12

GRAVITATION

12.1. IDENTIFY and SET UP: Use the law of gravitation, Eq.(12.1), to determine F_g.

EXECUTE: $F_{\text{S on M}} G\dfrac{m_S m_M}{r_{SM}^2}$ (S = sun, M = moon); $F_{\text{E on M}} = G\dfrac{m_E m_M}{r_{EM}^2}$ (E = earth)

$$\frac{F_{\text{S on M}}}{F_{\text{E on M}}} = \left(G\frac{m_S m_M}{r_{SM}^2}\right)\left(\frac{r_{EM}^2}{G m_E m_M}\right) = \frac{m_S}{m_E}\left(\frac{r_{EM}}{r_{SM}}\right)^2$$

r_{EM}, the radius of the moon's orbit around the earth is given in Appendix F as 3.84×10^8 m. The moon is much closer to the earth than it is to the sun, so take the distance r_{SM} of the moon from the sun to be r_{SE}, the radius of the earth's orbit around the sun.

$$\frac{F_{\text{S on M}}}{F_{\text{E on M}}} = \left(\frac{1.99\times10^{30}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}\right)\left(\frac{3.84\times10^8\ \text{m}}{1.50\times10^{11}\ \text{m}}\right)^2 = 2.18.$$

EVALUATE: The force exerted by the sun is larger than the force exerted by the earth. The moon's motion is a combination of orbiting the sun and orbiting the earth.

12.2. IDENTIFY: The gravity force between spherically symmetric spheres is $F_g = \dfrac{Gm_1m_2}{r^2}$, where r is the separation between their centers.

SET UP: $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$. The moment arm for the torque due to each force is 0.150 m.

EXECUTE: **(a)** For each pair of spheres, $F_g = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.10\ \text{kg})(25.0\ \text{kg})}{(0.120\ \text{m})^2} = 1.27\times10^{-7}\ \text{N}$. From Figure 12.4 in the textbook we see that the forces for each pair are in opposite directions, so $F_{\text{net}} = 0$.

(b) The net torque is $\tau_{\text{net}} = 2F_g l = 2(1.27\times10^{-7}\ \text{N})(0.150\ \text{m}) = 3.81\times10^{-8}\ \text{N}\cdot\text{m}$.

(c) The torque is very small and the apparatus must be very sensitive. The torque could be increased by increasing the mass of the spheres or by decreasing their separation.

EVALUATE: The quartz fiber must twist through a measurable angle when a small torque is applied to it.

12.3. IDENTIFY: The force exerted on the particle by the earth is $w = mg$, where m is the mass of the particle. The force exerted by the 100 kg ball is $F_g = \dfrac{Gm_1m_2}{r^2}$, where r is the distance of the particle from the center of the ball.

SET UP: $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$, $g = 9.80\ \text{m/s}^2$.

EXECUTE: $F_g = w$ gives $\dfrac{Gmm_{\text{ball}}}{r^2} = mg$ and

$r = \sqrt{\dfrac{Gm_{\text{ball}}}{g}} = \sqrt{\dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(100\ \text{kg})}{9.80\ \text{m/s}^2}} = 2.61\times10^{-5}\ \text{m} = 0.0261\ \text{mm}$. It is not feasible to do this; a 100 kg ball would have a radius much larger than 0.0261 mm.

EVALUATE: The gravitational force between ordinary objects is very small. The gravitational force exerted by the earth on objects near its surface is large enough to be important because the mass of the earth is very large.

12.4. IDENTIFY: Apply Eq.(12.2), generalized to any pair of spherically symmetric objects.

SET UP: The separation of the centers of the spheres is $2R$.

EXECUTE: The magnitude of the gravitational attraction is $GM^2/(2R)^2 = GM^2/4R^2$.

EVALUATE: Eq.(12.2) applies to any pair of spherically symmetric objects; one of the objects doesn't have to be the earth.

12.5. **IDENTIFY:** Use Eq.(12.1) to calculate F_g exerted by the earth and by the sun and add these forces as vectors.

(a) SET UP: The forces and distances are shown in Figure 12.5.

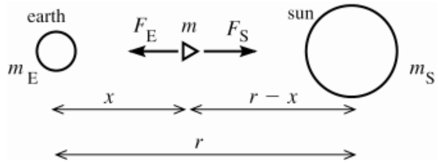

Figure 12.5

Let $\vec{F}_E$ and $\vec{F}_S$ be the gravitational forces exerted on the spaceship by the earth and by the sun.

EXECUTE: The distance from the earth to the sun is $r = 1.50\times10^{11}$ m. Let the ship be a distance x from the earth; it is then a distance $r - x$ from the sun.

$F_E = F_S$ says that $Gmm_E/x^2 = Gmm_S/(r-x)^2$

$m_E/x^2 = ms/\left(r-x\right)^2$ and $(r-x)^2 = x^2(m_S/m_E)$

$r - x = x\sqrt{m_S/m_E}$ and $r = x(1+\sqrt{m_S/m_E})$

$$x = \frac{r}{1+\sqrt{m_S/m_E}} = \frac{1.50\times10^{11}\text{ m}}{1+\sqrt{1.99\times10^{30}\text{ kg}/5.97\times10^{24}\text{ kg}}} = 2.59\times10^{8}\text{ m (from center of earth)}$$

(b) EVALUATE: At the instant when the spaceship passes through this point its acceleration is zero. Since $m_S \gg m_E$ this equal-force point is much closer to the earth than to the sun.

12.6. **IDENTIFY:** Apply Eq.(12.1) to calculate the magnitude of the gravitational force exerted by each sphere. Each force is attractive. The net force is the vector sum of the individual forces.

SET UP: Let +*x* be to the right.

EXECUTE: **(a)** $F_{gx} = \left(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(0.100\text{ kg}\right)\left[-\frac{(5.00\text{ kg})}{(0.400\text{ m})^2}+\frac{(10.0\text{ kg})}{(0.600\text{ m})^2}\right] = -2.32\times10^{-11}\text{N}$, with the minus sign indicating a net force to the left.

(b) No, the force found in part (a) is the *net* force due to the other two spheres.

EVALUATE: The force from the 5.00 kg sphere is greater than for the 10.0 kg sphere even though its mass is less, because *r* is smaller for this mass.

12.7. **IDENTIFY:** The force exerted by the moon is the gravitational force, $F_g = \frac{Gm_Mm}{r^2}$. The force exerted on the person by the earth is $w = mg$.

SET UP: The mass of the moon is $m_M = 7.35\times10^{22}$ kg. $G = 6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** $F_{moon} = F_g = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(7.35\times10^{22}\text{ kg})(70\text{ kg})}{(3.78\times10^{8}\text{ m})^2} = 2.4\times10^{-3}\text{ N}$.

(b) $F_{earth} = w = (70\text{ kg})(9.80\text{ m/s}^2) = 690\text{ N}$. $F_{moon}/F_{earth} = 3.5\times10^{-6}$.

EVALUATE: The force exerted by the earth is much greater than the force exerted by the moon. The mass of the moon is less than the mass of the earth and the center of the earth is much closer to the person than is the center of the moon.

12.8. **IDENTIFY:** Use Eq.(12.2) to find the force each point mass exerts on the particle, find the net force, and use Newton's second law to calculate the acceleration.

SET UP: Each force is attractive. The particle (mass *m*) is a distance $r_1 = 0.200$ m from $m_1 = 8.00$ kg and therefore a distance $r_2 = 0.300$ m from $m_2 = 15.0$ kg. Let +*x* be toward the 15.0 kg mass.

EXECUTE: $F_1 = \frac{Gm_1m}{r_1^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(8.00\text{ kg})m}{(0.200\text{ m})^2} = (1.334\times10^{-8}\text{ N/kg})m$, in the $-x$-direction.

$F_2 = \frac{Gm_2m}{r_2^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(15.0\text{ kg})m}{(0.300\text{ m})^2} = (1.112\times10^{-8}\text{ N/kg})m$, in the $+x$-direction. The net force is

$F_x = F_{1x} + F_{2x} = (-1.334\times10^{-8}\text{ N/kg} + 1.112\times10^{-8}\text{ N/kg})m = (-2.2\times10^{-9}\text{ N/kg})m$. $a_x = \frac{F_x}{m} = -2.2\times10^{-9}\text{ m/s}^2$. The acceleration is $2.2\times10^{-9}\text{ m/s}^2$, toward the 8.00 kg mass.

EVALUATE: The smaller mass exerts the greater force, because the particle is closer to the smaller mass.

12.9. **IDENTIFY:** Apply Eq.(12.1) to calculate the magnitude of each gravitational force. Each force is attractive.

SET UP: The masses are $m_M = 7.35\times10^{22}$ kg, $m_S = 1.99\times10^{30}$ kg and $m_E = 5.97\times10^{24}$ kg. Denote the earth-sun separation as r_1 and the earth-moon separation as r_2.

EXECUTE: **(a)** $(Gm_M)\left[\dfrac{m_S}{(r_1+r_2)^2}+\dfrac{m_E}{r_2^2}\right] = 6.30\times10^{20}$ N, toward the sun.

(b) The earth-moon distance is sufficiently small compared to the earth-sun distance ($r_2 << r_1$) that the vector from the earth to the moon can be taken to be perpendicular to the vector from the sun to the moon. The gravitational forces are then $\dfrac{Gm_M m_S}{r_1^2} = 4.34\times10^{20}$ N and $\dfrac{Gm_M m_E}{r_2^2} = 1.99\times10^{20}$ N, and so the force has magnitude 4.77×10^{20} N and is directed $24.6°$ from the direction toward the sun.

(c) $(Gm_M)\left[\dfrac{m_S}{(r_1-r_2)^2}-\dfrac{m_E}{r_2^2}\right] = 2.37\times10^{20}$ N, toward the sun.

EVALUATE: The net force is very different in each of these three positions, even though the magnitudes of the forces from the sun and earth change very little.

12.10. **IDENTIFY:** Apply Eq.(12.1) to calculate the magnitude of each gravitational force. Each force is attractive.

SET UP: The forces on one of the masses are sketched in Figure 12.10. The figure shows that the vector sum of the three forces is toward the center of the square.

EXECUTE: $F_{onA} = 2F_B\cos 45° + F_D = 2\dfrac{Gm_A m_B \cos 45°}{r_{AB}^2} + \dfrac{Gm_A m_D}{r_{AD}^2}$.

$$F_{onA} = \frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(800\ \text{kg})^2\cos 45°}{(0.10\ \text{m})^2} + \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(800\ \text{kg})^2}{(0.10\ \text{m})^2} = 8.2\times10^{-3}\ \text{N}$$ toward the center of the square.

EVALUATE: We have assumed each mass can be treated as a uniform sphere. Each mass must have an unusually large density in order to have mass 800 kg and still fit into a square of side length 10.0 cm.

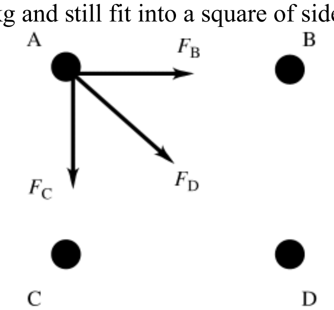

Figure 12.10

12.11. **IDENTIFY:** Use Eq.(12.2) to calculate the gravitational force each particle exerts on the third mass. The equilibrium is stable when for a displacement from equilibrium the net force is directed toward the equilibrium position and it is unstable when the net force is directed away from the equilibrium position.

SET UP: For the net force to be zero, the two forces on M must be in opposite directions. This is the case only when M is on the line connecting the two particles and between them. The free-body diagram for M is given in Figure 12.11. $m_1 = 3m$ and $m_2 = m$. If M is a distance x from m_1, it is a distance $1.00\ \text{m} - x$ from m_2.

EXECUTE: **(a)** $F_x = F_{1x} + F_{2x} = -G\dfrac{3mm}{x^2} + G\dfrac{mM}{(1.00\ \text{m}-x)^2} = 0$. $3(1.00\ \text{m}-x)^2 = x^2$. $1.00\ \text{m} - x = \pm x/\sqrt{3}$. Since M is between the two particles, x must be less than 1.00 m and $x = \dfrac{1.00\ \text{m}}{1+1/\sqrt{3}} = 0.634$ m. M must be placed at a point that is 0.634 m from the particle of mass $3m$ and 0.366 m from the particle of mass m.

(b) (i) If M is displaced slightly to the right in Figure 12.11, the attractive force from m is larger than the force from $3m$ and the net force is to the right. If M is displaced slightly to the left in Figure 12.11, the attractive force from $3m$ is larger than the force from m and the net force is to the left. In each case the net force is away from equilibrium and the equilibrium is unstable.

(ii) If M is displaced a very small distance along the y axis in Figure 12.11, the net force is directed opposite to the direction of the displacement and therefore the equilibrium is stable.

EVALUATE: The point where the net force on M is zero is closer to the smaller mass.

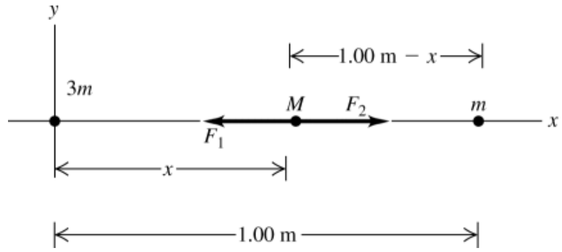

Figure 12.11

12.12. **IDENTIFY:** The force $\vec{F}_1$ exerted by m on M and the force $\vec{F}_2$ exerted by $2m$ on M are each given by Eq.(12.2) and the net force is the vector sum of these two forces.

SET UP: Each force is attractive. The forces on M in each region are sketched in Figure 12.12a. Let M be at coordinate x on the x-axis.

EXECUTE: **(a)** For the net force to be zero, $\vec{F}_1$ and $\vec{F}_2$ must be in opposite directions and this is the case only for $0 < x < L$. $\vec{F}_1 + \vec{F}_2 = 0$ then requires $F_1 = F_2$. $\dfrac{GmM}{x^2} = \dfrac{G(2m)M}{(L-x)^2}$. $2x^2 = (L-x)^2$ and $L - x = \pm\sqrt{2}x$. x must be less than L, so $x = \dfrac{L}{1+\sqrt{2}} = 0.414L$.

(b) For $x < 0$, $F_x > 0$. $F_x \to 0$ as $x \to -\infty$ and $F_x \to +\infty$ as $x \to 0$. For $x > L$, $F_x < 0$. $F_x \to 0$ as $x \to \infty$ and $F_x \to -\infty$ as $x \to L$. For $0 < x < 0.414L$, $F_x < 0$ and F_x increases from $-\infty$ to 0 as x goes from 0 to $0.414L$. For $0.414L < x < L$, $F_x > 0$ and F_x increases from 0 to $+\infty$ as x goes from $0.414L$ to L. The graph of F_x versus x is sketched in Figure 12.12b.

EVALUATE: Any real object is not exactly a point so it is not possible to have both m and M exactly at $x = 0$ or $2m$ and M both exactly at $x = L$. But the magnitude of the gravitational force between two objects approaches infinity as the objects get very close together.

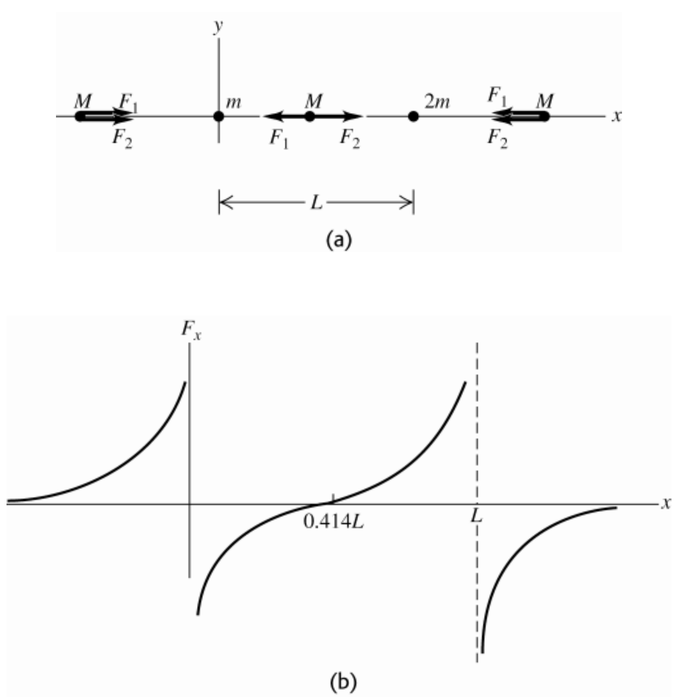

Figure 12.12

12.13. **IDENTIFY:** Use Eq.(12.1) to find the force exerted by each large sphere. Add these forces as vectors to get the net force and then use Newton's 2nd law to calculate the acceleration.

