6

WORK AND KINETIC ENERGY

6.1. **IDENTIFY:** Apply Eq.(6.2).

SET UP: The bucket rises slowly, so the tension in the rope may be taken to be the bucket's weight.

EXECUTE: **(a)** $W = Fs = mgs = (6.75 \text{ kg})\ (9.80 \text{ m/s}^2)(4.00 \text{ m}) = 265 \text{ J}.$

(b) Gravity is directed opposite to the direction of the bucket's motion, so Eq.(6.2) gives the negative of the result of part (a), or -265 J .

(c) The total work done on the bucket is zero.

EVALUATE: When the force is in the direction of the displacement, the force does positive work. When the force is directed opposite to the displacement, the force does negative work.

6.2. **IDENTIFY:** In each case the forces are constant and the displacement is along a straight line, so $W = Fs\cos\phi$.

SET UP: In part (a), when the cable pulls horizontally $\phi = 0°$ and when it pulls at $35.0°$ above the horizontal $\phi = 35.0°$. In part (b), if the cable pulls horizontally $\phi = 180°$. If the cable pulls on the car at $35.0°$ above the horizontal it pulls on the truck at $35.0°$ below the horizontal and $\phi = 145.0°$. For the gravity force $\phi = 90°$, since the force is vertical and the displacement is horizontal.

EXECUTE: **(a)** When the cable is horizontal, $W = (850 \text{ N})(5.00\times10^3 \text{ m})\cos 0° = 4.25\times10^6 \text{ J}$. When the cable is $35.0°$ above the horizontal, $W = (850 \text{ N})(5.00\times10^3 \text{ m})\cos 35.0° = 3.48\times10^6 \text{ J}$.

(b) $\cos 180° = -\cos 0°$ and $\cos 145.0° = -\cos 35.0°$, so the answers are -4.26×10^6 J and -3.48×10^6 J .

(c) Since $\cos\phi = \cos 90° = 0$, $W = 0$ in both cases.

EVALUATE: If the car and truck are taken together as the system, the tension in the cable does no net work.

6.3. **IDENTIFY:** Each force can be used in the relation $W = F_{\parallel}s = (F\cos\phi)s$ for parts (b) through (d). For part (e), apply the net work relation as $W_{\text{net}} = W_{\text{worker}} + W_{\text{grav}} + W_n + W_f$.

SET UP: In order to move the crate at constant velocity, the worker must apply a force that equals the force of friction, $F_{\text{worker}} = f_k = \mu_k n.$

EXECUTE: **(a)** The magnitude of the force the worker must apply is:

$$F_{\text{worker}} = f_k = \mu_k n = \mu_k mg = (0.25)(30.0 \text{ kg})(9.80 \text{ m/s}^2) = 74 \text{ N}$$

(b) Since the force applied by the worker is horizontal and in the direction of the displacement, $\phi = 0°$ and the work is:

$$W_{\text{worker}} = (F_{\text{worker}}\cos\phi)s = [(74 \text{ N})(\cos 0°)](4.5 \text{ m}) = +333 \text{ J}$$

(c) Friction acts in the direction opposite of motion, thus $\phi = 180°$ and the work of friction is:

$$W_f = (f_k\cos\phi)s = [(74 \text{ N})(\cos 180°)](4.5 \text{ m}) = -333 \text{ J}$$

(d) Both gravity and the normal force act perpendicular to the direction of displacement. Thus, neither force does any work on the crate and $W_{\text{grav}} = W_n = 0.0$ J.

(e) Substituting into the net work relation, the net work done on the crate is:

$$W_{\text{net}} = W_{\text{worker}} + W_{\text{grav}} + W_n + W_f = +333 \text{ J} + 0.0 \text{ J} + 0.0 \text{ J} - 333 \text{ J} = 0.0 \text{ J}$$

EVALUATE: The net work done on the crate is zero because the two contributing forces, F_{worker} and F_f, are equal in magnitude and opposite in direction.

6.4. **IDENTIFY:** The forces are constant so Eq.(6.2) can be used to calculate the work. Constant speed implies $a = 0.$ We must use $\sum\vec{F} = m\vec{a}$ applied to the crate to find the forces acting on it.

(a) SET UP: The free-body diagram for the crate is given in Figure 6.4.

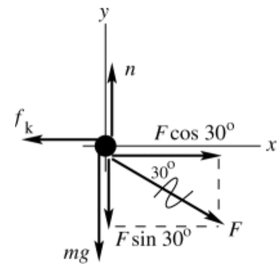


Figure 6.4

EXECUTE: $\sum F_y = ma_y$

$n - mg - F\sin 30° = 0$

$n = mg + F\sin 30°$

$f_k = \mu_k n = \mu_k mg + F\mu_k \sin 30°$

$\sum F_x = ma_x$

$F\cos 30° - f_k = 0$

$F\cos 30° - \mu_k mg - \mu_k \sin 30° F = 0$

$$F = \frac{\mu_k mg}{\cos 30° - \mu_k \sin 30°} = \frac{0.25(30.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 30° - (0.25)\sin 30°} = 99.2\text{ N}$$

(b) $W_F = (F\cos\phi)s = (99.2\text{ N})(\cos 30°)(4.5\text{ m}) = 387\text{ J}$

($F\cos 30°$ is the horizontal component of $\vec{F}$; the work done by $\vec{F}$ is the displacement times the component of $\vec{F}$ in the direction of the displacement.)

(c) We have an expression for f_k from part (a):

$$f_k = \mu_k(mg + F\sin 30°) = (0.250)[(30.0\text{ kg})(9.80\text{ m/s}^2) + (99.2\text{ N})(\sin 30°)] = 85.9\text{ N}$$

$\phi = 180°$ since f_k is opposite to the displacement. Thus $W_f = (f_k\cos\phi)s = (85.9\text{ N})(\cos 180°)(4.5\text{ m}) = -387\text{ J}$

(d) The normal force is perpendicular to the displacement so $\phi = 90°$ and $W_n = 0$. The gravity force (the weight) is perpendicular to the displacement so $\phi = 90°$ and $W_w = 0$

(e) $W_{tot} = W_F + W_f + W_n + W_w = +387\text{ J} + (-387\text{ J}) = 0$

EVALUATE: Forces with a component in the direction of the displacement do positive work, forces opposite to the displacement do negative work and forces perpendicular to the displacement do zero work. The total work, obtained as the sum of the work done by each force, equals the work done by the net force. In this problem, $F_{net} = 0$ since $a = 0$ and $W_{tot} = 0$, which agrees with the sum calculated in part (e).

6.5. IDENTIFY: The gravity force is constant and the displacement is along a straight line, so $W = Fs\cos\phi$.

SET UP: The displacement is upward along the ladder and the gravity force is downward, so $\phi = 180.0° - 30.0° = 150.0°$. $w = mg = 735\text{ N}$.

EXECUTE: **(a)** $W = (735\text{ N})(2.75\text{ m})\cos 150.0° = -1750\text{ J}$.

(b) No, the gravity force is independent of the motion of the painter.

EVALUATE: Gravity is downward and the vertical component of the displacement is upward, so the gravity force does negative work.

6.6. IDENTIFY and SET UP: $W_F = (F\cos\phi)s$, since the forces are constant. We can calculate the total work by summing the work done by each force. The forces are sketched in Figure 6.6.

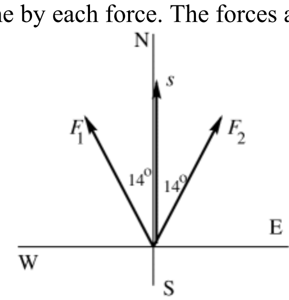


Figure 6.6

EXECUTE: $W_1 = F_1 s\cos\phi_1$

$W_1 = (1.80\times 10^6\text{ N})(0.75\times 10^3\text{ m})\cos 14°$

$W_1 = 1.31\times 10^9\text{ J}$

$W_2 = F_2 s\cos\phi_2 = W_1$

$W_{tot} = W_1 + W_2 = 2(1.31\times 10^9\text{ J}) = 2.62\times 10^9\text{ J}$

EVALUATE: Only the component $F\cos\phi$ of force in the direction of the displacement does work. These components are in the direction of $\vec{s}$ so the forces do positive work.

6.7. **IDENTIFY:** All forces are constant and each block moves in a straight line. so $W = Fs\cos\phi$. The only direction the system can move at constant speed is for the 12.0 N block to descend and the 20.0 N block to move to the right.

SET UP: Since the 12.0 N block moves at constant speed, $a = 0$ for it and the tension T in the string is $T = 12.0\text{ N}$. Since the 20.0 N block moves to the right at constant speed the friction force f_k on it is to the left and $f_k = T = 12.0\text{ N}$.

EXECUTE: **(a)** (i) $\phi = 0°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 0° = 9.00\text{ J}$. (ii) $\phi = 180°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 180° = -9.00\text{ J}$.

(b) (i) $\phi = 90°$ and $W = 0$. (ii) $\phi = 0°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 0° = 9.00\text{ J}$. (iii) $\phi = 180°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 180° = -9.00\text{ J}$. (iv) $\phi = 90°$ and $W = 0$.

(c) $W_{tot} = 0$ for each block.

EVALUATE: For each block there are two forces that do work, and for each block the two forces do work of equal magnitude and opposite sign. When the force and displacement are in opposite directions, the work done is negative.

6.8. **IDENTIFY:** Apply Eq.(6.5).

SET UP: $\hat{\boldsymbol{i}} \cdot \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}} \cdot \hat{\boldsymbol{j}} = 1$ and $\hat{\boldsymbol{i}} \cdot \hat{\boldsymbol{j}} = \hat{\boldsymbol{j}} \cdot \hat{\boldsymbol{i}} = 0$

EXECUTE: The work you do is $\vec{F} \cdot \vec{s} = ((30\text{ N})\hat{\boldsymbol{i}} - (40\text{ N})\hat{\boldsymbol{j}}) \cdot ((-9.0\text{ m})\hat{\boldsymbol{i}} - (3.0\text{ m})\hat{\boldsymbol{j}})$

$\vec{F} \cdot \vec{s} = (30\text{ N})(-9.0\text{ m}) + (-40\text{ N})(-3.0\text{ m}) = -270\text{ N}\cdot\text{m} + 120\text{ N}\cdot\text{m} = -150\text{ J}$.

EVALUATE: The x-component of $\vec{F}$ does negative work and the y-component of $\vec{F}$ does positive work. The total work done by $\vec{F}$ is the sum of the work done by each of its components.

6.9. **IDENTIFY:** Apply Eq.(6.2) or (6.3).

SET UP: The gravity force is in the $-y$-direction, so $\vec{F}_{mg} \cdot \vec{s} = -mg(y_2 - y_1)$

EXECUTE: **(a)** (i) Tension force is always perpendicular to the displacement and does no work. (ii) Work done by gravity is $-mg(y_2 - y_1)$. When $y_1 = y_2$, $W_{mg} = 0$.

(b) (i) Tension does no work. (ii) Let l be the length of the string. $W_{mg} = -mg(y_2 - y_1) = -mg(2l) = -25.1\text{ J}$

EVALUATE: In part (b) the displacement is upward and the gravity force is downward, so the gravity force does negative work.

6.10. **IDENTIFY:** $K = \frac{1}{2}mv^2$

SET UP: 65 mi/h = 29.1 m/s

EXECUTE: **(a)** $K = \frac{1}{2}(750\text{ kg})(29.1\text{ m/s})^2 = 3.18 \times 10^5\text{ J}$

(b) $K_1 = \frac{1}{2}mv_1^2$. $K_2 = \frac{1}{2}mv_2^2$, with $v_2 = v_1/2$, so $K_2 = \frac{1}{2}m(v_1/2)^2 = \frac{1}{4}(\frac{1}{2}mv_1^2) = K_1/4$. The change in kinetic energy is a decrease of $\frac{3}{4}K_1$.

(c) $K_2 = \frac{1}{2}K_1$. $\dfrac{K}{v^2} = \dfrac{m}{2} =$ constant, so $\dfrac{K_1}{v_1^2} = \dfrac{K_2}{v_2^2}$. $v_2 = v_1\sqrt{K_2/K_1} = (65\text{ mi/h})\sqrt{\frac{1}{2}K_1/K_1} = 46\text{ mi/h}$.

EVALUATE: Since $K \sim v^2$, to have half the kinetic energy the speed must be less than half of the original speed.

6.11. **IDENTIFY:** $K = \frac{1}{2}mv^2$. Since the meteor comes to rest the energy it delivers to the ground equals its original kinetic energy.

SET UP: $v = 12\text{ km/s} = 1.2 \times 10^4\text{ m/s}$. A 1.0 megaton bomb releases 4.184×10^{15} J of energy.

EXECUTE: **(a)** $K = \frac{1}{2}(1.4 \times 10^8\text{ kg})(1.2 \times 10^4\text{ m/s})^2 = 1.0 \times 10^{16}\text{ J}$.

(b) $\dfrac{1.0 \times 10^{16}\text{ J}}{4.184 \times 10^{15}\text{ J}} = 2.4$. The energy is equivalent to 2.4 one-megaton bombs.

EVALUATE: Part of the energy transferred to the ground lifts soil and rocks into the air and creates a large crater.

6.12. **IDENTIFY:** $K = \frac{1}{2}mv^2$. Use the equations for free-fall to find the speed of the weight when it reaches the ground.

SET UP: Estimate that a person has speed 2 m/s when walking and 6 m/s when running. The mass of an electron is 9.11×10^{-31} kg. In part (c) take $+y$ downward, so $a_y = +9.80\text{ m/s}^2$. Estimate a shoulder height of 1.6 m.

EXECUTE: **(a)** Walking: $K = \frac{1}{2}(75\text{ kg})(2\text{ m/s})^2 = 150\text{ J}$. Running: $K = \frac{1}{2}(75\text{ kg})(6\text{ m/s})^2 = 1400\text{ J}$.

(b) $K = \frac{1}{2}(9.11 \times 10^{-31}\text{ kg})(2.19 \times 10^6\text{ m/s})^2 = 2.2 \times 10^{-18}\text{ J}$.

(c) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(9.80\text{ m/s}^2)(1.6\text{ m})} = 5.6\text{ m/s}$. $K = \frac{1}{2}(1.0\text{ kg})(5.6\text{ m/s})^2 = 16\text{ J}$.

(d) $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(100\ \text{J})}{30\ \text{kg}}} = 2.6\ \text{m/s}$. Yes, this is reasonable.

EVALUATE: A walking speed of 2 m/s corresponds to walking a mile in about 13 min. A running speed of 6 m/s corresponds to running a 100 m dash in about 17 s.

6.13. **IDENTIFY:** $K = \frac{1}{2}mv^2$. Set up a ratio that relates *K*, *m* and *v*.

SET UP: $m_p = 1836 m_e$

EXECUTE: **(a)** $K_p = K_e$ gives $m_e v_e^2 = m_p v_p^2$. $v_e = v_p\sqrt{m_p/m_e} = V\sqrt{1836} = 42.85V$.

(b) $v_p = v_e$ gives $\dfrac{K_p}{m_p} = \dfrac{K_e}{m_e}$. $K_p = K_e(m_p/m_e) = 1836K$.

EVALUATE: The electron has less mass so must travel faster to have the same kinetic energy. And with equal speeds the proton has more kinetic energy.

6.14. **IDENTIFY:** Only gravity does work on the watermelon, so $W_{tot} = W_{grav}$. $W_{tot} = \Delta K$ and $K = \frac{1}{2}mv^2$.

SET UP: Since the watermelon is dropped from rest, $K_1 = 0$.

EXECUTE: **(a)** $W_{grav} = mgs = (4.80\ \text{kg})(9.80\ \text{m/s}^2)(25.0\ \text{m}) = 1180\ \text{J}$

(b) $W_{tot} = K_2 - K_1$ so $K_2 = 1180\ \text{J}$. $v = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(1180\ \text{J})}{4.80\ \text{kg}}} = 22.2\ \text{m/s}$.

(c) The work done by gravity would be the same. Air resistance would do negative work and W_{tot} would be less than W_{grav} . The answer in (a) would be unchanged and both answers in (b) would decrease.

EVALUATE: The gravity force is downward and the displacement is downward, so gravity does positive work.

6.15. **IDENTIFY:** $W_{tot} = K_2 - K_1$. In each case calculate W_{tot} from what we know about the force and the displacement.

SET UP: The gravity force is *mg*, downward. The friction force is $f_k = \mu_k n = \mu_k mg$ and is directed opposite to the displacement. The mass of the object isn't given, so we expect that it will divide out in the calculation.

EXECUTE: **(a)** $K_1 = 0$. $W_{tot} = W_{grav} = mgs$. $mgs = \frac{1}{2}mv_2^2$ and $v_2 = \sqrt{2gs} = \sqrt{2(9.80\ \text{m/s}^2)(95.0\ \text{m})} = 43.2\ \text{m/s}$.

(b) $K_2 = 0$ (at the maximum height). $W_{tot} = W_{grav} = -mgs$. $-mgs = -\frac{1}{2}mv_1^2$ and

$v_1 = \sqrt{2gs} = \sqrt{2(9.80\ \text{m/s}^2)(525\ \text{m})} = 101\ \text{m/s}$.

