# 8

# MOMENTUM, IMPULSE, AND COLLISIONS

**8.1.** **IDENTIFY** and **SET UP:** $p = mv$. $K = \frac{1}{2}mv^2$.

**EXECUTE:** **(a)** $p = (10{,}000 \text{ kg})(12.0 \text{ m/s}) = 1.20\times 10^5 \text{ kg}\cdot\text{m/s}$

**(b)** (i) $v = \dfrac{p}{m} = \dfrac{1.20\times 10^5 \text{ kg}\cdot\text{m/s}}{2000 \text{ kg}} = 60.0 \text{ m/s}$ . (ii) $\frac{1}{2}m_\text{T}v_\text{T}^2 = \frac{1}{2}m_\text{SUV}v_\text{SUV}^2$ , so

$$v_\text{SUV} = \sqrt{\frac{m_\text{T}}{m_\text{SUV}}}v_\text{T} = \sqrt{\frac{10{,}000 \text{ kg}}{2000 \text{ kg}}}(12.0 \text{ m/s}) = 26.8 \text{ m/s}$$

**EVALUATE:** The SUV must have less speed to have the same kinetic energy as the truck than to have the same momentum as the truck.

**8.2.** **IDENTIFY:** Example 8.1 shows that the two iceboats have the same kinetic energy at the finish line. $K = \frac{1}{2}mv^2$ . $p = mv$ .

**SET UP:** Let $A$ be the iceboat with mass $m$ and let $B$ be the iceboat with mass $2m$, so $m_B = 2m_A$ .

**EXECUTE:** $K_A = K_B$ gives $\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2$ . $v_A = \sqrt{\dfrac{m_B}{m_A}}v_B = \sqrt{2}v_B$ .

$$p_A = m_Av_A . \; p_B = m_Bv_B = (2m_A)\left(v_A/\sqrt{2}\right) = \sqrt{2}m_Av_A = \sqrt{2}p_A .$$

**EVALUATE:** The more massive boat must have less speed but greater momentum than the other boat in order to have the same kinetic energy.

**8.3.** **IDENTIFY** and **SET UP:** $p = mv$ . $K = \frac{1}{2}mv^2$ .

**EXECUTE:** **(a)** $v = \dfrac{p}{m}$ and $K = \frac{1}{2}m\left(\dfrac{p}{m}\right)^2 = \dfrac{p^2}{2m}$ .

**(b)** $K_\text{c} = K_\text{b}$ and the result from part (a) gives $\dfrac{p_\text{c}^2}{2m_\text{c}} = \dfrac{p_\text{b}^2}{2m_\text{b}}$ . $p_\text{b} = \sqrt{\dfrac{m_\text{b}}{m_\text{c}}}p_\text{c} = \sqrt{\dfrac{0.145 \text{ kg}}{0.040 \text{ kg}}}p_\text{c} = 1.90p_\text{c}$ . The baseball has the greater magnitude of momentum. $p_\text{c}/p_\text{b} = 0.526$ .

**(c)** $p^2 = 2mK$ so $p_\text{m} = p_\text{w}$ gives $2m_\text{m}K_\text{m} = 2m_\text{w}K_\text{w}$ . $w = mg$ , so $w_\text{m}K_\text{m} = w_\text{w}K_\text{w}$ .

$$K_\text{w} = \left(\frac{w_\text{m}}{w_\text{w}}\right)K_\text{m} = \left(\frac{700 \text{ N}}{450 \text{ N}}\right)K_\text{m} = 1.56K_\text{m} .$$

The woman has greater kinetic energy. $K_\text{m}/K_\text{w} = 0.641$ .

**EVALUATE:** For equal kinetic energy, the more massive object has the greater momentum. For equal momenta, the less massive object has the greater kinetic energy.

**8.4.** **IDENTIFY:** Each momentum component is the mass times the corresponding velocity component.

**SET UP:** Let $+x$ be along the horizontal motion of the shotput. Let $+y$ be vertically upward. $v_x = v\cos\theta$ , $v_y = v\sin\theta$ .

**EXECUTE:** The horizontal component of the initial momentum is

$$p_x = mv_x = mv\cos\theta = (7.30 \text{ kg})(15.0 \text{ m/s})\cos 40.0^\circ = 83.9 \text{ kg}\cdot\text{m/s} .$$

The vertical component of the initial momentum is $p_y = mv_y = mv\sin\theta = (7.30 \text{ kg})(15.0 \text{ m/s})\sin 40.0^\circ = 70.4 \text{ kg}\cdot\text{m/s}$

**EVALUATE:** The initial momentum is directed at $40.0^\circ$ above the horizontal.

**8.5.** **IDENTIFY:** For each object, $\vec{p} = m\vec{v}$ and $K = \frac{1}{2}mv^2$. The total momentum is the vector sum of the momenta of each object. The total kinetic energy is the scalar sum of the kinetic energies of each object.

**SET UP:** Let object $A$ be the 110 kg lineman and object $B$ the 125 kg lineman. Let $+x$ be the object to the right, so $v_{Ax} = +2.75$ m/s and $v_{Bx} = -2.60$ m/s.

**EXECUTE:** **(a)** $P_x = m_A v_{Ax} + m_B v_{Bx} = (110\text{ kg})(2.75\text{ m/s}) + (125\text{ kg})(-2.60\text{ m/s}) = -22.5\text{ kg}\cdot\text{m/s}$. The net momentum has magnitude $22.5\text{ kg}\cdot\text{m/s}$ and is directed to the left.

**(b)** $K = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(110\text{ kg})(2.75\text{ m/s})^2 + \frac{1}{2}(125\text{ kg})(2.60\text{ m/s})^2 = 838\text{ J}$

**EVALUATE:** The kinetic energy of an object is a scalar and is never negative. It depends only on the magnitude of the velocity of the object, not on its direction. The momentum of an object is a vector and has both magnitude and direction. When two objects are in motion, their total kinetic energy is greater than the kinetic energy of either one. But if they are moving in opposite directions, the net momentum of the system has a smaller magnitude than the magnitude of the momentum of either object.

**8.6.** **IDENTIFY:** For each object $\vec{p} = m\vec{v}$ and the net momentum of the system is $\vec{P} = \vec{p}_A + \vec{p}_B$. The momentum vectors are added by adding components. The magnitude and direction of the net momentum is calculated from its $x$ and $y$ components.

**SET UP:** Let object $A$ be the pickup and object $B$ be the sedan. $v_{Ax} = -14.0$ m/s, $v_{Ay} = 0$. $v_{Bx} = 0$, $v_{By} = +23.0$ m/s.

**EXECUTE:** **(a)** $P_x = p_{Ax} + p_{Bx} = m_A v_{Ax} + m_B v_{Bx} = (2500\text{ kg})(-14.0\text{ m/s}) + 0 = -3.50\times10^4\text{ kg}\cdot\text{m/s}$

$$P_y = p_{Ay} + p_{By} = m_A v_{Ay} + m_B v_{By} = (1500\text{ kg})(+23.0\text{ m/s}) = +3.45\times10^4\text{ kg}\cdot\text{m/s}$$

**(b)** $P = \sqrt{P_x^2 + P_y^2} = 4.91\times10^4\text{ kg}\cdot\text{m/s}$. From Figure 8.6, $\tan\theta = \dfrac{|P_x|}{|P_y|} = \dfrac{3.50\times10^4\text{ kg}\cdot\text{m/s}}{3.45\times10^4\text{ kg}\cdot\text{m/s}}$ and $\theta = 45.4°$. The net momentum has magnitude $4.91\times10^4\text{ kg}\cdot\text{m/s}$ and is directed at $45.4°$ west of north.

**EVALUATE:** The momenta of the two objects must be added as vectors. The momentum of one object is west and the other is north. The momenta of the two objects are nearly equal in magnitude, so the net momentum is directed approximately midway between west and north.

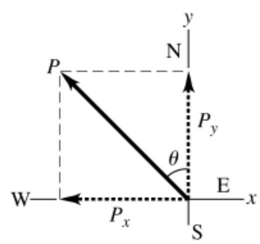


**Figure 8.6**

**8.7.** **IDENTIFY:** The average force on an object and the object's change in momentum are related by Eq. 8.9. The weight of the ball is $w = mg$.

**SET UP:** Let $+x$ be in the direction of the final velocity of the ball, so $v_{1x} = 0$ and $v_{2x} = 25.0$ m/s.

**EXECUTE:** $(F_{\text{av}})_x(t_2 - t_1) = mv_{2x} - mv_{1x}$ gives $(F_{\text{av}})_x = \dfrac{mv_{2x} - mv_{1x}}{t_2 - t_1} = \dfrac{(0.0450\text{ kg})(25.0\text{ m/s})}{2.00\times10^{-3}\text{ s}} = 562\text{ N}$.

$w = (0.0450\text{ kg})(9.80\text{ m/s}^2) = 0.441\text{ N}$. The force exerted by the club is much greater than the weight of the ball, so the effect of the weight of the ball during the time of contact is not significant.

**EVALUATE:** Forces exerted during collisions typically are very large but act for a short time.

**8.8.** **IDENTIFY:** The change in momentum, the impulse and the average force are related by Eq. 8.9.

**SET UP:** Let the direction in which the batted ball is traveling be the $+x$ direction, so $v_{1x} = -45.0$ m/s and $v_{2x} = 55.0$ m/s.

**EXECUTE:** **(a)** $\Delta p_x = p_{2x} - p_{1x} = m(v_{2x} - v_{1x}) = (0.145\text{ kg})(55.0\text{ m/s} - [-45.0\text{ m/s}]) = 14.5\text{ kg}\cdot\text{m/s}$. $J_x = \Delta p_x$, so $J_x = 14.5\text{ kg}\cdot\text{m/s}$. Both the change in momentum and the impulse have magnitude $14.5\text{ kg}\cdot\text{m/s}$.

**(b)** $(F_{\text{av}})_x = \dfrac{J_x}{\Delta t} = \dfrac{14.5\text{ kg}\cdot\text{m/s}}{2.00\times10^{-3}\text{ s}} = 7250\text{ N}$.

**EVALUATE:** The force is in the direction of the momentum change.

**8.9.** **IDENTIFY:** Use Eq. 8.9. We know the intial momentum and the impluse so can solve for the final momentum and then the final velocity.

**SET UP:** Take the $x$-axis to be toward the right, so $v_{1x} = +3.00\ \text{m/s}$. Use Eq. 8.5 to calculate the impulse, since the force is constant.

**EXECUTE:** **(a)** $J_x = p_{2x} - p_{1x}$

$$J_x = F_x(t_2 - t_1) = (+25.0\ \text{N})(0.050\ \text{s}) = +1.25\ \text{kg}\cdot\text{m/s}$$

Thus $p_{2x} = J_x + p_{1x} = +1.25\ \text{kg}\cdot\text{m/s} + (0.160\ \text{kg})(+3.00\ \text{m/s}) = +1.73\ \text{kg}\cdot\text{m/s}$

$$v_{2x} = \frac{p_{2x}}{m} = \frac{1.73\ \text{kg}\cdot\text{m/s}}{0.160\ \text{kg}} = +10.8\ \text{kg}\cdot\text{m/s (to the right)}$$

**(b)** $J_x = F_x(t_2 - t_1) = (-12.0\ \text{N})(0.050\ \text{s}) = -0.600\ \text{kg}\cdot\text{m/s}$ (negative since force is to left)

$$p_{2x} = J_x + p_{1x} = -0.600\ \text{kg}\cdot\text{m/s} + (0.160\ \text{kg})(+3.00\ \text{m/s}) = -0.120\ \text{kg}\cdot\text{m/s}$$

$$v_{2x} = \frac{p_{2x}}{m} = \frac{-0.120\ \text{kg}\cdot\text{m/s}}{0.160\ \text{kg}} = -0.75\ \text{m/s (to the left)}$$

**EVALUATE:** In part (a) the impulse and initial momentum are in the same direction and $v_x$ increases. In part (b) the impulse and initial momentum are in opposite directions and the velocity decreases.

**8.10.** **IDENTIFY:** The impulse, change in momentum and change in velocity are related by Eq. 8.9.

**SET UP:** $F_y = 26{,}700\ \text{N}$ and $F_x = 0$. The force is constant, so $(F_{\text{av}})_y = F_y$.

**EXECUTE:** **(a)** $J_y = F_y\Delta t = (26{,}700\ \text{N})(3.90\ \text{s}) = 1.04\times10^5\ \text{N}\cdot\text{s}$.

**(b)** $\Delta p_y = J_y = 1.04\times10^5\ \text{kg}\cdot\text{m/s}$.

**(c)** $\Delta p_y = m\Delta v_y$. $\Delta v_y = \dfrac{\Delta p_y}{m} = \dfrac{1.04\times10^5\ \text{kg}\cdot\text{m/s}}{95{,}000\ \text{kg}} = 1.09\ \text{m/s}$.

**(d)** The initial velocity of the shuttle isn't known. The change in kinetic energy is $\Delta K = K_2 - K_1 = \frac{1}{2}m(v_2^2 - v_1^2)$. It depends on the initial and final speeds and isn't determined solely by the change in speed.

**EVALUATE:** The force in the $+y$ direction produces an increase of the velocity in the $+y$ direction.

**8.11.** **IDENTIFY:** The force is not constant so $\vec{J} = \int_{t_1}^{t_2} \vec{F}dt$. The impulse is related to the change in velocity by Eq. 8.9.

**SET UP:** Only the $x$ component of the force is nonzero, so $J_x = \int_{t_1}^{t_2} F_x dt$ is the only nonzero component of $\vec{J}$.

$$J_x = m(v_{2x} - v_{1x}).\ t_1 = 2.00\ \text{s},\ t_2 = 3.50\ \text{s}.$$

**EXECUTE:** **(a)** $A = \dfrac{F_x}{t^2} = \dfrac{781.25\ \text{N}}{(1.25\ \text{s})^2} = 500\ \text{N/s}^2$.

**(b)** $J_x = \int_{t_1}^{t_2} At^2dt = \frac{1}{3}A(t_2^3 - t_1^3) = \frac{1}{3}(500\ \text{N/s}^2)([3.50\ \text{s}]^3 - [2.00\ \text{s}]^3) = 5.81\times10^3\ \text{N}\cdot\text{s}$.

**(c)** $\Delta v_x = v_{2x} - v_{1x} = \dfrac{J_x}{m} = \dfrac{5.81\times10^3\ \text{N}\cdot\text{s}}{2150\ \text{kg}} = 2.70\ \text{m/s}$. The $x$ component of the velocity of the rocket increases by 2.70 m/s.

**EVALUATE:** The change in velocity is in the same direction as the impulse, which in turn is in the direction of the net force. In this problem the net force equals the force applied by the engine, since that is the only force on the rocket.

**8.12.** **IDENTIFY:** Apply Eq. 8.9 to relate the change in momentum of the momentum to the components of the average force on it.

**SET UP:** Let $+x$ be to the right and $+y$ be upward.

**EXECUTE:** **(a)** $J_x = \Delta p_x = mv_{2x} - mv_{1x} = (0.145\ \text{kg})(-[65.0\ \text{m/s}]\cos30° - 50.0\ \text{m/s}) = -15.4\ \text{kg}\cdot\text{m/s}$.

$$J_y = \Delta p_y = mv_{2y} - mv_{1y} = (0.145\ \text{kg})([65.0\ \text{m/s}]\sin30° - 0) = 4.71\ \text{kg}\cdot\text{m/s}$$

The horizontal component is $15.4\ \text{kg}\cdot\text{m/s}$, to the left and the vertical component is $4.71\ \text{kg}\cdot\text{m/s}$, upward.

**(b)** $F_{\text{av-}x} = \dfrac{J_x}{\Delta t} = \dfrac{-15.4\ \text{kg}\cdot\text{m/s}}{1.75\times10^{-3}\ \text{s}} = -8800\ \text{N}$. $F_{\text{av-}y} = \dfrac{J_y}{\Delta t} = \dfrac{4.71\ \text{kg}\cdot\text{m/s}}{1.75\times10^{-3}\ \text{s}} = 2690\ \text{N}$.

The horizontal component is 8800 N, to the left, and the vertical component is 2690 N, upward.

**EVALUATE:** The ball gains momentum to the left and upward and the force components are in these directions.

**8.13.** **IDENTIFY:** The force is constant during the 1.0 ms interval that it acts, so $\vec{J} = \vec{F}\Delta t$. $\vec{J} = \vec{p}_2 - \vec{p}_1 = m(\vec{v}_2 - \vec{v}_1)$.

**SET UP:** Let $+x$ be to the right, so $v_{1x} = +5.00\ \text{m/s}$. Only the $x$ component of $\vec{J}$ is nonzero, and $J_x = m(v_{2x} - v_{1x})$.

**EXECUTE:** **(a)** The magnitude of the impulse is $J = F\Delta t = (2.50\times10^3\ \text{N})(1.00\times10^{-3}\ \text{s}) = 2.50\ \text{N}\cdot\text{s}$. The direction of the impulse is the direction of the force.

**(b)** (i) $v_{2x} = \dfrac{J_x}{m} + v_{1x}$. $J_x = +2.50\ \text{N}\cdot\text{s}$. $v_{2x} = \dfrac{+2.50\ \text{N}\cdot\text{s}}{2.00\ \text{kg}} + 5.00\ \text{m/s} = 6.25\ \text{m/s}$. The stone's velocity has magnitude 6.25 m/s and is directed to the right. (ii) Now $J_x = -2.50\ \text{N}\cdot\text{s}$ and $v_{2x} = \dfrac{-2.50\ \text{N}\cdot\text{s}}{2.00\ \text{kg}} + 5.00\ \text{m/s} = 3.75\ \text{m/s}$. The stone's velocity has magnitude 3.75 m/s and is directed to the right.

**EVALUATE:** When the force and initial velocity are in the same direction the speed increases and when they are in opposite directions the speed decreases.

**8.14.** **IDENTIFY:** Apply conservation of momentum to the system of the astronaut and tool.

**SET UP:** Let $A$ be the astronaut and $B$ be the tool. Let $+x$ be the direction in which she throws the tool, so $v_{B2x} = +3.20\ \text{m/s}$. Assume she is initially at rest, so $v_{A1x} = v_{B1x} = 0$. Solve for $v_{A2x}$.

**EXECUTE:** $P_{1x} = P_{2x}$. $P_{1x} = m_A v_{A1x} + m_B v_{B1x} = 0$. $P_{2x} = m_A v_{A2x} + m_B v_{B2x} = 0$ and $v_{A2x} = -\dfrac{m_B v_{A2x}}{m_A} = -\dfrac{(2.25\ \text{kg})(3.20\ \text{m/s})}{68.5\ \text{kg}} = -0.105\ \text{m/s}$. Her speed is 0.105 m/s and she moves opposite to the direction in which she throws the tool.

**EVALUATE:** Her mass is much larger than that of the tool so to have the same magnitude of momentum as the tool her speed is much less.

**8.15.** **IDENTIFY:** Since drag effects are neglected there is no net external force on the system of squid plus expelled water and the total momentum of the system is conserved. Since the squid is initially at rest, with the water in its cavity, the initial momentum of the system is zero. For each object, $K = \frac{1}{2}mv^2$.

**SET UP:** Let $A$ be the squid and $B$ be the water it expels, so $m_A = 6.50\ \text{kg} - 1.75\ \text{kg} = 4.75\ \text{kg}$. Let $+x$ be the direction in which the water is expelled. $v_{A2x} = -2.50\ \text{m/s}$. Solve for $v_{B2x}$.

**EXECUTE:** **(a)** $P_{1x} = 0$. $P_{2x} = P_{1x}$, so $0 = m_A v_{A2x} + m_B v_{B2x}$. $v_{B2x} = -\dfrac{m_A v_{A2x}}{m_B} = -\dfrac{(4.75\ \text{kg})(-2.50\ \text{m/s})}{1.75\ \text{kg}} = +6.79\ \text{m/s}$.

**(b)** $K_2 = K_{A2} + K_{B2} = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 = \frac{1}{2}(4.75\ \text{kg})(2.50\ \text{m/s})^2 + \frac{1}{2}(1.75\ \text{kg})(6.79\ \text{m/s})^2 = 55.2\ \text{J}$ The initial kinetic energy is zero, so the kinetic energy produced is $K_2 = 55.2\ \text{J}$.

**EVALUATE:** The two objects end up with momenta that are equal in magnitude and opposite in direction, so the total momentum of the system remains zero. The kinetic energy is created by the work done by the squid as it expels the water.

**8.16.** **IDENTIFY:** Apply conservation of momentum to the system of you and the ball. In part (a) both objects have the same final velocity.

**SET UP:** Let $+x$ be in the direction the ball is traveling initially. $m_A = 0.400\ \text{kg}$ (ball). $m_B = 70.0\ \text{kg}$ (you).

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$ gives $(0.400\ \text{kg})(10.0\ \text{m/s}) = (0.400\ \text{kg} + 70.0\ \text{kg})v_2$ and $v_2 = 0.0568\ \text{m/s}$.

**(b)** $P_{1x} = P_{2x}$ gives $(0.400\ \text{kg})(10.0\ \text{m/s}) = (0.400\ \text{kg})(-8.00\ \text{m/s}) + (70.0\ \text{kg})v_{B2}$ and $v_{B2} = 0.103\ \text{m/s}$.

**EVALUATE:** When the ball bounces off it has a greater change in momentum and you acquire a greater final speed.

**8.17.** **IDENTIFY:** Apply conservation of momentum to the system of the two pucks.

**SET UP:** Let $+x$ be to the right.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$ says $(0.250)v_{A1} = (0.250\ \text{kg})(-0.120\ \text{m/s}) + (0.350\ \text{kg})(0.650\ \text{m/s})$ and $v_{A1} = 0.790\ \text{m/s}$.

**(b)** $K_1 = \frac{1}{2}(0.250\ \text{kg})(0.790\ \text{m/s})^2 = 0.0780\ \text{J}$.

$K_2 = \frac{1}{2}(0.250\ \text{kg})(0.120\ \text{m/s})^2 + \frac{1}{2}(0.350\ \text{kg})(0.650\ \text{m/s})^2 = 0.0757\ \text{J}$ and $\Delta K = K_2 - K_1 = -0.0023\ \text{J}$.

**EVALUATE:** The total momentum of the system is conserved but the total kinetic energy decreases.

**8.18.** **IDENTIFY:** Since road friction is neglected, there is no net external force on the system of the two cars and the total momentum of the system is conserved. For each object, $K = \frac{1}{2}mv^2$.

**SET UP:** Let $A$ be the 1750 kg car and $B$ be the 1450 kg car. Let $+x$ be to the right, so $v_{A1x} = +1.50\ \text{m/s}$, $v_{B1x} = -1.10\ \text{m/s}$, and $v_{A2x} = +0.250\ \text{m/s}$. Solve for $v_{B2x}$.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$. $v_{B2x} = \dfrac{m_A v_{A1x} + m_B v_{B1x} - m_A v_{A2x}}{m_B}$.

$$v_{B2x} = \frac{(1750\ \text{kg})(1.50\ \text{m/s}) + (1450\ \text{kg})(-1.10\ \text{m/s}) - (1750\ \text{kg})(0.250\ \text{m/s})}{1450\ \text{kg}} = 0.409\ \text{m/s}.$$

After the collision the lighter car is moving to the right with a speed of 0.409 m/s.

**(b)** $K_1 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(1750\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}(1450\text{ kg})(1.10\text{ m/s})^2 = 2846\text{ J}$.

$$K_2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 = \frac{1}{2}(1750\text{ kg})(0.250\text{ m/s})^2 + \frac{1}{2}(1450\text{ kg})(0.409\text{ m/s})^2 = 176\text{ J}.$$

The change in kinetic energy is $\Delta K = K_2 - K_1 = 176\text{ J} - 2846\text{ J} = -2670\text{ J}$.