SET UP: The forces are shown in Figure 12.13.

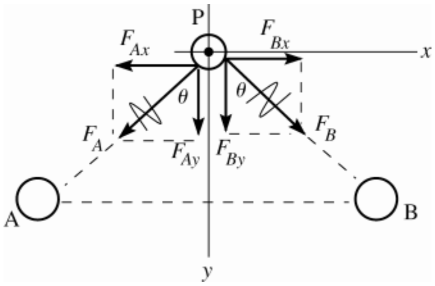

Figure 12.13

$\sin\theta = 0.80$
$\cos\theta = 0.60$
Take the origin of coordinate at point P.

EXECUTE: $F_A = G\dfrac{m_A m}{r^2} = G\dfrac{(0.26\text{ kg})(0.010\text{ kg})}{(0.100\text{ m})^2} = 1.735\times10^{-11}\text{ N}$

$F_B = G\dfrac{m_B m}{r^2} = 1.735\times10^{-11}\text{ N}$

$F_{Ax} = -F_A\sin\theta = -(1.735\times10^{-11}\text{ N})(0.80) = -1.39\times10^{-11}\text{ N}$

$F_{Ay} = -F_A\cos\theta = +(1.735\times10^{-11}\text{N})(0.60) = +1.04\times10^{-11}\text{ N}$

$F_{Bx} = +F_B\sin\theta = +1.39\times10^{-11}\text{ N}$

$F_{By} = +F_B\cos\theta = +1.04\times10^{-11}\text{ N}$

$\sum F_x = ma_x$ gives $F_{Ax} + F_{Bx} = ma_x$

$0 = ma_x$ so $a_x = 0$

$\sum F_y = ma_y$ gives $F_{Ay} + F_{By} = ma_y$

$2(1.04\times10^{-11}\text{ N}) = (0.010\text{ kg})a_y$

$a_y = 2.1\times10^{-9}\text{ m/s}^2$, directed downward midway between A and B

EVALUATE: For ordinary size objects the gravitational force is very small, so the initial acceleration is very small. By symmetry there is no x-component of net force and the y-component is in the direction of the two large spheres, since they attract the small sphere.

12.14. **IDENTIFY:** Apply Eq.(12.4) to Pluto.

SET UP: Pluto has mass $m = 1.5\times10^{22}$ kg and radius $R = 1.15\times10^6$ m.

EXECUTE: Equation (12.4) gives $g = \dfrac{\left(6.763\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.5\times10^{22}\text{kg}\right)}{\left(1.15\times10^6\text{m}\right)^2} = 0.757\text{ m}^2/\text{s}^2.$

EVALUATE: g at the surface of Pluto is much less than g at the surface of Earth. Eq.(12.4) applies to any spherically symmetric object.

12.15. **IDENTIFY:** $F_g = G\dfrac{mm_E}{r^2}$, so $a_g = G\dfrac{m_E}{r^2}$, where r is the distance of the object from the center of the earth.

SET UP: $r = h + R_E$, where h is the distance of the object above the surface of the earth and $R_E = 6.38\times10^6$ m is the radius of the earth.

EXECUTE: To decrease the acceleration due to gravity by one-tenth, the distance from the center of the earth must be increased by a factor of $\sqrt{10}$, and so the distance above the surface of the earth is

$\left(\sqrt{10}-1\right)R_E = 1.38\times10^7\text{m}.$

EVALUATE: This height is about twice the radius of the earth.

12.16. **IDENTIFY:** Apply Eq.(12.4) to the earth and to Venus. $w = mg$.

SET UP: $g = \dfrac{Gm_E}{R_E^2} = 9.80\text{ m/s}^2$. $m_V = 0.815m_E$ and $R_V = 0.949R_E$. $w_E = mg_E = 75.0$ N.

EXECUTE: **(a)** $g_V = \dfrac{Gm_V}{R_V^2} = \dfrac{G(0.815m_E)}{(0.949R_E)^2} = 0.905\dfrac{Gm_E}{R_E^2} = 0.905g_E$.

(b) $w_V = mg_V = 0.905mg_E = (0.905)(75.0\text{ N}) = 67.9$ N.

EVALUATE: The mass of the rock is independent of its location but its weight equals the gravitational force on it and that depends on its location.

12.17. **(a) IDENTIFY and SET UP:** Apply Eq.(12.4) to the earth and to Titania. The acceleration due to gravity at the surface of Titania is given by $g_T = Gm_T/R_T^2$, where m_T is its mass and R_T is its radius.

For the earth, $g_E = Gm_E/R_E^2$.

EXECUTE: For Titania, $m_T = m_E/1700$ and $R_T = R_E/8$, so $g_T = \dfrac{Gm_T}{R_T^2} = \dfrac{G(m_E/1700)}{(R_E/8)^2} = \left(\dfrac{64}{1700}\right)\dfrac{Gm_E}{R_E^2} = 0.0377 g_E$.

Since $g_E = 9.80\ \text{m/s}^2$, $g_T = (0.0377)(9.80\ \text{m/s}^2) = 0.37\ \text{m/s}^2$.

EVALUATE: g on Titania is much smaller than on earth. The smaller mass reduces g and is a greater effect than the smaller radius, which increases g.

(b) IDENTIFY and SET UP: Use density = mass/volume. Assume Titania is a sphere.

EXECUTE: From Section 12.2 we know that the average density of the earth is $5500\ \text{kg/m}^3$. For Titania

$$\rho_T = \frac{m_T}{\frac{4}{3}\pi R_T^3} = \frac{m_E/1700}{\frac{4}{3}\pi (R_E/8)^3} = \frac{512}{1700}\rho_E = \frac{512}{1700}(5500\ \text{kg/m}^3) = 1700\ \text{kg/m}^3$$

EVALUATE: The average density of Titania is about a factor of 3 smaller than for earth. We can write Eq.(12.4) for Titania as $g_T = \frac{4}{3}\pi G R_T \rho_T$. $g_T < g_E$ both because $\rho_T < \rho_E$ and $R_T < R_E$.

12.18. **IDENTIFY:** Apply Eq.(12.4) to Rhea.

SET UP: $\rho = m/V$. The volume of a sphere is $V = \frac{4}{3}\pi R^3$.

EXECUTE: $M = \dfrac{gR^2}{G} = 2.44\times10^{21}\ \text{kg}$ and $\rho = \dfrac{M}{(4\pi/3)R^3} = 1.30\times10^3\ \text{kg/m}^3$.

EVALUATE: The average density of Rhea is about one-fourth that of the earth.

12.19. **IDENTIFY:** Apply Eq.(12.2) to the astronaut.

SET UP: $m_E = 5.97\times10^{24}\ \text{kg}$ and $R_E = 6.38\times10^6\ \text{m}$.

EXECUTE: $F_g = G\dfrac{mm_E}{r^2}$. $r = 600\times10^3\ \text{m} + R_E$ so $F_g = 610\ \text{N}$. At the surface of the earth, $w = mg = 735\ \text{N}$. The gravity force is not zero in orbit. The satellite and the astronaut have the same acceleration so the astronaut's apparent weight is zero.

EVALUATE: In Eq.(12.2), r is the distance of the object from the center of the earth.

12.20. **IDENTIFY:** $g_n = G\dfrac{m_n}{R_n^2}$, where the subscript n refers to the neutron star. $w = mg$.

SET UP: $R_n = 10.0\times10^3\ \text{m}$. $m_n = 1.99\times10^{30}\ \text{kg}$. Your mass is $m = \dfrac{w}{g} = \dfrac{675\ \text{N}}{9.80\ \text{m/s}^2} = 68.9\ \text{kg}$.

EXECUTE: $g_n = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{1.99\times10^{30}\ \text{kg}}{(10.0\times10^3\ \text{m})^2} = 1.33\times10^{12}\ \text{m/s}^2$

Your weight on the neutron star would be $w_n = mg_n = (68.9\ \text{kg})(1.33\times10^{12}\ \text{m/s}^2) = 9.16\times10^{13}\ \text{N}$.

EVALUATE: Since R_n is much less than the radius of the sun, the gravitational force exerted by the neutron star on an object at its surface is immense.

12.21. **IDENTIFY and SET UP:** Use the measured gravitational force to calculate the gravitational constant G, using Eq.(12.1). Then use Eq.(12.4) to calculate the mass of the earth:

EXECUTE: $F_g = G\dfrac{m_1 m_2}{r^2}$ so $G = \dfrac{F_g r^2}{m_1 m_2} = \dfrac{(8.00\times10^{-10}\ \text{N})(0.0100\ \text{m})^2}{(0.400\ \text{kg})(3.00\times10^{-3}\ \text{kg})} = 6.667\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

$g = \dfrac{Gm_E}{R_E^2}$ gives $m_E = \dfrac{R_E^2 g}{G} = \dfrac{(6.38\times10^6\ \text{m})^2(9.80\ \text{m/s}^2)}{6.667\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 5.98\times10^{24}\ \text{kg}$.

EVALUATE: Our result agrees with the value given in Appendix F.

12.22. **IDENTIFY:** Use Eq.(12.4) to calculate g for Europa. The acceleration of a particle moving in a circular path is $a_{rad} = r\omega^2$.

SET UP: In $a_{rad} = r\omega^2$, ω must be in rad/s. For Europa, $R = 1.569\times10^6\ \text{m}$.

EXECUTE: $g = \dfrac{Gm}{R^2} = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(4.8\times10^{22}\ \text{kg})}{(1.569\times10^6\ \text{m})^2} = 1.30\ \text{m/s}^2$. $g = a_{\text{rad}}$ gives

$\omega = \sqrt{\dfrac{g}{r}} = \sqrt{\dfrac{1.30\ \text{m/s}^2}{4.25\ \text{m}}} = (0.553\ \text{rad/s})\left(\dfrac{60\ \text{s}}{1\ \text{min}}\right)\left(\dfrac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = 5.28\ \text{rpm}$.

EVALUATE: The radius of Europa is about one-fourth that of the earth and its mass is about one-hundredth that of earth, so g on Europa is much less than g on earth. The lander would have some spatial extent so different points on it would be different distances from the rotation axis and a_{rad} would have different values. For the ω we calculated, $a_{\text{rad}} = g$ at a point that is precisely 4.25 m from the rotation axis.

12.23. **IDENTIFY** and **SET UP:** Example 12.5 gives the escape speed as $v_1 = \sqrt{2GM/R}$, where M and R are the mass and radius of the astronomical object.

EXECUTE: $v_1 = \sqrt{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(3.6\times10^{12}\ \text{kg})/700\ \text{m}} = 0.83\ \text{m/s}.$

EVALUATE: At this speed a person can walk 100 m in 120 s; easily achieved for the average person. We can write the escape speed as $v_1 = \sqrt{\tfrac{4}{3}\pi\rho G R^2}$, where ρ is the average density of Dactyl. Its radius is much smaller than earth's and its density is about the same, so the escape speed is much less on Dactyl than on earth.

12.24. **IDENTIFY:** In part (a) use the expression for the escape speed that is derived in Example 12.5. In part (b) apply conservation of energy.

SET UP: $R = 4.5\times10^3\ \text{m}$. In part (b) let point 1 be at the surface of the comet.

EXECUTE: **(a)** The escape speed is $v = \sqrt{\dfrac{2GM}{R}}$ so $M = \dfrac{Rv^2}{2G} = \dfrac{(4.5\times10^3\ \text{m})(1.0\ \text{m/s})^2}{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 3.37\times10^{13}\ \text{kg}$.

(b) (i) $K_1 = \tfrac{1}{2}mv_1^2$. $K_2 = 0.100K_1$. $U_1 = -\dfrac{GMm}{R}$; $U_2 = -\dfrac{GMm}{r}$. $K_1 + U_1 = K_2 + U_2$ gives

$\tfrac{1}{2}mv_1^2 - \dfrac{GMm}{R} = (0.100)(\tfrac{1}{2}mv_1^2) - \dfrac{GMm}{r}$. Solving for r gives

$\dfrac{1}{r} = \dfrac{1}{R} - \dfrac{0.450v_1^2}{GM} = \dfrac{1}{4.5\times10^3\ \text{m}} - \dfrac{0.450(1.0\ \text{m/s})^2}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(3.37\times10^{13}\ \text{kg})}$ and $r = 45\ \text{km}$. (ii) The debris never loses all of its initial kinetic energy, but $K_2 \to 0$ as $r \to \infty$. The farther the debris are from the comet's center, the smaller is their kinetic energy.

EVALUATE: The debris will have lost 90.0% of their initial kinetic energy when they are at a distance from the comet's center of about ten times the radius of the comet.

12.25. **IDENTIFY:** The escape speed, from the results of Example 12.5, is $\sqrt{2GM/R}$.

SET UP: For Mars, $M = 6.42\times10^{23}\ \text{kg}$ and $R = 3.40\times10^6\ \text{m}$. For Jupiter, $M = 1.90\times10^{27}\ \text{kg}$ and $R = 6.91\times10^7\ \text{m}$.

EXECUTE: **(a)** $v = \sqrt{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.42\times10^{23}\ \text{kg})/(3.40\times10^6\ \text{m})} = 5.02\times10^3\ \text{m/s}.$

(b) $v = \sqrt{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2(1.90\times10^{27}\ \text{kg})/(6.91\times10^7\ \text{m})} = 6.06\times10^4\ \text{m/s}.$

(c) Both the kinetic energy and the gravitational potential energy are proportional to the mass of the spacecraft.

EVALUATE: Example 12.5 calculates the escape speed for earth to be $1.12\times10^4\ \text{m/s}$. This is larger than our result for Mars and less than our result for Jupiter.

12.26. **IDENTIFY:** The kinetic energy is $K = \tfrac{1}{2}mv^2$ and the potential energy is $U = -\dfrac{GMm}{r}$

SET UP: The mass of the earth is $M_{\text{E}} = 5.97\times10^{24}\ \text{kg}$.

EXECUTE: **(a)** $K = \tfrac{1}{2}(629\ \text{kg})(3.33\times10^3\ \text{m/s})^2 = 3.49\times10^9\ \text{J}$

(b) $U = -\dfrac{GM_{\text{E}}m}{r} = -\dfrac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})(629\ \text{kg})}{2.87\times10^9\ \text{m}} = -8.73\times10^7\ \text{J}$.

EVALUATE: The total energy $K + U$ is positive.

12.27. **IDENTIFY:** Apply Newton's 2nd law to the motion of the satellite and obtain and equation that relates the orbital speed v to the orbital radius r.

SET UP: The distances are shown in Figure 12.27a.

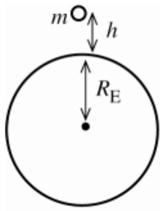

Figure 12.27a

The radius of the orbit is $r = h + R_E$.

$r = 7.80\times10^5\ \text{m} + 6.38\times10^6\ \text{m} = 7.16\times10^6\ \text{m}$.

The free-body diagram for the satellite is given in Figure 12.27b.

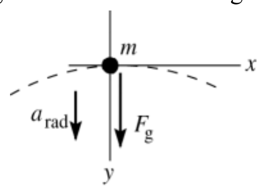

Figure 12.27b

(a) EXECUTE: $\sum F_y = ma_y$

$F_g = ma_{rad}$

$G\dfrac{mm_E}{r^2} = m\dfrac{v^2}{r}$

$$v = \sqrt{\frac{Gm_E}{r}} = \sqrt{\frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{7.16\times10^6\ \text{m}}} = 7.46\times10^3\ \text{m/s}$$

(b) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi(7.16\times10^6\ \text{m})}{7.46\times10^3\ \text{m/s}} = 6030\ \text{s} = 1.68\ \text{h}.$

EVALUATE: Note that $r = h + R_E$ is the radius of the orbit, measured from the center of the earth. For this satellite r is greater than for the satellite in Example 12.6, so its orbital speed is less.