(c) $K_1 = \frac{1}{2}mv_1^2$. $K_2 = 0$. $W_{tot} = W_f = -\mu_k mgs$. $-\mu_k mgs = -\frac{1}{2}mv_1^2$. $s = \dfrac{v_1^2}{2\mu_k g} = \dfrac{(5.00\ \text{m/s})^2}{2(0.220)(9.80\ \text{m/s}^2)} = 5.80\ \text{m}$.

(d) $K_1 = \frac{1}{2}mv_1^2$. $K_2 = \frac{1}{2}mv_2^2$. $W_{tot} = W_f = -\mu_k mgs$. $K_2 = W_{tot} + K_1$. $\frac{1}{2}mv_2^2 = -\mu_k mgs + \frac{1}{2}mv_1^2$

$v_2 = \sqrt{v_1^2 - 2\mu_k gs} = \sqrt{(5.00\ \text{m/s})^2 - 2(0.220)(9.80\ \text{m/s}^2)(2.90\ \text{m})} = 3.53\ \text{m/s}$.

(e) $K_1 = \frac{1}{2}mv_1^2$. $K_2 = 0$. $W_{grav} = -mgy_2$, where y_2 is the vertical height. $-mgy_2 = -\frac{1}{2}mv_1^2$ and

$y_2 = \dfrac{v_1^2}{2g} = \dfrac{(12.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 7.35\ \text{m}$.

EVALUATE: In parts (c) and (d), friction does negative work and the kinetic energy is reduced. In part (a), gravity does positive work and the speed increases. In parts (b) and (e), gravity does negative work and the speed decreases. The vertical height in part (e) is independent of the slope angle of the hill.

6.16. **IDENTIFY:** From the work-energy relation, $W = W_{grav} = \Delta K_{rock}$.

SET UP: As the rock rises, the gravitational force, $F = mg$, does work on the rock. Since this force acts in the direction opposite to the motion and displacement, *s*, the work is negative. Let *h* be the vertical distance the rock travels.

EXECUTE: **(a)** Applying $W_{grav} = K_2 - K_1$ we obtain $-mgh = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$. Dividing by *m* and solving for v_1 , $v_1 = \sqrt{v_2^2 + 2gh}$. Substituting $h = 15.0$ m and $v_2 = 25.0$ m/s,

$$v_1 = \sqrt{(25.0\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(15.0\ \text{m})} = 30.3\ \text{m/s}$$

(b) Solve the same work-energy relation for *h*. At the maximum height $v_2 = 0$.

$-mgh = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ and $h = \dfrac{v_1^2 - v_2^2}{2g} = \dfrac{(30.3\ \text{m/s})^2 - (0.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 46.8\ \text{m}$.

EVALUATE: Note that the weight of 20 N was never used in the calculations because both gravitational potential and kinetic energy are proportional to mass, *m*. Thus any object, that attains 25.0 m/s at a height of 15.0 m, must have an initial velocity of 30.3 m/s. As the rock moves upward gravity does negative work and this reduces the kinetic energy of the rock.

6.17. **IDENTIFY** and **SET UP:** Apply Eq.(6.6) to the box. Let point 1 be at the bottom of the incline and let point 2 be at the skier. Work is done by gravity and by friction. Solve for K_1 and from that obtain the required initial speed.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2,\;\; K_2 = 0$

Work is done by gravity and friction, so $W_{\text{tot}} = W_{mg} + W_f$.

$W_{mg} = -mg(y_2 - y_1) = -mgh$

$W_f = -fs$. The normal force is $n = mg\cos\alpha$ and $s = h/\sin\alpha$, where *s* is the distance the box travels along the incline.

$W_f = -(\mu_k mg\cos\alpha)(h/\sin\alpha) = -\mu_k mgh/\tan\alpha$

Substituting these expressions into the work-energy theorem gives

$-mgh - \mu_k mgh/\tan\alpha = -\frac{1}{2}mv_0^2$.

Solving for v_0 then gives $v_0 = \sqrt{2gh(1 + \mu_k/\tan\alpha)}$.

EVALUATE: The result is independent of the mass of the box. As $\alpha \to 90°$, $h = s$ and $v_0 = \sqrt{2gh}$, the same as throwing the box straight up into the air. For $\alpha = 90°$ the normal force is zero so there is no friction.

6.18. **IDENTIFY:** Apply $W = Fs\cos\phi$ and $W_{\text{tot}} = \Delta K$.

SET UP: Parallel to incline: force component $W_{\parallel} = mg\sin\alpha$, down incline; displacement $s = h/\sin\alpha$, down incline. Perpendicular to the incline: $s = 0$.

EXECUTE: **(a)** $W_{\parallel} = (mg\sin\alpha)(h/\sin\alpha) = mgh$. $W_{\perp} = 0$, since there is no displacement in this direction.

$W_{mg} = W_{\parallel} + W_{\perp} = mgh$, same as falling height *h*.

(b) $W_{\text{tot}} = K_2 - K_1$ gives $mgh = \frac{1}{2}mv^2$ and $v = \sqrt{2gh}$, same as if had been dropped from height *h*. The work done by gravity depends only on the vertical displacement of the object. When the slope angle is small, there is a small force component in the direction of the displacement but a large displacement in this direction. When the slope angle is large, the force component in the direction of the displacement along the incline is larger but the displacement in this direction is smaller.

(c) $h = 15.0\text{ m}$, so $v = \sqrt{2gh} = 17.1\text{ s}$.

EVALUATE: The acceleration and time of travel are different for an object sliding down an incline and an object in free-fall, but the final velocity is the same in these two cases.

6.19. **IDENTIFY:** $W_{\text{tot}} = K_2 - K_1$ with $W_{\text{tot}} = W_f$. The car stops, so $K_2 = 0$. In each case identify what is constant and set up a ratio.

SET UP: $W_f = -fs$, so $-fs = -\frac{1}{2}mv_0^2$.

EXECUTE: **(a)** $v_{0b} = 3v_{0a}$. $s_a = D$. *f* is constant. $\frac{v_0^2}{s} = \frac{2f}{m} = \text{constant}$, so $\frac{v_{0a}^2}{s_a} = \frac{v_{0b}^2}{s_b}$. $s_b = s_a\left(\frac{v_{0b}}{v_{0a}}\right)^2 = D(3)^2 = 9D$.

(b) $f_b = 3f_a$. v_0 is constant. $fs = \frac{1}{2}mv_0^2 = \text{constant}$, so $f_a s_a = f_b s_b$. $s_b = s_a\left(\frac{f_a}{f_b}\right) = D/3$.

EVALUATE: The stopping distance is proportional to the square of the initial speed. When the friction force increases, the stopping distance decreases.

6.20. **IDENTIFY** and **SET UP:** Apply Eq.(6.6). The relation between the speeds v_1 and v_2 tells us the relation between K_1 and K_2.

EXECUTE: **(a)** $W = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2,\;\; K_2 = \frac{1}{2}mv_2^2$

$v_2 = \frac{1}{4}v_1$ gives that $K_2 = \frac{1}{2}m\left(\frac{1}{4}v_1\right)^2 = \frac{1}{16}\left(\frac{1}{2}mv_1^2\right) = \frac{1}{16}K_1$

$W = K_2 - K_1 = \frac{1}{16}K_1 - K_1 = -\frac{15}{16}K_1$

(b) EVALUATE: *K* depends only on the magnitude of $\vec{v}$ not on its direction, so the answer for *W* in part (a) does *not* depend on the final direction of the electron's motion. The electron slows down, so its kinetic energy decreases and the total work done on it is negative.

6.21. **IDENTIFY:** Apply $W = Fs\cos\phi$ and $W_{\text{tot}} = \Delta K$.

SET UP: $\phi = 0°$

EXECUTE: From Equations (6.1), (6.5) and (6.6), and solving for *F*,

$$F = \frac{\Delta K}{s} = \frac{\frac{1}{2}m(v_2^2 - v_1^2)}{s} = \frac{\frac{1}{2}(8.00\ \text{kg})((6.00\ \text{m/s})^2 - (4.00\ \text{m/s})^2)}{(2.50\ \text{m})} = 32.0\ \text{N}.$$

EVALUATE: The force is in the direction of the displacement, so the force does positive work and the kinetic energy of the object increases.

6.22. **IDENTIFY** and **SET UP:** Use Eq.(6.6) to calculate the work done by the foot on the ball. Then use Eq.(6.2) to find the distance over which this force acts.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.420\ \text{kg})(2.00\ \text{m/s})^2 = 0.84\ \text{J}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.420\ \text{kg})(6.00\ \text{m/s})^2 = 7.56\ \text{J}$

$W_{\text{tot}} = K_2 - K_1 = 7.56\ \text{J} - 0.84\ \text{J} = 6.72\ \text{J}$

The 40.0 N force is the only force doing work on the ball, so it must do 6.72 J of work. $W_F = (F\cos\phi)s$ gives that

$$s = \frac{W}{F\cos\phi} = \frac{6.72\ \text{J}}{(40.0\ \text{N})(\cos 0)} = 0.168\ \text{m}$$

EVALUATE: The force is in the direction of the motion so positive work is done and this is consistent with an increase in kinetic energy.

6.23. **IDENTIFY:** Apply $W_{\text{tot}} = \Delta K$.

SET UP: $v_1 = 0$, $v_2 = v$. $f_k = \mu_k mg$ and f_k does negative work. The force $F = 36.0\ \text{N}$ is in the direction of the motion and does positive work.

EXECUTE: **(a)** If there is no work done by friction, the final kinetic energy is the work done by the applied force, and solving for the speed,

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2Fs}{m}} = \sqrt{\frac{2(36.0\ \text{N})(1.20\ \text{m})}{(4.30\ \text{kg})}} = 4.48\ \text{m/s}.$$

(b) The net work is $Fs - f_k s = (F - \mu_k mg)s$, so

$$v = \sqrt{\frac{2(F - \mu_k mg)s}{m}} = \sqrt{\frac{2(36.0\ \text{N} - (0.30)(4.30\ \text{kg})(9.80\ \text{m/s}^2))(1.20\ \text{m})}{(4.30\ \text{kg})}} = 3.61\ \text{m/s}$$

EVALUATE: The total work done is larger in the absence of friction and the final speed is larger in that case.

6.24. **IDENTIFY:** Apply $W = Fs\cos\phi$ and $W_{\text{tot}} = \Delta K$

SET UP: The gravity force has magnitude *mg* and is directed downward.

EXECUTE: **(a)** On the way up, gravity is opposed to the direction of motion, and so

$W = -mgs = -(0.145\ \text{kg})(9.80\ \text{m/s}^2)(20.0\ \text{m}) = -28.4\ \text{J}$.

(b) $v_2 = \sqrt{v_1^2 + 2\frac{W}{m}} = \sqrt{(25.0\ \text{m/s})^2 + \frac{2(-28.4\ \text{J})}{(0.145\ \text{kg})}} = 15.3\ \text{m/s}$.

(c) No; in the absence of air resistance, the ball will have the same speed on the way down as on the way up. On the way down, gravity will have done both negative and positive work on the ball, but the net work at this height will be the same.

EVALUATE: As the baseball moves upward, gravity does negative work and the speed of the baseball decreases.

6.25. **(a) IDENTIFY** and **SET UP:** Use Eq.(6.2) to find the work done by the positive force. Then use Eq.(6.6) to find the final kinetic energy, and then $K_2 = \frac{1}{2}mv_2^2$ gives the final speed.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$, so $K_2 = W_{\text{tot}} + K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(7.00\ \text{kg})(4.00\ \text{m/s})^2 = 56.0\ \text{J}$

The only force that does work on the wagon is the 10.0 N force. This force is in the direction of the displacement so $\phi = 0°$ and the force does positive work:

$$W_F = (F\cos\phi)s = (10.0\ \text{N})(\cos 0)(3.0\ \text{m}) = 30.0\ \text{J}$$

Then $K_2 = W_{\text{tot}} + K_1 = 30.0\ \text{J} + 56.0\ \text{J} = 86.0\ \text{J}$.

$$K_2 = \tfrac{1}{2}mv_2^2;\quad v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(86.0\ \text{J})}{7.00\ \text{kg}}} = 4.96\ \text{m/s}$$

(b) IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the wagon to calculate *a*. Then use a constant acceleration equation to calculate the final speed. The free-body diagram is given in Figure 6.25.

SET UP:

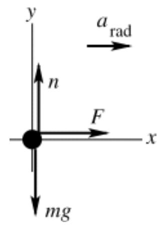


Figure 6.25

EXECUTE: $\sum F_x = ma_x$

$F = ma_x$

$a_x = \dfrac{F}{m} = \dfrac{10.0\text{ N}}{7.00\text{ kg}} = 1.43\text{ m/s}^2$

$v_{2x}^2 = v_{1x}^2 + 2a_2(x - x_0)$

$v_{2x} = \sqrt{v_{1x}^2 + 2a_x(x - x_0)} = \sqrt{(4.00\text{ m/s})^2 + 2(1.43\text{ m/s}^2)(3.0\text{ m})} = 4.96\text{ m/s}$

EVALUATE: This agrees with the result calculated in part (a). The force in the direction of the motion does positive work and the kinetic energy and speed increase. In part (b), the equivalent statement is that the force produces an acceleration in the direction of the velocity and this causes the magnitude of the velocity to increase.

6.26. IDENTIFY: Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $K_1 = 0$. The normal force does no work. The work *W* done by gravity is $W = mgh$, where $h = L\sin\theta$ is the vertical distance the block has dropped when it has traveled a distance *L* down the incline and θ is the angle the plane makes with the horizontal.

EXECUTE: The work-energy theorem gives $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2W}{m}} = \sqrt{2gh} = \sqrt{2gL\sin\theta}$. Using the given numbers,

$v = \sqrt{2(9.80\text{ m/s}^2)(0.75\text{ m})\sin 36.9°} = 2.97\text{ m/s}$.

EVALUATE: The final speed of the block is the same as if it had been dropped from a height *h*.

6.27. IDENTIFY: $W_{\text{tot}} = K_2 - K_1$. Only friction does work.

SET UP: $W_{\text{tot}} = W_{f_k} = -\mu_k mgs$. $K_2 = 0$ (car stops). $K_1 = \frac{1}{2}mv_0^2$.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$ gives $-\mu_k mgs = -\frac{1}{2}mv_0^2$. $s = \dfrac{v_0^2}{2\mu_k g}$.

(b) (i) $\mu_{kb} = 2\mu_{ka}$. $s\mu_k = \dfrac{v_0^2}{2g} = \text{constant}$ so $s_a\mu_{ka} = s_b\mu_{kb}$. $s_b = \left(\dfrac{\mu_{ka}}{\mu_{kb}}\right)s_a = s_a/2$. The minimum stopping distance would be halved. (ii) $v_{0b} = 2v_{0a}$. $\dfrac{s}{v_0^2} = \dfrac{1}{2\mu_k g} = \text{constant}$, so $\dfrac{s_a}{v_{0a}^2} = \dfrac{s_b}{v_{0b}^2}$. $s_b = s_a\left(\dfrac{v_{0b}}{v_{0a}}\right)^2 = 4s_a$. The stopping distance would become 4 times as great. (iii) $v_{0b} = 2v_{0a}$, $\mu_{kb} = 2\mu_{ka}$. $\dfrac{s\mu_k}{v_0^2} = \dfrac{1}{2g} = \text{constant}$, so $\dfrac{s_a\mu_{ka}}{v_{0a}^2} = \dfrac{s_b\mu_{kb}}{v_{0b}^2}$.

$s_b = s_a\left(\dfrac{\mu_{ka}}{\mu_{kb}}\right)\left(\dfrac{v_{0b}}{v_{0a}}\right)^2 = s_a\left(\dfrac{1}{2}\right)(2)^2 = 2s_a$. The stopping distance would double.

EVALUATE: The stopping distance is directly proportional to the square of the initial speed and indirectly proportional to the coefficient of kinetic friction.

6.28. IDENTIFY: The work that must be done to move the end of a spring from x_1 to x_2 is $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$. The force required to hold the end of the spring at displacement *x* is $F_x = kx$.

SET UP: When the spring is at its unstretched length, $x = 0$. When the spring is stretched, $x > 0$, and when the spring is compressed, $x < 0$.

EXECUTE: **(a)** $x_1 = 0$ and $W = \frac{1}{2}kx_2^2$. $k = \dfrac{2W}{x_2^2} = \dfrac{2(12.0\text{ J})}{(0.0300\text{ m})^2} = 2.67\times10^4\text{ N/m}$.

(b) $F_x = kx = (2.67\times10^4\text{ N/m})(0.0300\text{ m}) = 801\text{ N}$.

(c) $x_1 = 0$, $x_2 = -0.0400\text{ m}$. $W = \frac{1}{2}(2.67\times10^4\text{ N/m})(-0.0400\text{ m})^2 = 21.4\text{ J}$.

$F_x = kx = (2.67\times10^4\text{ N/m})(0.0400\text{ m}) = 1070\text{ N}$.

