

ROTATION OF RIGID BODIES

9.1. IDENTIFY: $s = r\theta$, with θ in radians.

SET UP: $\pi \text{ rad} = 180^\circ$.

EXECUTE: (a) $\theta = \frac{s}{r} = \frac{1.50 \text{ m}}{2.50 \text{ m}} = 0.600 \text{ rad} = 34.4^\circ$

(b) $r = \frac{s}{\theta} = \frac{14.0 \text{ cm}}{(128^\circ)(\pi \text{ rad}/180^\circ)} = 6.27 \text{ cm}$

(c) $s = r\theta = (1.50 \text{ m})(0.700 \text{ rad}) = 1.05 \text{ m}$

EVALUATE: An angle is the ratio of two lengths and is dimensionless. But, when $s = r\theta$ is used, θ must be in radians. Or, if $\theta = s/r$ is used to calculate θ , the calculation gives θ in radians.

9.2. IDENTIFY: $\theta - \theta_0 = \omega t$, since the angular velocity is constant.

SET UP: $1 \text{ rpm} = (2\pi/60) \text{ rad/s}$.

EXECUTE: (a) $\omega = (1900)(2\pi \text{ rad}/60 \text{ s}) = 199 \text{ rad/s}$

(b) $35^\circ = (35^\circ)(\pi/180^\circ) = 0.611 \text{ rad}$. $t = \frac{\theta - \theta_0}{\omega} = \frac{0.611 \text{ rad}}{199 \text{ rad/s}} = 3.1 \times 10^{-3} \text{ s}$

EVALUATE: In $t = \frac{\theta - \theta_0}{\omega}$ we must use the same angular measure (radians, degrees or revolutions) for both $\theta - \theta_0$ and ω .

9.3. IDENTIFY: $\alpha_z(t) = \frac{d\omega_z}{dt}$. Writing Eq.(2.16) in terms of angular quantities gives $\theta - \theta_0 = \int_{t_1}^{t_2} \omega_z dt$.

SET UP: $\frac{d}{dt} t^n = nt^{n-1}$ and $\int t^n dt = \frac{1}{n+1} t^{n+1}$

EXECUTE: (a) A must have units of rad/s and B must have units of rad/s^3 .

(b) $\alpha_z(t) = 2Bt = (3.00 \text{ rad/s}^3)t$. (i) For $t = 0$, $\alpha_z = 0$. (ii) For $t = 5.00 \text{ s}$, $\alpha_z = 15.0 \text{ rad/s}^2$.

(c) $\theta_2 - \theta_1 = \int_{t_1}^{t_2} (A + Bt^2) dt = A(t_2 - t_1) + \frac{1}{3}B(t_2^3 - t_1^3)$. For $t_1 = 0$ and $t_2 = 2.00 \text{ s}$,

$\theta_2 - \theta_1 = (2.75 \text{ rad/s})(2.00 \text{ s}) + \frac{1}{3}(1.50 \text{ rad/s}^3)(2.00 \text{ s})^3 = 9.50 \text{ rad}$.

EVALUATE: Both α_z and ω_z are positive and the angular speed is increasing.

9.4. IDENTIFY: $\alpha_z = d\omega_z / dt$. $\alpha_{\text{av-z}} = \frac{\Delta\omega_z}{\Delta t}$.

SET UP: $\frac{d}{dt}(t^2) = 2t$

EXECUTE: (a) $\alpha_z(t) = \frac{d\omega_z}{dt} = -2\beta t = (-1.60 \text{ rad/s}^3)t$.

(b) $\alpha_z(3.0 \text{ s}) = (-1.60 \text{ rad/s}^3)(3.0 \text{ s}) = -4.80 \text{ rad/s}^2$.

$\alpha_{\text{av-z}} = \frac{\omega_z(3.0 \text{ s}) - \omega_z(0)}{3.0 \text{ s}} = \frac{-2.20 \text{ rad/s} - 5.00 \text{ rad/s}}{3.0 \text{ s}} = -2.40 \text{ rad/s}^2$,

which is half as large (in magnitude) as the acceleration at $t = 3.0 \text{ s}$.

EVALUATE: $\alpha_z(t)$ increases linearly with time, so $\alpha_{\text{av-z}} = \frac{\alpha_z(0) + \alpha_z(3.0 \text{ s})}{2}$. $\alpha_z(0) = 0$.

- 9.5. IDENTIFY and SET UP:** Use Eq.(9.3) to calculate the angular velocity and Eq.(9.2) to calculate the average angular velocity for the specified time interval.

EXECUTE: $\theta = \gamma t + \beta t^3$; $\gamma = 0.400 \text{ rad/s}$, $\beta = 0.0120 \text{ rad/s}^3$

(a) $\omega_z = \frac{d\theta}{dt} = \gamma + 3\beta t^2$

(b) At $t = 0$, $\omega_z = \gamma = 0.400 \text{ rad/s}$

(c) At $t = 5.00 \text{ s}$, $\omega_z = 0.400 \text{ rad/s} + 3(0.0120 \text{ rad/s}^3)(5.00 \text{ s})^2 = 1.30 \text{ rad/s}$

$$\omega_{\text{av-z}} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

For $t_1 = 0$, $\theta_1 = 0$.

For $\theta_2 = 5.00 \text{ s}$, $\theta_2 = (0.400 \text{ rad/s})(5.00 \text{ s}) + (0.012 \text{ rad/s}^3)(5.00 \text{ s})^3 = 3.50 \text{ rad}$

So $\omega_{\text{av-z}} = \frac{3.50 \text{ rad} - 0}{5.00 \text{ s} - 0} = 0.700 \text{ rad/s}$.

EVALUATE: The average of the instantaneous angular velocities at the beginning and end of the time interval is $\frac{1}{2}(0.400 \text{ rad/s} + 1.30 \text{ rad/s}) = 0.850 \text{ rad/s}$. This is larger than $\omega_{\text{av-z}}$, because $\omega_z(t)$ is increasing faster than linearly.

- 9.6. IDENTIFY:** $\omega_z(t) = \frac{d\theta}{dt}$, $\alpha_z(t) = \frac{d\omega_z}{dt}$, $\omega_{\text{av-z}} = \frac{\Delta\theta}{\Delta t}$.

SET UP: $\omega_z = (250 \text{ rad/s}) - (40.0 \text{ rad/s}^2)t - (4.50 \text{ rad/s}^3)t^2$. $\alpha_z = -(40.0 \text{ rad/s}^2) - (9.00 \text{ rad/s}^3)t$.

EXECUTE: (a) Setting $\omega_z = 0$ results in a quadratic in t . The only positive root is $t = 4.23 \text{ s}$.

(b) At $t = 4.23 \text{ s}$, $\alpha_z = -78.1 \text{ rad/s}^2$.

(c) At $t = 4.23 \text{ s}$, $\theta = 586 \text{ rad} = 93.3 \text{ rev}$.

(d) At $t = 0$, $\omega_z = 250 \text{ rad/s}$.

(e) $\omega_{\text{av-z}} = \frac{586 \text{ rad}}{4.23 \text{ s}} = 138 \text{ rad/s}$.

EVALUATE: Between $t = 0$ and $t = 4.23 \text{ s}$, ω_z decreases from 250 rad/s to zero. ω_z is not linear in t , so $\omega_{\text{av-z}}$ is not midway between the values of ω_z at the beginning and end of the interval.

- 9.7. IDENTIFY:** $\omega_z(t) = \frac{d\theta}{dt}$, $\alpha_z(t) = \frac{d\omega_z}{dt}$. Use the values of θ and ω_z at $t = 0$ and α_z at 1.50 s to calculate a , b , and c .

SET UP: $\frac{d}{dt}t^n = nt^{n-1}$

EXECUTE: (a) $\omega_z(t) = b - 3ct^2$. $\alpha_z(t) = -6ct$. At $t = 0$, $\theta = a = \pi/4 \text{ rad}$ and $\omega_z = b = 2.00 \text{ rad/s}$. At $t = 1.50 \text{ s}$, $\alpha_z = -6c(1.50 \text{ s}) = 1.25 \text{ rad/s}^2$ and $c = -0.139 \text{ rad/s}^3$.

(b) $\theta = \pi/4 \text{ rad}$ and $\alpha_z = 0$ at $t = 0$.

(c) $\alpha_z = 3.50 \text{ rad/s}^2$ at $t = -\frac{\alpha_z}{6c} = -\frac{3.50 \text{ rad/s}^2}{6(-0.139 \text{ rad/s}^3)} = 4.20 \text{ s}$. At $t = 4.20 \text{ s}$,

$$\theta = \frac{\pi}{4} \text{ rad} + (2.00 \text{ rad/s})(4.20 \text{ s}) - (-0.139 \text{ rad/s}^3)(4.20 \text{ s})^3 = 19.5 \text{ rad}$$

$$\omega_z = 2.00 \text{ rad/s} - 3(-0.139 \text{ rad/s}^3)(4.20 \text{ s})^2 = 9.36 \text{ rad/s}$$

EVALUATE: θ , ω_z and α_z all increase as t increases.

- 9.8. IDENTIFY:** $\alpha_z = \frac{d\omega_z}{dt}$. $\theta - \theta_0 = \omega_{\text{av-z}}t$. When ω_z is linear in t , $\omega_{\text{av-z}}$ for the time interval t_1 to t_2 is

$$\omega_{\text{av-z}} = \frac{\omega_{z1} + \omega_{z2}}{t_2 - t_1}$$

SET UP: From the information given, $\omega_z(t) = -6.00 \text{ rad/s} + (2.00 \text{ rad/s}^2)t$

EXECUTE: (a) The angular acceleration is positive, since the angular velocity increases steadily from a negative value to a positive value.

(b) It takes 3.00 s for the wheel to stop ($\omega_z = 0$). During this time its speed is decreasing. For the next 4.00 s its speed is increasing from 0 rad/s to $+8.00 \text{ rad/s}$.

(c) The average angular velocity is $\frac{-6.00 \text{ rad/s} + 8.00 \text{ rad/s}}{2} = 1.00 \text{ rad/s}$. $\theta - \theta_0 = \omega_{\text{av-z}} t$ then leads to displacement of 7.00 rad after 7.00 s.

EVALUATE: When α_z and ω_z have the same sign, the angular speed is increasing; this is the case for $t = 3.00 \text{ s}$ to $t = 7.00 \text{ s}$. When α_z and ω_z have opposite signs, the angular speed is decreasing; this is the case between $t = 0$ and $t = 3.00 \text{ s}$.

9.9. IDENTIFY: Apply the constant angular acceleration equations.

SET UP: Let the direction the wheel is rotating be positive.

EXECUTE: (a) $\omega_z = \omega_{0z} + \alpha_z t = 1.50 \text{ rad/s} + (0.300 \text{ rad/s}^2)(2.50 \text{ s}) = 2.25 \text{ rad/s}$.

(b) $\theta - \theta_0 = \omega_{0z} t + \frac{1}{2} \alpha_z t^2 = (1.50 \text{ rad/s})(2.50 \text{ s}) + \frac{1}{2}(0.300 \text{ rad/s}^2)(2.50 \text{ s})^2 = 4.69 \text{ rad}$.

EVALUATE: $\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t = \left(\frac{1.50 \text{ rad/s} + 2.25 \text{ rad/s}}{2}\right)(2.50 \text{ s}) = 4.69 \text{ rad}$, the same as calculated with another equation in part (b).

9.10. IDENTIFY: Apply the constant angular acceleration equations to the motion of the fan.

(a) **SET UP:** $\omega_{0z} = (500 \text{ rev/min})(1 \text{ min}/60 \text{ s}) = 8.333 \text{ rev/s}$, $\omega_z = (200 \text{ rev/min})(1 \text{ min}/60 \text{ s}) = 3.333 \text{ rev/s}$, $t = 4.00 \text{ s}$, $\alpha_z = ?$

$$\omega_z = \omega_{0z} + \alpha_z t$$

EXECUTE: $\alpha_z = \frac{\omega_z - \omega_{0z}}{t} = \frac{3.333 \text{ rev/s} - 8.333 \text{ rev/s}}{4.00 \text{ s}} = -1.25 \text{ rev/s}^2$

$$\theta - \theta_0 = ?$$

$$\theta - \theta_0 = \omega_{0z} t + \frac{1}{2} \alpha_z t^2 = (8.333 \text{ rev/s})(4.00 \text{ s}) + \frac{1}{2}(-1.25 \text{ rev/s}^2)(4.00 \text{ s})^2 = 23.3 \text{ rev}$$

(b) **SET UP:** $\omega_z = 0$ (comes to rest); $\omega_{0z} = 3.333 \text{ rev/s}$; $\alpha_z = -1.25 \text{ rev/s}^2$;

$$t = ?$$

$$\omega_z = \omega_{0z} + \alpha_z t$$

EXECUTE: $t = \frac{\omega_z - \omega_{0z}}{\alpha_z} = \frac{0 - 3.333 \text{ rev/s}}{-1.25 \text{ rev/s}^2} = 2.67 \text{ s}$

EVALUATE: The angular acceleration is negative because the angular velocity is decreasing. The average angular velocity during the 4.00 s time interval is 350 rev/min and $\theta - \theta_0 = \omega_{\text{av-z}} t$ gives $\theta - \theta_0 = 23.3 \text{ rev}$, which checks.

9.11. IDENTIFY: Apply the constant angular acceleration equations to the motion. The target variables are t and $\theta - \theta_0$.

SET UP: (a) $\alpha_z = 1.50 \text{ rad/s}^2$; $\omega_{0z} = 0$ (starts from rest); $\omega_z = 36.0 \text{ rad/s}$; $t = ?$

$$\omega_z = \omega_{0z} + \alpha_z t$$

EXECUTE: $t = \frac{\omega_z - \omega_{0z}}{\alpha_z} = \frac{36.0 \text{ rad/s} - 0}{1.50 \text{ rad/s}^2} = 24.0 \text{ s}$

(b) $\theta - \theta_0 = ?$

$$\theta - \theta_0 = \omega_{0z} t + \frac{1}{2} \alpha_z t^2 = 0 + \frac{1}{2}(1.50 \text{ rad/s}^2)(24.0 \text{ s})^2 = 432 \text{ rad}$$

$$\theta - \theta_0 = 432 \text{ rad}(1 \text{ rev}/2\pi \text{ rad}) = 68.8 \text{ rev}$$

EVALUATE: We could use $\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t$ to calculate $\theta - \theta_0 = \frac{1}{2}(0 + 36.0 \text{ rad/s})(24.0 \text{ s}) = 432 \text{ rad}$, which checks.

9.12. IDENTIFY: In part (b) apply the equation derived in part (a).

SET UP: Let the direction the propeller is rotating be positive.

EXECUTE: (a) Solving Eq. (9.7) for t gives $t = \frac{\omega_z - \omega_{0z}}{\alpha_z}$. Rewriting Eq. (9.11) as $\theta - \theta_0 = t(\omega_{0z} + \frac{1}{2}\alpha_z t)$ and

substituting for t gives

$$\theta - \theta_0 = \left(\frac{\omega_z - \omega_{0z}}{\alpha_z}\right)\left(\omega_{0z} + \frac{1}{2}(\omega_z - \omega_{0z})\right) = \frac{1}{\alpha_z}(\omega_z - \omega_{0z})\left(\frac{\omega_z + \omega_{0z}}{2}\right) = \frac{1}{2\alpha_z}(\omega_z^2 - \omega_{0z}^2),$$

which when rearranged gives Eq. (9.12).

(b) $\alpha_z = \frac{1}{2}\left(\frac{1}{\theta - \theta_0}\right)(\omega_z^2 - \omega_{0z}^2) = \frac{1}{2}\left(\frac{1}{7.00 \text{ rad}}\right)\left((16.0 \text{ rad/s})^2 - (12.0 \text{ rad/s})^2\right) = 8.00 \text{ rad/s}^2$

EVALUATE: We could also use $\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t$ to calculate $t = 0.500$ s. Then $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = 8.00$ rad/s², which agrees with our results in part (b).

9.13. IDENTIFY: Use a constant angular acceleration equation and solve for ω_{0z} .

SET UP: Let the direction of rotation of the flywheel be positive.

EXECUTE: $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ gives $\omega_{0z} = \frac{\theta - \theta_0}{t} - \frac{1}{2}\alpha_z = \frac{60.0 \text{ rad}}{4.00 \text{ s}} - \frac{1}{2}(2.25 \text{ rad/s}^2)(4.00 \text{ s}) = 10.5 \text{ rad/s}$.

EVALUATE: At the end of the 4.00 s interval, $\omega_z = \omega_{0z} + \alpha_z t = 19.5 \text{ rad/s}$.

$\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t = \left(\frac{10.5 \text{ rad/s} + 19.5 \text{ rad/s}}{2}\right)(4.00 \text{ s}) = 60.0 \text{ rad}$, which checks.

9.14. IDENTIFY: Apply the constant angular acceleration equations.

SET UP: Let the direction of the rotation of the blade be positive. $\omega_{0z} = 0$.