12.28. **IDENTIFY:** The time to complete one orbit is the period T, given by Eq.(12.12). The speed v of the satellite is given by $v = \dfrac{2\pi r}{T}$.

SET UP: If h is the height of the orbit above the earth's surface, the radius of the orbit is $r = h + R_E$. $R_E = 6.38\times10^6\ \text{m}$ and $m_E = 5.97\times10^{24}\ \text{kg}$.

EXECUTE: **(a)** $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}} = \dfrac{2\pi(7.05\times10^5\ \text{m} + 6.38\times10^6\ \text{m})^{3/2}}{\sqrt{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}} = 5.94\times10^3\ \text{s} = 99.0\ \text{min}$

(b) $v = \dfrac{2\pi(7.05\times10^5\ \text{m} + 6.38\times10^6\ \text{m})}{5.94\times10^3\ \text{s}} = 7.49\times10^3\ \text{m/s} = 7.49\ \text{km/s}$

EVALUATE: The satellite in Example 12.6 is at a lower altitude and therefore has a smaller orbit radius than the satellite in this problem. Therefore, the satellite in this problem has a larger period and a smaller orbital speed. But a large percentage change in h corresponds to a small percentage change in r and the values of T and v for the two satellites do not differ very much.

12.29. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the motion of the earth around the sun.

SET UP: For the earth, $T = 365.3\ \text{days} = 3.156\times10^7\ \text{s}$ and $r = 1.50\times10^{11}\ \text{m}$. $T = \dfrac{2\pi r}{v}$.

EXECUTE: $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(1.50\times10^{11}\ \text{m})}{3.156\times10^7\ \text{s}} = 2.99\times10^4\ \text{s}$. $F_g = ma_{rad}$ gives $G\dfrac{m_E m_S}{r^2} = m_E\dfrac{v^2}{r}$.

$$m_S = \frac{v^2 r}{G} = \frac{(2.99\times10^4\ \text{s})^2(1.50\times10^{11}\ \text{m})}{6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 2.01\times10^{30}\ \text{kg}$$

EVALUATE: Appendix F gives $m_S = 1.99\times10^{30}\ \text{kg}$, in good agreement with our calculation.

12.30. **IDENTIFY:** We can calculate the orbital period T from the number of revolutions per day. Then the period and the orbit radius are related by Eq.(12.12).

SET UP: $m_E = 5.97\times10^{24}\ \text{kg}$ and $R_E = 6.38\times10^6\ \text{m}$. The height h of the orbit above the surface of the earth is related to the orbit radius r by $r = h + R_E$. $1\ \text{day} = 8.64\times10^4\ \text{s}$.

EXECUTE: The satellite moves 15.65 revolutions in 8.64×10^4 s, so the time for 1.00 revolution is

$T=\dfrac{8.64\times10^4\text{s}}{15.65}=5.52\times10^3$ s. $T=\dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ gives

$r=\left(\dfrac{Gm_ET^2}{4\pi^2}\right)^{1/3}=\left(\dfrac{[6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2][5.97\times10^{24}\ \text{kg}][5.52\times10^3\text{s}]^2}{4\pi^2}\right)^{1/3}$. $r=6.75\times10^6$ m and

$h=r-R_E=3.7\times10^5\ \text{m}=370\ \text{km}$.

EVALUATE: The period of this satellite is slightly larger than the period for the satellite in Example 12.6 and the altitude of this satellite is therefore somewhat greater.

12.31. IDENTIFY: Apply $\sum\vec{F}=m\vec{a}$ to the motion of the baseball. $v=\dfrac{2\pi r}{T}$.

SET UP: $r_D=6\times10^3$ m.

EXECUTE: **(a)** $F_g=ma_{rad}$ gives $G\dfrac{m_Dm}{r_D^2}=m\dfrac{v^2}{r_D}$. $v=\sqrt{\dfrac{Gm_D}{r_D}}=\sqrt{\dfrac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{15}\ \text{kg})}{6\times10^3\ \text{m}}}=4.7$ m/s

$4.7\ \text{m/s}=11$ mph, which is easy to achieve.

(b) $T=\dfrac{2\pi r}{v}=\dfrac{2\pi(6\times10^3\ \text{m})}{4.7\ \text{m/s}}=8020\ \text{s}=134\ \text{min}$. The game would last a long time.

EVALUATE: The speed v is relative to the center of Deimos. The baseball would already have some speed before we throw it, because of the rotational motion of Deimos.

12.32. IDENTIFY: $T=\dfrac{2\pi r}{v}$ and $F_g=ma_{rad}$.

SET UP: The sun has mass $m_S=1.99\times10^{30}$ kg. The radius of Mercury's orbit is 5.79×10^{10} m, so the radius of Vulcan's orbit is 3.86×10^{10} m.

EXECUTE: $F_g=ma_{rad}$ gives $G\dfrac{m_Sm}{r^2}=m\dfrac{v^2}{r}$ and $v^2=\dfrac{Gm_S}{r}$.

$$T=2\pi r\sqrt{\frac{r}{Gm_S}}=\frac{2\pi r^{3/2}}{\sqrt{Gm_S}}=\frac{2\pi(3.86\times10^{10}\ \text{m})^{3/2}}{\sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}}=4.13\times10^6\ \text{s}=47.8\ \text{days}$$

EVALUATE: The orbital period of Mercury is 88.0 d, so we could calculate T for Vulcan as $T=(88.0\ \text{d})(2/3)^{3/2}=47.9$ days.

12.33. IDENTIFY: The orbital speed is given by $v=\sqrt{Gm/r}$, where m is the mass of the star. The orbital period is given by $T=\dfrac{2\pi r}{v}$.

SET UP: The sun has mass $m_S=1.99\times10^{30}$ kg. The orbit radius of the earth is 1.50×10^{11} m.

EXECUTE: **(a)** $v=\sqrt{Gm/r}$.

$v=\sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.85\times1.99\times10^{30}\ \text{kg})/((1.50\times10^{11}\ \text{m})(0.11))}=8.27\times10^4$ m/s.

(b) $2\pi r/v=1.25\times10^6\ \text{s}=14.5$ days (about two weeks).

EVALUATE: The orbital period is less than the 88 day orbital period of Mercury; this planet is orbiting very close to its star, compared to the orbital radius of Mercury.

12.34. IDENTIFY: The period of each satellite is given by Eq.(12.12). Set up a ratio involving T and r.

SET UP: $T=\dfrac{2\pi r^{3/2}}{\sqrt{Gm_p}}$ gives $\dfrac{T}{r^{3/2}}=\dfrac{2\pi}{\sqrt{Gm_p}}=$ constant, so $\dfrac{T_1}{r_1^{3/2}}=\dfrac{T_2}{r_2^{3/2}}$.

EXECUTE: $T_2=T_1\left(\dfrac{r_2}{r_1}\right)^{3/2}=(6.39\ \text{days})\left(\dfrac{48{,}000\ \text{km}}{19{,}600\ \text{km}}\right)^{3/2}=24.5$ days. For the other satellite,

$T_2=(6.39\ \text{days})\left(\dfrac{64{,}000\ \text{km}}{19{,}600\ \text{km}}\right)^{3/2}=37.7$ days.

EVALUATE: T increases when r increases.

12.35. IDENTIFY: In part (b) apply the results from part (a).

SET UP: For Pluto, $e=0.248$ and $a=5.92\times10^{12}$ m. For Neptune, $e=0.010$ and $a=4.50\times10^{12}$ m. The orbital period for Pluto is $T=247.9$ y.

EXECUTE: **(a)** The result follows directly from Figure 12.19 in the textbook.

(b) The closest distance for Pluto is $(1-0.248)(5.92\times10^{12}\text{m}) = 4.45\times10^{12}\text{m}$. The greatest distance for Neptune is $(1+0.010)(4.50\times10^{12}\text{m}) = 4.55\times10^{12}\text{m}$.

(c) The time is the orbital period of Pluto, $T = 248\text{ y}$.

EVALUATE: Pluto's closest distance calculated in part (a) is $0.10\times10^{12}\text{ m} = 1.0\times10^{8}\text{ km}$, so Pluto is about 100 million km closer to the sun than Neptune, as is stated in the problem. The eccentricity of Neptune's orbit is small, so its distance from the sun is approximately constant.

12.36. **IDENTIFY:** $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{star}}}}$, where m_{star} is the mass of the star. $v = \dfrac{2\pi r}{T}$.

SET UP: $3.09\text{ days} = 2.67\times10^{5}\text{ s}$. The orbit radius of Mercury is $5.79\times10^{10}\text{ m}$. The mass of our sun is $1.99\times10^{30}\text{ kg}$.

EXECUTE: **(a)** $T = 2.67\times10^{5}\text{ s}$. $r = (5.79\times10^{10}\text{ m})/9 = 6.43\times10^{9}\text{ m}$. $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{star}}}}$ gives

$$m_{\text{star}} = \frac{4\pi^2 r^3}{T^2 G} = \frac{4\pi^2(6.43\times10^{9}\text{ m})^3}{(2.67\times10^{5}\text{ s})^2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)} = 2.21\times10^{30}\text{ kg}.\quad \frac{m_{\text{star}}}{m_{\text{sun}}} = 1.11\text{, so } m_{\text{star}} = 1.11m_{\text{sun}}.$$

(b) $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(6.43\times10^{9}\text{ m})}{2.67\times10^{5}\text{ s}} = 1.51\times10^{5}\text{ m/s}$

EVALUATE: The orbital period of Mercury is 88.0 d. The period for this planet is much less primarily because the orbit radius is much less and also because the mass of the star is greater than the mass of our sun.

12.37. **(a) IDENTIFY:** If the orbit is circular, Newton’s 2nd law requires a particular relation between its orbit radius and orbital speed.

SET UP: The gravitational force exedrted on the spacecraft by the sun is $F_{\text{g}} = Gm_{\text{S}}m_{\text{H}}/r^2,$ where m_{S} is the mass of the sun and m_{H} is the mass of the Helios B spacecraft.

For a circular orbit, $a_{\text{rad}} = v^2/r$ and $\sum F = m_{\text{H}}v^2/r.$ If we neglect all forces on the spacecraft except for the force exerted by the sun, $F_{\text{g}} = \sum F = m_{\text{H}}v^2/r,$ so $Gm_{\text{S}}m_{\text{H}}/r^2 = m_{\text{H}}v^2/r$

EXECUTE: $v = \sqrt{Gm_{\text{S}}/r} = \sqrt{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})/43\times10^{9}\text{ m}} = 5.6\times10^{4}\text{ m/s} = 56\text{ km/s}$

EVALUATE: The actual speed is 71 km/s, so the orbit cannot be circular.

(b) IDENTIFY and **SET UP:** The orbit is a circle or an ellipse if it is closed, a parabola or hyperbola if open. The orbit is closed if the total energy (kinetic + potential) is negative, so that the object cannot reach $r\to\infty.$

EXECUTE: For Helios B,

$K = \tfrac{1}{2}m_{\text{H}}v^2 = \tfrac{1}{2}m_{\text{H}}(71\times10^{3}\text{ m/s})^2 = (2.52\times10^{9}\text{m}^2/\text{s}^2)m_{\text{H}}$

$U = -Gm_{\text{S}}m_{\text{H}}/r = m_{\text{H}}(-(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})/(43\times10^{9}\text{ m})) = -(3.09\times10^{9}\text{ m}^2/\text{s}^2)m_{\text{H}}$

$E = K + U = (2.52\times10^{9}\text{ m}^2/\text{s}^2)m_{\text{H}} - (3.09\times10^{9}\text{ m}^2/\text{s}^2)m_{\text{H}} = -(5.7\times10^{8}\text{ m}^2/\text{s}^2)m_{\text{H}}$

EVALUATE: The total energy *E* is negative, so the orbit is closed. We know from part (a) that it is not circular, so it must be elliptical.

12.38. **IDENTIFY:** Section 12.6 states that for a point mass outside a spherical shell the gravitational force is the same as if all the mass of the shell were concentrated at its center. It also states that for a point inside a spherical shell the force is zero.

SET UP: For $r = 5.01\text{ m}$ the point mass is outside the shell and for $r = 4.99\text{ m}$ and $r = 2.12\text{ m}$ the point mass is inside the shell.

EXECUTE: **(a)** (i) $F_{\text{g}} = \dfrac{Gm_1m_2}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(1000.0\text{ kg})(2.00\text{ kg})}{(5.01\text{ m})^2} = 5.31\times10^{-9}\text{ N}$. (ii) $F_{\text{g}} = 0$. (iii) $F_{\text{g}} = 0$.

(b) For $r < 5.00\text{ m}$ the force is zero and for $r > 5.00\text{ m}$ the force is proportional to $1/r^2$. The graph of F_{g} versus *r* is sketched in Figure 12.38.

EVALUATE: Inside the shell the gravitational potential energy is constant and the force on a point mass inside the shell is zero.

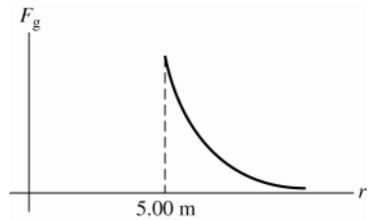

Figure 12.38

12.39. **IDENTIFY:** Section 12.6 states that for a point mass outside a uniform sphere the gravitational force is the same as if all the mass of the sphere were concentrated at its center. It also states that for a point mass a distance r from the center of a uniform sphere, where r is less than the radius of the sphere, the gravitational force on the point mass is the same as though we removed all the mass at points farther than r from the center and concentrated all the remaining mass at the center.

SET UP: The density of the sphere is $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$, where M is the mass of the sphere and R is its radius. The mass inside a volume of radius $r < R$ is $M_r = \rho V_r = \left(\dfrac{M}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = M\left(\dfrac{r}{R}\right)^3$. $r = 5.01$ m is outside the sphere and $r = 2.50$ m is inside the sphere.

EXECUTE: **(a)** (i) $F_g = \dfrac{GMm}{r^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(1000.0\ \text{kg})(2.00\ \text{kg})}{(5.01\ \text{m})^2} = 5.31\times10^{-9}\ \text{N}$.

(ii) $F_g = \dfrac{GM'm}{r^2}$. $M' = M\left(\dfrac{r}{R}\right)^3 = (1000.0\ \text{kg})\left(\dfrac{2.50\ \text{m}}{5.00\ \text{m}}\right)^3 = 125\ \text{kg}$.

$F_g = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(125\ \text{kg})(2.00\ \text{kg})}{(2.50\ \text{m})^2} = 2.67\times10^{-9}\ \text{N}$.

(b) $F_g = \dfrac{GM(r/R)^3 m}{r^2} = \left(\dfrac{GMm}{R^3}\right) r$ for $r < R$ and $F_g = \dfrac{GMm}{r^2}$ for $r > R$. The graph of F_g versus r is sketched in Figure 12.39.

EVALUATE: At points outside the sphere the force on a point mass is the same as for a shell of the same mass and radius. For $r < R$ the force is different in the two cases of uniform sphere versus hollow shell.

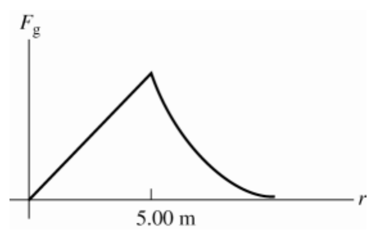

Figure 12.39

12.40. **IDENTIFY:** The gravitational potential energy of a point of point masses is $U = -G\dfrac{m_1 m_2}{r}$. Divide the rod into infinitesimal pieces and integrate to find U.

SET UP: Divide the rod into differential masses dm at position l, measured from the right end of the rod. $dm = dl(M/L)$.

EXECUTE: **(a)** $U = -\dfrac{Gm\ dm}{l+x} = -\dfrac{GmM}{L}\dfrac{dl}{l+x}$.

Integrating, $U - \dfrac{GmM}{L}\displaystyle\int_0^L \dfrac{dl}{l+x} = -\dfrac{GmM}{L}\ln\left(1+\dfrac{L}{x}\right)$. For $x >> L$, the natural logarithm is $\sim(L/x)$, and $U \to -GmM/x$.

(b) The x-component of the gravitational force on the sphere is $F_x = -\frac{\partial U}{\partial x} = \frac{GmM}{L}\frac{(-L/x^2)}{(1+(L/x))} = -\frac{GmM}{(x^2+Lx)}$, with the minus sign indicating an attractive force. As $x >> L$, the denominator in the above expression approaches x^2, and $F_x \to -GmM/x^2$, as expected.

