5

APPLYING NEWTON'S LAWS

5.1. **IDENTIFY:** $a = 0$ for each object. Apply $\sum F_y = ma_y$ to each weight and to the pulley.

SET UP: Take $+y$ upward. The pulley has negligible mass. Let T_r be the tension in the rope and let T_c be the tension in the chain.

EXECUTE: **(a)** The free-body diagram for each weight is the same and is given in Figure 5.1a. $\sum F_y = ma_y$ gives $T_r = w = 25.0\text{ N}$.

(b) The free-body diagram for the pulley is given in Figure 5.1b. $T_c = 2T_r = 50.0\text{ N}$.

EVALUATE: The tension is the same at all points along the rope.

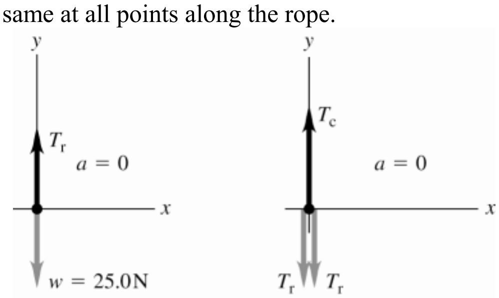


Figure 5.1a, b

5.2. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each weight.

SET UP: Two forces act on each mass**:** w down and $T(= w)$ up.

EXECUTE: In all cases, each string is supporting a weight w against gravity, and the tension in each string is w.

EVALUATE: The tension is the same in all three cases.

5.3. **IDENTIFY:** Both objects are at rest and $a = 0$. Apply Newton's first law to the appropriate object. The maximum tension T_{max} is at the top of the chain and the minimum tension is at the bottom of the chain.

SET UP: Let $+y$ be upward. For the maximum tension take the object to be the chain plus the ball. For the minimum tension take the object to be the ball. For the tension T three-fourths of the way up from the bottom of the chain, take the chain below this point plus the ball to be the object. The free-body diagrams in each of these three cases are sketched in Figures 5.3a, 5.3b and 5.3c. $m_{\text{b+c}} = 75.0\text{ kg} + 26.0\text{ kg} = 101.0\text{ kg}$. $m_{\text{b}} = 75.0\text{ kg}$. m is the mass of three-fourths of the chain: $m = \frac{3}{4}(26.0\text{ kg}) = 19.5\text{ kg}$.

EXECUTE: **(a)** From Figure 5.3a, $\sum F_y = 0$ gives $T_{\text{max}} - m_{\text{b+c}}g = 0$ and $T_{\text{max}} = (101.0\text{ kg})(9.80\text{ m/s}^2) = 990\text{ N}$.

From Figure 5.3b, $\sum F_y = 0$ gives $T_{\text{min}} - m_{\text{b}}g = 0$ and $T_{\text{min}} = (75.0\text{ kg})(9.80\text{ m/s}^2) = 735\text{ N}$.

(b) From Figure 5.3c, $\sum F_y = 0$ gives $T - (m + m_{\text{b}})g = 0$ and $T = (19.5\text{ kg} + 75.0\text{ kg})(9.80\text{ m/s}^2) = 926\text{ N}$.

EVALUATE: The tension in the chain increases linearly from the bottom to the top of the chain.

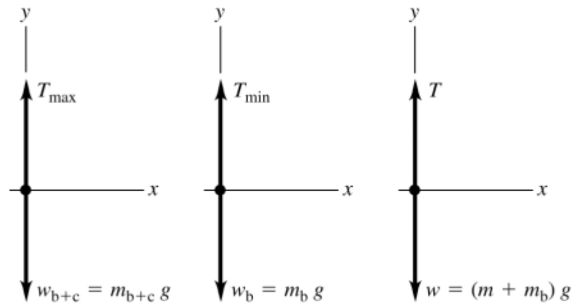


Figure 5.3a–c

5.4. **IDENTIFY:** Apply Newton's 1st law to the person. Each half of the rope exerts a force on him, directed along the rope and equal to the tension T in the rope.

SET UP: **(a)** The force diagram for the person is given in Figure 5.4

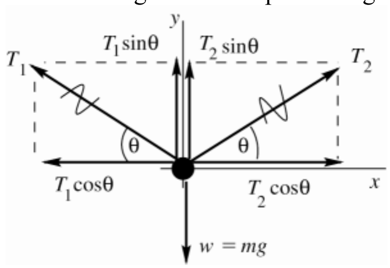


Figure 5.4

T_1 and T_2 are the tensions in each half of the rope.

EXECUTE: $\sum F_x = 0$

$T_2\cos\theta - T_1\cos\theta = 0$

This says that $T_1 = T_2 = T$ (The tension is the same on both sides of the person.)

$\sum F_y = 0$

$T_1\sin\theta + T_2\sin\theta - mg = 0$

But $T_1 = T_2 = T$, so $2T\sin\theta = mg$

$$T = \frac{mg}{2\sin\theta} = \frac{(90.0\text{ kg})(9.80\text{ m/s}^2)}{2\sin 10.0^\circ} = 2540\text{ N}$$

(b) The relation $2T\sin\theta = mg$ still applies but now we are given that $T = 2.50\times 10^4$ N (the breaking strength) and are asked to find θ.

$$\sin\theta = \frac{mg}{2T} = \frac{(90.0\text{ kg})(9.80\text{ m/s}^2)}{2(2.50\times 10^4\text{ N})} = 0.01764,\ \ \theta = 1.01^\circ.$$

EVALUATE: $T = mg/(2\sin\theta)$ says that $T = mg/2$ when $\theta = 90^\circ$ (rope is vertical).

$T \to \infty$ when $\theta \to 0$ since the upward component of the tension becomes a smaller fraction of the tension.

5.5. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the frame.

SET UP: Let w be the weight of the frame. Since the two wires make the same angle with the vertical, the tension is the same in each wire. $T = 0.75w$.

EXECUTE: The vertical component of the force due to the tension in each wire must be half of the weight, and this in turn is the tension multiplied by the cosine of the angle each wire makes with the vertical. $\dfrac{w}{2} = \dfrac{3w}{4}\cos\theta$ and $\theta = \arccos\frac{2}{3} = 48^\circ$.

EVALUATE: If $\theta = 0^\circ$, $T = w/2$ and $T \to \infty$ as $\theta \to 90^\circ$. Therefore, there must be an angle where $T = 3w/4$.

5.6. IDENTIFY: Apply Newton's 1st law to the car. The forces are the same as in Example 5.5.
SET UP: The free-body diagram is sketched in Figure 5.6.

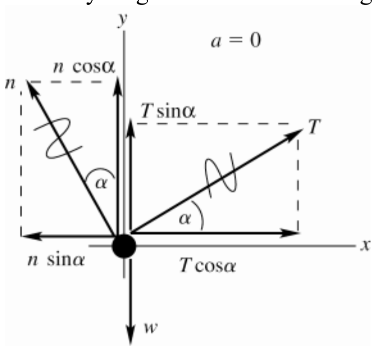


Figure 5.6

EXECUTE:
$\sum F_x = ma_x$
$T\cos\alpha - n\sin\alpha = 0$
$T\cos\alpha = n\sin\alpha$
$\sum F_y = ma_y$
$n\cos\alpha + T\sin\alpha - w = 0$
$n\cos\alpha + T\sin\alpha = w$

The first equation gives $n = T\left(\dfrac{\cos\alpha}{\sin\alpha}\right)$.

Use this in the second equation to eliminate n:

$$\left(T\frac{\cos\alpha}{\sin\alpha}\right)\cos\alpha + T\sin\alpha = w$$

Multiply this equation by $\sin\alpha$:

$T(\cos^2\alpha + \sin^2\alpha) = w\sin\alpha$

$T = w\sin\alpha$ (since $\cos^2\alpha + \sin^2\alpha = 1$).

Then $n = T\left(\dfrac{\cos\alpha}{\sin\alpha}\right) = w\sin\alpha\left(\dfrac{\cos\alpha}{\sin\alpha}\right) = w\cos\alpha.$

EVALUATE: These results are the same as obtained in Example 5.5. The choice of coordinate axes is up to us. Some choices may make the calculation easier, but the results are the same for any choice of axes.

5.7. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the car.

SET UP: Use coordinates with $+x$ parallel to the surface of the street.