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = \frac{\omega_z - \omega_{0z}}{t} = \frac{140 \text{ rad/s} - 0}{6.00 \text{ s}} = 23.3 \text{ rad/s}^2$.

$(\theta - \theta_0) = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t = \left(\frac{0 + 140 \text{ rad/s}}{2}\right)(6.00 \text{ s}) = 420 \text{ rad}$

EVALUATE: We could also use $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$. This equation gives

$\theta - \theta_0 = \frac{1}{2}(23.3 \text{ rad/s}^2)(6.00 \text{ s})^2 = 419 \text{ rad}$, in agreement with the result obtained above.

9.15. IDENTIFY: Apply constant angular acceleration equations.

SET UP: Let the direction the flywheel is rotating be positive.

$\theta - \theta_0 = 200 \text{ rev}$, $\omega_{0z} = 500 \text{ rev/min} = 8.333 \text{ rev/s}$, $t = 30.0 \text{ s}$.

EXECUTE: (a) $\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\omega_z = 5.00 \text{ rev/s} = 300 \text{ rpm}$

(b) Use the information in part (a) to find α_z : $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = -0.1111 \text{ rev/s}^2$. Then $\omega_z = 0$,

$\alpha_z = -0.1111 \text{ rev/s}^2$, $\omega_{0z} = 8.333 \text{ rev/s}$ in $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = 75.0 \text{ s}$ and $\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t$ gives

$\theta - \theta_0 = 312 \text{ rev}$.

EVALUATE: The mass and diameter of the flywheel are not used in the calculation.

9.16. IDENTIFY: Use the constant angular acceleration equations, applied to the first revolution and to the first two revolutions.

SET UP: Let the direction the disk is rotating be positive. $1 \text{ rev} = 2\pi \text{ rad}$. Let t be the time for the first revolution. The time for the first two revolutions is $t + 0.750 \text{ s}$.

EXECUTE: (a) $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ applied to the first and to the first two revolutions gives $2\pi \text{ rad} = \frac{1}{2}\alpha_z t^2$ and $4\pi \text{ rad} = \frac{1}{2}\alpha_z (t + 0.750 \text{ s})^2$. Eliminating α_z between these equations gives $4\pi \text{ rad} = \frac{2\pi \text{ rad}}{t^2}(t + 0.750 \text{ s})^2$.

$2t^2 = (t + 0.750 \text{ s})^2$. $\sqrt{2}t = \pm(t + 0.750 \text{ s})$. The positive root is $t = \frac{0.750 \text{ s}}{\sqrt{2} - 1} = 1.81 \text{ s}$.

(b) $2\pi \text{ rad} = \frac{1}{2}\alpha_z t^2$ and $t = 1.81 \text{ s}$ gives $\alpha_z = 3.84 \text{ rad/s}^2$

EVALUATE: At the start of the second revolution, $\omega_{0z} = (3.84 \text{ rad/s}^2)(1.81 \text{ s}) = 6.95 \text{ rad/s}$. The distance the disk rotates in the next 0.750 s is $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (6.95 \text{ rad/s})(0.750 \text{ s}) + \frac{1}{2}(3.84 \text{ rad/s}^2)(0.750 \text{ s})^2 = 6.29 \text{ rad}$, which is two revolutions.

9.17. IDENTIFY: Apply Eq.(9.12) to relate ω_z to $\theta - \theta_0$.

SET UP: Establish a proportionality.

EXECUTE: From Eq.(9.12), with $\omega_{0z} = 0$, the number of revolutions is proportional to the square of the initial angular velocity, so tripling the initial angular velocity increases the number of revolutions by 9, to 9.00 rev.

EVALUATE: We don't have enough information to calculate α_z ; all we need to know is that it is constant.

9.18. IDENTIFY: In each case we apply constant acceleration equations to determine $\theta(t)$ and $\omega_z(t)$.

SET UP: Let $\theta_0 = 0$. The following table gives the revolutions and the angle θ (in degrees) through which the wheel has rotated for each instant in time (in seconds) and each of the three situations:

t	(a)		(b)		(c)	
	rev	θ	rev	θ	rev	θ
0.05	0.50	180	0.03	11.3	0.44	158
0.10	1.00	360	0.13	45	0.75	270
0.15	1.50	540	0.28	101	0.94	338
0.20	2.00	720	0.50	180	1.00	360

EXECUTE: The θ and ω_z graphs for each case are given in Figures 9.18 a–c.

EVALUATE: The slope of the $\theta(t)$ graph is $\omega_z(t)$ and the slope of the $\omega_z(t)$ graph is $\alpha_z(t)$.

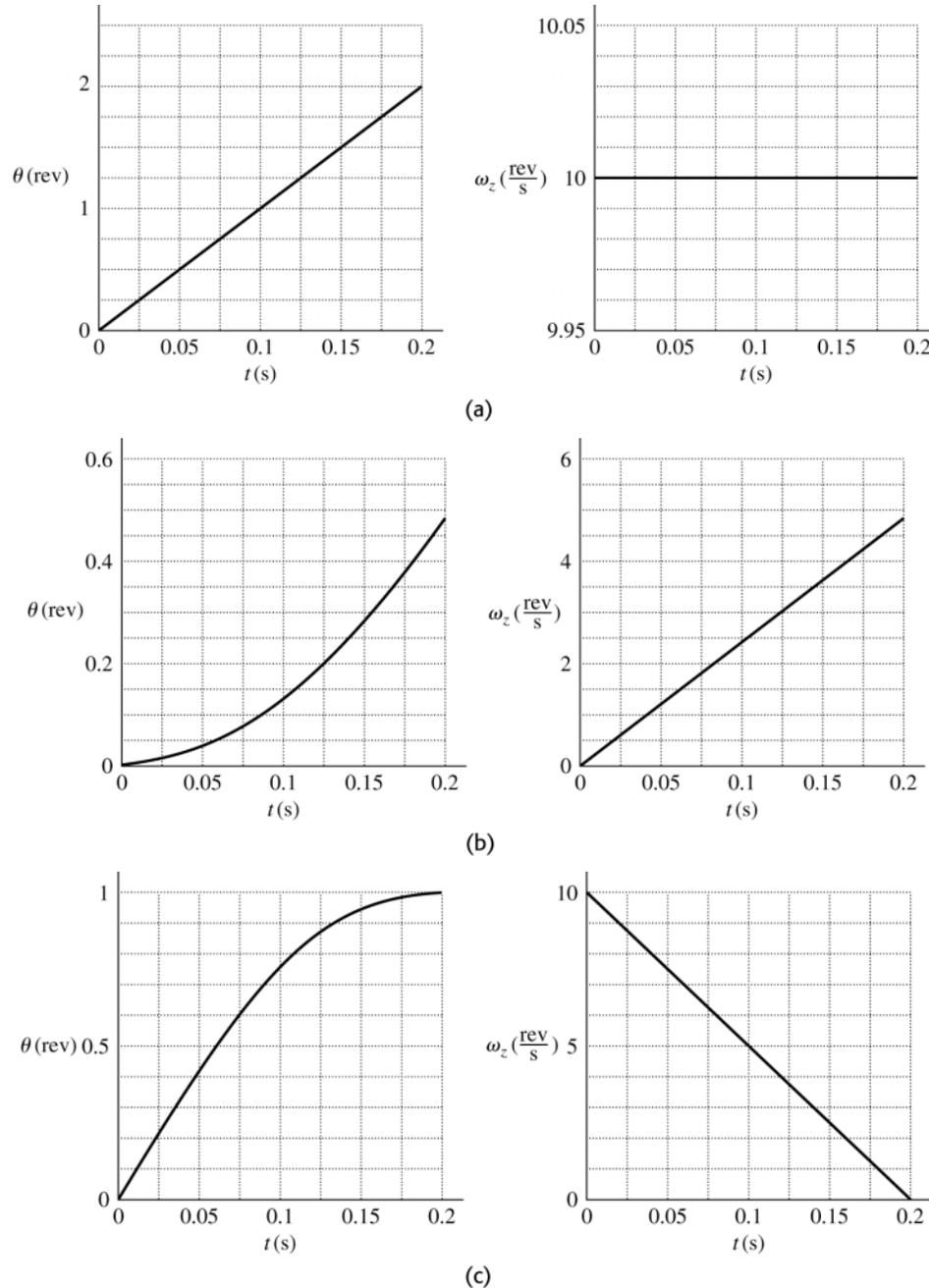


Figure 9.18

9.19. IDENTIFY: Apply the constant angular acceleration equations separately to the time intervals 0 to 2.00 s and 2.00 s until the wheel stops.

(a) SET UP: Consider the motion from $t = 0$ to $t = 2.00$ s:

$$\theta - \theta_0 = ?; \quad \omega_{0z} = 24.0 \text{ rad/s}; \quad \alpha_z = 30.0 \text{ rad/s}^2; \quad t = 2.00 \text{ s}$$

$$\text{EXECUTE: } \theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (24.0 \text{ rad/s})(2.00 \text{ s}) + \frac{1}{2}(30.0 \text{ rad/s}^2)(2.00 \text{ s})^2$$

$$\theta - \theta_0 = 48.0 \text{ rad} + 60.0 \text{ rad} = 108 \text{ rad}$$

Total angular displacement from $t = 0$ until stops: $108 \text{ rad} + 432 \text{ rad} = 540 \text{ rad}$

Note: At $t = 2.00$ s, $\omega_z = \omega_{0z} + \alpha_z t = 24.0 \text{ rad/s} + (30.0 \text{ rad/s}^2)(2.00 \text{ s}) = 84.0 \text{ rad/s}$; angular speed when breaker trips.

(b) SET UP: Consider the motion from when the circuit breaker trips until the wheel stops. For this calculation let $t = 0$ when the breaker trips.

$$t = ?; \quad \theta - \theta_0 = 432 \text{ rad}; \quad \omega_z = 0; \quad \omega_{0z} = 84.0 \text{ rad/s} \text{ (from part (a))}$$

$$\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2} \right) t$$

$$\text{EXECUTE: } t = \frac{2(\theta - \theta_0)}{\omega_{0z} + \omega_z} = \frac{2(432 \text{ rad})}{84.0 \text{ rad/s} + 0} = 10.3 \text{ s}$$

The wheel stops 10.3 s after the breaker trips so $2.00 \text{ s} + 10.3 \text{ s} = 12.3 \text{ s}$ from the beginning.

(c) SET UP: $\alpha_z = ?$; consider the same motion as in part (b):

$$\omega_z = \omega_{0z} + \alpha_z t$$

$$\text{EXECUTE: } \alpha_z = \frac{\omega_z - \omega_{0z}}{t} = \frac{0 - 84.0 \text{ rad/s}}{10.3 \text{ s}} = -8.16 \text{ rad/s}^2$$

EVALUATE: The angular acceleration is positive while the wheel is speeding up and negative while it is slowing down. We could also use $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ to calculate $\alpha_z = \frac{\omega_z^2 - \omega_{0z}^2}{2(\theta - \theta_0)} = \frac{0 - (84.0 \text{ rad/s})^2}{2(432 \text{ rad})} = -8.16 \text{ rad/s}^2$ for

the acceleration after the breaker trips.

9.20. IDENTIFY: The linear distance the elevator travels, its speed and the magnitude of its acceleration are equal to the tangential displacement, speed and acceleration of a point on the rim of the disk. $s = r\theta$, $v = r\omega$ and $a = r\alpha$. In these equations the angular quantities must be in radians.

SET UP: $1 \text{ rev} = 2\pi \text{ rad}$. $1 \text{ rpm} = 0.1047 \text{ rad/s}$. $\pi \text{ rad} = 180^\circ$. For the disk, $r = 1.25 \text{ m}$.

$$\text{EXECUTE: (a) } v = 0.250 \text{ m/s so } \omega = \frac{v}{r} = \frac{0.250 \text{ m/s}}{1.25 \text{ m}} = 0.200 \text{ rad/s} = 1.91 \text{ rpm}.$$

$$\text{(b) } a = \frac{1}{8}g = 1.225 \text{ m/s}^2. \quad \alpha = \frac{a}{r} = \frac{1.225 \text{ m/s}^2}{1.25 \text{ m}} = 0.980 \text{ rad/s}^2.$$

$$\text{(c) } s = 3.25 \text{ m}. \quad \theta = \frac{s}{r} = \frac{3.25 \text{ m}}{1.25 \text{ m}} = 2.60 \text{ rad} = 149^\circ.$$

EVALUATE: When we use $s = r\theta$, $v = r\omega$ and $a_{\text{tan}} = r\alpha$ to solve for θ , ω and α , the results are in rad, rad/s and rad/s^2 .

9.21. IDENTIFY: When the angular speed is constant, $\omega = \theta/t$. $v_{\text{tan}} = r\omega$, $a_{\text{tan}} = r\alpha$ and $a_{\text{rad}} = r\omega^2$. In these equations radians must be used for the angular quantities.

SET UP: The radius of the earth is $R_E = 6.38 \times 10^6 \text{ m}$ and the earth rotates once in $1 \text{ day} = 86,400 \text{ s}$. The orbit radius of the earth is $1.50 \times 10^{11} \text{ m}$ and the earth completes one orbit in $1 \text{ y} = 3.156 \times 10^7 \text{ s}$. When ω is constant, $\omega = \theta/t$.

$$\text{EXECUTE: (a) } \theta = 1 \text{ rev} = 2\pi \text{ rad in } t = 3.156 \times 10^7 \text{ s}. \quad \omega = \frac{2\pi \text{ rad}}{3.156 \times 10^7 \text{ s}} = 1.99 \times 10^{-7} \text{ rad/s}.$$

$$\text{(b) } \theta = 1 \text{ rev} = 2\pi \text{ rad in } t = 86,400 \text{ s}. \quad \omega = \frac{2\pi \text{ rad}}{86,400 \text{ s}} = 7.27 \times 10^{-5} \text{ rad/s}$$

$$\text{(c) } v = r\omega = (1.50 \times 10^{11} \text{ m})(1.99 \times 10^{-7} \text{ rad/s}) = 2.98 \times 10^4 \text{ m/s}.$$

$$\text{(d) } v = r\omega = (6.38 \times 10^6 \text{ m})(7.27 \times 10^{-5} \text{ rad/s}) = 464 \text{ m/s}.$$

$$\text{(e) } a_{\text{rad}} = r\omega^2 = (6.38 \times 10^6 \text{ m})(7.27 \times 10^{-5} \text{ rad/s})^2 = 0.0337 \text{ m/s}^2. \quad a_{\text{tan}} = r\alpha = 0. \quad \alpha = 0 \text{ since the angular velocity is constant.}$$

EVALUATE: The tangential speeds associated with these motions are large even though the angular speeds are very small, because the radius for the circular path in each case is quite large.

9.22. IDENTIFY: Linear and angular velocities are related by $v = r\omega$. Use $\omega_z = \omega_{0z} + \alpha_z t$ to calculate α_z .

SET UP: $\omega = v/r$ gives ω in rad/s.

EXECUTE: (a) $\frac{1.25 \text{ m/s}}{25.0 \times 10^{-3} \text{ m}} = 50.0 \text{ rad/s}$, $\frac{1.25 \text{ m/s}}{58.0 \times 10^{-3} \text{ m}} = 21.6 \text{ rad/s}$.

(b) $(1.25 \text{ m/s}) (74.0 \text{ min}) (60 \text{ s/min}) = 5.55 \text{ km}$.

(c) $\alpha_z = \frac{21.55 \text{ rad/s} - 50.0 \text{ rad/s}}{(74.0 \text{ min}) (60 \text{ s/min})} = -6.41 \times 10^{-3} \text{ rad/s}^2$.

EVALUATE: The width of the tracks is very small, so the total track length on the disc is huge.

9.23. IDENTIFY: Use constant acceleration equations to calculate the angular velocity at the end of two revolutions.

$v = r\omega$.

SET UP: 2 rev = 4π rad. $r = 0.200 \text{ m}$.

EXECUTE: (a) $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$. $\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(3.00 \text{ rad/s}^2)(4\pi \text{ rad})} = 8.68 \text{ rad/s}$.

$a_{\text{rad}} = r\omega^2 = (0.200 \text{ m})(8.68 \text{ rad/s})^2 = 15.1 \text{ m/s}^2$.

(b) $v = r\omega = (0.200 \text{ m})(8.68 \text{ rad/s}) = 1.74 \text{ m/s}$. $a_{\text{rad}} = \frac{v^2}{r} = \frac{(1.74 \text{ m/s})^2}{0.200 \text{ m}} = 15.1 \text{ m/s}^2$.

EVALUATE: $r\omega^2$ and v^2/r are completely equivalent expressions for a_{rad} .

9.24. IDENTIFY: $a_{\text{rad}} = r\omega^2$, with ω in rad/s. Solve for ω .

SET UP: 1 rpm = $(2\pi/60)$ rad/s

EXECUTE: $\omega = \sqrt{\frac{a_{\text{rad}}}{r}} = \sqrt{\frac{(400,000)(9.80 \text{ m/s}^2)}{0.0250 \text{ m}}} = 1.25 \times 10^4 \text{ rad/s} = 1.20 \times 10^5 \text{ rpm}$

EVALUATE: In $a_{\text{rad}} = r\omega^2$, ω must be in rad/s.