EVALUATE: When x is much larger than L the rod can be treated as a point mass, and our results for U and F_x do reduce to the correct expression when $x >> L$.

12.41. IDENTIFY: Find the potential due to a small segment of the ring and integrate over the entire ring to find the total U.

(a) SET UP:

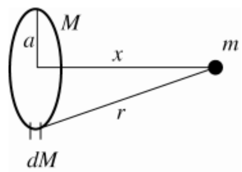

Figure 12.41

Divide the ring up into small segments dM, as indicated in Figure 12.41.

EXECUTE: The gravitational potential energy of dM and m is $dU = -GmdM/r$.

The total gravitational potential energy of the ring and particle is $U = \int dU = -Gm\int dM/r$.

But $r = \sqrt{x^2+a^2}$ is the same for all segments of the ring, so

$$U = -\frac{Gm}{r}\int dM = -\frac{GmM}{r} = -\frac{GmM}{\sqrt{x^2+a^2}}$$

(b) EVALUATE: When $x >> a$, $\sqrt{x^2+a^2} \to \sqrt{x^2} = x$ and $U = -GmM/x$. This is the gravitational potential energy of two point masses separated by a distance x. This is the expected result.

(c) IDENTIFY and SET UP: Use $F_x = -dU/dx$ with $U(x)$ from part (a) to calculate F_x.

EXECUTE: $F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left(-\frac{GmM}{\sqrt{x^2+a^2}}\right)$

$$F_x = +GmM\frac{d}{dx}(x^2+a^2)^{-1/2} = GmM(-\frac{1}{2}(2x)(x^2+a^2)^{-3/2})$$

$F_x = -GmMx/(x^2+a^2)^{3/2}$; the minus sign means the force is attractive.

EVALUATE: **(d)** For $x >> a$, $(x^2+a^2)^{3/2} \to (x^2)^{3/2} = x^3$

Then $F_x = -GmMx/x^3 = -GmM/x^2$. This is the force between two point masses separated by a distance x and is the expected result.

(e) For $x = 0$, $U = -GMm/a$. Each small segment of the ring is the same distance from the center and the potential is the same as that due to a point charge of mass M located at a distance a.

For $x = 0$, $F_x = 0$. When the particle is at the center of the ring, symmetrically placed segments of the ring exert equal and opposite forces and the total force exerted by the ring is zero.

12.42. IDENTIFY: At the equator the object has inward acceleration $\frac{v^2}{R_E}$ and the reading w of the balance is related to the true weight w_0 (the gravitational force exerted by the earth) by $w_0 - w = \frac{mv^2}{R_E}$. At the North Pole, $a_{rad} = 0$ and $w = w_0$.

SET UP: As shown in Section 12.7, $v = 465$ m/s. $R_E = 6.38\times10^6$ m.

EXECUTE: $w_0 = 875$ N and $m = \frac{w_0}{g} = 89.29$ kg. $w = w_0 - \frac{mv^2}{R_E} = 875\text{ N} - (89.29\text{ kg})\frac{(465\text{ m/s})^2}{6.38\times10^6\text{ m}} = 872\text{ N}$

EVALUATE: The rotation of the earth causes the scale reading to be slightly less than the true weight, since there must be a net inward force on the object.

12.43. IDENTIFY and SET UP: Ate the north pole, $F_g = w_0 = mg_0$, where g_0 is given by Eq.(12.4) applied to Neptune. At the equator, the apparent weight is given by Eq.(12.28). The orbital speed v is obtained from the rotational period using Eq.(12.12).

EXECUTE: **(a)** $g_0 = Gm/R^2 = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.0\times10^{26}\ \text{kg})/(2.5\times10^{7}\ \text{m})^2 = 10.7\ \text{m/s}^2$. This agrees with the value of g given in the problem.

$F = w_0 = mg_0 = (5.0\ \text{kg})(10.7\ \text{m/s}^2) = 53\ \text{N}$; this is the true weight of the object.

(b) From Eq.(23.28), $w = w_0 - mv^2/R$

$T = \dfrac{2\pi r}{v}$ gives $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(2.5\times10^7\ \text{m})}{(16\ \text{h})(3600\ \text{s/1 h})} = 2.727\times10^3\ \text{m/s}$

$v^2/R = (2.727\times10^3\ \text{s})^2/2.5\times10^7\ \text{m} = 0.297\ \text{m/s}^2$

Then $w = 53\ \text{N} - (5.0\ \text{kg})(0.297\ \text{m/s}^2) = 52\ \text{N}$.

EVALUATE: The apparent weight is less than the true weight. This effect is larger on Neptune than on earth.

12.44. **IDENTIFY:** The radius of a black hole and its mass are related by $R_S = \dfrac{2GM}{c^2}$.

SET UP: $R_S = 0.50\times10^{-15}\ \text{m}$, $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$ and $c = 3.00\times10^8\ \text{m/s}$.

EXECUTE: $M = \dfrac{c^2R_S}{2G} = \dfrac{(3.00\times10^8\ \text{m/s})^2(0.50\times10^{-15}\ \text{m})}{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 3.4\times10^{11}\ \text{kg}$

EVALUATE: The average density of the black hole would be

$\rho = \dfrac{M}{\frac{4}{3}\pi R_S^3} = \dfrac{3.4\times10^{11}\ \text{kg}}{\frac{4}{3}\pi(0.50\times10^{-15}\ \text{m})^3} = 6.49\times10^{56}\ \text{kg/m}^3$. We can combine $\rho = \dfrac{M}{\frac{4}{3}\pi R_S^3}$ and $R_S = \dfrac{2GM}{c^2}$ to give

$\rho = \dfrac{3c^6}{32\pi G^3M^2}$. The average density of a black hole increases when its mass decreases. The average density of this mini black hole is much greater than the average density of the much more massive black hole in Example 12.11.

12.45. **IDENTIFY** and **SET UP:** A black hole with the earth's mass M has the Schwarzschild radius R_S given by Eq.(12.30).

EXECUTE: $R_S = 2GM/c^2 = 2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})/(2.998\times 10^8\ \text{m/s})^2 = 8.865\times10^{-3}\ \text{m}$

The ratio of R_S to the current radius R is $R_S/R = 8.865\times10^{-3}\ \text{m}/6.38\times10^6\ \text{m} = 1.39\times10^{-9}$.

EVALUATE: A black hole with the earth's radius is very small.

12.46. **IDENTIFY:** Apply Eq.(12.1) to calculate the gravitational force. For a black hole, the mass M and Schwarzschild radius R_S are related by Eq.(12.30).

SET UP: The speed of light is $c = 3.00\times10^8\ \text{m/s}$.

EXECUTE: **(a)** $\dfrac{GMm}{r^2} = \dfrac{\left(R_Sc^2/2\right)}{r^2} = \dfrac{mc^2R_S}{2r^2}$.

(b) $\dfrac{(5.00\ \text{kg})\left(3.00\times10^8\ \text{m/s}\right)^2\left(1.4\times10^{-2}\text{m}\right)}{2\left(3.00\times10^6\ \text{m}\right)^2} = 350\ \text{N}$.

(c) Solving Eq.(12.30) for M, $M = \dfrac{R_Sc^2}{2G} = \dfrac{\left(14.00\times10^{-3}\ \text{m}\right)\left(3.00\times10^8\ \text{m/s}\right)^2}{2\left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)} = 9.44\times10^{24}\ \text{kg}$.

EVALUATE: The mass of the black hole is about twice the mass of the earth.

12.47. **IDENTIFY:** The orbital speed for an object a distance r from an object of mass M is $v = \dfrac{GM}{r}$. The mass M of a black hole and its Schwarzschild radius R_S are related by Eq.(12.30).

SET UP: $c = 3.00\times10^8\ \text{m/s}$. $1\ \text{ly} = 9.461\times10^{15}\ \text{m}$.

EXECUTE: **(a)**

$$M = \frac{Rv^2}{G} = \frac{(7.5\ \text{ly})\left(9.461\times10^{15}\ \text{m/ly}\right)\left(200\times10^3\ \text{m/s}\right)^2}{\left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)} = 4.3\times10^{37}\ \text{kg} = 2.1\times10^7\ M_S.$$

(b) No, the object has a mass very much greater than 50 solar masses.

(c) $R_S = \dfrac{2GM}{c^2} = \dfrac{2v^2r}{c^2} = 6.32\times10^{10}\ \text{m}$, which does fit.

EVALUATE: The Schwarzschild radius of a black hole is approximately the same as the radius of Mercury's orbit around the sun.

12.48. **IDENTIFY:** The clumps orbit the black hole. Their speed, orbit radius and orbital period are related by $v = \frac{2\pi r}{T}$. Their orbit radius and period are related to the mass M of the black hole by $T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$. The radius of the black hole's event horizon is related to the mass of the black hole by $R_S = \frac{2GM}{c^2}$.

SET UP: $v = 3.00 \times 10^7$ m/s. $T = 27$ h $= 9.72 \times 10^4$ s. $c = 3.00 \times 10^8$ m/s.

EXECUTE: **(a)** $r = \frac{vT}{2\pi} = \frac{(3.00 \times 10^7 \text{ m/s})(9.72 \times 10^4 \text{ s})}{2\pi} = 4.64 \times 10^{11}$ m.

(b) $T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$ gives $M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (4.64 \times 10^{11} \text{ m})^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(9.72 \times 10^4 \text{ s})^2} = 6.26 \times 10^{36}$ kg.

(c) $R_S = \frac{2GM}{c^2} = \frac{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(6.26 \times 10^{36} \text{ kg})}{(3.00 \times 10^8 \text{ m/s})^2} = 9.28 \times 10^9$ m

EVALUATE: The black hole has a mass that is about 3×10^6 solar masses.

12.49. **IDENTIFY:** Use Eq.(12.1) to find each gravitational force. Each force is attractive. In part (b) apply conservation of energy.

SET UP: For a pair of masses m_1 and m_2 with separation r, $U = -G\frac{m_1 m_2}{r}$.

EXECUTE: **(a)** From symmetry, the net gravitational force will be in the direction 45° from the x-axis (bisecting the x and y axes), with magnitude

$$F = (6.673 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2)(0.0150 \text{ kg})\left[\frac{(2.0 \text{ kg})}{(2(0.50 \text{ m})^2)} + 2\frac{(1.0 \text{ kg})}{(0.50 \text{ m})^2} \sin 45°\right] = 9.67 \times 10^{-12} \text{ N}$$

(b) The initial displacement is so large that the initial potential may be taken to be zero. From the work-energy theorem, $\frac{1}{2}mv^2 = Gm\left[\frac{(2.0 \text{ kg})}{\sqrt{2}\ (0.50 \text{ m})} + 2\frac{(1.0 \text{ kg})}{(0.50 \text{ m})}\right]$. Canceling the factor of m and solving for v, and using the numerical values gives $v = 3.02 \times 10^{-5}$ m/s.

EVALUATE: The result in part (b) is independent of the mass of the particle. It would take the particle a long time to reach point P.

12.50. **IDENTIFY:** Use Eq.(12.1) to calculate each gravitational force and add the forces as vectors.

(a) SET UP: The locations of the masses are sketched in Figure 12.50a.

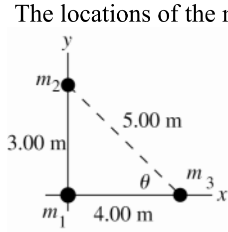

Figure 12.50a

Section 12.6 proves that any two spherically symmetric masses interact as though they were point masses with all the mass concentrated at their centers.

The force diagram for m_3 is given in Figure 12.50b

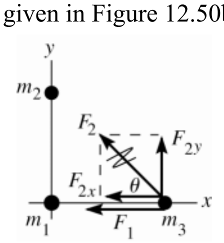

Figure 12.50b

$\cos\theta = 0.800$
$\sin\theta = 0.600$

EXECUTE: $F_1 = G\frac{m_1 m_3}{r_{13}^2} = \frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(60.0 \text{ kg})(0.500 \text{ kg})}{(4.00 \text{ m})^2} = 1.251 \times 10^{-10}$ N

$F_2 = G\frac{m_2 m_3}{r_{23}^2} = \frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(80.0 \text{ kg})(0.500 \text{ kg})}{(5.00 \text{ m})^2} = 1.068 \times 10^{-10}$ N

$F_{1x} = -1.251 \times 10^{-10}$ N, $F_{1y} = 0$

$F_{2x} = -F_2\cos\theta = -(1.068\times10^{-10}\ \text{N})(0.800) = -8.544\times10^{-11}\ \text{N}$

$F_{2y} = +F_2\sin\theta = +(1.068\times10^{-10}\ \text{N})(0.600) = +6.408\times10^{-11}\ \text{N}$

$F_x = F_{1x} + F_{2x} = -1.251\times10^{-10}\ \text{N} - 8.544\times10^{-11}\ \text{N} = -2.105\times10^{-10}\ \text{N}$

$F_y = F_{1y} + F_{2y} = 0 + 6.408\times10^{-11}\ \text{N} = +6.408\times10^{-11}\ \text{N}$

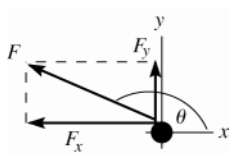

Figure 12.50c

F and its components are sketched in Figure 12.50c.

$F = \sqrt{F_x^2 + F_y^2}$

$F = \sqrt{(-2.105\times10^{-10}\ \text{N})^2 + (+6.408\times10^{-11}\ \text{N})^2}$

$F = 2.20\times10^{-10}\ \text{N}$

$\tan\theta = \dfrac{F_y}{F_x} = \dfrac{+6.408\times10^{-11}\ \text{N}}{-2.105\times10^{-10}\ \text{N}};\quad \theta = 163°$

EVALUATE: Both spheres attract the third sphere and the net force is in the second quadrant.

(b) SET UP: For the net force to be zero the forces from the two spheres must be equal in magnitude and opposite in direction. For the forces on it to be opposite in direction the third sphere must be on the y-axis and between the other two spheres. The forces on the third sphere are shown in Figure 12.50d.

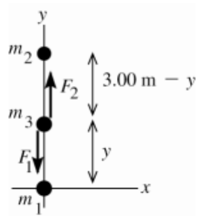

Figure 12.50d

EXECUTE: $F_{\text{net}} = 0$ if $F_1 = F_2$

$G\dfrac{m_1m_3}{y^2} = G\dfrac{m_2m_3}{(3.00\ \text{m} - y)^2}$

$\dfrac{60.0}{y^2} = \dfrac{80.0}{(3.00\ \text{m} - y)^2}$

$\sqrt{80.0}\,y = \sqrt{60.0}(3.00\ \text{m} - y)$

$(\sqrt{80.0} + \sqrt{60.0})y = (3.00\ \text{m})\sqrt{60.0}$ and $y = 1.39\ \text{m}$

Thus the sphere would have to be placed at the point $x = 0,\ \ y = 1.39\ \text{m}$

EVALUATE: For the forces to have the same magnitude the third sphere must be closer to the sphere that has smaller mass.

12.51. **IDENTIFY:** $\tau = Fr\sin\phi$. The net torque is the sum of the torques due to each force.

SET UP: From Example 12.3, using Newton's third law, the forces of the small star on each large star are $F_1 = 6.67\times10^{25}\ \text{N}$ and $F_2 = 1.33\times10^{26}\ \text{N}$. Let counterclockwise torques be positive.

EXECUTE: **(a)** The direction from the origin to the point midway between the two large stars is $\arctan(\frac{0.100\ \text{m}}{0.200\ \text{m}}) = 26.6°$, which is not the angle $(14.6°)$ found in the example.

(b) The common lever arm is 0.100 m, and the force on the upper mass is at an angle of 45° from the lever arm. The net torque is $\tau = +F_1(1.00\times10^{12}\ \text{m})\sin45° - F_2(1.00\times10^{12}\ \text{m}) = -8.58\times10^{37}\ \text{N}\cdot\text{m}$, with the minus sign indicating a clockwise torque.

EVALUATE: **(c)** There can be no net torque due to gravitational fields with respect to the center of gravity, and so the center of gravity in this case is not at the center of mass. For the center of gravity to be the same point as the center of mass, the gravity force on each mass must be proportional to the mass, with the same constant of proportionality, and that is not the case here.