**EVALUATE:** The total momentum of the system is constant because there is no net external force during the collision. The kinetic energy of the system decreases because of negative work done by the forces the cars exert on each other during the collision.

**8.19.** **IDENTIFY:** Since the rifle is loosely held there is no net external force on the system consisting of the rifle, bullet and propellant gases and the momentum of this system is conserved. Before the rifle is fired everything in the system is at rest and the initial momentum of the system is zero.

**SET UP:** Let $+x$ be in the direction of the bullet's motion. The bullet has speed $601\text{ m/s} - 1.85\text{ m/s} = 599\text{ m/s}$ relative to the earth. $P_{2x} = p_{rx} + p_{bx} + p_{gx}$, the momenta of the rifle, bullet and gases. $v_{rx} = -1.85\text{ m/s}$ and $v_{bx} = +599\text{ m/s}$.

**EXECUTE:** $P_{2x} = P_{1x} = 0$. $p_{rx} + p_{bx} + p_{gx} = 0$. $p_{gx} = -p_{rx} - p_{bx} = -(2.80\text{ kg})(-1.85\text{ m/s}) - (0.00720\text{ kg})(599\text{ m/s})$ and $p_{gx} = +5.18\text{ kg}\cdot\text{m/s} - 4.31\text{ kg}\cdot\text{m/s} = 0.87\text{ kg}\cdot\text{m/s}$. The propellant gases have momentum $0.87\text{ kg}\cdot\text{m/s}$, in the same direction as the bullet is traveling.

**EVALUATE:** The magnitude of the momentum of the recoiling rifle equals the magnitude of the momentum of the bullet plus that of the gases as both exit the muzzle.

**8.20.** **IDENTIFY:** In part (a) no horizontal force implies $P_x$ is constant. In part (b) use the energy expression, Eq. 7.14, to find the potential energy intially in the spring.

**SET UP:** Initially both blocks are at rest.

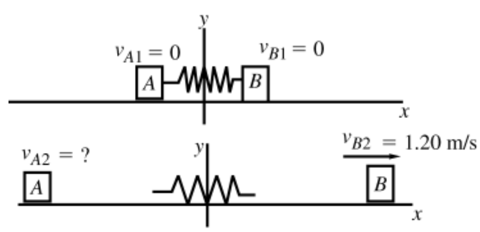


**Figure 8.20**

**EXECUTE:** **(a)** $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$

$$0 = m_A v_{A2x} + m_B v_{B2x}$$

$$v_{A2x} = -\left(\frac{m_B}{m_A}\right)v_{B2x} = -\left(\frac{3.00\text{ kg}}{1.00\text{ kg}}\right)(+1.20\text{ m/s}) = -3.60\text{ m/s}$$

Block *A* has a final speed of 3.60 m/s, and moves off in the opposite direction to *B*.

**(b)** Use energy conservation: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

Only the spring force does work so $W_{\text{other}} = 0$ and $U = U_{\text{el}}$.

$K_1 = 0$ (the blocks initially are at rest)

$U_2 = 0$ (no potential energy is left in the spring)

$$K_2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 = \frac{1}{2}(1.00\text{ kg})(3.60\text{ m/s})^2 + \frac{1}{2}(3.00\text{ kg})(1.20\text{ m/s})^2 = 8.64\text{ J}$$

$U_1 = U_{1,\text{el}}$ the potential energy stored in the compressed spring.

Thus $U_{1,\text{el}} = K_2 = 8.64\text{ J}$

**EVALUATE:** The blocks have equal and opposite momenta as they move apart, since the total momentum is zero. The kinetic energy of each block is positive and doesn't depend on the direction of the block's velocity, just on its magnitude.

**8.21.** **IDENTIFY:** Since friction at the pond surface is neglected, there is no net external horizontal force and the horizontal component of the momentum of the system of hunter plus bullet is conserved. Both objects are initially at rest, so the initial momentum of the system is zero. Gravity and the normal force exerted by the ice together produce a net vertical force while the rifle is firing, so the vertical component of momentum is not conserved.

**SET UP:** Let object *A* be the hunter and object *B* be the bullet. Let $+x$ be the direction of the horizontal component of velocity of the bullet. Solve for $v_{A2x}$.

**EXECUTE:** **(a)** $v_{B2x} = +965$ m/s . $P_{1x} = P_{2x} = 0$ . $0 = m_A v_{A2x} + m_B v_{B2x}$ and

$$v_{A2x} = -\frac{m_B}{m_A} v_{B2x} = -\left(\frac{4.20\times10^{-3}\text{ kg}}{72.5\text{ kg}}\right)(965\text{ m/s}) = -0.0559\text{ m/s}.$$

**(b)** $v_{B2x} = v_{B2}\cos\theta = (965\text{ m/s})\cos 56.0° = 540$ m/s . $v_{A2x} = -\left(\frac{4.20\times10^{-3}\text{ kg}}{72.5\text{ kg}}\right)(540\text{ m/s}) = -0.0313\text{ m/s}$ .

**EVALUATE:** The mass of the bullet is much less than the mass of the hunter, so the final mass of the hunter plus gun is still 72.5 kg, to three significant figures. Since the hunter has much larger mass, her final speed is much less than the speed of the bullet.

**8.22.** **IDENTIFY:** Assume the nucleus is initially at rest. $K = \frac{1}{2}mv^2$ .

**SET UP:** Let +*x* be to the right. $v_{A2x} = -v_A$ and $v_{B2x} = +v_B$ .

**EXECUTE:** **(a)** $P_{2x} = P_{1x} = 0$ gives $m_A v_{A2x} + m_B v_{B2x} = 0$ . $v_B = \left(\frac{m_A}{m_B}\right) v_A$ .

**(b)** $\frac{K_A}{K_B} = \frac{\frac{1}{2}m_A v_A^2}{\frac{1}{2}m_B v_B^2} = \frac{m_A v_A^2}{m_B\left(m_A v_A/m_B\right)^2} = \frac{m_B}{m_A}$ .

**EVALUATE:** The lighter fragment has the greater kinetic energy.

**8.23.** **IDENTIFY:** Apply conservation of momentum to the nucleus and its fragments. The initial momentum is zero. The $^{214}$Po nucleus has mass $214(1.67\times10^{-27}\text{ kg}) = 3.57\times10^{-25}$ kg , where $1.67\times10^{-27}$ kg is the mass of a nucleon (proton or neutron). $K = \frac{1}{2}mv^2$ .

**SET UP:** Let +*x* be the direction in which the alpha particle is emitted. The nucleus that is left after the decay has mass $m_n = 3.75\times10^{-25}\text{ kg} - m_\alpha = 3.57\times10^{-25}\text{ kg} - 6.65\times10^{-27}\text{ kg} = 3.50\times10^{-25}$ kg .

**EXECUTE:** $P_{2x} = P_{1x} = 0$ gives $m_\alpha v_\alpha + m_n v_n = 0$ . $v_n = \frac{m_\alpha}{m_n} v_\alpha$ . $v_\alpha = \sqrt{\frac{2K_\alpha}{m_\alpha}} = \sqrt{\frac{2(1.23\times10^{-12}\text{ J})}{6.65\times10^{-27}\text{ kg}}} = 1.92\times10^7$ m/s .

$$v_n = \left(\frac{6.65\times10^{-27}\text{ kg}}{3.50\times10^{-25}\text{ kg}}\right)(1.92\times10^7\text{ m/s}) = 3.65\times10^5\text{ m/s}.$$

**EVALUATE:** The recoil velocity of the more massive nucleus is much less than the speed of the emitted alpha particle.

**8.24.** **IDENTIFY** and **SET UP:** Let the +*x*-direction be horizontal, along the direction the rock is thrown. There is no net horizontal force, so $P_x$ is constant. Let object *A* be you and object *B* be the rock.

**EXECUTE:** $0 = -m_A v_A + m_B v_B \cos 35.0°$

$$v_A = \frac{m_B v_B \cos 35.0°}{m_A} = 2.11\text{ m/s}$$

**EVALUATE:** $P_y$ is not conserved because there is a net external force in the vertical direction; as you throw the rock the normal force exerted on you by the ice is larger than the total weight of the system.

**8.25.** **IDENTIFY:** Each horizontal component of momentum is conserved. $K = \frac{1}{2}mv^2$ .

**SET UP:** Let +*x* be the direction of Rebecca's initial velocity and let the +*y* axis make an angle of 36.9° with respect to the direction of her final velocity. $v_{D1x} = v_{D1y} = 0$ . $v_{R1x} = 13.0$ m/s ; $v_{R1y} = 0$ .

$v_{R2x} = (8.00\text{ m/s})\cos 53.1° = 4.80$ m/s ; $v_{R2y} = (8.00\text{ m/s})\sin 53.1° = 6.40$ m/s . Solve for $v_{D2x}$ and $v_{D2y}$ .

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$ gives $m_R v_{R1x} = m_R v_{R2x} + m_D v_{D2x}$ .

$$v_{D2x} = \frac{m_R(v_{R1x} - v_{R2x})}{m_D} = \frac{(45.0\text{ kg})(13.0\text{ m/s} - 4.80\text{ m/s})}{65.0\text{ kg}} = 5.68\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $0 = m_R v_{R2y} + m_D v_{D2y}$ . $v_{D2y} = -\frac{m_R}{m_D} v_{R2y} = -\left(\frac{45.0\text{ kg}}{65.0\text{ kg}}\right)(6.40\text{ m/s}) = -4.43\text{ m/s}$ .

The directions of $\vec{v}_{R1}$ , $\vec{v}_{R2}$ and $\vec{v}_{D2}$ are sketched in Figure 8.25. $\tan\theta = \left|\frac{v_{D2y}}{v_{D2x}}\right| = \frac{4.43\text{ m/s}}{5.68\text{ m/s}}$ and $\theta = 38.0°$ .

$$v_D = \sqrt{v_{D2x}^2 + v_{D2y}^2} = 7.20\text{ m/s}.$$

**(b)** $K_1 = \frac{1}{2}m_R v_{R1}^2 = \frac{1}{2}(45.0\text{ kg})(13.0\text{ m/s})^2 = 3.80\times 10^3\text{ J}$.

$K_2 = \frac{1}{2}m_R v_{R2}^2 + \frac{1}{2}m_D v_{D2}^2 = \frac{1}{2}(45.0\text{ kg})(8.00\text{ m/s})^2 + \frac{1}{2}(65.0\text{ kg})(7.20\text{ m/s})^2 = 3.12\times 10^3\text{ J}$.

$\Delta K = K_2 - K_1 = -680\text{ J}$.

**EVALUATE:** Each component of momentum is separately conserved. The kinetic energy of the system increases.

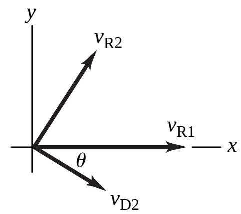


**Figure 8.25**

**8.26.** **IDENTIFY:** There is no net external force on the system of astronaut plus canister, so the momentum of the system is conserved.

**SET UP:** Let object *A* be the astronaut and object *B* be the canister. Assume the astronaut is initially at rest. After the collision she must be moving in the same direction as the canister. Let +*x* be the direction in which the canister is traveling initially, so $v_{A1x} = 0$, $v_{A2x} = +2.40\text{ m/s}$, $v_{B1x} = +3.50\text{ m/s}$, and $v_{B2x} = +1.20\text{ m/s}$. Solve for $m_B$.

**EXECUTE:** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$. $m_B = \dfrac{m_A(v_{A2x} - v_{A1x})}{v_{B1x} - v_{B2x}} = \dfrac{(78.4\text{ kg})(2.40\text{ m/s} - 0)}{3.50\text{ m/s} - 1.20\text{ m/s}} = 81.8\text{ kg}$.

**EVALUATE:** She must exert a force on the canister in the $-x$ direction to reduce its velocity component in the +*x* direction. By Newton's third law, the canister exerts a force on her that is in the +*x* direction and she gains velocity in that direction.

**8.27.** **IDENTIFY:** The horizontal component of the momentum of the system of the rain and freight car is conserved.

**SET UP:** Let +*x* be the direction the car is moving initially. Before it lands in the car the rain has no momentum along the *x* axis.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$ says $(24{,}000\text{ kg})(4.00\text{ m/s}) = (27{,}000\text{ kg})v_{2x}$ and $v_{2x} = 3.56\text{ m/s}$.

**(b)** After it lands in the car the water must gain horizontal momentum, so the car loses horizontal momentum.

**EVALUATE:** The vertical component of the momentum is not conserved, because of the vertical external force exerted by the track.

**8.28.** **IDENTIFY:** The *x* and *y* components of the momentum of the system of the two asteroids are separately conserved.

**SET UP:** The before and after diagrams are given in Figure 8.28 and the choice of coordinates is indicated. Each asteroid has mass *m*.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$ gives $mv_{A1} = mv_{A2}\cos 30.0° + mv_{B2}\cos 45.0°$. $40.0\text{ m/s} = 0.866v_{A2} + 0.707v_{B2}$ and $0.707v_{B2} = 40.0\text{ m/s} - 0.866v_{A2}$.

$$P_{2y} = P_{2y} \text{ gives } 0 = mv_{A2}\sin 30.0° - mv_{B2}\sin 45.0° \text{ and } 0.500v_{A2} = 0.707v_{B2}.$$

Combining these two equations gives $0.500v_{A2} = 40.0\text{ m/s} - 0.866v_{A2}$ and $v_{A2} = 29.3\text{ m/s}$. Then

$$v_{B2} = \left(\frac{0.500}{0.707}\right)(29.3\text{ m/s}) = 20.7\text{ m/s}.$$

**(b)** $K_1 = \frac{1}{2}mv_{A1}^2$. $K_2 = \frac{1}{2}mv_{A2}^2 + \frac{1}{2}mv_{B2}^2$. $\dfrac{K_2}{K_1} = \dfrac{v_{A2}^2 + v_{B2}^2}{v_{A1}^2} = \dfrac{(29.3\text{ m/s})^2 + (20.7\text{ m/s})^2}{(40.0\text{ m/s})^2} = 0.804$.

$$\frac{\Delta K}{K_1} = \frac{K_2 - K_1}{K_1} = \frac{K_2}{K_1} - 1 = -0.196.$$

19.6% of the original kinetic energy is dissipated during the collision.

**EVALUATE:** We could use any directions we wish for the *x* and *y* coordinate directions, but the particular choice we have made is especially convenient.

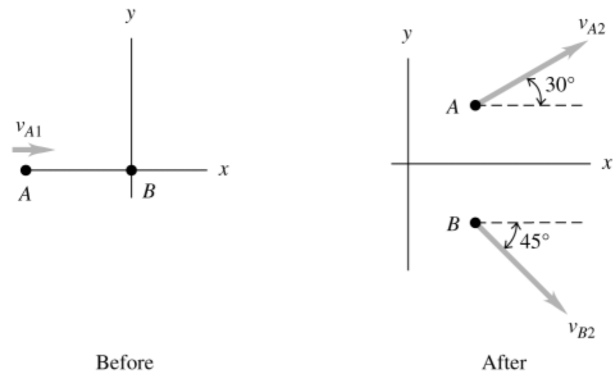


**Figure 8.28**

**8.29.** **IDENTIFY:** Since drag effects are neglected there is no net external force on the system of two fish and the momentum of the system is conserved. The mechanical energy equals the kinetic energy, which is $K = \frac{1}{2}mv^2$ for each object.

**SET UP:** Let object $A$ be the 15.0 kg fish and $B$ be the 4.50 kg fish. Let $+x$ be the direction the large fish is moving initially, so $v_{A1x} = 1.10$ m/s and $v_{B1x} = 0$. After the collision the two objects are combined and move with velocity $\vec{v}_2$. Solve for $v_{2x}$.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(15.0\text{ kg})(1.10\text{ m/s}) + 0}{15.0\text{ kg} + 4.50\text{ kg}} = 0.846\text{ m/s}.$$

**(b)** $K_1 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(15.0\text{ kg})(1.10\text{ m/s})^2 = 9.08\text{ J}$. $K_2 = \frac{1}{2}(m_A + m_B)v_2^2 = \frac{1}{2}(19.5\text{ kg})(0.846\text{ m/s})^2 = 6.98\text{ J}$.

$\Delta K = K_2 - K_1 = -2.10\text{ J}$. 2.10 J of mechanical energy is dissipated.

**EVALUATE:** The total kinetic energy always decreases in a collision where the two objects become combined.

**8.30.** **IDENTIFY:** There is no net external force on the system of the two otters and the momentum of the system is conserved. The mechanical energy equals the kinetic energy, which is $K = \frac{1}{2}mv^2$ for each object.

**SET UP:** Let $A$ be the 7.50 kg otter and $B$ be the 5.75 kg otter. After the collision their combined velocity is $\vec{v}_2$. Let $+x$ be to the right, so $v_{A1x} = -5.00$ m/s and $v_{B1x} = +6.00$ m/s. Solve for $v_{2x}$.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(7.50\text{ kg})(-5.00\text{ m/s}) + (5.75)(+6.00\text{ m/s})}{7.50\text{ kg} + 5.75\text{ kg}} = -0.226\text{ m/s}.$$

**(b)** $K_1 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(7.50\text{ kg})(5.00\text{ m/s})^2 + \frac{1}{2}(5.75\text{ kg})(6.00\text{ m/s})^2 = 197.2\text{ J}$.

$K_2 = \frac{1}{2}(m_A + m_B)v_2^2 = \frac{1}{2}(13.25\text{ kg})(0.226\text{ m/s})^2 = 0.338\text{ J}$.

$\Delta K = K_2 - K_1 = -197\text{ J}$. 197 J of mechanical energy is dissipated.

**EVALUATE:** The total kinetic energy always decreases in a collision where the two objects become combined.

**8.31.** **IDENTIFY:** Treat the comet and probe as an isolated system for which momentum is conserved.

**SET UP:** In part (a) let object $A$ be the probe and object $B$ be the comet. Let $-x$ be the direction the probe is traveling just before the collision. After the collision the combined object moves with speed $v_2$. The change in velocity is $\Delta v = v_{2x} - v_{B1x}$. In part (a) the impact speed of 37,000 km/h is the speed of the probe relative to the comet just before impact: $v_{A1x} - v_{B1x} = -37{,}000$ km/h. In part (b) let object $A$ be the comet and object $B$ be the earth. Let $-x$ be the direction the comet is traveling just before the collision. The impact speed is 40,000 km/h, so $v_{A1x} - v_{B1x} = -40{,}000$ km/h.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$. $v_{2x} = \dfrac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B}$.

$$\Delta v = v_{2x} - v_{B1x} = \left(\frac{m_A}{m_A + m_B}\right)v_{A1x} + \left(\frac{m_B - m_A - m_B}{m_A + m_B}\right)v_{B1x} = \left(\frac{m_A}{m_A + m_B}\right)(v_{A1x} - v_{B1x}).$$

$$\Delta v = \left(\frac{372\text{ kg}}{372\text{ kg} + 0.10\times10^{14}\text{ kg}}\right)(-37{,}000\text{ km/h}) = -1.4\times10^{-6}\text{ km/h}.$$

The speed of the comet decreased by $1.4\times10^{-6}$ km/h. This change is not noticeable.

**(b)** $\Delta v = \left(\dfrac{0.10\times 10^{14}\ \text{kg}}{0.10\times 10^{14}\ \text{kg} + 5.97\times 10^{24}\ \text{kg}}\right)(-40{,}000\ \text{km/h}) = -6.7\times 10^{-8}\ \text{km/h}$. The speed of the earth would change by $6.7\times 10^{-8}$ km/h. This change is not noticeable.

**EVALUATE:** $v_{A1x} - v_{B1x}$ is the velocity of the projectile (probe or comet) relative to the target (comet or earth). The expression for $\Delta v$ can be derived directly by applying momentum conservation in coordinates in which the target is initially at rest.

**8.32. IDENTIFY:** The forces the two vehicles exert on each other during the collision are much larger than the horizontal forces exerted by the road, and it is a good approximation to assume momentum conservation.

**SET UP:** Let $+x$ be eastward. After the collision two vehicles move with a common velocity $\vec{v}_2$.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$ gives $m_{\text{SC}}v_{\text{SC}x} + m_{\text{T}}v_{\text{T}x} = (m_{\text{SC}} + m_{\text{T}})v_{2x}$.

$$v_{2x} = \frac{m_{\text{SC}}v_{\text{SC}x} + m_{\text{T}}v_{\text{T}x}}{m_{\text{SC}} + m_{\text{T}}} = \frac{(1050\ \text{kg})(-15.0\ \text{m/s}) + (6320\ \text{kg})(+10.0\ \text{m/s})}{1050\ \text{kg} + 6320\ \text{kg}} = 6.44\ \text{m/s}.$$

The final velocity is 6.44 m/s, eastward.

**(b)** $P_{1x} = P_{2x} = 0$ gives $m_{\text{SC}}v_{\text{SC}x} + m_{\text{T}}v_{\text{T}x} = 0$. $v_{\text{T}x} = -\left(\dfrac{m_{\text{SC}}}{m_{\text{T}}}\right)v_{\text{SC}x} = -\left(\dfrac{1050\ \text{kg}}{6320\ \text{kg}}\right)(-15.0\ \text{m/s}) = 2.50\ \text{m/s}$. The truck would need to have initial speed 2.50 m/s.

**(c)** part (a): $\Delta K = \frac{1}{2}(7370\ \text{kg})(6.44\ \text{m/s})^2 - \frac{1}{2}(1050\ \text{kg})(15.0\ \text{m/s})^2 - \frac{1}{2}(6320\ \text{kg})(10.0\ \text{m/s})^2 = -2.81\times 10^5\ \text{J}$

part (b): $\Delta K = 0 - \frac{1}{2}(1050\ \text{kg})(15.0\ \text{m/s})^2 - \frac{1}{2}(6320\ \text{kg})(2.50\ \text{m/s})^2 = -1.38\times 10^5\ \text{J}$. The change in kinetic energy has the greater magnitude in part (a).

**EVALUATE:** In part (a) the eastward momentum of the truck has a greater magnitude than the westward momentum of the car and the wreckage moves eastward after the collision. In part (b) the two vehicles have equal magnitudes of momentum, the total momentum of the system is zero, and the wreckage is at rest after the collision.

**8.33. IDENTIFY:** The forces the two players exert on each other during the collision are much larger than the horizontal forces exerted by the slippery ground and it is a good approximation to assume momentum conservation. Each component of momentum is separately conserved.

**SET UP:** Let $+x$ be east and $+y$ be north. After the collision the two players have velocity $\vec{v}_2$. Let the linebacker be object $A$ and the halfback be object $B$, so $v_{A1x} = 0$, $v_{A1y} = 8.8$ m/s, $v_{B1x} = 7.2$ m/s and $v_{B1y} = 0$. Solve for $v_{2x}$ and $v_{2y}$.

**EXECUTE:** $P_{1x} = P_{2x}$ gives $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(85\ \text{kg})(7.2\ \text{m/s})}{110\ \text{kg} + 85\ \text{kg}} = 3.14\ \text{m/s}.$$

$P_{1y} = P_{2y}$ gives $m_A v_{A1y} + m_B v_{B1y} = (m_A + m_B)v_{2y}$.

$$v_{2y} = \frac{m_A v_{A1y} + m_B v_{B1y}}{m_A + m_B} = \frac{(110\ \text{kg})(8.8\ \text{m/s})}{110\ \text{kg} + 85\ \text{kg}} = 4.96\ \text{m/s}.$$

$v = \sqrt{v_{2x}^2 + v_{2y}^2} = 5.9$ m/s.

$$\tan\theta = \frac{v_{2y}}{v_{2x}} = \frac{4.96\ \text{m/s}}{3.14\ \text{m/s}}\ \text{and}\ \theta = 58^\circ.$$

The players move with a speed of 5.9 m/s and in a direction $58^\circ$ north of east.