EXECUTE: $\sum F_x = 0$ gives $T = w\sin\alpha$. $F = mg\sin\theta = (1390\text{ kg})(9.80\text{ m/s}^2)\sin 17.5° = 4.10\times 10^3\text{ N}$.

EVALUATE: The force required is less than the weight of the car by the factor $\sin\alpha$.

5.8. IDENTIFY: Apply Newton's 1st law to the wrecking ball. Each cable exerts a force on the ball, directed along the cable.
SET UP: The force diagram for the wrecking ball is sketched in Figure 5.8.

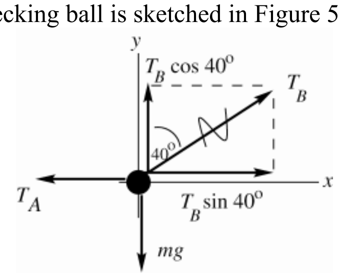


Figure 5.8

EXECUTE:
(a) $\sum F_y = ma_y$

$T_B\cos 40° - mg = 0$

$T_B = \dfrac{mg}{\cos 40°} = \dfrac{(4090\text{ kg})(9.80\text{ m/s}^2)}{\cos 40°} = 5.23\times 10^4\text{ N}$

(b) $\sum F_x = ma_x$

$T_B\sin 40° - T_A = 0$

$T_A = T_B\sin 40° = 3.36\times 10^4\text{ N}$

EVALUATE: If the angle $40°$ is replaces by $0°$ (cable B is vertical), then $T_B = mg$ and $T_A = 0$.

5.9. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the object and to the knot where the cords are joined.

SET UP: Let $+y$ be upward and $+x$ be to the right.

EXECUTE: **(a)** $T_C = w$, $T_A \sin 30° + T_B \sin 45° = T_C = w$, and $T_A \cos 30° - T_B \cos 45° = 0$. Since $\sin 45° = \cos 45°$, adding the last two equations gives $T_A(\cos 30° + \sin 30°) = w$, and so $T_A = \dfrac{w}{1.366} = 0.732w$. Then, $T_B = T_A \dfrac{\cos 30°}{\cos 45°} = 0.897w$.

(b) Similar to part (a), $T_C = w$, $-T_A \cos 60° + T_B \sin 45° = w$, and $T_A \sin 60° - T_B \cos 45° = 0$.

Adding these two equations, $T_A = \dfrac{w}{(\sin 60° - \cos 60°)} = 2.73w$, and $T_B = T_A \dfrac{\sin 60°}{\cos 45°} = 3.35w$.

EVALUATE: In part (a), $T_A + T_B > w$ since only the vertical components of T_A and T_B hold the object against gravity. In part (b), since T_A has a downward component T_B is greater than w.

5.10. **IDENTIFY:** Apply Newton's first law to the car.

SET UP: Use x and y coordinates that are parallel and perpendicular to the ramp.

EXECUTE: **(a)** The free-body diagram for the car is given in Figure 5.10. The vertical weight w and the tension T in the cable have each been replaced by their x and y components.

(b) $\sum F_x = 0$ gives $T \cos 31.0° - w \sin 25.0° = 0$ and $T = w\dfrac{\sin 25.0°}{\cos 31.0°} = (1130\text{ kg})(9.80\text{ m/s}^2)\dfrac{\sin 25.0°}{\cos 31.0°} = 5460\text{ N}$.

(c) $\sum F_y = 0$ gives $n + T \sin 31.0° - w \cos 25.0° = 0$ and

$n = w \cos 25.0° - T \sin 31.0° = (1130\text{ kg})(9.80\text{ m/s}^2)\cos 25.0° - (5460\text{ N})\sin 31.0° = 7220\text{ N}$

EVALUATE: We could also use coordinates that are horizontal and vertical and would obtain the same values of n and T.

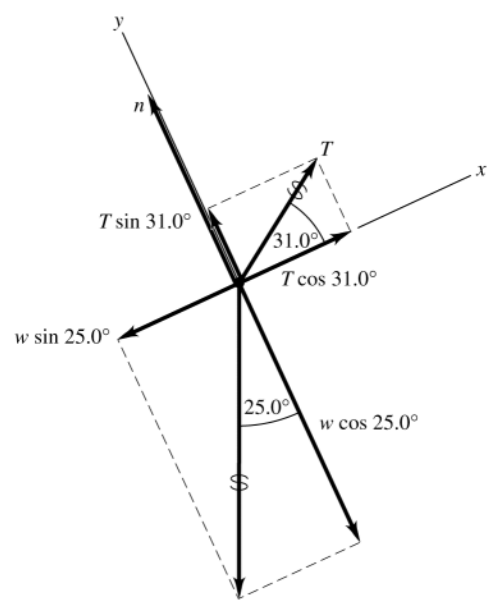


Figure 5.10

5.11. **IDENTIFY:** Since the velocity is constant, apply Newton's first law to the piano. The push applied by the man must oppose the component of gravity down the incline.

SET UP: The free-body diagrams for the two cases are shown in Figures 5.11a and b. $\vec{F}$ is the force applied by the man. Use the coordinates shown in the figure.

EXECUTE: **(a)** $\sum F_x = 0$ gives $F - w \sin 11.0° = 0$ and $F = (180\text{ kg})(9.80\text{ m/s}^2)\sin 11.0° = 337\text{ N}$.

(b) $\sum F_y = 0$ gives $n \cos 11.0° - w = 0$ and $n = \dfrac{w}{\cos 11.0°}$. $\sum F_x = 0$ gives $F - n \sin 11.0° = 0$ and

$F = \left(\dfrac{w}{\cos 11.0°}\right)\sin 11.0° = w \tan 11.0° = 343\text{ N}$.

EVALUATE: A slightly greater force is required when the man pushes parallel to the floor. If the slope angle of the incline were larger, $\sin\alpha$ and $\tan\alpha$ would differ more and there would be more difference in the force needed in each case.

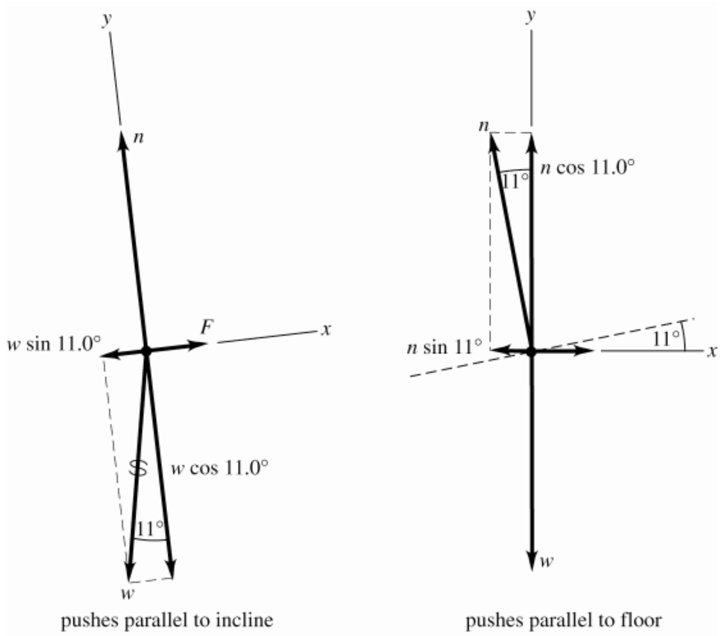


Figure 5.11a, b

5.12. **IDENTIFY:** Apply Newton's 1st law to the hanging weight and to each knot. The tension force at each end of a string is the same.

(a) Let the tensions in the three strings be T, T', and T'', as shown in Figure 5.12a.

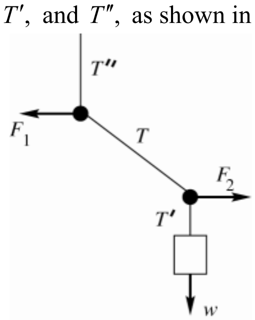


Figure 5.12a

SET UP: The free-body diagram for the block is given in Figure 5.12b.

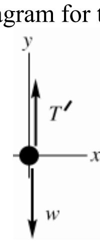


Figure 5.12b

EXECUTE:

$\sum F_y = 0$

$T' - w = 0$

$T' = w = 60.0\text{ N}$

SET UP: The free-body diagram for the lower knot is given in Figure 5.12c.