9.25. IDENTIFY and SET UP: Use constant acceleration equations to find ω and α after each displacement. The use Eqs.(9.14) and (9.15) to find the components of the linear acceleration.

EXECUTE: (a) at the start $t = 0$

flywheel starts from rest so $\omega_z = \omega_{0z} = 0$

$a_{\text{tan}} = r\alpha = (0.300 \text{ m})(0.600 \text{ rad/s}^2) = 0.180 \text{ m/s}^2$

$a_{\text{rad}} = r\omega^2 = 0$

$a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2} = 0.180 \text{ m/s}^2$

(b) $\theta - \theta_0 = 60^\circ$

$a_{\text{tan}} = r\alpha = 0.180 \text{ m/s}^2$

Calculate ω :

$\theta - \theta_0 = 60^\circ(\pi \text{ rad}/180^\circ) = 1.047 \text{ rad}$; $\omega_{0z} = 0$; $\alpha_z = 0.600 \text{ rad/s}^2$; $\omega_z = ?$

$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$

$\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.600 \text{ rad/s}^2)(1.047 \text{ rad})} = 1.121 \text{ rad/s}$ and $\omega = \omega_z$.

Then $a_{\text{rad}} = r\omega^2 = (0.300 \text{ m})(1.121 \text{ rad/s})^2 = 0.377 \text{ m/s}^2$.

$a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2} = \sqrt{(0.377 \text{ m/s}^2)^2 + (0.180 \text{ m/s}^2)^2} = 0.418 \text{ m/s}^2$

(c) $\theta - \theta_0 = 120^\circ$

$a_{\text{tan}} = r\alpha = 0.180 \text{ m/s}^2$

Calculate ω :

$\theta - \theta_0 = 120^\circ(\pi \text{ rad}/180^\circ) = 2.094 \text{ rad}$; $\omega_{0z} = 0$; $\alpha_z = 0.600 \text{ rad/s}^2$; $\omega_z = ?$

$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$

$\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.600 \text{ rad/s}^2)(2.094 \text{ rad})} = 1.585 \text{ rad/s}$ and $\omega = \omega_z$.

Then $a_{\text{rad}} = r\omega^2 = (0.300 \text{ m})(1.585 \text{ rad/s})^2 = 0.754 \text{ m/s}^2$.

$a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2} = \sqrt{(0.754 \text{ m/s}^2)^2 + (0.180 \text{ m/s}^2)^2} = 0.775 \text{ m/s}^2$

EVALUATE: α is constant so a_{tan} is constant. ω increases so a_{rad} increases.

- 9.26. IDENTIFY:** Apply constant angular acceleration equations. $v = r\omega$. A point on the rim has both tangential and radial components of acceleration.

SET UP: $a_{\text{tan}} = r\alpha$ and $a_{\text{rad}} = r\omega^2$.

EXECUTE: (a) $\omega_z = \omega_{0z} + \alpha_z t = 0.250 \text{ rev/s} + (0.900 \text{ rev/s}^2)(0.200 \text{ s}) = 0.430 \text{ rev/s}$

(Note that since ω_{0z} and α_z are given in terms of revolutions, it's not necessary to convert to radians).

(b) $\omega_{\text{av-z}} \Delta t = (0.340 \text{ rev/s})(0.2 \text{ s}) = 0.068 \text{ rev}$.

(c) Here, the conversion to radians must be made to use Eq. (9.13), and

$$v = r\omega = \left(\frac{0.750 \text{ m}}{2}\right)(0.430 \text{ rev/s})(2\pi \text{ rad/rev}) = 1.01 \text{ m/s}.$$

(d) Combining equations (9.14) and (9.15),

$$a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2}.$$

$$a = \sqrt{\left[((0.430 \text{ rev/s})(2\pi \text{ rad/rev}))^2 (0.375 \text{ m}) \right]^2 + \left[(0.900 \text{ rev/s}^2)(2\pi \text{ rad/rev})(0.375 \text{ m}) \right]^2}.$$

$$a = 3.46 \text{ m/s}^2.$$

EVALUATE: If the angular acceleration is constant, a_{tan} is constant but a_{rad} increases as ω increases.

- 9.27. IDENTIFY:** Use Eq.(9.15) and solve for r .

SET UP: $a_{\text{rad}} = r\omega^2$ so $r = a_{\text{rad}} / \omega^2$, where ω must be in rad/s

EXECUTE: $a_{\text{rad}} = 3000g = 3000(9.80 \text{ m/s}^2) = 29,400 \text{ m/s}^2$

$$\omega = (5000 \text{ rev/min}) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 523.6 \text{ rad/s}$$

$$\text{Then } r = \frac{a_{\text{rad}}}{\omega^2} = \frac{29,400 \text{ m/s}^2}{(523.6 \text{ rad/s})^2} = 0.107 \text{ m}.$$

EVALUATE: The diameter is then 0.214 m, which is larger than 0.127 m, so the claim is *not* realistic.

- 9.28. IDENTIFY:** In part (b) apply the result derived in part (a).

SET UP: $a_{\text{rad}} = r\omega^2$ and $v = r\omega$; combine to eliminate r .

EXECUTE: (a) $a_{\text{rad}} = \omega^2 r = \omega^2 \left(\frac{v}{\omega} \right) = \omega v$.

(b) From the result of part (a), $\omega = \frac{a_{\text{rad}}}{v} = \frac{0.500 \text{ m/s}^2}{2.00 \text{ m/s}} = 0.250 \text{ rad/s}$.

EVALUATE: $a_{\text{rad}} = r\omega^2$ and $v = r\omega$ both require that ω be in rad/s, so in $a_{\text{rad}} = \omega v$, ω is in rad/s.

- 9.29. IDENTIFY:** $v = r\omega$ and $a_{\text{rad}} = r\omega^2 = v^2 / r$.

SET UP: $2\pi \text{ rad} = 1 \text{ rev}$, so $\pi \text{ rad/s} = 30 \text{ rev/min}$.

EXECUTE: (a) $\omega r = (1250 \text{ rev/min}) \left(\frac{\pi \text{ rad/s}}{30 \text{ rev/min}} \right) \left(\frac{12.7 \times 10^{-3} \text{ m}}{2} \right) = 0.831 \text{ m/s}$.

(b) $\frac{v^2}{r} = \frac{(0.831 \text{ m/s})^2}{(12.7 \times 10^{-3} \text{ m})/2} = 109 \text{ m/s}^2$.

EVALUATE: In $v = r\omega$, ω must be in rad/s.

- 9.30. IDENTIFY:** $a_{\text{tan}} = r\alpha$, $v = r\omega$ and $a_{\text{rad}} = v^2 / r$. $\theta - \theta_0 = \omega_{\text{av-z}} t$.

SET UP: When α_z is constant, $\omega_{\text{av-z}} = \frac{\omega_{0z} + \omega_z}{2}$. Let the direction the wheel is rotating be positive.

EXECUTE: (a) $\alpha = \frac{a_{\text{tan}}}{r} = \frac{-10.0 \text{ m/s}^2}{0.200 \text{ m}} = -50.0 \text{ rad/s}^2$

(b) At $t = 3.00 \text{ s}$, $v = 50.0 \text{ m/s}$ and $\omega = \frac{v}{r} = \frac{50.0 \text{ m/s}}{0.200 \text{ m}} = 250 \text{ rad/s}$ and at $t = 0$,

$$v = 50.0 \text{ m/s} + (-10.0 \text{ m/s}^2)(0 - 3.00 \text{ s}) = 80.0 \text{ m/s}, \quad \omega = 400 \text{ rad/s}.$$

(c) $\omega_{\text{av-z}} t = (325 \text{ rad/s})(3.00 \text{ s}) = 975 \text{ rad} = 155 \text{ rev}$.

(d) $v = \sqrt{a_{\text{rad}} r} = \sqrt{(9.80 \text{ m/s}^2)(0.200 \text{ m})} = 1.40 \text{ m/s}$. This speed will be reached at time $\frac{50.0 \text{ m/s} - 1.40 \text{ m/s}}{10.0 \text{ m/s}} = 4.86 \text{ s}$ after $t = 3.00 \text{ s}$, or at $t = 7.86 \text{ s}$. (There are many equivalent ways to do this calculation.)

EVALUATE: At $t = 0$, $a_{\text{rad}} = r\omega^2 = 3.20 \times 10^4 \text{ m/s}^2$. At $t = 3.00 \text{ s}$, $a_{\text{rad}} = 1.25 \times 10^4 \text{ m/s}^2$. For $a_{\text{rad}} = g$ the wheel must be rotating more slowly than at 3.00 s so it occurs some time after 3.00 s.

9.31. IDENTIFY and SET UP: Use Eq.(9.15) to relate ω to a_{rad} and $\sum \vec{F} = m\vec{a}$ to relate a_{rad} to F_{rad} . Use Eq.(9.13) to relate ω and v , where v is the tangential speed.

EXECUTE: (a) $a_{\text{rad}} = r\omega^2$ and $F_{\text{rad}} = ma_{\text{rad}} = mr\omega^2$

$$\frac{F_{\text{rad},2}}{F_{\text{rad},1}} = \left(\frac{\omega_2}{\omega_1}\right)^2 = \left(\frac{640 \text{ rev/min}}{423 \text{ rev/min}}\right)^2 = 2.29$$

(b) $v = r\omega$

$$\frac{v_2}{v_1} = \frac{\omega_2}{\omega_1} = \frac{640 \text{ rev/min}}{423 \text{ rev/min}} = 1.51$$

(c) $v = r\omega$

$$\omega = (640 \text{ rev/min}) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 67.0 \text{ rad/s}$$

Then $v = r\omega = (0.235 \text{ m})(67.0 \text{ rad/s}) = 15.7 \text{ m/s}$.

$$a_{\text{rad}} = r\omega^2 = (0.235 \text{ m})(67.0 \text{ rad/s})^2 = 1060 \text{ m/s}^2$$

$$\frac{a_{\text{rad}}}{g} = \frac{1060 \text{ m/s}^2}{9.80 \text{ m/s}^2} = 108; \quad a = 108g$$

EVALUATE: In parts (a) and (b), since a ratio is used the units cancel and there is no need to convert ω to rad/s. In part (c), v and a_{rad} are calculated from ω , and ω must be in rad/s.

9.32. IDENTIFY: $v = r\omega$ and $a_{\text{tan}} = r\alpha$.

SET UP: The linear acceleration of the bucket equals a_{tan} for a point on the rim of the axle.

EXECUTE: (a) $v = R\omega$. $2.00 \text{ cm/s} = R \left(\frac{7.5 \text{ rev}}{\text{min}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$ gives $R = 2.55 \text{ cm}$.

$$D = 2R = 5.09 \text{ cm}.$$

(b) $a_{\text{tan}} = R\alpha$. $\alpha = \frac{a_{\text{tan}}}{R} = \frac{0.400 \text{ m/s}^2}{0.0255 \text{ m}} = 15.7 \text{ rad/s}^2$.

EVALUATE: In $v = R\omega$ and $a_{\text{tan}} = R\alpha$, ω and α must be in radians.

9.33. IDENTIFY: Apply $v = r\omega$.

SET UP: Points on the chain all move at the same speed, so $r_f\omega_f = r_r\omega_r$.

EXECUTE: The angular velocity of the rear wheel is $\omega_r = \frac{v_r}{r} = \frac{5.00 \text{ m/s}}{0.330 \text{ m}} = 15.15 \text{ rad/s}$.

The angular velocity of the front wheel is $\omega_f = 0.600 \text{ rev/s} = 3.77 \text{ rad/s}$. $r_f = r_r(\omega_r/\omega_f) = 2.99 \text{ cm}$.

EVALUATE: The rear sprocket and wheel have the same angular velocity and the front sprocket and wheel have the same angular velocity. $r\omega$ is the same for both, so the rear sprocket has a smaller radius since it has a larger angular velocity. The speed of a point on the chain is $v = r_f\omega_f = (2.99 \times 10^{-2} \text{ m})(15.15 \text{ rad/s}) = 0.453 \text{ m/s}$. The linear speed of the bicycle is 5.50 m/s.

9.34. IDENTIFY and SET UP: Use Eq.(9.16). Treat the spheres as point masses and ignore I of the light rods.

EXECUTE: The object is shown in Figure 9.34a.

(a)

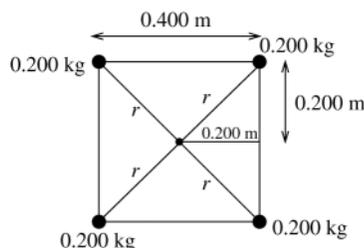


Figure 9.34a

$$r = \sqrt{(0.200 \text{ m})^2 + (0.200 \text{ m})^2} = 0.2828 \text{ m}$$

$$I = \sum m_i r_i^2 = 4(0.200 \text{ kg})(0.2828 \text{ m})^2$$

$$I = 0.0640 \text{ kg} \cdot \text{m}^2$$

(b) The object is shown in Figure 9.34b.

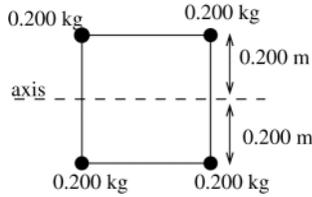


Figure 9.34b

$$r = 0.200 \text{ m}$$

$$I = \sum m_i r_i^2 = 4(0.200 \text{ kg})(0.200 \text{ m})^2$$

$$I = 0.0320 \text{ kg} \cdot \text{m}^2$$

(c) The object is shown in Figure 9.34c.

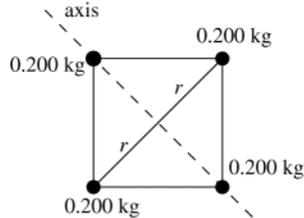


Figure 9.34c

$$r = 0.2828 \text{ m}$$

$$I = \sum m_i r_i^2 = 2(0.200 \text{ kg})(0.2828 \text{ m})^2$$

$$I = 0.0320 \text{ kg} \cdot \text{m}^2$$

EVALUATE: In general I depends on the axis and our answer for part (a) is larger than for parts (b) and (c). It just happens that I is the same in parts (b) and (c).

9.35. IDENTIFY: Use Table 9.2. The correct expression to use in each case depends on the shape of the object and the location of the axis.

SET UP: In each case express the mass in kg and the length in m, so the moment of inertia will be in $\text{kg} \cdot \text{m}^2$.

EXECUTE: (a) (i) $I = \frac{1}{3}ML^2 = \frac{1}{3}(2.50 \text{ kg})(0.750 \text{ m})^2 = 0.469 \text{ kg} \cdot \text{m}^2$.

(ii) $I = \frac{1}{12}ML^2 = \frac{1}{4}(0.469 \text{ kg} \cdot \text{m}^2) = 0.117 \text{ kg} \cdot \text{m}^2$. (iii) For a very thin rod, all of the mass is at the axis and $I = 0$.

(b) (i) $I = \frac{2}{5}MR^2 = \frac{2}{5}(3.00 \text{ kg})(0.190 \text{ m})^2 = 0.0433 \text{ kg} \cdot \text{m}^2$.

(ii) $I = \frac{2}{3}MR^2 = \frac{2}{3}(0.0433 \text{ kg} \cdot \text{m}^2) = 0.0722 \text{ kg} \cdot \text{m}^2$.

(c) (i) $I = MR^2 = (8.00 \text{ kg})(0.0600 \text{ m})^2 = 0.0288 \text{ kg} \cdot \text{m}^2$.

(ii) $I = \frac{1}{2}MR^2 = \frac{1}{2}(8.00 \text{ kg})(0.0600 \text{ m})^2 = 0.0144 \text{ kg} \cdot \text{m}^2$.

EVALUATE: I depends on how the mass of the object is distributed relative to the axis.

9.36. IDENTIFY: Treat each block as a point mass, so for each block $I = mr^2$, where r is the distance of the block from the axis. The total I for the object is the sum of the I for each of its pieces.

SET UP: In part (a) two blocks are a distance $L/2$ from the axis and the third block is on the axis. In part (b) two blocks are a distance $L/4$ from the axis and one is a distance $3L/4$ from the axis.

EXECUTE: (a) $I = 2m(L/2)^2 = \frac{1}{2}mL^2$.

(b) $I = 2m(L/4)^2 + m(3L/4)^2 = \frac{1}{16}mL^2(2+9) = \frac{11}{16}mL^2$.

EVALUATE: For the same object I is in general different for different axes.

9.37. IDENTIFY: I for the object is the sum of the values of I for each part.

SET UP: For the bar, for an axis perpendicular to the bar, use the appropriate expression from Table 9.2. For a point mass, $I = mr^2$, where r is the distance of the mass from the axis.

