10

DYNAMICS OF ROTATIONAL MOTION

10.1. **IDENTIFY:** Use Eq.(10.2) to calculate the magnitude of the torque and use the right-hand rule illustrated in Fig.(10.4) to calculate the torque direction.

(a) SET UP: Consider Figure 10.1a.

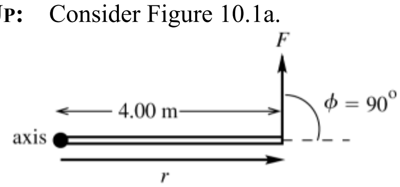


Figure 10.1a

EXECUTE: $\tau = Fl$

$l = r\sin\phi = (4.00 \text{ m})\sin 90°$

$l = 4.00 \text{ m}$

$\tau = (10.0 \text{ N})(4.00 \text{ m}) = 40.0 \text{ N}\cdot\text{m}$

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

(b) SET UP: Consider Figure 10.1b.

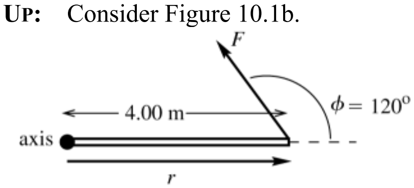


Figure 10.1b

EXECUTE: $\tau = Fl$

$l = r\sin\phi = (4.00 \text{ m})\sin 120°$

$l = 3.464 \text{ m}$

$\tau = (10.0 \text{ N})(3.464 \text{ m}) = 34.6 \text{ N}\cdot\text{m}$

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

(c) SET UP: Consider Figure 10.1c.

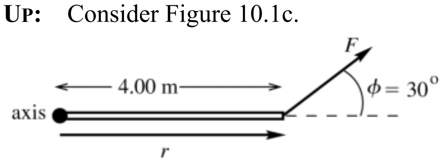


Figure 10.1c

EXECUTE: $\tau = Fl$

$l = r\sin\phi = (4.00 \text{ m})\sin 30°$

$l = 2.00 \text{ m}$

$\tau = (10.0 \text{ N})(2.00 \text{ m}) = 20.0 \text{ N}\cdot\text{m}$

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

(d) SET UP: Consider Figure 10.1d.

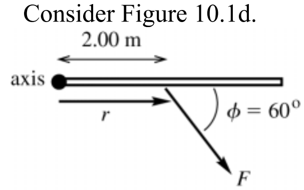


Figure 10.1d

EXECUTE: $\tau = Fl$

$l = r\sin\phi = (2.00 \text{ m})\sin 60° = 1.732 \text{ m}$

$\tau = (10.0 \text{ N})(1.732 \text{ m}) = 17.3 \text{ N}\cdot\text{m}$

This force tends to produce a clockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed into the plane of the figure.

(e) SET UP: Consider Figure 10.1e.

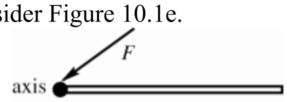


Figure 10.1e

EXECUTE: $\tau = Fl$

$r = 0$ so $l = 0$ and $\tau = 0$

(f) SET UP: Consider Figure 10.1f.

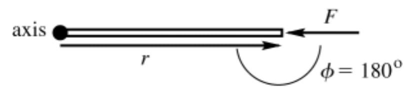


Figure 10.1f

EXECUTE: $\tau = Fl$
$l = r\sin\phi,\ \phi = 180°$,
so $l = 0$ and $\tau = 0$

EVALUATE: The torque is zero in parts (e) and (f) because the moment arm is zero; the line of action of the force passes through the axis.

10.2. **IDENTIFY:** $\tau = Fl$ with $l = r\sin\phi$. Add the two torques to calculate the net torque.

SET UP: Let counterclockwise torques be positive.

EXECUTE: $\tau_1 = -F_1 l_1 = -(8.00\ \text{N})(5.00\ \text{m}) = -40.0\ \text{N}\cdot\text{m}$. $\tau_2 = +F_2 l_2 = (12.0\ \text{N})(2.00\ \text{m})\sin 30.0° = +12.0\ \text{N}\cdot\text{m}$.

$\sum\tau = \tau_1 + \tau_2 = -28.0\ \text{N}\cdot\text{m}$. The net torque is $28.0\ \text{N}\cdot\text{m}$, clockwise.

EVALUATE: Even though $F_1 < F_2$, the magnitude of τ_1 is greater than the magnitude of τ_2, because F_1 has a larger moment arm.

10.3. **IDENTIFY** and **SET UP:** Use Eq.(10.2) to calculate the magnitude of each torque and use the right-hand rule (Fig.10.4) to determine the direction. Consider Figure 10.3

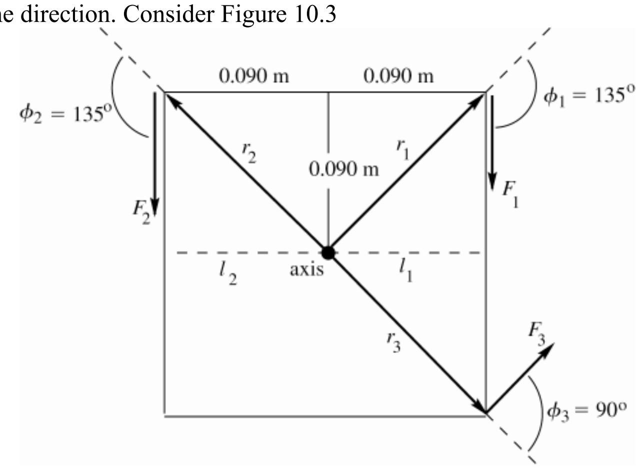


Figure 10.3

Let counterclockwise be the positive sense of rotation.

EXECUTE: $r_1 = r_2 = r_3 = \sqrt{(0.090\ \text{m})^2 + (0.090\ \text{m})^2} = 0.1273\ \text{m}$

$\tau_1 = -F_1 l_1$

$l_1 = r_1\sin\phi_1 = (0.1273\ \text{m})\sin 135° = 0.0900\ \text{m}$

$\tau_1 = -(18.0\ \text{N})(0.0900\ \text{m}) = -1.62\ \text{N}\cdot\text{m}$

$\vec{\tau}_1$ is directed into paper

$\tau_2 = +F_2 l_2$

$l_2 = r_2\sin\phi_2 = (0.1273\ \text{m})\sin 135° = 0.0900\ \text{m}$

$\tau_2 = +(26.0\ \text{N})(0.0900\ \text{m}) = +2.34\ \text{N}\cdot\text{m}$

$\vec{\tau}_2$ is directed out of paper

$\tau_3 = +F_3 l_3$

$l_3 = r_3\sin\phi_3 = (0.1273\ \text{m})\sin 90° = 0.1273\ \text{m}$

$\tau_3 = +(14.0\ \text{N})(0.1273\ \text{m}) = +1.78\ \text{N}\cdot\text{m}$

$\vec{\tau}_3$ is directed out of paper

$\sum\tau = \tau_1 + \tau_2 + \tau_3 = -1.62\ \text{N}\cdot\text{m} + 2.34\ \text{N}\cdot\text{m} + 1.78\ \text{N}\cdot\text{m} = 2.50\ \text{N}\cdot\text{m}$

EVALUATE: The net torque is positive, which means it tends to produce a counterclockwise rotation; the vector torque is directed out of the plane of the paper. In summing the torques it is important to include $+$ or $-$ signs to show direction.

10.4. **IDENTIFY:** Use $\tau = Fl = rF\sin\phi$ to calculate the magnitude of each torque and use the right-hand rule to determine the direction of each torque. Add the torques to find the net torque.

SET UP: Let counterclockwise torques be positive. For the 11.9 N force (F_1), $r = 0$. For the 14.6 N force (F_2), $r = 0.350$ m and $\phi = 40.0°$. For the 8.50 N force (F_3), $r = 0.350$ m and $\phi = 90.0°$

EXECUTE: $\tau_1 = 0$. $\tau_2 = -(14.6 \text{ N})(0.350 \text{ m})\sin 40.0° = -3.285 \text{ N}\cdot\text{m}$.

$\tau_3 = +(8.50 \text{ N})(0.350 \text{ m})\sin 90.0° = +2.975 \text{ N}\cdot\text{m}$. $\sum\tau = -3.285 \text{ N}\cdot\text{m} + 2.975 \text{ N}\cdot\text{m} = -0.31 \text{ N}\cdot\text{m}$. The net torque is $0.31 \text{ N}\cdot\text{m}$ and is clockwise.

EVALUATE: If we treat the torques as vectors, $\vec{\boldsymbol{\tau}}_2$ is into the page and $\vec{\boldsymbol{\tau}}_3$ is out of the page.

10.5. **IDENTIFY** and **SET UP:** Calculate the torque using Eq.(10.3) and also determine the direction of the torque using the right-hand rule.

(a) $\vec{\boldsymbol{r}} = (-0.450 \text{ m})\hat{\boldsymbol{i}} + (0.150 \text{ m})\hat{\boldsymbol{j}}$; $\vec{\boldsymbol{F}} = (-5.00 \text{ N})\hat{\boldsymbol{i}} + (4.00 \text{ N})\hat{\boldsymbol{j}}$. The sketch is given in Figure 10.5.

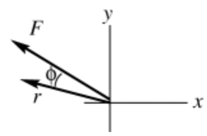


Figure 10.5

EXECUTE: **(b)** When the fingers of your right hand curl from the direction of $\vec{\boldsymbol{r}}$ into the direction of $\vec{\boldsymbol{F}}$ (through the smaller of the two angles, angle ϕ) your thumb points into the page (the direction of $\vec{\tau}$, the $-z$-direction).

(c) $\vec{\tau} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} = \left[(-0.450 \text{ m})\hat{\boldsymbol{i}} + (0.150 \text{ m})\hat{\boldsymbol{j}}\right] \times \left[(-5.00 \text{ N})\hat{\boldsymbol{i}} + (4.00 \text{ N})\hat{\boldsymbol{j}}\right]$

$\vec{\tau} = +(2.25 \text{ N}\cdot\text{m})\hat{\boldsymbol{i}} \times \hat{\boldsymbol{i}} - (1.80 \text{ N}\cdot\text{m})\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}} - (0.750 \text{ N}\cdot\text{m})\hat{\boldsymbol{j}} \times \hat{\boldsymbol{i}} + (0.600 \text{ N}\cdot\text{m})\hat{\boldsymbol{j}} \times \hat{\boldsymbol{j}}$

$\hat{\boldsymbol{i}} \times \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}} \times \hat{\boldsymbol{j}} = \boldsymbol{0}$

$\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}}$, $\hat{\boldsymbol{j}} \times \hat{\boldsymbol{i}} = -\hat{\boldsymbol{k}}$

Thus $\vec{\tau} = -(1.80 \text{ N}\cdot\text{m})\hat{\boldsymbol{k}} - (0.750 \text{ N}\cdot\text{m})(-\hat{\boldsymbol{k}}) = (-1.05 \text{ N}\cdot\text{m})\hat{\boldsymbol{k}}$.

EVALUATE: The calculation gives that $\vec{\tau}$ is in the $-z$-direction. This agrees with what we got from the right-hand rule.

10.6. **IDENTIFY:** Use $\tau = Fl = rF\sin\phi$ for the magnitude of the torque and the right-hand rule for the direction.

SET UP: In part (a), $r = 0.250$ m and $\phi = 37°$

EXECUTE: (a) $\tau = (17.0 \text{ N})(0.250 \text{ m})\sin 37° = 2.56 \text{ N}\cdot\text{m}$. The torque is counterclockwise.

(b) The torque is maximum when $\phi = 90°$ and the force is perpendicular to the wrench. This maximum torque is $(17.0 \text{ N})(0.250 \text{ m}) = 4.25 \text{ N}\cdot\text{m}$.

EVALUATE: If the force is directed along the handle then the torque is zero. The torque increases as the angle between the force and the handle increases.

10.7. **IDENTIFY:** Apply $\sum\tau_z = I\alpha_z$.

SET UP: $\omega_{0z} = 0$. $\omega_z = (400 \text{ rev/min})\left(\dfrac{2\pi \text{ rad/rev}}{60 \text{ s/min}}\right) = 41.9 \text{ rad/s}$

EXECUTE: $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t} = \left(2.50 \text{ kg}\cdot\text{m}^2\right)\dfrac{41.9 \text{ rad/s}}{8.00 \text{ s}} = 13.1 \text{ N}\cdot\text{m}.$

EVALUATE: In $\tau_z = I\alpha_z$, α_z must be in rad/s^2.

10.8. **IDENTIFY:** Use a constant acceleration equation to calculate α_z and then apply $\sum\tau_z = I\alpha_z$.

SET UP: $I = \frac{2}{3}MR^2 + 2mR^2$, where $M = 8.40$ kg, $m = 2.00$ kg, so $I = 0.600 \text{ kg}\cdot\text{m}^2$.

$\omega_{0z} = 75.0 \text{ rpm} = 7.854 \text{ rad/s}$; $\omega_z = 50.0 \text{ rpm} = 5.236 \text{ rad/s}$; $t = 30.0$ s.

EXECUTE: $\omega_z = \omega_{0_z} + \alpha_z t$ gives $\alpha_z = -0.08726 \text{ rad/s}^2$. $\tau_z = I\alpha_z = -0.0524 \text{ N}\cdot\text{m}$

EVALUATE: The torque is negative because its direction is opposite to the direction of rotation, which must be the case for the speed to decrease.

10.9. **IDENTIFY:** Use $\sum\tau_z = I\alpha_z$ to calculate α. Use a constant angular acceleration kinematic equation to relate α_z, ω_z and t.

SET UP: For a solid uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$. Let the direction the sphere is spinning be the positive sense of rotation. The moment arm for the friction force is $l = 0.0150$ m and the torque due to this force is negative.

EXECUTE: **(a)** $\alpha_z = \dfrac{\tau_z}{I} = \dfrac{-(0.0200\text{ N})(0.0150\text{ m})}{\frac{2}{5}(0.225\text{ kg})(0.0150\text{ m})^2} = -14.8\text{ rad/s}^2$

(b) $\omega_z - \omega_{0z} = -22.5\text{ rad/s}$. $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{-22.5\text{ rad/s}}{-14.8\text{ rad/s}^2} = 1.52\text{ s}$.

EVALUATE: The fact that α_z is negative means its direction is opposite to the direction of spin. The negative α_z causes ω_z to decrease.

10.10. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ to the wheel. The acceleration a of a point on the cord and the angular acceleration α of the wheel are related by $a = R\alpha$.

SET UP: Let the direction of rotation of the wheel be positive. The wheel has the shape of a disk and $I = \frac{1}{2}MR^2$. The free-body diagram for the wheel is sketched in Figure 10.10a for a horizontal pull and in Figure 10.10b for a vertical pull. P is the pull on the cord and F is the force exerted on the wheel by the axle.

EXECUTE: **(a)** $\alpha_z = \dfrac{\tau_z}{I} = \dfrac{(40.0\text{ N})(0.250\text{ m})}{\frac{1}{2}(9.20\text{ kg})(0.250\text{ m})^2} = 34.8\text{ rad/s}^2$. $a = R\alpha = (0.250\text{ m})(34.8\text{ rad/s}^2) = 8.70\text{ m/s}^2$.

(b) $F_x = -P$, $F_y = -Mg$. $F = \sqrt{P^2 + (Mg)^2} = \sqrt{(40.0\text{ N})^2 + ([9.20\text{ kg}][9.80\text{ m/s}^2])^2} = 98.6\text{ N}$.

$\tan\phi = \dfrac{|F_y|}{|F_x|} = \dfrac{Mg}{P} = \dfrac{(9.20\text{ kg})(9.80\text{ m/s}^2)}{40.0\text{ N}}$ and $\phi = 66.1°$. The force exerted by the axle has magnitude 98.6 N and is directed at $66.1°$ above the horizontal, away from the direction of the pull on the cord.

(c) The pull exerts the same torque as in part (a), so the answers to part (a) don't change. In part (b), $F + P = Mg$ and $F = Mg - P = (9.20\text{ kg})(9.80\text{ m/s}^2) - 40.0\text{ N} = 50.2\text{ N}$. The force exerted by the axle has magnitude 50.2 N and is upward.

EVALUATE: The weight of the wheel and the force exerted by the axle produce no torque because they act at the axle.

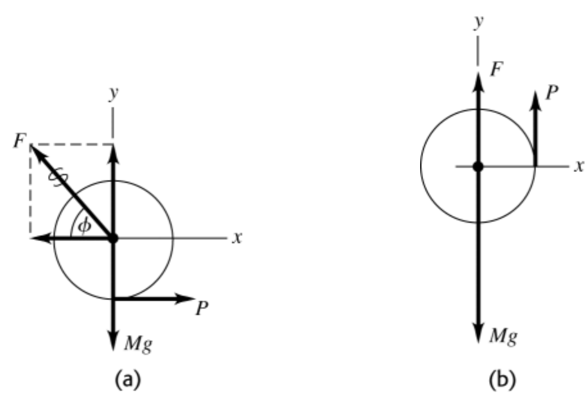


Figure 10.10

10.11. **IDENTIFY:** Use a constant angular acceleration equation to calculate α_z and then apply $\sum \tau_z = I\alpha_z$ to the motion of the cylinder. $f_k = \mu_k n$.

SET UP: $I = \frac{1}{2}mR^2 = \frac{1}{2}(8.25\text{ kg})(0.0750\text{ m})^2 = 0.02320\text{ kg}\cdot\text{m}^2$. Let the direction the cylinder is rotating be positive. $\omega_{0z} = 220\text{ rpm} = 23.04\text{ rad/s}$; $\omega_z = 0$; $\theta - \theta_0 = 5.25\text{ rev} = 33.0\text{ rad}$.

EXECUTE: $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ gives $\alpha_z = -8.046\text{ rad/s}^2$. $\sum \tau_z = \tau_f = -f_k R = -\mu_k nR$. Then $\sum \tau_z = I\alpha_z$ gives $-\mu_k nR = I\alpha_z$ and $n = \dfrac{I\alpha_z}{\mu_k R} = 7.47\text{ N}$.

EVALUATE: The friction torque is directed opposite to the direction of rotation and therefore produces an angular acceleration that slows the rotation.

10.12. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the stone and $\sum \tau_z = I\alpha_z$ to the pulley. Use a constant acceleration equation to find a for the stone.

SET UP: For the motion of the stone take $+y$ to be downward. The pulley has $I = \frac{1}{2}MR^2$. $a = R\alpha$.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $12.6\text{ m} = \frac{1}{2}a_y(3.00\text{ s})^2$ and $a_y = 2.80\text{ m/s}^2$. Then $\sum F_y = ma_y$ applied to the stone gives $mg - T = ma$. $\sum \tau_z = I\alpha_z$ applied to the pulley gives $TR = \frac{1}{2}MR^2\alpha = \frac{1}{2}MR^2(a/R)$. $T = \frac{1}{2}Ma$. Combining these two equations to eliminate T gives

$$M = \frac{M}{2}\left(\frac{a}{g-a}\right) = \left(\frac{10.0\text{ kg}}{2}\right)\left(\frac{2.80\text{ m/s}^2}{9.80\text{ m/s}^2 - 2.80\text{ m/s}^2}\right) = 2.00\text{ kg}.$$

(b) $T = \frac{1}{2}Ma = \frac{1}{2}(10.0\text{ kg})(2.80\text{ m/s}^2) = 14.0\text{ N}$

EVALUATE: The tension in the wire is less than the weight $mg = 19.6$ N of the stone, because the stone has a downward acceleration.

10.13. **IDENTIFY:** Use the kinematic information to solve for the angular acceleration of the grindstone. Assume that the grindstone is rotating counterclockwise and let that be the positive sense of rotation. Then apply Eq.(10.7) to calculate the friction force and use $f_k = \mu_k n$ to calculate μ_k.

SET UP: $\omega_{0z} = 850\text{ rev/min}(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 89.0\text{ rad/s}$

$t = 7.50$ s; $\omega_z = 0$ (comes to rest); $\alpha_z = ?$

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z t$

$$\alpha_z = \frac{0 - 89.0\text{ rad/s}}{7.50\text{ s}} = -11.9\text{ rad/s}^2$$

SET UP: Apply $\sum \tau_z = I\alpha_z$ to the grindstone. The free-body diagram is given in Figure 10.13.

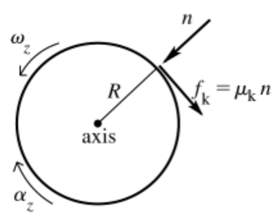


Figure 10.13

The normal force has zero moment arm for rotation about an axis at the center of the grindstone, and therefore zero torque. The only torque on the grindstone is that due to the friction force f_k exerted by the ax; for this force the moment arm is $l = R$ and the torque is negative.

EXECUTE: $\sum \tau_z = -f_k R = -\mu_k nR$

$I = \frac{1}{2}MR^2$ (solid disk, axis through center)

Thus $\sum \tau_z = I\alpha_z$ gives $-\mu_k nR = \left(\frac{1}{2}MR^2\right)\alpha_z$

$$\mu_k = -\frac{MR\alpha_z}{2n} = -\frac{(50.0\text{ kg})(0.260\text{ m})(-11.9\text{ rad/s}^2)}{2(160\text{ N})} = 0.483$$

EVALUATE: The friction torque is clockwise and slows down the counterclockwise rotation of the grindstone.