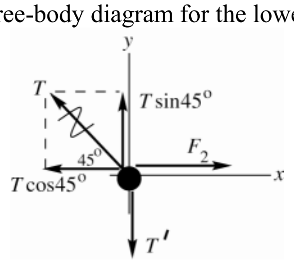


Figure 5.12c

EXECUTE:

$\sum F_y = 0$

$T\sin 45° - T' = 0$

$$T = \frac{T'}{\sin 45°} = \frac{60.0\text{ N}}{\sin 45°} = 84.9\text{ N}$$

(b) Apply $\sum F_x = 0$ to the force diagram for the lower knot:

$\sum F_x = 0$

$F_2 = T\cos 45° = (84.9\text{ N})\cos 45° = 60.0\text{ N}$

SET UP: The free-body diagram for the upper knot is given in Figure 5.12d.

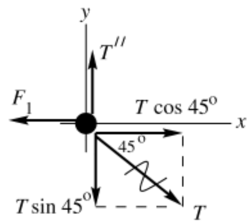


Figure 5.12d

EXECUTE:

$\sum F_x = 0$

$T\cos 45° - F_1 = 0$

$F_1 = (84.9\text{ N})\cos 45°$

$F_1 = 60.0\text{ N}$

Note that $F_1 = F_2$.

EVALUATE: Applying $\sum F_y = 0$ to the upper knot gives $T'' = T\sin 45° = 60.0\text{ N} = w$. If we treat the whole system as a single object, the force diagram is given in Figure 5.12e.

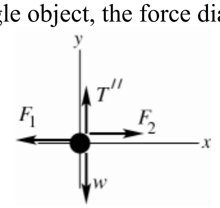


Figure 5.12e

$\sum F_x = 0$ gives $F_2 = F_1$, which checks

$\sum F_y = 0$ gives $T'' = w$, which checks

5.13. **IDENTIFY:** Apply Newton's first law to the ball. The force of the wall on the ball and the force of the ball on the wall are related by Newton's third law.

SET UP: The forces on the ball are its weight, the tension in the wire, and the normal force applied by the wall.

To calculate the angle ϕ that the wire makes with the wall, use Figure 5.13a. $\sin\phi = \dfrac{16.0\text{ cm}}{46.0\text{ cm}}$ and $\phi = 20.35°$

EXECUTE: **(a)** The free-body diagram is shown in Figure 5.13b. Use the x and y coordinates shown in the figure.

$\sum F_y = 0$ gives $T\cos\phi - w = 0$ and $T = \dfrac{w}{\cos\phi} = \dfrac{(45.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 20.35°} = 470\text{ N}$

(b) $\sum F_x = 0$ gives $T\sin\phi - n = 0$. $n = (470\text{ N})\sin 20.35° = 163\text{ N}$. By Newton's third law, the force the ball exerts on the wall is 163 N, directed to the right.

EVALUATE: $n = \left(\dfrac{w}{\cos\phi}\right)\sin\phi = w\tan\phi$. As the angle ϕ decreases (by increasing the length of the wire), T decreases and n decreases.

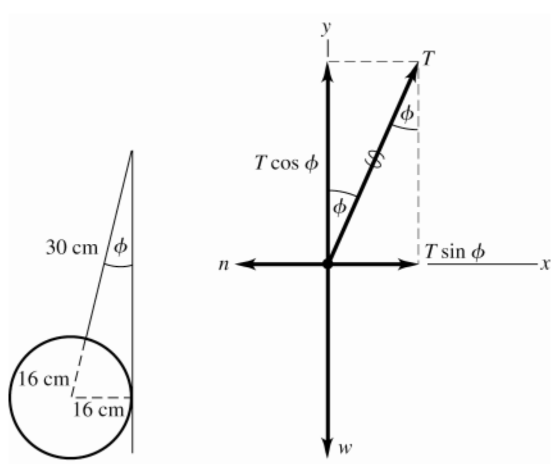


Figure 5.13a, b

5.14. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. $a = 0$.

SET UP: Take $+y$ perpendicular to the incline and $+x$ parallel to the incline.

EXECUTE: The free-body diagrams for each block, A and B, are given in Figure 5.14.

(a) For B, $\sum F_x = ma_x$ gives $T_1 - w\sin\alpha = 0$ and $T_1 = w\sin\alpha$.

(b) For block A, $\sum F_x = ma_x$ gives $T_1 - T_2 - w\sin\alpha = 0$ and $T_2 = 2w\sin\alpha$.

(c) $\sum F_y = ma_y$ for each block gives $n_A = n_B = w\cos\alpha$.

(d) For $\alpha \to 0$, $T_1 = T_2 \to 0$ and $n_A = n_B \to w$. For $\alpha \to 90°$, $T_1 = w$, $T_2 = 2w$ and $n_A = n_B = 0$.

EVALUATE: The two tensions are different but the two normal forces are the same.

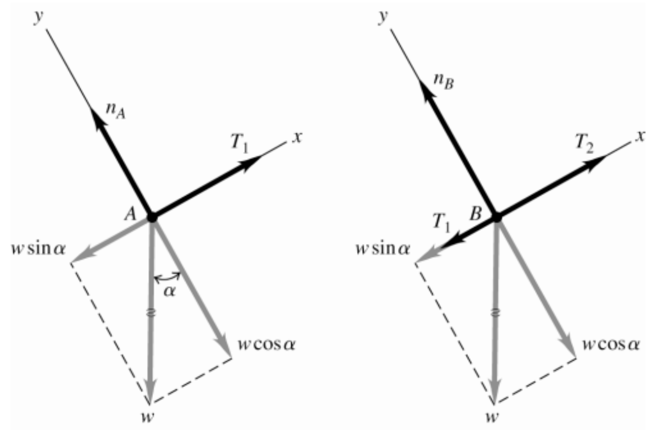


Figure 5.14a, b

5.15. **IDENTIFY:** Apply Newton's first law to the ball. Treat the ball as a particle.

SET UP: The forces on the ball are gravity, the tension in the wire and the normal force exerted by the surface. The normal force is perpendicular to the surface of the ramp. Use x and y axes that are horizontal and vertical.

EXECUTE: **(a)** The free-body diagram for the ball is given in Figure 5.15. The normal force has been replaced by its x and y components.

(b) $\sum F_y = 0$ gives $n\cos 35.0° - w = 0$ and $n = \dfrac{mg}{\cos 35.0°} = 1.22mg$.

(c) $\sum F_x = 0$ gives $T - n\sin 35.0° = 0$ and $T = (1.22mg)\sin 35.0° = 0.700mg$.

EVALUATE: Note that the normal force is greater than the weight, and increases without limit as the angle of the ramp increases towards $90°$. The tension in the wire is $w\tan\phi$, where ϕ is the angle of the ramp and T also increases without limit as $\phi \to 90°$.

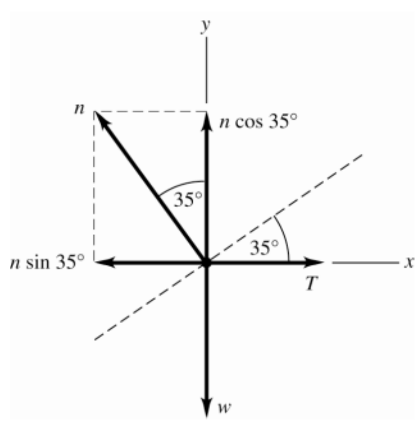


Figure 5.15

5.16. **IDENTIFY:** Apply Newton's second law to the rocket plus its contents and to the power supply. Both the rocket and the power supply have the same acceleration.

SET UP: The free-body diagrams for the rocket and for the power supply are given in Figures 5.16a and b. Since the highest altitude of the rocket is 120 m, it is near to the surface of the earth and there is a downward gravity force on each object. Let $+y$ be upward, since that is the direction of the acceleration. The power supply has mass $m_{ps} = (15.5\text{ N})/(9.80\text{ m/s}^2) = 1.58\text{ kg}$

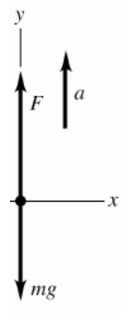
EXECUTE: **(a)** $\sum F_y = ma_y$ applied to the rocket gives $F - m_r g = m_r a$.

$$a = \frac{F - m_r g}{m_r} = \frac{1720\text{ N} - (125\text{ kg})(9.80\text{ m/s}^2)}{125\text{ kg}} = 3.96\text{ m/s}^2.$$

(b) $\sum F_y = ma_y$ applied to the power supply gives $n - m_{ps} g = m_{ps} a$.

$$n = m_{ps}(g + a) = (1.58\text{ kg})(9.80\text{ m/s}^2 + 3.96\text{ m/s}^2) = 21.7\text{ N}.$$

EVALUATE: The acceleration is constant while the thrust is constant and the normal force is constant while the acceleration is constant. The altitude of 120 m is not used in the calculation.