**EVALUATE:** Each component of momentum is separately conserved.

**8.34. IDENTIFY:** There is no net external force on the system of the two skaters and the momentum of the system is conserved.

**SET UP:** Let object $A$ be the skater with mass 70.0 kg and object $B$ be the skater with mass 65.0 kg. Let $+x$ be to the right, so $v_{A1x} = +2.00$ m/s and $v_{B1x} = -2.50$ m/s. After the collision the two objects are combined and move with velocity $\vec{v}_2$. Solve for $v_{2x}$.

**EXECUTE:** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(70.0\ \text{kg})(2.00\ \text{m/s}) + (65.0)(-2.50\ \text{m/s})}{70.0\ \text{kg} + 65.0\ \text{kg}} = -0.167\ \text{m/s}.$$

The two skaters move to the left at 0.167 m/s.

**EVALUATE:** There is a large decrease in kinetic energy.

**8.35.** **IDENTIFY:** Neglect external forces during the collision. Then the momentum of the system of the two cars is conserved.

**SET UP:** $m_S = 1200$ kg , $m_L = 3000$ kg . The small car has velocity $v_S$ and the large car has velocity $v_L$ .

**EXECUTE:** **(a)** The total momentum of the system is conserved, so the momentum lost by one car equals the momentum gained by the other car. They have the same magnitude of change in momentum. Since $\vec{p} = m\vec{v}$ and $\Delta\vec{p}$ is the same, the car with the smaller mass has a greater change in velocity.

$$m_S \Delta v_S = m_L \Delta v_L \text{ and } \Delta v_S = \left(\frac{m_L}{m_S}\right)\Delta v_L = \left(\frac{3000 \text{ kg}}{1200 \text{ kg}}\right)\Delta v = 2.50\Delta v .$$

**(b)** The acceleration of the small car is greater, since it has a greater change in velocity during the collision. The large acceleration means a large force on the occupants of the small car and they would sustain greater injuries.

**EVALUATE:** Each car exerts the same magnitude of force on the other car but the force on the compact has a greater effect on its velocity since its mass is less.

**8.36.** **IDENTIFY:** The collision forces are large so gravity can be neglected during the collision. Therefore, the horizontal and vertical components of the momentum of the system of the two birds are conserved.

**SET UP:** The system before and after the collision is sketched in Figure 8.36. Use the coordinates shown.

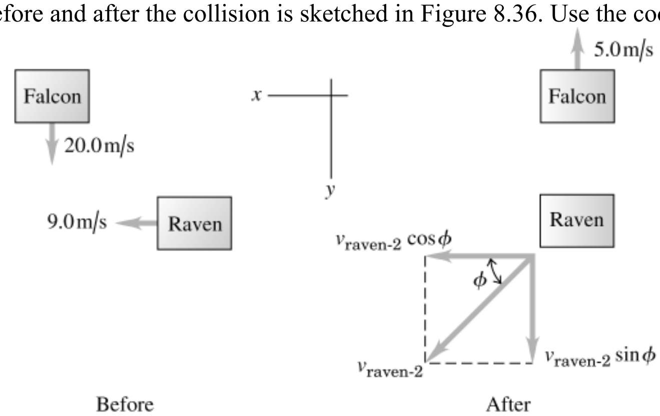


**Figure 8.36**

**EXECUTE:** There is no external force on the system so $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$ .

$P_{1x} = P_{2x}$ gives $(1.5 \text{ kg})(9.0 \text{ m/s}) = (1.5 \text{ kg})v_{\text{raven-2}}\cos\phi$ and $v_{\text{raven-2}}\cos\phi = 9.0$ m/s .

$P_{1y} = P_{2y}$ gives $(0.600 \text{ kg})(20.0 \text{ m/s}) = (0.600 \text{ kg})(-5.0 \text{ m/s}) + (1.5 \text{ kg})v_{\text{raven-2}}\sin\phi$ and $v_{\text{raven-2}}\sin\phi = 10.0$ m/s .

Combining these two equations gives $\tan\phi = \dfrac{10.0 \text{ m/s}}{9.0 \text{ m/s}}$ and $\phi = 48°$ .

**EVALUATE:** Due to its large initial speed the lighter falcon was able to produce a large change in the raven's direction of motion.

**8.37.** **IDENTIFY:** Since friction forces from the road are ignored, the *x* and *y* components of momentum are conserved.

**SET UP:** Let object *A* be the subcompact and object *B* be the truck. After the collision the two objects move together with velocity $\vec{v}_2$ . Use the *x* and *y* coordinates given in the problem. $v_{A1y} = v_{B1y} = 0$ .

$v_{2x} = (16.0 \text{ m/s})\sin 24.0° = 6.5$ m/s ; $v_{2y} = (16.0 \text{ m/s})\cos 24.0° = 14.6$ m/s .

**EXECUTE:** $P_{1x} = P_{2x}$ gives $m_A v_{A1x} = (m_A + m_B)v_{2x}$ .

$$v_{A1x} = \left(\frac{m_A + m_B}{m_A}\right)v_{2x} = \left(\frac{950 \text{ kg} + 1900 \text{ kg}}{950 \text{ kg}}\right)(6.5 \text{ m/s}) = 19.5 \text{ m/s} .$$

$P_{1y} = P_{2y}$ gives $m_A v_{B1y} = (m_A + m_B)v_{2y}$ .

$$v_{B1y} = \left(\frac{m_A + m_B}{m_A}\right)v_{2y} = \left(\frac{950 \text{ kg} + 1900 \text{ kg}}{1900 \text{ kg}}\right)(14.6 \text{ m/s}) = 21.9 \text{ m/s} .$$

Before the collision the subcompact car has speed 19.5 m/s and the truck has speed 21.9 m/s.

**EVALUATE:** Each component of momentum is independently conserved.

**8.38.** **IDENTIFY:** Apply conservation of momentum to the collision. Apply conservation of energy to the motion of the block after the collision.

**SET UP:** Conservation of momentum applied to the collision between the bullet and the block: Let object $A$ be the bullet and object $B$ be the block. Let $v_A$ be the speed of the bullet before the collision and let $V$ be the speed of the block with the bullet inside just after the collision.

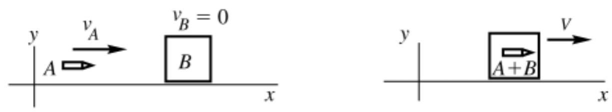


**Figure 8.38a**

$P_x$ is constant gives $m_A v_A = (m_A + m_B)V$.

Conservation of energy applied to the motion of the block after the collision:

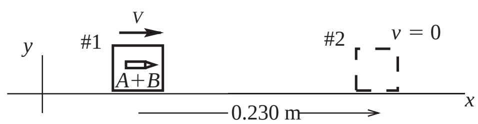


**Figure 8.38b**

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

**EXECUTE:** Work is done by friction so $W_{\text{other}} = W_f = (f_k \cos\phi)s = -f_k s = -\mu_k mgs$

$U_1 = U_2 = 0$ (no work done by gravity)

$K_1 = \frac{1}{2}mV^2$; $K_2 = 0$ (block has come to rest)

Thus $\frac{1}{2}mV^2 - \mu_k mgs = 0$

$V = \sqrt{2\mu_k gs} = \sqrt{2(0.20)(9.80\text{ m/s}^2)(0.230\text{ m})} = 0.9495\text{ m/s}$

Use this in the conservation of momentum equation

$$v_A = \left(\frac{m_A + m_B}{m_A}\right)V = \left(\frac{5.00\times10^{-3}\text{ kg} + 1.20\text{ kg}}{5.00\times10^{-3}\text{ kg}}\right)(0.9495\text{ m/s}) = 229\text{ m/s}$$

**EVALUATE:** When we apply conservation of momentum to the collision we are ignoring the impulse of the friction force exerted by the surface during the collision. This is reasonable since this force is much smaller than the forces the bullet and block exert on each other during the collision. This force does work as the block moves after the collision, and takes away all the kinetic energy.

**8.39.** **IDENTIFY:** Apply conservation of momentum to the collision and conservation of energy to the motion after the collision. After the collision the kinetic energy of the combined object is converted to gravitational potential energy.

**SET UP:** Immediately after the collision the combined object has speed $V$. Let $h$ be the vertical height through which the pendulum rises.

**EXECUTE:** **(a)** Conservation of momentum applied to the collision gives

$(12.0\times10^{-3}\text{ kg})(380\text{ m/s}) = (6.00\text{ kg} + 12.0\times10^{-3}\text{ kg})V$ and $V = 0.758\text{ m/s}$.

Conservation of energy applied to the motion after the collision gives $\frac{1}{2}m_{\text{tot}}V^2 = m_{\text{tot}}gh$ and

$$h = \frac{V^2}{2g} = \frac{(0.758\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 0.0293\text{ m} = 2.93\text{ cm}.$$

**(b)** $K = \frac{1}{2}m_b v_b^2 = \frac{1}{2}(12.0\times10^{-3}\text{ kg})(380\text{ m/s})^2 = 866\text{ J}$.

**(c)** $K = \frac{1}{2}m_{\text{tot}}V^2 = \frac{1}{2}(6.00\text{ kg} + 12.0\times10^{-3}\text{ kg})(0.758\text{ m/s})^2 = 1.73\text{ J}$.

**EVALUATE:** Most of the initial kinetic energy of the bullet is dissipated in the collision.

**8.40.** **IDENTIFY:** Each component of horizontal momentum is conserved.

**SET UP:** Let $+x$ be east and $+y$ be north. $v_{S1y} = v_{A1x} = 0$. $v_{S2x} = (6.00\text{ m/s})\cos37.0° = 4.79\text{ m/s}$, $v_{S2y} = (6.00\text{ m/s})\sin37.0° = 3.61\text{ m/s}$, $v_{A2x} = (9.00\text{ m/s})\cos23.0° = 8.28\text{ m/s}$ and $v_{A2y} = -(9.00\text{ m/s})\sin23.0° = -3.52\text{ m/s}$.

**EXECUTE:** $P_{1x} = P_{2x}$ gives $m_S v_{S1x} = m_S v_{S2x} + m_A v_{A2x}$.

$$v_{S1x} = \frac{m_S v_{S2x} + m_A v_{A2x}}{m_S} = \frac{(80.0\text{ kg})(4.79\text{ m/s}) + (50.0\text{ kg})(8.28\text{ m/s})}{80.0\text{ kg}} = 9.97\text{ m/s}.$$

Sam's speed before the collision was 9.97 m/s.

$P_{1y} = P_{2y}$ gives $m_A v_{A1y} = m_S v_{S2y} + m_A v_{A2y}$.

$$v_{A1y} = \frac{m_S v_{S2y} + m_A v_{A2y}}{m_S} = \frac{(80.0\text{ kg})(3.61\text{ m/s}) + (50.0\text{ kg})(-3.52\text{ m/s})}{50.0\text{ kg}} = 2.26\text{ m/s}.$$

Abigail's speed before the collision was 2.26 m/s.

(b) $\Delta K = \frac{1}{2}(80.0\,\text{kg})(6.00\,\text{m/s})^2 + \frac{1}{2}(50.0\,\text{kg})(9.00\,\text{m/s})^2 - \frac{1}{2}(80.0\,\text{kg})(9.97\,\text{m/s})^2 - \frac{1}{2}(50.0\,\text{kg})(2.26\,\text{m/s})^2$. $\Delta K = -639$ J.

**EVALUATE:** The total momentum is conserved because there is no net external horizontal force. The kinetic energy decreases because the forces between the objects do negative work during the collision.

**8.41.** **IDENTIFY:** When the spring is compressed the maximum amount the two blocks aren't moving relative to each other and have the same velocity $\vec{V}$ relative to the surface. Apply conservation of momentum to find $V$ and conservation of energy to find the energy stored in the spring. Since the collision is elastic, Eqs. 8.24 and 8.25 give the final velocity of each block after the collision.

**SET UP:** Let $+x$ be the direction of the initial motion of $A$.

**EXECUTE:** **(a)** Momentum conservation gives $(2.00\text{ kg})(2.00\text{ m/s}) = (12.0\text{ kg})V$ and $V = 0.333$ m/s. Both blocks are moving at 0.333 m/s, in the direction of the initial motion of block $A$. Conservation of energy says the initial kinetic energy of $A$ equals the total kinetic energy at maximum compression plus the potential energy $U_b$ stored in the bumpers: $\frac{1}{2}(2.00\text{ kg})(2.00\text{ m/s})^2 = U_b + \frac{1}{2}(12.0\text{ kg})(0.333\text{ m/s})^2$ and $U_b = 3.33$ J.

**(b)** $v_{A2x} = \left(\frac{m_A - m_B}{m_A + m_B}\right)v_{A1x} = \left(\frac{2.00\text{ kg} - 10.0\text{ kg}}{12.0\text{ kg}}\right)(2.00\text{ m/s}) = -1.33$ m/s. Block $A$ is moving in the $-x$ direction at 1.33 m/s.

$v_{B2x} = \left(\frac{2m_A}{m_A + m_B}\right)v_{A1x} = \frac{2(2.00\text{ kg})}{12.0\text{ kg}}(2.00\text{ m/s}) = +0.667$ m/s. Block $B$ is moving in the $+x$ direction at 0.667 m/s.

**EVALUATE:** When the spring is compressed the maximum amount the system must still be moving in order to conserve momentum.

**8.42.** **IDENTIFY:** No net external horizontal force so $P_x$ is conserved. Elastic collision so $K_1 = K_2$ and can use Eq. 8.27.

**SET UP:**

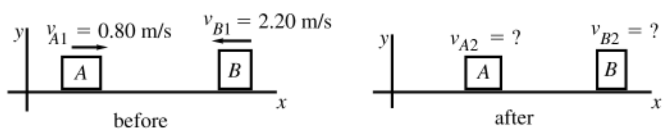


**Figure 8.42**

**EXECUTE:** From conservation of $x$-component of momentum:

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

$$m_A v_{A1} - m_B v_{B1} = m_A v_{A2x} + m_B v_{B2x}$$

$$(0.150\text{ kg})(0.80\text{ m/s}) - (0.300\text{ kg})(2.20\text{ m/s}) = (0.150\text{ kg})v_{A2x} + (0.300\text{ kg})v_{B2x}$$

$$-3.60\text{ m/s} = v_{A2x} + 2v_{B2x}$$

From the relative velocity equation for an elastic collision Eq. 8.27:

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x}) = -(-2.20\text{ m/s} - 0.80\text{ m/s}) = +3.00\text{ m/s}$$

$$3.00\text{ m/s} = -v_{A2x} + v_{B2x}$$

Adding the two equations gives $-0.60\text{ m/s} = 3v_{B2x}$ and $v_{B2x} = -0.20$ m/s. Then $v_{A2x} = v_{B2x} - 3.00\text{ m/s} = -3.20$ m/s. The 0.150 kg glider ($A$) is moving to the left at 3.20 m/s and the 0.300 kg glider ($B$) is moving to the left at 0.20 m/s.

**EVALUATE:** We can use our $v_{A2x}$ and $v_{B2x}$ to show that $P_x$ is constant and $K_1 = K_2$

**8.43.** **IDENTIFY:** Since the collision is elastic, both momentum conservation and Eq. 8.27 apply.

**SET UP:** Let object $A$ be the 30.0 kg marble and let object $B$ be the 10.0 g marble. Let $+x$ be to the right.

**EXECUTE:** **(a)** Conservation of momentum gives

$(0.0300\text{ kg})(0.200\text{ m/s}) + (0.0100\text{ kg})(-0.400\text{ m/s}) = (0.0300\text{ kg})v_{A2x} + (0.0100\text{ kg})v_{B2x}$.

$3v_{A2x} + v_{B2x} = 0.200$ m/s. Eq. 8.27 says $v_{B2x} - v_{A2x} = -(-0.400\text{ m/s} - 0.200\text{ m/s}) = +0.600$ m/s. Solving this pair of equations gives $v_{A2x} = -0.100$ m/s and $v_{B2x} = +0.500$ m/s. The 30.0 g marble is moving to the left at 0.100 m/s and the 10.0 g marble is moving to the right at 0.500 m/s.

**(b)** For marble $A$, $\Delta P_{Ax} = m_A v_{A2x} - m_A v_{A1x} = (0.0300\text{ kg})(-0.100\text{ m/s} - 0.200\text{ m/s}) = -0.00900\text{ kg}\cdot\text{m/s}$.

For marble $B$, $\Delta P_{Bx} = m_B v_{B2x} - m_B v_{B1x} = (0.0100\text{ kg})(0.500\text{ m/s} - [-0.400\text{ m/s}]) = +0.00900\text{ kg}\cdot\text{m/s}$.

The changes in momentum have the same magnitude and opposite sign.

**(c)** For marble $A$, $\Delta K_A = \frac{1}{2}m_A v_{A2}^2 - \frac{1}{2}m_A v_{A1}^2 = \frac{1}{2}(0.0300\text{ kg})([0.100\text{ m/s}]^2 - [0.200\text{ m/s}]^2) = -4.5\times10^{-4}\text{ J}$.

For marble $B$, $\Delta K_B = \frac{1}{2}m_B v_{B2}^2 - \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(0.0100\text{ kg})([0.500\text{ m/s}]^2 - [0.400\text{ m/s}]^2) = +4.5\times10^{-4}\text{ J}$.

The changes in kinetic energy have the same magnitude and opposite sign.

**EVALUATE:** The results of parts **(b)** and **(c)** show that momentum and kinetic energy are conserved in the collision.

**8.44.** **IDENTIFY** and **SET UP:** Without rounding, the calculation in Example 8.12 gives $v_{B2} = \sqrt{20}\text{ m/s}$.

**EXECUTE:** The two equations in Example 8.12 for $\alpha$ and $\beta$ are

$$(0.500\text{ kg})(4.00\text{ m/s}) = (0.500\text{ kg})(2.00\text{ m/s})(\cos\alpha) + (0.300\text{ kg})(\sqrt{20}\text{ m/s})(\cos\beta)\ \text{ Eq. 1}$$

and

$$0 = (0.500\text{ kg})(2.00\text{ m/s})(\sin\alpha) - (0.300\text{ kg})(\sqrt{20}\text{ m/s})\sin\beta\ \text{ Eq. 2.}$$

Dividing each equation by $(0.500\text{ kg})(1.00\text{ m/s})$ gives

$$4.00 = 2.00\cos\alpha + 0.6\sqrt{20}\cos\beta\ \text{ Eq. 3}$$

and

$$0 = 2.00\sin\alpha - 0.6\sqrt{20}\sin\beta\ \text{ Eq. 4.}$$

Eq. 3 gives $\cos\beta = \dfrac{4.00 - 2.00\cos\alpha}{0.6\sqrt{20}}$ and $\cos^2\beta = 2.222 - 2.222\cos\alpha + 0.5556\cos^2\alpha$.

Eq. 4 gives $\sin\beta = 0.7454\sin\alpha$ and $\sin^2\beta = 0.5556\sin^2\alpha = 0.5556 - 0.5556\cos^2\alpha$.

Adding the two equations and using $\sin^2\beta + \cos^2\beta = 1$ gives $1 = 2.778 - 2.222\cos\alpha$ and $\cos\alpha = 0.8002$. $\alpha = 36.9°$. Then $\sin\beta = 0.7454\sin\alpha$ gives $\beta = 26.6°$.

**EVALUATE:** For these values of $\alpha$ and $\beta$, the $x$ component of momentum, the $y$ component of momentum and the kinetic energy are all conserved in the collision.

**8.45.** **IDENTIFY:** Eqs. 8.24 and 8.25 apply, with object $A$ being the neutron.

**SET UP:** Let $+x$ be the direction of the initial momentum of the neutron. The mass of a neutron is $m_n = 1.0\text{ u}$.

**EXECUTE:** **(a)** $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x} = \dfrac{1.0\text{ u} - 2.0\text{ u}}{1.0\text{ u} + 2.0\text{ u}}v_{A1x} = -v_{A1x}/3.0$. The speed of the neutron after the collision is one-third its initial speed.

**(b)** $K_2 = \frac{1}{2}m_n v_n^2 = \frac{1}{2}m_n(v_{A1}/3.0)^2 = \dfrac{1}{9.0}K_1$.

**(c)** After $n$ collisions, $v_{A2} = \left(\dfrac{1}{3.0}\right)^n v_{A1}$. $\left(\dfrac{1}{3.0}\right)^n = \dfrac{1}{59{,}000}$, so $3.0^n = 59{,}000$. $n\log 3.0 = \log 59{,}000$ and $n = 10$.

**EVALUATE:** Since the collision is elastic, in each collision the kinetic energy lost by the neutron equals the kinetic energy gained by the deuteron.

**8.46.** **IDENTIFY:** Elastic collision. Solve for mass and speed of target nucleus.

**SET UP:** (a) Let $A$ be the proton and $B$ be the target nucleus. The collision is elastic, all velocities lie along a line, and $B$ is at rest before the collision. Hence the results of Eqs. 8.24 and 8.25 apply.

**EXECUTE:** Eq. 8.24: $m_B(v_x + v_{Ax}) = m_A(v_x - v_{Ax})$, where $v_x$ is the velocity component of $A$ before the collision and $v_{Ax}$ is the velocity component of $A$ after the collision. Here, $v_x = 1.50\times10^7\text{ m/s}$ (take direction of incident beam to be positive) and $v_{Ax} = -1.20\times10^7\text{ m/s}$ (negative since traveling in direction opposite to incident beam).

$$m_B = m_A\left(\frac{v_x - v_{Ax}}{v_x + v_{Ax}}\right) = m\left(\frac{1.50\times10^7\text{ m/s} + 1.20\times10^7\text{ m/s}}{1.50\times10^7\text{ m/s} - 1.20\times10^7\text{ m/s}}\right) = m\left(\frac{2.70}{0.30}\right) = 9.00m.$$

**(b)** Eq. 8.25: $v_{Bx} = \left(\dfrac{2m_A}{m_A + m_B}\right)v = \left(\dfrac{2m}{m + 9.00m}\right)(1.50\times10^7\text{ m/s}) = 3.00\times10^6\text{ m/s.}$

**EVALUATE:** Can use our calculated $v_{Bx}$ and $m_B$ to show that $P_x$ is constant and that $K_1 = K_2$.

**8.47. IDENTIFY:** Apply Eq. 8.28.

**SET UP:** $m_A = 0.300\text{ kg}$, $m_B = 0.400\text{ kg}$, $m_C = 0.200\text{ kg}$.

**EXECUTE:** $x_{\text{cm}} = \dfrac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C}$.

$$x_{\text{cm}} = \frac{(0.300\text{ kg})(0.200\text{ m}) + (0.400\text{ kg})(0.100\text{ m}) + (0.200\text{ kg})(-0.300\text{ m})}{0.300\text{ kg} + 0.400\text{ kg} + 0.200\text{ kg}} = 0.0444\text{ m}.$$

$$y_{\text{cm}} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C}.$$

$$y_{\text{cm}} = \frac{(0.300\text{ kg})(0.300\text{ m}) + (0.400\text{ kg})(-0.400\text{ m}) + (0.200\text{ kg})(0.600\text{ m})}{0.300\text{ kg} + 0.400\text{ kg} + 0.200\text{ kg}} = 0.0556\text{ m}.$$

**EVALUATE:** There is mass at both positive and negative $x$ and at positive and negative $y$ and therefore the center of mass is close to the origin.

**8.48. IDENTIFY:** Calculate $x_{\text{cm}}$.

**SET UP:** Apply Eq. 8.28 with the sun as mass 1 and Jupiter as mass 2. Take the origin at the sun and let Jupiter lie on the positive $x$-axis.