12.52. **IDENTIFY:** The gravity force for each pair of objects is given by Eq.(12.1). The work done is $W = -\Delta U$.

SET UP: The simplest way to approach this problem is to find the force between the spacecraft and the center of mass of the earth-moon system, which is 4.67×10^6 m from the center of the earth. The distance from the spacecraft to the center of mass of the earth-moon system is 3.82×10^8 m (Figure 12.52). $m_{\text{E}} = 5.97\times10^{24}\ \text{kg}$, $m_{\text{M}} = 7.35\times10^{22}\ \text{kg}$.

EXECUTE: **(a)** Using the Law of Gravitation, the force on the spacecraft is 3.4 N, an angle of $0.61°$ from the earth-spacecraft line.

(b) $U = -G\dfrac{m_A m_B}{r}$. $U_2 = 0$ and $r_1 = 3.84\times10^8$ m for the spacecraft and the earth, and the spacecraft and the moon.

$W = U_2 - U_1 = +\dfrac{GMm}{r_1} = +\dfrac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg} + 7.35\times10^{22}\ \text{kg})(1250\ \text{kg})}{3.84\times10^8\ \text{m}}$. $W = -1.31\times10^9$ J.

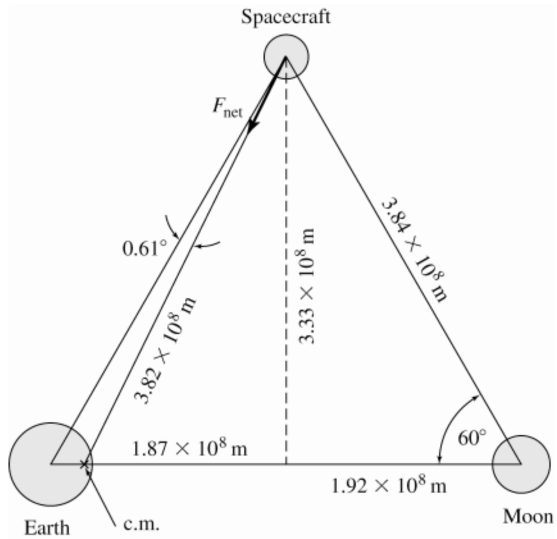

Figure 12.52

12.53. **IDENTIFY:** Apply conservation of energy and conservation of linear momentum to the motion of the two spheres.
SET UP: Denote the 25-kg sphere by a subscript 1 and the 100-kg sphere by a subscript 2.
EXECUTE: **(a)** Linear momentum is conserved because we are ignoring all other forces, that is, the net external force on the system is zero. Hence, $m_1 v_1 = m_2 v_2$.

(b) From the work-energy theorem in the form $K_\text{i} + U_\text{i} = K_\text{f} + U_\text{f}$, with the initial kinetic energy $K_\text{i} = 0$ and

$U = -G\dfrac{m_1 m_2}{r}$, $Gm_1 m_2\left[\dfrac{1}{r_\text{f}} - \dfrac{1}{r_\text{i}}\right] = \dfrac{1}{2}\left(m_1 v_1^2 + m_2 v_2^2\right)$. Using the conservation of momentum relation $m_1 v_1 = m_2 v_2$ to

eliminate v_2 in favor of v_1 and simplifying yields $v_1^2 = \dfrac{2Gm_2^2}{m_1 + m_2}\left[\dfrac{1}{r_\text{f}} - \dfrac{1}{r_\text{i}}\right]$, with a similar expression for v_2.

Substitution of numerical values gives $v_1 = 1.63\times10^{-5}$ m/s, $v_2 = 4.08\times10^{-6}$ m/s. The magnitude of the relative velocity is the sum of the speeds, 2.04×10^{-5} m/s.

(c) The distance the centers of the spheres travel $(x_1 \text{ and } x_2)$ is proportional to their acceleration, and

$\dfrac{x_1}{x_2} = \dfrac{a_1}{a_2} = \dfrac{m_2}{m_1}$, or $x_1 = 4x_2$. When the spheres finally make contact, their centers will be a distance of

$2r$ apart, or $x_1 + x_2 + 2r = 40$ m, or $x_2 + 4x_2 + 2r = 40$ m. Thus, $x_2 = 8\ \text{m} - 0.4r$, and $x_1 = 32\ \text{m} - 1.6r$. The point of contact of the surfaces is $32\ \text{m} - 0.6r = 31.9$ m from the initial position of the center of the 25.0 kg sphere.

EVALUATE: The result $x_1/x_2 = 4$ can also be obtained from the conservation of momentum result that $\dfrac{v_1}{v_2} = \dfrac{m_2}{m_1}$, at every point in the motion.

12.54. **IDENTIFY:** Apply Eq.(12.12).
SET UP: $m_\text{E} = 5.97\times10^{24}$ kg

EXECUTE: Solving Eq. (12.14) for *R*, $R^3 = Gm_\text{E}\left(\dfrac{T}{2\pi}\right)^2$.

$R = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})\left(\dfrac{(27.3\ \text{d})(86,400\ \text{s/d})}{2\pi}\right)^2 = 5.614\times10^{25}\ \text{m}^3$,

from which $r = 3.83\times10^8$ m.
EVALUATE: The result we calculated is in very good agreement with the orbit radius given in Appendix F.

12.55. **IDENTIFY** and **SET UP:** **(a)** To stay above the same point on the surface of the earth the orbital period of the satellite must equal the orbital period of the earth:

$T = 1\text{ d}(24\text{ h/1 d})(3600\text{ s/1 h}) = 8.64\times10^4\text{ s}$

Eq.(12.14) gives the relation between the orbit radius and the period:

EXECUTE: $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ and $T^2 = \dfrac{4\pi^2 r^3}{Gm_E}$

$$r = \left(\frac{T^2 Gm_E}{4\pi^2}\right)^{1/3} = \left(\frac{(8.64\times10^4\text{ s})^2(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}{4\pi^2}\right)^{1/3} = 4.23\times10^7\text{ m}$$

This is the radius of the orbit; it is related to the height h above the earth's surface and the radius R_E of the earth by $r = h + R_E$. Thus $h = r - R_E = 4.23\times10^7\text{ m} - 6.38\times10^6\text{ m} = 3.59\times10^7\text{ m}$.

EVALUATE: The orbital speed of the geosynchronous satellite is $2\pi r/T = 3080\text{ m/s}$. The altitude is much larger and the speed is much less than for the satellite in Example 12.6.

(b) Consider Figure 12.55.

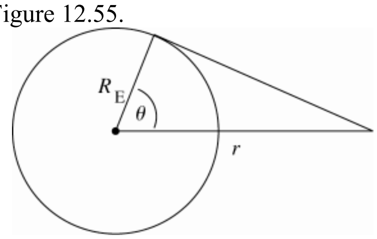

Figure 12.55

$$\cos\theta = \frac{R_E}{r} = \frac{6.38\times10^6\text{ m}}{4.23\times10^7\text{ m}}$$

$\theta = 81.3°$

A line from the satellite is tangent to a point on the earth that is at an angle of $81.3°$ above the equator. The sketch shows that points at higher latitudes are blocked by the earth from viewing the satellite.

12.56. **IDENTIFY:** Apply Eq.(12.12) to relate the orbital period T and M_P, the planet's mass, and then use Eq.(12.2) applied to the planet to calculate the astronaut's weight.

SET UP: The radius of the orbit of the lander is $5.75\times10^5\text{ m} + 4.80\times10^6\text{ m}$.

EXECUTE: From Eq.(12.14), $T^2 = \dfrac{4\pi^2 r^3}{GM_P}$ and

$$M_P = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(5.75\times10^5\text{ m} + 4.80\times10^6\text{ m})^3}{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.8\times10^3\text{ s})^2} = 2.731\times10^{24}\text{ kg},$$

or about half the earth's mass. Now we can find the astronaut's weight on the surface from Eq.(12.2). (The landing on the north pole removes any need to account for centripetal acceleration.)

$$w = \frac{GM_P m_a}{r_P^2} = \frac{\left(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.731\times10^{24}\text{ kg}\right)\left(85.6\text{ kg}\right)}{\left(4.80\times10^6\text{ m}\right)^2} = 677\text{ N}.$$

EVALUATE: At the surface of the earth the weight of the astronaut would be 839 N.

12.57. **IDENTIFY:** From Example 12.5, the escape speed is $v = \sqrt{\dfrac{2GM}{R}}$. Use $\rho = M/V$ to write this expression in terms of ρ.

SET UP: For a sphere $V = \frac{4}{3}\pi R^3$.

EXECUTE: In terms of the density ρ, the ratio M/R is $(4\pi/3)\rho R^2$, and so the escape speed is

$$v = \sqrt{(8\pi/3)\left(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(2500\text{ kg/m}^3\right)\left(150\times10^3\text{ m}\right)^2} = 177\text{ m/s}.$$

EVALUATE: This is much less than the escape speed for the earth, 11,200 m/s.

12.58. **IDENTIFY:** From Example 12.5, the escape speed is $v = \sqrt{\dfrac{2GM}{R}}$. Use $\rho = M/V$ to write this expression in terms of ρ. On earth, the height h you can jump is related to your jump speed by $v = \sqrt{2gh}$ For part (b), apply Eq.(12.4) to Europa.

SET UP: For a sphere $V = \frac{4}{3}\pi R^3$

EXECUTE: $\rho = M/(\frac{4}{3}\pi R^3)$, so the escape speed can be written as $v = \sqrt{\dfrac{8\pi G\rho R^2}{3}}$. Equating the two expressions for v and squaring gives $2gh = \dfrac{8\pi}{3}\rho G R^2$, or $R^2 = \dfrac{3}{4\pi}\dfrac{gh}{\rho G}$, where $g = 9.80\ \text{m/s}^2$ is for the surface of the earth, not the asteroid. Estimate $h = 1$ m (variable for different people, of course), $R = 3.7$ km. For Europa,

$$g = \frac{GM}{R^2} = \frac{4\pi\rho RG}{3}.\quad \rho = \frac{3g}{4\pi RG} = \frac{3(1.33\ \text{m/s}^2)}{4\pi(1.57\times10^6\ \text{m})(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 3.03\times10^3\ \text{kg/m}^3.$$

EVALUATE: The earth has average density $5500\ \text{kg/m}^3$. The average density of Europa is about half that of the earth but a little larger than the average density of most asteroids.

12.59. **IDENTIFY** and **SET UP:** The observed period allows you to calculate the angular velocity of the satellite relative to you. You know your angular velocity as you rotate with the earth, so you can find the angular velocity of the satellite in a space-fixed reference frame. $v = r\omega$ gives the orbital speed of the satellite and Newton's second law relates this to the orbit radius of the satellite.

EXECUTE: **(a)** The satellite is revolving west to east, in the same direction the earth is rotating. If the angular speed of the satellite is ω_S and the angular speed of the earth is ω_E, the angular speed ω_{rel} of the satellite relative to you is $\omega_{rel} = \omega_s - \omega_E$.

$\omega_{rel} = (1\ \text{rev})/(12\ \text{h}) = (\frac{1}{12})\ \text{rev/h}$

$\omega_E = (\frac{1}{24})\ \text{rev/h}$

$\omega_s = \omega_{rel} + \omega_E = (\frac{1}{8})\ \text{rev/h} = 2.18\times10^{-4}\ \text{rad/s}$

$\sum \vec{F} = m\vec{a}$ says $G\dfrac{mm_E}{r^2} = m\dfrac{v^2}{r}$

$v^2 = \dfrac{Gm_E}{r}$ and with $v = r\omega$ this gives $r^3 = \dfrac{Gm_E}{\omega^2}$; $r = 2.03\times10^7$ m

This is the radius of the satellite's orbit. Its height h above the surface of the earth is $h = r - R_E = 1.39\times10^7$ m.

EVALUATE: In part (a) the satellite is revolving faster than the earth's rotation and in part (b) it is revolving slower. Slower v and ω means larger orbit radius r.

(b) Now the satellite is revolving opposite to the rotation of the earth. If west to east is positive, then

$\omega_{rel} = (-\frac{1}{12})\ \text{rev/h}$

$\omega_s = \omega_{rel} + \omega_E = (-\frac{1}{24})\ \text{rev/h} = -7.27\times10^{-5}\ \text{rad/s}$

$r^3 = \dfrac{Gm_E}{\omega^2}$ gives $r = 4.22\times10^7$ m and $h = 3.59\times10^7$ m

12.60. **IDENTIFY:** Apply the law of gravitation to the astronaut at the north pole to calculate the mass of planet . Then apply $\sum \vec{F} = m\vec{a}$ to the astronaut, with $a_{rad} = \dfrac{4\pi^2 R}{T^2}$, toward the center of the planet, to calculate the period T. Apply Eq.(12.12) to the satellite in order to calculate its orbital period.

SET UP: Get radius of X: $\frac{1}{4}(2\pi R) = 18{,}850$ km and $R = 1.20\times10^7$ m . Astronaut mass:

$$m = \frac{\omega}{g} = \frac{943\ \text{N}}{9.80\ \text{m/s}^2} = 96.2\ \text{kg}.$$

EXECUTE: $\dfrac{GmM_X}{R^2} = w$, where $w = 915.0$ N . $M_x = \dfrac{mg_x R^2}{Gm} = \dfrac{(915\ \text{N})(1.20\times10^7\ \text{m})^2}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(96.2\ \text{kg})} = 2.05\times10^{25}$ kg

Apply Newton's second law to astronaut on a scale at the equator of X. $F_{grav} - F_{scale} = ma_{rad}$, so

$$F_{grav} - F_{scale} = \frac{4\pi^2 mR}{T^2}.\ 915.0\ \text{N} - 850.0\ \text{N} = \frac{4\pi^2(96.2\ \text{kg})(1.20\times10^7\ \text{m})}{T^2}\ \text{and}\ T = 2.65\times10^4\ \text{s}\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 7.36\ \text{h}.$$

(b) For the satellite, $T = \sqrt{\dfrac{4\pi^2 r^3}{Gm_X}} = \sqrt{\dfrac{4\pi^2(1.20\times10^7\ \text{m} + 2.0\times10^6\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.05\times10^{25}\ \text{kg})}} = 8.90\times10^3\ \text{s} = 2.47$ hours .

EVALUATE: The acceleration of gravity at the surface of the planet is $g_X = \dfrac{915.0\ \text{N}}{96.2\ \text{kg}} = 9.51\ \text{m/s}^2$, similar to the value on earth. The radius of the planet is about twice that of earth. The planet rotates more rapidly than earth and the length of a day is about one-third what it is on earth.

12.61. IDENTIFY: Use $g=\dfrac{Gm_E}{R_E^2}$ and follow the procedure specified in the problem.

SET UP: $R_E = 6.38\times10^6\ \text{m}$

EXECUTE: The fractional error is $1-\dfrac{mgh}{Gmm_E\left(1/R_E-1/(R_E+h)\right)}=1-\dfrac{g}{Gm_E}(R_E+h)(R_E)$.

Using Eq.(12.4) for g the fractional difference is $1-(R_E+h)/R_E=-h/R_E$, so if the fractional difference is -1%. $h=(0.01)R_E=6.4\times10^4\ \text{m}$.

EVALUATE: For $h=1\ \text{km}$, the fractional error is only 0.016%. Eq.(7.2) is very accurate for the motion of objects near the earth's surface.

12.62. IDENTIFY: Use the measurements of the motion of the rock to calculate g_M, the value of g on Mongo. Then use this to calculate the mass of Mongo. For the ship, $F_g = ma_{rad}$ and $T=\dfrac{2\pi r}{v}$.

SET UP: Take $+y$ upward. When the stone returns to the ground its velocity is 12.0 m/s, downward. $g_M = G\dfrac{m_M}{R_M^2}$.

The radius of Mongo is $R_M=\dfrac{c}{2\pi}=\dfrac{2.00\times10^8\ \text{m}}{2\pi}=3.18\times10^7\ \text{m}$. The ship moves in an orbit of radius $r=3.18\times10^7\ \text{m}+3.00\times10^7\ \text{m}=6.18\times10^7\ \text{m}$.