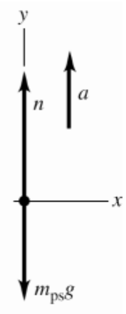


Figure 5.16a, b

5.17. **IDENTIFY:** Use the kinematic information to find the acceleration of the capsule and the stopping time. Use Newton's second law to find the force *F* that the ground exerted on the capsule during the crash.
SET UP: Let $+y$ be upward. $311\text{ km/h} = 86.4\text{ m/s}$. The free-body diagram for the capsule is given in Figure 15.17.

EXECUTE: $y - y_0 = -0.810\text{ m}$, $v_{0y} = -86.4\text{ m/s}$, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{0 - (-86.4\text{ m/s})^2}{2(-0.810)\text{ m}} = 4610\text{ m/s}^2 = 470g.$$

(b) $\sum F_y = ma_y$ applied to the capsule gives $F - mg = ma$ and

$$F = m(g + a) = (210\text{ kg})(9.80\text{ m/s}^2 + 4610\text{ m/s}^2) = 9.70 \times 10^5\text{ N} = 471w.$$

(c) $y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t$ gives $t = \frac{2(y - y_0)}{v_{0y} + v_y} = \frac{2(-0.810\text{ m})}{-86.4\text{ m/s}^2 + 0} = 0.0187\text{ s}$

EVALUATE: The upward force exerted by the ground is much larger than the weight of the capsule and stops the capsule in a short amount of time. After the capsule has come to rest, the ground still exerts a force *mg* on the capsule, but the large 9.00×10^5 N force is exerted only for 0.0187 s.

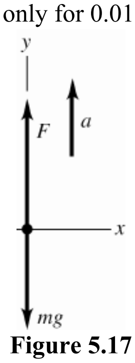


Figure 5.17

5.18. **IDENTIFY:** Apply Newton's second law to the three sleds taken together as a composite object and to each individual sled. All three sleds have the same horizontal acceleration *a*.
SET UP: The free-body diagram for the three sleds taken as a composite object is given in Figure 5.18a and for each individual sled in Figure 5.18b-d. Let $+x$ be to the right, in the direction of the acceleration. $m_{tot} = 60.0\text{ kg}$.

EXECUTE: **(a)** $\sum F_x = ma_x$ for the three sleds as a composite object gives $P = m_{tot} a$ and

$$a = \frac{P}{m_{tot}} = \frac{125\text{ N}}{60.0\text{ kg}} = 2.08\text{ m/s}^2.$$

(b) $\sum F_x = ma_x$ applied to the 10.0 kg sled gives $P - T_A = m_{10}a$ and

$T_A = P - m_{10}a = 125\text{ N} - (10.0\text{ kg})(2.08\text{ m/s}^2) = 104\text{ N}$. $\sum F_x = ma_x$ applied to the 30.0 kg sled gives

$T_B = m_{30}a = (30.0\text{ kg})(2.08\text{ m/s}^2) = 62.4\text{ N}$.

EVALUATE: If we apply $\sum F_x = ma_x$ to the 20.0 kg sled and calculate a from T_A and T_B found in part (b), we get $T_A - T_B = m_{20}a$. $a = \dfrac{T_A - T_B}{m_{20}} = \dfrac{104\text{ N} - 62.4\text{ N}}{20.0\text{ kg}} = 2.08\text{ m/s}^2$, which agrees with the value we calculated in part (a).

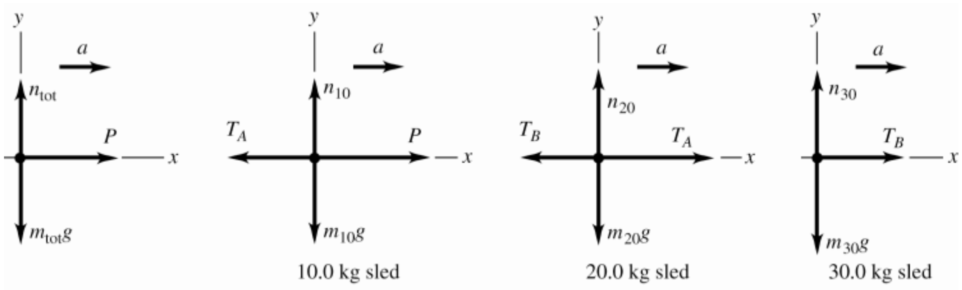


Figure 5.18a–d

5.19. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the load of bricks and to the counterweight. The tension is the same at each end of the rope. The rope pulls up with the same force (T) on the bricks and on the counterweight. The counterweight accelerates downward and the bricks accelerate upward; these accelerations have the same magnitude.

(a) SET UP: The free-body diagrams for the bricks and counterweight are given in Figure 5.19.

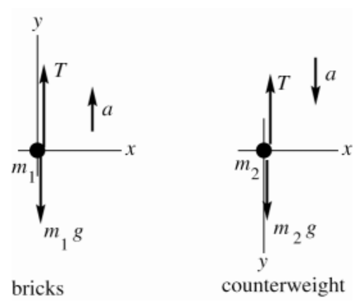


Figure 5.19

(b) EXECUTE: Apply $\sum F_y = ma_y$ to each object. The acceleration magnitude is the same for the two objects. For the bricks take $+y$ to be upward since $\vec{a}$ for the bricks is upward. For the counterweight take $+y$ to be downward since $\vec{a}$ is downward.

<u>bricks</u>: $\sum F_y = ma_y$

$T - m_1 g = m_1 a$

<u>counterweight</u>: $\sum F_y = ma_y$

$m_2 g - T = m_2 a$

Add these two equations to eliminate T:

$$(m_2 - m_1)g = (m_1 + m_2)a$$

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g = \left(\frac{28.0\text{ kg} - 15.0\text{ kg}}{15.0\text{ kg} + 28.0\text{ kg}}\right)(9.80\text{ m/s}^2) = 2.96\text{ m/s}^2$$

(c) $T - m_1 g = m_1 a$ gives $T = m_1(a + g) = (15.0\text{ kg})(2.96\text{ m/s}^2 + 9.80\text{ m/s}^2) = 191\text{ N}$

As a check, calculate T using the other equation.

$m_2 g - T = m_2 a$ gives $T = m_2(g - a) = 28.0\text{ kg}(9.80\text{ m/s}^2 - 2.96\text{ m/s}^2) = 191\text{ N}$, which checks.

EVALUATE: The tension is 1.30 times the weight of the bricks; this causes the bricks to accelerate upward. The tension is 0.696 times the weight of the counterweight; this causes the counterweight to accelerate downward. If $m_1 = m_2$, $a = 0$ and $T = m_1 g = m_2 g$. In this special case the objects don't move. If $m_1 = 0$, $a = g$ and $T = 0$; in this special case the counterweight is in free-fall. Our general result is correct in these two special cases.

5.20. **IDENTIFY:** In part (a) use the kinematic information and the constant acceleration equations to calculate the acceleration of the ice. Then apply $\sum \vec{F} = m\vec{a}$. In part (b) use $\sum \vec{F} = m\vec{a}$ to find the acceleration and use this in the constant acceleration equations to find the final speed.

SET UP: Figures 5.20a and b give the free-body diagrams for the ice both with and without friction. Let $+x$ be directed down the ramp, so $+y$ is perpendicular to the ramp surface. Let ϕ be the angle between the ramp and the horizontal. The gravity force has been replaced by its x and y components.

EXECUTE: **(a)** $x - x_0 = 1.50\text{ m}$, $v_{0x} = 0$, $v_x = 2.50\text{ m/s}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$a_x = \dfrac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \dfrac{(2.50\text{ m/s})^2 - 0}{2(1.50\text{ m})} = 2.08\text{ m/s}^2$. $\sum F_x = ma_x$ gives $mg\sin\phi = ma$ and $\sin\phi = \dfrac{a}{g} = \dfrac{2.08\text{ m/s}^2}{9.80\text{ m/s}^2}$.