10.14. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the bucket, with $+y$ downward. Apply $\sum \tau_z = I\alpha_z$ to the cylinder, with the direction the cylinder rotates positive.

SET UP: The free-body diagram for the bucket is given in Fig.10.14a and the free-body diagram for the cylinder is given in Fig.10.14b. $I = \frac{1}{2}MR^2$. $a(\text{bucket}) = R\alpha(\text{cylinder})$

EXECUTE: **(a)** For the bucket, $mg - T = ma$. For the cylinder, $\sum \tau_z = I\alpha_z$ gives $TR = \frac{1}{2}MR^2\alpha$. $\alpha = a/R$ then gives $T = \frac{1}{2}Ma$. Combining these two equations gives $mg - \frac{1}{2}Ma = ma$ and

$$a = \frac{mg}{m + M/2} = \left(\frac{15.0\text{ kg}}{15.0\text{ kg} + 6.0\text{ kg}}\right)(9.80\text{ m/s}^2) = 7.00\text{ m/s}^2.$$

$T = m(g - a) = (15.0\text{ kg})(9.80\text{ m/s}^2 - 7.00\text{ m/s}^2) = 42.0\text{ N}$.

(b) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(7.00\text{ m/s}^2)(10.0\text{ m})} = 11.8\text{ m/s}$.

(c) $a_y = 7.00\text{ m/s}^2$, $v_{0y} = 0$, $y - y_0 = 10.0$ m. $y - y_0 = v_{0y}t + \frac{1}{2}\alpha_y t^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(10.0\text{ m})}{7.00\text{ m/s}^2}} = 1.69\text{ s}$

(d) $\sum F_y = ma_y$ applied to the cylinder gives $n - T - Mg = 0$ and

$n = T + mg = 42.0\text{ N} + (12.0\text{ kg})(9.80\text{ m/s}^2) = 160\text{ N}$.

EVALUATE: The tension in the rope is less than the weight of the bucket, because the bucket has a downward acceleration. If the rope were cut, so the bucket would be in free-fall, the bucket would strike the water in $t = \sqrt{\dfrac{2(10.0\text{ m})}{9.80\text{ m/s}^2}} = 1.43$ s and would have a final speed of 14.0 m/s. The presence of the cylinder slows the fall of the bucket.

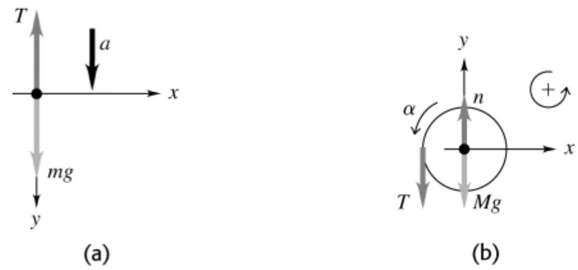


Figure 10.14

10.15. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to each book and apply $\sum \tau_z = I\alpha_z$ to the pulley. Use a constant acceleration equation to find the common acceleration of the books.

SET UP: $m_1 = 2.00\text{ kg}$, $m_2 = 3.00\text{ kg}$. Let T_1 be the tension in the part of the cord attached to m_1 and T_2 be the tension in the part of the cord attached to m_2. Let the $+x$-direction be in the direction of the acceleration of each book. $a = R\alpha$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $a_x = \dfrac{2(x - x_0)}{t^2} = \dfrac{2(1.20\text{ m})}{(0.800\text{ s})^2} = 3.75\text{ m/s}^2$. $a_1 = 3.75\text{ m/s}^2$ so

$T_1 = m_1 a_1 = 7.50$ N and $T_2 = m_2(g - a_1) = 18.2$ N.

(b) The torque on the pulley is $(T_2 - T_1)R = 0.803\text{ N}\cdot\text{m}$, and the angular acceleration is

$\alpha = a_1/R = 50\text{ rad/s}^2$, so $I = \tau/\alpha = 0.016\text{ kg}\cdot\text{m}^2$.

EVALUATE: The tensions in the two parts of the cord must be different, so there will be a net torque on the pulley.

10.16. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to each box and $\sum \tau_z = I\alpha_z$ to the pulley. The magnitude a of the acceleration of each box is related to the magnitude of the angular acceleration α of the pulley by $a = R\alpha$.

SET UP: The free-body diagrams for each object are shown in Figure 10.16a-c. For the pulley, $R = 0.250$ m and $I = \frac{1}{2}MR^2$. T_1 and T_2 are the tensions in the wire on either side of the pulley. $m_1 = 12.0\text{ kg}$, $m_2 = 5.00\text{ kg}$ and $M = 2.00\text{ kg}$. $\vec{F}$ is the force that the axle exerts on the pulley. For the pulley, let clockwise rotation be positive.

EXECUTE: **(a)** $\sum F_x = ma_x$ for the 12.0 kg box gives $T_1 = m_1 a$. $\sum F_y = ma_y$ for the 5.00 kg weight gives $m_2 g - T_2 = m_2 a$. $\sum \tau_z = I\alpha_z$ for the pulley gives $(T_2 - T_1)R = (\frac{1}{2}MR^2)\alpha$. $a = R\alpha$ and $T_2 - T_1 = \frac{1}{2}Ma$. Adding these three equations gives $m_2 g = (m_1 + m_2 + \frac{1}{2}M)a$ and

$$a = \left(\frac{m_2}{m_1 + m_2 + \frac{1}{2}M}\right)g = \left(\frac{5.00\text{ kg}}{12.0\text{ kg} + 5.00\text{ kg} + 1.00\text{ kg}}\right)(9.80\text{ m/s}^2) = 2.72\text{ m/s}^2.$$ Then

$T_1 = m_1 a = (12.0\text{ kg})(2.72\text{ m/s}^2) = 32.6\text{ N}$. $m_2 g - T_2 = m_2 a$ gives

$T_2 = m_2(g - a) = (5.00\text{ kg})(9.80\text{ m/s}^2 - 2.72\text{ m/s}^2) = 35.4\text{ N}$. The tension to the left of the pulley is 32.6 N and below the pulley it is 35.4 N.

(b) $a = 2.72\text{ m/s}^2$

(c) For the pulley, $\sum F_x = ma_x$ gives $F_x = T_1 = 32.6$ N and $\sum F_y = ma_y$ gives

$F_y = Mg + T_2 = (2.00\text{ kg})(9.80\text{ m/s}^2) + 35.4\text{ N} = 55.0\text{ N}$.

EVALUATE: The equation $m_2g = (m_1 + m_2 + \frac{1}{2}M)a$ says that the external force m_2g must accelerate all three objects.

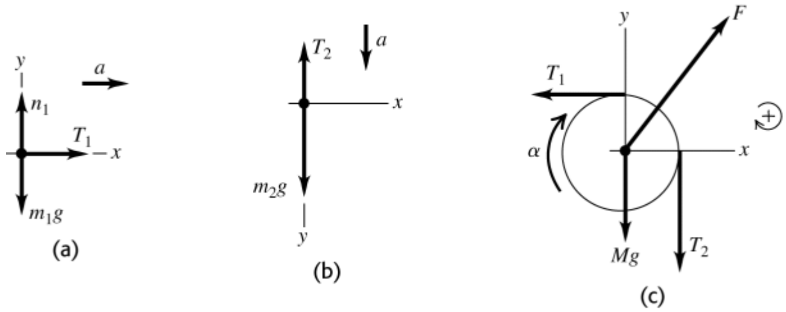


Figure 10.16

10.17. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ to the post and $\sum \vec{F} = m\vec{a}$ to the hanging mass. The acceleration $\vec{a}$ of the mass has the same magnitude as the tangential acceleration $a_{tan} = r\alpha$ of the point on the post where the string is attached; $r = 1.75\text{ m} - 0.500\text{ m} = 1.25\text{ m}$.

SET UP: The free-body diagrams for the post and mass are given in Figures 10.17a and b. The post has $I = \frac{1}{3}ML^2$, with $M = 15.0\text{ kg}$ and $L = 1.75\text{ m}$.

EXECUTE: **(a)** $\sum \tau_z = I\alpha_z$ for the post gives $Tr = \left(\frac{1}{3}ML^2\right)\alpha$. $a = r\alpha$ so $\alpha = \dfrac{a}{r}$ and $T = \left(\dfrac{ML^2}{3r^2}\right)a$. $\sum F_y = ma_y$ for the mass gives $mg - T = ma$. These two equations give $mg = (m + ML^2/[3r^2])a$ and

$$a = \left(\frac{m}{m + ML^2/[3r^2]}\right)g = \left(\frac{5.00\text{ kg}}{5.00\text{ kg} + [15.0\text{ kg}][1.75\text{ m}]^2/3[1.25\text{ m}]^2}\right)(9.80\text{ m/s}^2) = 3.31\text{ m/s}^2 .$$

$$\alpha = \frac{a}{r} = \frac{3.31\text{ m/s}^2}{1.25\text{ m}} = 2.65\text{ rad/s}^2 .$$

(b) No. As the post rotates and the point where the string is attached moves in an arc of a circle, the string is no longer perpendicular to the post. The torque due to this tension changes and the acceleration due to this torque is not constant.

(c) From part (a), $a = 3.31\text{ m/s}^2$. The acceleration of the mass is not constant. It changes as α for the post changes.

EVALUATE: At the instant the cable breaks the tension in the string is less than the weight of the mass because the mass accelerates downward and there is a net downward force on it.

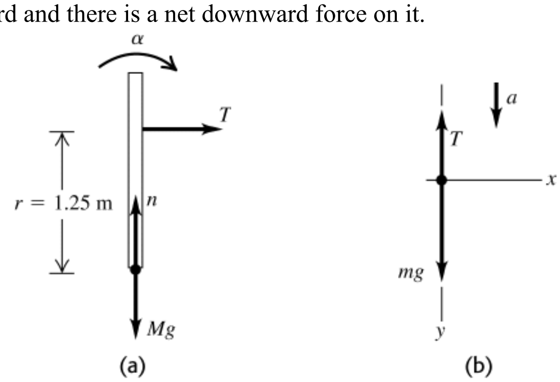


Figure 10.17

10.18. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ to the rod.

SET UP: For the rod and axis at one end, $I = \frac{1}{3}Ml^2$.

EXECUTE: $\alpha = \dfrac{\tau}{I} = \dfrac{Fl}{\frac{1}{3}Ml^2} = \dfrac{3F}{Ml}$.

EVALUATE: Note that α decreases with the length of the rod, even though the torque increases.

10.19. **IDENTIFY:** Since there is rolling without slipping, $v_{cm} = R\omega$. The kinetic energy is given by Eq.(10.8). The velocities of points on the rim of the hoop are as described in Figure 10.13 in chapter 10.

SET UP: $\omega = 3.00\text{ rad/s}$ and $R = 0.600\text{ m}$. For a hoop rotating about an axis at its center, $I = MR^2$.

EXECUTE: **(a)** $v_{cm} = R\omega = (0.600\text{ m})(3.00\text{ rad/s}) = 1.80\text{ m/s}$.

(b) $K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}(MR^2)(v_{cm}/R^2) = Mv_{cm}^2 = (2.20\text{ kg})(1.80\text{ m/s})^2 = 7.13\text{ J}$

(c) (i) $v = 2v_{cm} = 3.60\text{ m/s}$. $\vec{v}$ is to the right. (ii) $v = 0$

(iii) $v = \sqrt{v_{cm}^2 + v_{tan}^2} = \sqrt{v_{cm}^2 + (R\omega)^2} = \sqrt{2}v_{cm} = 2.55\text{ m/s}$. $\vec{v}$ at this point is at $45°$ below the horizontal.

(d) To someone moving to the right at $v = v_{cm}$, the hoop appears to rotate about a stationary axis at its center.

(i) $v = R\omega = 1.80\text{ m/s}$, to the right. (ii) $v = 1.80\text{ m/s}$, to the left. (iii) $v = 1.80\text{ m/s}$, downward.

EVALUATE: For the special case of a hoop, the total kinetic energy is equally divided between the motion of the center of mass and the rotation about the axis through the center of mass. In the rest frame of the ground, different points on the hoop have different speed.

10.20. **IDENTIFY:** Only gravity does work, so $W_{other} = 0$ and conservation of energy gives $K_i + U_i = K_f + U_f$. $K_f = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$.

SET UP: Let $y_f = 0$, so $U_f = 0$ and $y_i = 0.750\text{ m}$. The hoop is released from rest so $K_i = 0$. $v_{cm} = R\omega$. For a hoop with an axis at its center, $I_{cm} = MR^2$.

EXECUTE: **(a)** Conservation of energy gives $U_i = K_f$. $K_f = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}(MR^2)\omega^2 = MR^2\omega^2$, so $MR^2\omega^2 = Mgy_i$.

$$\omega = \frac{\sqrt{gy_i}}{R} = \frac{\sqrt{(9.80\text{ m/s}^2)(0.750\text{ m})}}{0.0800\text{ m}} = 33.9\text{ rad/s}.$$

(b) $v = R\omega = (0.0800\text{ m})(33.9\text{ rad/s}) = 2.71\text{ m/s}$

EVALUATE: An object released from rest and falling in free-fall for 0.750 m attains a speed of $\sqrt{2g(0.750\text{ m})} = 3.83\text{ m/s}$. The final speed of the hoop is less than this because some of its energy is in kinetic energy of rotation. Or, equivalently, the upward tension causes the magnitude of the net force of the hoop to be less than its weight.

10.21. **IDENTIFY:** Apply Eq.(10.8).

SET UP: For an object that is rolling without slipping, $v_{cm} = R\omega$.

EXECUTE: The fraction of the total kinetic energy that is rotational is

$$\frac{(1/2)I_{cm}\omega^2}{(1/2)Mv_{cm}^2 + (1/2)I_{cm}\omega^2} = \frac{1}{1(M/I_{cm})v_{cm}^2/\omega^2} = \frac{1}{1 + (MR^2/I_{cm})}$$

(a) $I_{cm} = (1/2)MR^2$, so the above ratio is $1/3$.

(b) $I_{cm} = (2/5)MR^2$ so the above ratio is $2/7$.

(c) $I_{cm} = (2/3)MR^2$ so the ratio is $2/5$.

(d) $I_{cm} = (5/8)MR^2$ so the ratio is $5/13$.

EVALUATE: The moment of inertia of each object takes the form $I = \beta MR^2$. The ratio of rotational kinetic energy to total kinetic energy can be written as $\frac{1}{1 + 1/\beta} = \frac{\beta}{1 + \beta}$. The ratio increases as β increases.

10.22. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the translational motion of the center of mass and $\sum \tau_z = I\alpha_z$ to the rotation about the center of mass.

SET UP: Let $+x$ be down the incline and let the shell be turning in the positive direction. The free-body diagram for the shell is given in Fig.10.22. From Table 9.2, $I_{cm} = \frac{2}{3}mR^2$.

EXECUTE: $\sum F_x = ma_x$ gives $mg\sin\beta - f = ma_{cm}$. $\sum \tau_z = I\alpha_z$ gives $fR = (\frac{2}{3}mR^2)\alpha$. With $\alpha = a_{cm}/R$ this becomes $f = \frac{2}{3}ma_{cm}$. Combining the equations gives $mg\sin\beta - \frac{2}{3}ma_{cm} = ma_{cm}$ and

$a_{cm} = \frac{3g\sin\beta}{5} = \frac{3(9.80\text{ m/s}^2)(\sin 38.0°)}{5} = 3.62\text{ m/s}^2$. $f = \frac{2}{3}ma_{cm} = \frac{2}{3}(2.00\text{ kg})(3.62\text{ m/s}^2) = 4.83\text{ N}$. The friction is static since there is no slipping at the point of contact. $n = mg\cos\beta = 15.45\text{ N}$. $\mu_s = \frac{f}{n} = \frac{4.83\text{ N}}{15.45\text{ N}} = 0.313$.

(b) The acceleration is independent of *m* and doesn't change. The friction force is proportional to *m* so will double; $f = 9.66\text{ N}$. The normal force will also double, so the minimum μ_s required for no slipping wouldn't change.

EVALUATE: If there is no friction and the object slides without rolling, the acceleration is $g\sin\beta$. Friction and rolling without slipping reduce a to 0.60 times this value.

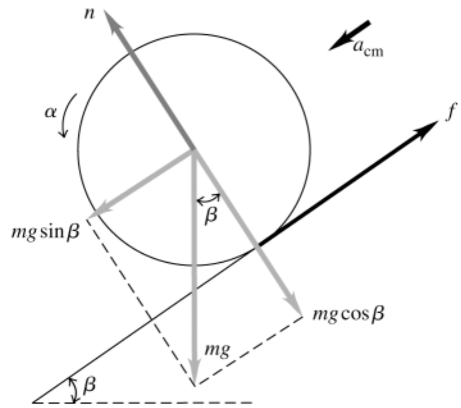


Figure 10.22

10.23. IDENTIFY: Apply $\sum \vec{F}_{ext} = m\vec{a}_{cm}$ and $\sum \tau_z = I_{cm}\alpha_z$ to the motion of the ball.

(a) SET UP: The free-body diagram is given in Figure 10.23a.

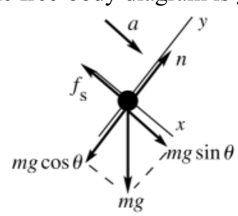


Figure 10.23a

EXECUTE: $\sum F_y = ma_y$

$n = mg\cos\theta$ and $f_s = \mu_s mg\cos\theta$

$\sum F_x = ma_x$

$mg\sin\theta - \mu_s mg\cos\theta = ma$

$g(\sin\theta - \mu_s\cos\theta) = a$ (eq. 1)

SET UP: Consider Figure 10.23b.

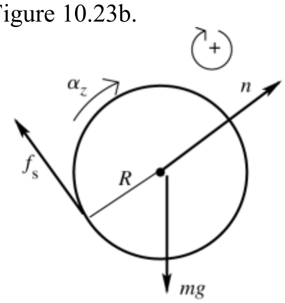


Figure 10.23b

n and *mg* act at the center of the ball and provide no torque

EXECUTE: $\sum \tau = \tau_f = \mu_s mg\cos\theta R$; $I = \frac{2}{5}mR^2$

$\sum \tau_z = I_{cm}\alpha_z$ gives $\mu_s mg\cos\theta R = \frac{2}{5}mR^2\alpha$

No slipping means $\alpha = a/R$, so $\mu_s g\cos\theta = \frac{2}{5}a$ (eq.2)

We have two equations in the two unknowns a and μ_s. Solving gives $a = \frac{5}{7}g\sin\theta$ and $\mu_s = \frac{2}{7}\tan\theta = \frac{2}{7}\tan 65.0° = 0.613$

(b) Repeat the calculation of part (a), but now $I = \frac{2}{3}mR^2$. $a = \frac{3}{5}g\sin\theta$ and $\mu_s = \frac{2}{5}\tan\theta = \frac{2}{5}\tan 65.0° = 0.858$

The value of μ_s calculated in part (a) is not large enough to prevent slipping for the hollow ball.

(c) EVALUATE: There is no slipping at the point of contact. More friction is required for a hollow ball since for a given *m* and *R* it has a larger *I* and more torque is needed to provide the same α. Note that the required μ_s is independent of the mass or radius of the ball and only depends on how that mass is distributed.

10.24. IDENTIFY: Apply conservation of energy to the motion of the marble.

SET UP: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, with $I = \frac{2}{5}MR^2$. $v_{cm} = R\omega$ for no slipping. Let $y = 0$ at the bottom of the bowl. The marble at its initial and final locations is sketched in Figure 10.24.

EXECUTE: **(a)** Motion from the release point to the bottom of the bowl: $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$ and $v = \sqrt{\frac{10}{7}gh}$.

Motion along the smooth side: The rotational kinetic energy does not change, since there is no friction torque on the marble, $\frac{1}{2}mv^2 + K_{\text{rot}} = mgh' + K_{\text{rot}}$. $h' = \frac{v^2}{2g} = \frac{\frac{10}{7}gh}{2g} = \frac{5}{7}h$

(b) $mgh = mgh'$ so $h' = h$.

EVALUATE: **(c)** With friction on both halves, all the initial potential energy gets converted back to potential energy. Without friction on the right half some of the energy is still in rotational kinetic energy when the marble is at its maximum height.

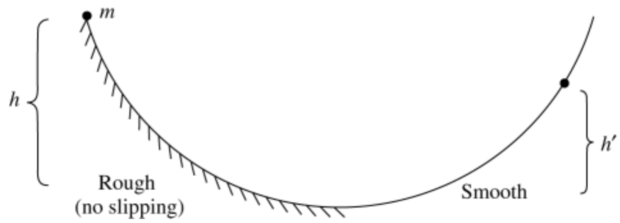
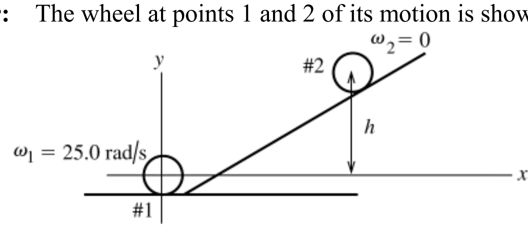


Figure 10.24

10.25. **IDENTIFY:** Apply conservation of energy to the motion of the wheel.