EXECUTE: (a) $v_{0y}=+12.0\ \text{m/s}$, $v_y=-12.0\ \text{m/s}$, $a_y=-g_M$ and $t=8.00\ \text{s}$. $v_y=v_{0y}+a_yt$ gives

$$-g_M=\frac{v_y-v_{0y}}{t}=\frac{-12.0\ \text{m/s}-12.0\ \text{m/s}}{8.00\ \text{s}}\text{ and } g_M=3.00\ \text{m/s}^2.$$

$$m_M=\frac{g_M R_M^2}{G}=\frac{(3.00\ \text{m/s}^2)(3.18\times10^7\ \text{m})^2}{6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}=4.55\times10^{25}\ \text{kg}$$

(b) $F_g=ma_{rad}$ gives $G\dfrac{m_M m}{r^2}=m\dfrac{v^2}{r}$ and $v^2=\dfrac{Gm_M}{r}$.

$$T=\frac{2\pi r}{v}=2\pi r\sqrt{\frac{r}{Gm_M}}=\frac{2\pi r^{3/2}}{\sqrt{Gm_M}}=\frac{2\pi(6.18\times10^7\ \text{m})^{3/2}}{\sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(4.55\times10^{25}\ \text{kg})}}$$

$T=5.54\times10^4\ \text{s}=15.4\ \text{h}$

EVALUATE: $R_M=5.0R_E$ and $m_M=7.6m_E$, so $g_M=\dfrac{7.6}{(5.0)^2}g_E=0.30g_E$, which agrees with the value calculated in part (a).

12.63. IDENTIFY and **SET UP:** Use Eq.(12.2) to calculate the gravity force at each location. For the top of Mount Everest write $r=h+R_E$ and use the fact that $h \ll R_E$ to obtain an expression for the difference in the two forces.

EXECUTE: At Sacramento, the gravity force on you is $F_1=G\dfrac{mm_E}{R_E^2}$.

At the top of Mount Everest, a height of $h=8800\ \text{m}$ above seal level, the gravity force on you is

$$F_2=G\frac{mm_E}{(R_E+h)^2}=G\frac{mm_E}{R_E^2(1+h/R_E)^2}$$

$$(1+h/R_E)^{-2}\approx1-\frac{2h}{R_E},\quad F_2=F_1\left(1-\frac{2h}{R_E}\right)$$

$$\frac{F_1-F_2}{F_1}=\frac{2h}{R_E}=0.28\%$$

EVALUATE: The change in the gravitational force is very small, so for objects near the surface of the earth it is a good approximation to treat it as a constant.

12.64. IDENTIFY: Apply Eq.(12.9) to the particle-earth and particle-moon systems.

SET UP: When the particle is a distance r from the center of the earth, it is a distance $R_{EM}-r$ from the center of the moon.

EXECUTE: **(a)** The total gravitational potential energy in this model is $U = -Gm\left[\frac{m_E}{r} + \frac{m_M}{R_{EM} - r}\right]$.

(b) See Exercise 12.5. The point where the net gravitational force vanishes is $r = \frac{R_{EM}}{1 + \sqrt{m_M/m_E}} = 3.46\times10^8$ m.

Using this value for r in the expression in part (a) and the work-energy theorem, including the initial potential energy of $-Gm(m_E/R_E + m_M/(R_{EM} - R_E))$ gives 11.1 km/s.

(c) The final distance from the earth is not R_M, but the Earth-moon distance minus the radius of the moon, or 3.823×10^8 m. From the work-energy theorem, the rocket impacts the moon with a speed of 2.9 km/s.

EVALUATE: The spacecraft has a greater gravitational potential energy at the surface of the moon than at the surface of the earth, so it reaches the surface of the moon with a speed that is less than its launch speed on earth.

12.65. **IDENTIFY** and **SET UP:** First use the radius of the orbit to find the initial orbital speed, from Eq.(12.10) applied to the moon.

EXECUTE: $v = \sqrt{Gm/r}$ and $r = R_M + h = 1.74\times10^6\ \text{m} + 50.0\times10^3\ \text{m} = 1.79\times10^6\ \text{m}$

Thus $v = \sqrt{\frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.35\times10^{22}\ \text{kg})}{1.79\times10^6\ \text{m}}} = 1.655\times10^3\ \text{m/s}$

After the speed decreases by 20.0 m/s it becomes 1.655×10^3 m/s – 20.0 m/s = 1.635×10^3 m/s.

IDENTIFY and **SET UP:** Use conservation of energy to find the speed when the spacecraft reaches the lunar surface.

$K_1 + U_1 + W_{other} = K_2 + U_2$

Gravity is the only force that does work so $W_{other} = 0$ and $K_2 = K_1 + U_1 - U_2$

EXECUTE: $U_1 = -Gm_m m/r$; $U_2 = -Gm_m m/R_m$

$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 + Gmm_m(1/R_m - 1/r)$

And the mass m divides out to give $v_2 = \sqrt{v_1^2 + 2Gm_m(1/R_m - 1/r)}$

$v_2 = 1.682\times10^3\ \text{m/s}(1\ \text{km}/1000\ \text{m})(3600\ \text{s}/1\ \text{h}) = 6060\ \text{km/h}$

EVALUATE: After the thruster fires the spacecraft is moving too slowly to be in a stable orbit; the gravitational force is larger than what is needed to maintain a circular orbit. The spacecraft gains energy as it is accelerated toward the surface.

12.66. **IDENTIFY:** $g = 0$ means the apparent weight is zero, so $a_{rad} = 9.80\ \text{m/s}^2$. $a_{rad} = \frac{4\pi^2 R}{T^2}$.

SET UP: The radius of the earth is $R_E = 6.38\times10^6$ m.

EXECUTE: $T = 2\pi\sqrt{\frac{R}{a_{rad}}} = 5.07\times10^3$ s, which is 84.5 min, or about an hour and a half.

EVALUATE: At the poles, g would still be $9.80\ \text{m/s}^2$.

12.67. **IDENTIFY** and **SET UP:** Apply conservation of energy. Must use Eq.(12.9) for the gravitational potential energy since h is not small compared to R_E.

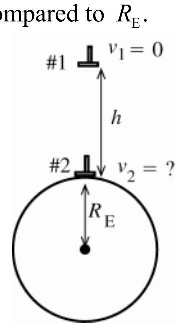

Figure 12.67

As indicated in Figure 12.67, take point 1 to be where the hammer is released and point 2 to be just above the surface of the earth, so $r_1 = R_E + h$ and $r_2 = R_E$.

EXECUTE: $K_1 + U_1 + W_{other} = K_2 + U_2$

Only gravity does work, so $W_{other} = 0$.

$K_1 = 0$, $K_2 = \frac{1}{2}mv_2^2$

$U_1 = -G\frac{mm_E}{r_1} = -\frac{Gmm_E}{h + R_E}$, $U_2 = -G\frac{mm_E}{r_2} = -\frac{Gmm_E}{R_E}$

Thus, $-G\dfrac{mm_E}{h+R_E}=\dfrac{1}{2}mv_2^2-G\dfrac{mm_E}{R_E}$

$$v_2^2=2Gm_E\left(\frac{1}{R_E}-\frac{1}{R_E+h}\right)=\frac{2Gm_E}{R_E(R_E+h)}(R_E+h-R_E)=\frac{2Gm_Eh}{R_E(R_E+h)}$$

$$v_2=\sqrt{\frac{2Gm_Eh}{R_E(R_E+h)}}$$

EVALUATE: If $h\to\infty$, $v_2\to\sqrt{2Gm_E/R_E}$, which equals the escape speed. In this limit this event is the reverse of an object being projected upward from the surface with the escape speed. If $h\ll R_E$, then $v_2=\sqrt{2Gm_Eh/R_E^2}=\sqrt{2gh}$, the same result if used Eq.(7.2) for U.

12.68. **IDENTIFY:** In orbit the total mechanical energy of the satellite is $E=-\dfrac{Gm_Em}{2R_E}$. $U=-G\dfrac{m_Em}{r}$. $W=E_f-E_i$.

SET UP: $U\to 0$ as $r\to\infty$.

EXECUTE: **(a)** The energy the satellite has as it sits on the surface of the Earth is $E_i=\dfrac{-GmM_E}{R_E}$. The energy it has when it is in orbit at a radius $R\approx R_E$ is $E_f=\dfrac{-GmM_E}{2R_E}$. The work needed to put it in orbit is the difference between these: $W=E_f-E_i=\dfrac{GmM_E}{2R_E}$.

(b) The total energy of the satellite far away from the Earth is zero, so the additional work needed is $0-\left(\dfrac{-GmM_E}{2R_E}\right)=\dfrac{GmM_E}{2R_E}$.

EVALUATE: **(c)** The work needed to put the satellite into orbit was the same as the work needed to put the satellite from orbit to the edge of the universe.

12.69. **IDENTIFY:** At the escape speed, $E=K+U=0$.

SET UP: At the surface of the earth the satellite is a distance $R_E=6.38\times10^6$ m from the center of the earth and a distance $R_{ES}=1.50\times10^{11}$ m from the sun. The orbital speed of the earth is $\dfrac{2\pi R_{ES}}{T}$, where $T=3.156\times10^7$ s is the orbital period. The speed of a point on the surface of the earth at an angle ϕ from the equator is $v=\dfrac{2\pi R_E\cos\phi}{T}$, where $T=86{,}400$ s is the rotational period of the earth.

EXECUTE: **(a)** The escape speed will be $v=\sqrt{2G\left[\dfrac{m_E}{R_E}+\dfrac{m_s}{R_{ES}}\right]}=4.35\times10^4$ m/s. Making the simplifying assumption that the direction of launch is the direction of the earth's motion in its orbit, the speed relative to the center of the earth is $v-\dfrac{2\pi R_{ES}}{T}=4.35\times10^4\text{ m/s}-\dfrac{2\pi(1.50\times10^{11}\text{ m})}{(3.156\times10^7\text{s})}=1.37\times10^4$ m/s.

(b) The rotational speed at Cape Canaveral is $\dfrac{2\pi(6.38\times10^6\text{ m})\cos 28.5°}{86{,}400\text{ s}}=4.09\times10^2$ m/s, so the speed relative to the surface of the earth is 1.33×10^4 m/s.

(c) In French Guiana, the rotational speed is 4.63×10^2 m/s, so the speed relative to the surface of the earth is 1.32×10^4 m/s.

EVALUATE: The orbital speed of the earth is a large fraction of the escape speed, but the rotational speed of a point on the surface of the earth is much less.

12.70. **IDENTIFY:** From the discussion of Section 12.6, the force on a point mass at a distance r from the center of a spherically symmetric mass distribution is the same as though we removed all the mass at points farther than r from the center and concentrated all the remaining mass at the center.

SET UP: The mass M of a hollow sphere of density ρ, inner radius R_1 and outer radius R_2 is $M=\rho\frac{4}{3}\pi(R_2^3-R_1^3)$. From Figure 12.9 in the textbook, the inner core has outer radius 1.2×10^6 m, inner radius zero and density 1.3×10^4 kg/m^3. The outer core has inner radius 1.2×10^6 m, outer radius 3.6×10^6 m and density 1.1×10^4 kg/m^3. The total mass of the earth is $m_E=5.97\times10^{24}$ kg and its radius is $R_E=6.38\times10^6$ m.

EXECUTE: **(a)** $F_g = G\dfrac{m_E m}{R_E^2} = mg = (10.0\text{ kg})(9.80\text{ m/s}^2) = 98.0\text{ N}$.

(b) The mass of the inner core is $m_{\text{inner}} = \rho_{\text{inner}}\frac{4}{3}\pi(R_2^3 - R_1^3) = (1.3\times10^4\text{ kg/m}^3)\frac{4}{3}\pi(1.2\times10^6\text{ m})^3 = 9.4\times10^{22}\text{ kg}$. The mass of the outer core is $m_{\text{outer}} = (1.1\times10^4\text{ kg/m}^3)\frac{4}{3}\pi([3.6\times10^6\text{ m}]^3 - [1.2\times10^6\text{ m}]^3) = 2.1\times10^{24}\text{ kg}$. Only the inner and outer cores contribute to the force. $F_g = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(9.4\times10^{22}\text{ kg} + 2.1\times10^{24}\text{ kg})(10.0\text{ kg})}{(3.6\times10^6\text{ m})^2} = 110\text{ N}$.

(c) Only the inner core contributes to the force and $F_g = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(9.4\times10^{22}\text{ kg})(10.0\text{ kg})}{(1.2\times10^6\text{ m})^2} = 44\text{ N}$.

(d) At $r = 0$, $F_g = 0$.

EVALUATE: In this model the earth is spherically symmetric but not uniform, so the result of Example 12.10 doesn't apply. In particular, the force at the surface of the outer core is greater than the force at the surface of the earth.

12.71. **IDENTIFY:** Eq.(12.12) relates orbital period and orbital radius for a circular orbit.

SET UP: The mass of the sun is $M = 1.99\times10^{30}\text{ kg}$.

EXECUTE: **(a)** The period of the asteroid is $T = \dfrac{2\pi a^{3/2}}{GM}$. Inserting $3\times10^{11}\text{ m}$ for a gives 2.84 y and 5×10^{11} m gives a period of 6.11 y.

(b) If the period is 5.93 y, then $a = 4.90\times10^{11}$ m.

(c) This happens because $0.4 = 2/5$, another ratio of integers. So once every 5 orbits of the asteroid and 2 orbits of Jupiter, the asteroid is at its perijove distance. Solving when $T = 4.74\ y$, $a = 4.22\times10^{11}$ m.

EVALUATE: The orbit radius for Jupiter is 7.78×10^{11} m and for Mars it is 2.21×10^{11} m. The asteroid belt lies between Mars and Jupiter. The mass of Jupiter is about 3000 times that of Mars, so the effect of Jupiter on the asteroids is much larger.

12.72. **IDENTIFY:** Apply the work-energy relation in the form $W = \Delta E$, where $E = K + U$. The speed v is related to the orbit radius by Eq.(12.10).

SET UP: $m_E = 5.97\times10^{24}\text{ kg}$

EXECUTE: **(a)** In moving to a lower orbit by whatever means, gravity does positive work, and so the speed does increase.

(b) $v = (Gm_E)^{1/2} r^{-1/2}$, so $\Delta v = (Gm_E)^{1/2}\left(-\dfrac{-\Delta r}{2}\right)r^{-3/2} = \left(\dfrac{\Delta r}{2}\right)\sqrt{\dfrac{Gm_E}{r^3}}$. Note that a positive Δr is given as a decrease in radius. Similarly, the kinetic energy is $K = (1/2)mv^2 = (1/2)Gm_E m/r$, and so

$\Delta K = (1/2)(Gm_E m/r^2)\Delta r$ and $\Delta U = -(Gm_E m/r^2)\Delta r$.

$W = \Delta U + \Delta K = -(Gm_E m/2r^2)\ \Delta r$

(c) $v = \sqrt{Gm_E/r} = 7.72\times10^3\text{ m/s}$, \ $\Delta v = (\Delta r/2)\sqrt{Gm_E/r^3} = 28.9\text{ m/s}$, $E = -Gm_E m/2r = -8.95\times10^{10}$ J (from Eq.(12.15)),

$\Delta K = (Gm_E m/2r^2)(\Delta r) = 6.70\times10^8\text{ J}$, $\Delta U = -2\Delta K = -1.34\times10^9\text{ J}$, and $W = -\Delta K = -6.70\times10^8\text{ J}$.

(d) As the term "burns up" suggests, the energy is converted to heat or is dissipated in the collisions of the debris with the ground.

EVALUATE: When r decreases, K increases and U decreases (becomes more negative).

12.73. **IDENTIFY:** Use Eq.(12.2) to calculate F_g. Apply Newton's 2nd law to circular motion of each star to find the orbital speed and period. Apply the conservation of energy expression, Eq.(7.13), to calculate the energy input (work) required to separate the two stars to infinity.

(a) SET UP: The cm is midway between the two stars since they have equal masses. Let R be the orbit radius for each star, as sketched in Figure 12.73.

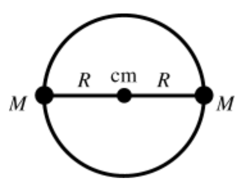

Figure 12.73

The two stars are separated by a distance $2R$, so $F_g = GM^2/(2R)^2 = GM^2/4R^2$

(b) EXECUTE: $F_g = ma_{rad}$

$GM^2/4R^2 = M(v^2/R)$ so $v = \sqrt{GM/4R}$

And $T = 2\pi R/v = 2\pi R\sqrt{4R/GM} = 4\pi\sqrt{R^3/GM}$

(c) SET UP: Apply $K_1 + U_1 + W_{other} = K_2 + U_2$ to the system of the two stars. Separate to infinity implies $K_2 = 0$ and $U_2 = 0$.