$\phi = 12.3°$.

(b) $\sum F_x = ma_x$ gives $mg\sin\phi - f = ma$ and

$$a = \frac{mg\sin\phi - f}{m} = \frac{(8.00\text{ kg})(9.80\text{ m/s}^2)\sin 12.3° - 10.0\text{ N}}{8.00\text{ kg}} = 0.838\text{ m/s}^2.$$

Then $x - x_0 = 1.50\text{ m}$, $v_{0x} = 0$, $a_x = 0.838\text{ m/s}^2$ and $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$v_x = \sqrt{2a_x(x - x_0)} = \sqrt{2(0.838\text{ m/s}^2)(1.50\text{ m})} = 1.59\text{ m/s}$$

EVALUATE: With friction present the speed at the bottom of the ramp is less.

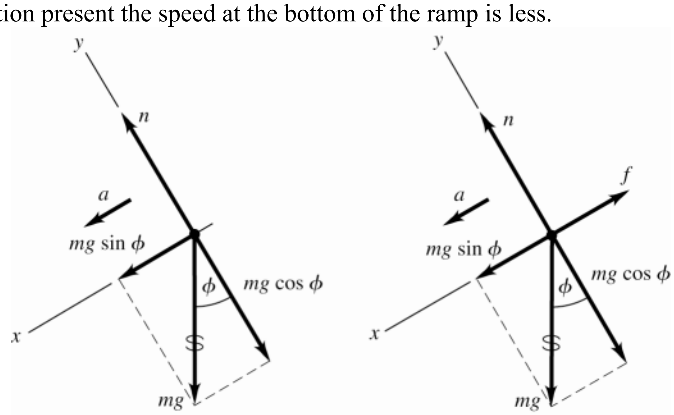


Figure 5.20a, b

5.21. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. Each block has the same magnitude of acceleration a.

SET UP: Assume the pulley is to the right of the 4.00 kg block. There is no friction force on the 4.00 kg block, the only force on it is the tension in the rope. The 4.00 kg block therefore accelerates to the right and the suspended block accelerates downward. Let $+x$ be to the right for the 4.00 kg block, so for it $a_x = a$, and let $+y$ be downward for the suspended block, so for it $a_y = a$.

EXECUTE: **(a)** The free-body diagrams for each block are given in Figures 5.21a and b.

(b) $\sum F_x = ma_x$ applied to the 4.00 kg block gives $T = (4.00\text{ kg})a$ and $a = \dfrac{T}{4.00\text{ kg}} = \dfrac{10.0\text{ N}}{4.00\text{ kg}} = 2.50\text{ m/s}^2$.

(c) $\sum F_y = ma_y$ applied to the suspended block gives $mg - T = ma$ and

$$m = \frac{T}{g - a} = \frac{10.0\text{ N}}{9.80\text{ m/s}^2 - 2.50\text{ m/s}^2} = 1.37\text{ kg}.$$

(d) The weight of the hanging block is $mg = (1.37\text{ kg})(9.80\text{ m/s}^2) = 13.4\text{ N}$. This is greater than the tension in the rope; $T = 0.75mg$.

EVALUATE: Since the hanging block accelerates downward, the net force on this block must be downward and the weight of the hanging block must be greater than the tension in the rope. Note that the blocks accelerate no matter how small m is. It is not necessary to have $m > 4.00\text{ kg}$, and in fact in this problem m is less than 4.00 kg.

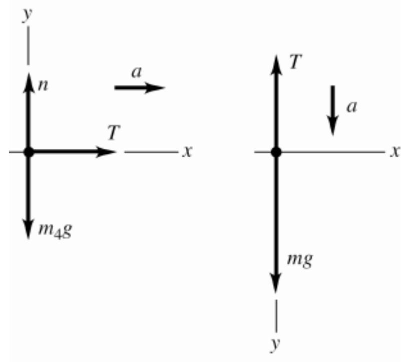


Figure 5.21a, b

5.22. **IDENTIFY:** (a) Consider both gliders together as a single object, apply $\sum \vec{F} = m\vec{a}$, and solve for a. Use a in a constant acceleration equation to find the required runway length.

(b) Apply $\sum \vec{F} = m\vec{a}$ to the second glider and solve for the tension T_g in the towrope that connects the two gliders.

SET UP: In part (a), set the tension T_t in the towrope between the plane and the first glider equal to its maximum value, $T_t = 12{,}000\text{ N}$.

EXECUTE: **(a)** The free-body diagram for both gliders as a single object of mass $2m = 1400\text{ kg}$ is given in Figure 5.22a. $\sum F_x = ma_x$ gives $T_t - 2f = (2m)a$ and $a = \dfrac{T_t - 2f}{2m} = \dfrac{12{,}000\text{ N} - 5000\text{ N}}{1400\text{ kg}} = 5.00\text{ m/s}^2$. Then $a_x = 5.00\text{ m/s}^2$, $v_{0x} = 0$ and $v_x = 40\text{ m/s}$ in $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $(x - x_0) = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = 160\text{ m}$.

(b) The free-body diagram for the second glider is given in Figure 5.22b.

$\sum F_x = ma_x$ gives $T_g - f = ma$ and $T = f + ma = 2500\text{ N} + (700\text{ kg})(5.00\text{ m/s}^2) = 6000\text{ N}$.

EVALUATE: We can verify that $\sum F_x = ma_x$ is also satisfied for the first glider.

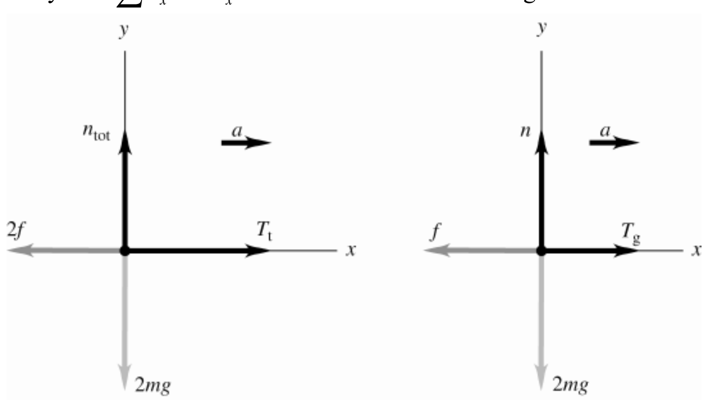


Figure 5.22a, b

5.23. **IDENTIFY:** The maximum tension in the chain is at the top of the chain. Apply $\sum \vec{F} = m\vec{a}$ to the composite object of chain and boulder. Use the constant acceleration kinematic equations to relate the acceleration to the time.

SET UP: Let $+y$ be upward. The free-body diagram for the composite object is given in Figure 5.23. $T = 2.50w_{\text{chain}}$. $m_{\text{tot}} = m_{\text{chain}} + m_{\text{boulder}} = 1325\text{ kg}$.

EXECUTE: **(a)** $\sum F_y = ma_y$ gives $T - m_{\text{tot}}g = m_{\text{tot}}a$. $a = \dfrac{T - m_{\text{tot}}g}{m_{\text{tot}}} = \dfrac{2.50m_{\text{chain}}g - m_{\text{tot}}g}{m_{\text{tot}}} = \left(\dfrac{2.50m_{\text{chain}}}{m_{\text{tot}}} - 1\right)g$

$$a = \left(\frac{2.50[575\text{ kg}]}{1325\text{ kg}} - 1\right)(9.80\text{ m/s}^2) = 0.832\text{ m/s}^2.$$

(b) Assume the acceleration has its maximum value: $a_y = 0.832\ \text{m/s}^2$, $y - y_0 = 125$ m and $v_{0y} = 0$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(125\ \text{m})}{0.832\ \text{m/s}^2}} = 17.3\ \text{s}$

EVALUATE: The tension in the chain is $T = 1.41\times10^4$ N and the total weight is 1.30×10^4 N. The upward force exceeds the downward force and the acceleration is upward.