SET UP: The wheel at points 1 and 2 of its motion is shown in Figure 10.25.



Take $y = 0$ at the center of the wheel when it is at the bottom of the hill.

Figure 10.25

The wheel has both translational and rotational motion so its kinetic energy is $K = \frac{1}{2}I_{\text{cm}}\omega^2 + \frac{1}{2}Mv_{\text{cm}}^2$.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$W_{\text{other}} = W_{\text{fric}} = -3500$ J (the friction work is negative)

$K_1 = \frac{1}{2}I\omega_1^2 + \frac{1}{2}Mv_1^2$; $v = R\omega$ and $I = 0.800MR^2$ so

$K_1 = \frac{1}{2}(0.800)MR^2\omega_1^2 + \frac{1}{2}MR^2\omega_1^2 = 0.900MR^2\omega_1^2$

$K_2 = 0$, $U_1 = 0$, $U_2 = Mgh$

Thus $0.900MR^2\omega_1^2 + W_{\text{fric}} = Mgh$

$M = w/g = 392\ \text{N}/(9.80\ \text{m/s}^2) = 40.0\ \text{kg}$

$$h = \frac{0.900MR^2\omega_1^2 + W_{\text{fric}}}{Mg}$$

$$h = \frac{(0.900)(40.0\ \text{kg})(0.600\ \text{m})^2(25.0\ \text{rad/s})^2 - 3500\ \text{J}}{(40.0\ \text{kg})(9.80\ \text{m/s}^2)} = 11.7\ \text{m}$$

EVALUATE: Friction does negative work and reduces h.

10.26. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ and $\sum \vec{F} = m\vec{a}$ to the motion of the bowling ball.

SET UP: $a_{\text{cm}} = R\alpha$. $f_s = \mu_s n$. Let $+x$ be directed down the incline.

EXECUTE: **(a)** The free-body diagram is sketched in Figure 10.26.
The angular speed of the ball must decrease, and so the torque is provided by a friction force that acts up the hill.
(b) The friction force results in an angular acceleration, given by $I\alpha = fR$. $\sum \vec{F} = m\vec{a}$ applied to the motion of the center of mass gives $mg\sin\beta - f = ma_{\text{cm}}$, and the acceleration and angular acceleration are related by $a_{\text{cm}} = R\alpha$.

Combining, $mg\sin\beta = ma\left(1 + \frac{I}{mR^2}\right) = ma(7/5)$. $a_{\text{cm}} = (5/7)g\sin\beta$.

(c) From either of the above relations between *f* and a_{cm}, $f = \frac{2}{5}ma_{\text{cm}} = \frac{2}{7}mg\sin\beta \le \mu_s n = \mu_s mg\cos\beta$.

$\mu_s \ge (2/7)\tan\beta.$

EVALUATE: If $\mu_s = 0$, $a_{\text{cm}} = mg\sin\beta$. a_{cm} is less when friction is present. The ball rolls farther uphill when friction is present, because the friction removes the rotational kinetic energy and converts it to gravitational potential energy. In the absence of friction the ball retains the rotational kinetic energy that is has initially.

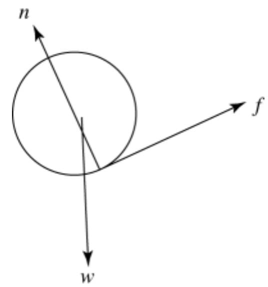


Figure 10.26

10.27. **(a) IDENTIFY:** Use Eq.(10.7) to find α_z and then use a constant angular acceleration equation to find ω_z.

SET UP: The free-body diagram is given in Figure 10.27.

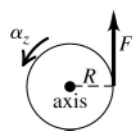


Figure 10.27

EXECUTE: Apply $\sum\tau_z = I\alpha_z$ to find the angular acceleration:

$FR = I\alpha_z$

$$\alpha_z = \frac{FR}{I} = \frac{(18.0\text{ N})(2.40\text{ m})}{2100\text{ kg}\cdot\text{m}^2} = 0.02057\text{ rad/s}^2$$

SET UP: Use the constant α_z kinematic equations to find ω_z.

$\omega_z = ?$; ω_{0z} (initially at rest); $\alpha_z = 0.02057\text{ rad/s}^2$; $t = 15.0\text{ s}$

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z t = 0 + (0.02057\text{ rad/s}^2)(15.0\text{ s}) = 0.309\text{ rad/s}$

(b) IDENTIFY and **SET UP:** Calculate the work from Eq.(10.21), using a constant angular acceleration equation to calculate $\theta - \theta_0$, or use the work-energy theorem. We will do it both ways.

EXECUTE: (1) $W = \tau_z\Delta\theta$ (Eq.(10.21))

$\Delta\theta = \theta - \theta_0 = \omega_{0z}t + \tfrac{1}{2}\alpha_z t^2 = 0 + \tfrac{1}{2}(0.02057\text{ rad/s}^2)(15.0\text{ s})^2 = 2.314\text{ rad}$

$t_z = FR = (18.0\text{ N})(2.40\text{ m}) = 43.2\text{ N}\cdot\text{m}$

Then $W = \tau_z\Delta\theta = (43.2\text{ N}\cdot\text{m})(2.314\text{ rad}) = 100\text{ J}$.

or

(2) $W_{\text{tot}} = K_2 - K_1$ (the work-energy relation from chapter 6)

$W_{\text{tot}} = W$, the work done by the child

$K_1 = 0$; $K_2 = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(2100\text{ kg}\cdot\text{m}^2)(0.309\text{ rad/s})^2 = 100\text{ J}$

Thus $W = 100\text{ J}$, the same as before.

EVALUATE: Either method yields the same result for *W*.

(c) IDENTIFY and **SET UP:** Use Eq.(6.15) to calculate P_{av}

EXECUTE: $P_{\text{av}} = \dfrac{\Delta W}{\Delta t} = \dfrac{100\text{ J}}{15.0\text{ s}} = 6.67\text{ W}$

EVALUATE: Work is in joules, power is in watts.

10.28. **IDENTIFY:** Apply $P = \tau\omega$ and $W = \tau\Delta\theta$.

SET UP: *P* must be in watts, $\Delta\theta$ must be in radians, and ω must be in rad/s. $1\text{ rev} = 2\pi\text{ rad}$. $1\text{ hp} = 746\text{ W}$. $\pi\text{ rad/s} = 30\text{ rev/min}$.

EXECUTE: **(a)** $\tau = \dfrac{P}{\omega} = \dfrac{(175\text{ hp})(746\text{ W/hp})}{(2400\text{ rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)} = 519\text{ N}\cdot\text{m}.$

(b) $W = \tau\Delta\theta = (519\text{ N}\cdot\text{m})(2\pi\text{ rad}) = 3260\text{ J}$

EVALUATE: $\omega = 40$ rev/s, so the time for one revolution is 0.025 s. $P = 1.306 \times 10^5$ W, so in one revolution, $W = Pt = 3260$ J, which agrees with our previous result.

10.29. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ and constant angular acceleration equations to the motion of the wheel.

SET UP: 1 rev $= 2\pi$ rad. π rad/s $= 30$ rev/min.

EXECUTE: **(a)** $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t}$.

$$\tau_z = \frac{\left((1/2)(1.50\text{ kg})(0.100\text{ m})^2\right)(1200\text{ rev/min})\left(\dfrac{\pi\text{ rad/s}}{30\text{ rev/min}}\right)}{2.5\text{ s}} = 0.377\text{ N}\cdot\text{m}$$

(b) $\omega_{\text{av}}\Delta t = \dfrac{(600\text{ rev/min})(2.5\text{ s})}{60\text{ s/min}} = 25.0\text{ rev} = 157\text{ rad}.$

(c) $W = \tau\Delta\theta = (0.377\text{ N}\cdot\text{m})(157\text{ rad}) = 59.2\text{ J}$.

(d) $K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\left((1/2)(1.5\text{ kg})(0.100\text{ m})^2\right)\left((1200\text{ rev/min})\left(\dfrac{\pi\text{ rad/s}}{30\text{ rev/min}}\right)\right)^2 = 59.2\text{ J}$.

the same as in part (c).

EVALUATE: The agreement between the results of parts (c) and (d) illustrates the work-energy theorem

10.30. **IDENTIFY:** The power output of the motor is related to the torque it produces and to its angular velocity by $P = \tau_z\omega_z$, where ω_z must be in rad/s.

SET UP: The work output of the motor in 60.0 s is $\dfrac{2}{3}(9.00\text{ kJ}) = 6.00\text{ kJ}$, so $P = \dfrac{6.00\text{ kJ}}{60.0\text{ s}} = 100\text{ W}$.

$\omega_z = 2500\text{ rev/min} = 262\text{ rad/s}$.

EXECUTE: $\tau_z = \dfrac{P}{\omega_z} = \dfrac{100\text{ W}}{262\text{ rad/s}} = 0.382\text{ N}\cdot\text{m}$

EVALUATE: For a constant power output, the torque developed decreases and the rotation speed of the motor increases.

10.31. **IDENTIFY:** Apply $\tau = FR$ and $P = \tau\omega$.

SET UP: 1 hp $= 746$ W. π rad/s $= 30$ rev/min

EXECUTE: **(a)** With no load, the only torque to be overcome is friction in the bearings (neglecting air friction), and the bearing radius is small compared to the blade radius, so any frictional torque can be neglected.

(b) $F = \dfrac{\tau}{R} = \dfrac{P/\omega}{R} = \dfrac{(1.9\text{ hp})(746\text{ W/hp})}{(2400\text{ rev/min})\left(\dfrac{\pi\text{ rad/s}}{30\text{ rev/min}}\right)(0.086\text{ m})} = 65.6\text{ N}.$

EVALUATE: In $P = I\omega$, τ must be in watts and ω must be in rad/s.

10.32. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ to the motion of the propeller and then use constant acceleration equations to analyze the motion. $W = \tau\Delta\theta$.

SET UP: $I = \frac{1}{2}mL^2 = \frac{1}{2}(117\text{ kg})(2.08\text{ m})^2 = 42.2\text{ kg}\cdot\text{m}^2$.

EXECUTE: **(a)** $\alpha = \dfrac{\tau}{I} = \dfrac{1950\text{ N}\cdot\text{m}}{42.2\text{ kg}\cdot\text{m}^2} = 46.2\text{ rad/s}^2.$

(b) $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ gives $\omega = \sqrt{2\alpha\theta} = \sqrt{2(46.2\text{ rad/s}^2)(5.0\text{ rev})(2\pi\text{ rad/rev})} = 53.9\text{ rad/s}.$

(c) $W = \tau\theta = (1950\text{ N}\cdot\text{m})(5.00\text{ rev})(2\pi\text{ rad/rev}) = 6.13\times 10^4\text{ J}.$

(d) $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{53.9\text{ rad/s}}{46.2\text{ rad/s}^2} = 1.17\text{ s}$. $P_{\text{av}} = \dfrac{W}{\Delta t} = \dfrac{6.13\times 10^4\text{ J}}{1.17\text{ s}} = 52.5\text{ kW}$.

EVALUATE: $P = \tau\omega$. τ is constant and ω is linear in *t*, so P_{av} is half the instantaneous power at the end of the 5.00 revolutions. We could also calculate W from $W = \Delta K = \frac{1}{2}I\omega^2 = \frac{1}{2}(42.2\text{ kg}\cdot\text{m}^2)(53.9\text{ rad/s})^2 = 6.13\times 10^4\text{ J}$.

10.33. **(a) IDENTIFY** and **SET UP:** Use Eq.(10.23) and solve for τ_z.

$P = \tau_z\omega_z$, where ω_z must be in rad/s

EXECUTE: $\omega_z = (4000\text{ rev/min})(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 418.9\text{ rad/s}$

$\tau_z = \dfrac{P}{\omega_z} = \dfrac{1.50\times 10^5\text{ W}}{418.9\text{ rad/s}} = 358\text{ N}\cdot\text{m}$

(b) IDENTIFY and SET UP: Apply $\sum \vec{F} = m\vec{a}$ to the drum. Find the tension T in the rope using τ_z from part (a). The system is sketched in Figure 10.33.

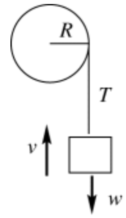


Figure 10.33

EXECUTE: v constant implies $a = 0$ and $T = w$

$\tau_z = TR$ implies

$T = \tau_z / R = 358\ \text{N}\cdot\text{m}/0.200\ \text{m} = 1790\ \text{N}$

Thus a weight $w = 1790\ \text{N}$ can be lifted.

(c) IDENTIFY and SET UP: Use $v = R\omega$.

EXECUTE: The drum has $\omega = 418.9$ rad/s, so $v = (0.200\ \text{m})(418.9\ \text{rad/s}) = 83.8\ \text{m/s}$

EVALUATE: The rate at which T is doing work on the drum is $P = Tv = (1790\ \text{N})(83.8\ \text{m/s}) = 150\ \text{kW}$. This agrees with the work output of the motor.

10.34. IDENTIFY: $L = I\omega$ and $I = I_{\text{disk}} + I_{\text{woman}}$.

SET UP: $\omega = 0.50\ \text{rev/s} = 3.14\ \text{rad/s}$. $I_{\text{disk}} = \frac{1}{2} m_{\text{disk}} R^2$ and $I_{\text{woman}} = m_{\text{woman}} R^2$.

EXECUTE: $I = (55\ \text{kg} + 50.0\ \text{kg})(4.0\ \text{m})^2 = 1680\ \text{kg}\cdot\text{m}^2$. $L = (1680\ \text{kg}\cdot\text{m}^2)(3.14\ \text{rad/s}) = 5.28\times 10^3\ \text{kg}\cdot\text{m}^2/\text{s}$

EVALUATE: The disk and the woman have similar values of I, even though the disk has twice the mass.

10.35. (a) IDENTIFY: Use $L = mvr\sin\phi$ (Eq.(10.25)):

SET UP: Consider Figure 10.35.

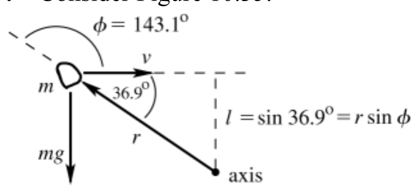


Figure 10.35

EXECUTE: $L = mvr\sin\phi =$

$(2.00\ \text{kg})(12.0\ \text{m/s})(8.00\ \text{m})\sin 143.1°$

$L = 115\ \text{kg}\cdot\text{m}^2/\text{s}$

To find the direction of $\vec{L}$ apply the right-hand rule by turning $\vec{r}$ into the direction of $\vec{v}$ by pushing on it with the fingers of your right hand. Your thumb points into the page, in the direction of $\vec{L}$.

(b) IDENTIFY and SET UP: By Eq.(10.26) the rate of change of the angular momentum of the rock equals the torque of the net force acting on it.

EXECUTE: $\tau = mg(8.00\ \text{m})\cos 36.9° = 125\ \text{kg}\cdot\text{m}^2/\text{s}^2$

To find the direction of $\vec{\tau}$ and hence of $d\vec{L}/dt$, apply the right-hand rule by turning $\vec{r}$ into the direction of the gravity force by pushing on it with the fingers of your right hand. Your thumb points out of the page, in the direction of $d\vec{L}/dt$.

EVALUATE: $\vec{L}$ and $d\vec{L}/dt$ are in opposite directions, so L is decreasing. The gravity force is accelerating the rock downward, toward the axis. Its horizontal velocity is constant but the distance l is decreasing and hence L is decreasing.

10.36. IDENTIFY: $L_z = I\omega_z$

SET UP: For a particle of mass m moving in a circular path at a distance r from the axis, $I = mr^2$ and $v = r\omega$. For a uniform sphere of mass M and radius R and an axis through its center, $I = \frac{2}{5}MR^2$. The earth has mass $m_{\text{E}} = 5.97\times 10^{24}\ \text{kg}$, radius $R_{\text{E}} = 6.38\times 10^6\ \text{m}$ and orbit radius $r = 1.50\times 10^{11}\ \text{m}$. The earth completes one rotation on its axis in $24\ \text{h} = 86{,}400\ \text{s}$ and one orbit in $1\ \text{y} = 3.156\times 10^7\ \text{s}$.

EXECUTE: **(a)** $L_z = I\omega_z = mr^2\omega_z = (5.97\times 10^{24}\ \text{kg})(1.50\times 10^{11}\ \text{m})^2\left(\frac{2\pi\ \text{rad}}{3.156\times 10^7\ \text{s}}\right) = 2.67\times 10^{40}\ \text{kg}\cdot\text{m}^2/\text{s}$.

The radius of the earth is much less than its orbit radius, so it is very reasonable to model it as a particle for this calculation.

(b) $L_z = I\omega_z = \left(\frac{2}{5}MR^2\right)\omega = \frac{2}{5}(5.97\times 10^{24}\ \text{kg})(6.38\times 10^6\ \text{m})^2\left(\frac{2\pi\ \text{rad}}{86{,}400\ \text{s}}\right) = 7.07\times 10^{33}\ \text{kg}\cdot\text{m}^2/\text{s}$

EVALUATE: The angular momentum associated with each of these motions is very large.

10.37. **IDENTIFY** and **SET UP:** Use $L = I\omega$

EXECUTE: The second hand makes 1 revolution in 1 minute, so
$\omega = (1.00 \text{ rev/min})(2\pi \text{ rad/1 rev})(1 \text{ min/60 s}) = 0.1047 \text{ rad/s}$

For a slender rod, with the axis about one end,
$I = \frac{1}{3}ML^2 = \frac{1}{3}(6.00\times10^{-3} \text{ kg})(0.150 \text{ m})^2 = 4.50\times10^{-5} \text{ kg}\cdot\text{m}^2$

Then $L = I\omega = (4.50\times10^{-5} \text{ kg}\cdot\text{m}^2)(0.1047 \text{ rad/s}) = 4.71\times10^{-6} \text{ kg}\cdot\text{m}^2/\text{s}.$

EVALUATE: $\vec{L}$ is clockwise.

10.38. **IDENTIFY:** $\omega_z = d\theta/dt$. $L_z = I\omega_z$ and $\tau_z = dL_z dt$.

SET UP: For a hollow, thin-walled sphere rolling about an axis through its center, $I = \frac{2}{3}MR^2$. $R = 0.240 \text{ m}$.

EXECUTE: **(a)** $A = 1.50 \text{ rad/s}^2$ and $B = 1.10 \text{ rad/s}^4$, so that $\theta(t)$ will have units of radians.

(b) (i) $\omega_z = \dfrac{d\theta}{dt} = 2At + 4Bt^3$. At $t = 3.00 \text{ s}$, $\omega_z = 2(1.50 \text{ rad/s}^2)(3.00 \text{ s}) + 4(1.10 \text{ rad/s}^4)(3.00 \text{ s})^3 = 128 \text{ rad/s}$.

$L_z = (\frac{2}{3}MR^2)\omega_z = \frac{2}{3}(12.0 \text{ kg})(0.240 \text{ m})^2(128 \text{ rad/s}) = 59.0 \text{ kg}\cdot\text{m}^2\text{/s}$.

(ii) $\tau_z = \dfrac{dL_z}{dt} = I\dfrac{d\omega_z}{dt} = I(2A + 12Bt^2)$ and

$\tau_z = \frac{2}{3}(12.0 \text{ kg})(0.240 \text{ m})^2(2[1.50 \text{ rad/s}^2] + 12[1.10 \text{ rad/s}^4][3.00 \text{ s}]^2) = 56.1 \text{ N}\cdot\text{m}$.

EVALUATE: The angular speed of rotation is increasing. This increase is due to an acceleration α_z that is produced by the torque on the sphere. When *I* is constant, as it is here, $\tau_z = dL_z dt = Id\omega_z/dt = I\alpha_z$ and Equations (10.29) and (10.7) are identical.

10.39. **IDENTIFY:** Apply conservation of angular momentum.

SET UP: For a uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$.

EXECUTE: The moment of inertia is proportional to the square of the radius, and so the angular velocity will be proportional to the inverse of the square of the radius, and the final angular velocity is

$$\omega_2 = \omega_1\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{2\pi \text{ rad}}{(30 \text{ d})(86{,}400 \text{ s/d})}\right)\left(\frac{7.0\times10^5 \text{ km}}{16 \text{ km}}\right)^2 = 4.6\times10^3 \text{ rad/s}.$$

EVALUATE: $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega$. *L* is constant and ω increases by a large factor, so there is a large increase in the rotational kinetic energy of the star. This energy comes from potential energy associated with the gravity force within the star.

10.40. **IDENTIFY** and **SET UP:** $\vec{L}$ is conserved if there is no net external torque.
Use conservation of angular momentum to find ω at the new radius and use $K = \frac{1}{2}I\omega^2$ to find the change in kinetic energy, which is equal to the work done on the block.

EXECUTE: **(a)** Yes, angular momentum is conserved. The moment arm for the tension in the cord is zero so this force exerts no torque and there is no net torque on the block.

(b) $L_1 = L_2$ so $I_1\omega_1 = I_2\omega_2$. Block treated as a point mass, so $I = mr^2$, where *r* is the distance of the block from the hole.