EXECUTE: (a) $I = I_{\text{bar}} + I_{\text{balls}} = \frac{1}{12}M_{\text{bar}}L^2 + 2m_{\text{balls}}\left(\frac{L}{2}\right)^2$.

$$I = \frac{1}{12}(4.00 \text{ kg})(2.00 \text{ m})^2 + 2(0.500 \text{ kg})(1.00 \text{ m})^2 = 2.33 \text{ kg} \cdot \text{m}^2$$

(b) $I = \frac{1}{3}m_{\text{bar}}L^2 + m_{\text{ball}}L^2 = \frac{1}{3}(4.00 \text{ kg})(2.00 \text{ m})^2 + (0.500 \text{ kg})(2.00 \text{ m})^2 = 7.33 \text{ kg} \cdot \text{m}^2$

(c) $I = 0$ because all masses are on the axis.

(d) All the mass is a distance $d = 0.500 \text{ m}$ from the axis and

$$I = m_{\text{bar}}d^2 + 2m_{\text{ball}}d^2 = M_{\text{Total}}d^2 = (5.00 \text{ kg})(0.500 \text{ m})^2 = 1.25 \text{ kg} \cdot \text{m}^2$$

EVALUATE: I for an object depends on the location and direction of the axis.

- 9.38. IDENTIFY and SET UP:** According to Eq.(9.16), I for the entire object equals the sum of I for each piece, the rod plus the end caps. The object is shown in Figure 9.38.

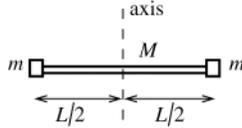


Figure 9.38

EXECUTE: $I = I_{\text{rod}} + 2I_{\text{cap}}$

$$I = \frac{1}{12}ML^2 + 2(m)(L/2)^2 = \left(\frac{1}{12}M + \frac{1}{2}m\right)L^2$$

EVALUATE: Table 9.2 was used for I_{rod} and $I = mr^2$ for the end caps, since they are treated as point particles.

- 9.39. IDENTIFY and SET UP:** $I = \sum m_i r_i^2$ implies $I = I_{\text{rim}} + I_{\text{spokes}}$

EXECUTE: $I_{\text{rim}} = MR^2 = (1.40 \text{ kg})(0.300 \text{ m})^2 = 0.126 \text{ kg} \cdot \text{m}^2$

Each spoke can be treated as a slender rod with the axis through one end, so

$$I_{\text{spokes}} = 8\left(\frac{1}{3}ML^2\right) = \frac{8}{3}(0.280 \text{ kg})(0.300 \text{ m})^2 = 0.0672 \text{ kg} \cdot \text{m}^2$$

$$I = I_{\text{rim}} + I_{\text{spokes}} = 0.126 \text{ kg} \cdot \text{m}^2 + 0.0672 \text{ kg} \cdot \text{m}^2 = 0.193 \text{ kg} \cdot \text{m}^2$$

EVALUATE: Our result is smaller than $m_{\text{tot}}R^2 = (3.64 \text{ kg})(0.300 \text{ m})^2 = 0.328 \text{ kg} \cdot \text{m}^2$, since the mass of each spoke is distributed between $r = 0$ and $r = R$.

- 9.40. IDENTIFY:** Compare this object to a uniform disk of radius R and mass $2M$.

SET UP: With an axis perpendicular to the round face of the object at its center, I for a uniform disk is the same as for a solid cylinder.

EXECUTE: (a) The total I for a disk of mass $2M$ and radius R , $I = \frac{1}{2}(2M)R^2 = MR^2$. Each half of the disk has the same I , so for the half-disk, $I = \frac{1}{2}MR^2$.

(b) The same mass M is distributed the same way as a function of distance from the axis.

(c) The same method as in part (a) says that I for a quarter-disk of radius R and mass M is half that of a half-disk of radius R and mass $2M$, so $I = \frac{1}{2}\left(\frac{1}{2}[2M]R^2\right) = \frac{1}{2}MR^2$.

EVALUATE: I depends on how the mass of the object is distributed relative to the axis, and this is the same for any segment of a disk.

- 9.41. IDENTIFY:** I for the compound disk is the sum of I of the solid disk and of the ring.

SET UP: For the solid disk, $I = \frac{1}{2}m_d r_d^2$. For the ring, $I_r = \frac{1}{2}m_r(r_1^2 + r_2^2)$, where $r_1 = 50.0 \text{ cm}$, $r_2 = 70.0 \text{ cm}$. The mass of the disk and ring is their area times their area density.

EXECUTE: $I = I_d + I_r$.

Disk: $m_d = (3.00 \text{ g/cm}^2)\pi r_d^2 = 23.56 \text{ kg}$. $I_d = \frac{1}{2}m_d r_d^2 = 2.945 \text{ kg} \cdot \text{m}^2$.

Ring: $m_r = (2.00 \text{ g/cm}^2)\pi(r_2^2 - r_1^2) = 15.08 \text{ kg}$. $I_r = \frac{1}{2}m_r(r_1^2 + r_2^2) = 5.580 \text{ kg} \cdot \text{m}^2$.

$$I = I_d + I_r = 8.52 \text{ kg} \cdot \text{m}^2$$

EVALUATE: Even though $m_r < m_d$, $I_r > I_d$ since the mass of the ring is farther from the axis.

- 9.42. IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2b to calculate I .

SET UP: $I = \frac{1}{12}ML^2$. 1 rpm = 0.1047 rad/s

EXECUTE: (a) $I = \frac{1}{12}(117 \text{ kg})(2.08 \text{ m})^2 = 42.2 \text{ kg} \cdot \text{m}^2$. $\omega = (2400 \text{ rev/min})\left(\frac{0.1047 \text{ rad/s}}{1 \text{ rev/min}}\right) = 251 \text{ rad/s}$.

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(42.2 \text{ kg} \cdot \text{m}^2)(251 \text{ rad/s})^2 = 1.33 \times 10^6 \text{ J}$$

(b) $K_1 = \frac{1}{2}M_1 L_1^2 \omega_1^2$, $K_2 = \frac{1}{2}M_2 L_2^2 \omega_2^2$. $L_1 = L_2$ and $K_1 = K_2$, so $M_1 \omega_1^2 = M_2 \omega_2^2$.

$$\omega_2 = \omega_1 \sqrt{\frac{M_1}{M_2}} = (2400 \text{ rpm})\sqrt{\frac{M_1}{0.750M_1}} = 2770 \text{ rpm}$$

EVALUATE: The rotational kinetic energy is proportional to the square of the angular speed and directly proportional to the mass of the object.

- 9.43. IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2 to calculate I .

SET UP: $I = \frac{2}{5}MR^2$. For the moon, $M = 7.35 \times 10^{22} \text{ kg}$ and $R = 1.74 \times 10^6 \text{ m}$. The moon moves through 1 rev = 2π rad in 27.3 d. 1 d = $8.64 \times 10^4 \text{ s}$.

EXECUTE: (a) $I = \frac{2}{5}(7.35 \times 10^{22} \text{ kg})(1.74 \times 10^6 \text{ m})^2 = 8.90 \times 10^{34} \text{ kg} \cdot \text{m}^2$.

$$\omega = \frac{2\pi \text{ rad}}{(27.3 \text{ d})(8.64 \times 10^4 \text{ s/d})} = 2.66 \times 10^{-6} \text{ rad/s}.$$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(8.90 \times 10^{34} \text{ kg} \cdot \text{m}^2)(2.66 \times 10^{-6} \text{ rad/s})^2 = 3.15 \times 10^{23} \text{ J}.$$

(b) $\frac{3.15 \times 10^{23} \text{ J}}{5(4.0 \times 10^{20} \text{ J})} = 158 \text{ years}$. Considering the expense involved in tapping the moon's rotational energy, this

does not seem like a worthwhile scheme for only 158 years worth of energy.

EVALUATE: The moon has a very large amount of kinetic energy due to its motion. The earth has even more, but changing the rotation rate of the earth would change the length of a day.

9.44. IDENTIFY: $K = \frac{1}{2}I\omega^2$. Use Table 9.2 to relate I to the mass M of the disk.

SET UP: $45.0 \text{ rpm} = 4.71 \text{ rad/s}$. For a uniform solid disk, $I = \frac{1}{2}MR^2$.

EXECUTE: (a) $I = \frac{2K}{\omega^2} = \frac{2(0.250 \text{ J})}{(4.71 \text{ rad/s})^2} = 0.0225 \text{ kg} \cdot \text{m}^2$.

(b) $I = \frac{1}{2}MR^2$ and $M = \frac{2I}{R^2} = \frac{2(0.0225 \text{ kg} \cdot \text{m}^2)}{(0.300 \text{ m})^2} = 0.500 \text{ kg}$.

EVALUATE: No matter what the shape is, the rotational kinetic energy is proportional to the mass of the object.

9.45. IDENTIFY: $K = \frac{1}{2}I\omega^2$, with ω in rad/s. Solve for I .

SET UP: $1 \text{ rev/min} = (2\pi/60) \text{ rad/s}$. $\Delta K = -500 \text{ J}$

EXECUTE: $\omega_i = 650 \text{ rev/min} = 68.1 \text{ rad/s}$. $\omega_f = 520 \text{ rev/min} = 54.5 \text{ rad/s}$. $\Delta K = K_f - K_i = \frac{1}{2}I(\omega_f^2 - \omega_i^2)$ and

$$I = \frac{2(\Delta K)}{\omega_f^2 - \omega_i^2} = \frac{2(-500 \text{ J})}{(54.5 \text{ rad/s})^2 - (68.1 \text{ rad/s})^2} = 0.600 \text{ kg} \cdot \text{m}^2.$$

EVALUATE: In $K = \frac{1}{2}I\omega^2$, ω must be in rad/s.

9.46. IDENTIFY: The work done on the cylinder equals its gain in kinetic energy.

SET UP: The work done on the cylinder is PL , where L is the length of the rope. $K_1 = 0$. $K_2 = \frac{1}{2}I\omega^2$.

$$I = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{w}{g}\right)r^2.$$

EXECUTE: $PL = \frac{1}{2}wv^2$, or $P = \frac{1}{2} \frac{w}{g} \frac{v^2}{L} = \frac{(40.0 \text{ N})(6.00 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(5.00 \text{ m})} = 14.7 \text{ N}$.

EVALUATE: The linear speed v of the end of the rope equals the tangential speed of a point on the rim of the cylinder. When K is expressed in terms of v , the radius r of the cylinder doesn't appear.

9.47. IDENTIFY and SET UP: Combine Eqs.(9.17) and (9.15) to solve for K . Use Table 9.2 to get I .

EXECUTE: $K = \frac{1}{2}I\omega^2$

$$a_{\text{rad}} = R\omega^2, \text{ so } \omega = \sqrt{a_{\text{rad}}/R} = \sqrt{(3500 \text{ m/s}^2)/1.20 \text{ m}} = 54.0 \text{ rad/s}$$

$$\text{For a disk, } I = \frac{1}{2}MR^2 = \frac{1}{2}(70.0 \text{ kg})(1.20 \text{ m})^2 = 50.4 \text{ kg} \cdot \text{m}^2$$

$$\text{Thus } K = \frac{1}{2}I\omega^2 = \frac{1}{2}(50.4 \text{ kg} \cdot \text{m}^2)(54.0 \text{ rad/s})^2 = 7.35 \times 10^4 \text{ J}$$

EVALUATE: The limit on a_{rad} limits ω which in turn limits K .

9.48. IDENTIFY: Repeat the calculation in Example 9.9, but with a different expression for I .

SET UP: For the solid cylinder in Example 9.9, $I = \frac{1}{2}MR^2$. For the thin-walled, hollow cylinder, $I = MR^2$.

EXECUTE: (a) With $I = MR^2$, the expression for v is $v = \sqrt{\frac{2gh}{1 + M/m}}$.

(b) This expression is smaller than that for the solid cylinder; more of the cylinder's mass is concentrated at its edge, so for a given speed, the kinetic energy of the cylinder is larger. A larger fraction of the potential energy is converted to the kinetic energy of the cylinder, and so less is available for the falling mass.

EVALUATE: When M is much larger than m , v is very small. When M is much less than m , v becomes $v = \sqrt{2gh}$, the same as for a mass that falls freely from a height h .

- 9.49. IDENTIFY:** Apply conservation of energy to the system of stone plus pulley. $v = r\omega$ relates the motion of the stone to the rotation of the pulley.
- SET UP:** For a uniform solid disk, $I = \frac{1}{2}MR^2$. Let point 1 be when the stone is at its initial position and point 2 be when it has descended the desired distance. Let $+y$ be upward and take $y = 0$ at the initial position of the stone, so $y_1 = 0$ and $y_2 = -h$, where h is the distance the stone descends.
- EXECUTE:** (a) $K_p = \frac{1}{2}I_p\omega^2$. $I_p = \frac{1}{2}M_pR^2 = \frac{1}{2}(2.50 \text{ kg})(0.200 \text{ m})^2 = 0.0500 \text{ kg}\cdot\text{m}^2$.
- $$\omega = \sqrt{\frac{2K_p}{I_p}} = \sqrt{\frac{2(4.50 \text{ J})}{0.0500 \text{ kg}\cdot\text{m}^2}} = 13.4 \text{ rad/s}$$
- The stone has speed $v = R\omega = (0.200 \text{ m})(13.4 \text{ rad/s}) = 2.68 \text{ m/s}$. The stone has kinetic energy $K_s = \frac{1}{2}mv^2 = \frac{1}{2}(1.50 \text{ kg})(2.68 \text{ m/s})^2 = 5.39 \text{ J}$. $K_1 + U_1 = K_2 + U_2$ gives $0 = K_2 + U_2$.
- $$0 = 4.50 \text{ J} + 5.39 \text{ J} + mg(-h) \quad h = \frac{9.89 \text{ J}}{(1.50 \text{ kg})(9.80 \text{ m/s}^2)} = 0.673 \text{ m}$$
- (b) $K_{\text{tot}} = K_p + K_s = 9.89 \text{ J}$. $\frac{K_p}{K_{\text{tot}}} = \frac{4.50 \text{ J}}{9.89 \text{ J}} = 45.5\%$.
- EVALUATE:** The gravitational potential energy of the pulley doesn't change as it rotates. The tension in the wire does positive work on the pulley and negative work of the same magnitude on the stone, so no net work on the system.
- 9.50. IDENTIFY:** $K_p = \frac{1}{2}I\omega^2$ for the pulley and $K_b = \frac{1}{2}mv^2$ for the bucket. The speed of the bucket and the rotational speed of the pulley are related by $v = R\omega$.
- SET UP:** $K_p = \frac{1}{2}K_b$
- EXECUTE:** $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}mv^2) = \frac{1}{4}mR^2\omega^2$. $I = \frac{1}{2}mR^2$.
- EVALUATE:** The result is independent of the rotational speed of the pulley and the linear speed of the mass.
- 9.51. IDENTIFY:** The general expression for I is Eq.(9.16). $K = \frac{1}{2}I\omega^2$.
- SET UP:** R will be multiplied by f .
- EXECUTE:** (a) In the expression of Eq. (9.16), each term will have the mass multiplied by f^3 and the distance multiplied by f , and so the moment of inertia is multiplied by $f^3(f)^2 = f^5$.
- (b) $(2.5 \text{ J})(48)^5 = 6.37 \times 10^8 \text{ J}$.
- EVALUATE:** Mass and volume are proportional to each other so both scale by the same factor.
- 9.52. IDENTIFY:** The work the person does is the negative of the work done by gravity. $W_{\text{grav}} = U_{\text{grav},1} - U_{\text{grav},2}$.
- $$U_{\text{grav}} = Mgy_{\text{cm}}$$
- SET UP:** The center of mass of the ladder is at its center, 1.00 m from each end.
- $$y_{\text{cm},1} = (1.00 \text{ m})\sin 53.0^\circ = 0.799 \text{ m} \quad y_{\text{cm},2} = 1.00 \text{ m}$$
- EXECUTE:** $W_{\text{grav}} = (9.00 \text{ kg})(9.80 \text{ m/s}^2)(0.799 \text{ m} - 1.00 \text{ m}) = -17.7 \text{ J}$. The work done by the person is 17.7 J.
- EVALUATE:** The gravity force is downward and the center of mass of the ladder moves upward, so gravity does negative work. The person pushes upward and does positive work.
- 9.53. IDENTIFY:** $U = Mgy_{\text{cm}}$. $\Delta U = U_2 - U_1$.
- SET UP:** Half the rope has mass 1.50 kg and length 12.0 m. Let $y = 0$ at the top of the cliff and take $+y$ to be upward. The center of mass of the hanging section of rope is at its center and $y_{\text{cm},2} = -6.00 \text{ m}$.
- EXECUTE:** $\Delta U = U_2 - U_1 = mg(y_{\text{cm},2} - y_{\text{cm},1}) = (1.50 \text{ kg})(9.80 \text{ m/s}^2)(-6.00 \text{ m} - 0) = -88.2 \text{ J}$.
- EVALUATE:** The potential energy of the rope decreases when part of the rope moves downward.
- 9.54. IDENTIFY:** Apply Eq.(9.19), the parallel-axis theorem.
- SET UP:** The center of mass of the hoop is at its geometrical center.
- EXECUTE:** In Eq. (9.19), $I_{\text{cm}} = MR^2$ and $d = R$, so $I_p = 2MR^2$.
- EVALUATE:** I is larger for an axis at the edge than for an axis at the center. Some mass is closer than distance R from the axis but some is also farther away. Since I for each piece of the hoop is proportional to the square of the distance from the axis, the increase in distance has a larger effect.
- 9.55. IDENTIFY:** Use Eq.(9.19) to relate I for the wood sphere about the desired axis to I for an axis along a diameter.
- SET UP:** For a thin-walled hollow sphere, axis along a diameter, $I = \frac{2}{3}MR^2$.
- For a solid sphere with mass M and radius R , $I_{\text{cm}} = \frac{2}{5}MR^2$, for an axis along a diameter.