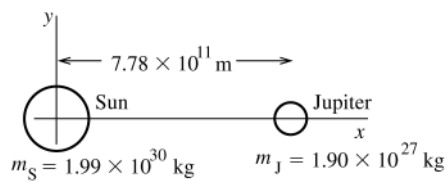


**Figure 8.48**

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

**EXECUTE:** $x_1 = 0$ and $x_2 = 7.78 \times 10^{11}\text{ m}$

$$x_{\text{cm}} = \frac{(1.90\times 10^{27}\text{ kg})(7.78\times 10^{11}\text{ m})}{1.99\times 10^{30}\text{ kg} + 1.90\times 10^{27}\text{ kg}} = 7.42\times 10^{8}\text{ m}$$

The center of mass is $7.42\times 10^8\text{ m}$ from the center of the sun and is on the line connecting the centers of the sun and Jupiter. The sun's radius is $6.96\times 10^8\text{ m}$ so the center of mass lies just outside the sun.

**EVALUATE:** The mass of the sun is much greater than the mass of Jupiter so the center of mass is much closer to the sun. For each object we have considered all the mass as being at the center of mass (geometrical center) of the object.

**8.49. IDENTIFY:** The location of the center of mass is given by Eq. 8.48. The mass can be expressed in terms of the diameter. Each object can be replaced by a point mass at its center.

**SET UP:** Use coordinates with the origin at the center of Pluto and the $+x$ direction toward Charon, so $x_{\text{P}} = 0$ $x_{\text{C}} = 19{,}700\text{ km}$. $m = \rho V = \rho \frac{4}{3}\pi r^3 = \frac{1}{6}\rho\pi d^3$.

**EXECUTE:** $x_{\text{cm}} = \dfrac{m_{\text{P}} x_{\text{P}} + m_{\text{C}} x_{\text{C}}}{m_{\text{P}} + m_{\text{C}}} = \left(\dfrac{m_{\text{C}}}{m_{\text{P}} + m_{\text{C}}}\right) x_{\text{C}} = \left(\dfrac{\frac{1}{6}\rho\pi d_{\text{C}}^3}{\frac{1}{6}\rho\pi d_{\text{P}}^3 + \frac{1}{6}\rho\pi d_{\text{C}}^3}\right) x_{\text{C}} = \left(\dfrac{d_{\text{C}}^3}{d_{\text{P}}^3 + d_{\text{C}}^3}\right) x_{\text{C}}$.

$$x_{\text{cm}} = \left(\frac{[1250\text{ km}]^3}{[2370\text{ km}]^3 + [1250\text{ km}]^3}\right)(19{,}700\text{ km}) = 2.52\times 10^3\text{ km}.$$

The center of mass of the system is $2.52\times 10^3\text{ km}$ from the center of Pluto.

**EVALUATE:** The center of mass is closer to Pluto because Pluto has more mass than Charon.

**8.50. IDENTIFY:** Apply Eqs. 8.28, 8.30 and 8.32. There is only one component of position and velocity.

**SET UP:** $m_A = 1200\text{ kg}$, $m_B = 1800\text{ kg}$. $M = m_A + m_B = 3000\text{ kg}$. Let $+x$ be to the right and let the origin be at the center of mass of the station wagon.

**EXECUTE:** **(a)** $x_{\text{cm}} = \dfrac{m_A x_A + m_B x_B}{m_A + m_B} = \dfrac{0 + (1800\text{ kg})(40.0\text{ m})}{1200\text{ kg} + 1800\text{ kg}} = 24.0\text{ m}.$

The center of mass is between the two cars, 24.0 m to the right of the station wagon and 16.0 m behind the lead car.

**(b)** $P_x = m_A v_{A1} + m_B v_{B1} = (1200 \text{ kg})(12.0 \text{ m/s}) + (1800 \text{ kg})(20.0 \text{ m/s}) = 5.04\times 10^4 \text{ kg}\cdot\text{m/s}.$

**(c)** $v_{\text{cm},x} = \dfrac{m_A v_{A,x} + m_B v_{B,x}}{m_A + m_B} = \dfrac{(1200 \text{ kg})(12.0 \text{ m/s}) + (1800 \text{ kg})(20.0 \text{ m/s})}{1200 \text{ kg} + 1800 \text{ kg}} = 16.8 \text{ m/s}.$

**(d)** $P_x = Mv_{\text{cm-}x} = (3000 \text{ kg})(16.8 \text{ m/s}) = 5.04\times 10^4 \text{ kg}\cdot\text{m/s}$, the same as in part (b).

**EVALUATE:** The total momentum can be calculated either as the vector sum of the momenta of the individual objects in the system, or as the total mass of the system times the velocity of the center of mass.

**8.51.** **IDENTIFY:** Use Eq. 8.28 to find the *x* and *y* coordinates of the center of mass of the machine part for each configuration of the part. In calculating the center of mass of the machine part, each uniform bar can be represented by a point mass at its geometrical center.

**SET UP:** Use coordinates with the axis at the hinge and the +*x* and +*y* axes along the horizontal and vertical bars in the figure in the problem. Let $(x_\text{i}, y_\text{i})$ and $(x_\text{f}, y_\text{f})$ be the coordinates of the bar before and after the vertical bar is pivoted. Let object 1 be the horizontal bar, object 2 be the vertical bar and 3 be the ball.

**EXECUTE:** $x_\text{i} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \dfrac{(4.00 \text{ kg})(0.750 \text{ m}) + 0 + 0}{4.00 \text{ kg} + 3.00 \text{ kg} + 2.00 \text{ kg}} = 0.333 \text{ m}$.

$$y_\text{i} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{0 + (3.00 \text{ kg})(0.900 \text{ m}) + (2.00 \text{ kg})(1.80 \text{ m})}{9.00 \text{ kg}} = 0.700 \text{ m}.$$

$$x_\text{f} = \frac{(4.00 \text{ kg})(0.750 \text{ m}) + (3.00 \text{ kg})(-0.900 \text{ m}) + (2.00 \text{ kg})(-1.80 \text{ m})}{9.00 \text{ kg}} = -0.366 \text{ m}.$$

$y_\text{f} = 0$. $x_\text{f} - x_\text{i} = -0.700 \text{ m}$ and $y_\text{f} - y_\text{i} = -0.700 \text{ m}$. The center of mass moves 0.700 m to the right and 0.700 m upward.

**EVALUATE:** The vertical bar moves upward and to the right so it is sensible for the center of mass of the machine part to move in these directions.

**8.52.** **(a) IDENTIFY:** Use Eq. 8.28.

**SET UP:** The target variable is $m_1$.

**EXECUTE:** $x_\text{cm} = 2.0 \text{ m}$, $y_\text{cm} = 0$

$$x_\text{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{m_1(0) + (0.10 \text{ kg})(8.0 \text{ m})}{m_1 + (0.10 \text{ kg})} = \frac{0.80 \text{ kg}\cdot\text{m}}{m_1 + 0.10 \text{ kg}}.$$

$$x_\text{cm} = 2.0 \text{ m gives } 2.0 \text{ m} = \frac{0.80 \text{ kg}\cdot\text{m}}{m_1 + 0.10 \text{ kg}}.$$

$$m_1 + 0.10 \text{ kg} = \frac{0.80 \text{ kg}\cdot\text{m}}{2.0 \text{ m}} = 0.40 \text{ kg}.$$

$$m_1 = 0.30 \text{ kg}.$$

**EVALUATE:** The cm is closer to $m_1$ so its mass is larger then $m_2$.

**(b) IDENTIFY:** Use Eq. 8.32 to calculate $\vec{\boldsymbol{P}}$.

**SET UP:** $\vec{\boldsymbol{v}}_\text{cm} = (5.0 \text{ m/s})\hat{\boldsymbol{j}}$.

$$\vec{\boldsymbol{P}} = M\vec{\boldsymbol{v}}_\text{cm} = (0.10 \text{ kg} + 0.30 \text{ kg})(5.0 \text{ m/s})\hat{\boldsymbol{i}} = (2.0 \text{ kg}\cdot\text{m/s})\hat{\boldsymbol{i}}.$$

**(c) IDENTIFY:** Use Eq. 8.31.

**SET UP:** $\vec{\boldsymbol{v}}_\text{cm} = \dfrac{m_1\vec{\boldsymbol{v}}_1 + m_2\vec{\boldsymbol{v}}_2}{m_1 + m_2}$. The target variable is $\vec{\boldsymbol{v}}_1$. Particle 2 at rest says $v_2 = 0$.

**EXECUTE:** $\vec{\boldsymbol{v}}_1 = \left(\dfrac{m_1 + m_2}{m_1}\right)\vec{\boldsymbol{v}}_\text{cm} = \left(\dfrac{0.30 \text{ kg} + 0.10 \text{ kg}}{0.30 \text{ kg}}\right)(5.00 \text{ m/s})\hat{\boldsymbol{i}} = (6.7 \text{ m/s})\hat{\boldsymbol{i}}.$

**EVALUATE:** Using the result of part (c) we can calculate $\vec{\boldsymbol{p}}_1$ and $\vec{\boldsymbol{p}}_2$ and show that $\vec{\boldsymbol{P}}$ as calculated in part (b) does equal $\vec{\boldsymbol{p}}_1 + \vec{\boldsymbol{p}}_2$.

**8.53.** **IDENTIFY:** There is no net external force on the system of James, Ramon and the rope and the momentum of the system is conserved and the velocity of its center of mass is constant. Initially there is no motion, and the velocity of the center of mass remains zero after Ramon has started to move.

**SET UP:** Let $+x$ be in the direction of Ramon's motion. Ramon has mass $m_R = 60.0$ kg and James has mass $m_J = 90.0$ kg.

**EXECUTE:** $v_{\text{cm-}x} = \dfrac{m_R v_{Rx} + m_J v_{Jx}}{m_R + m_J} = 0$.

$$v_{Jx} = -\left(\frac{m_R}{m_J}\right) v_{Rx} = -\left(\frac{60.0 \text{ kg}}{90.0 \text{ kg}}\right)(0.700 \text{ m/s}) = -0.47 \text{ m/s}$$ . James' speed is 0.47 m/s.

**EVALUATE:** As they move, the two men have momenta that are equal in magnitude and opposite in direction, and the total momentum of the system is zero. Also, Example 8.14 shows that Ramon moves farther than James in the same time interval. This is consistent with Ramon having a greater speed.

**8.54.** **(a) IDENTIFY and SET UP:** Apply Eq. 8.28 and solve for $m_1$ and $m_2$.

**EXECUTE:** $y_{\text{cm}} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$

$$m_1 + m_2 = \frac{m_1 y_1 + m_2 y_2}{y_{\text{cm}}} = \frac{m_1(0) + (0.50 \text{ kg})(6.0 \text{ m})}{2.4 \text{ m}} = 1.25 \text{ kg} \text{ and } m_1 = 0.75 \text{ kg}.$$

**EVALUATE:** $y_{\text{cm}}$ is closer to $m_1$ since $m_1 > m_2$.

**(b) IDENTIFY and SET UP:** Apply $\vec{a} = d\vec{v}/dt$ for the cm motion.

**EXECUTE:** $\vec{a}_{\text{cm}} = \dfrac{d\vec{v}_{\text{cm}}}{dt} = (1.5 \text{ m/s}^3) t\hat{i}$.

**(c) IDENTIFY and SET UP:** Apply Eq. 8.34.

**EXECUTE:** $\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}} = (1.25 \text{ kg})(1.5 \text{ m/s}^3) t\hat{i}$.

At $t = 3.0$ s, $\sum \vec{F}_{\text{ext}} = (1.25 \text{ kg})(1.5 \text{ m/s}^3)(3.0 \text{ s})\hat{i} = (5.6 \text{ N})\hat{i}$.

**EVALUATE:** $v_{\text{cm-}x}$ is positive and increasing so $a_{\text{cm}-x}$ is positive and $\vec{F}_{\text{ext}}$ is in the $+x$-direction. There is no motion and no force component in the $y$-direction.

**8.55.** **IDENTIFY:** Apply $\sum \vec{F} = \dfrac{d\vec{P}}{dt}$ to the airplane.

**SET UP:** $\dfrac{d}{dt}(t^n) = nt^{n-1}$. $1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$.

**EXECUTE:** $\dfrac{d\vec{P}}{dt} = [-(1.50 \text{ kg} \cdot \text{m/s}^3)t]\vec{i} + (0.25 \text{ kg} \cdot \text{m/s}^2)\vec{j}$. $F_x = -(1.50 \text{ N/s})t$, $F_y = 0.25$ N, $F_z = 0$.

**EVALUATE:** There is no momentum or change in momentum in the $z$ direction and there is no force component in this direction.

**8.56.** **IDENTIFY:** Use Eq. 8.38, applied to a finite time interval.

**SET UP:** $v_{\text{ex}} = 1600$ m/s

**EXECUTE:** **(a)** $F = -v_{\text{ex}} \dfrac{\Delta m}{\Delta t} = -(1600 \text{ m/s}) \dfrac{-0.0500 \text{ kg}}{1.00 \text{ s}} = +80.0 \text{ N}$.

**(b)** The absence of atmosphere would not prevent the rocket from operating. The rocket could be steered by ejecting the gas in a direction with a component perpendicular to the rocket's velocity and braked by ejecting it in a direction parallel (as opposed to antiparallel) to the rocket's velocity.

**EVALUATE:** The thrust depends on the speed of the ejected gas relative to the rocket and on the mass of gas ejected per second.

**8.57.** **IDENTIFY:** $a = -\dfrac{v_{\text{ex}}}{m}\dfrac{dm}{dt}$. Assume that $dm/dt$ is constant over the 5.0 s interval, since $m$ doesn't change much during that interval. The thrust is $F = -v_{\text{ex}} \dfrac{dm}{dt}$.

**SET UP:** Take $m$ to have the constant value $110 \text{ kg} + 70 \text{ kg} = 180 \text{ kg}$. $dm/dt$ is negative since the mass of the MMU decreases as gas is ejected.

**EXECUTE:** **(a)** $\dfrac{dm}{dt} = -\dfrac{m}{v_{\text{ex}}} a = -\left(\dfrac{180 \text{ kg}}{490 \text{ m/s}}\right)(0.029 \text{ m/s}^2) = -0.0106 \text{ kg/s}$. In 5.0 s the mass that is ejected is $(0.0106 \text{ kg/s})(5.0 \text{ s}) = 0.053 \text{ kg}$.

**(b)** $F = -v_{\text{ex}} \dfrac{dm}{dt} = -(490 \text{ m/s})(-0.0106 \text{ kg/s}) = 5.19 \text{ N}$.

**EVALUATE:** The mass change in the 5.0 s is a very small fraction of the total mass *m*, so it is accurate to take *m* to be constant.

**8.58.** **IDENTIFY** and **SET UP:** Apply Eq. 8.39: $a=-\frac{v_{ex}}{m}\frac{dm}{dt}$. Solve for $dm/dt$.

**EXECUTE:**

$$\frac{dm}{dt}=-\frac{ma}{v_{ex}}=-\frac{(6000\text{ kg})(25.0\text{ m/s}^2)}{2000\text{ m/s}}=-75.0\text{ kg/s}.$$

So in 1 s the rocket must eject 75.0 kg of gas.

**EVALUATE:** We have approximated $dm/dt$ by $\Delta m/\Delta t$. We have assumed that $25.0\text{ m/s}^2$ is the average acceleration for the first second.

**8.59.** **IDENTIFY:** Use Eq. 8.39, applied to a finite time interval. Solve for $v_{ex}$.

**SET UP:** $\frac{\Delta m}{\Delta t}=-\frac{m}{160}$.

**EXECUTE:** $a=-\frac{v_{ex}}{m}\frac{\Delta m}{\Delta t}$. $v_{ex}=-\frac{a}{\left(\frac{\Delta m}{\Delta t}/m\right)}=\frac{-15.0\text{ m/s}^2}{\left(-\frac{m}{160}\right)/m}=2.40\times10^3\text{ m/s}=2.40\text{ km/s}$

**EVALUATE:** The acceleration is proportional to the speed of the exhaust gas and to the rate at which mass is ejected.

**8.60.** **IDENTIFY** and **SET UP:** $(F_{av})\Delta t=J$ relates the impulse *J* to the average thrust $F_{av}$. Eq. 8.38 applied to a finite time interval gives $F_{av}=-v_{ex}\frac{\Delta m}{\Delta t}$. $v-v_0=v_{ex}\ln\left(\frac{m_0}{m}\right)$. The remaining mass *m* after 1.70 s is 0.0133 kg.

**EXECUTE:** **(a)** $F=\frac{J}{\Delta t}=\frac{10.0\text{ N}\cdot\text{s}}{1.70\text{ s}}=5.88\text{ N}$. $F_{av}/F_{max}=0.442$.

**(b)** $v_{ex}=-\frac{F_{av}\Delta t}{-0.0125\text{ kg}}=800\text{ m/s}$.

**(c)** $v_0=0$ and $v=v_{ex}\ln\left(\frac{m_0}{m}\right)=(800\text{ m/s})\ln\left(\frac{0.0258\text{ kg}}{0.0133\text{ kg}}\right)=530\text{ m/s}$.

**EVALUATE:** The acceleration of the rocket is not constant. It increases as the mass remaining decreases.

**8.61.** **IDENTIFY:** $v-v_0=v_{ex}\ln\left(\frac{m_0}{m}\right)$.

**SET UP:** $v_0=0$.

**EXECUTE:** $\ln\left(\frac{m_0}{m}\right)=\frac{v}{v_{ex}}=\frac{8.00\times10^3\text{ m/s}}{2100\text{ m/s}}=3.81$ and $\frac{m_0}{m}=e^{3.81}=45.2$.

**EVALUATE:** Note that the final speed of the rocket is greater than the relative speed of the exhaust gas.

**8.62.** **IDENTIFY** and **SET UP:** Use Eq. 8.40: $v-v_0=v_{ex}\ln(m_0/m)$.

$v_0=0$ ("fired from rest"), so $v/v_{ex}=\ln(m_0/m)$.

Thus $m_0/m=e^{v/v_{ex}}$, or $m/m_0=e^{-v/v_{ex}}$.

If *v* is the final speed then *m* is the mass left when all the fuel has been expended; $m/m_0$ is the fraction of the initial mass that is not fuel.

**(a) EXECUTE:** $v=1.00\times10^{-3}c=3.00\times10^5\text{ m/s}$ gives

$$m/m_0=e^{-(3.00\times10^5\text{ m/s})/(2000\text{ m/s})}=7.2\times10^{-66}.$$

**EVALUATE:** This is clearly not feasible, for so little of the initial mass to not be fuel.

**(b) EXECUTE:** $v=3000\text{ m/s}$ gives $m/m_0=e^{-(3000\text{ m/s})/(2000\text{ m/s})}=0.223$.

**EVALUATE:** 22.3% of the total initial mass not fuel, so 77.7% is fuel; this is possible.

**8.63.** **IDENTIFY:** Use the heights to find $v_{1y}$ and $v_{2y}$, the velocity of the ball just before and just after it strikes the slab. Then apply $J_y=F_y\Delta t=\Delta p_y$.

**SET UP:** Let $+y$ be downward.

**EXECUTE:** **(a)** $\frac{1}{2}mv^2 = mgh$ so $v = \pm\sqrt{2gh}$.

$v_{1y} = +\sqrt{2(9.80\text{ m/s}^2)(2.00\text{ m})} = 6.26\text{ m/s}$. $v_{2y} = -\sqrt{2(9.80\text{ m/s}^2)(1.60\text{ m})} = -5.60\text{ m/s}$.

$$J_y = \Delta p_y = m(v_{2y} - v_{1y}) = (40.0\times 10^{-3}\text{ kg})(-5.60\text{ m/s} - 6.26\text{ m/s}) = -0.474\text{ kg}\cdot\text{m/s}.$$

The impulse is $0.474\text{ kg}\cdot\text{m/s}$, upward.

**(b)** $F_y = \dfrac{J_y}{\Delta t} = \dfrac{-0.474\text{ kg}\cdot\text{m/s}}{2.00\times 10^{-3}\text{ s}} = -237\text{ N}$. The average force on the ball is 237 N, upward.

**EVALUATE:** The upward force on the ball changes the direction of its momentum.

**8.64.** **IDENTIFY:** Momentum is conserved in the explosion. At the highest point the velocity of the boulder is zero. Since one fragment moves horizontally the other fragment also moves horizontally. Use projectile motion to relate the initial horizontal velocity of each fragment to its horizontal displacement.

**SET UP:** Use coordinates where $+x$ is north. Since both fragments start at the same height with zero vertical component of velocity, the time in the air, $t$, is the same for both. Call the fragments $A$ and $B$, with $A$ being the one that lands to the north. Therefore, $m_B = 3m_A$.

**EXECUTE:** Apply $P_{1x} = P_{2x}$ to the collision: $0 = m_A v_{Ax} + m_B v_{Bx}$. $v_{Bx} = -\dfrac{m_A}{m_B} v_{Ax} = -v_{Ax}/3$. Apply projectile motion to the motion after the collision: $x - x_0 = v_{0x}t$. Since $t$ is the same, $\dfrac{(x-x_0)_A}{v_{Ax}} = \dfrac{(x-x_0)_B}{v_{Bx}}$ and

$(x-x_0)_B = \left(\dfrac{v_{Bx}}{v_{Ax}}\right)(x-x_0)_A = \left(\dfrac{-v_{Ax}/3}{v_{Ax}}\right)(x-x_0)_A = -(274\text{ m})/3 = -91.3\text{ m}$. The other fragment lands 91.3 m directly south of the point of explosion.

**EVALUATE:** The fragment that has three times the mass travels one-third as far.

**8.65.** **IDENTIFY:** The impulse, force and change in velocity are related by Eq. 8.9

**SET UP:** $m = w/g = 0.0571\text{ kg}$. Since the force is constant, $\vec{F} = \vec{F}_{\text{av}}$.

**EXECUTE:** **(a)** $J_x = F_x\Delta t = (-380\text{ N})(3.00\times 10^{-3}\text{ s}) = -1.14\text{ N}\cdot\text{s}$. $J_y = F_y\Delta t = (110\text{ N})(3.00\times 10^{-3}\text{ s}) = 0.330\text{ N}\cdot\text{s}$.

**(b)** $v_{2x} = \dfrac{J_x}{m} + v_{1x} = \dfrac{-1.14\text{ N}\cdot\text{s}}{0.0571\text{ kg}} + 20.0\text{ m/s} = 0.04\text{ m/s}$. $v_{2y} = \dfrac{J_y}{m} - v_{1y} = \dfrac{0.330\text{ N}\cdot\text{s}}{0.0571\text{ kg}} + (-4.0\text{ m/s}) = +1.8\text{ m/s}$.

**EVALUATE:** The change in velocity $\Delta\vec{v}$ is in the same direction as the force, so $\Delta\vec{v}$ has a negative $x$ component and a positive $y$ component.

**8.66.** **IDENTIFY:** The horizontal component of the momentum of the system of cars is conserved.

**SET UP:** Let $+x$ be the direction the cars are traveling. Each car has mass $m$. Let $v_1$ be the initial speed of the three cars. $v_2 = \frac{1}{5}v_1$. Let $N$ be the number of cars in the final collection.

**EXECUTE:** $P_{1x} = P_{2x}$. $(3m)v_1 = (Nm)v_2$. $N = \dfrac{3v_1}{v_2} = 3\dfrac{v_1}{v_1/5} = 15$.

**EVALUATE:** In the complete absence of friction or other external horizontal forces this process of adding cars and slowing down continues forever.

**8.67.** **IDENTIFY:** $P_x = p_{Ax} + p_{Bx}$ and $P_y = p_{Ay} + p_{By}$.

**SET UP:** Let object $A$ be the convertible and object $B$ be the SUV. Let $+x$ be west and $+y$ be south, $p_{Ax} = 0$ and $p_{By} = 0$.