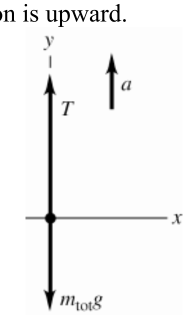


Figure 5.23

5.24. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the composite object of elevator plus student ($m_{\text{tot}} = 850$ kg) and also to the student ($w = 550$ N). The elevator and the student have the same acceleration.

SET UP: Let $+y$ be upward. The free-body diagrams for the composite object and for the student are given in Figure 5.24a and b. T is the tension in the cable and n is the scale reading, the normal force the scale exerts on the student. The mass of the student is $m = w/g = 56.1$ kg.

EXECUTE: **(a)** $\sum F_y = ma_y$ applied to the student gives $n - mg = ma_y$.

$a_y = \dfrac{n - mg}{m} = \dfrac{450\ \text{N} - 550\ \text{N}}{56.1\ \text{kg}} = -1.78\ \text{m/s}^2$. The elevator has a downward acceleration of $1.78\ \text{m/s}^2$.

(b) $a_y = \dfrac{670\ \text{N} - 550\ \text{N}}{56.1\ \text{kg}} = 2.14\ \text{m/s}^2$.

(c) $n = 0$ means $a_y = -g$. The student should worry; the elevator is in free-fall.

(d) $\sum F_y = ma_y$ applied to the composite object gives $T - m_{\text{tot}}g = m_{\text{tot}}a$. $T = m_{\text{tot}}(a_y + g)$. In part (a), $T = (850\ \text{kg})(-1.78\ \text{m/s}^2 + 9.80\ \text{m/s}^2) = 6820$ N. In part (c), $a_y = -g$ and $T = 0$.

EVALUATE: In part (b), $T = (850\ \text{kg})(2.14\ \text{m/s}^2 + 9.80\ \text{m/s}^2) = 10{,}150$ N. The weight of the composite object is 8330 N. When the acceleration is upward the tension is greater than the weight and when the acceleration is downward the tension is less than the weight.

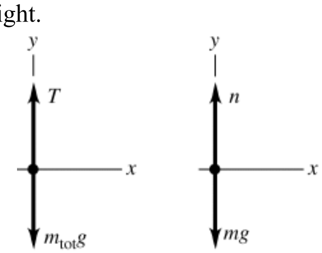


Figure 5.24a, b

5.25. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the puck. Use the information about the motion to calculate the acceleration. The table must slope downward to the right.

SET UP: Let α be the angle between the table surface and the horizontal. Let the $+x$-axis be to the right and parallel to the surface of the table.

EXECUTE: $\sum F_x = ma_x$ gives $mg\sin\alpha = ma_x$. The time of travel for the puck is L/v_0, where $L = 1.75$ m and $v_0 = 3.80$ m/s. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $a_x = \dfrac{2x}{t^2} = \dfrac{2xv_0^2}{L^2}$, where $x = 0.0250$ m. $\sin\alpha = \dfrac{a_x}{g} = \dfrac{2xv_0^2}{gL^2}$.

$$\alpha = \arcsin\left(\frac{2(2.50\times10^{-2}\,\text{m})(3.80\ \text{m/s})^2}{\left(9.80\ \text{m/s}^2\right)(1.75\ \text{m})^2}\right) = 1.38^\circ.$$

EVALUATE: The table is level in the direction along its length, since the velocity in that direction is constant. The angle of slope to the right is small, so the acceleration and deflection in that direction are small.

5.26. **IDENTIFY:** Acceleration and velocity are related by $a_y = \dfrac{dv_y}{dt}$. Apply $\sum \vec{F} = m\vec{a}$ to the rocket.

SET UP: Let $+y$ be upward. The free-body diagram for the rocket is sketched in Figure 5.26. $\vec{F}$ is the thrust force.

EXECUTE: **(a)** $v_y = At + Bt^2$. $a_y = A + 2Bt$. At $t = 0$, $a_y = 1.50\ \text{m/s}^2$ so $A = 1.50\ \text{m/s}^2$. Then $v_y = 2.00\ \text{m/s}$ at $t = 1.00\ \text{s}$ gives $2.00\ \text{m/s} = (1.50\ \text{m/s}^2)(1.00\ \text{s}) + B(1.00\ \text{s})^2$ and $B = 0.50\ \text{m/s}^3$.

(b) At $t = 4.00\ \text{s}$, $a_y = 1.50\ \text{m/s}^2 + 2(0.50\ \text{m/s}^3)(4.00\ \text{s}) = 5.50\ \text{m/s}^2$.

(c) $\sum F_y = ma_y$ applied to the rocket gives $T - mg = ma$ and

$T = m(a + g) = (2540\ \text{kg})(9.80\ \text{m/s}^2 + 5.50\ \text{m/s}^2) = 3.89 \times 10^4\ \text{N}$. $T = 1.56w$.

(d) When $a = 1.50\ \text{m/s}^2$, $T = (2540\ \text{kg})(9.80\ \text{m/s}^2 + 1.50\ \text{m/s}^2) = 2.87 \times 10^4\ \text{N}$

EVALUATE: During the time interval when $v(t) = At + Bt^2$ applies the magnitude of the acceleration is increasing, and the thrust is increasing.

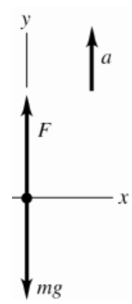


Figure 5.26

5.27. **IDENTIFY:** Consider the forces in each case. There is the force of gravity and the forces from objects that touch the object in question.
SET UP: A surface exerts a normal force perpendicular to the surface, and a friction force, parallel to the surface.
EXECUTE: The free-body diagrams are sketched in Figure 5.27a-c.
EVALUATE: Friction opposes relative motion between the two surfaces. When one surface is stationary the friction force on the other surface is directed opposite to its motion.

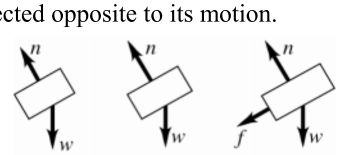


Figure 5.27a–c

5.28. **IDENTIFY:** $f_s \le \mu_s n$ and $f_k = \mu_k n$. The normal force n is determined by applying $\sum \vec{F} = m\vec{a}$ to the block. Normally, $\mu_k \le \mu_s$. f_s is only as large as it needs to be to prevent relative motion between the two surfaces.
SET UP: Since the table is horizontal, with only the block present $n = 135\ \text{N}$. With the brick on the block, $n = 270\ \text{N}$.
EXECUTE: **(a)** The friction is static for $P = 0$ to $P = 75.0\ \text{N}$. The friction is kinetic for $P > 75.0\ \text{N}$.
(b) The maximum value of f_s is $\mu_s n$. From the graph the maximum f_s is $f_s = 75.0\ \text{N}$, so

$\mu_s = \dfrac{\max f_s}{n} = \dfrac{75.0\ \text{N}}{135\ \text{N}} = 0.556$. $f_k = \mu_k n$. From the graph, $f_k = 50.0\ \text{N}$ and $\mu_k = \dfrac{f_k}{n} = \dfrac{50.0\ \text{N}}{135\ \text{N}} = 0.370$.

(c) When the block is moving the friction is kinetic and has the constant value $f_k = \mu_k n$, independent of P. This is why the graph is horizontal for $P > 75.0\ \text{N}$. When the block is at rest, $f_s = P$ since this prevents relative motion. This is why the graph for $P < 75.0\ \text{N}$ has slope $+1$.
(d) max f_s and f_k would double. The values of f on the vertical axis would double but the shape of the graph would be unchanged.
EVALUATE: The coefficients of friction are independent of the normal force.

5.29. **(a) IDENTIFY:** Constant speed implies $a = 0$. Apply Newton's 1st law to the box. The friction force is directed opposite to the motion of the box.

SET UP: Consider the free-body diagram for the box, given in Figure 5.29a. Let $\vec{F}$ be the horizontal force applied by the worker. The friction is kinetic friction since the box is sliding along the surface.