EVALUATE: When a spring, initially unstretched, is either compressed or stretched, positive work is done by the force that moves the end of the spring.

6.29. IDENTIFY and SET UP: Use Eq.(6.8) to calculate k for the spring. Then Eq.(6.10), with $x_1 = 0$, can be used to calculate the work done to stretch or compress the spring an amount x_2.

EXECUTE: Use the information given to calculate the force constant of the spring.

$F_x = kx$ gives $k = \dfrac{F_x}{x} = \dfrac{160\ \text{N}}{0.050\ \text{m}} = 3200\ \text{N/m}$

(a) $F_x = kx = (3200\ \text{N/m})(0.015\ \text{m}) = 48\ \text{N}$

$F_x = kx = (3200\ \text{N/m})(-0.020\ \text{m}) = -64\ \text{N}$ (magnitude 64 N)

(b) $W = \frac{1}{2}kx^2 = \frac{1}{2}(3200\ \text{N/m})(0.015\ \text{m})^2 = 0.36\ \text{J}$

$W = \frac{1}{2}kx^2 = \frac{1}{2}(3200\ \text{N/m})(-0.020\ \text{m})^2 = 0.64\ \text{J}$

Note that in each case the work done is positive.

EVALUATE: The force is not constant during the displacement so Eq.(6.2) *cannot* be used. A force in the $+x$ direction is required to stretch the spring and a force in the opposite direction to compress it. The force F_x is in the same direction as the displacement, so positive work is done in both cases.

6.30. IDENTIFY: The magnitude of the work can be found by finding the area under the graph.

SET UP: The area under each triangle is $1/2\ \text{base} \times \text{height}$. $F_x > 0$, so the work done is positive when x increases during the displacement.

EXECUTE: **(a)** $1/2\,(8\ \text{m})(10\ \text{N}) = 40\ \text{J}$.

(b) $1/2\,(4\ \text{m})(10\ \text{N}) = 20\ \text{J}$.

(c) $1/2\,(12\ \text{m})(10\ \text{N}) = 60\ \text{J}$.

EVALUATE: The sum of the answers to parts (a) and (b) equals the answer to part (c).

6.31. IDENTIFY: Use the work-energy theorem and the results of Problem 6.30.

SET UP: For $x = 0$ to $x = 8.0\ \text{m}$, $W_{\text{tot}} = 40\ \text{J}$. For $x = 0$ to $x = 12.0\ \text{m}$, $W_{\text{tot}} = 60\ \text{J}$.

EXECUTE: **(a)** $v = \sqrt{\dfrac{(2)(40\ \text{J})}{10\ \text{kg}}} = 2.83\ \text{m/s}$

(b) $v = \sqrt{\dfrac{(2)(60\ \text{J})}{10\ \text{kg}}} = 3.46\ \text{m/s}$.

EVALUATE: $\vec{F}$ is always in the $+x$-direction. For this motion $\vec{F}$ does positive work and the speed continually increases during the motion.

6.32. IDENTIFY: The force has only an x-component and the motion is along the x-direction, so $W = \int_{x_1}^{x_2} F_x dx$.

SET UP: $x_1 = 0$ and $x_2 = 6.9\ \text{m}$.

EXECUTE: The work you do with your changing force is

$W = \int_{x_1}^{x_2} F(x)dx = \int_{x_1}^{x_2} (-20.0\ \text{N})dx - \int_{x_1}^{x_2} (3.0\ \text{N/m})x dx = (-20.0\ \text{N})x\,|_{x_1}^{x_2} - (3.0\ \text{N/m})(x^2/2)\,|_{x_1}^{x_2}$

$W = -138\ \text{N}\cdot\text{m} - 71.4\ \text{N}\cdot\text{m} = -209\ \text{J}$.

EVALUATE: The work is negative because the cow continues to move forward (in the $+x$-direction) as you vainly attempt to push her backward.

6.33. IDENTIFY: Apply Eq.(6.6) to the box.

SET UP: Let point 1 be just before the box reaches the end of the spring and let point 2 be where the spring has maximum compression and the box has momentarily come to rest.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2$, $K_2 = 0$

Work is done by the spring force. $W_{\text{tot}} = -\frac{1}{2}kx_2^2$, where x_2 is the amount the spring is compressed.

$-\frac{1}{2}kx_2^2 = -\frac{1}{2}mv_0^2$ and $x_2 = v_0\sqrt{m/k} = (3.0\ \text{m/s})\sqrt{(6.0\ \text{kg})/(7500\ \text{N/m})} = 8.5\ \text{cm}$

EVALUATE: The compression of the spring increases when either v_0 or m increases and decreases when k increases (stiffer spring).

6.34. IDENTIFY: The force applied to the springs is $F_x = kx$. The work done on a spring to move its end from x_1 to x_2 is $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$. Use the information that is given to calculate k.

SET UP: When the springs are compressed 0.200 m from their uncompressed length, $x_1 = 0$ and $x_2 = -0.200\ \text{m}$. When the platform is moved 0.200 m farther, x_2 becomes $-0.400\ \text{m}$.

EVALUATE: **(a)** $k = \dfrac{2W}{x_2^2 - x_1^2} = \dfrac{2(80.0\text{ J})}{(0.200\text{ m})^2 - 0} = 4000\text{ N/m}$. $F_x = kx = (4000\text{ N/m})(-0.200\text{ m}) = -800\text{ N}$. The magnitude of force that is required is 800 N.

(b) To compress the springs from $x_1 = 0$ to $x_2 = -0.400\text{ m}$, the work required is $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 = \frac{1}{2}(4000\text{ N/m})(-0.400\text{ m})^2 = 320\text{ J}$. The additional work required is $320\text{ J} - 80\text{ J} = 240\text{ J}$. For $x = -0.400\text{ m}$, $F_x = kx = -1600\text{ N}$. The magnitude of force required is 1600 N.

EVALUATE: More work is required to move the end of the spring from $x = -0.200\text{ m}$ to $x = -0.400\text{ m}$ than to move it from $x = 0$ to $x = -0.200\text{ m}$, even though the displacement of the platform is the same in each case. The magnitude of the force increases as the compression of the spring increases.

6.35. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to calculate the μ_s required for the static friction force to equal the spring force.

SET UP: **(a)** The free-body diagram for the glider is given in Figure 6.35.

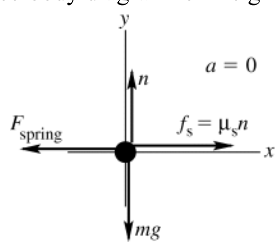


Figure 6.35

EXECUTE: $\sum F_y = ma_y$

$n - mg = 0$

$n = mg$

$f_s = \mu_s mg$

$\sum F_x = ma_x$

$f_s - F_{\text{spring}} = 0$

$\mu_s mg - kd = 0$

$$\mu_s = \frac{kd}{mg} = \frac{(20.0\text{ N/m})(0.086\text{ m})}{(0.100\text{ kg})(9.80\text{ m/s}^2)} = 1.76$$

(b) IDENTIFY and **SET UP:** Apply $\sum \vec{F} = m\vec{a}$ to find the maximum amount the spring can be compressed and still have the spring force balanced by friction. Then use $W_{\text{tot}} = K_2 - K_1$ to find the initial speed that results in this compression of the spring when the glider stops.

EXECUTE: $\mu_s mg = kd$

$$d = \frac{\mu_s mg}{k} = \frac{(0.60)(0.100\text{ kg})(9.80\text{ m/s}^2)}{20.0\text{ N/m}} = 0.0294\text{ m}$$

Now apply the work-energy theorem to the motion of the glider:

$W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2$, $K_2 = 0$ (instantaneously stops)

$W_{\text{tot}} = W_{\text{spring}} + W_{\text{fric}} = -\frac{1}{2}kd^2 - \mu_k mgd$ (as in Example 6.8)

$W_{\text{tot}} = -\frac{1}{2}(20.0\text{ N/m})(0.0294\text{ m})^2 - 0.47(0.100\text{ kg})(9.80\text{ m/s}^2)(0.0294\text{ m}) = -0.02218\text{ J}$

Then $W_{\text{tot}} = K_2 - K_1$ gives $-0.02218\text{ J} = -\frac{1}{2}mv_1^2$.

$$v_1 = \sqrt{\frac{2(0.02218\text{ J})}{0.100\text{ kg}}} = 0.67\text{ m/s}$$

EVALUATE: In Example 6.8 an initial speed of 1.50 m/s compresses the spring 0.086 m and in part (a) of this problem we found that the glider doesn't stay at rest. In part (b) we found that a smaller displacement of 0.0294 m when the glider stops is required if it is to stay at rest. And we calculate a smaller initial speed (0.67 m/s) to produce this smaller displacement.

6.36. **IDENTIFY:** For the spring, $W = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$. Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $x_1 = -0.025\text{ m}$ and $x_2 = 0$.

EXECUTE: **(a)** $W = \frac{1}{2}kx_1^2 = \frac{1}{2}(200\text{ N/m})(-0.025\text{ m})^2 = 0.060\text{ J}$.

(b) The work-energy theorem gives $v_2 = \sqrt{\dfrac{2W}{m}} = \sqrt{\dfrac{2(0.060\text{ J})}{(4.0\text{ kg})}} = 0.18\text{ m/s}$.

EVALUATE: The block moves in the direction of the spring force, the spring does positive work and the kinetic energy of the block increases.

6.37. **IDENTIFY** and **SET UP:** The magnitude of the work done by F_x equals the area under the F_x versus x curve. The work is positive when F_x and the displacement are in the same direction; it is negative when they are in opposite directions.

EXECUTE: **(a)** F_x is positive and the displacement Δx is positive, so $W > 0$.

$W = \frac{1}{2}(2.0 \text{ N})(2.0 \text{ m}) + (2.0 \text{ N})(1.0 \text{ m}) = +4.0 \text{ J}$

(b) During this displacement $F_x = 0$, so $W = 0$.

(c) F_x is negative, Δx is positive, so $W < 0$. $W = -\frac{1}{2}(1.0 \text{ N})(2.0 \text{ m}) = -1.0 \text{ J}$

(d) The work is the sum of the answers to parts (a), (b), and (c), so $W = 4.0 \text{ J} + 0 - 1.0 \text{ J} = +3.0 \text{ J}$

(e) The work done for $x = 7.0 \text{ m}$ to $x = 3.0 \text{ m}$ is $+1.0$ J. This work is positive since the displacement and the force are both in the $-x$-direction. The magnitude of the work done for $x = 3.0 \text{ m}$ to $x = 2.0 \text{ m}$ is 2.0 J, the area under F_x versus x. This work is negative since the displacement is in the $-x$-direction and the force is in the $+x$-direction. Thus $W = +1.0 \text{ J} - 2.0 \text{ J} = -1.0 \text{ J}$

EVALUATE: The work done when the car moves from $x = 2.0 \text{ m}$ to $x = 0$ is $-\frac{1}{2}(2.0 \text{ N})(2.0 \text{ m}) = -2.0 \text{ J}$. Adding this to the work for $x = 7.0 \text{ m}$ to $x = 2.0 \text{ m}$ gives a total of $W = -3.0 \text{ J}$ for $x = 7.0 \text{ m}$ to $x = 0$. The work for $x = 7.0 \text{ m}$ to $x = 0$ is the negative of the work for $x = 0$ to $x = 7.0 \text{ m}$.

6.38. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $K_1 = 0$. From Exercise 6.37, the work for $x = 0$ to $x = 3.0$ m is 4.0 J. W for $x = 0$ to $x = 4.0$ m is also 4.0 J. For $x = 0$ to $x = 7.0 \text{ m}$, $W = 3.0 \text{ J}$.

EXECUTE: **(a)** $K = 4.0 \text{ J}$, so $v = \sqrt{2K/m} = \sqrt{2(4.0 \text{ J})/(2.0 \text{ kg})} = 2.00 \text{ m/s}$.

(b) No work is done between $x = 3.0 \text{ m}$ and $x = 4.0 \text{ m}$, so the speed is the same, 2.00 m/s.

(c) $K = 3.0 \text{ J}$, so $v = \sqrt{2K/m} = \sqrt{2(3.0 \text{ J})/(2.0 \text{ kg})} = 1.73 \text{ m/s}$.

EVALUATE: In each case the work done by F is positive and the car gains kinetic energy.

6.39. **IDENTIFY** and **SET UP:** Apply Eq.(6.6). Let point 1 be where the sled is released and point 2 be at $x = 0$ for part (a) and at $x = -0.200 \text{ m}$ for part (b). Use Eq.(6.10) for the work done by the spring and calculate K_2. Then $K_2 = \frac{1}{2}mv_2^2$ gives v_2.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$ so $K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$ (released with no initial velocity), $K_2 = \frac{1}{2}mv_2^2$

The only force doing work is the spring force. Eq.(6.10) gives the work done *on* the spring to move its end from x_1 to x_2. The force the spring exerts on an object attached to it is $F = -kx$, so the work the spring does is

$W_{\text{spr}} = -\left(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right) = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$. Here $x_1 = -0.375 \text{ m}$ and $x_2 = 0$. Thus $W_{\text{spr}} = \frac{1}{2}(4000 \text{ N/m})(-0.375 \text{ m})^2 - 0 = 281 \text{ J}$.

$K_2 = K_1 + W_{\text{tot}} = 0 + 281 \text{ J} = 281 \text{ J}$

Then $K_2 = \frac{1}{2}mv_2^2$ implies $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(281 \text{ J})}{70.0 \text{ kg}}} = 2.83 \text{ m/s}$.

(b) $K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$

$W_{\text{tot}} = W_{\text{spr}} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$. Now $x_2 = 0.200 \text{ m}$, so

$W_{\text{spr}} = \frac{1}{2}(4000 \text{ N/m})(-0.375 \text{ m})^2 - \frac{1}{2}(4000 \text{ N/m})(-0.200 \text{ m})^2 = 281 \text{ J} - 80 \text{ J} = 201 \text{ J}$

Thus $K_2 = 0 + 201 \text{ J} = 201 \text{ J}$ and $K_2 = \frac{1}{2}mv_2^2$ gives $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(201 \text{ J})}{70.0 \text{ kg}}} = 2.40 \text{ m/s}$.

EVALUATE: The spring does positive work and the sled gains speed as it returns to $x = 0$. More work is done during the larger displacement in part (a), so the speed there is larger than in part (b).

6.40. **IDENTIFY:** $F_x = kx$

SET UP: When the spring is in equilibrium, the same force is applied to both ends of any segment of the spring.

EXECUTE: **(a)** When a force F is applied to each end of the original spring, the end of the spring is displaced a distance x. Each half of the spring elongates a distance x_h, where $x_h = x/2$. Since F is also the force applied to each half of the spring, $F = kx$ and $F = k_h x_h$. $kx = k_h x_h$ and $k_h = k\left(\dfrac{x}{x_h}\right) = 2k$.

(b) The same reasoning as in part (a) gives $k_{seg} = 3k$, where k_{seg} is the force constant of each segment.

EVALUATE: For half of the spring the same force produces less displacement than for the original spring. Since $k = F/x$, smaller x for the same F means larger k.

6.41. **IDENTIFY** and **SET UP:** Apply Eq.(6.6) to the glider. Work is done by the spring and by gravity. Take point 1 to be where the glider is released. In part (a) point 2 is where the glider has traveled 1.80 m and $K_2 = 0$. There two points are shown in Figure 6.41a. In part (b) point 2 is where the glider has traveled 0.80 m.

EXECUTE: **(a)** $W_{tot} = K_2 - K_1 = 0$. Solve for x_1, the amount the spring is initially compressed.

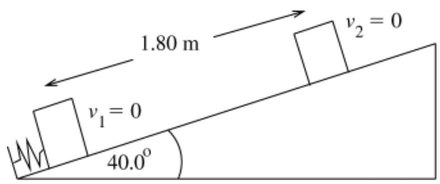


Figure 6.41a

$W_{tot} = W_{spr} + W_w = 0$

So $W_{spr} = -W_w$

(The spring does positive work on the glider since the spring force is directed up the incline, the same as the direction of the displacement.)

The directions of the displacement and of the gravity force are shown in Figure 6.41b.

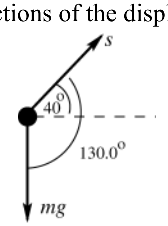


Figure 6.41b

$W_w = (w\cos\phi)s = (mg\cos 130.0°)s$

$W_w = (0.0900\text{ kg})(9.80\text{ m/s}^2)(\cos 130.0°)(1.80\text{ m}) = -1.020\text{ J}$

(The component of w parallel to the incline is directed down the incline, opposite to the displacement, so gravity does negative work.)

$W_{spr} = -W_w = +1.020\text{ J}$

$W_{spr} = \frac{1}{2}kx_1^2$ so $x_1 = \sqrt{\dfrac{2W_{spr}}{k}} = \sqrt{\dfrac{2(1.020\text{ J})}{640\text{ N/m}}} = 0.0565\text{ m}$

(b) The spring was compressed only 0.0565 m so at this point in the motion the glider is no longer in contact with the spring. Points 1 and 2 are shown in Figure 6.41c.