**EXECUTE:** $P_x = (8000\text{ kg}\cdot\text{m/s})\sin 60.0° = 6928\text{ kg}\cdot\text{m/s}$, so $p_{Bx} = 6928\text{ kg}\cdot\text{m/s}$ and

$v_{Bx} = \dfrac{6928\text{ kg}\cdot\text{m/s}}{2000\text{ kg}} = 3.46\text{ m/s}$.

$P_y = (8000\text{ kg}\cdot\text{m/s})\cos 60.0° = 4000\text{ kg}\cdot\text{m/s}$, so $p_{Bx} = 4000\text{ kg}\cdot\text{m/s}$ and $v_{Ay} = \dfrac{4000\text{ kg}\cdot\text{m/s}}{1500\text{ kg}} = 2.67\text{ m/s}$.

The convertible has speed 2.67 m/s and the SUV has speed 3.46 m/s.

**EVALUATE:** Each component of the total momentum arises from a single vehicle.

**8.68.** **IDENTIFY:** The total momentum of the system is conserved and is equal to zero, since the pucks are released from rest.

**SET UP:** Each puck has the same mass $m$. Let $+x$ be east and $+y$ be north. Let object $A$ be the puck that moves west. All three pucks have the same speed $v$.

**EXECUTE:** $P_{1x} = P_{2x}$ gives $0 = -mv + mv_{Bx} + mv_{Cx}$ and $v = v_{Bx} + v_{Cx}$. $P_{1y} = P_{2y}$ gives $0 = mv_{By} + mv_{Cy}$ and $v_{By} = -v_{Cy}$. Since $v_B = v_C$ and the *y* components are equal in magnitude, the *x* components must also be equal: $v_{Bx} = v_{Cx}$ and $v = v_{Bx} + v_{Cx}$ says $v_{Bx} = v_{Cx} = v/2$. If $v_{By}$ is positive then $v_{Cy}$ is negative. The angle $\theta$ that puck *B* makes with the *x* axis is given by $\cos\theta = \frac{v/2}{v}$ and $\theta = 60°$. One puck moves in a direction $60°$ north of east and the other puck moves in a direction $60°$ south of east.

**EVALUATE:** Each component of momentum is separately conserved.

**8.69.** **IDENTIFY:** The *x* and *y* components of the momentum of the system are conserved.

**Set Up:** After the collision the combined object with mass $m_{\text{tot}} = 0.100$ kg moves with velocity $\vec{v}_2$. Solve for $v_{Cx}$ and $v_{Cy}$.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$ gives $m_A v_{Ax} + m_B v_{Bx} + m_C v_{Cx} = m_{\text{tot}} v_{2x}$.

$$v_{Cx} = -\frac{m_A v_{Ax} + m_B v_{Bx} - m_{\text{tot}} v_{2x}}{m_C}$$

$$v_{Cx} = -\frac{(0.020\text{ kg})(-1.50\text{ m/s}) + (0.030\text{ kg})(-0.50\text{ m/s})\cos 60° - (0.100\text{ kg})(0.50\text{ m/s})}{0.050\text{ kg}}.$$

$$v_{Cx} = 1.75\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $m_A v_{Ay} + m_B v_{By} + m_C v_{Cy} = m_{\text{tot}} v_{2y}$.

$$v_{Cy} = -\frac{m_A v_{Ay} + m_B v_{By} - m_{\text{tot}} v_{2y}}{m_C} = -\frac{(0.030\text{ kg})(-0.50\text{ m/s})\sin 60°}{0.050\text{ kg}} = +0.260\text{ m/s}.$$

**(b)** $v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2} = 1.77$ m/s. $\Delta K = K_2 - K_1$.

$\Delta K = \frac{1}{2}(0.100\text{ kg})(0.50\text{ m/s})^2 - [\frac{1}{2}(0.020\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}(0.030)(0.50\text{ m/s})^2 + \frac{1}{2}(0.050\text{ kg})(1.77\text{ m/s})^2]$

$\Delta K = -0.092$ J.

**EVALUATE:** Since there is no horizontal external force the vector momentum of the system is conserved. The forces the spheres exert on each other do negative work during the collision and this reduces the kinetic energy of the system.

**8.70.** **IDENTIFY:** Use a coordinate system attached to the ground. Take the *x*-axis to be east (along the tracks) and the *y*-axis to be north (parallel to the ground and perpendicular to the tracks). Then $P_x$ is conserved and $P_y$ is *not* conserved, due to the sideways force exerted by the tracks, the force that keeps the handcar on the tracks.

**(a) SET UP:** Let *A* be the 25.0 kg mass and *B* be the car (mass 175 kg). After the mass is thrown sideways relative to the car it still has the same eastward component of velocity, 5.00 m/s, as it had before it was thrown.

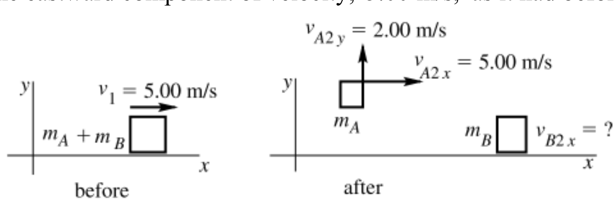


**Figure 8.70a**

$P_x$ is conserved so $(m_A + m_B)v_1 = m_A v_{A2x} + m_B v_{B2x}$

**EXECUTE:** $(200\text{ kg})(5.00\text{ m/s}) = (25.0\text{ kg})(5.00\text{ m/s}) + (175\text{ kg})v_{B2x}$.

$$v_{B2x} = \frac{1000\text{ kg}\cdot\text{m/s} - 125\text{ kg}\cdot\text{m/s}}{175\text{ kg}} = 5.00\text{ m/s}.$$

The final velocity of the car is 5.00 m/s, east (unchanged).

**EVALUATE:** The thrower exerts a force on the mass in the *y*-direction and by Newton's 3rd law the mass exerts an equal and opposite force in the $-y$-direction on the thrower and car.

**(b) SET UP:** We are applying $P_x$ = constant in coordinates attached to the ground, so we need the final velocity of *A* relative to the ground. Use the relative velocity addition equation. Then use $P_x$ = constant to find the final velocity of the car.

**EXECUTE:** $\vec{v}_{A/E} = \vec{v}_{A/B} + \vec{v}_{B/E}$

$v_{B/E} = +5.00$ m/s

$v_{A/B} = -5.00$ m/s (minus since the mass is moving west relative to the car). This gives $v_{A/E} = 0$; the mass is at rest relative to the earth after it is thrown backwards from the car.

As in part (a), $(m_A + m_B)v_1 = m_A v_{A2x} + m_B v_{B2x}$.

Now $v_{A2x} = 0$, so $(m_A + m_B)v_1 = m_B v_{B2x}$.

$$v_{B2x} = \left(\frac{m_A + m_B}{m_B}\right)v_1 = \left(\frac{200\text{ kg}}{175\text{ kg}}\right)(5.00\text{ m/s}) = 5.71\text{ m/s}.$$

The final velocity of the car is 5.71 m/s, east.

**EVALUATE:** The thrower exerts a force in the $-x$-direction so the mass exerts a force on him in the $+x$-direction and he and the car speed up.

**(c) SET UP:** Let *A* be the 25.0 kg mass and *B* be the car (mass $m_B = 200$ kg).

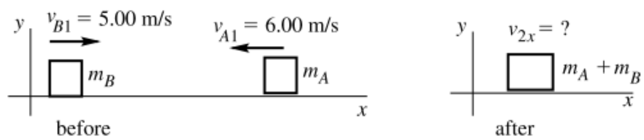


**Figure 8.70b**

$P_x$ is conserved so $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

**EXECUTE:** $-m_A v_{A1} + m_B v_{B1} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_B v_{B1} - m_A v_{A1}}{m_A + m_B} = \frac{(200\text{ kg})(5.00\text{ m/s}) - (25.0\text{ kg})(6.00\text{ m/s})}{200\text{ kg} + 25.0\text{ kg}} = 3.78\text{ m/s}.$$

The final velocity of the car is 3.78 m/s, east.

**EVALUATE:** The mass has negative $p_x$ so reduces the total $P_x$ of the system and the car slows down.

**8.71. IDENTIFY:** The horizontal component of the momentum of the sand plus railroad system is conserved.

**SET UP:** As the sand leaks out it retains its horizontal velocity of 15.0 m/s.

**EXECUTE:** The horizontal component of the momentum of the sand doesn't change when it leaks out so the speed of the railroad car doesn't change; it remains 15.0 m/s. In Exercise 8.27 the rain is falling vertically and initially has no horizontal component of momentum. Its momentum changes as it lands in the freight car. Therefore, in order to conserve the horizontal momentum of the system the freight car must slow down.

**EVALUATE:** The horizontal momentum of the sand does change when it strikes the ground, due to the force that is external to the system of sand plus railroad car.

**8.72. IDENTIFY:** Kinetic energy is $K = \frac{1}{2}mv^2$ and the magnitude of the momentum is $p = mv$. The force and the time *t* it acts are related to the change in momentum whereas the force and distance *d* it acts are related to the change in kinetic energy.

**SET UP:** Assume the net forces are constant and let the forces and the motion be along the *x* axis. The impulse-momentum theorem then says $Ft = \Delta p$ and the work-energy theorem says $Fd = \Delta K$.

**EXECUTE:** **(a)** $K_{\text{N}} = \frac{1}{2}(840\text{ kg})(9.0\text{ m/s})^2 = 3.40\times10^4\text{ J}$. $K_{\text{P}} = \frac{1}{2}(1620\text{ kg})(5.0\text{ m/s})^2 = 2.02\times10^4\text{ J}$. The Nash has the greater kinetic energy and $\frac{K_{\text{N}}}{K_{\text{P}}} = 1.68$.

**(b)** $p_{\text{N}} = (840\text{ kg})(9.0\text{ m/s}) = 7.56\times10^3\text{ kg}\cdot\text{m/s}$. $p_{\text{P}} = (1620\text{ kg})(5.0\text{ m/s}) = 8.10\times10^3\text{ kg}\cdot\text{m/s}$. The Packard has the greater magnitude of momentum and $\frac{p_{\text{N}}}{p_{\text{P}}} = 0.933$.

**(c)** Since the cars stop the magnitude of the change in momentum equals the initial momentum. Since $p_{\text{P}} > p_{\text{N}}$, $F_{\text{P}} > F_{\text{N}}$ and $\frac{F_{\text{N}}}{F_{\text{P}}} = \frac{p_{\text{N}}}{p_{\text{P}}} = 0.933$.

**(d)** Since the cars stop the magnitude of the change in kinetic energy equals the initial kinetic energy. Since $K_{\text{N}} > K_{\text{P}}$, $F_{\text{N}} > F_{\text{P}}$ and $\frac{F_{\text{N}}}{F_{\text{P}}} = \frac{K_{\text{N}}}{K_{\text{P}}} = 1.68$.

**EVALUATE:** If the stopping forces were the same, the Packard would have a larger stopping time but would travel a shorter distance while stopping. This consistent with it having a smaller initial speed.

**8.73. IDENTIFY:** Use the impulse-momentum theorem to relate the average force on the bullets to their rate of change in momentum. By Newton's third law, the average force the weapon exerts on the bullets is equal in magnitude and opposite in direction to the recoil force the bullets exert on the weapon.

**SET UP:** Consider a time interval of 1.00 minute. Let $+x$ be the direction of motion of the bullets and use coordinated fixed to the ground. The bullets start from rest.

**EXECUTE:** $F_{av}\Delta t = \Delta p$ gives $F_{av} = \dfrac{(1000)(7.45\times10^{-3}\text{ kg})(293\text{ m/s})}{60.0\text{ s}} = 36.4\text{ N}$. The recoil force is 36.4 N.

**EVALUATE:** The change in momentum for each bullet is small since the mass is small, but over 16 bullets are fired per second.

**8.74. IDENTIFY:** Find $k$ for the spring from the forces when the frame hangs at rest, use constant acceleration equations to find the speed of the putty just before it strikes the frame, apply conservation of momentum to the collision between the putty and the frame and then apply conservation of energy to the motion of the frame after the collision.

**SET UP:** Use the free-body diagram for the frame when it hangs at rest on the end of the spring to find the force constant $k$ of the spring. Let $s$ be the amount the spring is stretched.

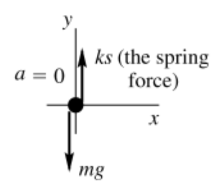


**Figure 8.74a**

**EXECUTE:** $\sum F_y = ma_y$.

$-mg + ks = 0$.

$$k = \frac{mg}{s} = \frac{(0.150\text{ kg})(9.80\text{ m/s}^2)}{0.050\text{ m}} = 29.4\text{ N/m}.$$

**SET UP:** Next find the speed of the putty when it reaches the frame. The putty falls with acceleration $a = g$, downward.

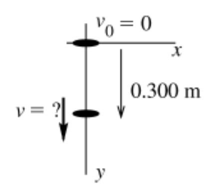


**Figure 8.74b**

$v_0 = 0$

$y - y_0 = 0.300\text{ m}$

$a = +9.80\text{ m/s}^2$

$v = ?$

$v^2 = v_0^2 + 2a(y - y_0)$

**EXECUTE:** $v = \sqrt{2a(y-y_0)} = \sqrt{2(9.80\text{ m/s}^2)(0.300\text{ m})} = 2.425\text{ m/s}$.

**SET UP:** Apply conservation of momentum to the collision between the putty ($A$) and the frame ($B$):

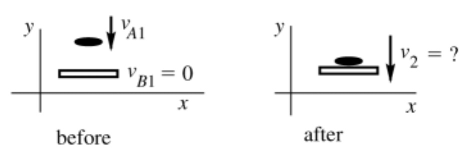


**Figure 8.74c**

$P_y$ is conserved, so $-m_A v_{A1} = -(m_A + m_B)v_2$.

**EXECUTE:** $v_2 = \left(\dfrac{m_A}{m_A + m_B}\right)v_{A1} = \left(\dfrac{0.200\text{ kg}}{0.350\text{ kg}}\right)(2.425\text{ m/s}) = 1.386\text{ m/s}$.

**SET UP:** Apply conservation of energy to the motion of the frame on the end of the spring after the collision. Let point 1 be just after the putty strikes and point 2 be when the frame has its maximum downward displacement. Let $d$ be the amount the frame moves downward.

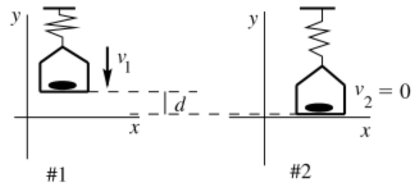


**Figure 8.74d**

When the frame is at position 1 the spring is stretched a distance $x_1 = 0.050$ m. When the frame is at position 2 the spring is stretched a distance $x_2 = 0.050\text{ m} + d$. Use coordinates with the $y$-direction upward and $y = 0$ at the lowest point reached by the frame, so that $y_1 = d$ and $y_2 = 0$. Work is done on the frame by gravity and by the spring force, so $W_{\text{other}} = 0$, and $U = U_{\text{el}} + U_{\text{gravity}}$.

**EXECUTE:** $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

$W_{\text{other}} = 0$.

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.350\text{ kg})(1.386\text{ m/s})^2 = 0.3362\text{ J}$.

$U_1 = U_{1,\text{el}} + U_{1,\text{grav}} = \frac{1}{2}kx_1^2 + mgy_1 = \frac{1}{2}(29.4\text{ N/m})(0.050\text{ m})^2 + (0.350\text{ kg})(9.80\text{ m/s}^2)d$.

$U_1 = 0.03675\text{ J} + (3.43\text{ N})d$.

$U_2 = U_{2,\text{el}} + U_{2,\text{grav}} = \frac{1}{2}kx_2^2 + mgy_2 = \frac{1}{2}(29.4\text{ N/m})(0.050\text{ m} + d)^2$.

$U_2 = 0.03675\text{ J} + (1.47\text{ N})d + (14.7\text{ N/m})d^2$.

Thus $0.3362\text{ J} + 0.03675\text{ J} + (3.43\text{ N})d = 0.03675\text{ J} + (1.47\text{ N})d + (14.7\text{ N/m})d^2$.

$(14.7\text{ N/m})d^2 - (1.96\text{ N})d - 0.3362\text{ J} = 0$.

$d = (1/29.4)\left[1.96 \pm \sqrt{(1.96)^2 - 4(14.7)(-0.3362)}\right]\text{ m} = 0.0667\text{ m} \pm 0.1653\text{ m}.$

The solution we want is a positive (downward) distance, so $d = 0.0667\text{ m} + 0.1653\text{ m} = 0.232\text{ m}$.

**EVALUATE:** The collision is inelastic and mechanical energy is lost. Thus the decrease in gravitational potential energy is *not* equal to the increase in potential energy stored in the spring.

**8.75. IDENTIFY:** Apply conservation of momentum to the collision and conservation of energy to the motion after the collision.

**SET UP:** Let $+x$ be to the right. The total mass is $m = m_{\text{bullet}} + m_{\text{block}} = 1.00\text{ kg}$. The spring has force constant $k = \dfrac{|F|}{|x|} = \dfrac{0.750\text{ N}}{0.250 \times 10^{-2}\text{ m}} = 300\text{ N/m}$. Let $V$ be the velocity of the block just after impact.

**EXECUTE:** **(a)** Conservation of energy for the motion after the collision gives $K_1 = U_{\text{el2}}$. $\frac{1}{2}mV^2 = \frac{1}{2}kx^2$ and

$$V = x\sqrt{\frac{k}{m}} = (0.150\text{ m})\sqrt{\frac{300\text{ N/m}}{1.00\text{ kg}}} = 2.60\text{ m/s}.$$

**(b)** Conservation of momentum applied to the collision gives $m_{\text{bullet}}v_1 = mV$.

$$v_1 = \frac{mV}{m_{\text{bullet}}} = \frac{(1.00\text{ kg})(2.60\text{ m/s})}{8.00 \times 10^{-3}\text{ kg}} = 325\text{ m/s}.$$

**EVALUATE:** The initial kinetic energy of the bullet is 422 J. The energy stored in the spring at maximum compression is 3.38 J. Most of the initial mechanical energy of the bullet is dissipated in the collision.

**8.76. IDENTIFY:** The horizontal components of momentum of the system of bullet plus stone are conserved. The collision is elastic if $K_1 = K_2$.

**SET UP:** Let $A$ be the bullet and $B$ be the stone.

**(a)**

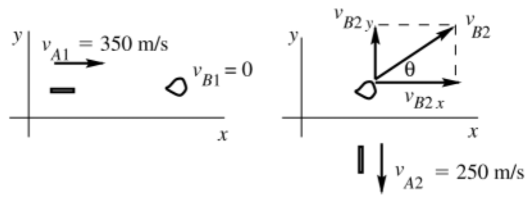


**Figure 8.76**

**EXECUTE:** $P_x$ is conserved so $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$.

$m_A v_{A1} = m_B v_{B2x}$.

$$v_{B2x} = \left(\frac{m_A}{m_B}\right) v_{A1} = \left(\frac{6.00\times10^{-3}\text{ kg}}{0.100\text{ kg}}\right)(350\text{ m/s}) = 21.0\text{ m/s}$$

$P_y$ is conserved so $m_A v_{A1y} + m_B v_{B1y} = m_A v_{A2y} + m_B v_{B2y}$.

$0 = -m_A v_{A2} + m_B v_{B2y}$.

$$v_{B2y} = \left(\frac{m_A}{m_B}\right) v_{A2} = \left(\frac{6.00\times10^{-3}\text{ kg}}{0.100\text{ kg}}\right)(250\text{ m/s}) = 15.0\text{ m/s}.$$

$$v_{B2} = \sqrt{v_{B2x}^2 + v_{B2y}^2} = \sqrt{(21.0\text{ m/s})^2 + (15.0\text{ m/s})^2} = 25.8\text{ m/s}.$$

$\tan\theta = \dfrac{v_{B2y}}{v_{B2x}} = \dfrac{15.0\text{ m/s}}{21.0\text{ m/s}} = 0.7143$; $\theta = 35.5°$ (defined in the sketch).

**(b)** To answer this question compare $K_1$ and $K_2$ for the system:

$$K_1 = \tfrac{1}{2} m_A v_{A1}^2 + \tfrac{1}{2} m_B v_{B1}^2 = \tfrac{1}{2}(6.00\times10^{-3}\text{ kg})(350\text{ m/s})^2 = 368\text{ J}.$$

$$K_2 = \tfrac{1}{2} m_A v_{A2}^2 + \tfrac{1}{2} m_B v_{B2}^2 = \tfrac{1}{2}(6.00\times10^{-3}\text{ kg})(250\text{ m/s})^2 + \tfrac{1}{2}(0.100\text{ kg})(25.8\text{ m/s})^2 = 221\text{ J}.$$

$$\Delta K = K_2 - K_1 = 221\text{ J} - 368\text{ J} = -147\text{ J}.$$

**EVALUATE:** The kinetic energy of the system decreases by 147 J as a result of the collision; the collision is *not* elastic. Momentum is conserved because $\sum F_{\text{ext},x} = 0$ and $\sum F_{\text{ext},y} = 0$. But there are internal forces between the bullet and the stone. These forces do negative work that reduces $K$.

**8.77.** **IDENTIFY:** Apply conservation of momentum to the collision between the two people. Apply conservation of energy to the motion of the stuntman before the collision and to the entwined people after the collision.

**SET UP:** For the motion of the stuntman, $y_1 - y_2 = 5.0\text{ m}$. Let $v_S$ be the magnitude of his horizontal velocity just before the collision. Let $V$ be the speed of the entwined people just after the collision. Let $d$ be the distance they slide along the floor.

**EXECUTE:** **(a)** Motion before the collision: $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$ and $\tfrac{1}{2} m v_S^2 = mg(y_1 - y_2)$.

$v_S = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.80\text{ m/s}^2)(5.0\text{ m})} = 9.90\text{ m/s}$.

Collision: $m_S v_S = m_{\text{tot}} V$. $V = \dfrac{m_S}{m_{\text{tot}}} v_S = \left(\dfrac{80.0\text{ kg}}{150.0\text{ kg}}\right)(9.90\text{ m/s}) = 5.28\text{ m/s}$.

**(b)** Motion after the collision: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $\tfrac{1}{2} m_{\text{tot}} V^2 - \mu_k m_{\text{tot}} g d = 0$.

$d = \dfrac{V^2}{2\mu_k g} = \dfrac{(5.28\text{ m/s})^2}{2(0.250)(9.80\text{ m/s}^2)} = 5.7\text{ m}$.

**EVALUATE:** Mechanical energy is dissipated in the inelastic collision, so the kinetic energy just after the collision is less than the initial potential energy of the stuntman.

**8.78.** **IDENTIFY:** Apply conservation of energy to the motion before and after the collision and apply conservation of momentum to the collision.

**SET UP:** Let $v$ be the speed of the mass released at the rim just before it strikes the second mass. Let each object have mass $m$.

**EXECUTE:** Conservation of energy says $\tfrac{1}{2} m v^2 = mgR$; $v = \sqrt{2gR}$.

**SET UP:** This is speed $v_1$ for the collision. Let $v_2$ be the speed of the combined object just after the collision.

**EXECUTE:** Conservation of momentum applied to the collision gives $mv_1 = 2mv_2$ so $v_2 = v_1/2 = \sqrt{gR/2}$

**SET UP:** Apply conservation of energy to the motion of the combined object after the collision. Let $y_3$ be the final height above the bottom of the bowl.

**EXECUTE:** $\frac{1}{2}(2m)v_2^2 = (2m)gy_3$.

$y_3 = \dfrac{v_2^2}{2g} = \dfrac{1}{2g}\left(\dfrac{gR}{2}\right) = R/4$.

**EVALUATE:** Mechanical energy is lost in the collision, so the final gravitational potential energy is less than the initial gravitational potential energy.

**8.79.** **IDENTIFY:** Eqs. 8.24 and 8.25 give the outcome of the elastic collision. Apply conservation of energy to the motion of the block after the collision.