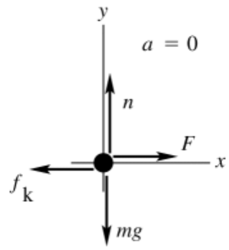


Figure 5.29a

EXECUTE:

$\sum F_y = ma_y$

$n - mg = 0$

$n = mg$

So $f_{\text{k}} = \mu_{\text{k}} n = \mu_{\text{k}} mg$

$\sum F_x = ma_x$

$F - f_{\text{k}} = 0$

$F = f_{\text{k}} = \mu_{\text{k}} mg = (0.20)(11.2 \text{ kg})(9.80 \text{ m/s}^2) = 22 \text{ N}$

(b) IDENTIFY: Now the only horizontal force on the box is the kinetic friction force. Apply Newton's 2nd law to the box to calculate its acceleration. Once we have the acceleration, we can find the distance using a constant acceleration equation. The friction force is $f_{\text{k}} = \mu_{\text{k}} mg$, just as in part (a).

SET UP: The free-body diagram is sketched in Figure 5.29b.

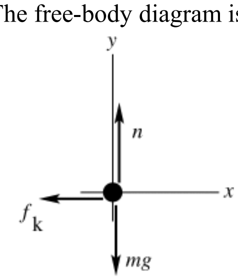


Figure 5.29b

EXECUTE:

$\sum F_x = ma_x$

$-f_{\text{k}} = ma_x$

$-\mu_{\text{k}} mg = ma_x$

$a_x = -\mu_{\text{k}} g = -(0.20)(9.80 \text{ m/s}^2) = -1.96 \text{ m/s}^2$

Use the constant acceleration equations to find the distance the box travels:

$v_x = 0,\ v_{0x} = 3.50 \text{ m/s},\ a_x = -1.96 \text{ m/s}^2,\ x - x_0 = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

$$x - x_0 = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - (3.50 \text{ m/s})^2}{2(-1.96 \text{ m/s}^2)} = 3.1 \text{ m}$$

EVALUATE: The normal force is the component of force exerted by a surface perpendicular to the surface. Its magnitude is determined by $\sum \vec{F} = m\vec{a}$. In this case *n* and *mg* are the only vertical forces and $a_y = 0$, so $n = mg$. Also note that f_{k} and *n* are proportional in magnitude but perpendicular in direction.

5.30. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the box.

SET UP: Since the only vertical forces are *n* and *w*, the normal force on the box equals its weight. Static friction is as large as it needs to be to prevent relative motion between the box and the surface, up to its maximum possible value of $f_{\text{s}}^{\text{max}} = \mu_{\text{s}} n$. If the box is sliding then the friction force is $f_{\text{k}} = \mu_{\text{k}} n$.

EXECUTE: **(a)** If there is no applied force, no friction force is needed to keep the box at rest.

(b) $f_{\text{s}}^{\text{max}} = \mu_{\text{s}} n = (0.40)(40.0 \text{ N}) = 16.0 \text{ N}$. If a horizontal force of 6.0 N is applied to the box, then $f_{\text{s}} = 6.0 \text{ N}$ in the opposite direction.

(c) The monkey must apply a force equal to $f_{\text{s}}^{\text{max}}$, 16.0 N.

(d) Once the box has started moving, a force equal to $f_{\text{k}} = \mu_{\text{k}} n = 8.0 \text{ N}$ is required to keep it moving at constant velocity.

EVALUATE: $\mu_{\text{k}} < \mu_{\text{s}}$ and less force must be applied to the box to maintain its motion than to start it moving.

5.31. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the crate. $f_s \le \mu_s n$ and $f_k = \mu_k n$.

SET UP: Let $+y$ be upward and let $+x$ be in the direction of the push. Since the floor is horizontal and the push is horizontal, the normal force equals the weight of the crate: $n = mg = 441\text{ N}$. The force it takes to start the crate moving equals $\max f_s$ and the force required to keep it moving equals f_k

EXECUTE: $\max f_s = 313\text{ N}$, so $\mu_s = \dfrac{313\text{ N}}{441\text{ N}} = 0.710$. $f_k = 208\text{ N}$, so $\mu_k = \dfrac{208\text{ N}}{441\text{ N}} = 0.472$.

(b) The friction is kinetic. $\sum F_x = ma_x$ gives $F - f_k = ma$ and $F = f_k + ma = 208 + (45.0\text{ kg})(1.10\text{ m/s}^2) = 258\text{ N}$.

(c) (i) The normal force now is $mg = 72.9\text{ N}$. To cause it to move, $F = \max f_s = \mu_s n = (0.710)(72.9\text{ N}) = 51.8\text{ N}$.

(ii) $F = f_k + ma$ and $a = \dfrac{F - f_k}{m} = \dfrac{258\text{ N} - (0.472)(72.9\text{ N})}{45.0\text{ kg}} = 4.97\text{ m/s}^2$

EVALUATE: The kinetic friction force is independent of the speed of the object. On the moon, the mass of the crate is the same as on earth, but the weight and normal force are less.

5.32. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the box and calculate the normal and friction forces. The coefficient of kinetic friction is the ratio $\dfrac{f_k}{n}$.

SET UP: Let $+x$ be in the direction of motion. $a_x = -0.90\text{ m/s}^2$. The box has mass 8.67 kg.

EXECUTE: The normal force has magnitude $85\text{ N} + 25\text{ N} = 110\text{ N}$. The friction force, from $F_H - f_k = ma$ is

$f_k = F_H - ma = 20\text{ N} - (8.67\text{ kg})(-0.90\text{ m/s}^2) = 28\text{ N}$. $\mu_k = \dfrac{28\text{ N}}{110\text{ N}} = 0.25$.

EVALUATE: The normal force is greater than the weight of the box, because of the downward component of the push force.

5.33. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the composite object consisting of the two boxes and to the top box. The friction the ramp exerts on the lower box is kinetic friction. The upper box doesn't slip relative to the lower box, so the friction between the two boxes is static. Since the speed is constant the acceleration is zero.

SET UP: Let $+x$ be up the incline. The free-body diagrams for the composite object and for the upper box are given in Figures 5.33a and b. The slope angle ϕ of the ramp is given by $\tan\phi = \dfrac{2.50\text{ m}}{4.75\text{ m}}$, so $\phi = 27.76°$. Since the boxes move down the ramp, the kinetic friction force exerted on the lower box by the ramp is directed up the incline. To prevent slipping relative to the lower box the static friction force on the upper box is directed up the incline. $m_{tot} = 32.0\text{ kg} + 48.0\text{ kg} = 80.0\text{ kg}$.

EXECUTE: **(a)** $\sum F_y = ma_y$ applied to the composite object gives $n_{tot} = m_{tot} g\cos\phi$ and $f_k = \mu_k m_{tot} g\cos\phi$.

$\sum F_x = ma_x$ gives $f_k + T - m_{tot} g\sin\phi = 0$ and

$T = (\sin\phi - \mu_k\cos\phi)m_{tot}g = (\sin 27.76° - [0.444]\cos 27.76°)(80.0\text{ kg})(9.80\text{ m/s}^2) = 57.1\text{ N}$.

The person must apply a force of 57.1 N, directed up the ramp.

(b) $\sum F_x = ma_x$ applied to the upper box gives $f_s = mg\sin\phi = (32.0\text{ kg})(9.80\text{ m/s}^2)\sin 27.76° = 146\text{ N}$, directed up the ramp.

EVALUATE: For each object the net force is zero.

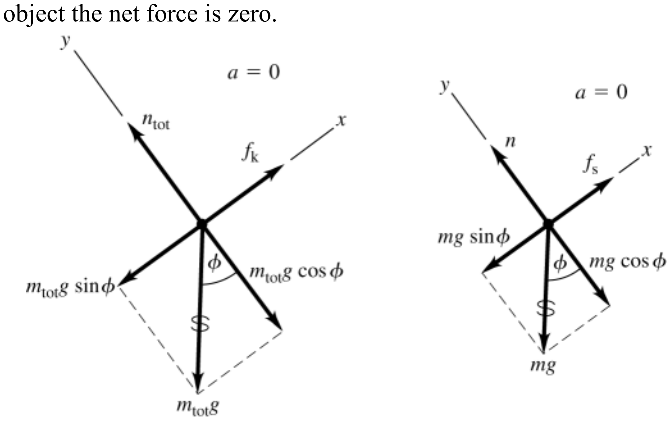


Figure 5.33a, b

5.34. **IDENTIFY:** Use $\sum \vec{F} = m\vec{a}$ to find the acceleration that can be given to the car by the kinetic friction force. Then use a constant acceleration equation.

SET UP: Take $+x$ in the direction the car is moving.