$mr_1^2\omega_1 = mr_2^2\omega_2$

$$\omega_2 = \left(\frac{r_1}{r_2}\right)^2\omega_1 = \left(\frac{0.300 \text{ m}}{0.150 \text{ m}}\right)^2(1.75 \text{ rad/s}) = 7.00 \text{ rad/s}$$

(c) $K_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}mr_1^2\omega_1^2 = \frac{1}{2}mv_1^2$

$v_1 = r_1\omega_1 = (0.300 \text{ m})(1.75 \text{ rad/s}) = 0.525 \text{ m/s}$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.0250 \text{ kg})(0.525 \text{ m/s})^2 = 0.00345 \text{ J}$

$K_2 = \frac{1}{2}mv_2^2$

$v_2 = r_2\omega_2 = (0.150 \text{ m})(7.00 \text{ rad/s}) = 1.05 \text{ m/s}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.0250 \text{ kg})(1.05 \text{ m/s})^2 = 0.01378 \text{ J}$

$\Delta K = K_2 - K_1 = 0.01378 \text{ J} - 0.00345 \text{ J} = 0.0103 \text{ J}$

(d) $W_{\text{tot}} = \Delta K$

But $W_{\text{tot}} = W$, the work done by the tension in the cord, so $W = 0.0103$ J

EVALUATE: Smaller *r* means smaller *I*. $L = I\omega$ is constant so ω increases and *K* increases. The work done by the tension is positive since it is directed inward and the block moves inward, toward the hole.

10.41. **IDENTIFY:** Apply conservation of angular momentum to the motion of the skater.

SET UP: For a thin-walled hollow cylinder $I = mR^2$. For a slender rod rotating about an axis through its center, $I = \frac{1}{12}ml^2$.

EXECUTE: $L_{\text{i}} = L_{\text{f}}$ so $I_{\text{i}}\omega_{\text{i}} = I_{\text{f}}\omega_{\text{f}}$.

$I_{\text{i}} = 0.40\ \text{kg}\cdot\text{m}^2 + \frac{1}{12}(8.0\ \text{kg})(1.8\ \text{m})^2 = 2.56\ \text{kg}\cdot\text{m}^2$. $I_{\text{f}} = 0.40\ \text{kg}\cdot\text{m}^2 + (8.0\ \text{kg})(0.25\ \text{m})^2 = 0.90\ \text{kg}\cdot\text{m}^2$.

$$\omega_{\text{f}} = \left(\frac{I_{\text{i}}}{I_{\text{f}}}\right)\omega_{\text{i}} = \left(\frac{2.56\ \text{kg}\cdot\text{m}^2}{0.90\ \text{kg}\cdot\text{m}^2}\right)(0.40\ \text{rev/s}) = 1.14\ \text{rev/s}.$$

EVALUATE: $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega$. ω increases and *L* is constant, so *K* increases. The increase in kinetic energy comes from the work done by the skater when he pulls in his hands.

10.42. **IDENTIFY:** Apply conservation of angular momentum to the diver.

SET UP: The number of revolutions she makes in a certain time is proportional to her angular velocity. The ratio of her untucked to tucked angular velocity is $(3.6\ \text{kg}\cdot\text{m}^2)/(18\ \text{kg}\cdot\text{m}^2)$.

EXECUTE: If she had tucked, she would have made $(2\ \text{rev})(3.6\ \text{kg}\cdot\text{m}^2)/(18\ \text{kg}\cdot\text{m}^2) = 0.40\ \text{rev}$ in the last 1.0 s, so she would have made $(0.40\ \text{rev})(1.5/1.0) = 0.60\ \text{rev}$ in the total 1.5 s.

EVALUATE: Untucked she rotates slower and completes fewer revolutions.

10.43. **IDENTIFY** and **SET UP:** There is no net external torque about the rotation axis so the angular momentum $L = I\omega$ is conserved.

EXECUTE: **(a)** $L_1 = L_2$ gives $I_1\omega_1 = I_2\omega_2$, so $\omega_2 = (I_1/I_2)\omega_1$

$I_1 = I_{\text{tt}} = \frac{1}{2}MR^2 = \frac{1}{2}(120\ \text{kg})(2.00\ \text{m})^2 = 240\ \text{kg}\cdot\text{m}^2$

$I_2 = I_{\text{tt}} + I_{\text{p}} = 240\ \text{kg}\cdot\text{m}^2 + mR^2 = 240\ \text{kg}\cdot\text{m}^2 + (70\ \text{kg})(2.00\ \text{m})^2 = 520\ \text{kg}\cdot\text{m}^2$

$\omega_2 = (I_1/I_2)\omega_1 = (240\ \text{kg}\cdot\text{m}^2/520\ \text{kg}\cdot\text{m}^2)(3.00\ \text{rad/s}) = 1.38\ \text{rad/s}$

(b) $K_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}(240\ \text{kg}\cdot\text{m}^2)(3.00\ \text{rad/s})^2 = 1080\ \text{J}$

$K_2 = \frac{1}{2}I_2\omega_2^2 = \frac{1}{2}(520\ \text{kg}\cdot\text{m}^2)(1.38\ \text{rad/s})^2 = 495\ \text{J}$

EVALUATE: The kinetic energy decreases because of the negative work done on the turntable and the parachutist by the friction force between these two objects.
The angular speed decreases because *I* increases when the parachutist is added to the system.

10.44. **IDENTIFY:** Apply conservation of angular momentum to the collision.

SET UP: Let the width of the door be *l*. The initial angular momentum of the mud is $mv(l/2)$, since it strikes the door at its center. For the axis at the hinge, $I_{\text{door}} = \frac{1}{3}Ml^2$ and $I_{\text{mud}} = m(l/2)^2$.

EXECUTE: $$\omega = \frac{L}{I} = \frac{mv(l/2)}{(1/3)Ml^2 + m(l/2)^2}.$$

$$\omega = \frac{(0.500\ \text{kg})(12.0\ \text{m/s})(0.500\ \text{m})}{(1/3)(40.0\ \text{kg})(1.00\ \text{m})^2 + (0.500\ \text{kg})(0.500\ \text{m})^2} = 0.223\ \text{rad/s}.$$

Ignoring the mass of the mud in the denominator of the above expression gives $\omega = 0.225\ \text{rad/s}$, so the mass of the mud in the moment of inertia does affect the third significant figure.

EVALUATE: Angular momentum is conserved but there is a large decrease in the kinetic energy of the system.

10.45. **(a) IDENTIFY** and **SET UP:** Apply conservation of angular momentum $\vec{L}$, with the axis at the nail. Let object *A* be the bug and object *B* be the bar. Initially, all objects are at rest and $L_1 = 0$. Just after the bug jumps, it has angular momentum in one direction of rotation and the bar is rotating with angular velocity ω_B in the opposite direction.

EXECUTE: $L_2 = m_A v_A r - I_B\omega_B$ where $r = 1.00$ m and $I_B = \frac{1}{3}m_B r^2$

$L_1 = L_2$ gives $m_A v_A r = \frac{1}{3}m_B r^2\omega_B$

$$\omega_B = \frac{3m_A v_A}{m_B r} = 0.120\ \text{rad/s}$$

(b) $K_1 = 0$; $K_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}I_B\omega_B^2 =$

$\frac{1}{2}(0.0100\text{ kg})(0.200\text{ m/s})^2 + \frac{1}{2}\left(\frac{1}{3}[0.0500\text{ kg}][1.00\text{ m}]^2\right)(0.120\text{ rad/s})^2 = 3.2\times10^{-4}\text{ J}.$

The increase in kinetic energy comes from work done by the bug when it pushes against the bar in order to jump.

EVALUATE: There is no external torque applied to the system and the total angular momentum of the system is constant. There are internal forces, forces the bug and bar exert on each other. The forces exert torques and change the angular momentum of the bug and the bar, but these changes are equal in magnitude and opposite in direction. These internal forces do positive work on the two objects and the kinetic energy of each object and of the system increases.

10.46. **IDENTIFY:** Apply conservation of angular momentum to the system of earth plus asteroid.

SET UP: Take the axis to be the earth's rotation axis. The asteroid may be treated as a point mass and it has zero angular momentum before the collision, since it is headed toward the center of the earth. For the earth, $L_z = I\omega_z$ and $I = \frac{2}{5}MR^2$, where M is the mass of the earth and R is its radius. The length of a day is $T = \frac{2\pi\text{ rad}}{\omega}$, where ω is the earth's angular rotation rate.

EXECUTE: Conservation of angular momentum applied to the collision between the earth and asteroid gives $\frac{2}{5}MR^2\omega_1 = (mR^2 + \frac{2}{5}MR^2)\omega_2$ and $m = \frac{2}{5}M\left(\frac{\omega_1 - \omega_2}{\omega_2}\right)$. $T_2 = 1.250T_1$ gives $\frac{1}{\omega_2} = \frac{1.250}{\omega_1}$ and $\omega_1 = 1.250\omega_2$.

$\frac{\omega_1 - \omega_2}{\omega_2} = 0.250$. $m = \frac{2}{5}(0.250)M = 0.100M$.

EVALUATE: If the asteroid hit the surface of the earth tangentially it could have some angular momentum with respect to the earth's rotation axis, and could either speed up or slow down the earth's rotation rate.

10.47. **IDENTIFY:** Apply conservation of angular momentum to the collision.

SET UP: The system before and after the collision is sketched in Figure 10.47. Let counterclockwise rotation be positive. The bar has $I = \frac{1}{3}m_2L^2$.

EXECUTE: **(a)** Conservation of angular momentum: $m_1v_0d = -m_1vd + \frac{1}{3}m_2L^2\omega$.

$$(3.00\text{ kg})(10.0\text{ m/s})(1.50\text{ m}) = -(3.00\text{ kg})(6.00\text{ m/s})(1.50\text{ m}) + \frac{1}{3}\left(\frac{90.0\text{ N}}{9.80\text{ m/s}^2}\right)(2.00\text{ m})^2\omega$$

$\omega = 5.88\text{ rad/s}$.

(b) There are no unbalanced torques about the pivot, so angular momentum is conserved. But the pivot exerts an unbalanced horizontal external force on the system, so the linear momentum is not conserved.

EVALUATE: Kinetic energy is not conserved in the collision.

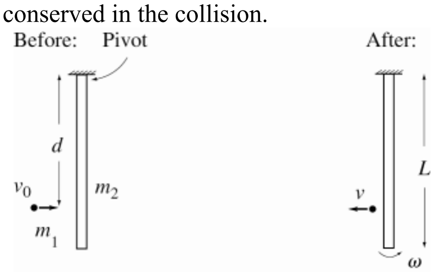


Figure 10.47

10.48. **IDENTIFY:** $d\vec{L} = \vec{\tau}dt$, so $d\vec{L}$ is in the direction of $\vec{\tau}$.

SET UP: The direction of $\vec{\omega}$ is given by the right-hand rule, as described in Figure 10.26 in the textbook.

EXECUTE: The sketches are given in Figures 10.48a–d.

EVALUATE: In figures (a) and (c) the precession is counterclockwise and in figures (b) and (d) it is clockwise. When the direction of either $\vec{\omega}$ or $\vec{\tau}$ reverses, the direction of precession reverses.

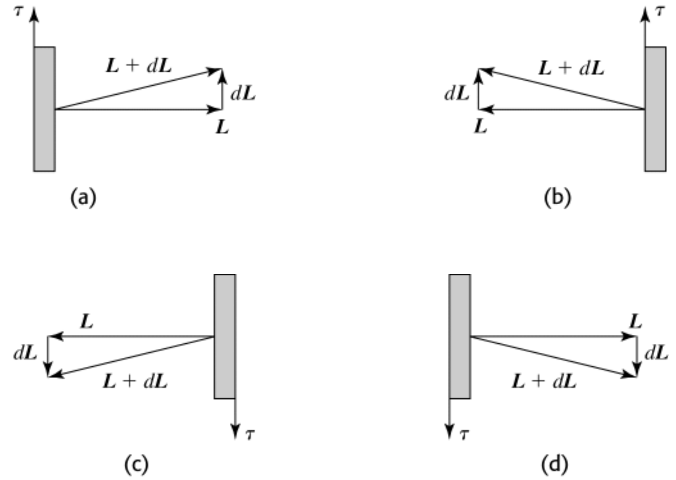


Figure 10.48

10.49. IDENTIFY: The precession angular velocity is $\Omega = \dfrac{wr}{I\omega}$, where ω is in rad/s. Also apply $\sum \vec{F} = m\vec{a}$ to the gyroscope.

SET UP: The total mass of the gyroscope is $m_r + m_f = 0.140\ \text{kg} + 0.0250\ \text{kg} = 0.165\ \text{kg}$.

$\Omega = \dfrac{2\pi\ \text{rad}}{T} = \dfrac{2\pi\ \text{rad}}{2.20\ \text{s}} = 2.856\ \text{rad/s}$.

EXECUTE: **(a)** $F_p = w_{tot} = (0.165\ \text{kg})(9.80\ \text{m/s}^2) = 1.62\ \text{N}$

(b) $\omega = \dfrac{wr}{I\Omega} = \dfrac{(0.165\ \text{kg})(9.80\ \text{m/s}^2)(0.0400\ \text{m})}{(1.20\times10^{-4}\ \text{kg}\cdot\text{m}^2)(2.856\ \text{rad/s})} = 189\ \text{rad/s} = 1.80\times10^3\ \text{rev/min}$

(c) If the figure in the problem is viewed from above, $\vec{\tau}$ is in the direction of the precession and $\vec{L}$ is along the axis of the rotor, away from the pivot.

EVALUATE: There is no vertical component of acceleration associated with the motion, so the force from the pivot equals the weight of the gyroscope. The larger ω is, the slower the rate of precession.

10.50. IDENTIFY: The precession angular speed is related to the acceleration due to gravity by Eq.(10.33), with $w = mg$.

SET UP: $\Omega_E = 0.50\ \text{rad/s}$, $g_E = g$ and $g_M = 0.165g$. For the gyroscope, *m*, *r*, *I*, and ω are the same on the moon as on the earth.

EXECUTE: $\Omega = \dfrac{mgr}{I\omega}$. $\dfrac{\Omega}{g} = \dfrac{mr}{I\omega} = \text{constant}$, so $\dfrac{\Omega_E}{g_E} = \dfrac{\Omega_M}{g_M}$.

$\Omega_M = \Omega_E\left(\dfrac{g_M}{g_E}\right) = 0.165\Omega_E = (0.165)(0.50\ \text{rad/s}) = 0.0825\ \text{rad/s}$.

EVALUATE: In the limit that $g \to 0$ the precession rate $\to 0$.

10.51. IDENTIFY and **SET UP:** Apply Eq.(10.33).

EXECUTE: **(a)** halved

(b) doubled (assuming that the added weight is distributed in such a way that *r* and *I* are not changed)

(c) halved (assuming that w and r are not changed)

(d) doubled

(e) unchanged.

EVALUATE: Ω is directly proportional to *w* and *r* and is inversely proportional to *I* and ω.

10.52. IDENTIFY: Apply Eq.(10.33), where $\tau = wr$.

SET UP: $1\ \text{day} = 86{,}400\ \text{s}$. $1\ \text{yr} = 3.156\times10^7\ \text{s}$. The earth has mass $M = 5.97\times10^{24}\ \text{kg}$ and radius $R = 6.38\times10^6\ \text{m}$. For a uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$.

EXECUTE: **(a)** $\tau = I\omega\Omega = (2/5)MR^2\omega\Omega$. Using $\omega = \dfrac{2\pi\ \text{rad}}{86{,}400\ \text{s}}$ and $\Omega = \dfrac{2\pi\ \text{rad}}{(26{,}000\ \text{y})(3.156\times10^7\ \text{s/y})}$, and the mass and radius of the earth from Appendix F, $\tau = 5.4\ \text{N}\cdot\text{m}$.

EVALUATE: If the torque is applied by the sun, $r = 1.5\times10^{11}\ \text{m}$ and $F_\perp = 3.6\times10^{11}\ \text{N}$.

10.53. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ and constant acceleration equations to the motion of the grindstone.

SET UP: Let the direction of rotation of the grindstone be positive. The friction force is $f = \mu_k n$ and produces torque fR. $\omega = \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 4\pi$ rad. $I = \frac{1}{2}MR^2 = 1.69 \text{ kg}\cdot\text{m}^2$.

EXECUTE: **(a)** The net torque must be

$$\tau = I\alpha = I\frac{\omega_z - \omega_{0z}}{t} = (1.69 \text{ kg}\cdot\text{m}^2)\frac{4\pi \text{ rad/s}}{9.00 \text{ s}} = 2.36 \text{ N}\cdot\text{m}.$$

This torque must be the sum of the applied force FR and the opposing frictional torques τ_f at the axle and $fR = \mu_k nR$ due to the knife. $F = \frac{1}{R}(\tau + \tau_f + \mu_k nR)$.

$$F = \frac{1}{0.500 \text{ m}}\big((2.36 \text{ N}\cdot\text{m}) + (6.50 \text{ N}\cdot\text{m}) + (0.60)(160 \text{ N})(0.260 \text{ m})\big) = 67.6 \text{ N}.$$

(b) To maintain a constant angular velocity, the net torque τ is zero, and the force F' is

$$F' = \frac{1}{0.500 \text{ m}}(6.50 \text{ N}\cdot\text{m} + 24.96 \text{ N}\cdot\text{m}) = 62.9 \text{ N}.$$

(c) The time t needed to come to a stop is found by taking the magnitudes in Eq.(10.27), with $\tau = \tau_f$ constant;

$$t = \frac{L}{\tau_f} = \frac{\omega I}{\tau_f} = \frac{(4\pi \text{ rad/s})\left(1.69 \text{ kg}\cdot\text{m}^2\right)}{6.50 \text{ N}\cdot\text{m}} = 3.27 \text{ s}.$$

EVALUATE: The time for a given change in ω is proportional to α, which is in turn proportional to the net torque, so the time in part (c) can also be found as $t = (9.00 \text{ s})\frac{2.36 \text{ N}\cdot\text{m}}{6.50 \text{ N}\cdot\text{m}}$.

10.54. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ and use the constant acceleration equations to relate α to the motion.

SET UP: Let the direction the wheel is rotating be positive. $100 \text{ rev/min} = 10.47 \text{ rad/s}$

EXECUTE: **(a)** $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = \frac{\omega_z - \omega_{0z}}{t} = \frac{10.47 \text{ rad/s} - 0}{2.00 \text{ s}} = 5.23 \text{ rad/s}^2$.

$$I = \frac{\sum \tau_z}{\alpha_z} = \frac{5.00 \text{ N}\cdot\text{m}}{5.23 \text{ rad/s}^2} = 0.956 \text{ kg}\cdot\text{m}^2$$

(b) $\omega_{0z} = 10.47$ rad/s, $\omega_z = 0$, $t = 125$ s. $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = \frac{\omega_z - \omega_{0z}}{t} = \frac{0 - 10.47 \text{ rad/s}}{125 \text{ s}} = -0.0838 \text{ rad/s}^2$

$$\sum \tau_z = I\alpha_z = (0.956 \text{ kg}\cdot\text{m}^2)(-0.0838 \text{ rad/s}^2) = -0.0801 \text{ N}\cdot\text{m}$$

(c) $\theta = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t = \left(\frac{10.47 \text{ rad/s} + 0}{2}\right)(125 \text{ s}) = 654 \text{ rad} = 104 \text{ rev}$

EVALUATE: The applied net torque ($5.00 \text{ N}\cdot\text{m}$) is much larger than the magnitude of the friction torque ($0.0801 \text{ N}\cdot\text{m}$), so the time of 2.00 s that it takes the wheel to reach an angular speed of 100 rev/min is much less than the 125 s it takes the wheel to be brought to rest by friction.

10.55. **IDENTIFY** and **SET UP:** Apply $v = r\omega$. v is the tangential speed of a point on the rim of the wheel and equals the linear speed of the car.

EXECUTE: **(a)** $v = 60 \text{ mph} = 26.82 \text{ m/s}$

$r = 12 \text{ in.} = 0.3048 \text{ m}$

$\omega = \frac{v}{r} = 88.0 \text{ rad/s} = 14.0 \text{ rev/s} = 840 \text{ rpm}$

(b) Same ω as in part (a) since speedometer reads same.

$r = 15 \text{ in.} = 0.381 \text{ m}$

$v = r\omega = (0.381 \text{ m})(88.0 \text{ rad/s}) = 33.5 \text{ m/s} = 75 \text{ mph}$

(c) $v = 50 \text{ mph} = 22.35 \text{ m/s}$

$r = 10 \text{ in.} = 0.254 \text{ m}$

$\omega = \frac{v}{r} = 88.0$ rad/s. This is the same as for 60 mph with correct tires, so speedometer read 60 mph.

EVALUATE: For a given ω, v increases when r increases.

10.56. **IDENTIFY:** The kinetic energy of the disk is $K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2$. As it falls its gravitational potential energy decreases and its kinetic energy increases. The only work done on the disk is the work done by gravity, so $K_1 + U_1 = K_2 + U_2$.

SET UP: $I_{cm} = \frac{1}{2}M(R_2^2 + R_1^2)$, where $R_1 = 0.300$ m and $R_2 = 0.500$ m. $v_{cm} = R_2\omega$. Take $y_1 = 0$, so $y_2 = -1.20$ m.

