rho_{\text{steel}}}{\rho_{\text{fluid}}} = \dfrac{w}{w - w_{\text{water}}}$. Dividing the second of these by the first gives

$\dfrac{\rho_{\text{fluid}}}{\rho_{\text{water}}} = \dfrac{w - w_{\text{fluid}}}{w - w_{\text{water}}}.$

(b) When w_{fluid} is greater than $w_{\text{water,}}$ the term on the right in the above expression is less than one, indicating that the fluids is less dense than water, and this is consistent with the buoyant force when suspended in liquid being less than that when suspended in water. If the density of the fluid is the same as that of water $w_{\text{fluid}} = w_{\text{water}}$, as expected. Similarly, if w_{fluid} is less than w_{water}, the term on the right in the above expression is greater than one, indicating that the fluid is denser than water.

(c) Writing the result of part (a) as $\frac{\rho_{\text{fluid}}}{\rho_{\text{water}}} = \frac{1-f_{\text{fluid}}}{1-f_{\text{water}}}$, and solving for f_{fluid},

$$f_{\text{fluid}} = 1 - \frac{\rho_{\text{fluid}}}{\rho_{\text{water}}}(1 - f_{\text{water}}) = 1 - (1.220)\ (0.128) = 0.844 = 84.4\%.$$

EVALUATE: Formic acid has density greater than the density of water. When the object is immersed in formic acid the buoyant force is greater and the apparent weight is less than when the object is immersed in water.

14.77. **IDENTIFY** and **SET UP:** Use Archimedes' principle for *B*.

(a) $B = \rho_{\text{water}} V_{\text{tot}} g$, where V_{tot} is the total volume of the object.

$V_{\text{tot}} = V_{\text{m}} + V_0$, where V_{m} is the volume of the metal.

EXECUTE: $V_{\text{m}} = w/g\rho_{\text{m}}$ so $V_{\text{tot}} = w/g\rho_{\text{m}} + V_0$

This gives $B = \rho_{\text{water}} g(w/g\rho_{\text{m}} + V_0)$

Solving for V_0 gives $V_0 = B/(\rho_{\text{water}} g) - w/(\rho_{\text{m}} g)$, as was to be shown.

(b) The expression derived in part (a) gives

$$V_0 = \frac{20\text{ N}}{(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)} - \frac{156\text{ N}}{(8.9\times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 2.52\times 10^{-4}\text{ m}^3$$

$$V_{\text{tot}} = \frac{B}{\rho_{\text{water}} g} = \frac{20\text{ N}}{(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 2.04\times 10^{-3}\text{ m}^3 \text{ and } V_0/V_{\text{tot}} = (2.52\times 10^{-4}\text{ m}^3)/(2.04\times 10^{-3}\text{ m}^3) = 0.124.$$

EVALUATE: When $V_0 \to 0$, the object is solid and $V_{\text{obj}} = V_{\text{m}} = w/(\rho_{\text{m}} g)$. For $V_0 = 0$, the result in part (a) gives $B = (w/\rho_{\text{m}})\rho_{\text{w}} = V_{\text{m}}\rho_{\text{w}} g = V_{\text{obj}}\rho_{\text{w}} g$, which agrees with Archimedes' principle. As V_0 increases with the weight kept fixed, the total volume of the object increases and there is an increase in *B*.

14.78. **IDENTIFY:** For a floating object the buoyant force equals the weight of the object. Archimedes's principle says the buoyant force equals the weight of fluid displaced by the object. $m = \rho V$.

SET UP: Let *d* be the depth of the oil layer, *h* the depth that the cube is submerged in the water, and *L* be the length of a side of the cube.

EXECUTE: **(a)** Setting the buoyant force equal to the weight and canceling the common factors of *g* and the cross-section area, $(1000)h + (750)d = (550)L$. *d*, *h* and *L* are related by $d + h + 0.35L = L$, so $h = 0.65L - d$.

Substitution into the first relation gives $d = L\frac{(0.65)(1000) - (550)}{(1000) - (750)} = \frac{2L}{5.00} = 0.040\text{ m}$.

(b) The gauge pressure at the lower face must be sufficient to support the block (the oil exerts only sideways forces directly on the block), and $p = \rho_{\text{wood}} gL = (550\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.100\text{ m}) = 539\text{ Pa}$.