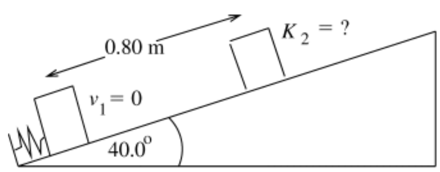


Figure 6.41c

$W_{tot} = K_2 - K_1$

$K_2 = K_1 + W_{tot}$

$K_1 = 0$

$W_{tot} = W_{spr} + W_w$

From part (a), $W_{spr} = 1.020\text{ J}$ and

$W_w = (mg\cos 130.0°)s = (0.0900\text{ kg})(9.80\text{ m/s}^2)(\cos 130.0°)(0.80\text{ m}) = -0.454\text{ J}$

Then $K_2 = W_{spr} + W_w = +1.020\text{ J} - 0.454\text{ J} = +0.57\text{ J}$.

EVALUATE: The kinetic energy in part (b) is positive, as it must be. In part (a), $x_2 = 0$ since the spring force is no longer applied past this point. In computing the work done by gravity we use the full 0.80 m the glider moves.

6.42. **IDENTIFY:** Apply $W_{tot} = K_2 - K_1$ to the brick. Work is done by the spring force and by gravity.

SET UP: At the maximum height. $v = 0$. Gravity does negative work, $W_{grav} = -mgh$. The work done by the spring is $\frac{1}{2}kd^2$, where d is the distance the spring is compressed initially.

EXECUTE: The initial and final kinetic energies of the brick are both zero, so the net work done on the brick by the spring and gravity is zero, so $(1/2)kd^2 - mgh = 0$, or

$d = \sqrt{2mgh/k} = \sqrt{2(1.80\text{ kg})(9.80\text{ m/s}^2)(3.6\text{ m})/(450\text{ N/m})} = 0.53\text{ m}$. The spring will provide an upward force while the spring and the brick are in contact. When this force goes to zero, the spring is at its uncompressed length. But when the spring reaches its uncompressed length the brick has an upward velocity and leaves the spring.

EVALUATE: Gravity does negative work because the gravity force is downward and the brick moves upward. The spring force does positive work on the brick because the spring force is upward and the brick moves upward.

6.43. **IDENTIFY:** Apply the relation between energy and power.

SET UP: Use $P = \dfrac{W}{\Delta t}$ to solve for W, the energy the bulb uses. Then set this value equal to $\frac{1}{2}mv^2$ and solve for the speed.

EXECUTE: $W = P\Delta t = (100\text{ W})(3600\text{ s}) = 3.6\times 10^5\text{ J}$

$$K = 3.6\times 10^5\text{ J so } v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(3.6\times 10^5\text{ J})}{70\text{ kg}}} = 100\text{ m/s}$$

EVALUATE: Olympic runners achieve speeds up to approximately 36 m/s, or roughly one third the result calculated.

6.44. **IDENTIFY:** Energy is power times time.

SET UP: $1\text{ W} = 1\text{ J/s}$. $1\text{ yr} = 3.16\times 10^7\text{ s}$.

EXECUTE: **(a)** $\dfrac{(1.0\times 10^{19}\text{ J/yr})}{(3.16\times 10^7\text{ s/yr})} = 3.2\times 10^{11}\text{ W}$.

(b) $\dfrac{3.2\times 10^{11}\text{ W}}{3.0\times 10^8\text{ folks}} = 1.1\text{ kW/person}$.

(c) $A = \dfrac{3.2\times 10^{11}\text{ W}}{(0.40)1.0\times 10^3\text{ W/m}^2} = 8.0\times 10^8\text{ m}^2 = 800\text{ km}^2$.

EVALUATE: The area in part (c) corresponds to a square about 28 km on a side, which is about 18 miles. The space required is not an impediment.

6.45. **IDENTIFY:** $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$. ΔW is the energy released.

SET UP: ΔW is to be the same. $1\text{ y} = 3.156\times 10^7\text{ s}$.

EXECUTE: $P_{\text{av}}\Delta t = \Delta W = \text{constant}$, so $P_{\text{av-sun}}\Delta t_{\text{sun}} = P_{\text{av-m}}\Delta t_{\text{m}}$.

$$P_{\text{av-m}} = P_{\text{av-sun}}\left(\frac{\Delta t_{\text{sun}}}{\Delta t_{\text{m}}}\right) = \left(\frac{[2.5\times 10^5\text{ y}][3.156\times 10^7\text{ s/y}]}{0.20\text{ s}}\right) = 3.9\times 10^{13}P.$$

EVALUATE: Since the power output of the magnetar is so much larger than that of our sun, the mechanism by which it radiates energy must be quite different.

6.46. **IDENTIFY:** The thermal energy is produced as a result of the force of friction, $F = \mu_{\text{k}}mg$. The average thermal power is thus the average rate of work done by friction or $P = F_{\|}v_{\text{av}}$.

SET UP: $v_{\text{av}} = \dfrac{v_2 + v_1}{2} = \left(\dfrac{8.00\text{ m/s} + 0}{2}\right) = 4.00\text{ m/s}$

EXECUTE: $P = Fv_{\text{av}} = \left[(0.200)(20.0\text{ kg})(9.80\text{ m/s}^2)\right](4.00\text{ m/s}) = 157\text{ W}$

EVALUATE: The power could also be determined as the rate of change of kinetic energy, $\Delta K/t$, where the time is calculated from $v_{\text{f}} = v_{\text{i}} + at$ and a is calculated from a force balance, $\sum F = ma = \mu_{\text{k}}mg$.

6.47. **IDENTIFY:** Use the relation $P = F_{\|}v$ to relate the given force and velocity to the total power developed.

SET UP: $1\text{ hp} = 746\text{ W}$

EXECUTE: The total power is $P = F_{\|}v = (165\text{ N})(9.00\text{ m/s}) = 1.49\times 10^3\text{ W}$. Each rider therefore contributes $P_{\text{each rider}} = (1.49\times 10^3\text{ W})/2 = 745\text{ W} \approx 1\text{ hp}$.

EVALUATE: The result of one horsepower is very large; a rider could not sustain this output for long periods of time.

6.48. **IDENTIFY** and **SET UP:** Calculate the power used to make the plane climb against gravity. Consider the vertical motion since gravity is vertical.

EXECUTE: The rate at which work is being done against gravity is

$$P = Fv = mgv = (700\text{ kg})(9.80\text{ m/s}^2)(2.5\text{ m/s}) = 17.15\text{ kW}.$$

This is the part of the engine power that is being used to make the airplane climb. The fraction this is of the total is $17.15\text{ kW}/75\text{ kW} = 0.23$.

EVALUATE: The power we calculate for making the airplane climb is considerably less than the power output of the engine.

6.49. **IDENTIFY:** $P_{av} = \dfrac{\Delta W}{\Delta t}$. The work you do in lifting mass *m* a height *h* is *mgh*.

SET UP: $1 \text{ hp} = 746 \text{ W}$

EXECUTE: **(a)** The number per minute would be the average power divided by the work (*mgh*) required to lift one box, $\dfrac{(0.50 \text{ hp})\,(746 \text{ W/hp})}{(30 \text{ kg})\,(9.80 \text{ m/s}^2)\,(0.90 \text{ m})} = 1.41\ /\text{s, or } 84.6\ /\text{min.}$

(b) Similarly, $\dfrac{(100 \text{ W})}{(30 \text{ kg})\,(9.80 \text{ m/s}^2)\,(0.90 \text{ m})} = 0.378\ /\text{s, or } 22.7\ /\text{min.}$

EVALUATE: A 30-kg crate weighs about 66 lbs. It is not possible for a person to perform work at this rate.

6.50. **IDENTIFY** and **SET UP:** Use Eq.(6.15) to relate the power provided and the amount of work done against gravity in 16.0 s. The work done against gravity depends on the total weight which depends on the number of passengers.

EXECUTE: Find the total mass that can be lifted:

$$P_{av} = \frac{\Delta W}{\Delta t} = \frac{mgh}{t}, \text{ so } m = \frac{P_{av}t}{gh}$$

$$P_{av} = (40 \text{ hp})\left(\frac{746 \text{ W}}{1 \text{ hp}}\right) = 2.984\times 10^4 \text{ W}$$

$$m = \frac{P_{av}t}{gh} = \frac{(2.984\times 10^4 \text{ W})(16.0 \text{ s})}{(9.80 \text{ m/s}^2)(20.0 \text{ m})} = 2.436\times 10^3 \text{ kg}$$

This is the total mass of elevator plus passengers. The mass of the passengers is $2.436\times 10^3 \text{ kg} - 600 \text{ kg} = 1.836\times 10^3 \text{ kg}$.

The number of passengers is $\dfrac{1.836\times 10^3 \text{ kg}}{65.0 \text{ kg}} = 28.2$. 28 passengers can ride.

EVALUATE: Typical elevator capacities are about half this, in order to have a margin of safety.

6.51. **IDENTIFY:** Calculate the gallons of gasoline consumed and from that the energy consumed. Find the time Δt for the trip and use $P_{av} = \dfrac{\Delta W}{\Delta t}$, where ΔW is the energy consumed.

SET UP: $200 \text{ km} = 124 \text{ mi}$

EXECUTE: **(a)** The gallons of gasoline consumed is $\dfrac{124 \text{ mi}}{30 \text{ mi/gal}} = 4.13 \text{ gal}$. The energy consumed is

$(4.13 \text{ gal})(1.3\times 10^9 \text{ J/gal}) = 5.4\times 10^9 \text{ J}$.

(b) The time for the trip is $\dfrac{124 \text{ mi}}{60 \text{ mi/h}} = 2.07 \text{ h} = 7450 \text{ s}$. $P_{av} = \dfrac{\Delta W}{\Delta t} = \dfrac{5.4\times 10^9 \text{ J}}{7450 \text{ s}} = 7.2\times 10^5 \text{ W} = 720 \text{ kW}$.

EVALUATE: The rate of energy consumption is $\dfrac{720\times 10^3 \text{ W}}{746 \text{ W/hp}} = 970 \text{ hp}$.

6.52. **IDENTIFY:** Apply $P = F_{\parallel}v$. $F_{\parallel}$ is the force *F* of water resistance.

SET UP: $1 \text{ hp} = 746 \text{ W}$. $1 \text{ km/h} = 0.228 \text{ m/s}$

EXECUTE: $F = \dfrac{(0.70)\,P}{v} = \dfrac{(0.70)\,(280{,}000 \text{ hp})(746 \text{ W/hp})}{(65 \text{ km/h})\,((0.228 \text{ m/s})/(1 \text{ km/h}))} = 8.1\times 10^6 \text{ N}.$

EVALUATE: The power required depends on speed, because of the factor of *v* in $P = F_{\parallel}v$ and also because the resistive force increases with speed.

6.53. **IDENTIFY:** To lift the skiers, the rope must do positive work to counteract the negative work developed by the component of the gravitational force acting on the total number of skiers,

$F_{rope} = Nmg\sin\alpha$.

SET UP: $P = F_{\parallel}v = F_{rope}v$

EXECUTE: $P_{rope} = F_{rope}v = \left[+Nmg\left(\cos\phi\right)\right]v$.

$$P_{rope} = \left[(50 \text{ riders})(70.0 \text{ kg})(9.80 \text{ m/s}^2)(\cos 75.0^\circ)\right]\left[(12.0 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.60 \text{ km/h}}\right)\right].$$

$P_{rope} = 2.96\times 10^4 \text{ W} = 29.6 \text{ kW}$.

EVALUATE: Some additional power would be needed to give the riders kinetic energy as they are accelerated from rest.

6.54. **IDENTIFY:** Relate power, work and time.

SET UP: Work done in each stroke is $W = Fs$ and $P_{av} = W/t$.

EXECUTE: 100 strokes per second means $P_{av} = 100Fs/t$ with $t = 1.00$ s, $F = 2mg$ and $s = 0.010$ m. $P_{av} = 0.20$ W.

EVALUATE: For a 70 kg person to apply a force of twice his weight through a distance of 0.50 m for 100 times per second, the average power output would be 7.0×10^5 W. This power output is very far beyond the capability of a person.

6.55. **IDENTIFY:** For mass dm located a distance x from the axis and moving with speed v, the kinetic energy is $K = \frac{1}{2}(dm)v^2$. Follow the procedure specified in the hint.

SET UP: The bar and an infinitesimal mass element along the bar are sketched in Figure 6.55. Let $M =$ total mass and $T =$ time for one revolution. $v = \dfrac{2\pi x}{T}$.

EXECUTE: $K = \int \frac{1}{2}(dm)v^2$. $dm = \dfrac{M}{L}dx$, so

$$K = \int_0^L \frac{1}{2}\left(\frac{M}{L}dx\right)\left(\frac{2\pi x}{T}\right)^2 = \frac{1}{2}\left(\frac{M}{L}\right)\left(\frac{4\pi^2}{T^2}\right)\int_0^L x^2dx = \frac{1}{2}\left(\frac{M}{L}\right)\left(\frac{4\pi^2}{T^2}\right)\left(\frac{L^3}{3}\right) = \frac{2}{3}\pi^2 ML^2/T^2$$

There are 5 revolutions in 3 seconds, so $T = 3/5\ \text{s} = 0.60\ \text{s}$

$$K = \frac{2}{3}\pi^2(12.0\ \text{kg})\ (2.00\ \text{m})^2/(0.60\ \text{s})^2 = 877\ \text{J}.$$

EVALUATE: If a point mass 12.0 kg is 2.00 m from the axis and rotates at the same rate as the bar, $v = \dfrac{2\pi(2.00\ \text{m})}{0.60\ \text{s}} = 20.9$ m/s and $K = \frac{1}{2}mv^2 = \frac{1}{2}(12\ \text{kg})(20.9\ \text{m/s})^2 = 2.62\times10^3$ J. K for the bar is smaller by a factor of 0.33. The speed of a segment of the bar decreases toward the axis.

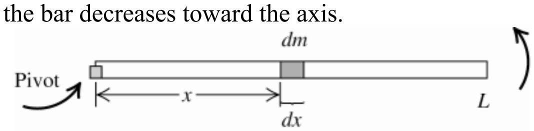


Figure 6.55

6.56. **IDENTIFY:** Density is mass per unit volume, $\rho = m/V$, so we can calculate the mass of the asteroid. $K = \frac{1}{2}mv^2$. Since the asteroid comes to rest, the kinetic energy it delivers equals its initial kinetic energy.

SET UP: The volume of a sphere is related to its diameter by $V = \dfrac{1}{6}\pi d^3$.

EXECUTE: **(a)** $V = \dfrac{\pi}{6}(320\ \text{m})^3 = 1.72\times10^7\ \text{m}^3$. $m = \rho V = (2600\ \text{kg/m}^3)(1.72\times10^7\ \text{m}^3) = 4.47\times10^{10}$ kg.

$K = \frac{1}{2}mv^2 = \frac{1}{2}(4.47\times10^{10}\ \text{kg})(12.6\times10^3\ \text{m/s})^2 = 3.55\times10^{18}$ J.

(b) The yield of a Castle/Bravo device is $(1\ \text{s})(4.184\times10^{15}\ \text{J}) = 6.28\times10^{16}$ J. $\dfrac{3.55\times10^{18}\ \text{J}}{6.28\times10^{16}\ \text{J}} = 56.5$ devices.

EVALUATE: If such an asteroid were to hit the earth the effect would be catastrophic.

6.57. **IDENTIFY** and **SET UP:** Since the forces are constant, Eq.(6.2) can be used to calculate the work done by each force. The forces on the suitcase are shown in Figure 6.57a.

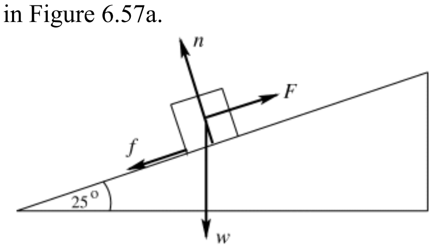


Figure 6.57a

In part (f), Eq.(6.6) is used to relate the total work to the initial and final kinetic energy.

EXECUTE: **(a)** $W_F = (F\cos\phi)s$

Both $\vec{F}$ and $\vec{s}$ are parallel to the incline and in the same direction, so $\phi = 90°$ and $W_F = Fs = (140\ \text{N})(3.80\ \text{m}) = 532$ J

(b) The directions of the displacement and of the gravity force are shown in Figure 6.57b.

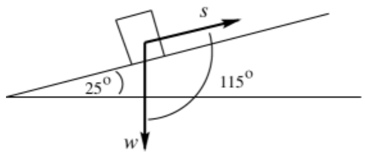


Figure 6.57b

$W_w = (w\cos\phi)s$

$\phi = 115°$, so

$W_w = (196\text{ N})(\cos 115°)(3.80\text{ m})$

$W_w = -315\text{ J}$

Alternatively, the component of w parallel to the incline is $w\sin 25°$. This component is down the incline so its angle with $\vec{s}$ is $\phi = 180°$. $W_{w\sin 25°} = (196\text{ N}\sin 25°)(\cos 180°)(3.80\text{ m}) = -315\text{ J}$. The other component of w, $w\cos 25°$, is perpendicular to $\vec{s}$ and hence does no work. Thus $W_w = W_{w\sin 25°} = -315\text{ J}$, which agrees with the above.