EXECUTE: $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$, $U_1 = 0$. $K_2 = -U_2$. $\frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = -Mgy_2$.

$\frac{1}{2}I_{cm}\omega^2 = \frac{1}{4}M\left(1 + [R_1/R_2]^2\right)v_{cm}^2 = 0.340Mv_{cm}^2$. Then $0.840Mv_{cm}^2 = -Mgy_2$ and

$$v_{cm} = \sqrt{\frac{-gy_2}{0.840}} = \sqrt{\frac{-(9.80\text{ m/s}^2)(-1.20\text{ m})}{0.840}} = 3.74\text{ m/s}$$

EVALUATE: A point mass in free-fall acquires a speed of 4.85 m/s after falling 1.20 m. The disk has a value of v_{cm} that is less than this, because some of the original gravitational potential energy has been converted to rotational kinetic energy.

10.57. **IDENTIFY:** Use $\sum\tau_z = I\alpha_z$ to find the angular acceleration just after the ball falls off and use conservation of energy to find the angular velocity of the bar as it swings through the vertical position.

SET UP: The axis of rotation is at the axle. For this axis the bar has $I = \frac{1}{12}m_{bar}L^2$, where $m_{bar} = 3.80$ kg and $L = 0.800$ m. Energy conservation gives $K_1 + U_1 = K_2 + U_2$. The gravitational potential energy of the bar doesn't change. Let $y_1 = 0$, so $y_2 = -L/2$.

EXECUTE: **(a)** $\tau_z = m_{ball}g(L/2)$ and $I = I_{ball} + I_{bar} = \frac{1}{12}m_{bar}L^2 + m_{ball}(L/2)^2$. $\sum\tau_z = I\alpha_z$ gives

$$\alpha_z = \frac{m_{ball}g(L/2)}{\frac{1}{12}m_{bar}L^2 + m_{ball}(L/2)^2} = \frac{2g}{L}\left(\frac{m_{ball}}{m_{ball} + m_{bar}/3}\right) \text{ and } \alpha_z = \frac{2(9.80\text{ m/s}^2)}{0.800\text{ m}}\left(\frac{2.50\text{ kg}}{2.50\text{ kg} + [3.80\text{ kg}]/3}\right) = 16.3\text{ rad/s}^2.$$

(b) As the bar rotates, the moment arm for the weight of the ball decreases and the angular acceleration of the bar decreases.

(c) $K_1 + U_1 = K_2 + U_2$. $0 = K_2 + U_2$. $\frac{1}{2}(I_{bar} + I_{ball})\omega^2 = -m_{ball}g(-L/2)$.

$$\omega = \sqrt{\frac{m_{ball}gL}{m_{ball}L^2/4 + m_{bar}L^2/12}} = \sqrt{\frac{g}{L}\left(\frac{4m_{ball}}{m_{ball} + m_{bar}/3}\right)} = \sqrt{\frac{9.80\text{ m/s}^2}{0.800\text{ m}}\left(\frac{4[2.50\text{ kg}]}{2.50\text{ kg} + [3.80\text{ kg}]/3}\right)}$$

$\omega = 5.70$ rad/s.

EVALUATE: As the bar swings through the vertical, the linear speed of the ball that is still attached to the bar is $v = (0.400\text{ m})(5.70\text{ rad/s}) = 2.28$ m/s. A point mass in free-fall acquires a speed of 2.80 m/s after falling 0.400 m; the ball on the bar acquires a speed less than this.

10.58. **IDENTIFY:** Use $\sum\tau_z = I\alpha_z$ to find α_z, and then use the constant α_z kinematic equations to solve for t.

SET UP: The door is sketched in Figure 10.58.

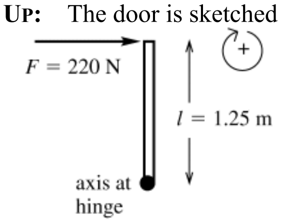


Figure 10.58

EXECUTE: $\sum\tau_z = Fl = (220\text{ N})(1.25\text{ m}) = 275\text{ N}\cdot\text{m}$

From Table 9.2(d), $I = \frac{1}{3}Ml^2$

$I = \frac{1}{3}(750\text{ N}/9.80\text{ m/s}^2)(1.25\text{ m})^2 = 39.9\text{ kg}\cdot\text{m}^2$

$$\sum\tau_z = I\alpha_z \text{ so } \alpha_z = \frac{\sum\tau_z}{I} = \frac{275\text{ N}\cdot\text{m}}{39.9\text{ kg}\cdot\text{m}^2} = 6.89\text{ rad/s}^2$$

SET UP: $\alpha_z = 6.89\text{ rad/s}^2$; $\theta - \theta_0 = 90°(\pi\text{ rad}/180°) = \pi/2$ rad; $\omega_{0z} = 0$ (door initially at rest); $t = ?$

$\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$

EXECUTE: $t = \sqrt{\dfrac{2(\theta - \theta_0)}{\alpha_z}} - \sqrt{\dfrac{2(\pi/2\text{ rad})}{6.89\text{ rad/s}^2}} = 0.675\text{ s}$

EVALUATE: The forces and the motion are connected through the angular acceleration.

10.59. **IDENTIFY:** $\tau = rF\sin\phi$

SET UP: Let x be the distance from the left end of the rod where the string is attached. For the value of x where $f(x)$ is a maximum, $df/dx = 0$.

EXECUTE: **(a)** From geometric consideration, the lever arm and the sine of the angle between $\vec{F}$ and $\vec{r}$ are both maximum if the string is attached at the end of the rod.

(b) In terms of the distance x where the string is attached, the magnitude of the torque is $Fxh/\sqrt{x^2+h^2}$. This function attains its maximum at the boundary, where $x = h$, so the string should be attached at the right end of the rod.

(c) As a function of x, l and h, the torque has magnitude $\tau = F\dfrac{xh}{\sqrt{(x-l/2)^2+h^2}}$. Differentiating τ with respect to x and setting equal to zero gives $x_{max} = (l/2)(1+(2h/l)^2)$. This will be the point at which to attach the string unless $2h > l$, in which case the string should be attached at the furthest point to the right, $x = l$.

EVALUATE: In part (a) the maximum torque is independent of h. In part (b) the maximum torque is independent of l. In part (c) the maximum torque depends on both h and l.

10.60. **IDENTIFY:** Apply $\sum\tau_z = I\alpha_z$, where τ_z is due to the gravity force on the object.

SET UP: $I = I_{rod} + I_{clay}$. $I_{rod} = \frac{1}{3}ML^2$. In part (b), $I_{clay} = ML^2$. In part (c), $I_{clay} = 0$.

EXECUTE: **(a)** A distance $L/4$ from the end with the clay.

(b) In this case $I = (4/3)ML^2$ and the gravitational torque is $(3L/4)(2Mg)\sin\theta = (3MgL/2)\sin\theta$, so $\alpha = (9g/8L)\sin\theta$.

(c) In this case $I = (1/3)ML^2$ and the gravitational torque is $(L/4)(2Mg)\sin\theta = (MgL/2)\sin\theta$, so $\alpha = (3g/2L)\sin\theta$. This is greater than in part (b).

(d) The greater the angular acceleration of the upper end of the cue, the faster you would have to react to overcome deviations from the vertical.

EVALUATE: In part (b), I is 4 times larger than in part (c) and τ is 3 times larger. $\alpha = \tau/I$, so the net effect is that α is smaller in (b) than in (c).

10.61. **IDENTIFY:** Calculate W using the procedure specified in the problem. In part (c) apply the work-energy theorem. In part (d), $a_{tan} = R\alpha$ and $\sum\tau_z = I\alpha_z$. $a_{rad} = R\omega^2$.

SET UP: Let θ be the angle the disk has turned through. The moment arm for F is $R\cos\theta$.

EXECUTE: **(a)** The torque is $\tau = FR\cos\theta$. $W = \int_0^{\pi/2} FR\cos\theta\, d\theta = FR$.

(b) In Eq.(6.14), dl is the horizontal distance the point moves, and so $W = F\int dl = FR$, the same as part (a).

(c) From $K_2 = W = (MR^2/4)\omega^2$, $\omega = \sqrt{4F/MR}$.

(d) The torque, and hence the angular acceleration, is greatest when $\theta = 0$, at which point $\alpha = (\tau/I) = 2F/MR$, and so the maximum tangential acceleration is $2F/M$.

(e) Using the value for ω found in part (c), $a_{rad} = \omega^2 R = 4F/M$.

EVALUATE: $a_{tan} = \omega^2 R$ is maximum initially, when the moment arm for F is a maximum, and it is zero after the disk has rotated one-quarter of a revolution. a_{rad} is zero initially and is a maximum at the end of the motion, after the disk has rotated one-quarter of a revolution.

10.62. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to the crate and $\sum\tau_z = I\alpha_z$ to the cylinder. The motions are connected by $a(\text{crate}) = R\alpha(\text{cylinder})$.

SET UP: The force diagram for the crate is given in Figure 10.62a.

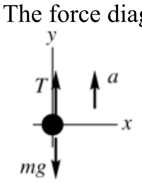


Figure 10.62a

EXECUTE: $\sum F_y = ma_y$

$T - mg = ma$

$T = m(g+a) = 50\text{ kg}(9.80\text{ m/s}^2 + 0.80\text{ m/s}^2) = 530\text{ N}$

SET UP: The force diagram for the cylinder is given in Figure 10.62b.

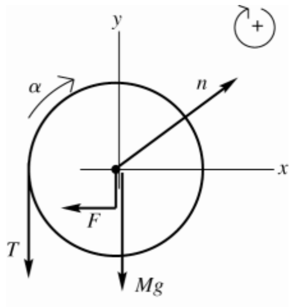


Figure 10.62b

EXECUTE: $\sum \tau_z = I\alpha_z$

$Fl - TR = I\alpha_z$, where $l = 0.12$ m and $R = 0.25$ m

$a = R\alpha$ so $\alpha_z = a/R$

$Fl = TR + Ia/R$

$$F = T\left(\frac{R}{l}\right) + \frac{Ia}{Rl} = 530\text{ N}\left(\frac{0.25\text{ m}}{0.12\text{ m}}\right) + \frac{(2.9\text{ kg}\cdot\text{m}^2)(0.80\text{ m/s}^2)}{(0.25\text{ m})(0.12\text{ m})} = 1200\text{ N}$$

EVALUATE: The tension in the rope is greater than the weight of the crate since the crate accelerates upward. If F were applied to the rim of the cylinder ($l = 0.25$ m), it would have the value $F = 567$ N. This is greater than T because it must accelerate the cylinder as well as the crate. And F is larger than this because it is applied closer to the axis than R so has a smaller moment arm and must be larger to give the same torque.

10.63. **IDENTIFY:** Apply $\sum \vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}$ and $\sum \tau_z = I_{\text{cm}}\alpha_z$ to the roll.

SET UP: At the point of contact, the wall exerts a friction force f directed downward and a normal force n directed to the right. This is a situation where the net force on the roll is zero, but the net torque is *not* zero.

EXECUTE: **(a)** Balancing vertical forces, $F_{\text{rod}}\cos\theta = f + w + F$, and balancing horizontal forces $F_{\text{rod}}\sin\theta = n$. With $f = \mu_k n$, these equations become $F_{\text{rod}}\cos\theta = \mu_k n + F + w$, $F_{\text{rod}}\sin\theta = n$. Eliminating n and solving for F_{rod} gives

$$F_{\text{rod}} = \frac{w + F}{\cos\theta - \mu_k \sin\theta} = \frac{(16.0\text{ kg})\,(9.80\text{ m/s}^2) + (40.0\text{ N})}{\cos 30° - (0.25)\sin 30°} = 266\text{ N}.$$

(b) With respect to the center of the roll, the rod and the normal force exert zero torque. The magnitude of the net torque is $(F - f)R$, and $f = \mu_k n$ may be found by insertion of the value found for F_{rod} into either of the above relations; *i.e.*, $f = \mu_k F_{\text{rod}}\sin\theta = 33.2$ N. Then,

$$\alpha = \frac{\tau}{I} = \frac{(40.0\text{ N} - 31.54\text{ N})(18.0\times 10^{-2}\text{ m})}{(0.260\text{ kg}\cdot\text{m}^2)} = 4.71\text{ rad/s}^2.$$

EVALUATE: If the applied force F is increased, F_{rod} increases and this causes n and f to increase. The angle ϕ changes as the amount of paper unrolls and this affects α for a given F.

10.64. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ to the flywheel and $\sum \vec{F} = m\vec{a}$ to the block. The target variables are the tension in the string and the acceleration of the block.

(a) SET UP: Apply $\sum \tau_z = I\alpha_z$ to the rotation of the flywheel about the axis. The free-body diagram for the flywheel is given in Figure 10.64a.

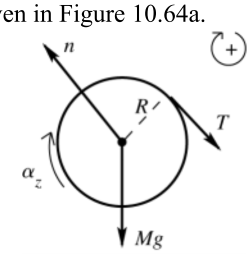


Figure 10.64a

EXECUTE: The forces n and Mg act at the axis so have zero torque.

$\sum \tau_z = TR$

$TR = I\alpha_z$

SET UP: Apply $\sum \vec{F} = m\vec{a}$ to the translational motion of the block. The free-body diagram for the block is given in Figure 10.64b.

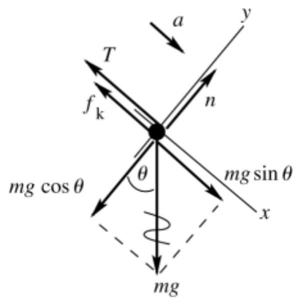


Figure 10.64b

EXECUTE: $\sum F_y = ma_y$

$n - mg\cos 36.9° = 0$

$n = mg\cos 36.9°$

$f_k = \mu_k n = \mu_k mg\cos 36.9°$

$\sum F_x = ma_x$

$mg\sin 36.9° - T - \mu_k mg\cos 36.9° = ma$

$mg(\sin 36.9° - \mu_k \cos 36.9°) - T = ma$

But we also know that $a_{\text{block}} = R\alpha_{\text{wheel}}$, so $\alpha = a/R$. Using this in the $\sum \tau_z = I\alpha_z$ equation gives $TR = Ia/R$ and $T = (I/R^2)a$. Use this to replace T in the $\sum F_x = ma_x$ equation:

$mg(\sin 36.9° - \mu_k \cos 36.9°) - (I/R^2)a = ma$

$$a = \frac{mg(\sin 36.9° - \mu_k \cos 36.9°)}{m + I/R^2}$$

$$a = \frac{(5.00\text{ kg})(9.80\text{ m/s}^2)(\sin 36.9° - (0.25)\cos 36.9°)}{5.00\text{ kg} + 0.500\text{ kg}\cdot\text{m}^2/(0.200\text{ m})^2} = 1.12\text{ m/s}^2$$

(b) $T = \dfrac{0.500\text{ kg}\cdot\text{m}^2}{(0.200\text{ m})^2}(1.12\text{ m/s}^2) = 14.0\text{ N}$

EVALUATE: If the string is cut the block will slide down the incline with $a = g\sin 36.9° - \mu_k g\cos 36.9° = 3.92\text{ m/s}^2$. The actual acceleration is less than this because $mg\sin 36.9°$ must also accelerate the flywheel. $mg\sin 36.9° - f_k = 19.6$ N. T is less than this; there must be more force on the block directed down the incline than up then incline since the block accelerates down the incline.

10.65. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the block and $\sum \tau_z = I\alpha_z$ to the combined disks.

SET UP: For a disk, $I_{\text{disk}} = \frac{1}{2}MR^2$, so I for the disk combination is $I = 2.25 \times 10^{-3}\text{ kg}\cdot\text{m}^2$.

EXECUTE: For a tension T in the string, $mg - T = ma$ and $TR = I\alpha = I\dfrac{a}{R}$. Eliminating T and solving for a gives

$a = g\dfrac{m}{m + I/R^2} = \dfrac{g}{1 + I/mR^2}$, where m is the mass of the hanging block and R is the radius of the disk to which the string is attached.

(a) With $m = 1.50$ kg and $R = 2.50 \times 10^{-2}$ m, $a = 2.88\text{ m/s}^2$.

(b) With $m = 1.50$ kg and $R = 5.00 \times 10^{-2}$ m, $a = 6.13\text{ m/s}^2$.

The acceleration is larger in case (b); with the string attached to the larger disk, the tension in the string is capable of applying a larger torque.

EVALUATE: $\omega = v/R$, where v is the speed of the block and ω is the angular speed of the disks. When R is larger, in part (b), a smaller fraction of the kinetic energy resides with the disks. The block gains more speed as it falls a certain distance and therefore has a larger acceleration.

10.66. IDENTIFY: Apply both $\sum \vec{F} = m\vec{a}$ and $\sum \tau_z = I\alpha_z$ to the motion of the roller. Rolling without slipping means $a_{\text{cm}} = R\alpha$. Target variables are a_{cm} and f.

SET UP: The free-body diagram for the roller is given in Figure 10.66.

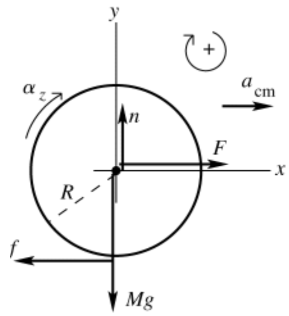


Figure 10.66

EXECUTE: Apply $\sum \vec{F} = m\vec{a}$ to the translational motion of the center of mass:

$\sum F_x = ma_x$

$F - f = Ma_{cm}$

Apply $\sum \tau_z = I\alpha_z$ to the rotation about the center of mass:

$\sum \tau_z = fR$

thin-walled hollow cylinder: $I = MR^2$

Then $\sum \tau_z = I\alpha_z$ implies $fR = MR^2\alpha.$

But $\alpha_{cm} = R\alpha,$ so $f = Ma_{cm}.$

Using this in the $\sum F_x = ma_x$ equation gives $F - Ma_{cm} = Ma_{cm}$

$a_{cm} = F/2M,$ and then $f = Ma_{cm} = M(F/2M) = F/2.$

EVALUATE: If the surface were frictionless the object would slide without rolling and the acceleration would be $a_{cm} = F/M.$ The acceleration is less when the object rolls.

10.67. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each object and apply $\sum \tau_z = I\alpha_z$ to the pulley.

SET UP: Call the 75.0 N weight *A* and the 125 N weight *B*. Let T_A and T_B be the tensions in the cord to the left and to the right of the pulley. For the pulley, $I = \frac{1}{2}MR^2$, where $Mg = 50.0\text{ N}$ and $R = 0.300\text{ m}$. The 125 N weight accelerates downward with acceleration *a*, the 75.0 N weight accelerates upward with acceleration *a* and the pulley rotates clockwise with angular acceleration α, where $a = R\alpha$.

EXECUTE: $\sum \vec{F} = m\vec{a}$ applied to the 75.0 N weight gives $T_A - w_A = m_A a$. $\sum \vec{F} = m\vec{a}$ applied to the 125.0 N weight gives $w_B - T_B = m_B a$. $\sum \tau_z = I\alpha_z$ applied to the pulley gives $(T_B - T_A)R = (\frac{1}{2}MR^2)\alpha_z$ and $T_B - T_A = \frac{1}{2}M$.
Combining these three equations gives $w_B - w_A = (m_A + m_B + M/2)a$ and

$a = \left(\dfrac{w_B - w_A}{w_A + w_B + w_{\text{pulley}}/2}\right)g = \left(\dfrac{125\text{ N} - 75.0\text{ N}}{75.0\text{ N} + 125\text{ N} + 25.0\text{ N}}\right)g = 0.222g$. $T_A = w_A(1 + a/g) = 1.222w_A = 91.65\text{ N}$.

$T_B = w_B(1 - a/g) = 0.778w_B = 97.25\text{ N}$. $\sum \vec{F} = m\vec{a}$ applied to the pulley gives that the force *F* applied by the hook to the pulley is $F = T_A + T_B + w_{\text{pulley}} = 239\text{ N}$. The force the ceiling applies to the hook is 239 N.

EVALUATE: The force the hook exerts on the pulley is less than the total weight of the system, since the net effect of the motion of the system is a downward acceleration of mass.

10.68. **IDENTIFY:** This problem can be done either with conservation of energy or with $\sum \vec{F}_{\text{ext}} = m\vec{a}.$ We will do it both ways.

(a) SET UP: (1) *Conservation of energy:* $K_1 + U_1 + W_{\text{other}} = K_2 + U_2.$

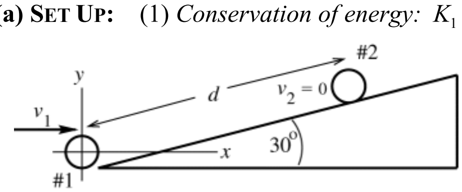


Figure 10.68a

Take position 1 to be the location of the disk at the base of the ramp and 2 to be where the disk momentarily stops before rolling back down, as shown in Figure 10.68a.