EVALUATE: As a check, the gauge pressure, found from the depths and densities of the fluids, is $[(0.040\text{ m})(750\text{ kg/m}^3) + (0.025\text{ m})(1000\text{ kg/m}^3)](9.80\text{ m/s}^2) = 539\text{ Pa}$.

14.79. **IDENTIFY** and **SET UP:** Apply the first condition of equilibrium to the barge plus the anchor. Use Archimedes' principle to relate the weight of the boat and anchor to the amount of water displaced. In both cases the total buoyant force must equal the weight of the barge plus the weight of the anchor. Thus the total amount of water displaced must be the same when the anchor is in the boat as when it is over the side. When the anchor is in the water the barge displaces less water, less by the amount the anchor displaces. Thus the barge rises in the water.

EXECUTE: The volume of the anchor is $V_{\text{anchor}} = m/\rho = (35.0\text{ kg})/(7860\text{ kg/m}^3) = 4.456\times 10^{-3}\text{ m}^3$. The barge rises in the water a vertical distance *h* given by $hA = 4.453\times 10^{-3}\text{ m}^3$, where *A* is the area of the bottom of the barge.

$h = (4.453\times 10^{-3}\text{ m}^3)/(8.00\text{ m}^2) = 5.57\times 10^{-4}\text{ m}$.

EVALUATE: The barge rises a very small amount. The buoyancy force on the barge plus the buoyancy force on the anchor must equal the weight of the barge plus the weight of the anchor. When the anchor is in the water, the buoyancy force on it is less than its weight (the anchor doesn't float on its own), so part of the buoyancy force on the barge is used to help support the anchor. If the rope is cut, the buoyancy force on the barge must equal only the weight of the barge and the barge rises still farther.

14.80. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the barrel, with $+y$ upward. The buoyant force on the barrel is given by Archimedes's principle.

SET UP: $\rho_{\text{av}} = m_{\text{tot}}/V$. An object floats in a fluid if its average density is less than the density of the fluid. The density of seawater is 1030 kg/m^3.

EXECUTE: **(a)** The average density of a filled barrel is $\frac{m_{\text{oil}} + m_{\text{steel}}}{V}\ \ \rho_{\text{oil}} + \frac{m_{\text{steel}}}{V} = 750\text{ kg/m}^3 + \frac{15.0\text{ kg}}{0.120\text{ m}^3} = 875\text{ kg/m}^3$, which is less than the density of seawater, so the barrel floats.

(b) The fraction above the surface (see Problem 14.29) is

$$1-\frac{\rho_{\text{av}}}{\rho_{\text{water}}}=1-\frac{875\ \text{kg/m}^3}{1030\ \text{kg/m}^3}=0.150=15.0\%.$$

(c) The average density is $910\ \text{kg/m}^3+\dfrac{32.0\ \text{kg}}{0.120\ \text{m}^3}=1172\ \text{kg/m}^3$, which means the barrel sinks. In order to lift it, a tension $T=w_{\text{tot}}-B=(1177\ \text{kg/m}^3)(0.120\ \text{m}^3)(9.80\ \text{m/s}^2)-(1030\ \text{kg/m}^3)(0.120\ \text{m}^3)(9.80\ \text{m/s}^2)=173\ \text{N}$ is required.

EVALUATE: When the barrel floats, the buoyant force B equals its weight, w. In part (c) the buoyant force is less than the weight and $T=w-B$.

14.81. IDENTIFY: Apply Newton's 2nd law to the block. In part (a), use Archimedes' principle for the buoyancy force. In part (b), use Eq.(14.6) to find the pressure at the lower face of the block and then use Eq.(14.3) to calculate the force the fluid exerts.

(a) SET UP: The free-body diagram for the block is given in Figure 14.81a.

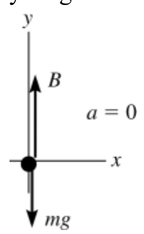

Figure 14.81a

EXECUTE: $\sum F_y=ma_y$

$B-mg=0$

$\rho_{\text{L}}V_{\text{sub}}g=\rho_{\text{B}}V_{\text{obj}}g$

The fraction of the volume that is submerged is $V_{\text{sub}}/V_{\text{obj}}=\rho_{\text{B}}/\rho_{\text{L}}$.