EXECUTE: Find d such that $I_p = I_{\text{cm}} + Md^2$ with $I_p = \frac{2}{3}MR^2$:

$$\frac{2}{3}MR^2 = \frac{2}{5}MR^2 + Md^2$$

The factors of M divide out and the equation becomes $(\frac{2}{3} - \frac{2}{5})R^2 = d^2$

$$d = \sqrt{(10-6)/15}R = 2R/\sqrt{15} = 0.516R.$$

The axis is parallel to a diameter and is $0.516R$ from the center.

EVALUATE: $I_{\text{cm}}(\text{lead}) > I_{\text{cm}}(\text{wood})$ even though M and R are the same since for a hollow sphere all the mass is a distance R from the axis. Eq.(9.19) says $I_p > I_{\text{cm}}$, so there must be a d where $I_p(\text{wood}) = I_{\text{cm}}(\text{lead})$.

- 9.56. IDENTIFY:** Using the parallel-axis theorem to find the moment of inertia of a thin rod about an axis through its end and perpendicular to the rod.

SET UP: The center of mass of the rod is at its center, and $I_{\text{cm}} = \frac{1}{12}ML^2$.

EXECUTE: $I_p = I_{\text{cm}} + Md^2 = \frac{M}{12}L^2 + M\left(\frac{L}{2}\right)^2 = \frac{M}{3}L^2$.

EVALUATE: I is larger when the axis is not at the center of mass.

- 9.57. IDENTIFY and SET UP:** Use Eq.(9.19). The cm of the sheet is at its geometrical center. The object is sketched in Figure 9.57.

EXECUTE: $I_p = I_{\text{cm}} + Md^2$.

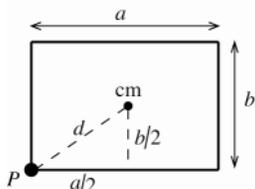


Figure 9.57

From part (c) of Table 9.2,

$$I_{\text{cm}} = \frac{1}{12}M(a^2 + b^2).$$

The distance d of P from the cm is

$$d = \sqrt{(a/2)^2 + (b/2)^2}.$$

Thus $I_p = I_{\text{cm}} + Md^2 = \frac{1}{12}M(a^2 + b^2) + M\left(\frac{1}{4}a^2 + \frac{1}{4}b^2\right) = \left(\frac{1}{12} + \frac{1}{4}\right)M(a^2 + b^2) = \frac{1}{3}M(a^2 + b^2)$

EVALUATE: $I_p = 4I_{\text{cm}}$. For an axis through P mass is farther from the axis.

- 9.58. IDENTIFY:** Consider the plate as made of slender rods placed side-by-side.

SET UP: The expression in Table 9.2(a) gives I for a rod and an axis through the center of the rod.

EXECUTE: (a) I is the same as for a rod with length a : $I = \frac{1}{12}Ma^2$.

(b) I is the same as for a rod with length b : $I = \frac{1}{12}Mb^2$.

EVALUATE: I is smaller when the axis is through the center of the plate than when it is along one edge.

- 9.59. IDENTIFY:** Use the equations in Table 9.2. I for the rod is the sum of I for each segment. The parallel-axis theorem says $I_p = I_{\text{cm}} + Md^2$.

SET UP: The bent rod and axes a and b are shown in Figure 9.59. Each segment has length $L/2$ and mass $M/2$.

EXECUTE: (a) For each segment the moment of inertia is for a rod with mass $M/2$, length $L/2$ and the axis

through one end. For one segment, $I_s = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{24}ML^2$. For the rod, $I_a = 2I_s = \frac{1}{12}ML^2$.

(b) The center of mass of each segment is at the center of the segment, a distance of $L/4$ from each end. For each

segment, $I_{\text{cm}} = \frac{1}{12}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{96}ML^2$. Axis b is a distance $L/4$ from the cm of each segment, so for each

segment the parallel axis theorem gives I for axis b to be $I_s = \frac{1}{96}ML^2 + \frac{M}{2}\left(\frac{L}{4}\right)^2 = \frac{1}{24}ML^2$ and $I_b = 2I_s = \frac{1}{12}ML^2$.

EVALUATE: I for these two axes are the same.

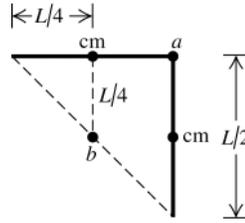


Figure 9.59

9.60. IDENTIFY: Apply the parallel-axis theorem.

SET UP: In Eq.(9.19), $I_{\text{cm}} = \frac{M}{12}L^2$ and $d = (L/2 - h)$.

$$\text{EXECUTE: } I_p = M \left[\frac{1}{12}L^2 + \left(\frac{L}{2} - h \right)^2 \right] = M \left[\frac{1}{12}L^2 + \frac{1}{4}L^2 - Lh + h^2 \right] = M \left[\frac{1}{3}L^2 - Lh + h^2 \right],$$

which is the same as found in Example 9.11.

EVALUATE: Example 9.11 shows that this result gives the expected result for $h = 0$, $h = L$ and $h = L/2$.

9.61. IDENTIFY: Apply Eq.(9.20).

SET UP: $dm = \rho dV = \rho(2\pi rL dr)$, where L is the thickness of the disk. $M = \pi L\rho R^2$.

EXECUTE: The analysis is identical to that of Example 9.12, with the lower limit in the integral being zero and the upper limit being R . The result is $I = \frac{1}{2}MR^2$.

EVALUATE: Our result agrees with Table 9.2(f).

9.62. IDENTIFY: Eq.(9.20), $I = \int r^2 dm$

SET UP:

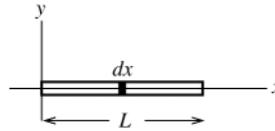


Figure 9.62

Take the x -axis to lie along the rod, with the origin at the left end. Consider a thin slice at coordinate x and width dx , as shown in Figure 9.62. The mass per unit length for this rod is M/L , so the mass of this slice is $dm = (M/L) dx$.

$$\text{EXECUTE: } I = \int_0^L x^2 (M/L) dx = (M/L) \int_0^L x^2 dx = (M/L)(L^3/3) = \frac{1}{3}ML^2$$

EVALUATE: This result agrees with Table 9.2.

9.63. IDENTIFY: Apply Eq.(9.20).

SET UP: For this case, $dm = \gamma dx$.

$$\text{EXECUTE: (a) } M = \int dm = \int_0^L \gamma x dx = \gamma \frac{x^2}{2} \Big|_0^L = \frac{\gamma L^2}{2}$$

(b) $I = \int_0^L x^2 (\gamma x) dx = \gamma \frac{x^4}{4} \Big|_0^L = \frac{\gamma L^4}{4} = \frac{M}{2} L^2$. This is larger than the moment of inertia of a uniform rod of the same mass and length, since the mass density is greater further away from the axis than nearer the axis.

$$\text{(c) } I = \int_0^L (L-x)^2 \gamma x dx = \gamma \int_0^L (L^2 x - 2Lx^2 + x^3) dx = \gamma \left(L^2 \frac{x^2}{2} - 2L \frac{x^3}{3} + \frac{x^4}{4} \right) \Big|_0^L = \gamma \frac{L^4}{12} = \frac{M}{6} L^2.$$

This is a third of the result of part (b), reflecting the fact that more of the mass is concentrated at the right end.

EVALUATE: For a uniform rod with an axis at one end, $I = \frac{1}{3}ML^2$. The result in (b) is larger than this and the result in (a) is smaller than this.

9.64. IDENTIFY: We know that $v = r\omega$ and \vec{v} is tangential. We know that $a_{\text{rad}} = r\omega^2$ and \vec{a}_{rad} is in toward the center of the wheel. See if the vector product expressions give these results.

SET UP: $|\vec{A} \times \vec{B}| = AB \sin \phi$, where ϕ is the angle between \vec{A} and \vec{B} .

EXECUTE: (a) For a counterclockwise rotation, $\vec{\omega}$ will be out of the page.

(b) The upward direction crossed into the radial direction is, by the right-hand rule, counterclockwise. $\vec{\omega}$ and \vec{r} are perpendicular, so the magnitude of $\vec{\omega} \times \vec{r}$ is $\omega r = v$.

(c) $\vec{\omega}$ is perpendicular to \vec{v} and so $\vec{\omega} \times \vec{v}$ has magnitude $\omega v = a_{\text{rad}}$, and from the right-hand rule, the upward direction crossed into the counterclockwise direction is inward, the direction of \vec{a}_{rad} .

EVALUATE: If the wheel rotates clockwise, the directions of $\vec{\omega}$ and \vec{v} are reversed, but \vec{a}_{rad} is still inward.

9.65. IDENTIFY: Apply $\theta = \omega t$.

SET UP: For alignment, the earth must move through 60° more than Mars, in the same time t . $\omega_e = 360^\circ/\text{yr}$.

$$\omega_M = 360^\circ/(1.9 \text{ yr}).$$

EXECUTE: $\theta_e = \theta_M + 60^\circ$. $\omega_e t = \omega_M t + 60^\circ$.

$$t = \frac{60^\circ}{\omega_e - \omega_M} = \frac{60^\circ}{\frac{360^\circ}{1 \text{ yr}} - \frac{360^\circ}{1.9 \text{ yr}}} = \frac{60^\circ}{360^\circ} (1/[0.9 \text{ yr}/1.9 \text{ yr}^2]) = 0.352 \text{ yr} = 128 \text{ days}.$$

EVALUATE: Earth has a larger angular velocity than Mars, and completes one orbit in less time.

9.66. IDENTIFY and SET UP: Use Eqs.(9.3) and (9.5). As long as $\alpha_z > 0$, ω_z increases. At the t when $\alpha_z = 0$, ω_z is at its maximum positive value and then starts to decrease when α_z becomes negative.

$$\theta(t) = \gamma t^2 - \beta t^3; \quad \gamma = 3.20 \text{ rad/s}^2, \quad \beta = 0.500 \text{ rad/s}^3$$

$$\text{EXECUTE: (a) } \omega_z(t) = \frac{d\theta}{dt} = \frac{d(\gamma t^2 - \beta t^3)}{dt} = 2\gamma t - 3\beta t^2$$

$$\text{(b) } \alpha_z(t) = \frac{d\omega_z}{dt} = \frac{d(2\gamma t - 3\beta t^2)}{dt} = 2\gamma - 6\beta t$$

(c) The maximum angular velocity occurs when $\alpha_z = 0$.

$$2\gamma - 6\beta t = 0 \text{ implies } t = \frac{2\gamma}{6\beta} = \frac{\gamma}{3\beta} = \frac{3.20 \text{ rad/s}^2}{3(0.500 \text{ rad/s}^3)} = 2.133 \text{ s}$$

$$\text{At this } t, \omega_z = 2\gamma t - 3\beta t^2 = 2(3.20 \text{ rad/s}^2)(2.133 \text{ s}) - 3(0.500 \text{ rad/s}^3)(2.133 \text{ s})^2 = 6.83 \text{ rad/s}$$

The maximum positive angular velocity is 6.83 rad/s and it occurs at 2.13 s.

EVALUATE: For large t both ω_z and α_z are negative and ω_z increases in magnitude. In fact, $\omega_z \rightarrow -\infty$ at $t \rightarrow \infty$. So the answer in (c) is not the largest angular speed, just the largest positive angular velocity.

9.67. IDENTIFY: The angular acceleration α of the disk is related to the linear acceleration a of the ball by $a = R\alpha$.

Since the acceleration is not constant, use $\omega_z - \omega_{0z} = \int_0^t \alpha_z dt$ and $\theta - \theta_0 = \int_0^t \omega_z dt$ to relate θ , ω_z , α_z and t for the disk. $\omega_{0z} = 0$.

SET UP: $\int t^n dt = \frac{1}{n+1} t^{n+1}$. In $a = R\alpha$, α is in rad/s^2 .

$$\text{EXECUTE: (a) } A = \frac{a}{t} = \frac{1.80 \text{ m/s}^2}{3.00 \text{ s}} = 0.600 \text{ m/s}^3$$

$$\text{(b) } \alpha = \frac{a}{R} = \frac{(0.600 \text{ m/s}^3)t}{0.250 \text{ m}} = (2.40 \text{ rad/s}^3)t$$

$$\text{(c) } \omega_z = \int_0^t (2.40 \text{ rad/s}^3)t dt = (1.20 \text{ rad/s}^3)t^2. \quad \omega_z = 15.0 \text{ rad/s for } t = \sqrt{\frac{15.0 \text{ rad/s}}{1.20 \text{ rad/s}^3}} = 3.54 \text{ s}.$$

$$\text{(d) } \theta - \theta_0 = \int_0^t \omega_z dt = \int_0^t (1.20 \text{ rad/s}^3)t^2 dt = (0.400 \text{ rad/s}^3)t^3. \text{ For } t = 3.54 \text{ s, } \theta - \theta_0 = 17.7 \text{ rad}.$$

EVALUATE: If the disk had turned at a constant angular velocity of 15.0 rad/s for 3.54 s it would have turned through an angle of 53.1 rad in 3.54 s. It actually turns through less than half this because the angular velocity is increasing in time and is less than 15.0 rad/s at all but the end of the interval.

9.68. IDENTIFY and SET UP: The translational kinetic energy is $K = \frac{1}{2}mv^2$ and the kinetic energy of the rotating flywheel is $K = \frac{1}{2}I\omega^2$. Use the scale speed to calculate the actual speed v . From that calculate K for the car and then solve for ω that gives this K for the flywheel.

EXECUTE: (a) $\frac{v_{\text{toy}}}{v_{\text{scale}}} = \frac{L_{\text{toy}}}{L_{\text{real}}}$

$$v_{\text{toy}} = v_{\text{scale}} \left(\frac{L_{\text{toy}}}{L_{\text{real}}} \right) = (700 \text{ km/h}) \left(\frac{0.150 \text{ m}}{3.0 \text{ m}} \right) = 35.0 \text{ km/h}$$

$$v_{\text{toy}} = (35.0 \text{ km/h})(1000 \text{ m/1 km})(1 \text{ h/3600 s}) = 9.72 \text{ m/s}$$

(b) $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.180 \text{ kg})(9.72 \text{ m/s})^2 = 8.50 \text{ J}$

(c) $K = \frac{1}{2}I\omega^2$ gives that $\omega = \sqrt{\frac{2K}{I}} = \sqrt{\frac{2(8.50 \text{ J})}{4.00 \times 10^{-5} \text{ kg} \cdot \text{m}^2}} = 652 \text{ rad/s}$

EVALUATE: $K = \frac{1}{2}I\omega^2$ gives ω in rad/s. $652 \text{ rad/s} = 6200 \text{ rev/min}$ so the rotation rate of the flywheel is very large.

9.69. IDENTIFY: $a_{\text{tan}} = r\alpha$, $a_{\text{rad}} = r\omega^2$. Apply the constant acceleration equations and $\sum \vec{F} = m\vec{a}$.

SET UP: a_{tan} and a_{rad} are perpendicular components of \vec{a} , so $a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2}$.

EXECUTE: (a) $\alpha = \frac{a_{\text{tan}}}{r} = \frac{3.00 \text{ m/s}^2}{60.0 \text{ m}} = 0.050 \text{ rad/s}^2$

(b) $\alpha t = (0.05 \text{ rad/s}^2)(6.00 \text{ s}) = 0.300 \text{ rad/s}$.

(c) $a_{\text{rad}} = \omega^2 r = (0.300 \text{ rad/s})^2 (60.0 \text{ m}) = 5.40 \text{ m/s}^2$.

(d) The sketch is given in Figure 9.69.

(e) $a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2} = \sqrt{(5.40 \text{ m/s}^2)^2 + (3.00 \text{ m/s}^2)^2} = 6.18 \text{ m/s}^2$, and the magnitude of the force is

$$F = ma = (1240 \text{ kg})(6.18 \text{ m/s}^2) = 7.66 \text{ kN}.$$

(f) $\arctan\left(\frac{a_{\text{rad}}}{a_{\text{tan}}}\right) = \arctan\left(\frac{5.40}{3.00}\right) = 60.9^\circ$.

EVALUATE: a_{tan} is constant and a_{rad} increases as ω increases. At $t = 0$, \vec{a} is parallel to \vec{v} . As t increases, \vec{a} moves toward the radial direction and the angle between \vec{a} increases toward 90° .