**SET UP:** Object *B* is the block, initially at rest. If *L* is the length of the wire and $\theta$ is the angle it makes with the vertical, the height of the block is $y = L(1-\cos\theta)$. Initially, $y_1 = 0$.

**EXECUTE:** Eq. 8.25 gives $v_B = \left(\dfrac{2m_A}{m_A + m_B}\right)v_A = \left(\dfrac{2M}{M+3M}\right)(5.00\text{ m/s}) = 2.50\text{ m/s}$. Conservation of energy gives

$\frac{1}{2}m_B v_B^2 = m_B gL(1-\cos\theta)$. $\cos\theta = 1 - \dfrac{v_B^2}{2gL} = 1 - \dfrac{(2.50\text{ m/s})^2}{2(9.80\text{ m/s}^2)(0.500\text{ m})} = 0.362$ and $\theta = 68.8°$.

**EVALUATE:** Only a portion of the initial kinetic energy of the ball is transferred to the block in the collision.

**8.80.** **IDENTIFY:** Apply conservation of energy to the motion before and after the collision. Apply conservation of momentum to the collision.

**SET UP:** First consider the motion after the collision. The combined object has mass $m_{tot} = 25.0$ kg. Apply $\sum \vec{F} = m\vec{a}$ to the object at the top of the circular loop, where the object has speed $v_3$. The acceleration is $a_{rad} = v_3^2/R$, downward.

**EXECUTE:** $T + mg = m\dfrac{v_3^2}{R}$.

The minimum speed $v_3$ for the object not to fall out of the circle is given by setting $T = 0$. This gives $v_3 = \sqrt{Rg}$, where $R = 3.50$ m.

**SET UP:** Next, use conservation of energy with point 2 at the bottom of the loop and point 3 at the top of the loop. Take $y = 0$ at point 2. Only gravity does work, so $K_2 + U_2 = K_3 + U_3$

**EXECUTE:** $\frac{1}{2}m_{tot}v_2^2 = \frac{1}{2}m_{tot}v_3^2 + m_{tot}g(2R)$.

Use $v_3 = \sqrt{Rg}$ and solve for $v_2$: $v_2 = \sqrt{5gR} = 13.1$ m/s.

**SET UP:** Now apply conservation of momentum to the collision between the dart and the sphere. Let $v_1$ be the speed of the dart before the collision.

**EXECUTE:** $(5.00\text{ kg})v_1 = (25.0\text{ kg})(13.1\text{ m/s})$.

$v_1 = 65.5$ m/s.

**EVALUATE:** The collision is inelastic and mechanical energy is removed from the system by the negative work done by the forces between the dart and the sphere.

**8.81.** **IDENTIFY:** Use Eq. 8.25 to find the speed of the hanging ball just after the collision. Apply $\sum \vec{F} = m\vec{a}$ to find the tension in the wire. After the collision the hanging ball moves in an arc of a circle with radius $R = 1.35$ m and acceleration $a_{rad} = v^2/R$.

**SET UP:** Let *A* be the 2.00 kg ball and *B* be the 8.00 kg ball. For applying $\sum \vec{F} = m\vec{a}$ to the hanging ball, let +*y* be upward, since $\vec{a}_{rad}$ is upward. The free-body force diagram for the 8.00 kg ball is given in Figure 8.81.

**EXECUTE:** $v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right)v_{A1x} = \left(\dfrac{2[2.00\text{kg}]}{2.00\text{ kg} + 8.00\text{ kg}}\right)(5.00\text{ m/s}) = 2.00\text{ m/s}$. Just after the collision the 8.00 kg ball has speed $v = 2.00$ m/s. Using the free-body diagram, $\sum F_y = ma_y$ gives $T - mg = ma_{rad}$.

$$T = m\left(g + \frac{v^2}{R}\right) = (8.00\text{ kg})\left(9.80\text{ m/s}^2 + \frac{[2.00\text{ m/s}]^2}{1.35\text{ m}}\right) = 102\text{ N}.$$

**EVALUATE:** The tension before the collision is the weight of the ball, 78.4 N. Just after the collision, when the ball has started to move, the tension is greater than this.

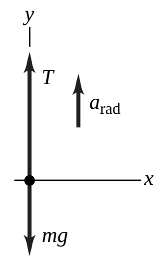


**Figure 8.81**

**8.82.** **IDENTIFY:** The impulse applied to the ball equals its change in momentum. The height of the ball and its speed are related by conservation of energy.

**SET UP:** Let $+y$ be upward.

**EXECUTE:** Applying conservation of energy to the motion of the ball from its height $h$ to the floor gives $\frac{1}{2}mv_1^2 = mgh$, where $v_1$ is its speed just before it hits the floor. Just before it hits, it is traveling downward, so the velocity of the ball just before it hits the floor is $v_{1y} = -\sqrt{2gh}$. Applying conservation of energy to the motion of the ball from just after it bounces off the floor with speed $v_2$ to its maximum height of $0.90h$ gives $\frac{1}{2}mv_2^2 = mg(0.90h)$. It is moving upward, so $v_{2y} = +\sqrt{2g(0.90h)}$. The impulse applied to the ball is $J_y = p_{2y} - p_{1y} = m(v_{2y} - v_{1y}) =$ $m\sqrt{2g(0.90h)} + m\sqrt{2gh} = 2.76m\sqrt{gh}$. The floor exerts an upward impulse of $2.76m\sqrt{gh}$ to the ball.

**EVALUATE:** The impulse increases when $m$ increases and when $h$ increases. The ball does not return to its initial height because some mechanical energy is dissipated during the collision with the floor.

**8.83.** **IDENTIFY:** Apply conservation of momentum to the collision between the bullet and the block and apply conservation of energy to the motion of the block after the collision.

**(a) SET UP:** Collision between the bullet and the block: Let object $A$ be the bullet and object $B$ be the block. Apply momentum conservation to find the speed $v_{B2}$ of the block just after the collision.

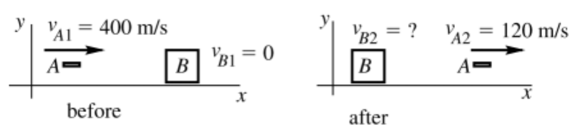


**Figure 8.83a**

**EXECUTE:** $P_x$ is conserved so $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$.

$m_A v_{A1} = m_A v_{A2} + m_B v_{B2x}$.

$$v_{B2x} = \frac{m_A(v_{A1} - v_{A2})}{m_B} = \frac{4.00\times10^{-3}\text{ kg}(400\text{ m/s} - 120\text{ m/s})}{0.800\text{ kg}} = 1.40\text{ m/s}.$$

**SET UP:** Motion of the block after the collision.

Let point 1 in the motion be just after the collision, where the block has the speed 1.40 m/s calculated above, and let point 2 be where the block has come to rest.

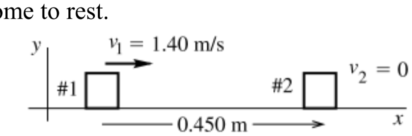


**Figure 8.83b**

$K_1 + U_1 + W_{other} = K_2 + U_2$.

**EXECUTE:** Work is done on the block by friction, so $W_{other} = W_f$.

$W_{other} = W_f = (f_k \cos\phi)s = -f_k s = -\mu_k mgs$, where $s = 0.450$ m

$U_1 = 0, \quad U_2 = 0$

$K_1 = \frac{1}{2}mv_1^2, \quad K_2 = 0$ (block has come to rest)

Thus $\frac{1}{2}mv_1^2 - \mu_k mgs = 0$.

$$\mu_k = \frac{v_1^2}{2gs} = \frac{(1.40\text{ m/s})^2}{2(9.80\text{ m/s}^2)(0.450\text{ m})} = 0.222.$$

**(b)** For the bullet,

$$K_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}\left(4.00\times10^{-3}\text{ kg}\right)\left(400\text{ m/s}\right)^2 = 320\text{ J}.$$

$$K_2 = \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}\left(4.00\times10^{-3}\text{ kg}\right)\left(120\text{ m/s}\right)^2 = 28.8\text{ J}.$$

$$\Delta K = K_2 - K_1 = 28.8\text{ J} - 320\text{ J} = -291\text{ J}.$$

The kinetic energy of the bullet decreases by 291 J.

**(c)** Immediately after the collision the speed of the block is $1.40\text{ m/s}$ so its kinetic energy is $K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(0.800\text{ kg}\right)\left(1.40\text{ m/s}\right)^2 = 0.784\text{ J}.$

**EVALUATE:** The collision is highly inelastic. The bullet loses 291 J of kinetic energy but only 0.784 J is gained by the block. But momentum is conserved in the collision. All the momentum lost by the bullet is gained by the block.

**8.84.** **IDENTIFY:** Apply conservation of momentum to the collision and conservation of energy to the motion of the block after the collision.

**SET UP:** Let $+x$ be to the right. Let the bullet be $A$ and the block be $B$. Let $V$ be the velocity of the block just after the collision.

**EXECUTE:** Motion of block after the collision: $K_1 = U_{\text{grav2}}$. $\tfrac{1}{2}m_BV^2 = m_Bgh$.

$$V = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(0.450\times10^{-2}\text{ m})} = 0.297\text{ m/s}.$$

Collision: $v_{B2} = 0.297\text{ m/s}$. $P_{1x} = P_{2x}$ gives $m_Av_{A1} = m_Av_{A2} + m_Bv_{B2}$.

$$v_{A2} = \frac{m_Av_{A1} - m_Bv_{B2}}{m_A} = \frac{(5.00\times10^{-3}\text{ kg})(450\text{ m/s}) - (1.00\text{ kg})(0.297\text{ m/s})}{5.00\times10^{-3}\text{ kg}} = 391\text{ m/s}.$$

**EVALUATE:** We assume the block moves very little during the time it takes the bullet to pass through it.

**8.85.** **IDENTIFY:** Eqs. 8.24 and 8.25 give the outcome of the elastic collision. The value of $M$ where the kinetic energy loss $K_{\text{loss}}$ of the neutron is a maximum satisfies $dK_{\text{loss}}/dM = 0$.

**SET UP:** Let object $A$ be the neutron and object $B$ be the nucleus. Let the initial speed of the neutron be $v_{A1}$. All motion is along the $x$-axis. $K_0 = \tfrac{1}{2}mv_{A1}^2$.

**EXECUTE:** **(a)** $v_{A2} = \dfrac{m-M}{m+M}v_{A1}$. $K_{\text{loss}} = \tfrac{1}{2}mv_{A1}^2 - \tfrac{1}{2}mv_{A2}^2 = \tfrac{1}{2}m\left(1-\left[\dfrac{m-M}{m+M}\right]^2\right)v_{A1}^2 = \dfrac{2m^2M}{(M+m)^2}v_{A1}^2 = \dfrac{4K_0mM}{(M+m)^2}$, as was to be shown.

**(b)** $\dfrac{dK_{\text{loss}}}{dM} = 4K_0m\left[\dfrac{1}{(M+m)^2} - \dfrac{2M}{(M+m)^3}\right] = 0$. $\dfrac{2M}{M+m} = 1$ and $M = m$. The incident neutron loses the most kinetic energy when the target has the same mass as the neutron.

**(c)** When $m_A = m_B$, Eq. 8.24 says $v_{A2} = 0$. The final speed of the neutron is zero and the neutron loses all of its kinetic energy.

**EVALUATE:** When $M >> m$, $v_{A2x} \approx -v_{A1x}$ and the neutron rebounds with speed almost equal to its initial speed. In this case very little kinetic energy is lost; $K_{\text{loss}} = 4K_0m/M$, which is very small.

**8.86.** **IDENTIFY:** Eqs. 8.24 and 8.25 give the outcome of the elastic collision.

**SET UP:** Let all the motion be along the $x$ axis. $v_{A1x} = v_0$.

**EXECUTE:** **(a)** $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_0$ and $v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right)v_0$. $K_1 = \tfrac{1}{2}m_Av_0^2$.

$$K_{A2} = \tfrac{1}{2}m_Av_{A2x}^2 = \tfrac{1}{2}m_A\left(\frac{m_A - m_B}{m_A + m_B}\right)^2 v_0^2 = \left(\frac{m_A - m_B}{m_A + m_B}\right)^2 K_1 \text{ and } \frac{K_{A2}}{K_1} = \left(\frac{m_A - m_B}{m_A + m_B}\right)^2.$$

$$K_{B2} = \tfrac{1}{2}m_Bv_{B2x}^2 = \tfrac{1}{2}m_B\left(\frac{2m_A}{m_A + m_B}\right)^2 v_0^2 = \frac{4m_Am_B}{\left(m_A + m_B\right)^2}K_1 \text{ and } \frac{K_{B2}}{K_1} = \frac{4m_Am_B}{\left(m_A + m_B\right)^2}.$$

**(b)** (i) For $m_A = m_B$, $\dfrac{K_{A2}}{K_1} = 0$ and $\dfrac{K_{B2}}{K_1} = 1$. (ii) For $m_A = 5m_B$, $\dfrac{K_{A2}}{K_1} = \dfrac{4}{9}$ and $\dfrac{K_{B2}}{K_1} = \dfrac{5}{9}$.

**(c)** Equal sharing of the kinetic energy means $\dfrac{K_{A2}}{K_1}=\dfrac{K_{B2}}{K_1}=\dfrac{1}{2}$. $\left(\dfrac{m_A-m_B}{m_A+m_B}\right)^2=\dfrac{1}{2}$.

$2m_A^2+2m_B^2-4m_Am_B=m_A^2+2m_Am_B+m_B^2$. $m_A^2-6m_Am_B+m_B^2=0$. The quadratic formula gives $\dfrac{m_A}{m_B}=5.83$ or $\dfrac{m_A}{m_B}=0.172$. We can also verify that these values give $\dfrac{K_{B2}}{K_1}=\dfrac{1}{2}$.

**EVALUATE:** When $m_A \ll m_B$ or when $m_A \gg m_B$, object *A* retains almost all of the original kinetic energy.

**8.87.** **IDENTIFY:** Apply conservation of energy to the motion of the package before the collision and apply conservation of the horizontal component of momentum to the collision.

**(a) SET UP:** Apply conservation of energy to the motion of the package from point 1 as it leaves the chute to point 2 just before it lands in the cart. Take $y=0$ at point 2, so $y_1=4.00$ m. Only gravity does work, so

$$K_1+U_1=K_2+U_2.$$

**EXECUTE:** $\frac{1}{2}mv_1^2+mgy_1=\frac{1}{2}mv_2^2$.

$v_2=\sqrt{v_1^2+2gy_1}=9.35\text{ m/s}$.

**(b) SET UP:** In the collision between the package and the cart momentum is conserved in the horizontal direction. (But not in the vertical direction, due to the vertical force the floor exerts on the cart.) Take $+x$ to be to the right. Let *A* be the package and *B* be the cart.

**EXECUTE:** $P_x$ is constant gives $m_Av_{A1x}+m_Bv_{B1x}=(m_A+m_B)v_{2x}$.

$v_{B1x}=-5.00\text{ m/s}$.

$v_{A1x}=(3.00\text{ m/s})\cos 37.0°$. (The horizontal velocity of the package is constant during its free-fall.)

Solving for $v_{2x}$ gives $v_{2x}=-3.29$ m/s. The cart is moving to the left at 3.29 m/s after the package lands in it.

**EVALUATE:** The cart is slowed by its collision with the package, whose horizontal component of momentum is in the opposite direction to the motion of the cart.

**8.88.** **IDENTIFY:** Eqs. 8.24, 8.25, and 8.27 give the outcome of the elastic collision.

**SET UP:** The blue puck is object *A* and the red puck is object *B*. Let +*x* be the direction of the initial motion of *A*. $v_{A1x}=0.200\text{ m/s}$, $v_{A2x}=0.050\text{ m/s}$ and $v_{B1x}=0$

**EXECUTE:** **(a)** Eq. 8.27 gives $v_{B2x}=v_{A2x}-v_{B1x}+v_{A1x}=0.250\text{ m/s}$.

**(b)** Eq. 8.25 gives $m_B=m_A\left(2\dfrac{v_{A1x}}{v_{B2x}}-1\right)=(0.0400\text{ kg})\left(2\left[\dfrac{0.200\text{ m/s}}{0.250\text{ m/s}}\right]-1\right)=0.024\text{ kg}$.

**EVALUATE:** We can verify that our results give $K_1=K_2$ and $P_{1x}=P_{2x}$, as required in an elastic collision.

**8.89.** **(a) IDENTIFY** and **SET UP:** $K=\frac{1}{2}m_Av_A^2+\frac{1}{2}m_Bv_B^2$.

Use $\vec{v}_A=\vec{v}'_A+\boldsymbol{v}_{\text{cm}}$ and $\vec{v}_B=\vec{v}'_B+\boldsymbol{v}_{\text{cm}}$ to replace $v_A$ and $v_B$ in this equation. Note $\vec{v}'_A$ and $\vec{v}'_B$ as defined in the problem are the velocities of *A* and *B* in coordinates moving with the center of mass. Note also that $m_A\vec{v}'_A+m_B\vec{v}'_B=M\vec{v}'_{\text{cm}}$ where $\vec{v}'_{\text{cm}}$ is the velocity of the car in these coordinates. But that's zero, so $m_A\vec{v}'_A+m_B\vec{v}'_B=0$; we can use this in the proof.

In part (b), use that $\vec{P}$ is conserved in a collision.

**EXECUTE:** $\vec{v}_A=\vec{v}'_A+\boldsymbol{v}_{\text{cm}}$, so $v_A^2=v_A'^2+v_{\text{cm}}^2+2\vec{v}'_A\cdot\vec{v}_{\text{cm}}$.

$$\vec{v}_B=\vec{v}'_B+\boldsymbol{v}_{\text{cm}},\ \text{so}\ v_B^2=v_B'^2+v_{\text{cm}}^2+2\vec{v}'_B\cdot\vec{v}_{\text{cm}}.$$

(We have used that for a vector $\vec{A}$, $A^2=\vec{A}\cdot\vec{A}$.)

Thus $K=\frac{1}{2}m_Av_A'^2+\frac{1}{2}m_Av_{\text{cm}}^2+m_A\vec{v}'_A\cdot\vec{v}_{\text{cm}}+\frac{1}{2}m_Bv_B'^2+\frac{1}{2}m_Bv_{\text{cm}}^2+m_B\vec{v}'_B\cdot\vec{v}_{\text{cm}}$.

$$K=\tfrac{1}{2}(m_A+m_B)v_{\text{cm}}^2+\tfrac{1}{2}\left(m_Av_A'^2+m_Bv_B'^2\right)+\left(m_A\vec{v}'_A+m_B\vec{v}'_B\right)\cdot\vec{v}_{\text{cm}}.$$

But $m_A+m_B=M$ and as noted earlier $m_A\vec{v}'_A+m_B\vec{v}'_B=0$, so $K=\frac{1}{2}Mv_{\text{cm}}^2+\frac{1}{2}\left(m_Av_A'^2+m_Bv_B'^2\right)$. This is the result the problem asked us to derive.

**(b) EVALUATE:** In the collision $\vec{P}=M\vec{v}_{\text{cm}}$ is constant, so $\frac{1}{2}Mv_{\text{cm}}^2$ stays constant. The asteroids can lose all their relative kinetic energy but the $\frac{1}{2}Mv_{\text{cm}}^2$ must remain.

**8.90.** **IDENTIFY:** Eq. 8.27 describes the elastic collision, with $x$ replaced by $y$. Speed and height are related by conservation of energy.

**SET UP:** Let $+y$ be upward. Let $A$ be the large ball and $B$ be the small ball, so $v_{B1y} = -v$ and $v_{A1y} = +v$. If the large ball has much greater mass than the small ball its speed is changed very little in the collision and $v_{A2y} = +v$.

**EXECUTE:** **(a)** $v_{B2y} - v_{A2y} = -(v_{B1y} - v_{A1y})$ gives $v_{B2y} = +v_{A2y} - v_{B1y} + v_{A1y} = v - (-v) + v = +3v$. The small ball moves upward with speed $3v$ after the collision.

**(b)** Let $h_1$ be the height the small ball fell before the collision. Conservation of energy applied to the motion from the release point to the floor gives $U_1 = K_2$ and $mgh_1 = \frac{1}{2}mv^2$. $h_1 = \dfrac{v^2}{2g}$. Conservation of energy applied to the motion of the small ball from immediately after the collision to its maximum height $h_2$ (rebound distance) gives $K_1 = U_2$ and $\frac{1}{2}m(3v)^2 = mgh_2$. $h_2 = \dfrac{9v^2}{2g} = 9h_1$. The ball's rebound distance is nine times the distance it fell.

**EVALUATE:** The mechanical energy gained by the small ball comes from the energy of the large ball. But since the large ball's mass is much larger it can give up this energy with very little decrease in speed.

**8.91.** **IDENTIFY:** Apply conservation of momentum to the system consisting of Jack, Jill and the crate. The speed of Jack or Jill relative to the ground will be different from 4.00 m/s.

**SET UP:** Use an inertial coordinate system attached to the ground. Let $+x$ be the direction in which the people jump. Let Jack be object $A$, Jill be $B$, and the crate be $C$.

**EXECUTE:** **(a)** If the final speed of the crate is $v$, $v_{C2x} = -v$, and $v_{A2x} = v_{B2x} = 4.00\text{ m/s} - v$. $P_{2x} = P_{1x}$ gives $m_A v_{A2x} + m_B v_{Bx2} + m_C v_{Cx2} = 0$. $(75.0\text{ kg})(4.00\text{ m/s} - v) + (45.0\text{ kg})(4.00\text{ m/s} - v) + (15.0\text{ kg})(-v) = 0$ and

$$v = \frac{(75.0\text{ kg} + 45.0\text{ kg})(4.00\text{ m/s})}{75.0\text{ kg} + 45.0\text{ kg} + 15.0\text{ kg}} = 3.56\text{ m/s}.$$

**(b)** Let $v'$ be the speed of the crate after Jack jumps. Apply momentum conservation to Jack jumping: $(75.0\text{ kg})(4.00\text{ m/s} - v') + (60.0\text{ kg})(-v') = 0$ and $v' = \dfrac{(75.0\text{ kg})(4.00\text{ m/s})}{135.0\text{ kg}} = 2.22\text{ m/s}$. Then apply momentum conservation to Jill jumping, with $v$ being the final speed of the crate: $P_{1x} = P_{2x}$ gives $(60.0\text{ kg})(-v') = (45.0\text{ kg})(4.00\text{ m/s} - v) + (15.0\text{ kg})(-v)$.

$$v = \frac{(45.0\text{ kg})(4.00\text{ m/s}) + (60.0\text{ kg})(2.22\text{ m/s})}{60.0\text{ kg}} = 5.22\text{ m/s}.$$

**(c)** Repeat the calculation in **(b)**, but now with Jill jumping first.
Jill jumps: $(45.0\text{ kg})(4.00\text{ m/s} - v') + (90.0\text{ kg})(-v') = 0$ and $v' = 1.33\text{ m/s}$.
Jack jumps: $(90.0\text{ kg})(-v') = (75.0\text{ kg})(4.00\text{ m/s} - v) + (15.0\text{ kg})(-v)$.

$$v = \frac{(75.0\text{ kg})(4.00\text{ m/s}) + (90.0\text{ kg})(1.33\text{ m/s})}{90.0\text{ kg}} = 4.66\text{ m/s}.$$

**EVALUATE:** The final speed of the crate is greater when Jack jumps first, then Jill. In this case Jack leaves with a speed of 1.78 m/s relative to the ground, whereas when they both jump simultaneously Jack and Jill each leave with a speed of only 0.44 m/s relative to the ground.

**8.92.** **IDENTIFY:** Momentum is conserved in the explosion. The total kinetic energy of the two fragments is $Q$.

**SET UP:** Let the final speed of the two fragments be $v_A$ and $v_B$. They must move in opposite directions after the explosion.