Thus the fraction that is *above* the surface is $V_{\text{above}}/V_{\text{obj}}=1-\rho_{\text{B}}/\rho_{\text{L}}$.

EVALUATE: If $\rho_{\text{B}}=\rho_{\text{L}}$ the block is totally submerged as it floats.

(b) SET UP: Let the water layer have depth d, as shown in Figure 14.81b.

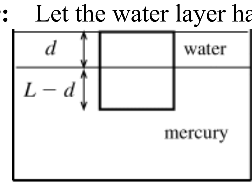

Figure 14.81b

EXECUTE: $p=p_0+\rho_{\text{w}}gd+\rho_{\text{L}}g(L-d)$

Applying $\sum F_y=ma_y$ to the block gives

$(p-p_0)A-mg=0.$

$[\rho_{\text{w}}gd+\rho_{\text{L}}g(L-d)]A=\rho_{\text{B}}LAg$

A and g divide out and $\rho_{\text{w}}d+\rho_{\text{L}}(L-d)=\rho_{\text{B}}L$

$d(\rho_{\text{w}}-\rho_{\text{L}})=(\rho_{\text{B}}-\rho_{\text{L}})L$

$$d=\left(\frac{\rho_{\text{L}}-\rho_{\text{B}}}{\rho_{\text{L}}-\rho_{\text{w}}}\right)L$$

(c) $d=\left(\dfrac{13.6\times10^3\ \text{kg/m}^3-7.8\times10^3\ \text{kg/m}^3}{13.6\times10^3\ \text{kg/m}^3-1000\ \text{kg/m}^3}\right)(0.100\ \text{m})=0.0460\ \text{m}=4.60\ \text{cm}$

EVALUATE: In the expression derived in part (b), if $\rho_{\text{B}}=\rho_{\text{L}}$ the block floats in the liquid totally submerged and no water needs to be added. If $\rho_{\text{L}}\to\rho_{\text{w}}$ the block continues to float with a fraction $1-\rho_{\text{B}}/\rho_{\text{w}}$ above the water as water is added, and the water never reaches the top of the block $(d\to\infty)$.

14.82. IDENTIFY: For the floating tanker, the buoyant force equals its total weight. The buoyant force is given by Archimedes's principle.

SET UP: When the metal is in the tanker, it displaces its weight of water and after it has been pushed overboard it displaces its volume of water.

EXECUTE: **(a)** The change in height Δy is related to the displaced volume ΔV by $\Delta y=\dfrac{\Delta V}{A}$, where A is the surface area of the water in the lock. ΔV is the volume of water that has the same weight as the metal, so

$$\Delta y=\frac{\Delta V}{A}=\frac{w/(\rho_{\text{water}}g)}{A}=\frac{w}{\rho_{\text{water}}gA}=\frac{(2.50\times10^6\ \text{N})}{(1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)[(60.0\ \text{m})(20.0\ \text{m})]}=0.213\ \text{m}.$$

(b) In this case, ΔV is the volume of the metal; in the above expression, ρ_{water} is replaced by $\rho_{\text{metal}} = 9.00\rho_{\text{water}}$, which gives $\Delta y' = \frac{\Delta y}{9}$, and $\Delta y - \Delta y' = \frac{8}{9}\Delta y = 0.189$ m; the water level falls this amount.

EVALUATE: The density of the metal is greater than the density of water, so the volume of water that has the same weight as the steel is greater than the volume of water that has the same volume as the steel.

14.83. **IDENTIFY:** Consider the fluid in the horizontal part of the tube. This fluid, with mass ρAl, is subject to a net force due to the pressure difference between the ends of the tube

SET UP: The difference between the gauge pressures at the bottoms of the ends of the tubes is $\rho g(y_L - y_R)$.

EXECUTE: The net force on the horizontal part of the fluid is $\rho g(y_L - y_R)A = \rho Ala$, or, $(y_L - y_R) = \frac{a}{g}l$.