(c) The normal force is perpendicular to the displacement $(\phi = 90°)$, so $W_n = 0$.

(d) $n = w\cos 25°$ so $f_k = \mu_k n = \mu_k w\cos 25° = (0.30)(196\text{ N})\cos 25° = 53.3\text{ N}$

$W_f = (f_k\cos\phi)x = (53.3\text{ N})(\cos 180°)(3.80\text{ m}) = -202\text{ J}$

(e) $W_{tot} = W_F + W_w + W_n + W_f = +532\text{ J} - 315\text{ J} + 0 - 202\text{ J} = 15\text{ J}$

(f) $W_{tot} = K_2 - K_1$, $K_1 = 0$, so $K_2 = W_{tot}$

$\frac{1}{2}mv_2^2 = W_{tot}$ so $v_2 = \sqrt{\frac{2W_{tot}}{m}} = \sqrt{\frac{2(15\text{ J})}{20.0\text{ kg}}} = 1.2\text{ m/s}$

EVALUATE: The total work done is positive and the kinetic energy of the suitcase increases as it moves up the incline.

6.58. **IDENTIFY:** The work he does to lift his body a distance h is $W = mgh$. The work per unit mass is $(W/m) = gh$.

SET UP: The quantity gh has units of N/kg.

EXECUTE: **(a)** The man does work, $(9.8\text{ N/kg})(0.4\text{ m}) = 3.92\text{ J/kg}$.

(b) $(3.92\text{ J/kg})/(70\text{ J/kg}) \times 100 = 5.6\%$.

(c) The child does work $(9.8\text{ N/kg})(0.2\text{ m}) = 1.96\text{ J/kg}$. $(1.96\text{ J/kg})/(70\text{ J/kg}) \times 100 = 2.8\%$.

(d) If both the man and the child can do work at the rate of 70 J/kg, and if the child only needs to use 1.96 J/kg instead of 3.92 J/kg, the child should be able to do more chin-ups.

EVALUATE: Since the child has arms half the length of his father's arms, the child must lift his body only 0.20 m to do a chin-up.

6.59. **IDENTIFY:** Apply the definitions of IMA and AMA given in the problem.

SET UP: When the object moves a distance L along the ramp, it rises a vertical distance $L\sin\alpha$.

EXECUTE: **(a)** $s_{in} = L$, $s_{out} = L\sin\alpha$, so $IMA = \dfrac{1}{\sin\alpha}$.

(b) If $AMA = IMA$, $(F_{out}/F_{in}) = (s_{in}/s_{out})$ and so $(F_{out})(s_{out}) = (F_{in})(s_{in})$, or $W_{out} = W_{in}$.

(c) The pulley is sketched in Figure 6.59.

(d) $e = \dfrac{W_{out}}{W_{in}} = \dfrac{(F_{out})(s_{out})}{(F_{in})(s_{in})} = \dfrac{F_{out}/F_{in}}{s_{in}/s_{out}} = \dfrac{AMA}{IMA}$.

EVALUATE: $F_{in} = w\sin\alpha$ and $F_{out} = w$. $(F_{in})(s_{in}) = (w\sin\alpha)L$. $(F_{out})(s_{out}) = w(\sin\alpha L)$. Therefore, $(F_{in})(s_{in}) = (F_{out})(s_{out})$. A smaller force acting over a larger distance does the same amount of work as a larger force acting over a smaller distance.

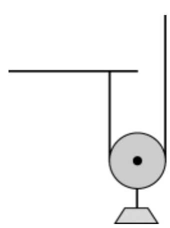


Figure 6.59

6.60. **IDENTIFY:** Apply $\sum\vec{F} = m\boldsymbol{a}$ to each block to find the tension in the string. Each force is constant and $W = Fs\cos\phi$.

SET UP: The free-body diagram for each block is given in Figure 6.60. $m_A = \dfrac{20.0\text{ N}}{g} = 2.04\text{ kg}$ and

$m_B = \dfrac{12.0\text{ N}}{g} = 1.22\text{ kg}$.

EXECUTE: $T - f_k = m_A a$. $w_B - T = m_B a$. $w_B - f_k = (m_A + m_B)a$.

$f_k = 0$. $a = \left(\frac{w_B}{m_A + m_B}\right)$ and $T = w_B\left(\frac{m_A}{m_A + m_B}\right) = w_B\left(\frac{w_A}{w_A + w_B}\right) = 7.50\text{ N}$.

20.0 N block: $W_{tot} = Ts = (7.50\text{ N})(0.750\text{ m}) = 5.62\text{ J}$.

12.0 N block: $W_{tot} = (w_B - T)s = (12.0\text{ N} - 7.50\text{ N})(0.750\text{ m}) = 3.38\text{ J}$

(b) $f_k = \mu_k w_A = 6.50\text{ N}$. $a = \frac{w_B - \mu_k w_A}{m_A + m_B}$. $T = f_k + (w_B - \mu_k w_A)\left(\frac{m_A}{m_A + m_B}\right) = \mu_k w_A + (w_B - \mu_k w_A)\left(\frac{w_A}{w_A + w_B}\right)$.

$T = 6.50\text{ N} + (5.50\text{ N})(0.625) = 9.94\text{ N}$.

20.0 N block: $W_{tot} = (T - f_k)s = (9.94\text{ N} - 6.50\text{ N})(0.750\text{ m}) = 2.58\text{ J}$.

12.0 N block: $W_{tot} = (w_B - T)s = (12.0\text{ N} - 9.94\text{ N})(0.750\text{ m}) = 1.54\text{ J}$.

EVALUATE: Since the two blocks move with equal speeds, for each block $W_{tot} = K_2 - K_1$ is proportional to the mass (or weight) of that block. With friction the gain in kinetic energy is less, so the total work on each block is less.

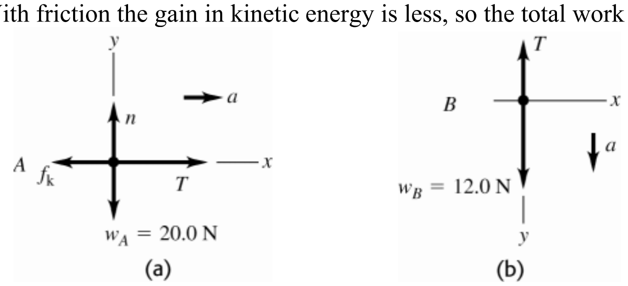


Figure 6.60

6.61. **IDENTIFY:** $K = \frac{1}{2}mv^2$. Find the speed of the shuttle relative to the earth and relative to the satellite.

SET UP: Velocity is distance divided by time. For one orbit the shuttle travels a distance $2\pi R$.

EXECUTE: **(a)** $\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{2\pi R}{T}\right)^2 = \frac{1}{2}(86{,}400\text{ kg})\left(\frac{2\pi(6.66\times10^6\text{ m})}{(90.1\text{ min})\,(60\text{ s/min})}\right)^2 = 2.59\times10^{12}\text{ J}.$

(b) $(1/2)\,mv^2 = (1/2)\,(86{,}400\text{ kg})\,((1.00\text{ m})/(3.00\text{ s}))^2 = 4.80\times10^3\text{ J}.$

EVALUATE: The kinetic energy of an object depends on the reference frame in which it is measured.

6.62. **IDENTIFY:** $W = Fs\cos\phi$. $W_{tot} = K_2 - K_1$.

SET UP: $f_k = \mu_k n$. The normal force is $n = mg\cos\theta$, with $\theta = 12.0°$. The component of the weight parallel to the incline is $mg\sin\theta$.

EXECUTE: **(a)** $\phi = 180°$ and $W_f = -f_k s = -(\mu_k mg\cos\theta)s = -(0.31)(5.00\text{ kg})(9.80\text{ m/s}^2)(\cos 12.0°)(1.50\text{ m}) = -22.3\text{ J}$

(b) $(5.00\text{ kg})(9.80\text{ m/s}^2)(\sin 12.0°)(1.50\text{ m}) = 15.3\text{ J}.$

(c) The normal force does no work.

(d) $W_{tot} = 15.3\text{ J} - 22.3\text{ J} = -7.0\text{ J}.$

(e) $K_2 = K_1 + W_{tot} = (1/2)(5.00\text{ kg})(2.2\text{ m/s})^2 - 7.0\text{ J} = 5.1\text{ J}$, and so $v_2 = \sqrt{2(5.1\text{ J})/(5.00\text{ kg})} = 1.4\text{ m/s}$.

EVALUATE: Friction does negative work and gravity does positive work. The net work is negative and the kinetic energy of the object decreases.

6.63. **IDENTIFY:** The effective force constant is defined by $k_{eff} = F/x$, where F is the force applied to each end of the spring combination and x is the amount the spring combination is stretched.

SET UP: Consider a force F applied to each end of the combination. Then F_1 and F_2 are the forces applied to each spring and $F = F_1 + F_2$. Each spring stretches the same amount x.

EXECUTE: **(a)** $F = k_{eff}x$. $F = F_1 + F_2 = k_1x + k_2x$. Equating the two expressions for F gives $k_{eff} = k_1 + k_2$.

(b) The same procedure as in part (a) gives $k_{eff} = k_1 + k_2 + \cdots + k_N$.

EVALUATE: The effective force constant of the configuration is greater than any of the force constants of the individual springs. More force is required to stretch the parallel combination that is required to stretch each separate spring the same amount.

6.64. **IDENTIFY:** The effective force constant is defined by $k_{\text{eff}} = F/x$, where F is the force applied to each end of the spring combination and x is the amount the spring combination is stretched.

SET UP: Consider a force F applied to each end of the combination. The same force F is applied to each spring. Spring 1 stretches a distance x_1 and spring 2 stretches a distance x_2, where $x_1 = F/k_1$ and $x_2 = F/k_2$. The total distance the combination stretches is $x = x_1 + x_2$.

EXECUTE: **(a)** $x = x_1 + x_2$ gives $\dfrac{F}{k_{\text{eff}}} = \dfrac{F}{k_1} = \dfrac{F}{k_2}$ and $\dfrac{1}{k_{\text{eff}}} = \dfrac{1}{k_1} + \dfrac{1}{k_2}$.

(b) The same procedure as in part (a) gives $\dfrac{1}{k_{\text{eff}}} = \dfrac{1}{k_1} + \dfrac{1}{k_2} + \cdots + \dfrac{1}{k_N}$.

EVALUATE: For two springs the result in part (a) can be written as $k_{\text{eff}} = \dfrac{k_1k_2}{k_1 + k_2}$. The effective force constant for the two springs in series is less than the force constant for each individual spring. It takes less force to stretch the combination an amount x than to stretch either separate spring an amount x.

6.65. **IDENTIFY:** Apply Eq.(6.7).

SET UP: $\displaystyle\int \frac{dx}{x^2} = -\frac{1}{x}$.

EXECUTE: **(a)** $W = \displaystyle\int_{x_1}^{x_2} F_x dx = -k\int_{x_1}^{x_2} \frac{dx}{x^2} = -k\left[-\frac{1}{x}\right]_{x_1}^{x_2} = k\left(\frac{1}{x_2} - \frac{1}{x_1}\right)$. The force is given to be attractive, so $F_x < 0$, and k must be positive. If $x_2 > x_1$, $\dfrac{1}{x_2} < \dfrac{1}{x_1}$, and $W < 0$.

(b) Taking "slowly" to be constant speed, the net force on the object is zero. The force applied by the hand is opposite F_x, and the work done is negative of that found in part (a), or $k\left(\dfrac{1}{x_1} - \dfrac{1}{x_2}\right)$, which is positive if $x_2 > x_1$.

(c) The answers have the same magnitude but opposite signs; this is to be expected, in that the net work done is zero.

EVALUATE: Your force is directed away from the origin, so when the object moves away from the origin your force does positive work.

6.66. **IDENTIFY:** Apply Eq.(6.6) to the motion of the asteroid.

SET UP: Let point 1 be at a great distance and let point 2 be at the surface of the earth. Assume $K_1 = 0$. From the information given about the gravitational force its magnitude as a function of distance r from the center of the earth must be $F = mg(R_{\text{E}}/r)^2$. This force is directed in the $-\hat{\boldsymbol{r}}$ direction since it is a "pull". F is not constant so Eq.(6.7) must be used to calculate the work it does.

EXECUTE: $W = -\displaystyle\int_1^2 F\ ds = -\int_\infty^{R_{\text{E}}} \left(\frac{mgR_{\text{E}}^2}{r^2}\right) dr = -mgR_{\text{E}}^2\left(-(1/r)\Big|_\infty^{R_{\text{E}}}\right) = mgR_{\text{E}}$

$W_{\text{tot}} = K_2 - K_1,\ \ K_1 = 0$

This gives $K_2 = mgR_{\text{E}} = 1.25\times10^{12}\ \text{J}$

$K_2 = \frac{1}{2}mv_2^2$ so $v_2 = \sqrt{2K_2/m} = 11{,}000\ \text{m/s}$

EVALUATE: Note that $v_2 = \sqrt{2gR_{\text{E}}}$, the impact speed is independent of the mass of the asteroid.

6.67. **IDENTIFY:** Calculate the work done by friction and apply $W_{\text{tot}} = K_2 - K_1$. Since the friction force is not constant, use Eq.(6.7) to calculate the work.

SET UP: Let x be the distance past P. Since μ_{k} increases linearly with x, $\mu_{\text{k}} = 0.100 + Ax$. When $x = 12.5\ \text{m}$, $\mu_{\text{k}} = 0.600$, so $A = 0.500/(12.5\ \text{m}) = 0.0400/\text{m}$

EXECUTE: **(a)** $W_{\text{tot}} = \Delta K = K_2 - K_1$ gives $-\displaystyle\int \mu_{\text{k}} mgdx = 0 - \frac{1}{2}mv_1^2$. Using the above expression for μ_{k},

$g\displaystyle\int_0^{x_2}(0.100 + Ax)dx = \frac{1}{2}v_1^2$ and $g\left[(0.100)x_2 + A\dfrac{x_2^2}{2}\right] = \dfrac{1}{2}v_1^2$. $(9.80\ \text{m/s}^2)\left[(0.100)x_{\text{f}} + (0.0400/\text{m})\dfrac{x_{\text{f}}^2}{2}\right] = \dfrac{1}{2}(4.50\ \text{m/s})^2$.

Solving for x_2 gives $x_2 = 5.11\ \text{m}$.

(b) $\mu_{\text{k}} = 0.100 + (0.0400/\text{m})(5.11\ \text{m}) = 0.304$

(c) $W_{\text{tot}} = K_2 - K_1$ gives $-\mu_k mgx_2 = 0 - \frac{1}{2}mv_1^2$. $x_2 = \frac{v_1^2}{2\mu_k g} = \frac{(4.50\text{ m/s})^2}{2(0.100)(9.80\text{ m/s}^2)} = 10.3\text{ m}$.

EVALUATE: The box goes farther when the friction coefficient doesn't increase.

6.68. **IDENTIFY:** Use Eq.(6.7) to calculate *W*.

SET UP: $x_1 = 0$. In part (a), $x_2 = 0.050\text{ m}$. In part (b), $x_2 = -0.050\text{ m}$.

EXECUTE: **(a)** $W = \int_0^{x_2} F dx = \int_0^{x_2}(kx - bx^2 + cx^3)dx = \frac{k}{2}x_2^2 - \frac{b}{3}x_2^3 + \frac{c}{4}x_2^4$.

$W = (50.0\text{ N/m})x_2^2 - (233\text{ N/m}^2)x_2^3 + (3000\text{ N/m}^3)x_2^4$. When $x_2 = 0.050\text{ m}$, $W = 0.12\text{ J}$.

(b) When $x_2 = -0.050$ m, $W = 0.17$ J.

(c) It's easier to stretch the spring; the quadratic $-bx^2$ term is always in the $-x$-direction, and so the needed force, and hence the needed work, will be less when $x_2 > 0$.

EVALUATE: When $x = 0.050\text{ m}$, $F_x = 4.75\text{ N}$. When $x = -0.050\text{ m}$, $F_x = 8.25\text{ N}$.

6.69. **IDENTIFY** and **SET UP:** Use $\sum \vec{F} = m\vec{a}$ to find the tension force *T*. The block moves in uniform circular motion and $\vec{a} = \vec{a}_{\text{rad}}$.

(a) The free-body diagram for the block is given in Figure 6.69.

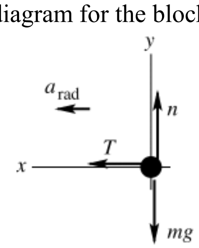


Figure 6.69

EXECUTE: $\sum F_x = ma_x$

$T = m\frac{v^2}{R}$

$T = (0.120\text{ kg})\frac{(0.70\text{ m/s})^2}{0.40\text{ m}} = 0.15\text{ N}$

(b) $T = m\frac{v^2}{R} = (0.120\text{ kg})\frac{(2.80\text{ m/s})^2}{0.10\text{ m}} = 9.4\text{ N}$

(c) SET UP: The tension changes as the distance of the block from the hole changes. We could use $W = \int_{x_1}^{x_2} F_x\, dx$ to calculate the work. But a much simpler approach is to use $W_{\text{tot}} = K_2 - K_1$.