**EXECUTE:** **(a)** Since the initial momentum of the system is zero, conservation of momentum says $m_A v_A = m_B v_B$ and $v_B = \left(\dfrac{m_A}{m_B}\right)v_A$. $K_A + K_B = Q$ gives $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B\left(\dfrac{m_A}{m_B}\right)^2 v_A^2 = Q$. $\frac{1}{2}m_A v_A^2\left(1 + \dfrac{m_A}{m_B}\right) = Q$.

$K_A = \dfrac{Q}{1 + m_A/m_B} = \left(\dfrac{m_B}{m_A + m_B}\right)Q$. $K_B = Q - K_A = Q\left(1 - \dfrac{m_B}{m_A + m_B}\right) = \left(\dfrac{m_A}{m_A + m_B}\right)Q$.

**(b)** If $m_B = 4m_A$, then $K_A = \dfrac{4}{5}Q$ and $K_B = \dfrac{1}{5}Q$. The lighter fragment gets 80% of the energy that is released.

**EVALUATE:** If $m_A = m_B$ the fragments share the energy equally. In the limit that $m_B >> m_A$, the lighter fragment gets almost all of the released energy.

**8.93.** **IDENTIFY:** Apply conservation of momentum to the system of the neutron and its decay products.

**SET UP:** Let the proton be moving in the +*x* direction with speed $v_p$ after the decay. The initial momentum of the neutron is zero, so to conserve momentum the electron must be moving in the $-x$ direction after the decay. Let the speed of the electron be $v_e$.

**EXECUTE:** $P_{1x} = P_{2x}$ gives $0 = m_p v_p - m_e v_e$ and $v_e = \left(\frac{m_p}{m_e}\right) v_p$. The total kinetic energy after the decay is

$$K_{tot} = \tfrac{1}{2} m_e v_e^2 + \tfrac{1}{2} m_p v_p^2 = \tfrac{1}{2} m_e \left(\frac{m_p}{m_e}\right)^2 v_p^2 + \tfrac{1}{2} m_p v_p^2 = \tfrac{1}{2} m_p v_p^2 \left(1 + \frac{m_p}{m_e}\right).$$

Thus, $\frac{K_p}{K_{tot}} = \frac{1}{1 + m_p/m_e} = \frac{1}{1 + 1836} = 5.44 \times 10^{-4} = 0.0544\%$.

**EVALUATE:** Most of the released energy goes to the electron, since it is much lighter than the proton.

**8.94.** **IDENTIFY:** Momentum is conserved in the decay. The results of Problem 8.92 give the kinetic energy of each fragment.

**SET UP:** Let *A* be the alpha particle and let *B* be the radium nucleus, so $m_A / m_B = 0.0176$. $Q = 6.54 \times 10^{-13}$ J.

**EXECUTE:** $K_A = \frac{Q}{1 + m_A / m_B} = \frac{6.54 \times 10^{-13}\text{ J}}{1 + 0.0176} = 6.43 \times 10^{-13}$ J and $K_B = 0.11 \times 10^{-13}$ J.

**EVALUATE:** The lighter particle receives most of the released energy.

**8.95.** **IDENTIFY:** The momentum of the system is conserved.

**SET UP:** Let +*x* be to the right. $P_{1x} = 0$. $p_{ex}$, $p_{nx}$ and $p_{anx}$ are the momenta of the electron, polonium nucleus and antineutrino, respectively.

**EXECUTE:** $P_{1x} = P_{2x}$ gives $p_{ex} + p_{nx} + p_{anx} = 0$. $p_{anx} = -(p_{ex} + p_{nx})$.

$p_{anx} = -(5.60 \times 10^{-22}\text{ kg} \cdot \text{m/s} + [3.50 \times 10^{-25}\text{ kg}][-1.14 \times 10^3\text{ m/s}]) = -1.66 \times 10^{-22}\text{ kg} \cdot \text{m/s}$.

The antineutrino has momentum to the left with magnitude $1.66 \times 10^{-22}\text{ kg} \cdot \text{m/s}$.

**EVALUATE:** The antineutrino interacts very weakly with matter and most easily shows its presence by the momentum it carries away.

**8.96.** **IDENTIFY:** Momentum components in the *x* and *y* directions are separately conserved. For an elastic collision $K_1 = K_2$.

**SET UP:** $v_{A1x} = +v_{A1}$, $v_{B1x} = 0$. $v_{A2x} = v_{A2}\cos\alpha$, $v_{A2y} = v_{A2}\sin\alpha$. $v_{B2x} = v_{B2}\cos\alpha$, $v_{B2y} = -v_{B2}\sin\alpha$.

$\sin^2\theta + \cos^2\theta = 1$, for any angle $\theta$. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$ gives $m_A v_{A1} = m_A v_{A2}\cos\alpha + m_B v_{B2}\cos\beta$.

$P_{1y} = P_{2y}$ gives $0 = m_A v_{A2}\sin\alpha - m_B v_{B2}\sin\beta$.

**(b)** $m_A^2 v_{A1}^2 = m_A^2 v_{A2}^2 \cos^2\alpha + m_B^2 v_{B2}^2 \cos^2\beta + 2 m_A m_B v_{A2} v_{B2} \cos\alpha\cos\beta$ and

$0 = m_A^2 v_{A2}^2 \sin^2\alpha + m_B^2 v_{B2}^2 \sin^2\beta - 2 m_A m_B v_{A2} v_{B2} \sin\alpha\sin\beta$. Adding these two equations and using the trig identities in the SET UP step gives $m_A^2 v_{A1}^2 = m_A^2 v_{A2}^2 + m_B^2 v_{B2}^2 + 2 m_A m_B v_{A2} v_{B2} \cos(\alpha + \beta)$.

**(c)** $K_1 = K_2$ says $\tfrac{1}{2} m_A v_{A1}^2 = \tfrac{1}{2} m_A v_{A2}^2 + \tfrac{1}{2} m_B v_{B2}^2$. The result in part (b) agrees with this expression only if

$\cos(\alpha + \beta) = 0$. This requires that $\alpha + \beta = 90° = \frac{\pi}{2}$ rad.

**EVALUATE:** The result of part (c) says that the two protons move in perpendicular directions after the collision.

**8.97.** **IDENTIFY** and **SET UP:**

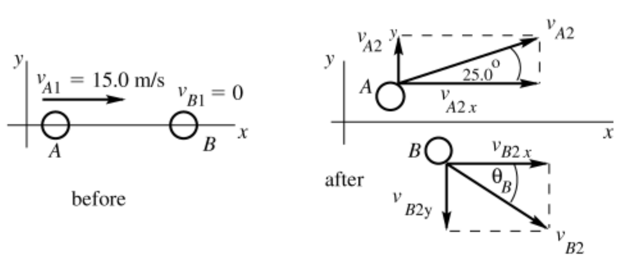


**Figure 8.97**

$P_x$ and $P_y$ are conserved in the collision since there is no external horizontal force.

The result of Problem 8.96 part (d) applies here since the collision is elastic This says that $25.0° + \theta_B = 90°$, so that $\theta_B = 65.0°$. (*A* and *B* move off in perpendicular directions.)

**EXECUTE:** $P_x$ is conserved so $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$.

But $m_A = m_B$ so $v_{A1} = v_{A2}\cos 25.0° + v_{B2}\cos 65.0°$.

$$P_y \text{ is conserved so } m_A v_{A1y} + m_B v_{B1y} = m_A v_{A2y} + m_B v_{B2y}.$$

$0 = v_{A2y} + v_{B2y}$.

$0 = v_{A2}\sin 25.0° - v_{B2}\sin 65.0°$.

$v_{B2} = (\sin 25.0° / \sin 65.0°) v_{A2}$.

This result in the first equation gives $v_{A1} = v_{A2}\cos 25.0° + \left(\dfrac{\sin 25.0° \cos 65.0°}{\sin 65.0°}\right) v_{A2}$.

$v_{A1} = 1.103 v_{A2}$.

$v_{A2} = v_{A1}/1.103 = (15.0 \text{ m/s})/1.103 = 13.6 \text{ m/s}$.

And then $v_{B2} = (\sin 25.0° / \sin 65.0°)(13.6 \text{ m/s}) = 6.34 \text{ m/s}$.

**EVALUATE:** We can use our numerical results to show that $K_1 = K_2$ and that $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.

**8.98.** **IDENTIFY:** Since there is no friction, the horizontal component of momentum of the system of Jonathan, Jane and the sleigh is conserved.

**SET UP:** Let +*x* be to the right. $w_A = 800 \text{ N}$, $w_B = 600 \text{ N}$ and $w_C = 1000 \text{ N}$.

**EXECUTE:** $P_{1x} = P_{2x}$ gives $0 = m_A v_{A2x} + m_B v_{B2x} + m_C v_{C2x}$. $v_{C2x} = \dfrac{m_A v_{A2x} + m_B v_{B2x}}{m_C} = \dfrac{w_A v_{A2x} + w_B v_{B2x}}{w_C}$.

$$v_{C2x} = \frac{(800 \text{ N})(-[5.00 \text{ m/s}]\cos 30.0°) + (600 \text{ N})(+[7.00 \text{ m/s}]\cos 36.9°)}{1000 \text{ N}} = -0.105 \text{ m/s}.$$

The sleigh's velocity is 0.105 m/s, to the left.

**EVALUATE:** The vertical component of the momentum of the system consisting of the two people and the sleigh is not conserved, because of the net force exerted on the sleigh by the ice while they jump.

**8.99.** **IDENTIFY:** In Eq. 8.28 treat each straight piece as an object in the system.

**SET UP:** The center of mass of each piece of length *L* is at its center.

**EXECUTE:** **(a)** From symmetry, the center of mass is on the vertical axis, a distance $(L/2)\cos(\alpha/2)$ below the apex.

**(b)** The center of mass is on the vertical axis of symmetry, a distance $2(L/2)/3 = L/3$ above the center of the horizontal segment.

**(c)** Using the wire frame as a coordinate system, the coordinates of the center of mass are equal and each is equal to $(L/2)/2 = L/4$. The center of mass is along the bisector of the angle, a distance $L/\sqrt{8}$ from the corner.

**(d)** By symmetry, the center of mass is at the center of the equilateral triangle, a distance $(L/3)\sin 60° = L/\sqrt{12}$ above the center of the horizontal segment.

**EVALUATE:** The center of mass need not lie on any point of the object, it can be in empty space.

**8.100.** **IDENTIFY:** There is no net horizontal external force so $v_{\text{cm}}$ is constant.

**SET UP:** Let +*x* be to the right, with the origin at the initial position of the left-hand end of the canoe. $m_A = 45.0 \text{ kg}$, $m_B = 60.0 \text{ kg}$. The center of mass of the canoe is at its center.

**EXECUTE:** Initially, $v_{\text{cm}} = 0$, so the center of mass doesn't move. Initially, $x_{\text{cm1}} = \dfrac{m_A x_{A1} + m_B x_{B1}}{m_A + m_B}$. After she walks, $x_{\text{cm2}} = \dfrac{m_A x_{A2} + m_B x_{B2}}{m_A + m_B}$. $x_{\text{cm1}} = x_{\text{cm2}}$ gives $m_A x_{A1} + m_B x_{B1} = m_A x_{A2} + m_B x_{B2}$. She walks to a point 1.00 m from the right-hand end of the canoe, so she is 1.50 m to the right of the center of mass of the canoe and $x_{A2} = x_{B2} + 1.50 \text{ m}$.

$$(45.0 \text{ kg})(1.00 \text{ m}) + (60.0 \text{ kg})(2.50 \text{ m}) = (45.0 \text{ kg})(x_{B2} + 1.50 \text{ m}) + (60.0 \text{ kg})x_{B2}.$$

$(105.0 \text{ kg})x_{B2} = 127.5 \text{ kg}\cdot\text{m}$ and $x_{B2} = 1.21 \text{ m}$. $x_{B2} - x_{B1} = 1.21 \text{ m} - 2.50 \text{ m} = -1.29 \text{ m}$. The canoe moves 1.29 m to the left.

**EVALUATE:** When the woman walks to the right, the canoe moves to the left. The woman walks 3.00 m to the right relative to the canoe and the canoe moves 1.29 m to the left, so she moves $3.00\text{ m} - 1.29\text{ m} = 1.71\text{ m}$ to the right relative to the water. Note that this distance is $(60.0\text{ kg}/45.0\text{ kg})(1.29\text{ m})$.

**8.101.** **IDENTIFY:** Take as the system you and the slab. There is no horizontal force, so horizontal momentum is conserved. By Eq. 8.32, $\vec{P}$ is constant $\vec{v}_{cm}$ is constant (for a system of constant mass). Use coordinates fixed to the ice, with the direction you walk as the $x$-direction. $\vec{v}_{cm}$ is constant and initially $\vec{v}_{cm} = 0$.

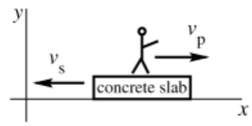


**Figure 8.101**

$$\vec{v}_{cm} = \frac{m_p\vec{v}_p + m_s\vec{v}_s}{m_p + m_s} = \mathbf{0}.$$

$$m_p\vec{v}_{cm} + m_s\vec{v}_s = \mathbf{0}.$$

$$m_p v_{px} + m_s v_{sx} = 0.$$

$$v_{sx} = -\left(m_p/m_s\right)v_{px} = -\left(m_p/5m_p\right)2.00\text{ m/s} = -0.400\text{ m/s}.$$

The slab moves at 0.400 m/s, in the direction opposite to the direction you are walking.

**EVALUATE:** The initial momentum of the system is zero. You gain momentum in the $+x$-direction so the slab gains momentum in the $-x$-direction. The slab exerts a force on you in the $+x$-direction so you exert a force on the slab in the $-x$-direction.

**8.102.** **IDENTIFY:** Conservation of $x$ and $y$ components of momentum applies to the collision. At the highest point of the trajectory the vertical component of the velocity of the projectile is zero.

**SET UP:** Let $+y$ be upward and $+x$ be horizontal and to the right. Let the two fragments be $A$ and $B$, each with mass $m$. For the projectile before the explosion and the fragments after the explosion. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$.

**EXECUTE:** **(a)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ with $v_y = 0$ gives that the maximum height of the projectile is $h = -\dfrac{v_{0y}^2}{2a_y} = -\dfrac{([80.0\text{ m/s}]\sin 60.0°)^2}{2(-9.80\text{ m/s}^2)} = 244.9\text{ m}$. Just before the explosion the projectile is moving to the right with horizontal velocity $v_x = v_{0x} = v_0\cos 60.0° = 40.0\text{ m/s}$. After the explosion $v_{Ax} = 0$ since fragment $A$ falls vertically. Conservation of momentum applied to the explosion gives $(2m)(40.0\text{ m/s}) = mv_{Bx}$ and $v_{Bx} = 80.0\text{ m/s}$. Fragment $B$ has zero initial vertical velocity so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives a time of fall of

$t = \sqrt{-\dfrac{2h}{a_y}} = \sqrt{-\dfrac{2(244.9\text{ m})}{-9.80\text{ m/s}^2}} = 7.07\text{ s}$. During this time the fragment travels horizontally a distance $(80.0\text{ m/s})(7.07\text{ s}) = 566\text{ m}$. It also took the projectile 7.07 s to travel from launch to maximum height and during this time it travels a horizontal distance of $([80.0\text{ m/s}]\cos 60.0°)(7.07\text{ s}) = 283\text{ m}$. The second fragment lands $283\text{ m} + 566\text{ m} = 849\text{ m}$ from the firing point.

**(b)** For the explosion, $K_1 = \frac{1}{2}(20.0\text{ kg})(40.0\text{ m/s})^2 = 1.60\times 10^4\text{ J}$. $K_2 = \frac{1}{2}(10.0\text{ kg})(80.0\text{ m/s})^2 = 3.20\times 10^4\text{ J}$. The energy released in the explosion is $1.60\times 10^4\text{ J}$.

**EVALUATE:** The kinetic energy of the projectile just after it is launched is $6.40\times 10^4\text{ J}$. We can calculate the speed of each fragment just before it strikes the ground and verify that the total kinetic energy of the fragments just before they strike the ground is $6.40\times 10^4\text{ J} + 1.60\times 10^4\text{ J} = 8.00\times 10^4\text{ J}$. Fragment $A$ has speed 69.3 m/s just before it strikes the ground, and hence has kinetic energy $2.40\times 10^4\text{ J}$. Fragment $B$ has speed $\sqrt{(80.0\text{ m/s})^2 + (69.3\text{ m/s})^2} = 105.8\text{ m/s}$ just before it strikes the ground, and hence has kinetic energy $5.60\times 10^4\text{ J}$.

Also, the center of mass of the system has the same horizontal range $R = \dfrac{v_0^2}{g}\sin(2\alpha_0) = 565\text{ m}$ that the projectile would have had if no explosion had occurred. One fragment lands at $R/2$ so the other, equal mass fragment lands at a distance $3R/2$ from the launch point.

**8.103.** **IDENTIFY:** The rocket moves in projectile motion before the explosion and its fragments move in projectile motion after the explosion. Apply conservation of energy and conservation of momentum to the explosion.

**SET UP:** Apply conservation of energy to the explosion. Just before the explosion the shell is at its maximum height and has zero kinetic energy. Let *A* be the piece with mass 1.40 kg and *B* be the piece with mass 0.28 kg. Let $v_A$ and $v_B$ be the speeds of the two pieces immediately after the collision.

**EXECUTE:** $\frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv_B^2 = 860 \text{ J}$

**SET UP:** Since the two fragments reach the ground at the same time, their velocities just after the explosion must be horizontal. The initial momentum of the shell before the explosion is zero, so after the explosion the pieces must be moving in opposite horizontal directions and have equal magnitude of momentum: $m_Av_A = m_Bv_B$.

**EXECUTE:** Use this to eliminate $v_A$ in the first equation and solve for $v_B$:

$$\tfrac{1}{2}m_Bv_B^2\left(1 + m_B/m_A\right) = 860 \text{ J} \text{ and } v_B = 71.6 \text{ m/s}.$$

Then $v_A = \left(m_B/m_A\right)v_B = 14.3 \text{ m/s}$.

**(b) SET UP:** Use the vertical motion from the maximum height to the ground to find the time it takes the pieces to fall to the ground after the explosion. Take $+y$ downward.

$$v_{0y} = 0,\quad a_y = +9.80 \text{ m/s}^2,\quad y - y_0 = 80.0 \text{ m},\quad t = ?$$

**EXECUTE:** $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $t = 4.04$ s.

During this time the horizontal distance each piece moves is $x_A = v_At = 57.8$ m and $x_B = v_Bt = 289.1$ m. They move in opposite directions, so they are $x_A + x_B = 347$ m apart when they land.

**EVALUATE:** Fragment *A* has more mass so it is moving slower right after the collision, and it travels horizontally a smaller distance as it falls to the ground.

**8.104.** **IDENTIFY:** Apply conservation of momentum to the collision. At the highest point of its trajectory the shell is moving horizontally. If one fragment received some upward momentum in the collision, the other fragment would have had to receive a downward component. Since they each the ground at the same time, each must have zero vertical velocity immediately after the explosion.

**SET UP:** Let +*x* be horizontal, along the initial direction of motion of the projectile and let +*y* be upward. At its maximum height the projectile has $v_x = v_0\cos 55.0° = 86.0$ m/s . Let the heavier fragment be *A* and the lighter fragment be *B*. $m_A = 9.00$ kg and $m_B = 3.00$ kg .

**EXECUTE:** Since fragment *A* returns to the launch point, immediately after the explosion it has $v_{Ax} = -86.0$ m/s . Conservation of momentum applied to the explosion gives $(12.0 \text{ kg})(86.0 \text{ m/s}) = (9.00 \text{ kg})(-86.0 \text{ m/s}) + (3.00 \text{ kg})v_{Bx}$ and $v_{Bx} = 602$ m/s . The horizontal range of the projectile, if no explosion occurred, would be $R = \dfrac{v_0^2}{g}\sin(2\alpha_0) = 2157$ m . The horizontal distance each fragment travels is proportional to its initial speed and the heavier fragment travels a horizontal distance $R/2 = 1078$ m after the explosion, so the lighter fragment travels a horizontal distance $\left(\dfrac{602 \text{ m}}{86 \text{ m}}\right)(1078 \text{ m}) = 7546$ m from the point of explosion and $1078 \text{ m} + 7546 \text{ m} = 8624$ m from the launch point. The energy released in the explosion is $K_2 - K_1 = \frac{1}{2}(9.00 \text{ kg})(86.0 \text{ m/s})^2 + \frac{1}{2}(3.00 \text{ kg})(602 \text{ m/s})^2 - \frac{1}{2}(12.0 \text{ kg})(86.0 \text{ m/s})^2 = 5.33\times10^5$ J .

**EVALUATE:** The center of mass of the system has the same horizontal range $R = 2157$ m as if the explosion didn't occur. This gives $(12.0 \text{ kg})(2157 \text{ m}) = (9.00 \text{ kg})(0) + (3.00 \text{ kg})d$ and $d = 8630$ m , where *d* is the distance from the launch point to where the lighter fragment lands. This agrees with our calculation.

**8.105.** **IDENTIFY:** No external force, so $\vec{P}$ is conserved in the collision.

**SET UP:** Apply momentum conservation in the *x* and *y* directions:

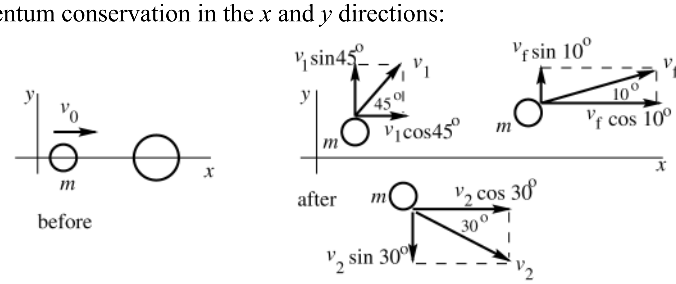


**Figure 8.105**

Solve for $v_1$ and $v_2$.

**EXECUTE:** $P_x$ is conserved so $mv_0 = m\left(v_1\cos 45° + v_\text{f}\cos 10° + v_2\cos 30°\right)$.

$v_0 - v_\text{f}\cos 10° = v_1\cos 45° + v_2\cos 30°$.

$1030.4\text{ m/s} = v_1\cos 45° + v_2\cos 30°$.

$P_x$ is conserved so $0 = m\left(v_1\sin 45° - v_2\sin 30° + v_\text{f}\sin 10°\right)$.

$v_1\sin 45° = v_2\sin 30° - 347.3\text{ m/s}$.

$\sin 45° = \cos 45°$ so

$1030.4\text{ m/s} = v_2\sin 30° - 347.3\text{ m/s} + v_2\cos 30°$.

$$v_2 = \frac{1030.4\text{ m/s} + 347.3\text{ m/s}}{\sin 30° + \cos 30.0°} = 1010\text{ m/s}.$$

And then $v_1 = \dfrac{v_2\sin 30° - 347.3\text{ m/s}}{\sin 45°} = 223\text{ m/s}$. Then two emitted neutrons have speeds of $223\text{ m/s}$ and $1010\text{ m/s}$.

The speeds of the Ba and Kr nuclei are related by $P_z$ conservation.

$P_z$ is constant implies that $0 = m_\text{Ba}v_\text{Ba} - m_\text{Kr}v_\text{Kr}$

$$v_\text{Kr} = \left(\frac{m_\text{Ba}}{m_\text{Kr}}\right)v_\text{Ba} = \left(\frac{2.3\times10^{-25}\text{ kg}}{1.5\times10^{-25}\text{ kg}}\right)v_\text{Ba} = 1.5v_\text{Ba}.$$

We can't say what these speeds are but they must satisfy this relation. The value of $v_\text{Ba}$ depends on energy considerations.