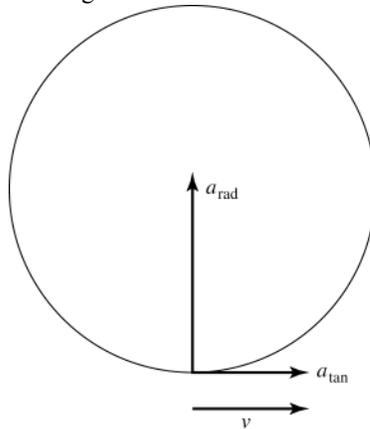


Figure 9.69

9.70. IDENTIFY: Apply conservation of energy to the system of drum plus falling mass, and compare the results for earth and for Mars.

SET UP: $K_{\text{drum}} = \frac{1}{2}I\omega^2$. $K_{\text{mass}} = \frac{1}{2}mv^2$. $v = R\omega$ so if K_{drum} is the same, ω is the same and v is the same on both planets. Therefore, K_{mass} is the same. Let $y = 0$ at the initial height of the mass and take $+y$ upward.

Configuration 1 is when the mass is at its initial position and 2 is when the mass has descended 5.00 m , so $y_1 = 0$ and $y_2 = -h$, where h is the height the mass descends.

EXECUTE: (a) $K_1 + U_1 = K_2 + U_2$ gives $0 = K_{\text{drum}} + K_{\text{mass}} - mgh$. $K_{\text{drum}} + K_{\text{mass}}$ are the same on both planets, so

$$mg_E h_E = mg_M h_M \cdot h_M = h_E \left(\frac{g_E}{g_M} \right) = (5.00 \text{ m}) \left(\frac{9.80 \text{ m/s}^2}{3.71 \text{ m/s}^2} \right) = 13.2 \text{ m}.$$

$$(b) \quad mg_M h_M = K_{\text{drum}} + K_{\text{mass}} \cdot \frac{1}{2}mv^2 = mg_M h_M - K_{\text{drum}} \quad \text{and}$$

$$v = \sqrt{2g_M h_M - \frac{2K_{\text{drum}}}{m}} = \sqrt{2(3.71 \text{ m/s}^2)(13.2 \text{ m}) - \frac{2(250.0 \text{ J})}{15.0 \text{ kg}}} = 8.04 \text{ m/s}$$

EVALUATE: We did the calculations without knowing the moment of inertia I of the drum, or the mass and radius of the drum.

- 9.71. IDENTIFY and SET UP:** All points on the belt move with the same speed. Since the belt doesn't slip, the speed of the belt is the same as the speed of a point on the rim of the shaft and on the rim of the wheel, and these speeds are related to the angular speed of each circular object by $v = r\omega$.

EXECUTE:



Figure 9.71

$$(a) \quad v_1 = r_1 \omega_1$$

$$\omega_1 = (60.0 \text{ rev/s})(2\pi \text{ rad/1 rev}) = 377 \text{ rad/s}$$

$$v_1 = r_1 \omega_1 = (0.45 \times 10^{-2} \text{ m})(377 \text{ rad/s}) = 1.70 \text{ m/s}$$

$$(b) \quad v_1 = v_2$$

$$r_1 \omega_1 = r_2 \omega_2$$

$$\omega_2 = (r_1 / r_2) \omega_1 = (0.45 \text{ cm} / 2.00 \text{ cm})(377 \text{ rad/s}) = 84.8 \text{ rad/s}$$

EVALUATE: The wheel has a larger radius than the shaft so turns slower to have the same tangential speed for points on the rim.

- 9.72. IDENTIFY:** The speed of all points on the belt is the same, so $r_1 \omega_1 = r_2 \omega_2$ applies to the two pulleys.

SET UP: The second pulley, with half the diameter of the first, must have twice the angular velocity, and this is the angular velocity of the saw blade. $\pi \text{ rad/s} = 30 \text{ rev/min}$.

$$\text{EXECUTE: (a)} \quad v_2 = (2(3450 \text{ rev/min})) \left(\frac{\pi \text{ rad/s}}{30 \text{ rev/min}} \right) \left(\frac{0.208 \text{ m}}{2} \right) = 75.1 \text{ m/s.}$$

$$(b) \quad a_{\text{rad}} = \omega^2 r = \left(2(3450 \text{ rev/min}) \left(\frac{\pi \text{ rad/s}}{30 \text{ rev/min}} \right) \right)^2 \left(\frac{0.208 \text{ m}}{2} \right) = 5.43 \times 10^4 \text{ m/s}^2,$$

so the force holding sawdust on the blade would have to be about 5500 times as strong as gravity.

EVALUATE: In $v = r\omega$ and $a_{\text{rad}} = r\omega^2$, ω must be in rad/s.

- 9.73. IDENTIFY and SET UP:** Use Eq.(9.15) to relate a_{rad} to ω and then use a constant acceleration equation to replace ω .

$$\text{EXECUTE: (a)} \quad a_{\text{rad}} = r\omega^2, \quad a_{\text{rad},1} = r\omega_1^2, \quad a_{\text{rad},2} = r\omega_2^2$$

$$\Delta a_{\text{rad}} = a_{\text{rad},2} - a_{\text{rad},1} = r(\omega_2^2 - \omega_1^2)$$

One of the constant acceleration equations can be written

$$\omega_2^2 = \omega_1^2 + 2\alpha_z(\theta_2 - \theta_1), \quad \text{or} \quad \omega_2^2 - \omega_1^2 = 2\alpha_z(\theta_2 - \theta_1)$$

Thus $\Delta a_{\text{rad}} = r2\alpha_z(\theta_2 - \theta_1) = 2r\alpha_z(\theta_2 - \theta_1)$, as was to be shown.

$$(b) \quad \alpha_z = \frac{\Delta a_{\text{rad}}}{2r(\theta_2 - \theta_1)} = \frac{85.0 \text{ m/s}^2 - 25.0 \text{ m/s}^2}{2(0.250 \text{ m})(15.0 \text{ rad})} = 8.00 \text{ rad/s}^2$$

$$\text{Then } a_{\text{tan}} = r\alpha = (0.250 \text{ m})(8.00 \text{ rad/s}^2) = 2.00 \text{ m/s}^2$$

EVALUATE: ω^2 is proportional to α_z and $(\theta_2 - \theta_1)$ so a_{rad} is also proportional to these quantities. a_{rad} increases while r stays fixed, ω_z increases, and α_z is positive.

IDENTIFY and SET UP: Use Eq.(9.17) to relate K and ω and then use a constant acceleration equation to replace ω .

$$\text{EXECUTE: (c)} \quad K = \frac{1}{2}I\omega^2; \quad K_2 = \frac{1}{2}I\omega_2^2, \quad K_1 = \frac{1}{2}I\omega_1^2$$

$$\Delta K = K_2 - K_1 = \frac{1}{2}I(\omega_2^2 - \omega_1^2) = \frac{1}{2}I(2\alpha_z(\theta_2 - \theta_1)) = I\alpha_z(\theta_2 - \theta_1), \quad \text{as was to be shown.}$$

$$(d) \quad I = \frac{\Delta K}{\alpha_z(\theta_2 - \theta_1)} = \frac{45.0 \text{ J} - 20.0 \text{ J}}{(8.00 \text{ rad/s}^2)(15.0 \text{ rad})} = 0.208 \text{ kg} \cdot \text{m}^2$$

EVALUATE: α_z is positive, ω increases, and K increases.

9.74. IDENTIFY: $I = I_{\text{wood}} + I_{\text{lead}}$. $m = \rho V$, where ρ is the volume density and $m = \sigma A$, where σ is the area density.

SET UP: For a solid sphere, $I = \frac{2}{5}mR^2$. For the hollow sphere (foil), $I = \frac{2}{3}mR^2$. For a sphere, $V = \frac{4}{3}\pi R^3$ and

$$A = 4\pi R^2. \quad m_w = \rho_w V_w = \rho_w \frac{4}{3}\pi R^3. \quad m_L = \sigma_L A_L = \sigma_L 4\pi R^2.$$

EXECUTE: $I = \frac{2}{5}m_w R^2 + \frac{2}{3}m_L R^2 = \frac{2}{5}\left(\rho_w \frac{4}{3}\pi R^3\right)R^2 + \frac{2}{3}(\sigma_L 4\pi R^2)R^2 = \frac{8}{3}\pi R^4\left(\frac{\rho_w R}{5} + \sigma_L\right).$

$$I = \frac{8\pi}{3}(0.20 \text{ m})^4 \left[\frac{(800 \text{ kg/m}^3)(0.20 \text{ m})}{5} + 20 \text{ kg/m}^2 \right] = 0.70 \text{ kg}\cdot\text{m}^2.$$

EVALUATE: $m_w = 26.8 \text{ kg}$ and $I_w = 0.429 \text{ kg}\cdot\text{m}^2$. $m_L = 10.1 \text{ kg}$ and $I_L = 0.268 \text{ kg}\cdot\text{m}^2$. Even though the foil is only 27% of the total mass its contribution to I is about 38% of the total.

9.75. IDENTIFY: Estimate the shape and dimensions of your body and apply the approximate expression from Table 9.2.

SET UP: I approximate my body as a vertical cylinder with mass 80 kg, length 1.7 m, and diameter 0.30 m (radius 0.15 m)

EXECUTE: $I = \frac{1}{2}mR^2 = \frac{1}{2}(80 \text{ kg})(0.15 \text{ m})^2 = 0.9 \text{ kg}\cdot\text{m}^2$

EVALUATE: I depends on your mass and width but not on your height.

9.76. IDENTIFY: Treat the V like two thin 0.160 kg bars, each 25 cm long.

SET UP: For a slender bar with the axis at one end, $I = \frac{1}{3}mL^2$.

EXECUTE: $I = 2\left(\frac{1}{3}mL^2\right) = 2\left(\frac{1}{3}\right)(0.160 \text{ kg})(0.250 \text{ m})^2 = 6.67 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

EVALUATE: The value of I is independent of the angle between the two sides of the V; the angle 70.0° didn't enter into the calculation.

9.77. IDENTIFY: $K = \frac{1}{2}I\omega^2$. $a_{\text{rad}} = r\omega^2$. $m = \rho V$.

SET UP: For a disk with the axis at the center, $I = \frac{1}{2}mR^2$. $V = t\pi R^2$, where $t = 0.100 \text{ m}$ is the thickness of the flywheel. $\rho = 7800 \text{ kg/m}^3$ is the density of the iron.

EXECUTE: (a) $\omega = 90.0 \text{ rpm} = 9.425 \text{ rad/s}$. $I = \frac{2K}{\omega^2} = \frac{2(10.0 \times 10^6 \text{ J})}{(9.425 \text{ rad/s})^2} = 2.252 \times 10^5 \text{ kg}\cdot\text{m}^2$.

$$m = \rho V = \rho \pi R^2 t. \quad I = \frac{1}{2}mR^2 = \frac{1}{2}\rho \pi t R^4. \quad \text{This gives } R = (2I/\rho \pi t)^{1/4} = 3.68 \text{ m} \text{ and the diameter is } 7.36 \text{ m}.$$

(b) $a_{\text{rad}} = R\omega^2 = 327 \text{ m/s}^2$

EVALUATE: In $K = \frac{1}{2}I\omega^2$, ω must be in rad/s. a_{rad} is about 33g; the flywheel material must have large cohesive strength to prevent the flywheel from flying apart.

9.78. IDENTIFY: $K = \frac{1}{2}I\omega^2$. To have the same K for any ω the two parts must have the same I . Use Table 9.2 for I .

SET UP: For a solid sphere, $I_{\text{solid}} = \frac{2}{5}M_{\text{solid}}R^2$. For a hollow sphere, $I_{\text{hollow}} = \frac{2}{3}M_{\text{hollow}}R^2$.

EXECUTE: $I_{\text{solid}} = I_{\text{hollow}}$ gives $\frac{2}{5}M_{\text{solid}}R^2 = \frac{2}{3}M_{\text{hollow}}R^2$ and $M_{\text{hollow}} = \frac{3}{5}M_{\text{solid}} = \frac{3}{5}M$.

EVALUATE: The hollow sphere has less mass since all its mass is distributed farther from the rotation axis.

9.79. IDENTIFY: $K = \frac{1}{2}I\omega^2$. $\omega = \frac{2\pi \text{ rad}}{T}$, where T is the period of the motion. For the earth's orbital motion it can be

treated as a point mass and $I = MR^2$.

SET UP: The earth's rotational period is 24 h = 86,164 s. Its orbital period is 1 yr = $3.156 \times 10^7 \text{ s}$.

$$M = 5.97 \times 10^{24} \text{ kg}. \quad R = 6.38 \times 10^6 \text{ m}.$$

EXECUTE: (a) $K = \frac{2\pi^2 I}{T^2} = \frac{2\pi^2 (0.3308)(5.97 \times 10^{24} \text{ kg})(6.38 \times 10^6 \text{ m})^2}{(86,164 \text{ s})^2} = 2.14 \times 10^{29} \text{ J}$.

(b) $\frac{1}{2}M\left(\frac{2\pi R}{T}\right)^2 = \frac{2\pi^2 (5.97 \times 10^{24} \text{ kg})(1.50 \times 10^{11} \text{ m})^2}{(3.156 \times 10^7 \text{ s})^2} = 2.66 \times 10^{33} \text{ J}$.

(c) Since the Earth's moment of inertia is less than that of a uniform sphere, more of the Earth's mass must be concentrated near its center.

EVALUATE: These kinetic energies are very large, because the mass of the earth is very large.

9.80. IDENTIFY: Using energy considerations, the system gains as kinetic energy the lost potential energy, mgR .

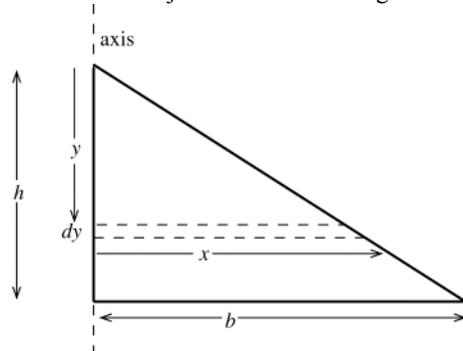
SET UP: The kinetic energy is $K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$, with $I = \frac{1}{2}mR^2$ for the disk. $v = R\omega$.

EXECUTE: $K = \frac{1}{2}I\omega^2 + \frac{1}{2}m(\omega R)^2 = \frac{1}{2}(I + mR^2)$. Using $I = \frac{1}{2}mR^2$ and solving for ω , $\omega^2 = \frac{4g}{3R}$ and $\omega = \sqrt{\frac{4g}{3R}}$.

EVALUATE: The small object has speed $v = \sqrt{\frac{2}{3}}\sqrt{2gR}$. If it was not attached to the disk and was dropped from a height h , it would attain a speed $\sqrt{2gR}$. Being attached to the disk reduces its final speed by a factor of $\sqrt{\frac{2}{3}}$.

9.81. IDENTIFY: Use Eq.(9.20) to calculate I . Then use $K = \frac{1}{2}I\omega^2$ to calculate K .

(a) SET UP: The object is sketched in Figure 9.81.



Consider a small strip of width dy and a distance y below the top of the triangle.

The length of the strip is $x = (y/h)b$.

Figure 9.81

EXECUTE: The strip has area $x dy$ and the area of the sign is $\frac{1}{2}bh$, so the mass of the strip is

$$dm = M \left(\frac{x dy}{\frac{1}{2}bh} \right) = M \left(\frac{yb}{h} \right) \left(\frac{2 dy}{bh} \right) = \left(\frac{2M}{h^2} \right) y dy$$

$$dI = \frac{1}{3}(dm)x^2 = \left(\frac{2Mb^2}{3h^4} \right) y^3 dy$$

$$I = \int_0^h dI = \frac{2Mb^2}{3h^4} \int_0^h y^3 dy = \frac{2Mb^2}{3h^4} \left(\frac{1}{h} y^4 \Big|_0^h \right) = \frac{1}{6} Mb^2$$

(b) $I = \frac{1}{6} Mb^2 = 2.304 \text{ kg} \cdot \text{m}^2$

$$\omega = 2.00 \text{ rev/s} = 4.00\pi \text{ rad/s}$$

$$K = \frac{1}{2} I \omega^2 = 182 \text{ J}$$

EVALUATE: From Table (9.2), if the sign were rectangular, with length b , then $I = \frac{1}{3} Mb^2$. Our result is one-half this, since mass is closer to the axis for the triangular than for the rectangular shape.

9.82. IDENTIFY: Apply conservation of energy to the system.

SET UP: For the falling mass $K = \frac{1}{2}mv^2$. For the wheel $K = \frac{1}{2}I\omega^2$.

EXECUTE: (a) The kinetic energy of the falling mass after 2.00 m is $K = \frac{1}{2}mv^2 = \frac{1}{2}(8.00 \text{ kg})(5.00 \text{ m/s})^2 = 100 \text{ J}$.