Take the origin of coordinates at the center of the disk at position 1 and take $+y$ to be upward. Then $y_1 = 0$ and $y_2 = d\sin 30°,$ where *d* is the distance that the disk rolls up the ramp. "Rolls without slipping" and neglect rolling friction says $W_f = 0;$ only gravity does work on the disk, so $W_{\text{other}} = 0$

EXECUTE: $U_1 = Mgy_1 = 0$

$K_1 = \frac{1}{2}Mv_1^2 + \frac{1}{2}I_{cm}\omega_1^2$ (Eq.10.11). But $\omega_1 = v_1/R$ and $I_{cm} = \frac{1}{2}MR^2$, so $\frac{1}{2}I_{cm}\omega_1^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)(v_1/R)^2 = \frac{1}{4}Mv_1^2$. Thus

$K_1 = \frac{1}{2}Mv_1^2 + \frac{1}{4}Mv_1^2 = \frac{3}{4}Mv_1^2$.

$U_2 = Mgy_2 = Mgd\sin 30°$

$K_2 = 0$ (disk is at rest at point 2).

Thus $\frac{3}{4}Mv_1^2 = Mgd\sin 30°$

$$d = \frac{3v_1^2}{4g\sin 30°} = \frac{3(2.50\text{ m/s})^2}{4(9.80\text{ m/s}^2)\sin 30°} = 0.957\text{ m}$$

SET UP: (2) *force and acceleration* The free-body diagram is given in Figure 10.68b.

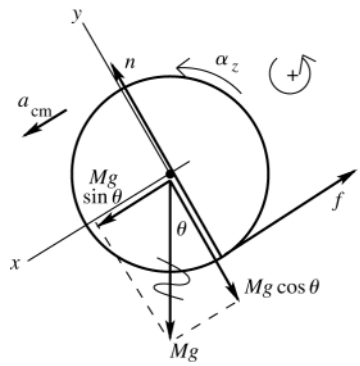


Figure 10.68b

EXECUTE: Apply $\sum F_x = ma_x$ to the translational motion of the center of mass:

$Mg\sin\theta - f = Ma_{cm}$

Apply $\sum \tau_z = I\alpha_z$ to the rotation about the center of mass:

$fR = \left(\frac{1}{2}MR^2\right)\alpha_z$

$f = \frac{1}{2}MR\alpha_z$

But $a_{cm} = R\alpha$ in this equation gives $f = \frac{1}{2}Ma_{cm}$. Use this in the $\sum F_x = ma_x$ equation to eliminate *f*.

$Mg\sin\theta - \frac{1}{2}Ma_{cm} = Ma_{cm}$

M divides out and $\frac{3}{2}a_{cm} = g\sin\theta$. $a_{cm} = \frac{2}{3}g\sin\theta = \frac{2}{3}(9.80\text{ m/s}^2)\sin 30° = 3.267\text{ m/s}^2$

SET UP: Apply the constant acceleration equations to the motion of the center of mass. Note that in our coordinates the positive *x*-direction is down the incline.

$v_{0x} = -2.50$ m/s (directed up the incline); $a_x = +3.267\text{ m/s}^2$;

$v_x = 0$ (momentarily comes to rest); $x - x_0 = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

EXECUTE: $x - x_0 = -\dfrac{v_{0x}^2}{2a_x} = -\dfrac{(-2.50\text{ m/s})^2}{2(3.267\text{ m/s}^2)} = -0.957\text{ m}$

(b) EVALUATE: The results from the two methods agree; the disk rolls 0.957 m up the ramp before it stops. The mass *M* enters both in the linear inertia and in the gravity force so divides out. The mass *M* and radius *R* enter in both the rotational inertia and the gravitational torque so divide out.

10.69. IDENTIFY: Apply $\sum \vec{F}_{ext} = m\vec{a}_{cm}$ to the motion of the center of mass and apply $\sum \tau_z = I_{cm}\alpha_z$ to the rotation about the center of mass.

SET UP: $I = 2\left(\frac{1}{2}MR^2\right) = MR^2$. The moment arm for *T is b*.

EXECUTE: The tension is related to the acceleration of the yo-yo by $(2m)g - T = (2m)a$, and to the angular acceleration by $Tb = I\alpha = I\frac{a}{b}$. Dividing the second equation by *b* and adding to the first to eliminate *T* yields

$a = g\dfrac{2m}{(2m + I/b^2)} = g\dfrac{2}{2 + (R/b)^2}$, $\alpha = g\dfrac{2}{2b + R^2/b}$. The tension is found by substitution into either of the two equations:

$$T = (2m)(g - a) = (2mg)\left(1 - \frac{2}{2 + (R/b)^2}\right) = 2mg\frac{(R/b)^2}{2 + (R/b)^2} = \frac{2mg}{(2(b/R)^2 + 1)}.$$

EVALUATE: $a \to 0$ when $b \to 0$. As $b \to R$, $a \to 2g/3$.

10.70. **IDENTIFY:** Apply conservation of energy to the motion of the shell, to find its linear speed v at points A and B. Apply $\sum \vec{F} = m\vec{a}$ to the circular motion of the shell in the circular part of the track to find the normal force exerted by the track at each point. Since $r << R$ the shell can be treated as a point mass moving in a circle of radius R when applying $\sum \vec{F} = m\vec{a}$. But as the shell rolls along the track, it has both translational and rotational kinetic energy.

SET UP: $K_1 + U_1 = K_2 + U_2$. Let 1 be at the starting point and take $y = 0$ to be at the bottom of the track, so $y_1 = h_0$. $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $I = \frac{2}{3}mr^2$ and $\omega = v/r$, so $K = \frac{5}{6}mv^2$. During the circular motion, $a_{rad} = v^2/R$.

EXECUTE: **(a)** $\sum \vec{F} = m\vec{a}$ at point A gives $n + mg = m\frac{v^2}{R}$. The minimum speed for the shell not to fall off the track is when $n \to 0$ and $v^2 = gR$. Let point 2 be A, so $y_2 = 2R$ and $v_2^2 = mR$. Then $K_1 + U_1 = K_2 + U_2$ gives $mgh_0 = 2mgR + \frac{5}{6}m(gR)$. $h_0 = (2 + \frac{5}{6})R = \frac{17}{6}R$.

(b) Let point 2 be B, so $y_2 = R$. Then $K_1 + U_1 = K_2 + U_2$ gives $mgh_0 = mgR + \frac{5}{6}mv_2^2$. With $h = \frac{17}{6}R$ this gives $v^2 = \frac{11}{5}gR$. Then $\sum \vec{F} = m\vec{a}$ at B gives $n = m\frac{v^2}{R} = \frac{11}{5}mg$.

(c) Now $K = \frac{1}{2}mv^2$ instead of $\frac{5}{6}mv^2$. The shell would be moving faster at A than with friction and would still make the complete loop.

(d) In part (c): $mgh_0 = mg(2R) + \frac{1}{2}mv^2$. $h_0 = \frac{17}{6}R$ gives $v^2 = \frac{5}{3}gR$. $\sum \vec{F} = m\vec{a}$ at point A gives $mg + n = m\frac{v^2}{R}$ and $n = m\left(\frac{v^2}{R} - g\right) = \frac{2}{3}mg$. In part (a), $n = 0$, since at this point gravity alone supplies the net downward force that is required for the circular motion.

EVALUATE: The normal force at A is greater when friction is absent because the speed of the shell at A is greater when friction is absent than when there is rolling without slipping.

10.71. **IDENTIFY:** Consider the direction of the net force and the sense of the net torque in each case.

SET UP: The free-body diagram in each case is shown in Figure 10.71.

EXECUTE: In the first case, $\vec{F}$ and the friction force act in opposite directions, and the friction force causes a larger torque to tend to rotate the yo-yo to the right. The net force to the right is the difference $F - f$, so the net force is to the right while the net torque causes a clockwise rotation. For the second case, both the torque and the friction force tend to turn the yo-yo clockwise, and the yo-yo moves to the right. In the third case, friction tends to move the yo-yo to the right, and since the applied force is vertical, the yo-yo moves to the right.

EVALUATE: In the first case the torque due to friction must be larger than the torque due to F, so the net torque is clockwise. In the third case the torque due to F must be larger than the torque due to f, so the net torque will be clockwise.

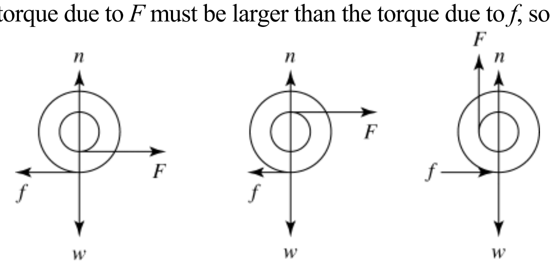


Figure 10.71

10.72. **IDENTIFY:** Apply $\sum \vec{F}_{ext} = m\vec{a}_{cm}$ to the motion of the center of mass and $\sum \tau_z = I_{cm}\alpha_z$ to the rotation about the center of mass.

SET UP: For a hoop, $I = MR^2$. For a solid disk, $I = \frac{1}{2}MR^2$.

EXECUTE: **(a)** Because there is no vertical motion, the tension is just the weight of the hoop: $T = Mg = (0.180\text{ kg})(9.8\text{ N/kg}) = 1.76\text{ N}$.

(b) Use $\tau = I\alpha$ to find α. The torque is RT, so $\alpha = RT/I = RT/MR^2 = T/MR = Mg/MR$, so $\alpha = g/R = (9.8\text{ m/s}^2)/(0.08\text{ m}) = 122.5\text{ rad/s}^2$.

(c) $a = R\alpha = 9.8\text{ m/s}^2$

(d) T would be unchanged because the mass M is the same, α and a would be twice as great because I is now $\frac{1}{2}MR^2$.

EVALUATE: a_{tan} for a point on the rim of the hoop or disk equals a for the free end of the string. Since I is smaller for the disk, the same value of T produces a greater angular acceleration.

10.73. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ to the cylinder or hoop. Find *a* for the free end of the cable and apply constant acceleration equations.

SET UP: a_{tan} for a point on the rim equals *a* for the free end of the cable, and $a_{\text{tan}} = R\alpha$.

EXECUTE: **(a)** $\sum \tau_z = I\alpha_z$ and $a_{\text{tan}} = R\alpha$ gives $FR = \dfrac{1}{2}MR^2\alpha = \dfrac{1}{2}MR^2\left(\dfrac{a_{\text{tan}}}{R}\right)$. $a_{\text{tan}} = \dfrac{2F}{M} = \dfrac{200\text{ N}}{4.00\text{ kg}} = 50\text{ m/s}^2$.

Distance the cable moves: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $50\text{ m} = \dfrac{1}{2}\left(50\text{ m/s}^2\right)t^2$ and $t = 1.41\text{ s}$.

$v_x = v_{0x} + a_x t = 0 + \left(50\text{ m/s}^2\right)\left(1.41\text{ s}\right) = 70.5\text{ m/s}$.

(b) For a hoop, $I = MR^2$, which is twice as large as before, so α and a_{tan} would be half as large. Therefore the time would be longer by a factor of $\sqrt{2}$. For the speed, $v_x^2 = v_{0x}^2 + 2a_x x$, in which *x* is the same, so v_x would be half as large since a_x is smaller.

EVALUATE: The acceleration *a* that is produced depends on the mass of the object but is independent of its radius. But *a* depends on how the mass is distributed and is different for a hoop versus a cylinder.

10.74. **IDENTIFY:** Use projectile motion to find the speed *v* the marble needs at the edge of the pit to make it to the level ground on the other side. Apply conservation of energy to the motion down the hill in order to relate the initial height to the speed *v* at the edge of the pit. $W_{\text{other}} = 0$ so conservation of energy gives $K_{\text{i}} + U_{\text{i}} = K_{\text{f}} + U_{\text{f}}$.

SET UP: In the projectile motion the marble must travel 36 m horizontally while falling vertically 20 m. Let $+y$ be downward. For the motion down the hill, let $y_{\text{f}} = 0$ so $U_{\text{f}} = 0$ and $y_{\text{i}} = h$. $K_{\text{i}} = 0$. Rolling without slipping means $v = R\omega$. $K = \frac{1}{2}I_{\text{cm}}\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}(\frac{2}{5}mR^2)\omega^2 + \frac{1}{2}mv^2 = \frac{7}{10}mv^2$.

EXECUTE: **(a)** Projectile motion: $v_{0y} = 0$. $a_y = 9.80\text{ m/s}^2$. $y - y_0 = 20\text{ m}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = 2.02\text{ s}$. Then $x - x_0 = v_{0x}t$ gives $v = v_{0x} = \dfrac{x - x_0}{t} = \dfrac{36\text{ m}}{2.02\text{ s}} = 17.8\text{ m/s}$.

Motion down the hill: $U_{\text{i}} = K_{\text{f}}$. $mgh = \frac{7}{10}mv^2$. $h = \dfrac{7v^2}{10g} = \dfrac{7(17.8\text{ m/s})^2}{10(9.80\text{ m/s}^2)} = 22.6\text{ m}$.

(b) $\frac{1}{2}I\omega^2 = \frac{1}{5}mv^2$, independent of *R*. *I* is proportional to R^2 but ω^2 is proportional to $1/R^2$ for a given translational speed *v*.

(c) The object still needs $v = 17.8\text{ m/s}$ at the bottom of the hill in order to clear the pit. But now $K_{\text{f}} = \frac{1}{2}mv^2$ and

$h = \dfrac{v^2}{2g} = 16.6\text{ m}$.

EVALUATE: The answer to part (a) also does not depend on the mass of the marble. But, it does depend on how the mass is distributed within the object. The answer would be different if the object were a hollow spherical shell. In part (c) less height is needed to give the object the same translational speed because in (c) none of the energy goes into rotational motion.

10.75. **IDENTIFY:** Apply conservation of energy to the motion of the boulder.

SET UP: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ and $v = R\omega$ when there is rolling without slipping. $I = \frac{2}{5}mR^2$.

EXECUTE: Break into 2 parts, the rough and smooth sections.

Rough: $mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $mgh_1 = \dfrac{1}{2}mv^2 + \dfrac{1}{2}\left(\dfrac{2}{5}mR^2\right)\left(\dfrac{v}{R}\right)^2$. $v^2 = \dfrac{10}{7}gh_1$.

Smooth: Rotational kinetic energy does not change. $mgh_2 + \dfrac{1}{2}mv^2 + K_{\text{rot}} = \dfrac{1}{2}mv_{\text{Bottom}}^2 + K_{\text{rot}}$. $gh_2 + \dfrac{1}{2}\left(\dfrac{10}{7}gh_1\right) = \dfrac{1}{2}v_{\text{B}}^2$.

$v_{\text{B}} = \sqrt{\dfrac{10}{7}gh_1 + 2gh_2} = \sqrt{\dfrac{10}{7}(9.80\text{ m/s}^2)(25\text{ m}) + 2(9.80\text{ m/s}^2)(25\text{ m})} = 29.0\text{ m/s}$.

EVALUATE: If all the hill was rough enough to cause rolling without slipping, $v_{\text{B}} = \sqrt{\dfrac{10}{7}g(50\text{ m})} = 26.5\text{ m/s}$. A smaller fraction of the initial gravitational potential energy goes into translational kinetic energy of the center of mass than if part of the hill is smooth. If the entire hill is smooth and the boulder slides without slipping, $v_{\text{B}} = \sqrt{2g(50\text{ m})} = 31.3\text{ m/s}$. In this case all the initial gravitational potential energy goes into the kinetic energy of the translational motion.

10.76. **IDENTIFY:** Apply conservation of energy to the motion of the ball as it rolls up the hill. After the ball leaves the edge of the cliff it moves in projectile motion and constant acceleration equations can be used.

(a) SET UP: Use conservation of energy to find the speed v_2 of the ball just before it leaves the top of the cliff. Let point 1 be at the bottom of the hill and point 2 be at the top of the hill. Take $y = 0$ at the bottom of the hill, so $y_1 = 0$ and $y_2 = 28.0$ m.

EXECUTE: $K_1 = U_1 = K_2 + U_2$

$\frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 = mgy_2 + \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2$

Rolling without slipping means $\omega = v/r$ and $\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}mr^2\right)(v/r)^2 = \frac{1}{5}mv^2$

$\frac{7}{10}mv_1^2 = mgy_2 + \frac{7}{10}mv_2^2$

$v_2 = \sqrt{v_1^2 - \frac{10}{7}gy_2} = 15.26 \text{ m/s}$

SET UP: Consider the projectile motion of the ball, from just after it leaves the top of the cliff until just before it lands. Take $+y$ to be downward. Use the vertical motion to find the time in the air:

$v_{0y} = 0,\ a_y = 9.80 \text{ m/s}^2,\ y - y_0 = 28.0 \text{ m},\ t = ?$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $t = 2.39$ s

During this time the ball travels horizontally
$x - x_0 = v_{0x}t = (15.26 \text{ m/s})(2.39 \text{ s}) = 36.5 \text{ m}.$

Just before it lands, $v_y = v_{0y} + a_yt = 23.4$ m/s and $v_x = v_{0x} = 15.3$ m/s

$v = \sqrt{v_x^2 + v_y^2} = 28.0 \text{ m/s}$

(b) EVALUATE: At the bottom of the hill, $\omega = v/r = (25.0 \text{ m/s})/r.$ The rotation rate doesn't change while the ball is in the air, after it leaves the top of the cliff, so just before it lands $\omega = (15.3 \text{ m/s})/r.$ The total kinetic energy is the same at the bottom of the hill and just before it lands, but just before it lands less of this energy is rotational kinetic energy, so the translational kinetic energy is greater.

10.77. **IDENTIFY:** Apply conservation of energy to the motion of the wheel. $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

SET UP: No slipping means that $\omega = v/R$. Uniform density means $m_r = \lambda 2\pi R$ and $m_s = \lambda R$, where m_r is the mass of the rim and m_s is the mass of each spoke. For the wheel, $I = I_{rim} + I_{spokes}$. For each spoke, $I = \frac{1}{3}m_sR^2$.

EXECUTE: **(a)** $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $I = I_{rim} + I_{spokes} = m_rR^2 + 6\left(\frac{1}{3}m_sR^2\right)$

Also, $m = m_r + m_s = 2\pi R\lambda + 6R\lambda = 2R\lambda(\pi + 3)$. Substituting into the conservation of energy equation gives

$$2R\lambda(\pi+3)gh = \frac{1}{2}(2R\lambda)(\pi+3)(R\omega)^2 + \frac{1}{2}\left[2\pi R\lambda R^2 + 6\left(\frac{1}{3}\pi RR^2\right)\right]\omega^2.$$

$$\omega = \sqrt{\frac{(\pi+3)gh}{R^2(\pi+2)}} = \sqrt{\frac{(\pi+3)(9.80 \text{ m/s}^2)(58.0 \text{ m})}{(0.210 \text{ m})^2(\pi+2)}} = 124 \text{ rad/s and } v = R\omega = 26.0 \text{ m/s}$$

(b) Doubling the density would have no effect because it does not appear in the answer. ω is inversely proportional to *R* so doubling the diameter would double the radius which would reduce ω by half, but $v = R\omega$ would be unchanged.

EVALUATE: Changing the masses of the rim and spokes by different amounts would alter the speed *v* at the bottom of the hill.

10.78. **IDENTIFY:** Apply $v = R\omega$.

SET UP: For the antique bike, *v* is the same for points on the rim of each wheel and equals the linear speed of the bike. $1 \text{ rev} = 2\pi \text{ rad}$.

EXECUTE: **(a)** The front wheel is turning at $\omega = 1.00 \text{ rev/s} = 2\pi \text{ rad/s}$. $v = r\omega = (0.330 \text{ m})(2\pi \text{ rad/s}) = 2.07 \text{ s}$.

(b) $\omega = v/r = (2.07 \text{ m/s})/(0.655 \text{ m}) = 3.16 \text{ rad/s} = 0.503 \text{ rev/s}$

(c) $\omega = v/r = (2.07 \text{ m/s})/(0.220 \text{ m}) = 9.41 \text{ rad/s} = 1.50 \text{ rev/s}$

EVALUATE: Since the front wheel has a larger radius for the antique bike, that wheel doesn't have to rotate at as many rev/s to achieve the same linear speed of the bike.

10.79. **IDENTIFY:** Apply conservation of energy to the motion of the ball. Once the ball leaves the track the ball moves in projectile motion.

SET UP: The ball has $I = \frac{2}{5}mR^2$; the silver dollar has $I = \frac{1}{2}mR^2$. For the projectile motion take $+y$ downward, so $a_x = 0$ and $a_y = +g$.

EXECUTE: **(a)** The kinetic energy of the ball when it leaves the track (when it is still rolling without slipping) is $(7/10)mv^2$ and this must be the work done by gravity, $W = mgh$, so $v = \sqrt{10gh/7}$. The ball is in the air for a time $t = \sqrt{2y/g}$, so $x = vt = \sqrt{20hy/7}$.

(b) The answer does not depend on g, so the result should be the same on the moon.

(c) The presence of rolling friction would decrease the distance.

(e) For the dollar coin, modeled as a uniform disc, $K = (3/4)mv^2$, and so $x = \sqrt{8hy/3}$.

EVALUATE: The sphere travels a little farther horizontally, because its moment of inertia is a smaller fraction of MR^2 than for the disk. The result is independent of the mass and radius of the object but it does depend on how that mass is distributed within the object.