EXECUTE: The only force doing work on the block is the tension in the cord, so $W_{\text{tot}} = W_T$.

$$K_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(0.120\text{ kg})(0.70\text{ m/s})^2 = 0.0294\text{ J}$$

$$K_2 = \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}(0.120\text{ kg})(2.80\text{ m/s})^2 = 0.470\text{ J}$$

$$W_{\text{tot}} = K_2 - K_1 = 0.470\text{ J} - 0.029\text{ J} = 0.44\text{ J}$$

This is the amount of work done by the person who pulled the cord.

EVALUATE: The block moves inward, in the direction of the tension, so *T* does positive work and the kinetic energy increases.

6.70. **IDENTIFY:** Use Eq.(6.7) to find the work done by *F*. Then apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $\int \frac{dx}{x^2} = -\frac{1}{x}$.

EXECUTE: $W = \int_{x_1}^{x_2} \frac{\alpha}{x^2}dx = \alpha\left(\frac{1}{x_1} - \frac{1}{x_2}\right)$. $W = (2.12\times10^{-26}\text{ N}\cdot\text{m}^2)((0.200\text{ m}^{-1}) - (1.25\times10^9\text{ m}^{-1})) = -2.65\times10^{-17}\text{ J}$.

Note that x_1 is so large compared to x_2 that the term $1/x_1$ is negligible. Then, using Eq. (6.13)) and solving for v_2,

$$v_2 = \sqrt{v_1^2 + \frac{2W}{m}} = \sqrt{(3.00\times10^5\text{ m/s})^2 + \frac{2(-2.65\times10^{-17}\text{ J})}{(1.67\times10^{-27}\text{ kg})}} = 2.41\times10^5\text{ m/s}.$$

(b) With $K_2 = 0, W = -K_1$. Using $W = -\frac{\alpha}{x_2}$,

$$x_2 = \frac{\alpha}{K_1} = \frac{2\alpha}{mv_1^2} = \frac{2(2.12\times10^{-26}\text{ N}\cdot\text{m}^2)}{(1.67\times10^{-27}\text{ kg})(3.00\times10^5\text{ m/s})^2} = 2.82\times10^{-10}\text{ m}.$$

(c) The repulsive force has done no net work, so the kinetic energy and hence the speed of the proton have their original values, and the speed is 3.00×10^5 m/s .

EVALUATE: As the proton moves toward the uranium nucleus the repulsive force does negative work and the kinetic energy of the proton decreases. As the proton moves away from the uranium nucleus the repulsive force does positive work and the kinetic energy of the proton increases.

6.71. **IDENTIFY** and **SET UP:** Use $v_x = dx/dt$ and $a_x = dv_x/dt$. Use $\sum \vec{F} = m\vec{a}$ to calculate $\vec{F}$ from $\vec{a}$.

EXECUTE: **(a)** $x(t) = \alpha t^2 + \beta t^3,\ \ v_x(t) = \dfrac{dx}{dt} = 2\alpha t + 3\beta t^2$

$t = 4.00$ s: $v_x = 2(0.200\ \text{m/s}^2)(4.00\ \text{s}) + 3(0.0200\ \text{m/s}^3)(4.00\ \text{s})^2 = 2.56$ m/s.

(b) $a_x(t) = \dfrac{dv_x}{dt} = 2\alpha + 6\beta t$

$F_x = ma_x = m(2\alpha + 6\beta t)$

$t = 4.00$ s: $F_x = 6.00\ \text{kg}(2(0.200\ \text{m/s}^2) + 6(0.0200\ \text{m/s}^3)(4.00\ \text{s})) = 5.28$ N

(c) IDENTIFY and **SET UP:** Use Eq.(6.6) to calculate the work.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

At $t_1 = 0,\ v_1 = 0$ so $K_1 = 0.$

$W_{\text{tot}} = W_F$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(6.00\ \text{kg})(2.56\ \text{m/s})^2 = 19.7$ J

Then $W_{\text{tot}} = K_2 - K_1$ gives that $W_F = 19.7$ J

EVALUATE: v increases with t so the kinetic energy increases and the work done is positive. We can also calculate W_F directly from Eq.(6.7), by writing dx as $v_x\ dt$ and performing the integral.

6.72. **IDENTIFY:** Since the capsule comes to rest, the amount of work the capsule does on the ground equals its original kinetic energy. Use constant acceleration kinematic equations to calculate the stopping time t; $\Delta t = t$.

SET UP: 311 km/h = 86.4 m/s . Let $+y$ be the direction the capsule is traveling before the crash.

EXECUTE: $\Delta W = K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(210\ \text{kg})(86.4\ \text{m/s})^2 = 7.84\times10^5$ J . $y - y_0 = 0.810$ m , $v_{0y} = 86.4$ m/s and $v_y = 0$.

$y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $t = \dfrac{2(y - y_0)}{v_{0y}} = \dfrac{2(0.810\ \text{m})}{86.4\ \text{m/s}} = 0.01875$ s . $\dfrac{\Delta W}{\Delta t} = \dfrac{7.84\times10^5\ \text{J}}{0.01875\ \text{s}} = 4.18\times10^7$ W

EVALUATE: A large amount of work is done in a very small amount of time.

6.73. **IDENTIFY** and **SET UP:** Use Eq.(6.6). You do positive work and gravity does negative work. Let point 1 be at the base of the bridge and point 2 be at the top of the bridge.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(80.0\ \text{kg})(5.00\ \text{m/s})^2 = 1000$ J

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(80.0\ \text{kg})(1.50\ \text{m/s})^2 = 90$ J

$W_{\text{tot}} = 90\ \text{J} - 1000\ \text{J} = -910$ J

(b) Neglecting friction, work is done by you (with the force you apply to the pedals) and by gravity: $W_{\text{tot}} = W_{\text{you}} + W_{\text{gravity}}$. The gravity force is $w = mg = (80.0\ \text{kg})(9.80\ \text{m/s}^2) = 784$ N, downward. The displacement is 5.20 m, upward. Thus $\phi = 180°$ and

$$W_{\text{gravity}} = (F\cos\phi)s = (784\ \text{N})(5.20\ \text{m})\cos180° = -4077\ \text{J}$$

Then $W_{\text{tot}} = W_{\text{you}} + W_{\text{gravity}}$ gives

$$W_{\text{you}} = W_{\text{tot}} - W_{\text{gravity}} = -910\ \text{J} - (-4077\ \text{J}) = +3170\ \text{J}$$

EVALUATE: The total work done is negative and you lose kinetic energy.

6.74. **IDENTIFY:** Use Eq.(6.7) to calculate W.

SET UP: $\int x^{-n}dx = -\dfrac{1}{n-1}x^{-(n-1)}$

EXECUTE: **(a)** $W = \int_{x_0}^{\infty} \dfrac{b}{x^n}dx = \dfrac{b}{(n-1))x^{n-1}}\Bigg|_{x_0}^{\infty} = \dfrac{b}{(n-1)x_0^{n-1}}$. Note that for this part, for $n > 1,\ x^{1-n} \to 0$ as $x \to \infty$.

(b) When $0 < n < 1$, the improper integral must be used, $W = \lim_{x_2 \to \infty}\left[\frac{b}{(n-1)}(x_2^{n-1} - x_0^{n-1})\right]$, and because the exponent on the x_2^{n-1} is positive, the limit does not exist, and the integral diverges. This is interpreted as the force F doing an infinite amount of work, even though $F \to 0$ as $x_2 \to \infty$.

EVALUATE: The work-energy theorem says that an object gains an infinite amount of kinetic energy when an infinite amount of work is done on it.

6.75. **IDENTIFY:** The negative work done by the spring equals the change in kinetic energy of the car.

SET UP: The work done by a spring when it is compressed a distance x from equilibrium is $-\frac{1}{2}kx^2$. $K_2 = 0$.

EXECUTE: $-\frac{1}{2}kx^2 = K_2 - K_1$ gives $\frac{1}{2}kx^2 = \frac{1}{2}mv_1^2$ and

$k = (mv_1^2)/x^2 = \left[(1200\ \text{kg})(0.65\ \text{m/s})^2\right]/(0.070\ \text{m})^2 = 1.0 \times 10^5\ \text{N/m}$.

EVALUATE: When the spring is compressed, the spring force is directed opposite to the displacement of the object and the work done by the spring is negative.

6.76. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: Let x_0 be the initial distance the spring is compressed. The work done by the spring is $\frac{1}{2}kx_0^2 - \frac{1}{2}kx^2$, where x is the final distance the spring is compressed.

EXECUTE: **(a)** Equating the work done by the spring to the gain in kinetic energy, $\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2$, so

$$v = \sqrt{\frac{k}{m}}x_0 = \sqrt{\frac{400\ \text{N/m}}{0.0300\ \text{kg}}}(0.060\ \text{m}) = 6.93\ \text{m/s}.$$

(b) W_{tot} must now include friction, so $\frac{1}{2}mv^2 = W_{\text{tot}} = \frac{1}{2}kx_0^2 - fx_0$, where f is the magnitude of the friction force. Then,

$$v = \sqrt{\frac{k}{m}x_0^2 - \frac{2f}{m}x_0} = \sqrt{\frac{400\ \text{N/m}}{0.0300\ \text{kg}}(0.060\ \text{m})^2 - \frac{2(6.00\ \text{N})}{(0.0300\ \text{kg})}(0.060\ \text{m})} = 4.90\ \text{m/s}.$$

(c) The greatest speed occurs when the acceleration (and the net force) are zero. Let x be the amount the spring is still compressed, so the distance the ball has moved is $x_0 - x$. $kx = f$, $x = \frac{f}{k} = \frac{6.00\ \text{N}}{400\ \text{N/m}} = 0.0150\ \text{m}$. To find the speed, the net work is $W_{\text{tot}} = \frac{1}{2}k(x_0^2 - x^2) - f(x_0 - x)$, so the maximum speed is $v_{\max} = \sqrt{\frac{k}{m}(x_0^2 - x^2) - \frac{2f}{m}(x_0 - x)}$.

$$v_{\max} = \sqrt{\frac{400\ \text{N/m}}{(0.0300\ \text{kg})}((0.060\ \text{m})^2 - (0.0150\ \text{m})^2) - \frac{2(6.00\ \text{N})}{(0.0300\ \text{kg})}(0.060\ \text{m} - 0.0150\ \text{m})} = 5.20\ \text{m/s}$$

EVALUATE: The maximum speed with friction present (part (c)) is larger than the result of part (b) but smaller than the result of part (a).

6.77. **IDENTIFY** and **SET UP:** Use Eq.(6.6). Work is done by the spring and by gravity. Let point 1 be where the textbook is released and point 2 be where it stops sliding. $x_2 = 0$ since at point 2 the spring is neither stretched nor compressed. The situation is sketched in Figure 6.77.

EXECUTE:

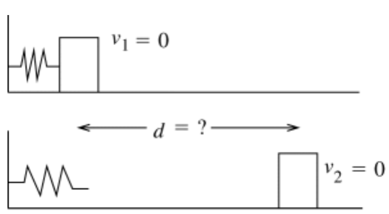


Figure 6.77

$W_{\text{tot}} = K_2 - K_1$

$K_1 = 0,\ \ K_2 = 0$

$W_{\text{tot}} = W_{\text{fric}} + W_{\text{spr}}$

$W_{\text{spr}} = \frac{1}{2}kx_1^2$, where $x_1 = 0.250$ m (Spring force is in direction of motion of block so it does positive work.)

$W_{\text{fric}} = -\mu_k mgd$

Then $W_{\text{tot}} = K_2 - K_1$ gives $\frac{1}{2}kx_1^2 - \mu_k mgd = 0$

$$d = \frac{kx_1^2}{2\mu_k mg} = \frac{(250\ \text{N/m})(0.250\ \text{m})^2}{2(0.30)(2.50\ \text{kg})(9.80\ \text{m/s}^2)} = 1.1\ \text{m},$$ measured from the point where the block was released.

EVALUATE: The positive work done by the spring equals the magnitude of the negative work done by friction. The total work done during the motion between points 1 and 2 is zero and the textbook starts and ends with zero kinetic energy.

6.78. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to the cat.

SET UP: Let point 1 be at the bottom of the ramp and point 2 be at the top of the ramp.

EXECUTE: The work done by gravity is $W_{\text{g}} = -mgL\sin\theta$ (negative since the cat is moving up), and the work done by the applied force is *FL*, where *F* is the magnitude of the applied force. The total work is

$$W_{\text{tot}} = (100\text{ N})(2.00\text{ m}) - (7.00\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})\sin 30° = 131.4\text{ J}.$$

The cat's initial kinetic energy is $\frac{1}{2}mv_1^2 = \frac{1}{2}(7.00\text{ kg})(2.40\text{ m/s})^2 = 20.2\text{ J}$, and

$$v_2 = \sqrt{\frac{2(K_1 + W)}{m}} = \sqrt{\frac{2(20.2\text{ J} + 131.4\text{ J})}{(7.00\text{ kg})}} = 6.58\text{ m/s}.$$

EVALUATE: The net work done on the cat is positive and the cat gains speed. Without your push, $W_{\text{tot}} = W_{\text{grav}} = -68.6\text{ J}$ and the cat wouldn't have enough initial kinetic energy to reach the top of the ramp.

6.79. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to the vehicle.

SET UP: Call the bumper compression *x* and the initial speed v_0. The work done by the spring is $-\frac{1}{2}kx^2$ and $K_2 = 0$.

EXECUTE: **(a)** The necessary relations are $\frac{1}{2}kx^2 = \frac{1}{2}mv_0^2$, $kx < 5\,mg$. Combining to eliminate *k* and then *x*, the two inequalities are $x > \frac{v^2}{5g}$ and $k < 25\frac{mg^2}{v^2}$. Using the given numerical values, $x > \frac{(20.0\text{ m/s})^2}{5(9.80\text{ m/s}^2)} = 8.16\text{ m}$ and $k < 25\frac{(1700\text{ kg})(9.80\text{ m/s}^2)^2}{(20.0\text{ m/s})^2} = 1.02\times10^4\text{ N/m}$.

(b) A distance of 8 m is not commonly available as space in which to stop a car. Also, the car stops only momentarily and then returns to its original speed when the spring returns to its original length.

EVALUATE: If *k* were doubled, to $2.04\times10^4\text{ N/m}$, then $x = 5.77\text{ m}$. The stopping distance is reduced by a factor of $1/\sqrt{2}$, but the maximum acceleration would then be $kx/m = 69.2\text{ m/s}^2$, which is $7.07g$.

6.80. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$. $W = Fs\cos\phi$.

SET UP: The students do positive work, and the force that they exert makes an angle of $30.0°$ with the direction of motion. Gravity does negative work, and is at an angle of $120.0°$ with the chair's motion,

EXECUTE: The total work done is $W_{\text{tot}} = ((600\text{ N})\cos 30.0° + (85.0\text{ kg})(9.80\text{ m/s}^2)\cos 120.0°)(2.50\text{ m}) = 257.8\text{ J}$, and so the speed at the top of the ramp is $v_2 = \sqrt{v_1^2 + \frac{2W_{\text{tot}}}{m}} = \sqrt{(2.00\text{ m/s})^2 + \frac{2(257.8\text{ J})}{(85.0\text{ kg})}} = 3.17\text{ m/s}.$

EVALUATE: The component of gravity down the incline is $mg\sin 30° = 417\text{ N}$ and the component of the push up the incline is $(600\text{ N})\cos 30° = 520\text{ N}$. The force component up the incline is greater than the force component down the incline, the net work done is positive and the speed increases.

6.81. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to the blocks.

SET UP: If *X* is the distance the spring is compressed, the work done by the spring is $-\frac{1}{2}kX^2$. At maximum compression, the spring (and hence the block) is not moving, so the block has no kinetic energy and $x_2 = 0$.

EXECUTE: **(a)** The work done *by* the block is equal to its initial kinetic energy, and the maximum compression is found from $\frac{1}{2}kX^2 = \frac{1}{2}mv_0^2$ and $X = \sqrt{\frac{m}{k}}v = \sqrt{\frac{5.00\text{ kg}}{500\text{ N/m}}}(6.00\text{ m/s}) = 0.600\text{ m}.$

(b) Solving for v_0 in terms of a known *X*, $v_0 = \sqrt{\frac{k}{m}}X = \sqrt{\frac{500\text{ N/m}}{5.00\text{ kg}}}(0.150\text{ m}) = 1.50\text{ m/s}.$

EVALUATE: The negative work done by the spring removes the kinetic energy of the block.

6.82. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to the system of the two blocks. The total work done is the sum of that done by gravity (on the hanging block) and that done by friction (on the block on the table).