(b) Again consider the fluid in the horizontal part of the tube. As in part (a), the fluid is accelerating; the center of mass has a radial acceleration of magnitude $a_{\text{rad}} = \omega^2 l/2$, and so the difference in heights between the columns is $(\omega^2 l/2)(l/g) = \omega^2 l^2/2g$. An equivalent way to do part (b) is to break the fluid in the horizontal part of the tube into elements of thickness dr; the pressure difference between the sides of this piece is $dp = \rho(\omega^2 r)dr$ and integrating from $r = 0$ to $r = l$ gives $\Delta p = \rho\omega^2 l^2/2$, the same result.

EVALUATE: **(c)** The pressure at the bottom of each arm is proportional to ρ and the mass of fluid in the horizontal portion of the tube is proportional to ρ, so ρ divides out and the results are independent of the density of the fluid. The pressure at the bottom of a vertical arm is independent of the cross-sectional area of the arm. Newton's second law could be applied to a cross-section of fluid smaller than that of the tubes. Therefore, the results are independent and of the size and shape of all parts of the tube.

14.84. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to a small fluid element located a distance r from the axis.

SET UP: For rotational motion, $a = \omega^2 r$.

EXECUTE: **(a)** The change in pressure with respect to the vertical distance supplies the force necessary to keep a fluid element in vertical equilibrium (opposing the weight). For the rotating fluid, the change in pressure with respect to radius supplies the force necessary to keep a fluid element accelerating toward the axis; specifically, $dp = \frac{\partial p}{\partial r}dr = \rho a\, dr$, and using $a = \omega^2 r$ gives $\frac{\partial p}{\partial r} = \rho\omega^2 r$.

(b) Let the pressure at $y = 0$, $r = 0$ be p_a (atmospheric pressure); integrating the expression for $\frac{\partial p}{\partial r}$ from part (a) gives $p(r, y = 0) = p_a + \frac{\rho\omega^2}{2}r^2$

(c) In Eq. (14.5), $p_2 = p_a$, $p = p_1 = p(r, y = 0)$ as found in part (b), $y_1 = 0$ and $y_2 = h(r)$, the height of the liquid above the $y = 0$ plane. Using the result of part (b) gives $h(r) = \omega^2 r^2/2g$.

EVALUATE: The curvature of the surface increases as the speed of rotation increases.

14.85. **IDENTIFY:** Follow the procedure specified in part (a) and integrate this result for part (b).

SET UP: A rotating particle a distance r' from the rotation axis has inward acceleration $\omega^2 r'$.

EXECUTE: **(a)** The net inward force is $(p + dp)A - pA = Adp$, and the mass of the fluid element is $\rho Adr'$. Using Newton's second law, with the inward radial acceleration of $\omega^2 r'$, gives $dp = \rho\omega^2 r'dr'$.

(b) Integrating the above expression, $\int_{p_0}^{p} dp = \int_{r_0}^{r} \rho\omega^2 r'dr'$ and $p - p_0 = \left(\frac{\rho\omega^2}{2}\right)(r^2 - r^2{}_0)$, which is the desired result.

(c) The net force on the object must be the same as that on a fluid element of the same shape. Such a fluid element is accelerating inward with an acceleration of magnitude $\omega^2 R_{\text{cm},}$ and so the force on the object is $\rho V\omega^2 R_{\text{cm}}$.

(d) If $\rho R_{\text{cm}} > \rho_{\text{ob}} R_{\text{cm ob}}$, the inward force is greater than that needed to keep the object moving in a circle with radius $R_{\text{cm ob}}$ at angular frequency ω, and the object moves inward. If $\rho R_{\text{cm}} < \rho_{\text{ob}} R_{\text{cm ob}}$, the net force is insufficient to keep the object in the circular motion at that radius, and the object moves outward.

(e) Objects with lower densities will tend to move toward the center, and objects with higher densities will tend to move away from the center.

EVALUATE: The pressure in the fluid increases as the distance r from the rotation axis increases.

14.86. **IDENTIFY:** Follow the procedure specified in the problem.

SET UP: Let increasing x correspond to moving toward the back of the car.

EXECUTE: **(a)** The mass of air in the volume element is $\rho dV = \rho A dx$, and the net force on the element in the forward direction is $(p + dp)A - pA = Adp$. From Newton's second law, $Adp = (\rho A dx)a$, from which $dp = \rho a dx$.