10.80. **IDENTIFY** and **SET UP:** Apply conservation of energy to the motion of the ball. The ball ends up with both translational and rotational kinetic energy. Use Fig.(10.13) in the textbook to relate the speed of different points on the ball to v_{cm}.

EXECUTE: **(a)** $U_{el} = \frac{1}{2}kx^2 = \frac{1}{2}(400\ \text{N}\cdot\text{m})(0.15\ \text{m})^2 = 4.50\ \text{J}$ and $K_1 = 0.800U_{el} = 3.60\ \text{J}$

$K_1 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$ rolling without slipping says $\omega = v_{cm}/R$

$I_{cm} = \frac{2}{5}mR^2$

Thus $K_1 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)(v_{cm}/R)^2 = mv_{cm}^2\left(\frac{1}{2}+\frac{1}{5}\right) = \frac{7}{10}mv_{cm}^2$

and $v_{cm} = \sqrt{\dfrac{10K_1}{7m}} = \sqrt{\dfrac{10(3.60\ \text{J})}{7(0.0590\ \text{kg})}} = 9.34\ \text{m/s}.$

(b) Consider Figure 10.80a.

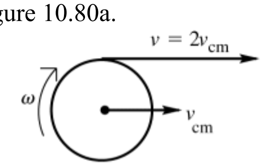


Figure 10.80a

From Fig.(10.13) in the textbook, at the top of the ball $v = 2v_{cm} = 18.7\ \text{m/s}$

(c)

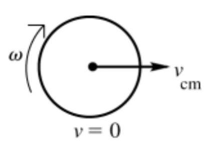


Figure 10.80b

From Fig.(10.13) in the textbook, $v = 0$ at the bottom of the ball.

(d) The problem says that $U_2 = 0.900K_1 = 3.24\ \text{J}$. Thus $U_2 = mgh = 3.24\ \text{J}$ and

$$h = \frac{3.24\ \text{J}}{mg} = \frac{3.24\ \text{J}}{(0.0590\ \text{kg})(9.80\ \text{m/s}^2)} = 5.60\ \text{m}$$

EVALUATE: Not all the potential energy stored in the spring goes into kinetic energy at the base of the ramp or into gravitational potential energy at the top of the ramp because of loss of mechanical energy due to negative work done by friction. If the ball slides without rolling, then $K_1 = \frac{1}{2}mv_{cm}^2$ and $v_{cm} = 11.0\ \text{m/s}$. v_{cm} is less than this when the ball rolls and some of its total kinetic energy is rotational.

10.81. **IDENTIFY:** $v_x = dx/dt$, $v_y = dy/dt$. $a_x = dv_x/dt$, $a_y = dv_y/dt$.

SET UP: $d\cos(\omega t)/dt = -\omega\sin(\omega t)$. $d\sin(\omega t)/dt = \omega\cos(\omega t)$.

EXECUTE: **(a)** The sketch is shown in Figure 10.81.

(b) R is the radius of the wheel (y varies from 0 to $2R$) and T is the period of the wheel's rotation.

(c) Differentiating, $v_x = \dfrac{2\pi R}{T}\left[1-\cos\left(\dfrac{2\pi t}{T}\right)\right]$, $a_x = \left(\dfrac{2\pi}{T}\right)^2 R\sin\left(\dfrac{2\pi t}{T}\right)$ and $v_y = \dfrac{2\pi R}{T}\sin\left(\dfrac{2\pi t}{T}\right)$,

$a_y = \left(\dfrac{2\pi}{T}\right)^2 R\cos\left(\dfrac{2\pi t}{T}\right).$

(d) $v_x = v_y = 0$ when $\left(\frac{2\pi t}{T}\right) = 2\pi$ or any multiple of 2π, so the times are integer multiples of the period *T*. The acceleration components at these times are $a_x = 0,\ a_y = \frac{4\pi^2 R}{T^2}$.

(e) $a = \sqrt{a_x^2 + a_y^2} = \left(\frac{2\pi}{T}\right)^2 R\sqrt{\cos^2\left(\frac{2\pi t}{T}\right) + \sin^2\left(\frac{2\pi t}{T}\right)} = \frac{4\pi^2 R}{T^2}$, independent of time. This is the magnitude of the radial acceleration for a point moving on a circle of radius *R* with constant angular velocity $2\pi/T$. For motion that consists of this circular motion superimposed on motion with constant velocity $(\vec{a} = 0)$, the acceleration due to the circular motion will be the total acceleration.

EVALUATE: *a* is independent of time, but *v* does depend on time.

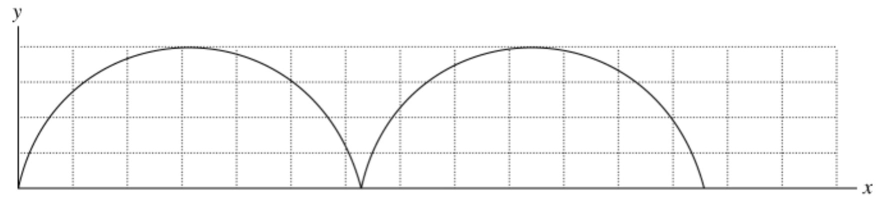


Figure 10.81

10.82. IDENTIFY: Apply the work-energy theorem to the motion of the basketball. $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ and $v = R\omega$.

SET UP: For a thin-walled, hollow sphere $I = \frac{2}{3}mR^2$.

EXECUTE: For rolling without slipping, the kinetic energy is $(1/2)(m + I/R^2)v^2 = (5/6)mv^2$; initially, this is 32.0 J and at the return to the bottom it is 8.0 J. Friction has done -24.0 J of work, -12.0 J each going up and down. The potential energy at the highest point was 20.0 J, so the height above the ground was $\frac{20.0\text{ J}}{(0.600\text{ kg})(9.80\text{ m/s}^2)} = 3.40\text{ m}$.

EVALUATE: All of the kinetic energy of the basketball, translational and rotational, has been removed at the point where the basketball is at its maximum height up the ramp.

10.83. IDENTIFY: Use conservation of energy to relate the speed of the block to the distance it has descended. Then use a constant acceleration equation to relate these quantities to the acceleration.

SET UP: For the cylinder, $I = \frac{1}{2}M(2R)^2$, and for the pulley, $I = \frac{1}{2}MR^2$.

EXECUTE: Doing this problem using kinematics involves four unknowns (six, counting the two angular accelerations), while using energy considerations simplifies the calculations greatly. If the block and the cylinder both have speed *v*, the pulley has angular velocity *v*/*R* and the cylinder has angular velocity *v*/2*R*, the total kinetic energy is

$$K = \frac{1}{2}\left[Mv^2 + \frac{M(2R)^2}{2}(v/2R)^2 + \frac{MR^2}{2}(v/R)^2 + Mv^2\right] = \frac{3}{2}Mv^2.$$

This kinetic energy must be the work done by gravity; if the hanging mass descends a distance *y*, $K = Mgy$, or $v^2 = (2/3)gy$. For constant acceleration, $v^2 = 2ay$, and comparison of the two expressions gives $a = g/3$.

EVALUATE: If the pulley were massless and the cylinder slid without rolling, $Mg = 2Ma$ and $a = g/2$. The rotation of the objects reduces the acceleration of the block.

10.84. IDENTIFY: Apply $\sum \tau_z = I\alpha_z$ to the drawbridge and calculate α_z. For part (c) use conservation of energy.

SET UP: The free-body diagram for the drawbridge is given in Fig.10.84. For an axis at the lower end, $I = \frac{1}{3}ml^2$.

EXECUTE: **(a)** $\sum \tau_z = I\alpha_z$ gives $mg(4.00\text{ m})(\cos 60.0°) = \frac{1}{3}ml^2\alpha_z$ and $\alpha_z = \frac{3g(4.00\text{ m})(\cos 60.0°)}{(8.00\text{ m})^2} = 0.919\text{ rad/s}^2$.

(b) α_z depends on the angle the bridge makes with the horizontal. α_z is not constant during the motion and $\omega_z = \omega_{0z} + \alpha_z t$ cannot be used.

(c) Use conservation of energy. Take $y = 0$ at the lower end of the drawbridge, so $y_i = (4.00\text{ m})(\sin 60.0°)$ and $y_f = 0$. $K_f + U_f = K_i + U_i + W_{other}$ gives $U_i = K_f$, $mgy_i = \frac{1}{2}I\omega^2$. $mgy_i = \frac{1}{2}(\frac{1}{3}ml^2)\omega^2$ and

$$\omega = \frac{\sqrt{6gy_i}}{l} = \frac{\sqrt{6(9.80\text{ m/s}^2)(4.00\text{ m})(\sin 60.0°)}}{8.00\text{ m}} = 1.78\text{ rad/s}.$$

EVALUATE: If we incorrectly assume that α_z is constant and has the value calculated in part (a), then $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ gives $\omega = 139$ rad/s . The angular acceleration increases as the bridge rotates and the actual angular velocity is larger than this.

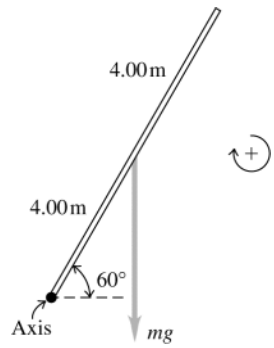


Figure 10.84

10.85. **IDENTIFY:** Apply conservation of energy to the motion of the first ball before the collision and to the motion of the second ball after the collision. Apply conservation of angular momentum to the collision between the first ball and the bar.

SET UP: The speed of the ball just before it hits the bar is $v = \sqrt{2gy} = 15.34$ m/s. Use conservation of angular momentum to find the angular velocity ω of the bar just after the collision. Take the axis at the center of the bar.

EXECUTE: $L_1 = mvr = (5.00\text{ kg})(15.34\text{ m/s})(2.00\text{ m}) = 153.4\text{ kg}\cdot\text{m}^2$

Immediately after the collision the bar and both balls are rotating together.

$L_2 = I_{\text{tot}}\omega$

$I_{\text{tot}} = \frac{1}{12}Ml^2 + 2mr^2 = \frac{1}{12}(8.00\text{ kg})(4.00\text{ m})^2 + 2(5.00\text{ kg})(2.00\text{ m})^2 = 50.67\text{ kg}\cdot\text{m}^2$

$L_2 = L_1 = 153.4\text{ kg}\cdot\text{m}^2$

$\omega = L_2/I_{\text{tot}} = 3.027$ rad/s

Just after the collision the second ball has linear speed $v = r\omega = (2.00\text{ m})(3.027\text{ rad/s}) = 6.055\text{ m/s}$ and is moving upward. $\frac{1}{2}mv^2 = mgy$ gives $y = 1.87$ m for the height the second ball goes.

EVALUATE: Mechanical energy is lost in the inelastic collision and some of the final energy is in the rotation of the bar with the first ball stuck to it. As a result, the second ball does not reach the height from which the first ball was dropped.

10.86. **IDENTIFY:** The rings and the rod exert forces on each other, but there is no net force or torque on the system, and so the angular momentum will be constant.

SET UP: For the rod, $I = \frac{1}{12}ML^2$. For each ring, $I = mr^2$, where *r* is their distance from the axis.

EXECUTE: **(a)** As the rings slide toward the ends, the moment of inertia changes, and the final angular velocity is given by $\omega_2 = \omega_1 \dfrac{I_1}{I_2} = \omega_1 \left[\dfrac{\frac{1}{12}ML^2 + 2mr_1^2}{\frac{1}{12}ML^2 + 2mr_2^2}\right] = \omega_1 \dfrac{5.00\times10^{-4}\text{ kg}\cdot\text{m}^2}{2.00\times10^{-3}\text{ kg}\cdot\text{m}^2} = \dfrac{\omega_1}{4}$, so $\omega_2 = 7.5$ rev/min.

(b) The forces and torques that the rings and the rod exert on each other will vanish, but the common angular velocity will be the same, 7.5 rev/min.

EVALUATE: Note that conversion from rev/min to rad/s was not necessary. The angular velocity of the rod decreases as the rings move away from the rotation axis.

10.87. **IDENTIFY:** Apply conservation of angular momentum to the collision. Linear momentum is not conserved because of the force applied to the rod at the axis. But since this external force acts at the axis, it produces no torque and angular momentum is conserved.

SET UP: The system before and after the collision is sketched in Figure 10.87.

EXECUTE: **(a)** $m_b = \frac{1}{4}m_{\text{rod}}$

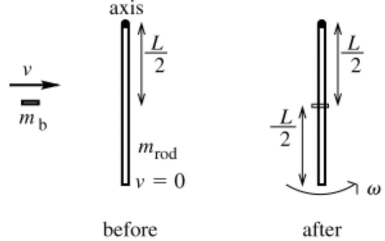


Figure 10.87

EXECUTE: $L_1 = m_b vr = \frac{1}{4}m_{\text{rod}}v(L/2)$

$L_1 = \frac{1}{8}m_{\text{rod}}vL$

$L_2 = (I_{\text{rod}} + I_b)\omega$

$I_{\text{rod}} = \frac{1}{3}m_{\text{rod}}L^2$

$I_b = m_b r^2 = \frac{1}{4}m_{\text{rod}}(L/2)^2$

$I_b = \frac{1}{16}m_{\text{rod}}L^2$

Thus $L_1 = L_2$ gives $\frac{1}{8}m_{\text{rod}}vL = \left(\frac{1}{3}m_{\text{rod}}L^2 + \frac{1}{16}m_{\text{rod}}L^2\right)\omega$

$\frac{1}{8}v = \frac{19}{48}L\omega$

$\omega = \frac{6}{19}v/L$

(b) $K_1 = \frac{1}{2}mv^2 = \frac{1}{8}m_{\text{rod}}v^2$

$K_2 = \frac{1}{2}I\omega^2 = \frac{1}{2}(I_{\text{rod}} + I_{\text{b}})\omega^2 = \frac{1}{2}\left(\frac{1}{3}m_{\text{rod}}L^2 + \frac{1}{16}m_{\text{rod}}L^2\right)(6v/19L)^2$

$K_2 = \frac{1}{2}\left(\frac{19}{48}\right)\left(\frac{6}{19}\right)^2 m_{\text{rod}}v^2 = \frac{3}{152}m_{\text{rod}}v^2$

Then $\dfrac{K_2}{K_1} = \dfrac{\frac{3}{152}m_{\text{rod}}v^2}{\frac{1}{8}m_{\text{rod}}v^2} = 3/19.$

EVALUATE: The collision is inelastic and $K_2 < K_1$.

10.88. **IDENTIFY:** Apply Eq.(10.29).

SET UP: The door has $I = \frac{1}{3}ml^2$. The torque applied by the force is rF_{av}, where $r = l/2$.

EXECUTE: $\Sigma\tau_{\text{av}} = rF_{\text{av},}$ and $\Delta L = rF_{\text{av}}\Delta t = rJ$. The angular velocity ω is then

$\omega = \dfrac{\Delta L}{I} = \dfrac{rF_{\text{av}}\Delta t}{I} = \dfrac{(l/2)F_{\text{av}}\Delta t}{\frac{1}{3}ml^2} = \dfrac{3}{2}\dfrac{F_{\text{av}}\Delta t}{ml}$, where l is the width of the door. Substitution of the given numeral values gives $\omega = 0.514\ \text{rad/s}$.

EVALUATE: The final angular velocity of the door is proportional to both the magnitude of the average force and also to the time it acts.

10.89. **(a) IDENTIFY:** Apply conservation of angular momentum to the collision between the bullet and the board:

SET UP: The system before and after the collision is sketched in Figure 10.89a.

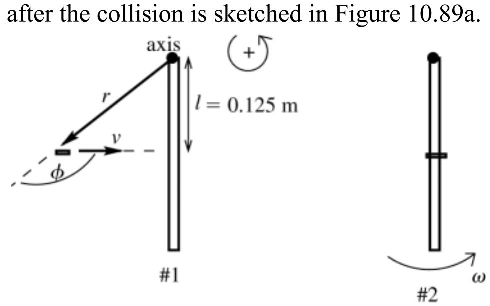


Figure 10.89a

EXECUTE: $L_1 = L_2$

$L_1 = mvr\sin\phi = mvl = (1.90\times10^{-3}\ \text{kg})(360\ \text{m/s})(0.125\ \text{m}) = 0.0855\ \text{kg}\cdot\text{m}^2/\text{s}$

$L_2 = I_2\omega_2$

$I_2 = I_{\text{board}} + I_{\text{bullet}} = \frac{1}{3}ML^2 + mr^2$

$I_2 = \frac{1}{3}(0.750\ \text{kg})(0.250\ \text{m})^2 + (1.90\times10^{-3}\ \text{kg})(0.125\ \text{m})^2 = 0.01565\ \text{kg}\cdot\text{m}^2$

Then $L_1 = L_2$ gives that $\omega_2 = \dfrac{L_1}{I_2} = \dfrac{0.0855\ \text{kg}\cdot\text{m}^2/\text{s}}{0.1565\ \text{kg}\cdot\text{m}^2} = 5.46\ \text{rad/s}$

(b) IDENTIFY: Apply conservation of energy to the motion of the board after the collision.

SET UP: The position of the board at points 1 and 2 in its motion is shown in Figure 10.89b. Take the origin of coordinates at the center of the board and $+y$ to be upward, so $y_{\text{cm},1} = 0$ and $y_{\text{cm},2} = h$, the height being asked for.

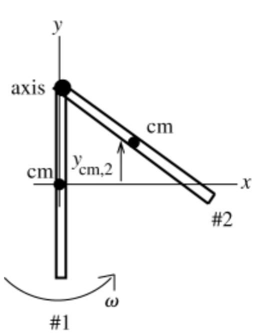


Figure 10.89b

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

EXECUTE: Only gravity does work, so $W_{\text{other}} = 0$.

$K_1 = \frac{1}{2}I\omega^2$

$U_1 = mgy_{\text{cm},1} = 0$

$K_2 = 0$

$U_2 = mgy_{\text{cm},2} = mgh$

Thus $\frac{1}{2}I\omega^2 = mgh.$

$$h = \frac{I\omega^2}{2mg} = \frac{(0.01565\ \text{kg}\cdot\text{m}^2)(5.46\ \text{rad/s})^2}{2(0.750\ \text{kg} + 1.90\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)} = 0.0317\ \text{m} = 3.17\ \text{cm}$$

(c) IDENTIFY and **SET UP:** The position of the board at points 1 and 2 in its motion is shown in Figure 10.89c.

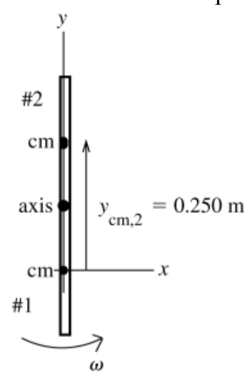


Figure 10.89c

Apply conservation of energy as in part (b), except now we want $y_{\text{cm},2} = h = 0.250$ m. Solve for the ω after the collision that is required for this to happen.

EXECUTE: $\frac{1}{2}I\omega^2 = mgh$

$$\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2(0.750\ \text{kg} + 1.90\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)(0.250\ \text{m})}{0.01565\ \text{kg}\cdot\text{m}^2}}$$

$\omega = 15.34$ rad/s

Now go back to the equation that results from applying conservation of angular momentum to the collision and solve for the initial speed of the bullet. $L_1 = L_2$ implies $m_{\text{bullet}}vl = I_2\omega_2$

$$v = \frac{I_2\omega_2}{m_{\text{bullet}}l} = \frac{(0.01565\ \text{kg}\cdot\text{m}^2)(15.34\ \text{rad/s})}{(1.90\times10^{-3}\ \text{kg})(0.125\ \text{m})} = 1010\ \text{m/s}$$

EVALUATE: We have divided the motion into two separate events: the collision and the motion after the collision. Angular momentum is conserved in the collision because the collision happens quickly. The board doesn't move much until after the collision is over, so there is no gravity torque about the axis. The collision is inelastic and mechanical energy is lost in the collision. Angular momentum of the system is not conserved during this motion, due to the external gravity torque. Our answer to parts (b) and (c) say that a bullet speed of 360 m/s causes the board to swing up only a little and a speed of 1010 m/s causes it to swing all the way over.

10.90. IDENTIFY: Angular momentum is conserved, so $I_0\omega_0 = I_2\omega_2$.

SET UP: For constant mass the moment of inertia is proportional to the square of the radius.

EXECUTE: $R_0^2\omega_0 = R_2^2\omega_2$, or $R_0^2\omega_0 = (R_0 + \Delta R)^2(\omega_0 + \Delta\omega) = R_0^2\omega_0 + 2R_0\Delta R\omega_0 + R_0^2\Delta\omega$, where the terms in $\Delta R\Delta\omega$ and $(\Delta\omega)^2$ have been omitted. Canceling the $R_0^2\omega_0$ term gives

$$\Delta R = -\frac{R_0}{2}\frac{\Delta\omega}{\omega_0} = -1.1\ \text{cm}.$$

EVALUATE: $\Delta R/R_0$ and $\Delta\omega/\omega_0$ are each very small so the neglect of terms containing $\Delta R\Delta\omega$ or $(\Delta\omega)^2$ is an accurate simplifying approximation.

10.91. IDENTIFY: Apply conservation of angular momentum to the collision between the bird and the bar and apply conservation of energy to the motion of the bar after the collision.