SET UP: Let *h* be the distance the 6.00 kg block descends. The work done by gravity is (6.00 kg)*gh* and the work done by friction is $-\mu_{\text{k}}(8.00\text{ kg})gh$.

EXECUTE: $W_{\text{tot}} = (6.00\text{ kg} - (0.25)(8.00\text{ kg}))\,(9.80\text{ m/s}^2)\,(1.50\text{ m}) = 58.8\text{ J}$. This work increases the kinetic energy of both blocks: $W_{\text{tot}} = \frac{1}{2}(m_1 + m_2)v^2$, so $v = \sqrt{\frac{2(58.8\text{ J})}{(14.00\text{ kg})}} = 2.90\text{ m/s}$.

EVALUATE: Since the two blocks are connected by the rope, they move the same distance h and have the same speed v.

6.83. **IDENTIFY** and **SET UP:** Apply $W_{\text{tot}} = K_2 - K_1$ to the system consisting of both blocks. Since they are connected by the cord, both blocks have the same speed at every point in the motion. Also, when the 6.00-kg block has moved downward 1.50 m, the 8.00-kg block has moved 1.50 m to the right. The target variable, μ_k, will be a factor in the work done by friction. The forces on each block are shown in Figure 6.83.

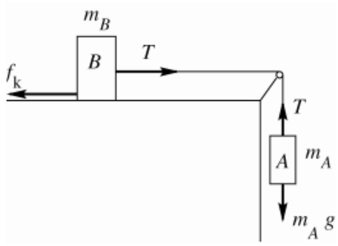


Figure 6.83

EXECUTE: $K_1 = \frac{1}{2}m_A v_1^2 + \frac{1}{2}m_B v_1^2 = \frac{1}{2}(m_A + m_B)v_1^2$

$K_2 = 0$

The tension T in the rope does positive work on block B and the same magnitude of negative work on block A, so T does no net work on the system. Gravity does work $W_{mg} = m_A g d$ on block A, where $d = 2.00\text{ m}$. (Block B moves horizontally, so no work is done on it by gravity.) Friction does work $W_{\text{fric}} = -\mu_k m_B g d$ on block B. Thus $W_{\text{tot}} = W_{mg} + W_{\text{fric}} = m_A g d - \mu_k m_B g d$. Then $W_{\text{tot}} = K_2 - K_1$ gives $m_A g d - \mu_k m_B g d = -\frac{1}{2}(m_A + m_B)v_1^2$ and

$$\mu_k = \frac{m_A}{m_B} + \frac{\frac{1}{2}(m_A + m_B)v_1^2}{m_B g d} = \frac{6.00\text{ kg}}{8.00\text{ kg}} + \frac{(6.00\text{ kg} + 8.00\text{ kg})(0.900\text{ m/s})^2}{2(8.00\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})} = 0.786$$

EVALUATE: The weight of block A does positive work and the friction force on block B does negative work, so the net work is positive and the kinetic energy of the blocks increases as block A descends. Note that K_1 includes the kinetic energy of both blocks. We could have applied the work-energy theorem to block A alone, but then W_{tot} includes the work done on block A by the tension force.

6.84. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$. The work done by the force from the bow is the area under the graph of F_x versus the draw length.

SET UP: One possible way of estimating the work is to approximate the F versus x curve as a parabola which goes to zero at $x = 0$ and $x = x_0$, and has a maximum of F_0 at $x = x_0/2$, so that $F(x) = (4F_0/x_0^2)x(x_0 - x)$. This may seem like a crude approximation to the figure, but it has the advantage of being easy to integrate.

EXECUTE: $\int_0^{x_0} F dx = \frac{4F_0}{x_0^2}\int_0^{x_0}(x_0 x - x^2)\,dx = \frac{4F_0}{x_0^2}\left(x_0\frac{x_0^2}{2} - \frac{x_0^3}{3}\right) = \frac{2}{3}F_0 x_0$. With $F_0 = 200\text{ N}$ and $x_0 = 0.75\text{ m}$, $W = 100\text{ J}$. The speed of the arrow is then $\sqrt{\frac{2W}{m}} = \sqrt{\frac{2(100\text{ J})}{(0.025\text{ kg})}} = 89\text{ m/s}$.

EVALUATE: We could alternatively represent the area as that of a rectangle 180 N by 0.55 m. This gives $W = 99\text{ J}$, in close agreement with our more elaborate estimate.

6.85. **IDENTIFY:** Apply Eq.(6.6) to the skater.

SET UP: Let point 1 be just before she reaches the rough patch and let point 2 be where she exits from the patch. Work is done by friction. We don't know the skater's mass so can't calculate either friction or the initial kinetic energy. Leave her mass m as a variable and expect that it will divide out of the final equation.

EXECUTE: $f_k = 0.25mg$ so $W_f = W_{\text{tot}} = -(0.25mg)s$, where s is the length of the rough patch.

$W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2,\quad K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}m(0.45v_0)^2 = 0.2025\left(\frac{1}{2}mv_0^2\right)$

The work-energy relation gives $-(0.25mg)s = (0.2025 - 1)\frac{1}{2}mv_0^2$

The mass divides out, and solving gives $s = 1.5\text{ m}$.

EVALUATE: Friction does negative work and this reduces her kinetic energy.

6.86. **IDENTIFY:** $P_{av} = F_{\|}v_{av}$. Use $F = ma$ to calculate the force.

SET UP: $v_{av} = \dfrac{0 + 6.00\text{ m/s}}{2} = 3.00\text{ m/s}$

EXECUTE: Your friend's average acceleration is $a = \dfrac{v - v_0}{t} = \dfrac{6.00\text{ m/s}}{3.00\text{ s}} = 2.00\text{ m/s}^2$. Since there are no other horizontal forces acting, the force you exert on her is given by $F_{net} = ma = (65.0\text{ kg})(2.00\text{ m/s}^2) = 130\text{ N}$.

$P_{av} = (130\text{ N})(3.00\text{ m/s}) = 390\text{ W}$.

EVALUATE: We could also use the work-energy theorem: $W = K_2 - K_1 = \frac{1}{2}(65.0\text{ kg})(6.00\text{ m/s})^2 = 1170\text{ J}$.

$P_{av} = \dfrac{W}{t} = \dfrac{1170\text{ J}}{3.00\text{ s}} = 390\text{ W}$, the same as obtained by our other approach.

6.87. **IDENTIFY:** To lift a mass *m* a height *h* requires work $W = mgh$. To accelerate mass *m* from rest to speed *v* requires $W = K_2 - K_1 = \frac{1}{2}mv^2$. $P_{av} = \dfrac{\Delta W}{\Delta t}$.

SET UP: $t = 60\text{ s}$

EXECUTE: **(a)** $(800\text{ kg})(9.80\text{ m/s}^2)(14.0\text{ m}) = 1.10\times10^5\text{ J}$

(b) $(1/2)(800\text{ kg})(18.0\text{ m/s}^2) = 1.30\times10^5\text{ J}$.

(c) $\dfrac{1.10\times10^5\text{ J} + 1.30\times10^5\text{ J}}{60\text{ s}} = 3.99\text{ kW}$.

EVALUATE: Approximately the same amount of work is required to lift the water against gravity as to accelerate it to its final speed.

6.88. **IDENTIFY:** $P = F_{\|}v$ and $F_{\|} = ma$.

SET UP: From Problem 6.71, $v = 2\alpha t + 3\beta t^2$ and $a = 2\alpha + 6\beta t$.

EXECUTE: $P = F_{\|}v = mav = m(2\alpha + 6\beta t)(2\alpha t + 3\beta t^2) = m(4\alpha^2 t + 18\alpha\beta t^2 + 18\beta^2 t^3)$.

$P = (0.96\text{ N/s})t + (0.43\text{ N/s}^2)t^2 + (0.043\text{ N/s}^3)t^3$. At $t = 4.00\text{ s}$, the power output is 13.5 W.

EVALUATE: *P* increases in time because *v* increase and because *a* increases.

6.89. **IDENTIFY** and **SET UP:** Energy is $P_{av}t$. The total energy expended in one day is the sum of the energy expended in each type of activity.

EXECUTE: $1\text{ day} = 8.64\times10^4\text{ s}$

Let t_{walk} be the time she spends walking and t_{other} be the time she spends in other activities; $t_{other} = 8.64\times10^4\text{ s} - t_{walk}$. The energy expended in each activity is the power output times the time, so

$E = Pt = (280\text{ W})t_{walk} + (100\text{ W})t_{other} = 1.1\times10^7\text{ J}$

$(280\text{ W})t_{walk} + (100\text{ W})(8.64\times10^4\text{ s} - t_{walk}) = 1.1\times10^7\text{ J}$

$(180\text{ W})t_{walk} = 2.36\times10^6\text{ J}$

$t_{walk} = 1.31\times10^4\text{ s} = 218\text{ min} = 3.6\text{ h}$.

EVALUATE: Her average power for one day is $(1.1\times10^7\text{ J})/([24][3600\text{ s}]) = 127\text{ W}$. This is much closer to her 100 W rate than to her 280 W rate, so most of her day is spent at the 100 W rate.

6.90. **IDENTIFY** and **SET UP:** $W = Pt$

EXECUTE: **(a)** The hummingbird produces energy at a rate of 0.7 J/s to 1.75 J/s. At 10 beats/s, the bird must expend between 0.07 J/beat and 0.175 J/beat.

(b) The steady output of the athlete is (500 W)/(70 kg) = 7 W/kg, which is below the 10 W/kg necessary to stay aloft. Though the athlete can expend 1400 W/70 kg = 20 W/kg for short periods of time, no human-powered aircraft could stay aloft for very long.

EVALUATE: Movies of early attempts at human-powered flight bear out our results.

6.91. **IDENTIFY** and **SET UP:** Use Eq.(6.15). The work done on the water by gravity is *mgh*, where $h = 170\text{ m}$. Solve for the mass *m* of water for 1.00 s and then calculate the volume of water that has this mass.

EXECUTE: The power output is $P_{av} = 2000 \text{ MW} = 2.00\times10^9 \text{ W}$. $P_{av} = \dfrac{\Delta W}{\Delta t}$ and 92% of the work done on the water by gravity is converted to electrical power output, so in 1.00 s the amount of work done on the water by gravity is

$$W = \frac{P_{av}\Delta t}{0.92} = \frac{(2.00\times10^9 \text{ W})(1.00 \text{ s})}{0.92} = 2.174\times10^9 \text{ J}$$

$W = mgh$, so the mass of water flowing over the dam in 1.00 s must be

$$m = \frac{W}{gh} = \frac{2.174\times10^9 \text{ J}}{(9.80 \text{ m/s}^2)(170 \text{ m})} = 1.30\times10^6 \text{ kg}$$

$$\text{density} = \frac{m}{V} \text{ so } V = \frac{m}{\text{density}} = \frac{1.30\times10^6 \text{ kg}}{1.00\times10^3 \text{ kg/m}^3} = 1.30\times10^3 \text{ m}^3.$$

EVALUATE: The dam is 1270 m long, so this volume corresponds to about a m^3 flowing over each 1 m length of the dam, a reasonable amount.

6.92. **IDENTIFY:** $P = \dfrac{W}{t}$ and $W = \tfrac{1}{2}mv^2$, if the object starts from rest. $a = \dfrac{dv}{dt}$ and $x - x_0 = \int v dt$.

SET UP: $\dfrac{d}{dt}t^{1/2} = \tfrac{1}{2}t^{-1/2}$. $\int t^{1/2}dt = \tfrac{2}{3}t^{3/2}$.

EXECUTE: **(a)** The power P is related to the speed by $Pt = K = \tfrac{1}{2}mv^2$, so $v = \sqrt{\dfrac{2Pt}{m}}$.

(b) $a = \dfrac{dv}{dt} = \dfrac{d}{dt}\sqrt{\dfrac{2Pt}{m}} = \sqrt{\dfrac{2P}{m}}\dfrac{d}{dt}\sqrt{t} = \sqrt{\dfrac{2P}{m}}\dfrac{1}{2\sqrt{t}} = \sqrt{\dfrac{P}{2mt}}.$

(c) $x - x_0 = \int v\, dt = \sqrt{\dfrac{2P}{m}}\int t^{\frac{1}{2}}\, dt = \sqrt{\dfrac{2P}{m}}\dfrac{2}{3}t^{\frac{3}{2}} = \sqrt{\dfrac{8P}{9m}}t^{\frac{3}{2}}.$

EVALUATE: v, a, and $x - x_0$ at a particular time are all proportional to $P^{1/2}$. The result in part (b) could also be obtained from $P = Fv$ and $a = F/m$, so $a = \dfrac{P}{vm}$.

6.93. **IDENTIFY** and **SET UP:** For part (a) calculate m from the volume of blood pumped by the heart in one day. For part (b) use W calculated in part (a) in Eq.(6.15).

EXECUTE: **(a)** $W = mgh$, as in Example 6.11. We need the mass of blood lifted; we are given the volume

$$V = (7500 \text{ L})\left(\frac{1\times10^{-3} \text{ m}^3}{1 \text{ L}}\right) = 7.50 \text{ m}^3.$$

$$m = \text{density}\times\text{volume} = (1.05\times10^3 \text{ kg/m}^3)(7.50 \text{ m}^3) = 7.875\times10^3 \text{ kg}$$

Then $W = mgh = (7.875\times10^3 \text{ kg})(9.80 \text{ m/s}^2)(1.63 \text{ m}) = 1.26\times10^5 \text{ J}.$

(b) $P_{av} = \dfrac{\Delta W}{\Delta t} = \dfrac{1.26\times10^5 \text{ J}}{(24 \text{ h})(3600 \text{ s/h})} = 1.46 \text{ W}.$

EVALUATE: Compared to light bulbs or common electrical devices, the power output of the heart is rather small.

6.94. **IDENTIFY:** $P = F_{\|}v = Mav$. To overcome gravity on a slope that is at an angle α above the horizontal, $P = (Mg\sin\alpha)v$.

SET UP: $1 \text{ MW} = 10^6 \text{ W}$. $1 \text{ kN} = 10^3 \text{ N}$. When α is small, $\tan\alpha \approx \sin\alpha$.

EXECUTE: **(a)** The number of cars is the total power available divided by the power needed per car,

$\dfrac{13.4\times10^6 \text{ W}}{(2.8\times10^3 \text{ N})(27 \text{ m/s})} = 177$, rounding down to the nearest integer.

(b) To accelerate a total mass M at an acceleration a and speed v, the extra power needed is Mav. To climb a hill of angle α, the extra power needed is $(Mg\sin\alpha)v$. This will be nearly the same if $a \sim g\sin\alpha$; if $g\sin\alpha \sim g\tan\alpha \sim 0.10 \text{ m/s}^2$, the power is about the same as that needed to accelerate at 0.10 m/s^2.

(c) $P = (Mg\sin\alpha)v$, where M is the total mass of the diesel units.

$P = (1.10\times10^6 \text{ kg})(9.80 \text{ m/s}^2)(0.010)(27 \text{ m/s}) = 2.9 \text{ MW}.$

(d) The power available to the cars is 13.4 MW, minus the 2.9 MW needed to maintain the speed of the diesel units on the incline. The total number of cars is then $\dfrac{13.4\times10^6 \text{ W} - 2.9\times10^6 \text{ W}}{(2.8\times10^3 \text{ N} + (8.2\times10^4 \text{ kg})(9.80 \text{ m/s}^2)(0.010))(27 \text{ m/s})} = 36,$ rounding to the nearest integer.

EVALUATE: For a single car, $Mg\sin\alpha = (8.2\times10^4\ \text{kg})(9.80\ \text{m/s}^2)(0.010) = 8.0\times10^3\ \text{N}$, which is over twice the 2.8 kN required to pull the car at 27 m/s on level tracks. Even a slope as gradual as 1.0% greatly increases the power requirements, or for constant power greatly decreases the number of cars that can be pulled.

6.95. IDENTIFY: $P = F_{\|}v$. The force required to give mass m an acceleration a is $F = ma$. For an incline at an angle α above the horizontal, the component of mg down the incline is $mg\sin\alpha$.

SET UP: For small α, $\sin\alpha \approx \tan\alpha$.

EXECUTE: **(a)** $P_0 = Fv = (53\times10^3\ \text{N})(45\ \text{m/s}) = 2.4\ \text{MW}$.

(b) $P_1 = mav = (9.1\times10^5\ \text{kg})(1.5\ \text{m/s}^2)(45\ \text{m/s}) = 61\ \text{MW}$.

(c) Approximating $\sin\alpha$, by $\tan\alpha$, and using the component of gravity down the incline as $mg\sin\alpha$,

$P_2 = (mg\sin\alpha)v = (9.1\times10^5\ \text{kg})(9.80\ \text{m/s}^2)(0.015)(45\ \text{m/s}) = 6.0\ \text{MW}$.

EVALUATE: From Problem 6.94, we would expect that a $0.15\ \text{m/s}^2$ acceleration and a 1.5% slope would require the same power. We found that a $1.5\ \text{m/s}^2$ acceleration requires ten times more power than a 1.5% slope, which is consistent.