(b) With ρ given to be constant, and with $p = p_0$ at $x = 0$, $p = p_0 + \rho a x$.

(c) Using $\rho = 1.2 \text{ kg/m}^3$ in the result of part (b) gives $(1.2 \text{ kg/m}^3)(5.0 \text{ m/s}^2)(2.5 \text{ m}) = 15.0 \text{ Pa} = 15 \times 10^{-5} p_{\text{atm}}$, so the fractional pressure difference is negligible.

(d) Following the argument in Section 14.4, the force on the balloon must be the same as the force on the same volume of air; this force is the product of the mass ρV and the acceleration, or $\rho V a$.

(e) The acceleration of the balloon is the force found in part (d) divided by the mass $\rho_{\text{bal}} V$, or $(\rho/\rho_{\text{bal}})a$. The acceleration relative to the car is the difference between this acceleration and the car's acceleration, $a_{\text{rel}} = [(\rho/\rho_{\text{bal}}) - 1]a$.

(f) For a balloon filled with air, $(\rho/\rho_{\text{bal}}) < 1$ (air balloons tend to sink in still air), and so the quantity in square brackets in the result of part (e) is negative; the balloon moves to the back of the car. For a helium balloon, the quantity in square brackets is positive, and the balloon moves to the front of the car.

EVALUATE: The pressure in the air inside the car increases with distance from the windshield toward the rear of the car. This pressure increase is proportional to the acceleration of the car.

14.87. **IDENTIFY:** After leaving the tank, the water is in free fall, with $a_x = 0$ and $a_y = +g$.

SET UP: From Example 14.8, the speed of efflux is $\sqrt{2gh}$.

EXECUTE: **(a)** The time it takes any portion of the water to reach the ground is $t = \sqrt{\dfrac{2(H-h)}{g}}$, in which time the water travels a horizontal distance $R = vt = 2\sqrt{h(H-h)}$.

(b) Note that if $h' = H - h, h'(H - h') = (H - h)h$, and so $h' = H - h$ gives the same range. A hole $H - h$ below the water surface is a distance h above the bottom of the tank.

EVALUATE: For the special case of $h = H/2$, $h = h'$ and the two points coincide. For the upper hole the speed of efflux is less but the time in the air during the free-fall is greater.

14.88. **IDENTIFY:** Use Bernoulli's equation to find the velocity with which the water flows out the hole.

SET UP: The water level in the vessel will rise until the volume flow rate into the vessel, $2.40 \times 10^{-4} \text{ m}^3/\text{s}$, equals the volume flow rate out the hole in the bottom.

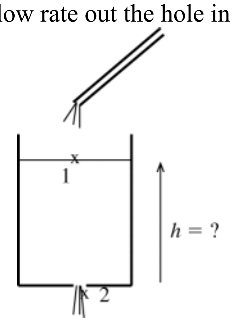

Figure 14.88

Let points 1 and 2 be chosen as in Figure 14.88.

EXECUTE: Bernoulli's equation: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

Volume flow rate out of hole equals volume flow rate from tube gives that $v_2 A_2 = 2.40 \times 10^{-4} \text{ m}^3/\text{s}$ and

$$v_2 = \frac{2.40 \times 10^{-4} \text{ m}^3/\text{s}}{1.50 \times 10^{-4} \text{ m}^2} = 1.60 \text{ m/s}$$

$A_1 \gg A_2$ and $v_1 A_1 = v_2 A_2$ says that $\frac{1}{2}\rho v_1^2 \ll \frac{1}{2}\rho v_2^2$; neglect the $\frac{1}{2}\rho v_1^2$ term.

Measure y from the bottom of the bucket, so $y_2 = 0$ and $y_1 = h$.

$p_1 = p_2 = p_{\text{a}}$ (air pressure)

Then $p_{\text{a}} + \rho g h = p_{\text{a}} + \frac{1}{2}\rho v_2^2$ and $h = v_2^2/2g = (1.60 \text{ m/s})^2/2(9.80 \text{ m/s}^2) = 0.131 \text{ m} = 13.1 \text{ cm}$

EVALUATE: The greater the flow rate into the bucket, the larger v_2 will be at equilibrium and the higher the water will rise in the bucket.

14.89. **IDENTIFY:** Apply Bernoulli's equation and the equation of continuity.