SET UP: For conservation of angular momentum take the axis at the hinge. For this axis the initial angular momentum of the bird is $m_{\text{bird}}(0.500\ \text{m})v$, where $m_{\text{bird}} = 0.500$ kg and $v = 2.25$ m/s. For this axis the moment of inertia is $I = \frac{1}{3}m_{\text{bar}}L^2 = \frac{1}{3}(1.50\ \text{kg})(0.750\ \text{m})^2 = 0.281\ \text{kg}\cdot\text{m}^2$. For conservation of energy, the gravitational potential energy of the bar is $U = m_{\text{bar}}gy_{\text{cm}}$, where y_{cm} is the height of the center of the bar. Take $y_{\text{cm},1} = 0$, so $y_{\text{cm},2} = -0.375$ m.

EXECUTE: (a) $L_1 = L_2$ gives $m_{\text{bird}}(0.500\ \text{m})v = (\frac{1}{3}m_{\text{bar}}L^2)\omega$.

$$\omega = \frac{3m_{\text{bird}}(0.500\ \text{m})v}{m_{\text{bar}}L^2} = \frac{3(0.500\ \text{kg})(0.500\ \text{m})(2.25\ \text{m/s})}{(1.50\ \text{kg})(0.750\ \text{m})^2} = 2.00\ \text{rad/s}.$$

(b) $U_1 + K_1 = U_2 + K_2$ applied to the motion of the bar after the collision gives $\frac{1}{2}I\omega_1^2 = m_{\text{bar}}g(-0.375\text{ m}) + \frac{1}{2}I\omega_2^2$.

$\omega_2 = \sqrt{\omega_1^2 + \frac{2}{I}m_{\text{bar}}g(0.375\text{ m})}$. $\omega_2 = \sqrt{(2.00\text{ rad/s})^2 + \frac{2}{0.281\text{ kg}\cdot\text{m}^2}(1.50\text{ kg})(9.80\text{ m/s}^2)(0.375\text{ m})} = 6.58\text{ rad/s}$

EVALUATE: Mechanical energy is not conserved in the collision. The kinetic energy of the bar just after the collision is less than the kinetic energy of the bird just before the collision.

10.92. IDENTIFY: Angular momentum is conserved, since the tension in the string is in the radial direction and therefore produces no torque. Apply $\sum \vec{F} = m\vec{a}$ to the block, with $a = a_{\text{rad}} = v^2/r$.

SET UP: The block's angular momentum with respect to the hole is $L = mvr$.

EXECUTE: The tension is related to the block's mass and speed, and the radius of the circle, by $T = m\frac{v^2}{r}$.

$T = mv^2\frac{1}{r} = \frac{m^2v^2}{m}\frac{r^2}{r^3} = \frac{(mvr)^2}{mr^3} = \frac{L^2}{mr^3}$. The radius at which the string breaks is

$r^3 = \frac{L^2}{mT_{\max}} = \frac{(mv_1r_1)^2}{mT_{\max}} = \frac{\left((0.250\text{ kg})(4.00\text{ m/s})(0.800\text{ m})\right)^2}{(0.250\text{ kg})(30.0\text{ N})}$, from which $r = 0.440$ m.

EVALUATE: Just before the string breaks the speed of the rock is $(4.00\text{ m/s})\left(\frac{0.800\text{ m}}{0.440\text{ m}}\right) = 7.27\text{ m/s}$. We can verify that $v = 7.27$ m/s and $r = 0.440$ m do give $T = 30.0$ N.

10.93. IDENTIFY and SET UP: Apply conservation of angular momentum to the system consisting of the disk and train.

SET UP: $L_1 = L_2$, counterclockwise positive. The motion is sketched in Figure 10.93.

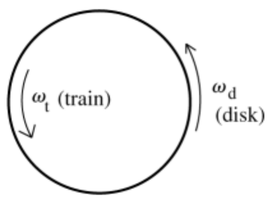


Figure 10.93

$L_1 = 0$ (before you switch on the train's engine; both the train and the platform are at rest)
$L_2 = L_{\text{train}} + L_{\text{disk}}$

EXECUTE: The train is $\frac{1}{2}(0.95\text{ m}) = 0.475$ m from the axis of rotation, so for it

$I_t = m_tR_t^2 = (1.20\text{ kg})(0.475\text{ m})^2 = 0.2708\text{ kg}\cdot\text{m}^2$

$\omega_{\text{rel}} = v_{\text{rel}}/R_t = (0.600\text{ m/s})/0.475\text{ s} = 1.263\text{ rad/s}$

This is the angular velocity of the train relative to the disk. Relative to the earth $\omega_t = \omega_{\text{rel}} + \omega_d$.

Thus $L_{\text{train}} = I_t\omega_t = I_t(\omega_{\text{rel}} + \omega_d)$.

$L_2 = L_1$ says $L_{\text{disk}} = -L_{\text{train}}$

$L_{\text{disk}} = I_d\omega_d$, where $I_d = \frac{1}{2}m_dR_d^2$

$\frac{1}{2}m_dR_d^2\omega_d = -I_t(\omega_{\text{rel}} + \omega_d)$

$\omega_d = -\frac{I_t\omega_{\text{rel}}}{\frac{1}{2}m_dR_d^2 + I_t} = -\frac{(0.2708\text{ kg}\cdot\text{m}^2)(1.263\text{ rad/s})}{\frac{1}{2}(7.00\text{ kg})(0.500\text{ m})^2 + 0.2708\text{ kg}\cdot\text{m}^2} = -0.30\text{ rad/s}.$

EVALUATE: The minus sign tells us that the disk is rotating clockwise relative to the earth. The disk and train rotate in opposite directions, since the total angular momentum of the system must remain zero. Note that we applied $L_1 = L_2$ in an inertial frame attached to the earth.

10.94. IDENTIFY: I for the wheel is the sum of the values of I for each of its parts, the rim and each spoke. The total length of wire is constant. The motion is related to the friction torque by $\sum\tau_z = I\alpha_z$.

SET UP: $4R + 2\pi R = L_0$, where R is the radius of the wheel and therefore the length of each of the four spokes. The mass of a piece is proportional to the length of that piece.

EXECUTE: **(a)** $R = \frac{L_0}{4 + 2\pi}$. $I_{\text{rim}} = m_{\text{rim}}R^2$. $m_{\text{rim}} = \frac{2\pi R}{L_0}M_0 = \left(\frac{2\pi}{4 + 2\pi}\right)M_0$.

$I_{\text{rim}} = M_0L_0^2\frac{2\pi}{(2\pi + 4)^3} = (5.778\times10^{-3})M_0L_0^2$. $I_{\text{spoke}} = \frac{1}{3}m_{\text{spoke}}R^2$. $m_{\text{spoke}} = \frac{R}{L_0}M_0 = \frac{M_0}{2\pi + 4}$ and

$I_{\text{spoke}} = M_0L_0^2\frac{1}{3(2\pi + 4)^3} = (3.065\times10^{-4})M_0L_0^2$. $I = I_{\text{rim}} + 4I_{\text{spoke}} = (7.00\times10^{-3})M_0L_0^2$.

(b) $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = -\dfrac{\omega_0}{T}$. Then $\sum \tau_z = I\alpha_z$ gives $\tau_f = (7.00\times10^{-3})M_0 L_0^2 \dfrac{\omega_0}{T}$

EVALUATE: If the wire were bent into a circle, without spokes, the moment of inertia would be $M_0 R^2 = \dfrac{M_0 L_0^2}{(4+2\pi)^2} = (9.46\times10^{-3})M_0 L_0^2$. The actual value of I for the wheel is less than this because the mass in the spokes is closer to the axis than the rim.

10.95. IDENTIFY and **SET UP:** Use the methods stipulated in the problem.

EXECUTE: **(a)** The initial angular momentum with respect to the pivot is mvr, and the final total moment of inertia is $I + mr^2$, so the final angular velocity is $\omega = mvr/\left(mr^2 + I\right)$.

(b) The kinetic energy after the collision is converted to gravitational potential energy, so

$\dfrac{1}{2}\omega^2\left(mr^2 + I\right) = \left(M + m\right)gh$, or $\omega = \sqrt{\dfrac{2\left(M+m\right)gh}{\left(mr^2 + I\right)}}$.

(c) Substitution of $I = Mr^2$ into the result of part (a) gives $\omega = \left(\dfrac{m}{m+M}\right)\left(v/r\right)$, and into the result of part (b), $\omega = \sqrt{2gh}\,(1/r)$, which are consistent with the forms for v.

EVALUATE: $I = Mr^2$ applies approximately when the pendulum consists of a heavy catcher mounted on a light arm. In the actual apparatus some of the mass is distributed closer to the axis and $I < Mr^2$.

10.96. IDENTIFY: Apply conservation of momentum to the system of the runner and turntable

SET UP: Let the positive sense of rotation be the direction the turntable is rotating initially.

EXECUTE: The initial angular momentum is $I\omega_1 - mRv_1$, with the minus sign indicating that runner's motion is opposite the motion of the part of the turntable under his feet. The final angular momentum is $\omega_2(I + mR^2)$, so $\omega_2 = \dfrac{I\omega_1 - mRv_1}{I + mR^2}$.

$$\omega_2 = \frac{(80\ \text{kg}\cdot\text{m}^2)(0.200\ \text{rad/s}) - (55.0\ \text{kg})(3.00\ \text{m})(2.8\ \text{m/s})}{(80\ \text{kg}\cdot\text{m}^2) + (55.0\ \text{kg})(3.00\ \text{m})^2} = -0.776\ \text{rad/s}.$$

EVALUATE: The minus sign indicates that the turntable has reversed its direction of motion. This happened because the man had the larger magnitude of angular momentum initially.

10.97. IDENTIFY: Treat the moon as a point mass, so $L = I\omega = mr^2\omega$, where r is the distance of the moon from the center of the earth. Conservation of angular momentum says $dL/dt = 0$.

SET UP: $dr/dt = 3.0\ \text{cm/y} = 3.0\times10^{-2}\ \text{m/y}$. The period of the moon's orbital motion is $27.3\ \text{d} = 2.36\times10^6\ \text{s}$. $r = 3.84\times10^8\ \text{m}$.

EXECUTE: $dL/dt = \dfrac{d}{dt}(mr^2\omega) = m\omega(2r)\dfrac{dr}{dt} + mr^2\dfrac{d\omega}{dt} = 0$, so $\dfrac{d\omega}{dt} = -\dfrac{2\omega}{r}\dfrac{dr}{dt}$.

$\omega = \dfrac{2\pi\ \text{rad}}{T} = \dfrac{2\pi\ \text{rad}}{2.36\times10^6\ \text{s}} = 2.66\times10^{-6}\ \text{rad/s}$. $\dfrac{d\omega}{dt} = -\dfrac{2(2.66\times10^{-6}\ \text{rad/s})}{3.84\times10^8\ \text{m}}(3.0\times10^{-2}\ \text{m/y}) = -4.2\times10^{-16}\ \text{rad/s per year}$.

$\dfrac{d\omega}{dt}$ is negative, so the angular velocity is decreasing.

EVALUATE: $L = mr^2\omega$. If L is constant, then ω decreases when r increases. The fractional changes in r and ω are very, very small.

10.98. IDENTIFY: Follow the method outlined in the hint.

SET UP: $J = m\Delta v_{cm}$. $\Delta L = J(x - x_{cm})$.

EXECUTE: The velocity of the center of mass will change by $\Delta v_{cm} = J/m$ and the angular velocity will change by $\Delta\omega = \dfrac{J(x - x_{cm})}{I}$. The change is velocity of the end of the bat will then be $\Delta v_{end} = \Delta v_{cm} - \Delta\omega x_{cm} = \dfrac{J}{m} - \dfrac{J(x - x_{cm})x_{cm}}{I}$. Setting $\Delta v_{end} = 0$ allows cancellation of J and gives $I = (x - x_{cm})x_{cm}m$, which when solved for x is

$x = \dfrac{I}{x_{cm}m} + x_{cm} = \dfrac{(5.30\times10^{-2}\text{kg}\cdot\text{m}^2)}{(0.600\ \text{m})(0.800\ \text{kg})} + (0.600\ \text{m}) = 0.710\ \text{m}.$

EVALUATE: The center of percussion is farther from the handle than the center of mass.

10.99. **IDENTIFY** and **SET UP:** Follow the analysis that led to Eq.(10.33).

EXECUTE: In Figure 10.33a in the textbook, if the vector $\vec{r}$ and hence the vector $\vec{L}$ are not horizontal but make an angle β with the horizontal, the torque will still be horizontal (the torque must be perpendicular to the vertical weight). The magnitude of the torque will be $\omega r\cos\beta$, and this torque will change the direction of the horizontal component of the angular momentum, which has magnitude $L\cos\beta$. Thus, the situation of Figure 10.35 in the textbook is reproduced, but with $\vec{L}_{\text{horiz}}$ instead of $\vec{L}$. Then, the expression found in Eq. (10.33) becomes

$$\Omega=\frac{d\phi}{dt}=\frac{\left|d\vec{L}\right|/\left|\vec{L}_{\text{horiz}}\right|}{dt}=\frac{\tau}{\left|\vec{L}_{\text{horiz}}\right|}=\frac{mgr\cos\beta}{L\cos\beta}=\frac{wr}{I\omega}.$$

EVALUATE: The torque and the horizontal component of $\vec{L}$ both depend on β by the same factor, $\cos\beta$.

10.100. **IDENTIFY:** Apply conservation of energy to the motion of the ball.

SET UP: In relating $\frac{1}{2}mv_{\text{cm}}^2$ and $\frac{1}{2}I\omega^2$, instead of $v_{\text{cm}}=R\omega$ use the relation derived in part (a). $I=\frac{2}{5}mR^2$.

EXECUTE: **(a)** Consider the sketch in Figure 10.100.
The distance from the center of the ball to the midpoint of the line joining the points where the ball is in contact with the rails is $\sqrt{R^2-(d/2)^2}$, so $v_{\text{cm}}=\omega\sqrt{R^2-d^2/4}$. When $d=0$, this reduces to $v_{\text{cm}}=\omega R$, the same as rolling on a flat surface. When $d=2R$, the rolling radius approaches zero, and $v_{\text{cm}}\to 0$ for any ω.

(b) $K=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2=\frac{1}{2}\left[mv_{\text{cm}}^2+(2/5)mR^2\left(\frac{v_{\text{cm}}}{\sqrt{R^2-(d^2/4)}}\right)^2\right]=\frac{mv_{\text{cm}}^2}{10}\left[5+\frac{2}{(1-d^2/4R^2)}\right]$

Setting this equal to mgh and solving for v_{cm} gives the desired result.

(c) The denominator in the square root in the expression for v_{cm} is larger than for the case $d=0$, so v_{cm} is smaller. For a given speed, ω is larger than in the $d=0$ case, so a larger fraction of the kinetic energy is rotational, and the translational kinetic energy, and hence v_{cm}, is smaller.

(d) Setting the expression in part (b) equal to 0.95 of that of the $d=0$ case and solving for the ratio d/R gives $d/R=1.05$. Setting the ratio equal to 0.995 gives $d/R=0.37$.

EVALUATE: If we set $d=0$ in the expression in part (b), $v_{\text{cm}}=\sqrt{\frac{10gh}{7}}$, the same as for a sphere rolling down a ramp. When $d\to 2R$, the expression gives $v_{\text{cm}}=0$, as it should.

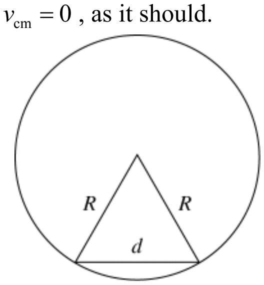


Figure 10.100

10.101. **IDENTIFY:** Apply $\sum\vec{F}_{\text{ext}}=m\vec{a}_{\text{cm}}$ and $\sum\tau_z=I_{\text{cm}}\alpha_z$ to the motion of the cylinder. Use constant acceleration equations to relate a_x to the distance the object travels. Use the work-energy theorem to find the work done by friction.

SET UP: The cylinder has $I_{\text{cm}}=\frac{1}{2}MR^2$.

EXECUTE: **(a)** The free-body diagram is sketched in Figure 10.101. The friction force is $f=\mu_{\text{k}}n=\mu_{\text{k}}Mg$, so $a=\mu_{\text{k}}g$. The magnitude of the angular acceleration is $\frac{fR}{I}=\frac{\mu_{\text{k}}MgR}{(1/2)MR^2}=\frac{2\mu_{\text{k}}g}{R}$.

(b) Setting $v=at=\omega R=(\omega_0-\omega t)R$ and solving for t gives $t=\frac{R\omega_0}{a+R\alpha}=\frac{R\omega_0}{\mu_{\text{k}}g+2\mu_{\text{k}}g}=\frac{R\omega_0}{3\mu_{\text{k}}g}$,

and $d=\frac{1}{2}at^2=\frac{1}{2}(\mu_{\text{k}}g)\left(\frac{R\omega_0}{3\mu_{\text{k}}g}\right)^2=\frac{R^2\omega_0^2}{18\mu_{\text{k}}g}$.

(c) The final kinetic energy is $(3/4)Mv^2 = (3/4)M(at)^2$, so the change in kinetic energy is

$$\Delta K = \frac{3}{4}M\left(\mu_k g \frac{R\omega_0}{3\mu_k g}\right)^2 - \frac{1}{4}MR^2\omega_0^2 = -\frac{1}{6}MR^2\omega_0^2.$$

EVALUATE: The fraction of the initial kinetic energy that is removed by friction work is $\frac{\frac{1}{6}MR\omega_0^2}{\frac{1}{4}MR\omega_0^2} = \frac{2}{3}$. This fraction is independent of the initial angular speed ω_0.

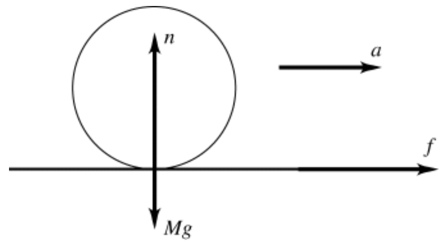


Figure 10.101

10.102. IDENTIFY: The vertical forces must sum to zero. Apply Eq.(10.33).

SET UP: Denote the upward forces that the hands exert as F_L and F_R. $\tau = (F_L - F_R)r$, where $r = 0.200$ m.

EXECUTE: The conditions that F_L and F_R must satisfy are $F_L + F_R = w$ and $F_L - F_R = \Omega\frac{I\omega}{r}$, where the second equation is $\tau = \Omega L$, divided by *r*. These two equations can be solved for the forces by first adding and then subtracting, yielding $F_L = \frac{1}{2}\left(w + \Omega\frac{I\omega}{r}\right)$ and $F_R = \frac{1}{2}\left(w - \Omega\frac{I\omega}{r}\right)$. Using the values $w = mg = (8.00\text{ kg})(9.80\text{ m/s}^2) = 78.4$ N and

$$\frac{I\omega}{r} = \frac{(8.00\text{ kg})(0.325\text{ m})^2(5.00\text{ rev/s}\times 2\pi\text{ rad/rev})}{(0.200\text{ m})} = 132.7\text{ kg}\cdot\text{m/s} \text{ gives}$$

$F_L = 39.2\text{ N} + \Omega(66.4\text{ N}\cdot\text{s})$, $F_R = 39.2\text{ N} - \Omega(66.4\text{ N}\cdot\text{s})$.

(a) $\Omega = 0, F_L = F_R = 39.2$ N.

(b) $\Omega = 0.05\text{ rev/s} = 0.314\text{ rad/s}, F_L = 60.0\text{ N}, F_R = 18.4\text{ N}.$

(c) $\Omega = 0.3\text{ rev/s} = 1.89\text{ rad/s}, F_L = 165\text{ N}, F_R = -86.2$ N, with the minus sign indicating a downward force.

(e) $F_R = 0$ gives $\Omega = \frac{39.2\text{ N}}{66.4\text{ N}\cdot\text{s}} = 0.575\text{ rad/s}$, which is 0.0916 rev/s.

EVALUATE: The larger the precession rate Ω, the greater the torque on the wheel and the greater the difference between the forces exerted by the two hands.

10.103. IDENTIFY: The answer to part (a) can be taken from the solution to Problem 10.92. The work-energy theorem says $W = \Delta K$.

SET UP: Problem 10.92 uses conservation of angular momentum to show that $r_1 v_1 = r_2 v_2$.

EXECUTE: **(a)** $T = mv_1^2 r_1^2 / r^3$.

(b) $\vec{T}$ and $d\vec{r}$ are always antiparallel. $\vec{T}\cdot d\vec{r} = -T dr$.

$$W = -\int_{r_1}^{r_2} T\,dr = mv_1^2 r_1^2 \int_{r_2}^{r_1} \frac{dr}{r^3} = \frac{mv_1^2}{2} r_1^2 \left[\frac{1}{r_2^2} - \frac{1}{r_1^2}\right].$$

(c) $v_2 = v_1(r_1/r_2)$, so $\Delta K = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{mv_1^2}{2}\left[(r_1/r_2)^2 - 1\right]$, which is the same as the work found in part (b).

EVALUATE: The work done by *T* is positive, since $\vec{T}$ is toward the hole in the surface and the block moves toward the hole. Positive work means the kinetic energy of the object increases.