6.96. IDENTIFY: $W = \int_{x_1}^{x_2} F_x dx$, and F_x depends on both x and y.

SET UP: In each case, use the value of y that applies to the specified path. $\int x dx = \frac{1}{2}x^2$. $\int x^2 dx = \frac{1}{3}x^3$

EXECUTE: **(a)** Along this path, y is constant, with the value $y = 3.00\ \text{m}$.

$W = \alpha y\int_{x_1}^{x_2} x dx = (2.50\ \text{N/m}^2)(3.00\ \text{m})\left(\dfrac{2.00\ \text{m}}{2}\right) = 15.0\ \text{J}$, since $x_1 = 0$ and $x_2 = 2.00\ \text{m}$.

(b) Since the force has no y-component, no work is done moving in the y-direction.

(c) Along this path, y varies with position along the path, given by $y = 1.5x$, so $F_x = \alpha(1.5x)x = 1.5\alpha x^2$, and

$$W = \int_{x_1}^{x_2} F dx = 1.5\alpha\int_{x_1}^{x_2} x^2 dx = 1.5(2.50\ \text{N/m}^2)\frac{(2.00\ \text{m})^3}{3} = 10.0\ \text{J}.$$

EVALUATE: The force depends on the position of the object along its path.

6.97. IDENTIFY and **SET UP:** Use Eq.(6.18) to relate the forces to the power required. The air resistance force is $F_{\text{air}} = \frac{1}{2}CA\rho v^2$, where C is the drag coefficient.

EXECUTE: **(a)** $P = F_{\text{tot}}v$, with $F_{\text{tot}} = F_{\text{roll}} + F_{\text{air}}$

$F_{\text{air}} = \frac{1}{2}CA\rho v^2 = \frac{1}{2}(1.0)(0.463\ \text{m}^3)(1.2\ \text{kg/m}^3)(12.0\ \text{m/s})^2 = 40.0\ \text{N}$

$F_{\text{roll}} = \mu_r n = \mu_r w = (0.0045)(490\ \text{N} + 118\ \text{N}) = 2.74\ \text{N}$

$P = (F_{\text{roll}} + F_{\text{air}})v = (2.74\ \text{N} + 40.0\ \text{N})(12.0\ \text{s}) = 513\ \text{W}$

(b) $F_{\text{air}} = \frac{1}{2}CA\rho v^2 = \frac{1}{2}(0.88)(0.366\ \text{m}^3)(1.2\ \text{kg/m}^3)(12.0\ \text{m/s})^2 = 27.8\ \text{N}$

$F_{\text{roll}} = \mu_r n = \mu_r w = (0.0030)(490\ \text{N} + 88\ \text{N}) = 1.73\ \text{N}$

$P = (F_{\text{roll}} + F_{\text{air}})v = (1.73\ \text{N} + 27.8\ \text{N})(12.0\ \text{s}) = 354\ \text{W}$

(c) $F_{\text{air}} = \frac{1}{2}CA\rho v^2 = \frac{1}{2}(0.88)(0.366\ \text{m}^3)(1.2\ \text{kg/m}^3)(6.0\ \text{m/s})^2 = 6.96\ \text{N}$

$F_{\text{roll}} = \mu_r n = 1.73\ \text{N}$ (unchanged)

$P = (F_{\text{roll}} + F_{\text{air}})v = (1.73\ \text{N} + 6.96\ \text{N})(6.0\ \text{s}) = 52.1\ \text{W}$

EVALUATE: Since F_{air} is proportional to v^2 and $P = Fv$, reducing the speed greatly reduces the power required.

6.98. IDENTIFY: $P = F_{\|}v$

SET UP: $1\ \text{m/s} = 3.6\ \text{km/h}$

EXECUTE: **(a)** $F = \dfrac{P}{v} = \dfrac{28.0\times10^3\ \text{W}}{(60.0\ \text{km/h})((1\ \text{m/s})/(3.6\ \text{km/h}))} = 1.68\times10^3\ \text{N}$.

(b) The speed is lowered by a factor of one-half, and the resisting force is lowered by a factor of $(0.65 + 0.35/4)$, and so the power at the lower speed is $(28.0\ \text{kW})(0.50)(0.65 + 0.35/4) = 10.3\ \text{kW} = 13.8\ \text{hp}$.

(c) Similarly, at the higher speed, $(28.0\ \text{kW})(2.0)(0.65 + 0.35\times4) = 114.8\ \text{kW} = 154\ \text{hp}$.

EVALUATE: At low speeds rolling friction dominates the power requirement but at high speeds air resistance dominates.

6.99. **IDENTIFY** and **SET UP:** Use Eq.(6.18) to relate F and P. In part (a), F is the retarding force. In parts (b) and (c), F includes gravity.

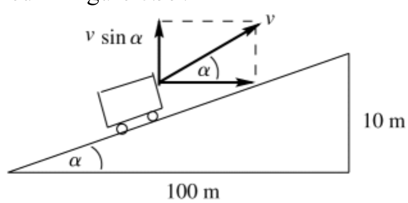
EXECUTE: **(a)** $P = Fv$, so $F = P/v$.

$$P = (8.00\text{ hp})\left(\frac{746\text{ W}}{1\text{ hp}}\right) = 5968\text{ W}$$

$$v = (60.0\text{ km/h})\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 16.67\text{ m/s}$$

$$F = \frac{P}{v} = \frac{5968\text{ W}}{16.67\text{ m/s}} = 358\text{ N}.$$

(b) The power required is the 8.00 hp of part (a) plus the power P_g required to lift the car against gravity. The situation is sketched in Figure 6.99.



$$\tan\alpha = \frac{10\text{ m}}{100\text{ m}} = 0.10$$
$$\alpha = 5.71°$$

Figure 6.99

The vertical component of the velocity of the car is $v\sin\alpha = (16.67\text{ m/s})\sin 5.71° = 1.658\text{ m/s}.$

Then $P_g = F(v\sin a) = mgv\sin\alpha = (1800\text{ kg})(9.80\text{ m/s}^2)(1.658\text{ m/s}) = 2.92\times10^4\text{ W}$

$$P_g = 2.92\times10^4\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = 39.1\text{ hp}$$

The total power required is $8.00\text{ hp} + 39.1\text{ hp} = 47.1\text{ hp}.$

(c) The power required from the engine is *reduced* by the rate at which gravity does positive work. The road incline angle α is given by $\tan\alpha = 0.0100$, so $\alpha = 0.5729°.$

$P_g = mg(v\sin\alpha) = (1800\text{ kg})(9.80\text{ m/s}^2)(16.67\text{ m/s})\sin 0.5729° = 2.94\times10^3\text{ W} = 3.94\text{ hp}.$

The power required from the engine is then $8.00\text{ hp} - 3.94\text{ hp} = 4.06\text{ hp}.$

(d) No power is needed from the engine if gravity does work at the rate of $P_g = 8.00\text{ hp} = 5968\text{ W}$

$$P_g = mgv\sin\alpha,\ \text{so}\ \sin\alpha = \frac{P_g}{mgv} = \frac{5968\text{ W}}{(1800\text{ kg})(9.80\text{ m/s}^2)(16.67\text{ m/s})} = 0.02030$$

$\alpha = 1.163°$ and $\tan\alpha = 0.0203$, a 2.03% grade.

EVALUATE: More power is required when the car goes uphill and less when it goes downhill. In part (d), at this angle the component of gravity down the incline is $mg\sin\alpha = 358\text{ N}$ and this force cancels the retarding force and no force from the engine is required. The retarding force depends on the speed so it is the same in parts (a), (b), and (c).

6.100. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to relate the initial speed v_0 to the distance x along the plank that the box moves before coming to rest.

SET UP: The component of weight down the incline is $mg\sin\alpha$, the normal force is $mg\cos\alpha$ and the friction force is $f = \mu mg\cos\alpha$.

EXECUTE: $\Delta K = 0 - \frac{1}{2}mv_0^2$ and $W = \int_0^x(-mg\sin\alpha - \mu mg\cos\alpha)dx.$ Then,

$$W = -mg\int_0^x(\sin\alpha + Ax\cos\alpha)dx,\ W = -mg\left[\sin\alpha x + \frac{Ax^2}{2}\cos\alpha\right].$$

Set $W = \Delta K$: $-\frac{1}{2}mv_0^2 = -mg\left[\sin\alpha x + \frac{Ax^2}{2}\cos\alpha\right]$. To eliminate x, note that the box comes to a rest when the force of static friction balances the component of the weight directed down the plane. So, $mg\sin\alpha = Ax\, mg\cos\alpha$. Solve this for x and substitute into the previous equation: $x = \frac{\sin\alpha}{A\cos\alpha}$. Then, $\frac{1}{2}v_0^2 = +g\left[\sin\alpha\frac{\sin\alpha}{A\cos\alpha} + \frac{1}{2}A\left(\frac{\sin\alpha}{A\cos\alpha}\right)^2\cos\alpha\right]$, and upon canceling factors and collecting terms, $v_0^2 = \frac{3g\sin^2\alpha}{A\cos\alpha}$. The box will remain stationary whenever $v_0^2 \geq \frac{3g\sin^2\alpha}{A\cos\alpha}$.

EVALUATE: If v_0 is too small the box stops at a point where the friction force is too small to hold the box in place. $\sin\alpha$ increases and $\cos\alpha$ decreases as α increases, so the v_0 required increases as α increases.

6.101. IDENTIFY: In part (a) follow the steps outlined in the problem. For parts (b), (c) and (d) apply the work-energy theorem.

SET UP: $\int x^2 dx = \frac{1}{3}x^3$

EXECUTE: **(a)** Denote the position of a piece of the spring by *l*; $l = 0$ is the fixed point and $l = L$ is the moving end of the spring. Then the velocity of the point corresponding to *l*, denoted *u*, is $u(l) = v(\mathrm{I}/L)$ (when the spring is moving, *l* will be a function of time, and so *u* is an implicit function of time). The mass of a piece of length *dl* is $dm = (M/L)dl$, and so $dK = \frac{1}{2}(dm)u^2 = \frac{1}{2}\frac{Mv^2}{L^3}l^2 dl$, and $K = \int dK = \frac{Mv^2}{2L^3}\int_0^L l^2 dl = \frac{Mv^2}{6}$.

(b) $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, so $v = \sqrt{(k/m)x} = \sqrt{(3200\ \text{N/m})/(0.053\ \text{kg})}(2.50\times10^{-2}\ \text{m}) = 6.1\ \text{m/s}.$

(c) With the mass of the spring included, the work that the spring does goes into the kinetic energies of both the ball and the spring, so $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{6}Mv^2$. Solving for *v*,

$$v = \sqrt{\frac{k}{m + M/3}}x = \sqrt{\frac{(3200\ \text{N/m})}{(0.053\ \text{kg}) + (0.243\ \text{kg})/3}}(2.50\times10^{-2}\text{m}) = 3.9\ \text{m/s}.$$

(d) Algebraically, $\frac{1}{2}mv^2 = \frac{(1/2)kx^2}{(1 + M/3m)} = 0.40\ \text{J}$ and $\frac{1}{6}Mv^2 = \frac{(1/2)kx^2}{(1 + 3m/M)} = 0.60\ \text{J}.$

EVALUATE: For this ball and spring, $\frac{K_{\text{ball}}}{K_{\text{spring}}} = \frac{3m}{M} = 3\left(\frac{0.053\ \text{kg}}{0.243\ \text{kg}}\right) = 0.65$. The percentage of the final kinetic energy that ends up with each object depends on the ratio of the masses of the two objects. As expected, when the mass of the spring is a small fraction of the mass of the ball, the fraction of the kinetic energy that ends up in the spring is small.

6.102. IDENTIFY: In both cases, a given amount of fuel represents a given amount of work W_0 that the engine does in moving the plane forward against the resisting force. Write W_0 in terms of the range *R* and speed *v* and in terms of the time of flight *T* and *v*.

SET UP: In both cases assume *v* is constant, so $W_0 = RF$ and $R = vT$.

EXECUTE: In terms of the range *R* and the constant speed *v*, $W_0 = RF = R\left(\alpha v^2 + \frac{\beta}{v^2}\right)$.

In terms of the time of flight $T, R = vt$, so $W_0 = vTF = T\left(\alpha v^3 + \frac{\beta}{v}\right)$.

(a) Rather than solve for *R* as a function of *v*, differentiate the first of these relations with respect to *v*, setting $\frac{dW_0}{dv} = 0$ to obtain $\frac{dR}{dv}F + R\frac{dF}{dv} = 0$. For the maximum range, $\frac{dR}{dv} = 0$, so $\frac{dF}{dv} = 0$. Performing the differentiation, $\frac{dF}{dv} = 2\alpha v - 2\beta/v^3 = 0$, which is solved for

$$v = \left(\frac{\beta}{\alpha}\right)^{1/4} = \left(\frac{3.5\times10^5\ \text{N}\cdot\text{m}^2/\text{s}^2}{0.30\ \text{N}\cdot\text{s}^2/\text{m}^2}\right)^{1/4} = 32.9\ \text{m/s} = 118\ \text{km/h}.$$

(b) Similarly, the maximum time is found by setting $\frac{d}{dv}(Fv) = 0$; performing the differentiation, $3\alpha v^2 - \beta/v^2 = 0$.

$$v = \left(\frac{\beta}{3\alpha}\right)^{1/4} = \left(\frac{3.5\times10^5\ \text{N}\cdot\text{m}^2/\text{s}^2}{3(0.30\ \text{N}\cdot\text{s}^2/\text{m}^2)}\right)^{1/4} = 25\ \text{m/s} = 90\ \text{km/h}.$$

EVALUATE: When $v = (\beta/\alpha)^{1/4}$, F_{air} has its minimum value $F_{\text{air}} = 2\sqrt{\alpha\beta}$. For this *v*, $R_1 = (0.50)\frac{W_0}{\sqrt{\alpha\beta}}$ and $T_1 = (0.50)\alpha^{-1/4}\beta^{-3/4}$. When $v = (\beta/3\alpha)^{1/4}$, $F_{\text{air}} = 2.3\sqrt{\alpha\beta}$. For this *v*, $R_2 = (0.43)\frac{W_0}{\sqrt{\alpha\beta}}$ and $T_2 = (0.57)\alpha^{-1/4}\beta^{-3/4}$.

$R_1 > R_2$ and $T_2 > T_1$, as they should be.

6.103. IDENTIFY: For each speed, calculate the time. Then use the graph to find the oxygen consumption and from that the energy consumption.

SET UP: $t = d/v$

EXECUTE: **(a)** The walk will take one-fifth of an hour, 12 min. From the graph, the oxygen consumption rate appears to be about $12\ \text{cm}^3/\text{kg}\cdot\text{min},$ and so the total energy is

$$(12\ \text{cm}^3/\text{kg}\cdot\text{min})\ (70\ \text{kg})\ (12\ \text{min})\ (20\ \text{J/cm}^3) = 2.0\times10^5\ \text{J}.$$

(b) The run will take 6 min. Using an estimation of the rate from the graph of about $33\ \text{cm}^3/\text{kg}\cdot\text{min}$ gives an energy consumption of about $2.8\times10^5\ \text{J}.$

(c) The run takes 4 min, and with an estimated rate of about $50\ \text{cm}^3/\text{kg}\cdot\text{min},$ the energy used is about $2.8\times10^5\ \text{J}.$

(d) Walking is the most efficient way to go. In general, the point where the slope of the line from the origin to the point on the graph is the smallest is the most efficient speed; about 5 km/h.

EVALUATE: In an exercise program, for a fixed distance, running burns more energy than walking.

6.104. **IDENTIFY:** Write equations similar to (6.11) for each component. Eq.(6.12) will now involve the sum of three integrals, one for each component.

SET UP: $v^2 = v_x^2 + v_y^2 + v_z^2$

EXECUTE: From $\vec{F} = m\vec{a},\ F_x = ma_x, F_y = ma_y$ and $F_z = ma_z.$ The generalization of Eq. (6.11) is then $a_x = v_x \dfrac{dv_x}{dx},\ a_y = v_y \dfrac{dv_y}{dy},\ \ a_z = v_z \dfrac{dv_z}{dz}.$ The total work is then

$$W_{\text{tot}} = \int_{(x_1,y_1,z_1)}^{(x_2,y_2,z_2)} F_x dx + F_y dy + F_z dz = m\left(\int_{x_1}^{x_2} v_x \frac{dv_x}{dx}dx + \int_{y_1}^{y_2} v_y \frac{dv_y}{dy}dy + \int_{z_1}^{z_2} v_z \frac{dv_z}{dz}dz\right).$$

$$W_{\text{tot}} = m\left(\int_{v_{x1}}^{v_{x2}} v_x dv_x + \int_{v_{y1}}^{v_{y2}} v_y dv_y + \int_{v_{z1}}^{v_{z2}} v_z dv_z\right) = \frac{1}{2}m(v_{x2}^2 - v_{x1}^2 + v_{y2}^2 - v_{y1}^2 + v_{z2}^2 - v_{z1}^2 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2.$$

EVALUATE: $\vec{F}$ and $d\vec{l}$ are vectors and have components. *W* and *K* are scalars and we never speak of their components.