SET UP: Example 14.8 says the speed of efflux is $\sqrt{2gh}$, where h is the distance of the hole below the surface of the fluid.

EXECUTE: **(a)** $v_3A_3 = \sqrt{2g(y_1 - y_3)}A_3 = \sqrt{2(9.80\text{ m/s}^2)(8.00\text{ m})}(0.0160\text{ m}^2) = 0.200\text{ m}^3\text{/s}.$

(b) Since p_3 is atmospheric, the gauge pressure at point 2 is $p_2 = \frac{1}{2}\rho(v_3^2 - v_2^2) = \frac{1}{2}\rho v_3^2\left(1 - \left(\frac{A_3}{A_2}\right)^2\right) = \frac{8}{9}\rho g(y_1 - y_3),$

using the expression for v_3 found above. Substitution of numerical values gives $p_2 = 6.97\times10^4$ Pa.

EVALUATE: We could also calculate p_2 by applying Bernoulli's equation to points 1 and 2.

14.90. **IDENTIFY:** Apply Bernoulli's equation to the air in the hurricane.

SET UP: For a particle a distance r from the axis, the angular momentum is $L = mvr$.

EXECUTE: **(a)** Using the constancy of angular momentum, the product of the radius and speed is constant, so the speed at the rim is about $(200\text{ km/h})\left(\frac{30}{350}\right) = 17\text{ km/h}.$

(b) The pressure is lower at the eye, by an amount

$$\Delta p = \frac{1}{2}(1.2\text{ kg/m}^3)((200\text{ km/h})^2 - (17\text{ km/h})^2)\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)^2 = 1.8\times10^3\text{ Pa}.$$

(c) $\frac{v^2}{2g} = 160\text{ m}$.

(d) The pressure difference at higher altitudes is even greater.

EVALUATE: According to Bernoulli's equation, the pressure decreases when the fluid velocity increases.

14.91. **IDENTIFY:** Apply Bernoulli's equation and the equation of continuity.

SET UP: Example 14.8 shows that the speed of efflux at point D is $\sqrt{2gh_1}$.

EXECUTE: Applying the equation of continuity to points at C and D gives that the fluid speed is $\sqrt{8gh_1}$ at C. Applying Bernoulli's equation to points A and C gives that the gauge pressure at C is $\rho gh_1 - 4\rho gh_1 = -3\rho gh_1$, and this is the gauge pressure at the surface of the fluid at E. The height of the fluid in the column is $h_2 = 3h_1$.

EVALUATE: The gauge pressure at C is less than the gauge pressure ρgh_1 at the bottom of tank A because of the speed of the fluid at C.

14.92. **IDENTIFY:** Apply Bernoulli's equation to points 1 and 2. Apply $p = p_0 + \rho gh$ to both arms of the U-shaped tube in order to calculate h.

SET UP: The discharge rate is $v_1A_1 = v_2A_2$. The density of mercury is $\rho_m = 13.6\times10^3\text{ kg/m}^3$ and the density of water is $\rho_w = 1.00\times10^3\text{ kg/m}^3$. Let point 1 be where $A_1 = 40.0\times10^{-4}\text{ m}^2$ and point 2 is where $A_2 = 10.0\times10^{-4}\text{ m}^2$. $y_1 = y_2$.

EXECUTE: **(a)** $v_1 = \frac{6.00\times10^{-3}\text{ kg/m}^3}{40.0\times10^{-4}\text{ m}^2} = 1.50\text{ m/s}$. $v_2 = \frac{6.00\times10^{-3}\text{ kg/m}^3}{10.0\times10^{-4}\text{ m}^2} = 6.00\text{ m/s}$

(b) $p_1 + \rho gy_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2$.

$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}(1000\text{ kg/m}^3)([6.00\text{ m/s}]^2 - [1.50\text{ m/s}]^2) = 1.69\times10^4\text{ Pa}$

(c) $p_1 + \rho_w gh = p_2 + \rho_m gh$ and $h = \frac{p_1 - p_2}{(\rho_m - \rho_w)g} = \frac{1.69\times10^4\text{ Pa}}{(13.6\times10^3\text{ kg/m}^3 - 1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 0.137\text{ m} = 13.7\text{ cm}.$

EVALUATE: The pressure in the fluid decreases when the speed of the fluid increases.