EXECUTE: $K_1 = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = 2(\frac{1}{2}M)(GM/4R) = GM^2/4R$

$U_1 = -GM^2/2R$

Thus the energy required is $W_{other} = -(K_1 + U_1) = -(GM^2/4R - GM^2/2R) = GM^2/4R.$

EVALUATE: The closer the stars are and the greater their mass, the larger their orbital speed, the shorter their orbital period and the greater the energy required to separate them.

12.74. IDENTIFY: In the center of mass coordinate system, $r_{cm} = 0$. Apply $\vec{F} = m\vec{a}$ to each star, where F is the gravitational force of one star on the other and $a = a_{rad} = \dfrac{4\pi^2 R}{T^2}$.

SET UP: $v = \dfrac{2\pi R}{T}$ allows R to be calculated from v and T.

IDENTIFY: **(a)** The radii R_1 and R_2 are measured with respect to the center of mass, and so $M_1R_1 = M_2R_2$, and $R_1/R_2 = M_2/M_1$.

(b) The forces on each star are equal in magnitude, so the product of the mass and the radial accelerations are equal: $\dfrac{4\pi^2 M_1R_1}{T_1^2} = \dfrac{4\pi^2 M_2R_2}{T_2^2}$. From the result of part (a), the numerators of these expressions are equal, and so the denominators are equal, and the periods are the same. To find the period in the symmetric form desired, there are many possible routes. An elegant method, using a bit of hindsight, is to use the above expressions to relate the periods to the force $F_g = \dfrac{GM_1M_2}{(R_1+R_2)^2}$, so that equivalent expressions for the period are $M_2T^2 = \dfrac{4\pi^2 R_1(R_1+R_2)^2}{G}$ and $M_1T^2 = \dfrac{4\pi^2 R_2(R_1+R_2)^2}{G}$. Adding the expressions gives $(M_1+M_2)T^2 = \dfrac{4\pi^2(R_1+R_2)^3}{G}$ or $T = \dfrac{2\pi(R_1+R_2)^{3/2}}{\sqrt{G(M_1+M_2)}}$.

(c) First we must find the radii of each orbit given the speed and period data. In a circular orbit, $v = \dfrac{2\pi R}{T}$, or $R = \dfrac{vT}{2\pi}$. Thus $R_\alpha = \dfrac{(36\times10^3\text{ m/s})(137\text{ d})(86{,}400\text{ s/d})}{2\pi} = 6.78\times10^{10}\text{ m}$ and

$R_\beta = \dfrac{(12\times10^3\text{ m/s})(137\text{ d})(86{,}400\text{ s/d})}{2\pi} = 2.26\times10^{10}\text{ m}$. Now find the sum of the masses. Use $M_\alpha R_\alpha = M_\beta R_\beta$, and the fact that $R_\alpha = 3R_\beta (M_\alpha + M_\beta) = \dfrac{4\pi^2(R_\alpha+R_\beta)^3}{T^2G}$, inserting the values of T, and the radii. This gives

$(M_\alpha + M_\beta) = \dfrac{4\pi^2(6.78\times10^{10}\text{m} + 2.26\times10^{10}\text{ m})^3}{[(137\text{ d})(86{,}400\text{ s/d})]^2(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)}$. $M_\alpha + M_\beta = 3.12\times10^{30}\text{ kg}$. Since

$M_\beta = M_\alpha R_\alpha/R_\beta = 3M_\alpha$, $4M_\alpha = 3.12\times10^{30}\text{ kg}$, or $M_\alpha = 7.80\times10^{29}\text{ kg}$ and $M_\beta = 2.34\times10^{30}\text{ kg}$.

(d) Let α refer to the star and β refer to the black hole. Use the relationships derived in parts (a) and (b):

$R_\beta = (M_\alpha/M_\beta)R_\alpha = (0.67/3.8)R_\alpha = (0.176)R_\alpha$, $R_\alpha + R_\beta = \sqrt[3]{\dfrac{(M_\alpha+M_\beta)T^2G}{4\pi^2}}$. For Monocerotis, inserting the values for M and T and R_β gives $R_\alpha = 1.9\times10^9$ m, $v_\alpha = 4.4\times10^2$ km/s and for the black hole $R_\beta = 34\times10^8$ m, $v_\beta = 77$ km/s.

EVALUATE: Since T is the same, v is smaller when R is smaller.

12.75. IDENTIFY and SET UP: Use conservation of energy, $K_1 + U_1 + W_{other} = K_2 + U_2$. The gravity force exerted by the sun is the only force that does work on the comet, so $W_{other} = 0$.

EXECUTE: $K_1 = \frac{1}{2}mv_1^2$, $v_1 = 2.0\times10^4$ m/s

$U_1 = -Gm_Sm/r_1$, $r_1 = 2.5\times10^{11}$ m

$K_2 = \frac{1}{2}mv_2^2$

$U_2 = -Gm_Sm/r_2$, $r_2 = 5.0\times10^{10}$ m

$\frac{1}{2}mv_1^2 - Gm_Sm/r_1 = \frac{1}{2}mv_2^2 - Gm_Sm/r_2$

$v_2^2 = v_1^2 + 2Gm_S\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = v_1^2 + 2Gm_S\left(\frac{r_1 - r_2}{r_1r_2}\right)$

$v_2 = 6.8\times10^4$ m/s

EVALUATE: The comet has greater speed when it is closer to the sun.

12.76. **IDENTIFY:** Apply conservation of energy.

SET UP: Let m_M be the mass of Mars and M_S be the mass of the sun. The subscripts a and p denote aphelion and perihelion.

EXECUTE: $\frac{1}{2}m_M v_a^2 - \frac{GM_Sm_M}{r_a} = \frac{1}{2}m_M v_p^2 - \frac{GM_Sm_M}{r_p}$, or $v_p = \sqrt{v_a^2 - 2GM_S\left(\frac{1}{r_a} - \frac{1}{r_p}\right)} = 2.650\times10^4$ m/s.

EVALUATE: We could instead use conservation of angular momentum. Note that at the extremes of distance (periheleion and aphelion), Mars' velocity vector must be perpendicular to its radius vector, and so the magnitude of the angular momentum is $L = mrv$. Since L is constant, the product rv must be a constant, and so

$v_p = v_a\frac{r_a}{r_p} = (2.198\times10^4 \text{ m/s})\frac{(2.492\times10^{11}\text{ m})}{(2.067\times10^{11}\text{ m})} = 2.650\times10^4$ m/s . Mars has larger speed when it is closer to the sun.

12.77. **(a) IDENTIFY** and **SET UP:** Use Eq.(12.17), applied to the satellites orbiting the earth rather than the sun.

EXECUTE: Find the value of a for the elliptical orbit:

$2a = r_a + r_p = R_E + h_a + R_E + h_p$, where h_a and h_p are the heights at apogee and perigee, respectively.

$a = R_E + (h_a + h_p)/2$

$a = 6.38\times10^6 \text{ m} + (400\times10^3 \text{ m} + 4000\times10^3 \text{ m})/2 = 8.58\times10^6$ m

$T = \frac{2\pi a^{3/2}}{\sqrt{GM_E}} = \frac{2\pi(8.58\times10^6\text{ m})^{3/2}}{\sqrt{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}} = 7.91\times10^3$ s

(b) Conservation of angular momentum gives $r_av_a = r_pv_p$

$\frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{6.38\times10^6\text{ m} + 4.00\times10^6\text{ m}}{6.38\times10^6\text{ m} + 4.00\times10^5\text{ m}} = 1.53$

(c) Conservation of energy applied to apogee and perigee gives $K_a + U_a = K_p + U_p$

$\frac{1}{2}mv_a^2 - Gm_Em/r_a = \frac{1}{2}mv_P^2 - Gm_Em/r_p$

$v_P^2 - v_a^2 = 2Gm_E(1/r_p - 1/r_a) = 2Gm_E(r_a - r_p)/r_ar_p$

But $v_p = 1.532v_a$, so $1.347v_a^2 = 2Gm_E(r_a - r_p)/r_ar_p$

$v_a = 5.51\times10^3$ m/s, $v_p = 8.43\times10^3$ m/s

(d) Need v so that $E = 0$, where $E = K + U$.

at perigee: $\frac{1}{2}mv_p^2 - Gm_Em/r_p = 0$

$v_p = \sqrt{2Gm_E/r_p} = \sqrt{2(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})/6.78\times10^6\text{ m}} = 1.084\times10^4$ m/s

This means an increase of $1.084\times10^4 \text{ m/s} - 8.43\times10^3 \text{ m/s} = 2.41\times10^3$ m/s.

at apogee: $v_a = \sqrt{2Gm_E/r_a} = \sqrt{2(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})/1.038\times10^7\text{ m}} = 8.761\times10^3$ m/s

This means an increase of $8.761\times10^3 \text{ m/s} - 5.51\times10^3 \text{ m/s} = 3.25\times10^3$ m/s.

EVALUATE: Perigee is more efficient. At this point r is smaller so v is larger and the satellite has more kinetic energy and more total energy.

12.78. **IDENTIFY:** $g = \frac{GM}{R^2}$, where M and R are the mass and radius of the planet.

SET UP: Let m_U and R_U be the mass and radius of Uranus and let g_U be the acceleration due to gravity at its poles. The orbit radius of Miranda is $r = h + R_U$, where $h = 1.04\times10^8$ m is the altitude of Miranda above the surface of Uranus.

EXECUTE: **(a)** From the value of g at the poles,

$m_U = \frac{g_UR_U^2}{G} = \frac{(11.1\text{ m/s}^2)\ (2.556\times10^7\text{ m})^2}{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)} = 1.09\times10^{26}$ kg.

(b) $Gm_{\text{U}}/r^2 = g_{\text{U}}\left(R_{\text{U}}/r\right)^2 = 0.432\ \text{m/s}^2$.

(c) $Gm_{\text{M}}/R_{\text{M}}^2 = 0.080\ \text{m/s}^2$.

EVALUATE: (d) No. Both the object and Miranda are in orbit together around Uranus, due to the gravitational force of Uranus. The object has additional force toward Miranda.

12.79. IDENTIFY and **SET UP:** Apply conservation of energy (Eq.7.13) and solve for W_{other}. Only $r = h + R_{\text{E}}$ is given, so use Eq.(12.10) to relate *r* and *v*.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$U_1 = -Gm_{\text{M}}m/r_1$, where m_{M} is the mass of Mars and $r_1 = R_{\text{M}} + h$, where R_{M} is the radius of Mars and $h = 2000\times10^3\ \text{m}$

$$U_1 = -(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(6.42\times10^{23}\ \text{kg})(3000\ \text{kg})}{3.40\times10^6\ \text{m} + 2000\times10^3\ \text{m}} = -2.380\times10^{10}\ \text{J}$$

$U_2 = -Gm_{\text{M}}m/r_2$, where r_2 is the new orbit radius.

$$U_2 = -(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(6.42\times10^{23}\ \text{kg})(3000\ \text{kg})}{3.40\times10^6\ \text{m} + 4000\times10^3\ \text{m}} = -1.737\times10^{10}\ \text{J}$$

For a circular orbit $v = \sqrt{Gm_{\text{M}}/r}$ (Eq.(12.10), with the mass of Mars rather than the mass of the earth).

Using this gives $K = \frac{1}{2}mv^2 = \frac{1}{2}m(Gm_{\text{M}}/r) = \frac{1}{2}Gm_{\text{M}}m/r$, so $K = -\frac{1}{2}U$

$K_1 = -\frac{1}{2}U_1 = +1.190\times10^{10}\ \text{J}$ and $K_2 = -\frac{1}{2}U_2 = +8.685\times10^9\ \text{J}$

Then $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$W_{\text{other}} = (K_2 - K_1) + (U_2 - U_1) = (8.685\times10^9\ \text{J} - 1.190\times10^{10}\ \text{J}) + (-2.380\times10^{10}\ \text{J} + 1.737\times10^{10}\ \text{J})$

$W_{\text{other}} = -3.215\times10^9\ \text{J} + 6.430\times10^9\ \text{J} = 3.22\times10^9\ \text{J}$

EVALUATE: When the orbit radius increases the kinetic energy decreases and the gravitational potential energy increases. $K = -U/2$ so $E = K + U = -U/2$ and the total energy also increases (becomes less negative). Positive work must be done to increase the total energy of the satellite.

12.80. IDENTIFY and **SET UP:** Use Eq.(12.17) to calculate a. $T = 30{,}000\ \text{y}(3.156\times10^7\ \text{s/1 y}) = 9.468\times10^{11}\ \text{s}$

EXECUTE: Eq.(12.17): $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_{\text{S}}}}$, $T^2 = \dfrac{4\pi^2 a^3}{Gm_{\text{S}}}$

$$a = \left(\frac{Gm_{\text{S}}T^2}{4\pi^2}\right)^{1/3} = 1.4\times10^{14}\ \text{m}.$$

EVALUATE: The average orbit radius of Pluto is $5.9\times10^{12}\ \text{m}$ (Appendix F); the semi-major axis for this comet is larger by a factor of 24.

$4.3\ \text{light years} = 4.3\ \text{light years}(9.461\times10^{15}\ \text{m/1 light year}) = 4.1\times10^{16}\ \text{m}$

The distance of Alpha Centauri is larger by a factor of 300.

The orbit of the comet extends well past Pluto but is well within the distance to Alpha Centauri.

12.81. IDENTIFY: Integrate $dm = \rho dV$ to find the mass of the planet. Outside the planet, the planet behaves like a point mass, so at the surface $g = GM/R^2$.

SET UP: A thin spherical shell with thickness dr has volume $dV = 4\pi r^2 dr$. The earth has radius $R_{\text{E}} = 6.38\times10^6\ \text{m}$.

EXECUTE: Get $M : M = \int dm = \int \rho dV = \int \rho 4\pi r^2 dr$. The density is $\rho = \rho_0 - br$, where $\rho_0 = 15.0\times10^3\ \text{kg/m}^3$ at the center and at the surface, $\rho_{\text{S}} = 2.0\times10^3\ \text{kg/m}^3$, so $b = \dfrac{\rho_0 - \rho_s}{R}$.

$$M = \int_0^R \left(\rho_0 - br\right) 4\pi r^2 dr = \frac{4\pi}{3}\rho_0 R^3 - \pi b R^4 = \frac{4}{3}\pi R^3\rho_0 - \pi R^4\left(\frac{\rho_0 - \rho_{\text{s}}}{R}\right) = \pi R^3\left(\frac{1}{3}\rho_0 + \rho_s\right)$$ and $M = 5.71\times10^{24}\ \text{kg}$.

Then $g = \dfrac{GM}{R^2} = \dfrac{G\pi R^3\left(\frac{1}{3}\rho_0 + \rho_s\right)}{R^2} = \pi RG\left(\dfrac{1}{3}\rho_0 + \rho_s\right)$.

$$g = \pi\left(6.38\times10^6\text{m}\right)\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(\frac{15.0\times10^3\ \text{kg/m}^3}{3} + 2.0\times10^3\ \text{kg/m}^3\right).$$

$g = 9.36\ \text{m/s}^2$.

EVALUATE: The average density of the planet is $\rho_{\text{av}} = \dfrac{M}{V} = \dfrac{M}{\frac{4}{3}\pi R^3} = \dfrac{3(5.71\times10^{24}\text{ kg})}{4\pi(6.38\times10^6\text{ m})^3} = 5.25\times10^3\text{ kg/m}^3$. Note that this is not $(\rho_0 + \rho_s)/2$.

12.82. IDENTIFY and **SET UP:** Use Eq.(12.1) to calculate the force between the point mass and a small segment of the semicircle.

EXECUTE: The radius of the semicircle is $R = L/\pi$

Divide the semicircle up into small segments of length $R\,d\theta$, as shown in Figure 12.82.