The change in its potential energy while falling is $mgh = (8.00 \text{ kg})(9.8 \text{ m/s}^2)(2.00 \text{ m}) = 156.8 \text{ J}$. The wheel must have the “missing” 56.8 J in the form of rotational kinetic energy. Since its outer rim is moving at the same speed

as the falling mass, 5.00 m/s, $v = r\omega$ gives $\omega = \frac{v}{r} = \frac{5.00 \text{ m/s}}{0.370 \text{ m}} = 13.51 \text{ rad/s}$. $K = \frac{1}{2}I\omega^2$; therefore

$$I = \frac{2K}{\omega^2} = \frac{2(56.8 \text{ J})}{(13.51 \text{ rad/s})^2} = 0.622 \text{ kg} \cdot \text{m}^2.$$

(b) The wheel’s mass is $(280 \text{ N})/(9.8 \text{ m/s}^2) = 28.6 \text{ kg}$. The wheel with the largest possible moment of inertia would have all this mass concentrated in its rim. Its moment of inertia would be

$$I = MR^2 = (28.6 \text{ kg})(0.370 \text{ m})^2 = 3.92 \text{ kg} \cdot \text{m}^2. \text{ The boss’s wheel is physically impossible.}$$

EVALUATE: If the mass falls from rest in free-fall its speed after it has descended 2.00 m is

$v = \sqrt{2g(2.00 \text{ m})} = 6.26 \text{ m/s}$. Its actual speed is less because some of the energy of the system is in the form of rotational kinetic energy of the wheel.

- 9.83. IDENTIFY:** Use conservation of energy. The stick rotates about a fixed axis so $K = \frac{1}{2}I\omega^2$. Once we have ω use $v = r\omega$ to calculate v for the end of the stick.

SET UP: The object is sketched in Figure 9.83.

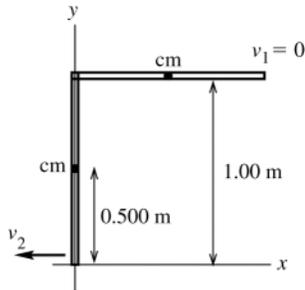


Figure 9.83

Take the origin of coordinates at the lowest point reached by the stick and take the positive y -direction to be upward.

EXECUTE: (a) Use Eq.(9.18): $U = Mgy_{\text{cm}}$

$$\Delta U = U_2 - U_1 = Mg(y_{\text{cm}2} - y_{\text{cm}1})$$

The center of mass of the meter stick is at its geometrical center, so $y_{\text{cm}1} = 1.00 \text{ m}$ and $y_{\text{cm}2} = 0.50 \text{ m}$

$$\text{Then } \Delta U = (0.160 \text{ kg})(9.80 \text{ m/s}^2)(0.50 \text{ m} - 1.00 \text{ m}) = -0.784 \text{ J}$$

(b) Use conservation of energy: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

Gravity is the only force that does work on the meter stick, so $W_{\text{other}} = 0$.

$$K_1 = 0.$$

Thus $K_2 = U_1 - U_2 = -\Delta U$, where ΔU was calculated in part (a).

$$K_2 = \frac{1}{2}I\omega_2^2 \text{ so } \frac{1}{2}I\omega_2^2 = -\Delta U \text{ and } \omega_2 = \sqrt{2(-\Delta U)/I}$$

For stick pivoted about one end, $I = \frac{1}{3}ML^2$ where $L = 1.00 \text{ m}$, so

$$\omega_2 = \sqrt{\frac{6(-\Delta U)}{ML^2}} = \sqrt{\frac{6(0.784 \text{ J})}{(0.160 \text{ kg})(1.00 \text{ m})^2}} = 5.42 \text{ rad/s}$$

(c) $v = r\omega = (1.00 \text{ m})(5.42 \text{ rad/s}) = 5.42 \text{ m/s}$

(d) For a particle in free-fall, with $+y$ upward,

$$v_{0y} = 0; \quad y - y_0 = -1.00 \text{ m}; \quad a_y = -9.80 \text{ m/s}^2; \quad v_y = ?$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$v_y = -\sqrt{2a_y(y - y_0)} = -\sqrt{2(-9.80 \text{ m/s}^2)(-1.00 \text{ m})} = -4.43 \text{ m/s}$$

EVALUATE: The magnitude of the answer in part (c) is larger. $U_{1,\text{grav}}$ is the same for the stick as for a particle

falling from a height of 1.00 m. For the stick $K = \frac{1}{2}I\omega_2^2 = \frac{1}{2}\left(\frac{1}{3}ML^2\right)(v/L)^2 = \frac{1}{6}Mv^2$. For the stick and for the

particle, K_2 is the same but the same K gives a larger v for the end of the stick than for the particle. The reason is that all the other points along the stick are moving slower than the end opposite the axis.

- 9.84. IDENTIFY:** Apply conservation of energy to the system of cylinder and rope.

SET UP: Taking the zero of gravitational potential energy to be at the axle, the initial potential energy is zero (the rope is wrapped in a circle with center on the axle). When the rope has unwound, its center of mass is a distance πR below the axle, since the length of the rope is $2\pi R$ and half this distance is the position of the center of the mass. Initially, every part of the rope is moving with speed $\omega_0 R$, and when the rope has unwound, and the cylinder has angular speed ω , the speed of the rope is ωR (the upper end of the rope has the same tangential speed at the edge of the cylinder). $I = (1/2)MR^2$ for a uniform cylinder,

EXECUTE: $K_1 = K_2 + U_2 \cdot \left(\frac{M}{4} + \frac{m}{2}\right) R^2 \omega_0^2 = \left(\frac{M}{4} + \frac{m}{2}\right) R^2 \omega^2 - mg\pi R$. Solving for ω gives

$$\omega = \sqrt{\omega_0^2 + \frac{(4\pi mg/R)}{(M+2m)}}, \text{ and the speed of any part of the rope is } v = \omega R.$$

EVALUATE: When $m \rightarrow 0$, $\omega \rightarrow \omega_0$. When $m \gg M$, $\omega = \sqrt{\omega_0^2 + \frac{2\pi g}{R}}$ and $v = \sqrt{v_0^2 + 2\pi gR}$. This is the final speed when an object with initial speed v_0 descends a distance πR .

9.85. IDENTIFY: Apply conservation of energy to the system consisting of blocks A and B and the pulley.

SET UP: The system at points 1 and 2 of its motion is sketched in Figure 9.85.

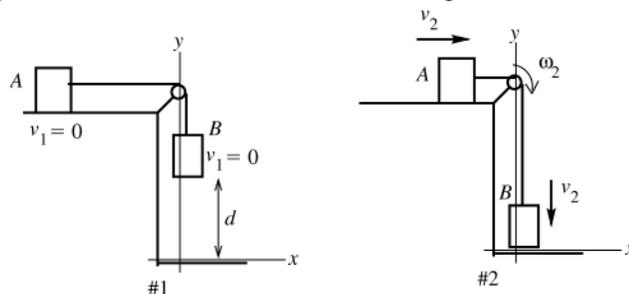


Figure 9.85

Use the work-energy relation $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Use coordinates where $+y$ is upward and where the origin is at the position of block B after it has descended. The tension in the rope does positive work on block A and negative work of the same magnitude on block B , so the net work done by the tension in the rope is zero. Both blocks have the same speed.

EXECUTE: Gravity does work on block B and kinetic friction does work on block A . Therefore

$$W_{\text{other}} = W_f = -\mu_k m_A g d.$$

$$K_1 = 0 \text{ (system is released from rest)}$$

$$U_1 = m_B g y_{B1} = m_B g d; \quad U_2 = m_B g y_{B2} = 0$$

$$K_2 = \frac{1}{2} m_A v_2^2 + \frac{1}{2} m_B v_2^2 + \frac{1}{2} I \omega_2^2.$$

But $v(\text{blocks}) = R\omega(\text{pulley})$, so $\omega_2 = v_2/R$ and

$$K_2 = \frac{1}{2} (m_A + m_B) v_2^2 + \frac{1}{2} I (v_2/R)^2 = \frac{1}{2} (m_A + m_B + I/R^2) v_2^2$$

Putting all this into the work-energy relation gives

$$m_B g d - \mu_k m_A g d = \frac{1}{2} (m_A + m_B + I/R^2) v_2^2$$

$$(m_A + m_B + I/R^2) v_2^2 = 2gd(m_B - \mu_k m_A)$$

$$v_2 = \sqrt{\frac{2gd(m_B - \mu_k m_A)}{m_A + m_B + I/R^2}}$$

EVALUATE: If $m_B \gg m_A$ and I/R^2 , then $v_2 = \sqrt{2gd}$; block B falls freely. If I is very large, v_2 is very small.

Must have $m_B > \mu_k m_A$ for motion, so the weight of B will be larger than the friction force on A . I/R^2 has units of mass and is in a sense the “effective mass” of the pulley.

9.86. IDENTIFY: Apply conservation of energy to the system of two blocks and the pulley.

SET UP: Let the potential energy of each block be zero at its initial position. The kinetic energy of the system is the sum of the kinetic energies of each object. $v = R\omega$, where v is the common speed of the blocks and ω is the angular velocity of the pulley.

EXECUTE: The amount of gravitational potential energy which has become kinetic energy is

$$K = (4.00 \text{ kg} - 2.00 \text{ kg})(9.80 \text{ m/s}^2)(5.00 \text{ m}) = 98.0 \text{ J. In terms of the common speed } v \text{ of the blocks, the kinetic}$$

energy of the system is $K = \frac{1}{2} (m_1 + m_2) v^2 + \frac{1}{2} I \left(\frac{v}{R}\right)^2$.

$$K = v^2 \frac{1}{2} \left(4.00 \text{ kg} + 2.00 \text{ kg} + \frac{(0.480 \text{ kg} \cdot \text{m}^2)}{(0.160 \text{ m})^2} \right) = v^2 (12.4 \text{ kg}). \text{ Solving for } v \text{ gives } v = \sqrt{\frac{98.0 \text{ J}}{12.4 \text{ kg}}} = 2.81 \text{ m/s.}$$

EVALUATE: If the pulley is massless, $98.0 \text{ J} = \frac{1}{2}(4.00 \text{ kg} + 2.00 \text{ kg})v^2$ and $v = 5.72 \text{ m/s}$. The moment of inertia of the pulley reduces the final speed of the blocks.

- 9.87. IDENTIFY and SET UP:** Apply conservation of energy to the motion of the hoop. Use Eq.(9.18) to calculate U_{grav} . Use $K = \frac{1}{2}I\omega^2$ for the kinetic energy of the hoop. Solve for ω . The center of mass of the hoop is at its geometrical center.

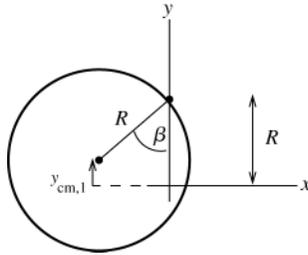


Figure 9.87

Take the origin to be at the original location of the center of the hoop, before it is rotated to one side, as shown in Figure 9.87.

$$y_{\text{cm}1} = R - R \cos \beta = R(1 - \cos \beta)$$

$$y_{\text{cm}2} = 0 \text{ (at equilibrium position hoop is at original position)}$$

$$\text{EXECUTE: } K_1 + U_1 + W_{\text{other}} = K_2 + U_2$$

$$W_{\text{other}} = 0 \text{ (only gravity does work)}$$

$$K_1 = 0 \text{ (released from rest), } K_2 = \frac{1}{2}I\omega_2^2$$

For a hoop, $I_{\text{cm}} = MR^2$, so $I = Md^2 + MR^2$ with $d = R$ and $I = 2MR^2$, for an axis at the edge. Thus

$$K_2 = \frac{1}{2}(2MR^2)\omega_2^2 = MR^2\omega_2^2.$$

$$U_1 = Mgy_{\text{cm}1} = MgR(1 - \cos \beta), \quad U_2 = mgy_{\text{cm}2} = 0$$

Thus $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$$MgR(1 - \cos \beta) = MR^2\omega_2^2 \text{ and } \omega = \sqrt{g(1 - \cos \beta)/R}$$

EVALUATE: If $\beta = 0$, then $\omega_2 = 0$. As β increases, ω_2 increases.

- 9.88. IDENTIFY:** $K = \frac{1}{2}I\omega^2$, with ω in rad/s. $P = \frac{\text{energy}}{t}$

SET UP: For a solid cylinder, $I = \frac{1}{2}MR^2$. $1 \text{ rev/min} = (2\pi/60) \text{ rad/s}$

$$\text{EXECUTE: (a) } \omega = 3000 \text{ rev/min} = 314 \text{ rad/s}. \quad I = \frac{1}{2}(1000 \text{ kg})(0.900 \text{ m})^2 = 405 \text{ kg} \cdot \text{m}^2$$

$$K = \frac{1}{2}(405 \text{ kg} \cdot \text{m}^2)(314 \text{ rad/s})^2 = 2.00 \times 10^7 \text{ J}.$$

$$\text{(b) } t = \frac{K}{P} = \frac{2.00 \times 10^7 \text{ J}}{1.86 \times 10^4 \text{ W}} = 1.08 \times 10^3 \text{ s} = 17.9 \text{ min}.$$

EVALUATE: In $K = \frac{1}{2}I\omega^2$, we must use ω in rad/s.

- 9.89. IDENTIFY:** $I = I_1 + I_2$. Apply conservation of energy to the system. The calculation is similar to Example 9.9.

SET UP: $\omega = \frac{v}{R_1}$ for part (b) and $\omega = \frac{v}{R_2}$ for part (c).

$$\text{EXECUTE: (a) } I = \frac{1}{2}M_1R_1^2 + \frac{1}{2}M_2R_2^2 = \frac{1}{2}((0.80 \text{ kg})(2.50 \times 10^{-2} \text{ m})^2 + (1.60 \text{ kg})(5.00 \times 10^{-2} \text{ m})^2)$$

$$I = 2.25 \times 10^{-3} \text{ kg} \cdot \text{m}^2.$$

$$\text{(b) The method of Example 9.9 yields } v = \sqrt{\frac{2gh}{1 + (I/mR_1^2)}}.$$

$$v = \sqrt{\frac{2(9.80 \text{ m/s}^2)(2.00 \text{ m})}{1 + ((2.25 \times 10^{-3} \text{ kg} \cdot \text{m}^2)/(1.50 \text{ kg})(0.025 \text{ m})^2)}} = 3.40 \text{ m/s}.$$

The same calculation, with R_2 instead of R_1 gives $v = 4.95 \text{ m/s}$.

EVALUATE: The final speed of the block is greater when the string is wrapped around the larger disk. $v = R\omega$, so when $R = R_2$ the factor that relates v to ω is larger. For $R = R_2$ a larger fraction of the total kinetic energy resides

with the block. The total kinetic energy is the same in both cases (equal to mgh), so when $R = R_2$ the kinetic energy and speed of the block are greater.

9.90. IDENTIFY: Apply conservation of energy to the motion of the mass after it hits the ground.

SET UP: From Example 9.9, the speed of the mass just before it hits the ground is $v = \sqrt{\frac{2gh}{1 + M/2m}}$.

EXECUTE: (a) In the case that no energy is lost, the rebound height h' is related to the speed v by $h' = \frac{v^2}{2g}$, and

with the form for v given in Example 9.9, $h' = \frac{h}{1 + M/2m}$.

(b) Considering the system as a whole, some of the initial potential energy of the mass went into the kinetic energy of the cylinder. Considering the mass alone, the tension in the string did work on the mass, so its total energy is not conserved.

EVALUATE: If $m \gg M$, $h' = h$ and the mass does rebound to its initial height.

9.91. IDENTIFY: Apply conservation of energy to relate the height of the mass to the kinetic energy of the cylinder.

SET UP: First use $K(\text{cylinder}) = 250 \text{ J}$ to find ω for the cylinder and v for the mass.

EXECUTE: $I = \frac{1}{2}MR^2 = \frac{1}{2}(10.0 \text{ kg})(0.150 \text{ m})^2 = 0.1125 \text{ kg} \cdot \text{m}^2$

$K = \frac{1}{2}I\omega^2$ so $\omega = \sqrt{2K/I} = 66.67 \text{ rad/s}$

$v = R\omega = 10.0 \text{ m/s}$

SET UP: Use conservation of energy $K_1 + U_1 = K_2 + U_2$ to solve for the distance the mass descends. Take $y = 0$ at lowest point of the mass, so $y_2 = 0$ and $y_1 = h$, the distance the mass descends.

EXECUTE: $K_1 = U_2 = 0$ so $U_1 = K_2$.

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, where $m = 12.0 \text{ kg}$

For the cylinder, $I = \frac{1}{2}MR^2$ and $\omega = v/R$, so $\frac{1}{2}I\omega^2 = \frac{1}{4}Mv^2$.

$mgh = \frac{1}{2}mv^2 + \frac{1}{4}Mv^2$

$h = \frac{v^2}{2g} \left(1 + \frac{M}{2m}\right) = 7.23 \text{ m}$

EVALUATE: For the cylinder $K_{\text{cyl}} = \frac{1}{2}I\omega^2 = \frac{1}{2} \left(\frac{1}{2}MR^2\right) (v/R)^2 = \frac{1}{4}Mv^2$.