14.93. **(a) IDENTIFY:** Apply constant acceleration equations to the falling liquid to find its speed as a function of the distance below the outlet. Then apply Eq.(14.10) to relate the speed to the radius of the stream.

SET UP:

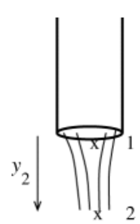

Figure 14.93

Let point 1 be at the end of the pipe and let point 2 be in the stream of liquid at a distance y_2 below the end of the tube, as shown in Figure 14.93.

Consider the free-fall of the liquid. Take $+y$ to be downward.

Free-fall implies $a_y = g$. v_y is positive, so replace it by the speed v.

EXECUTE: $v_2^2 = v_1^2 + 2a(y - y_0)$ gives $v_2^2 = v_1^2 + 2gy_2$ and $v_2 = \sqrt{v_1^2 + 2gy_2}$.

Equation of continuity says $v_1 A_1 = v_2 A_2$

And since $A = \pi r^2$ this becomes $v_1 \pi r_1^2 = v_2 \pi r_2^2$ and $v_2 = v_1 (r_1/r_2)^2$.

Use this in the above to eliminate v_2: $v_1(r_1^2/r_2^2) = \sqrt{v_1^2 + 2gy_2}$

$r_2 = r_1\sqrt{v_1}/(v_1^2 + 2gy_2)^{1/4}$

To correspond to the notation in the problem, let $v_1 = v_0$ and $r_1 = r_0$, since point 1 is where the liquid first leaves the pipe, and let r_2 be *r* and y_2 be *y*. The equation we have derived then becomes $r = r_0\sqrt{v_0}/(v_0^2 + 2gy)^{1/4}$

(b) $v_0 = 1.20$ m/s

We want the value of *y* that gives $r = \frac{1}{2}r_0$, or $r_0 = 2r$

The result obtained in part (a) says $r^4(v_0^2 + 2gy) = r_0^4 v_0^2$

Solving for *y* gives $y = \dfrac{[(r_0/r)^4 - 1]v_0^2}{2g} = \dfrac{(16-1)(1.20\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 1.10\text{ m}$

EVALUATE: The equation derived in part (a) says that *r* decreases with distance below the end of the pipe.

14.94. IDENTIFY: Apply $\sum F_y = ma_y$ to the rock.

SET UP: In the accelerated frame, all of the quantities that depend on *g* (weights, buoyant forces, gauge pressures and hence tensions) may be replaced by $g' = g + a$, with the positive direction taken upward.

EXECUTE: **(a)** The volume *V* of the rock is

$$V = \frac{B}{\rho_{\text{water}} g} = \frac{w - T}{\rho_{\text{water}} g} = \frac{((3.00\text{ kg})(9.80\text{ m/s}^2) - 21.0\text{ N})}{(1.00\times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 8.57\times 10^{-4}\text{ m}^3.$$

(b) The tension is $T = mg' - B' = (m - \rho V)g' = T_0 \dfrac{g'}{g}$, where $T_0 = 21.0\text{N}$. $g' = g + a$. For $a = 2.50\text{ m/s}^2$,

$T = (21.0\text{ N})\dfrac{9.80 + 2.50}{9.80} = 26.4\text{ N}$.

(c) For $a = -2.50\text{ m/s}^2$, $T = (21.0\text{ N})\dfrac{9.80 - 2.50}{9.80} = 15.6\text{ N}$.

(d) If $a = -g$, $g' = 0$ and $T = 0$.

EVALUATE: The acceleration of the water alters the buoyant force it exerts.

14.95. IDENTIFY: The sum of the vertical forces on the object must be zero.

SET UP: The depth of the bottom of the styrofoam is not given; let this depth be h_0. Denote the length of the piece of foam by *L* and the length of the two sides by *l*. The volume of the object is $\frac{1}{2}l^2 L$.

EXECUTE: **(a)** The tension in the cord plus the weight must be equal to the buoyant force, so

$T = Vg(\rho_{\text{water}} - \rho_{\text{foam}}) = \frac{1}{2}(0.20\text{ m})^2(0.50\text{ m})(9.80\text{ m/s}^2)(1000\text{ kg/m}^3 - 180\text{ kg/m}^3) = 80.4\text{ N}$.