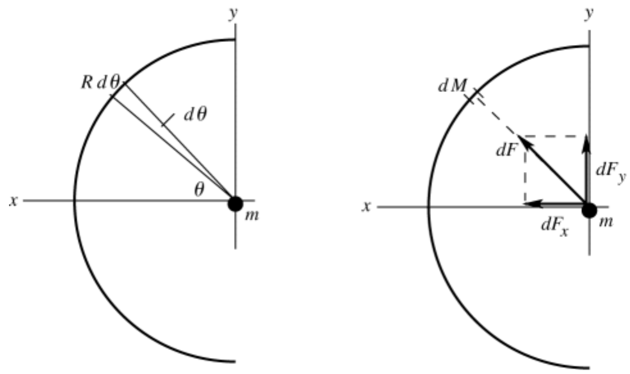

Figure 12.82

$dM = (M/L)R\,d\theta = (M/\pi)\,d\theta$

$d\vec{F}$ is the gravity force on m exerted by dM

$\int dF_y = 0$; the y-components from the upper half of the semicircle cancel the y-components from the lower half.

The x-components are all in the $+x$-direction and all add.

$$dF = G\frac{mdM}{R^2}$$

$$dF_x = G\frac{mdM}{R^2}\cos\theta = \frac{Gm\pi M}{L^2}\cos\theta\,d\theta$$

$$F_x = \int_{-\pi/2}^{\pi/2} dF_x = \frac{Gm\pi M}{L^2}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = \frac{Gm\pi M}{L^2}(2)$$

$$F = \frac{2\pi GmM}{L^2}$$

EVALUATE: If the semicircle were replaced by a point mass M at $x = R$, the gravity force would be $GmM/R^2 = \pi^2 GmM/L^2$. This is $\pi/2$ times larger than the force exerted by the semicirclar wire. For the semicircle it is the x-components that add, and the sum is less than if the force magnitudes were added.

12.83. IDENTIFY: The direct calculation of the force that the sphere exerts on the ring is slightly more involved than the calculation of the force that the ring exerts on the sphere. These forces are equal in magnitude but opposite in direction, so it will suffice to do the latter calculation. By symmetry, the force on the sphere will be along the axis of the ring in Figure 12.35 in the textbook, toward the ring.

SET UP: Divide the ring into infinitesimal elements with mass dM.

EXECUTE: Each mass element dM of the ring exerts a force of magnitude $\dfrac{(Gm)dM}{a^2+x^2}$ on the sphere, and the x-component of this force is $\dfrac{GmdM}{a^2+x^2}\dfrac{x}{\sqrt{a^2+x^2}} = \dfrac{GmdMx}{\left(a^2+x^2\right)^{3/2}}$.

Therefore, the force on the sphere is $GmMx/\left(a^2+x^2\right)^{3/2}$, in the $-x$-direction. The sphere attracts the ring with a force of the same magnitude.

EVALUATE: As $x >> a$ the denominator approaches x^3 and $F \to \dfrac{GMm}{x^2}$, as expected.

12.84. IDENTIFY: Use Eq.(12.1) for the force between a small segment of the rod and the particle. Integrate over the length of the rod to find the total force.

SET UP: Use a coordinate system with the origin at the left-hand end of the rod and the x'-axis along the rod, as shown in Figure 12.84. Divide the rod into small segments of length dx'. (Use x' for the coordinate so not to confuse with the distance x from the end of the rod to the particle.)

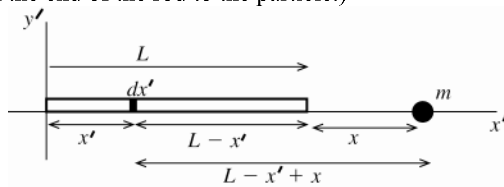

Figure 12.84

EXECUTE: The mass of each segment is $dM = dx'(M/L)$. Each segment is a distance $L - x' + x$ from mass m, so the force on the particle due to a segment is $dF = \dfrac{Gm\,dM}{(L-x'+x)^2} = \dfrac{GMm}{L}\dfrac{dx'}{(L-x'+x)^2}$.

$$F = \int_L^0 dF = \frac{GMm}{L}\int_L^0 \frac{dx'}{(L-x'+x)^2} = \frac{GMm}{L}\left(-\frac{1}{L-x'+x}\Big|_L^0\right)$$

$$F = \frac{GMm}{L}\left(\frac{1}{x} - \frac{1}{L+x}\right) = \frac{GMm}{L}\frac{(L+x-x)}{x(L+x)} = \frac{GMm}{x(L+x)}$$

EVALUATE: For $x >> L$ this result become $F = GMm/x^2$, the same as for a pair of point masses.

12.85. **IDENTIFY:** Compare F_E to Hooke's law.

SET UP: The earth has mass $m_E = 5.97 \times 10^{24}$ kg and radius $R_E = 6.38 \times 10^6$ m.

EXECUTE: For $F_x = -kx$, $U = \frac{1}{2}kx^2$. The force here is in the same form, so by analogy $U(r) = \dfrac{Gm_E m}{2R_E^3}r^2$. This is also given by the integral of F_g from 0 to r with respect to distance.

(b) From part (a), the initial gravitational potential energy is $\dfrac{Gm_E m}{2R_E}$. Equating initial potential energy and final kinetic energy (initial kinetic energy and final potential energy are both zero) gives $v^2 = \dfrac{Gm_E}{R_E}$, so $v = 7.90 \times 10^3$ m/s.

EVALUATE: When $r = 0$, $U(r) = 0$, as specified in the problem.

12.86. **IDENTIFY:** In Eqs.(12.12) and (12.16) replace T by $T + \Delta T$ and r by $r + \Delta r$. Use the expression in the hint to simplifying the resulting equations.

SET UP: The earth has $m_E = 5.97 \times 10^{24}$ kg and $R = 6.38 \times 10^6$ m. $r = h + R_E$, where h is the altitude above the surface of the earth.

EXECUTE: **(a)** $T = \dfrac{2\pi\, r^{3/2}}{\sqrt{GM_E}}$ therefore

$$T + \Delta T = \frac{2\pi}{\sqrt{GM_E}}(r+\Delta r)^{3/2} = \frac{2\pi\, r^{3/2}}{\sqrt{GM_E}}\left(1+\frac{\Delta r}{r}\right)^{3/2} \approx \frac{2\pi\, r^{3/2}}{\sqrt{GM_E}}\left(1+\frac{3\Delta r}{2r}\right) = T + \frac{3\pi\, r^{1/2}\Delta r}{\sqrt{GM_E}}.$$

Since $v = \sqrt{\dfrac{GM_E}{r}}, \Delta T = \dfrac{3\pi\,\Delta r}{v}$. $v = \sqrt{GM_E}\; r^{-1/2}$, and therefore

$$v - \Delta v = \sqrt{GM_E}\;(r+\Delta r)^{-1/2} = \sqrt{GM_E}\; r^{-1/2}\left(1+\frac{\Delta r}{r}\right)^{-1/2} \text{ and } v \approx \sqrt{GM_E}\; r^{-1/2}\left(1-\frac{\Delta r}{2r}\right) = v - \frac{\sqrt{GM_E}}{2r^{3/2}}\Delta r. \text{ Since}$$

$T = \dfrac{2\pi\, r^{3/2}}{\sqrt{GM_E}}$, $\Delta v = \dfrac{\pi \Delta r}{T}$.

(b) Starting with $T = \dfrac{2\pi\, r^{3/2}}{\sqrt{GM}}$ (Eq.(12.12)), $T = 2\pi\, r/v$, and $v = \sqrt{\dfrac{GM}{r}}$ (Eq.(12.10)), find the velocity and period of the initial orbit: $v = \sqrt{\dfrac{(6.673 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\,\text{kg})}{6.776 \times 10^6\,\text{m}}} = 7.672 \times 10^3$ m/s, and $T = 2\pi\, r/v = 5549\ \text{s} = 92.5$ min. We then can use the two derived equations to approximate ΔT and Δv:

$\Delta T = \frac{3\pi\Delta r}{v} = \frac{3\pi\ (100\text{ m})}{7.672\times10^3\text{ m/s}} = 0.1228\text{ s}$ and $\Delta v = \frac{\pi\Delta r}{T} = \frac{\pi(100\text{ m})}{(5549\text{ s})} = 0.05662\text{ m/s}$. Before the cable breaks, the shuttle will have traveled a distance *d*, $d = \sqrt{(125\text{ m}^2)-(100\text{ m}^2)} = 75\text{ m}$.

$t = (75\text{ m})/(0.05662\text{ m/s}) = 1324.7\text{ s} = 22\text{ min}$. It will take 22 minutes for the cable to break.

(c) The ISS is moving faster than the space shuttle, so the total angle it covers in an orbit must be 2π radians more than the angle that the space shuttle covers before they are once again in line. Mathematically, $\frac{vt}{r} - \frac{(v-\Delta v)t}{(r+\Delta r)} = 2\pi$.

Using the binomial theorem and neglecting terms of order $\Delta v\Delta r$, $\frac{vt}{r} - \frac{(v-\Delta v)t}{r}\left(1+\frac{\Delta r}{r}\right)^{-1} \approx t\left(\frac{\Delta v}{r}+\frac{v\Delta r}{r^2}\right) = 2\pi$.

Therefore, $t = \frac{2\pi r}{\left(\Delta v + \frac{v\Delta r}{r}\right)} = \frac{vT}{\frac{\pi\ \Delta r}{T}+\frac{v\Delta r}{r}}$. Since $2\pi\ r = vT$ and $\Delta r = \frac{v\Delta T}{3\pi}$, $t = \frac{vT}{\frac{\pi}{t}\left(\frac{v\Delta T}{3\pi}\right)+\frac{2\pi}{T}\left(\frac{v\Delta T}{3\pi}\right)} = \frac{T^2}{\Delta T}$, as was to be shown. $t = \frac{T^2}{\Delta T} = \frac{(5549\text{ s})^2}{(0.1228\text{ s})} = 2.5\times10^8\text{ s} = 2900\text{ d} = 7.9\text{ y}$. It is highly doubtful the shuttle crew would survive the congressional hearings if they miss!

EVALUATE: When the orbit radius increases, the orbital period increases and the orbital speed decreases.

12.87. IDENTIFY: Apply Eq.(12.19) to the transfer orbit.

SET UP: The orbit radius for Earth is $r_E = 1.50\times10^{11}$ m and for Mars it is $r_M = 2.28\times10^{11}$ m. From Figure 12.19 in the textbook, $a = \frac{1}{2}(r_E + r_M)$

EXECUTE: **(a)** To get from the circular orbit of the earth to the transfer orbit, the spacecraft's energy must increase, and the rockets are fired in the direction opposite that of the motion, that is, in the direction that increases the speed. Once at the orbit of Mars, the energy needs to be increased again, and so the rockets need to be fired in the direction opposite that of the motion. From Figure 12.38 in the textbook, the semimajor axis of the transfer orbit is the arithmetic average of the orbit radii of the earth and Mars, and so from Eq.(12.13), the energy of the spacecraft while in the transfer orbit is intermediate between the energies of the circular orbits. Returning from Mars to the earth, the procedure is reversed, and the rockets are fired against the direction of motion.

(b) The time will be half the period as given in Eq. (12.17), with the semimajor axis equal to

$a = \frac{1}{2}(r_E + r_M) = 1.89\times10^{11}\text{ m}$ so $t = \frac{T}{2} = \frac{\pi\ (1.89\times10^{11}\text{ m})^{3/2}}{\sqrt{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}} = 2.24\times10^7\text{ s} = 263\text{ days}$,

which is more than $8\frac{1}{2}$ months.

(c) During this time, Mars will pass through an angle of $(360°)\frac{(2.24\times10^7\text{ s})}{(687\text{ d})(86,400\text{ s/d})} = 135.9°$, and the spacecraft passes through an angle of $180°$, so the angle between the earth-sun line and the Mars-sun line must be $44.1°$.

EVALUATE: The period *T* for the transfer orbit is 526 days, the average of the orbital periods for Earth and Mars.

12.88. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to each ear.

SET UP: Denote the orbit radius as r and the distance from this radius to either ear as δ. Each ear, of mass m, can be modeled as subject to two forces, the gravitational force from the black hole and the tension force (actually the force from the body tissues), denoted by F.

EXECUTE: The force equation for either ear is $\frac{GMm}{(r+\delta)^2} - F = m\omega^2(r+\delta)$, where δ can be of either sign.

Replace the product $m\omega^2$ with the value for $\delta = 0$, $m\omega^2 = GMm/r^3$, and solve for *F*:

$$F = (GMm)\left[\frac{r+\delta}{r^3} - \frac{1}{(r+\delta)^2}\right] = \frac{GMm}{r^3}\left[r+\delta - r\left(1+(\delta/r)\right)^{-2}\right].$$

Using the binomial theorem to expand the term in square brackets in powers of δ/r,

$$F \approx \frac{GMm}{r^3}\left[r+\delta-r\left(1-2(\delta/r)\right)\right] = \frac{GMm}{r^3}(3\delta) = 2.1\text{ kN}.$$

This tension is much larger than that which could be sustained by human tissue, and the astronaut is in trouble.

(b) The center of gravity is not the center of mass. The gravity force on the two ears is not the same.

EVALUATE: The tension between her ears is proportional to their separation.

12.89. IDENTIFY: As suggested in the problem, divide the disk into rings of radius *r* and thickness *dr*.

SET UP: Each ring has an area $dA = 2\pi r\ dr$ and mass $dM = \frac{M}{\pi\ a^2}dA = \frac{2M}{a^2}r\ dr$.

EXECUTE: The magnitude of the force that this small ring exerts on the mass m is then $(G\ m\ dM)(x/(r^2+x^2)^{3/2})$. The contribution dF to the force is $dF = \dfrac{2GMmx}{a^2}\dfrac{rdr}{(x^2+r^2)^{3/2}}$.

The total force F is then the integral over the range of r;

$$F = \int dF = \frac{2GMmx}{a^2}\int_0^a \frac{r}{(x^2+r^2)^{3/2}}dr.$$

The integral (either by looking in a table or making the substitution $u = r^2 + a^2$) is

$$\int_0^a \frac{r}{(x^2+r^2)^{3/2}}dr = \left[\frac{1}{x} - \frac{1}{\sqrt{a^2+x^2}}\right] = \frac{1}{x}\left[1 - \frac{x}{\sqrt{a^2+x^2}}\right].$$

Substitution yields the result $F = \dfrac{2GMm}{a^2}\left[1 - \dfrac{x}{\sqrt{a^2+x^2}}\right]$. The force on m is directed toward the center of the ring.

The second term in brackets can be written as

$$\frac{1}{\sqrt{1+(a/x)^2}} = (1+(a/x)^2)^{-1/2} \approx 1 - \frac{1}{2}\left(\frac{a}{x}\right)^2$$

if $x >> a$, where the binomial expansion has been used. Substitution of this into the above form gives $F \approx \dfrac{GMm}{x^2}$, as it should.

EVALUATE: As $x \to 0$, the force approaches a constant.

12.90. IDENTIFY: Divide the rod into infinitesimal segments. Calculate the force each segment exerts on m and integrate over the rod to find the total force.

SET UP: From symmetry, the component of the gravitational force parallel to the rod is zero. To find the perpendicular component, divide the rod into segments of length dx and mass $dm = dx\dfrac{M}{2L}$, positioned at a distance x from the center of the rod.

EXECUTE: The magnitude of the gravitational force from each segment is

$dF = \dfrac{Gm\ dM}{x^2+a^2} = \dfrac{GmM}{2L}\dfrac{dx}{x^2+a^2}$. The component of dF perpendicular to the rod is $dF\dfrac{a}{\sqrt{x^2+a^2}}$ and so the net gravitational force is $F = \displaystyle\int_{-L}^{L} dF = \frac{GmMa}{2L}\int_{-L}^{L}\frac{dx}{(x^2+a^2)^{3/2}}$.

The integral can be found in a table, or found by making the substitution $x = a\ \tan\theta$. Then, $dx = a\ \sec^2\theta\ d\theta,\ (x^2+a^2) = a^2\ \sec^2\theta,$ and so

$$\int\frac{dx}{(x^2+a^2)^{3/2}} = \int\frac{a\ \sec^2\theta\ d\theta}{a^3\ \sec^3\theta} = \frac{1}{a^2}\int\cos\theta\ d\theta = \frac{1}{a^2}\sin\theta = \frac{x}{a^2\sqrt{x^2+a^2}},$$

and the definite integral is $F = \dfrac{GmM}{a\sqrt{a^2+L^2}}$.

EVALUATE: When a >> L, the term in the square root approaches a^2 and $F \to \dfrac{GmM}{a^2}$, as expected.