$K_{\text{mass}} = \frac{1}{2}mv^2$, so $K_{\text{mass}} = (2m/M)K_{\text{cyl}} = [2(12.0 \text{ kg})/10.0 \text{ kg}](250 \text{ J}) = 600 \text{ J}$. The mass has 600 J of kinetic energy when the cylinder has 250 J of kinetic energy and at this point the system has total energy 850 J since $U_2 = 0$.

Initially the total energy of the system is $U_1 = mgy_1 = mgh = 850 \text{ J}$, so the total energy is shown to be conserved.

9.92. IDENTIFY: Energy conservation: Loss of U of box equals gain in K of system. Both the cylinder and pulley have

kinetic energy of the form $K = \frac{1}{2}I\omega^2$. $m_{\text{box}}gh = \frac{1}{2}m_{\text{box}}v_{\text{box}}^2 + \frac{1}{2}I_{\text{pulley}}\omega_{\text{pulley}}^2 + \frac{1}{2}I_{\text{cylinder}}\omega_{\text{cylinder}}^2$.

SET UP: $\omega_{\text{pulley}} = \frac{v_{\text{Box}}}{r_p}$ and $\omega_{\text{cylinder}} = \frac{v_{\text{Box}}}{r_{\text{cylinder}}}$.

EXECUTE: $m_{\text{B}}gh = \frac{1}{2}m_{\text{B}}v_{\text{B}}^2 + \frac{1}{2} \left(\frac{1}{2}m_{\text{p}}r_{\text{p}}^2\right) \left(\frac{v_{\text{B}}}{r_{\text{p}}}\right)^2 + \frac{1}{2} \left(\frac{1}{2}m_{\text{c}}r_{\text{c}}^2\right) \left(\frac{v_{\text{B}}}{r_{\text{c}}}\right)^2$. $m_{\text{B}}gh = \frac{1}{2}m_{\text{B}}v_{\text{B}}^2 + \frac{1}{4}m_{\text{p}}v_{\text{B}}^2 + \frac{1}{4}m_{\text{c}}v_{\text{B}}^2$ and

$v_{\text{B}} = \sqrt{\frac{m_{\text{B}}gh}{\frac{1}{2}m_{\text{B}} + \frac{1}{4}m_{\text{p}} + \frac{1}{4}m_{\text{c}}}} = \sqrt{\frac{(3.00 \text{ kg})(9.80 \text{ m/s}^2)(1.50 \text{ m})}{1.50 \text{ kg} + \frac{1}{4}(7.00 \text{ kg})}} = 3.68 \text{ m/s}$.

EVALUATE: If the box was disconnected from the rope and dropped from rest, after falling 1.50 m its speed would be $v = \sqrt{2g(1.50 \text{ m})} = 5.42 \text{ m/s}$. Since in the problem some of the energy of the system goes into kinetic energy of the cylinder and of the pulley, the final speed of the box is less than this.

9.93. IDENTIFY: $I = I_{\text{disk}} - I_{\text{hole}}$, where I_{hole} is I for the piece punched from the disk. Apply the parallel-axis theorem to calculate the required moments of inertia.

SET UP: For a uniform disk, $I = \frac{1}{2}MR^2$.

EXECUTE: (a) The initial moment of inertia is $I_0 = \frac{1}{2}MR^2$. The piece punched has a mass of $\frac{M}{16}$ and a moment of inertia with respect to the axis of the original disk of

$$\frac{M}{16} \left[\frac{1}{2} \left(\frac{R}{4} \right)^2 + \left(\frac{R}{2} \right)^2 \right] = \frac{9}{512} MR^2.$$

The moment of inertia of the remaining piece is then $I = \frac{1}{2}MR^2 - \frac{9}{512}MR^2 = \frac{247}{512}MR^2$.

(b) $I = \frac{1}{2}MR^2 + M(R/2)^2 - \frac{1}{2}(M/16)(R/4)^2 = \frac{383}{512}MR^2$.

EVALUATE: For a solid disk and an axis at a distance $R/2$ from the disk's center, the parallel-axis theorem gives $I = \frac{1}{2}MR^2 = \frac{3}{4}MR^2 = \frac{384}{512}MR^2$. For both choices of axes the presence of the hole reduces I , but the effect of the hole is greater in part (a), when it is farther from the axis.

9.94. IDENTIFY: In part (a) use the parallel-axis theorem to relate the moment of inertia I_{cm} for an axis through the center of the sphere to I_p , the moment of inertia for an axis at the pivot.

SET UP: I for a uniform solid sphere and the axis through its center is $\frac{2}{5}MR^2$. I for a slender rod and an axis at one end is $\frac{1}{3}mL^2$, where m is the mass of the rod and L is its length.

EXECUTE: (a) From the parallel-axis theorem, the moment of inertia is $I_p = (2/5)MR^2 + ML^2$, and

$$\frac{I_p}{ML^2} = \left(1 + \left(\frac{2}{5} \right) \left(\frac{R}{L} \right)^2 \right). \text{ If } R = (0.05)L, \text{ the difference is } (2/5)(0.05)^2 = 0.001 = 0.1\%.$$

(b) $(I_{\text{rod}}/ML^2) = (m_{\text{rod}}/3M)$, which is 0.33% when $m_{\text{rod}} = (0.01)M$.

EVALUATE: In both these cases the correction to $I = ML^2$ is very small.

9.95. IDENTIFY: Follow the instructions in the problem to derive the perpendicular-axis theorem. Then apply that result in part (b).

SET UP: $I = \sum_i m_i r_i^2$. The moment of inertia for the washer and an axis perpendicular to the plane of the washer

at its center is $\frac{1}{2}M(R_1^2 + R_2^2)$. In part (b), I for an axis perpendicular to the plane of the square at its center is $\frac{1}{12}M(L^2 + L^2) = \frac{1}{6}ML^2$.

EXECUTE: (a) With respect to O , $r_i^2 = x_i^2 + y_i^2$, and so

$$I_O = \sum_i m_i r_i^2 = \sum_i m_i (x_i^2 + y_i^2) = \sum_i m_i x_i^2 + \sum_i m_i y_i^2 = I_x + I_y.$$

(b) Two perpendicular axes, both perpendicular to the washer's axis, will have the same moment of inertia about those axes, and the perpendicular-axis theorem predicts that they will sum to the moment of inertia about the washer axis, which is $I = \frac{1}{2}M(R_1^2 + R_2^2)$, and so $I_x = I_y = \frac{1}{4}M(R_1^2 + R_2^2)$.

(c) $I_0 = \frac{1}{6}mL^2$. Since $I_0 = I_x + I_y$, and $I_x = I_y$, both I_x and I_y must be $\frac{1}{12}mL^2$.

EVALUATE: The result in part (c) says that I is the same for an axis that bisects opposite sides of the square as for an axis along the diagonal of the square, even though the distribution of mass relative to the two axes is quite different in these two cases.

9.96. IDENTIFY: Apply the parallel-axis theorem to each side of the square.

SET UP: Each side has length a and mass $M/4$, and the moment of inertia of each side about an axis perpendicular to the side and through its center is $\frac{1}{12} \frac{1}{4} Ma^2 = \frac{1}{48} Ma^2$.

EXECUTE: The moment of inertia of each side about the axis through the center of the square is, from the perpendicular axis theorem, $\frac{Ma^2}{48} + \frac{M}{4} \left(\frac{a}{2} \right)^2 = \frac{Ma^2}{12}$. The total moment of inertia is the sum of the contributions from the four sides, or $4 \times \frac{Ma^2}{12} = \frac{Ma^2}{3}$.

EVALUATE: If all the mass of a side were at its center, a distance $a/2$ from the axis, we would have

$$I = 4 \left(\frac{M}{4} \right) \left(\frac{a}{2} \right)^2 = \frac{1}{4} Ma^2. \text{ If all the mass was divided equally among the four corners of the square, a distance}$$

$a/\sqrt{2}$ from the axis, we would have $I = 4 \left(\frac{M}{4} \right) \left(\frac{a}{\sqrt{2}} \right)^2 = \frac{1}{2} Ma^2$. The actual I is between these two values.

9.97. **IDENTIFY:** Use Eq.(9.20) to calculate I .

(a) **SET UP:** Let L be the length of the cylinder. Divide the cylinder into thin cylindrical shells of inner radius r and outer radius $r + dr$. An end view is shown in Figure 9.97.

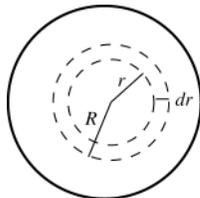


Figure 9.97

$$\rho = \alpha r$$

The mass of the thin cylindrical shell is

$$dm = \rho dV = \rho(2\pi r dr)L = 2\pi\alpha Lr^2 dr$$

EXECUTE: $I = \int r^2 dm = 2\pi\alpha L \int_0^R r^4 dr = 2\pi\alpha L \left(\frac{1}{5}R^5\right) = \frac{2}{5}\pi\alpha LR^5$

Relate M to α : $M = \int dm = 2\pi\alpha L \int_0^R r^2 dr = 2\pi\alpha L \left(\frac{1}{3}R^3\right) = \frac{2}{3}\pi\alpha LR^3$, so $\pi\alpha LR^3 = 3M/2$.

Using this in the above result for I gives $I = \frac{2}{5}(3M/2)R^2 = \frac{3}{5}MR^2$.

(b) **EVALUATE:** For a cylinder of uniform density $I = \frac{1}{2}MR^2$. The answer in (a) is larger than this. Since the density increases with distance from the axis the cylinder in (a) has more mass farther from the axis than for a cylinder of uniform density.

9.98. **IDENTIFY:** Write K in terms of the period T and take derivatives of both sides of this equation to relate dK/dt to dT/dt .

SET UP: $\omega = \frac{2\pi}{T}$ and $K = \frac{1}{2}I\omega^2$. The speed of light is $c = 3.00 \times 10^8$ m/s.

EXECUTE: (a) $K = \frac{2\pi^2 I}{T^2}$. $\frac{dK}{dt} = -\frac{4\pi^2 I}{T^3} \frac{dT}{dt}$. The rate of energy loss is $\frac{4\pi^2 I}{T^3} \frac{dT}{dt}$. Solving for the moment of inertia I in terms of the power P ,

$$I = \frac{PT^3}{4\pi} \frac{1}{dT/dt} = \frac{(5 \times 10^{31} \text{ W})(0.0331 \text{ s})^3}{4\pi^2} \frac{1 \text{ s}}{4.22 \times 10^{-13} \text{ s}} = 1.09 \times 10^{38} \text{ kg} \cdot \text{m}^2$$

(b) $R = \sqrt{\frac{5I}{2M}} = \sqrt{\frac{5(1.08 \times 10^{38} \text{ kg} \cdot \text{m}^2)}{2(1.4)(1.99 \times 10^{30} \text{ kg})}} = 9.9 \times 10^3 \text{ m}$, about 10 km.

(c) $v = \frac{2\pi R}{T} = \frac{2\pi(9.9 \times 10^3 \text{ m})}{(0.0331 \text{ s})} = 1.9 \times 10^6 \text{ m/s} = 6.3 \times 10^{-3} c$.

(d) $\rho = \frac{M}{V} = \frac{M}{(4\pi/3)R^3} = 6.9 \times 10^{17} \text{ kg/m}^3$, which is much higher than the density of ordinary rock by 14 orders of magnitude, and is comparable to nuclear mass densities.

EVALUATE: I is huge because M is huge. A small rate of change in the period corresponds to a large release of energy.

9.99. **IDENTIFY:** In part (a), do the calculations as specified in the hint. In part (b) calculate the mass of each shell of inner radius R_1 and outer radius R_2 and sum to get the total mass. In part (c) use the expression in part (a) to calculate I for each shell and sum to get the total I .

SET UP: $m = \rho V$. For a solid sphere, $V = \frac{4}{3}\pi R^3$.

EXECUTE: (a) Following the hint, the moment of inertia of a uniform sphere in terms of the mass density is $I = \frac{2}{5}MR^2 = \frac{8}{15}\pi\rho R^5$, and so the difference in the moments of inertia of two spheres with the same density ρ but different radii R_2 and R_1 is $I = \rho(8\pi/15)(R_2^5 - R_1^5)$.

(b) A rather tedious calculation, summing the product of the densities times the difference in the cubes of the radii that bound the regions and multiplying by $4\pi/3$, gives $M = 5.97 \times 10^{24} \text{ kg}$.

(c) A similar calculation, summing the product of the densities times the difference in the fifth powers of the radii that bound the regions and multiplying by $8\pi/15$, gives $I = 8.02 \times 10^{22} \text{ kg} \cdot \text{m}^2 = 0.334MR^2$.

EVALUATE: The calculated value of $I = 0.334MR^2$ agrees closely with the measured value of $0.3308MR^2$. This simple model is fairly accurate.

9.100. IDENTIFY: Apply Eq.(9.20)

SET UP: Let z be the coordinate along the vertical axis. $r(z) = \frac{zR}{h}$. $dm = \pi\rho \frac{R^2 z^2}{h^2}$ and $dI = \frac{\pi\rho R^4}{2 h^4} z^4 dz$.

EXECUTE: $I = \int dI = \frac{\pi\rho R^4}{2 h} \int_0^h z^4 dz = \frac{\pi\rho R^4}{10 h^4} [z^5]_0^h = \frac{1}{10} \pi\rho R^4 h$. The volume of a right circular cone is

$$V = \frac{1}{3} \pi R^2 h, \text{ the mass is } \frac{1}{3} \pi\rho R^2 h \text{ and so } I = \frac{3}{10} \left(\frac{\pi\rho R^2 h}{3} \right) R^2 = \frac{3}{10} MR^2.$$

EVALUATE: For a uniform cylinder of radius R and for an axis through its center, $I = \frac{1}{2} MR^2$. I for the cone is less, as expected, since the cone is constructed from a series of parallel discs whose radii decrease from R to zero along the vertical axis of the cone.

9.101. IDENTIFY: Follow the steps outlined in the problem.

SET UP: $\omega_z = d\theta/dt$. $\alpha_z = d^2\omega_z/dt^2$.

EXECUTE: (a) $ds = r d\theta = r_0 d\theta + \beta\theta d\theta$ so $s(\theta) = r_0\theta + \frac{\beta}{2}\theta^2$. θ must be in radians.

(b) Setting $s = vt = r_0\theta + \frac{\beta}{2}\theta^2$ gives a quadratic in θ . The positive solution is

$$\theta(t) = \frac{1}{\beta} \left[\sqrt{r_0^2 + 2\beta vt} - r_0 \right].$$

(The negative solution would be going backwards, to values of r smaller than r_0 .)

(c) Differentiating, $\omega_z(t) = \frac{d\theta}{dt} = \frac{v}{\sqrt{r_0^2 + 2\beta vt}}$, $\alpha_z = \frac{d\omega_z}{dt} = -\frac{\beta v^2}{(r_0^2 + 2\beta vt)^{3/2}}$. The angular acceleration α_z is not

constant.

(d) $r_0 = 25.0$ mm. θ must be measured in radians, so $\beta = (1.55 \mu\text{m/rev})(1 \text{ rev}/2\pi \text{ rad}) = 0.247 \mu\text{m/rad}$. Using $\theta(t)$ from part (b), the total angle turned in 74.0 min = 4440 s is

$$\theta = \frac{1}{2.47 \times 10^{-7} \text{ m/rad}} \left(\sqrt{2(2.47 \times 10^{-7} \text{ m/rad})(1.25 \text{ m/s})(4440 \text{ s}) + (25.0 \times 10^{-3} \text{ m})^2} - 25.0 \times 10^{-3} \text{ m} \right)$$

$\theta = 1.337 \times 10^5$ rad, which is 2.13×10^4 rev.

(e) The graphs are sketched in Figure 9.101.

EVALUATE: ω_z must decrease as r increases, to keep $v = r\omega$ constant. For ω_z to decrease in time, α_z must be negative.

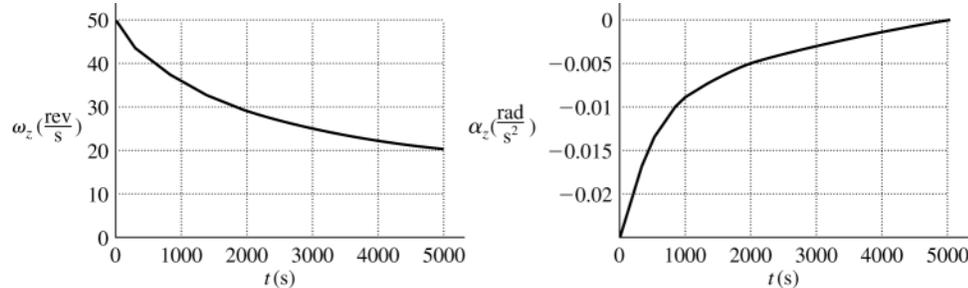


Figure 9.101