(b) The pressure force on the bottom of the foam is $(p_0 + \rho g h_0)L\left(\sqrt{2}l\right)$ and is directed up. The pressure on each side is not constant; the force can be found by integrating, or using the results of Problem 14.49 or Problem 14.51. Although these problems found forces on vertical surfaces, the result that the force is the product of the average pressure and the area is valid. The average pressure is $p_0 + \rho g(h_0 - (l/(2\sqrt{2})))$, and the force on one side has magnitude $(p_0 + \rho g(h_0 - l/(2\sqrt{2})))Ll$

and is directed perpendicular to the side, at an angle of $45.0°$ from the vertical. The force on the other side has the same magnitude, but has a horizontal component that is opposite that of the other side. The horizontal component of the net buoyant force is zero, and the vertical component is

$B = (p_0 + \rho g h_0)Ll\sqrt{2} - 2(\cos 45.0°)(p_0 + \rho g(h_0 - l/(2\sqrt{2})))Ll = \rho g \dfrac{Ll^2}{2}$,

the weight of the water displaced.

EVALUATE: The density of the object is less than the density of water, so if the cord were cut the object would float. When the object is fully submerged, the upward buoyant force is greater than its weight and the cord must pull downward on the object to hold it beneath the surface.

14.96. **IDENTIFY:** Use the efflux speed to calculate the volume flow rate and integrate to find the time for the entire volume of water to flow out of the tank.

SET UP: When the level of the water is a height y above the opening, the efflux speed is $\sqrt{2gy}$, and $\frac{dV}{dt} = \pi(d/2)^2\sqrt{2gy}$.

EXECUTE: As the tank drains, the height decreases, and $\frac{dy}{dt} = -\frac{dV/dt}{A} = -\frac{\pi(d/2)^2\sqrt{2gy}}{\pi(D/2)^2} = -\left(\frac{d}{D}\right)^2\sqrt{2gy}$. This is a separable differential equation, and the time T to drain the tank is found from $\frac{dy}{\sqrt{y}} = -\left(\frac{d}{D}\right)^2\sqrt{2g}\,dt$, which integrates to $\left[2\sqrt{y}\,\right]_H^0 = -\left(\frac{d}{D}\right)^2\sqrt{2g}T$, or $T = \left(\frac{D}{d}\right)^2\frac{2\sqrt{H}}{\sqrt{2g}} = \left(\frac{D}{d}\right)^2\sqrt{\frac{2H}{g}}$.

EVALUATE: Even though the volume flow rate approaches zero as the tank drains, it empties in a finite amount of time. Doubling the height of the tank doubles the volume of water in the tank but increases the time to drain by only a factor of $\sqrt{2}$.

14.97. **IDENTIFY:** Apply Bernoulli's equation to the fluid in the siphon.

SET UP: Example 14.8 shows that the efflux speed from a small hole a distance h below the surface of fluid in a large open tank is $\sqrt{2gh}$.

EXECUTE: **(a)** The fact that the water first moves upwards before leaving the siphon does not change the efflux speed, $\sqrt{2gh}$.

(b) Water will not flow if the absolute (not gauge) pressure would be negative. The hose is open to the atmosphere at the bottom, so the pressure at the top of the siphon is $p_a - \rho g(H + h)$, where the assumption that the cross-section area is constant has been used to equate the speed of the liquid at the top and bottom. Setting $p = 0$ and solving for H gives $H = (p_a/\rho g) - h$.

EVALUATE: The analysis shows that $H + h < \frac{p_a}{\rho g}$, so there is also a limitation on $H + h$. For water and normal atmospheric pressure, $\frac{p_a}{\rho g} = 10.3\text{ m}$.

14.98. **IDENTIFY** and **SET UP:** Apply $p = p_0 + \rho gh$.

EXECUTE: Any bubbles will cause inaccuracies. At the bubble, the pressure at the surfaces of the water will be the same, but the levels need not be the same. The use of a hose as a level assumes that pressure is the same at all points that are at the same level, an assumption that is invalidated by the bubble.

EVALUATE: Larger bubbles can cause larger inaccuracies, because there can be greater changes in height across the length of the bubble.