**EVALUATE:** $K_1 = \tfrac{1}{2}m_\text{n}\left(3.0\times10^3\text{ m/s}\right)^2 = \left(4.5\times10^6\text{ J/kg}\right)m_\text{n}$.

$$K_2 = \tfrac{1}{2}m_\text{n}\left(2.0\times10^3\text{ m/s}\right)^2 + \tfrac{1}{2}m_\text{n}\left(223\text{ m/s}\right)^2 + \tfrac{1}{2}m_\text{n}\left(1010\text{ m/s}\right)^2 + K_\text{Ba} + K_\text{Kr} = \left(2.5\times10^6\text{ J/kg}\right)m_\text{n} + K_\text{Ba} + K_\text{Kr}.$$

We don't know what $K_\text{Ba}$ and $K_\text{Kr}$ are, but they are positive. We will study such nuclear reactions further in Chapter 43 and will find that energy is released in this process; $K_2 > K_1$. Some of the potential energy stored in the $^{235}\text{U}$ nucleus is released as kinetic energy and shared by the collision fragments.

**8.106.** **IDENTIFY:** The velocity of the center of mass of the system of the two blocks is given by Eq. 8.30. Conservation of momentum says the center of mass moves at constant speed.

**SET UP:** $v_{A1x} = v_{A1}$, $v_{B1x} = 0$. The velocity $\vec{u}$ in the center of mass frame is related to the velocity $\vec{v}$ in the stationary frame by $\vec{u} = \vec{v} - \vec{v}_\text{cm}$. We can express kinetic energy as $K = \dfrac{p^2}{2m}$.

**EXECUTE:** **(a)** $v_{\text{cm-}x} = \dfrac{m_A v_{A1}}{m_A + m_B}$.

**(b)** The center of mass moves with constant speed so this coordinate system is an inertial frame.

**(c)** $u_{A1x} = v_{A1x} - v_{\text{cm-}x} = \dfrac{m_B v_{A1}}{m_A + m_B}$. $u_{B1x} = v_{B1x} - v_{\text{cm-}x} = -\dfrac{m_A v_{A1}}{m_A + m_B}$. In this frame $P_{1x} = m_A u_{A1x} + m_B u_{B1x} = 0$.

**(d)** $P_{2x} = P_{1x} = 0$ gives $p_{A1x} + p_{B1x} = 0$ and $p_{A2x} + p_{B2x} = 0$, so $p_{B1x} = -p_{A1x}$ and $p_{B2x} = -p_{A2x}$. Conservation of kinetic energy gives $\dfrac{p_{A2x}^2}{2m_A} + \dfrac{p_{B2x}^2}{2m_B} = \dfrac{p_{A1x}^2}{2m_A} + \dfrac{p_{B1x}^2}{2m_B}$. Using $p_{B2x} = -p_{A2x}$ and $p_{B1x} = -p_{A1x}$ gives $p_{A2x}^2 = p_{A1x}^2$ and $p_{A2x} = \pm p_{A1x}$. If a collision occurs $p_{Ax}$ changes and $p_{A2x} = -p_{A1x}$. But $p_{B2x} = -p_{A2x}$ and $p_{B1x} = -p_{A1x}$, so $p_{B2x} = -p_{B1x}$. In the center of mass frame the momentum and hence the velocity of each puck keeps the same magnitude and reverses direction.

**(e)** $v_{\text{cm-}x} = \left(\dfrac{0.400\text{ kg}}{0.600\text{ kg}}\right)(6.00\text{ m/s}) = 4.00\text{ m/s}$. $u_{A1x} = 6.00\text{ m/s} - 4.00\text{ m/s} = 2.00\text{ m/s}$.

$u_{B1x} = 0 - 4.00\text{ m/s} = -4.00\text{ m/s}$. $u_{A2x} = -2.00\text{ m/s}$ and $u_{B2x} = +4.00\text{ m/s}$.

$v_{A2x} = u_{A2x} + v_{\text{cm-}x} = -2.00\text{ m/s} + 4.00\text{ m/s} = 2.00\text{ m/s}$. $v_{B2x} = u_{B2x} + v_{\text{cm-}x} = 4.00\text{ m/s} + 4.00\text{ m/s} = 8.00\text{ m/s}$.

Eq. 8.24 says $v_{A2x} = \left(\dfrac{0.400\text{ kg} - 0.200\text{ kg}}{0.400\text{ kg} + 0.200\text{ kg}}\right)(6.00\text{ m/s}) = 2.00\text{ m/s}$. Eq. 8.25 says

$v_{A2x} = \left(\dfrac{2[0.400\text{ kg}]}{0.400\text{ kg} + 0.200\text{ kg}}\right)(6.00\text{ m/s}) = 8.00\text{ m/s}$. Our result agrees with Eqs. 8.24 and 8.25.

**EVALUATE:** Eqs. 8.24 and 8.25 apply only when $v_{B1} = 0$. The result that the velocity of each puck in the center of mass frame reverses direction and retains the same magnitude applies to all elastic collisions, even when both are moving initially.

**8.107.** **IDENTIFY** and **SET UP:** Apply conservation of energy to find the total energy before and after the collision with the floor from the initial and final maximum heights.

**EXECUTE:** **(a)** Objects stick together says that the relative speed after the collision is zero, so $\epsilon = 0$.

**(b)** In an elastic collision the relative velocity of the two bodies has the same magnitude before and after the collision, so $\epsilon = 1$.

**(c)** Speed of ball just before collision: $mgh = \frac{1}{2}mv_1^2$.

$v_1 = \sqrt{2gh}$

Speed of ball just after collision: $mgH_1 = \frac{1}{2}mv_2^2$.

$v_2 = \sqrt{2gH_1}$

The second object (the surface) is stationary, so $\epsilon = v_2/v_1 = \sqrt{H_1/h}$.

**(d)** $\epsilon = \sqrt{H_1/h}$ implies $H_1 = h\epsilon^2 = (1.2\text{ m})(0.85)^2 = 0.87\text{ m}$.

**(e)** $H_1 = h\epsilon^2$.

$H_2 = H_1\epsilon^2 = h\epsilon^4$.

$H_3 = H_2\epsilon^2 = \left(h\epsilon^4\right)\epsilon^2 = h\epsilon^6$.

Generalize to $H_n = H_{n-1}\epsilon^2 = h\epsilon^{2(n-1)}\epsilon^2 = h\epsilon^{2n}$.

**(f)** 8th bounce implies $n = 8$.

$H_8 = h\epsilon^{16} = 1.2\text{ m}(0.85)^{16} = 0.089\text{ m}$.

**EVALUATE:** $\epsilon$ is a measure of the kinetic energy lost in the collision. The collision here is between a ball and the earth. Momentum lost by the ball is gained by the earth, but the velocity gained by the earth is very small and can be taken to be zero.

**8.108.** **IDENTIFY:** Momentum is conserved in the collision. Conservation of energy says $K_2 = K_1 + \Delta$.

**SET UP:** For part (b) let $v_0$ be the common speed of each atom before the collision and let $\vec{V}$ and $\vec{v}_3$ be the velocities after the collision of the molecule and the atom that remains. $m = 1.67 \times 10^{-27}\text{ kg}$ is the mass of one hydrogen atom.

**EXECUTE:** **(a)** In the center of mass frame $P_{1x} = 0$ so $P_{2x} = 0$ and $v_{\text{cm2}} = 0$. But in this frame the potential energy decreases and the kinetic energy increases. This is inconsistent with $K_{2\text{cm}} = \frac{1}{2}m_{\text{tot}}v_{\text{cm2}}^2 = 0$.

**(b)** Before the collision $v_{\text{cm}} = 0$. After the collision the molecule and remaining atom move in opposite directions and $(2m)V = mv_3$; $v_3 = 2V$. Conservation of energy gives $\frac{1}{2}(2m)V^2 + \frac{1}{2}mv_3^2 = 3(\frac{1}{2}mv_0^2)^2 + \Delta$. With $v_3 = 2V$ this becomes $V^2 = \frac{1}{2}v_0^2 + \dfrac{\Delta}{3m}$. $V = \sqrt{\frac{1}{2}(1.00\times 10^3\text{ m/s})^2 + \dfrac{7.23\times 10^{-19}\text{ J}}{3(1.67\times 10^{-27})}} = 1.20\times 10^4\text{ m/s}$ and $v_3 = 2V = 2.40\times 10^4\text{ m/s}$.

**EVALUATE:** $K = 3\left(\frac{1}{2}mv_0^2\right) = 2.50\times 10^{-21}\text{ J}$, which is much less than the binding energy of the molecule. Other initial conditions also lead to molecule formation; the one of zero initial momentum is just particularly simple to analyze.

**8.109.** **IDENTIFY:** Apply conservation of energy to the motion of the wagon before the collision. After the collision the combined object moves with constant speed on the level ground. In the collision the horizontal component of momentum is conserved.

**SET UP:** Let the wagon be object *A* and treat the two people together as object *B*. Let +*x* be horizontal and to the right. Let *V* be the speed of the combined object after the collision.

**EXECUTE:** **(a)** The speed $v_{A1}$ of the wagon just before the collision is given by conservation of energy applied to the motion of the wagon prior to the collision. $U_1 = K_2$ says $m_A g([50\text{ m}][\sin 6.0°]) = \frac{1}{2}m_A v_{A1}^2$. $v_{A1} = 10.12\text{ m/s}$.

$P_{1x} = P_{2x}$ for the collision says $m_A v_{A1} = (m_A + m_B)V$ and $V = \left(\dfrac{300\text{ kg}}{300\text{ kg} + 75.0\text{ kg} + 60.0\text{ kg}}\right)(10.12\text{ m/s}) = 6.98\text{ m/s}$.

In 5.0 s the wagon travels $(6.98\text{ m/s})(5.0\text{ s}) = 34.9\text{ m}$, and the people will have time to jump out of the wagon before it reaches the edge of the cliff.

**(b)** For the wagon, $K_1 = \frac{1}{2}(300\text{ kg})(10.12\text{ m/s})^2 = 1.54\times10^4\text{ J}$. Assume that the two heroes drop from a small height, so their kinetic energy just before the wagon can be neglected compared to $K_1$ of the wagon. $K_2 = \frac{1}{2}(435\text{ kg})(6.98\text{ m/s})^2 = 1.06\times10^4\text{ J}$. The kinetic energy of the system decreases by $K_1 - K_2 = 4.8\times10^3\text{ J}$.

**EVALUATE:** The wagon slows down when the two heroes drop into it. The mass that is moving horizontally increases, so the speed decreases to maintain the same horizontal momentum. In the collision the vertical momentum is not conserved, because of the net external force due to the ground.

**8.110.** **IDENTIFY:** Gravity gives a downward external force of magnitude *mg*. The impulse of this force equals the change in momentum of the rocket.

**SET UP:** Let $+y$ be upward. Consider an infinitesimal time interval *dt*. In Example 8.15, $v_{ex} = 2400\text{ m/s}$ and $\frac{dm}{dt} = -\frac{m_0}{120\text{ s}}$. In Example 8.16, $m = m_0/4$ after $t = 90\text{ s}$.

**EXECUTE:** **(a)** The impulse-momentum theorem gives $-mgdt = (m+dm)(v+dv)+(dm)(v-v_{ex})-mv$. This simplifies to $-mgdt = mdv + v_{ex}dm$ and $m\frac{dv}{dt} = -v_{ex}\frac{dm}{dt} - mg$.

**(b)** $a = \frac{dv}{dt} = -\frac{v_{ex}}{m}\frac{dm}{dt} - g$.

**(c)** At $t = 0$, $a = -\frac{v_{ex}}{m_0}\frac{dm}{dt} - g = -(2400\text{ m/s})\left(-\frac{1}{120\text{ s}}\right) - 9.80\text{ m/s}^2 = 10.2\text{ m/s}^2$.

**(d)** $dv = -\frac{v_{ex}}{m}dm - gdt$. Integrating gives $v - v_0 = +v_{ex}\ln\frac{m_0}{m} - gt$. $v_0 = 0$ and $v = +(2400\text{ m/s})\ln 4 - (9.80\text{ m/s}^2)(90\text{ s}) = 2445\text{ m/s}$.

**EVALUATE:** Both the initial acceleration in Example 8.15 and the final speed of the rocket in Example 8.16 are reduced by the presence of gravity.

**8.111.** **IDENTIFY** and **SET UP:** Apply Eq. 8.40 to the single-stage rocket and to each stage of the two-stage rocket.

**(a) EXECUTE:** $v - v_0 = v_{ex}\ln(m_0/m)$; $v_0 = 0$ so $v = v_{ex}\ln(m_0/m)$

The total initial mass of the rocket is $m_0 = 12{,}000\text{ kg} + 1000\text{ kg} = 13{,}000\text{ kg}$. Of this, $9000\text{ kg} + 700\text{ kg} = 9700\text{ kg}$ is fuel, so the mass *m* left after all the fuel is burned is $13{,}000\text{ kg} - 9700\text{ kg} = 3300\text{ kg}$.

$$v = v_{ex}\ln(13{,}000\text{ kg}/3300\text{ kg}) = 1.37v_{ex}.$$

**(b)** First stage: $v = v_{ex}\ln(m_0/m)$

$m_0 = 13{,}000\text{ kg}$

The first stage has 9000 kg of fuel, so the mass left after the first stage fuel has burned is $13{,}000\text{ kg} - 9000\text{ kg} = 4000\text{ kg}$.

$$v = v_{ex}\ln(13{,}000\text{ kg}/4000\text{ kg}) = 1.18v_{ex}.$$

**(c)** Second stage: $m_0 = 1000\text{ kg}$, $m = 1000\text{ kg} - 700\text{ kg} = 300\text{ kg}$.

$$v = v_0 + v_{ex}\ln(m_0/m) = 1.18v_{ex} + v_{ex}\ln(1000\text{ kg}/300\text{ kg}) = 2.38v_{ex}.$$

**(d)** $v = 7.00\text{ km/s}$

$$v_{ex} = v/2.38 = (7.00\text{ km/s})/2.38 = 2.94\text{ km/s}.\ \text{q}$$

**EVALUATE:** The two-stage rocket achieves a greater final speed because it jetisons the left-over mass of the first stage before the second-state fires and this reduces the final *m* and increases $m_0/m$.

**8.112.** **IDENTIFY:** During an interval where the mass is constant the speed of the rocket is constant. During an interval where the mass is changing at a constant rate, the equations of Section 8.6 apply.

**SET UP:** For $0 \le t \le 90\text{ s}$, $\frac{dm}{dt} = -\frac{m_0}{120\text{ s}}$. From Example 8.15, $v_{ex} = 2400\text{ m/s}$.

**EXECUTE:** **(a)** For $t \le 0$, $v = 0$. For $0 \le t \le 90\text{ s}$, Eq. 8.40 says $v = (2400\text{ m/s})\ln 4 = 3327\text{ m/s}$. For $t > 90\text{ s}$, *v* has the constant value 3327 m/s. The graph of $v(t)$ is given in Fig. 8.112a.

**(b)** For $0 \le t \le 90\text{ s}$, Eq. 8.39 gives $a = -\frac{v_{ex}}{m}\frac{dm}{dt} = -\frac{2400\text{ m/s}}{m_0(1-t/[120\text{ s}])}\left(-\frac{m_0}{120\text{ s}}\right) = \frac{20\text{ m/s}^2}{1-t/[120\text{ s}]}$. $a = 20\text{ m/s}^2$ at $t = 0$ (as in Example 8.15) and $a = 80\text{ m/s}^2$ at $t = 90\text{ s}$. For $t > 90\text{ s}$, $a = 0$. The graph of $a(t)$ is given in Fig. 8.112b.

**(c)** The astronaut has the same acceleration as the rocket. This is maximum at $t = 90\text{ s}$ and $F_{max} = m_{astronaut}a_{max} = (75\text{ kg})(80\text{ m/s}^2) = 6.0\times10^3\text{ N}$. This is 8.2 times her weight on earth, since $a_{max}$ is 8.2 times *g*.

**EVALUATE:** The acceleration increases because the mass decreases while the thrust $F = -v_{ex}\dfrac{dm}{dt}$ remains constant.

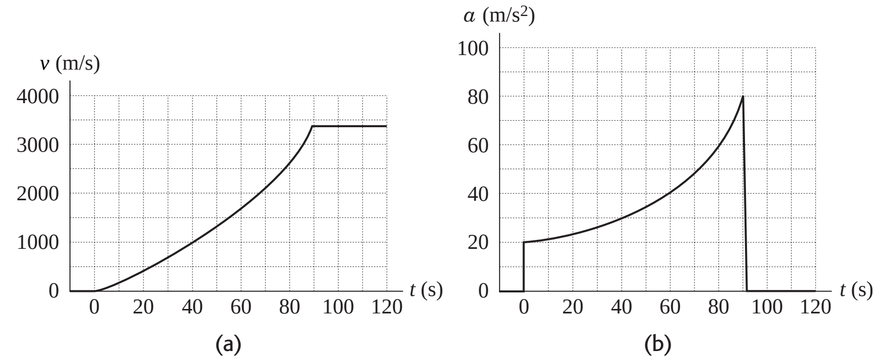


**Figure 8.112**

**8.113. IDENTIFY and SET UP:** $dm = \rho dV$ . $dV = Adx$ . Since the thin rod lies along the $x$ axis, $y_{cm} = 0$ . The mass of the rod is given by $M = \int dm$ .

**EXECUTE:** **(a)** $x_{cm} = \dfrac{1}{M}\int_0^L xdm = \dfrac{\rho}{M}A\int_0^L xdx = \dfrac{\rho A}{M}\dfrac{L^2}{2}$ . The volume of the rod is $AL$ and $M = \rho AL$ .

$x_{cm} = \dfrac{\rho AL^2}{2\rho AL} = \dfrac{L}{2}$ . The center of mass of the uniform rod is at its geometrical center, midway between its ends.

**(b)** $x_{cm} = \dfrac{1}{M}\int_0^L xdm = \dfrac{1}{M}\int_0^L x\rho Adx = \dfrac{A\alpha}{M}\int_0^L x^2dx = \dfrac{A\alpha L^3}{3M}.$ $M = \int dm = \int_0^L \rho Adx = \alpha A\int_0^L xdx = \dfrac{\alpha AL^2}{2}.$ Therefore,

$x_{cm} = \left(\dfrac{A\alpha L^3}{3}\right)\left(\dfrac{2}{\alpha AL^2}\right) = \dfrac{2L}{3}.$

**EVALUATE:** When the density increases with $x$, the center of mass is to the right of the center of the rod.

**8.114. IDENTIFY:** $x_{cm} = \dfrac{1}{M}\int xdm$ and $y_{cm} = \dfrac{1}{M}\int ydm.$ At the upper surface of the plate, $y^2 + x^2 = a^2.$

**SET UP:** To find $x_{cm}$ , divide the plate into thin strips parallel to the $y$-axis, as shown in Fig. 8.114a. To find $y_{cm}$ , divide the plate into thin strips parallel to the $x$-axis as shown in Fig. 8.114b. The plate has volume one-half that of a circular disk, so $V = \frac{1}{2}\pi a^2 t$ and $M = \frac{1}{2}\rho\pi a^2 t.$

**EXECUTE:** In Fig.114a each strip has length $y = \sqrt{a^2 - x^2}.$ $x_{cm} = \dfrac{1}{M}\int xdm,$ where $dm = \rho tydx = \rho t\sqrt{a^2 - x^2}dx.$

$x_{cm} = \dfrac{\rho t}{M}\int_{-a}^{a} x\sqrt{a^2 - x^2}dx = 0,$ since the integrand is an odd function of $x$. $x_{cm} = 0$ because of symmetry. In Fig.114b each strip has length $2x = 2\sqrt{a^2 - y^2}.$ $y_{cm} = \dfrac{1}{M}\int ydm,$ where $dm = 2\rho txdy = 2\rho t\sqrt{a^2 - y^2}dy.$

$y_{cm} = \dfrac{2\rho t}{M}\int_0^a y\sqrt{a^2 - y^2}dy$ . The integral can be evaluated using $u = a^2 - y^2$ , $du = -2ydy$ . This substitution gives

$y_{cm} = \dfrac{2\rho t}{M}\left(-\dfrac{1}{2}\right)\int_{a^2}^{0} u^{1/2}du = \dfrac{2\rho ta^3}{3M} = \left(\dfrac{2\rho ta^3}{3}\right)\left(\dfrac{2}{\rho\pi a^2 t}\right) = \dfrac{4a}{3\pi}.$

**EVALUATE:** $\dfrac{4}{3\pi} = 0.424.$ $y_{cm}$ is less than $a/2,$ as expected, since the plate becomes wider as $y$ decreases.

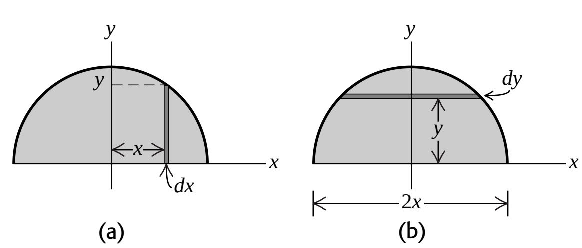


**Figure 8.114**

**8.115.** **IDENTIFY:** The work is related to the force by $W = \int_{x_1}^{x_2} F dx$. The force the person must apply equals the weight of the hanging portion. Since the rope is uniform, the center of mass of the hanging portion is at its geometrical center.

**SET UP:** Let $y$ be the length of the rope hanging over the edge and use coordinates where the origin is at the edge of the table and $+y$ is downward. When the rope is pulled onto the table, $y$ goes from $l/4$ to zero. A length $y$ of the rope has mass $\lambda y$.

**EXECUTE:** **(a)** When a length $y$ hangs over the edge, the person must apply an upward force

$F_y = -m(y)g = -\lambda y g$. $W = \int_{l/4}^{0} F_y(y)dy = -\lambda g \int_{l/4}^{0} y dy = \dfrac{\lambda g l^2}{32}$.

**(b)** Initially, $y_{\text{cm}} = l/8$. The work done to raise an object of mass $M$ a distance $y_{\text{cm}}$ is $W = Mgy_{\text{cm}}$.

$W = \left(\dfrac{\lambda l}{4}\right) g \left(\dfrac{l}{8}\right) = \dfrac{\lambda g l^2}{32}$.

**EVALUATE:** The answers from methods (a) and (b) agree. The change in gravitational potential energy of the rope can be calculated by considering all its mass acting at its center of mass, and the work done by the person equals the increase in gravitational potential energy of the rope.

**8.116.** **IDENTIFY:** From our analysis of motion with constant acceleration, if $v = at$ and $a$ is constant, then $x - x_0 = v_0 t + \frac{1}{2}at^2$.

**SET UP:** Take $v_0 = 0$, $x_0 = 0$ and let $+x$ downward.

**EXECUTE:** **(a)** $\dfrac{dv}{dt} = a$, $v = at$ and $x = \frac{1}{2}at^2$. Substituting into $xg = x\dfrac{dv}{dt} + v^2$ gives

$\frac{1}{2}at^2 g = \frac{1}{2}at^2 a + a^2t^2 = \frac{3}{2}a^2t^2$. The nonzero solution is $a = g/3$.

**(b)** $x = \frac{1}{2}at^2 = \frac{1}{6}gt^2 = \frac{1}{6}(9.80\text{ m/s}^2)(3.00\text{ s})^2 = 14.7\text{ m}$.

**(c)** $m = kx = (2.00\text{ g/m})(14.7\text{ m}) = 29.4\text{ g}$.

**EVALUATE:** The acceleration is less than $g$ because the small water droplets are initially at rest, before they adhere to the falling drop. The small droplets are suspended by buoyant forces that we ignore for the raindrops.