EXECUTE: **(a)** The free-body diagram for the car is shown in Figure 5.34. $\sum F_y = ma_y$ gives $n = mg$. $\sum F_x = ma_x$ gives $-\mu_k n = ma_x$. $-\mu_k mg = ma_x$ and $a_x = -\mu_k g$. Then $v_x = 0$ and $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $(x - x_0) = -\dfrac{v_{0x}^2}{2a_x} = +\dfrac{v_{0x}^2}{2\mu_k g} = \dfrac{(29.1\text{ m/s})^2}{2(0.80)(9.80\text{ m/s}^2)} = 54.0\text{ m}$.

(b) $v_{0x} = \sqrt{2\mu_k g(x - x_0)} = \sqrt{2(0.25)(9.80\text{ m/s}^2)(54.0\text{ m})} = 16.3\text{ m/s}$

EVALUATE: For constant stopping distance $\dfrac{v_{0x}^2}{\mu_k}$ is constant and v_{0x} is proportional to $\sqrt{\mu_k}$. The answer to part (b) can be calculated as $(29.1\text{ m/s})\sqrt{0.25/0.80} = 16.3\text{ m/s}$.

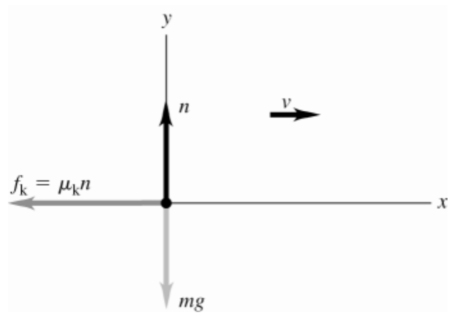


Figure 5.34

5.35. **IDENTIFY:** For a given initial speed, the distance traveled is inversely proportional to the coefficient of kinetic friction.

SET UP: From Table 5.1 the coefficient of kinetic friction is 0.04 for Teflon on steel and 0.44 for brass on steel.

EXECUTE: The ratio of the distances is $\dfrac{0.44}{0.04} = 11$.

EVALUATE: The smaller the coefficient of kinetic friction the smaller the retarding force of friction, and the greater the stopping distance.

5.36. **IDENTIFY:** Constant speed means zero acceleration for each block. If the block is moving the friction force the tabletop exerts on it is kinetic friction. Apply $\sum \vec{F} = m\vec{a}$ to each block.

SET UP: The free-body diagrams and choice of coordinates for each block are given by Figure 5.36. $m_A = 4.59\text{ kg}$ and $m_B = 2.55\text{ kg}$.

EXECUTE: **(a)** $\sum F_y = ma_y$ with $a_y = 0$ applied to block B gives $m_B g - T = 0$ and $T = 25.0\text{ N}$. $\sum F_x = ma_x$ with $a_x = 0$ applied to block A gives $T - f_k = 0$ and $f_k = 25.0\text{ N}$. $n_A = m_A g = 45.0\text{ N}$ and $\mu_k = \dfrac{f_k}{n_A} = \dfrac{25.0\text{ N}}{45.0\text{ N}} = 0.556$.

(b) Now let A be block A plus the cat, so $m_A = 9.18\text{ kg}$. $n_A = 90.0\text{ N}$ and $f_k = \mu_k n = (0.556)(90.0\text{ N}) = 50.0\text{ N}$. $\sum F_x = ma_x$ for A gives $T - f_k = m_A a_x$. $\sum F_y = ma_y$ for block B gives $m_B g - T = m_B a_y$. a_x for A equals a_y for B, so adding the two equations gives $m_B g - f_k = (m_A + m_B)a_y$ and $a_y = \dfrac{m_B g - f_k}{m_A + m_B} = \dfrac{25.0\text{ N} - 50.0\text{ N}}{9.18\text{ kg} + 2.55\text{ kg}} = -2.13\text{ m/s}^2$.

The acceleration is upward and block B slows down.

EVALUATE: The equation $m_B g - f_k = (m_A + m_B)a_y$ has a simple interpretation. If both blocks are considered together then there are two external forces: $m_B g$ that acts to move the system one way and f_k that acts oppositely. The net force of $m_B g - f_k$ must accelerate a total mass of $m_A + m_B$.

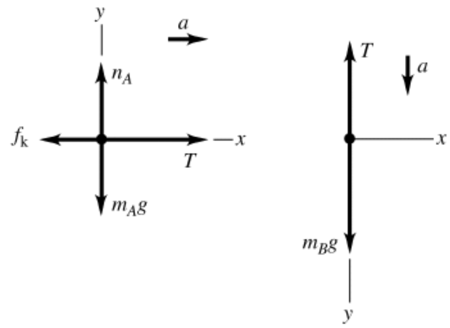


Figure 5.36

5.37. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each crate. The rope exerts force T to the right on crate A and force T to the left on crate B. The target variables are the forces T and F. Constant v implies $a = 0$.

SET UP: The free-body diagram for A is sketched in Figure 5.37a

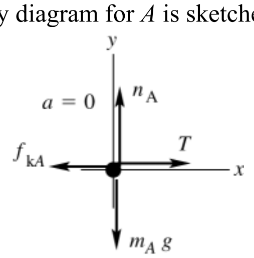


Figure 5.37a

EXECUTE:

$\sum F_y = ma_y$

$n_A - m_A g = 0$

$n_A = m_A g$

$f_{kA} = \mu_k n_A = \mu_k m_A g$

$\sum F_x = ma_x$

$T - f_{kA} = 0$

$T = \mu_k m_A g$

SET UP: The free-body diagram for B is sketched in Figure 5.37b.

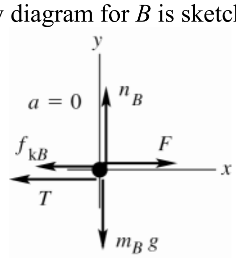


Figure 5.37b

EXECUTE:

$\sum F_y = ma_y$

$n_B - m_B g = 0$

$n_B = m_B g$

$f_{kB} = \mu_k n_B = \mu_k m_B g$

$\sum F_x = ma_x$

$F - T - f_{kB} = 0$

$F = T + \mu_k m_B g$

Use the first equation to replace T in the second:

$F = \mu_k m_A g + \mu_k m_B g.$

(a) $F = \mu_k (m_A + m_B)g$

(b) $T = \mu_k m_A g$

EVALUATE: We can also consider both crates together as a single object of mass $(m_A + m_B)$. $\sum F_x = ma_x$ for this combined object gives $F = f_k = \mu_k (m_A + m_B)g,$ in agreement with our answer in part (a).

5.38. **IDENTIFY:** $f = \mu_r n$. Apply $\sum \vec{F} = m\vec{a}$ to the tire.

SET UP: $n = mg$ and $f = ma$.

EXECUTE: $a_x = \dfrac{v^2 - v_0^2}{L}$, where L is the distance covered before the wheel's speed is reduced to half its original speed and $v = v_0/2$. $\mu_r = \dfrac{a}{g} = \dfrac{v_0^2 - v^2}{2Lg} = \dfrac{v_0^2 - \frac{1}{4}v_0^2}{2Lg} = \dfrac{3}{8}\dfrac{v_0^2}{Lg}$.

Low pressure, $L = 18.1$ m and $\dfrac{3}{8}\dfrac{(3.50\ \text{m/s})^2}{(18.1\ \text{m})(9.80\ \text{m/s}^2)} = 0.0259$.

High pressure, $L = 92.9$ m and $\dfrac{3}{8}\dfrac{(3.50\ \text{m/s})^2}{(3.50\ \text{m/s})^2} = 0.00505$.

EVALUATE: μ_r is inversely proportional to the distance L, so $\dfrac{\mu_{r1}}{\mu_{r2}} = \dfrac{L_2}{L_1}$.

5.39. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the box. Use the information about sliding to calculate the mass of the box.

SET UP: $f_k = \mu_k n$, $f_r = \mu_r n$ and $n = mg$.

EXECUTE: Without the dolly: $n = mg$ and $F - \mu_k n = 0$ ($a_x = 0$ since speed is constant).

$$m = \frac{F}{\mu_k g} = \frac{160\ \text{N}}{(0.47)\,(9.80\ \text{m/s}^2)} = 34.74\ \text{kg}$$

With the dolly: the total mass is $34.7\ \text{kg} + 5.3\ \text{kg} = 40.04\ \text{kg}$ and friction now is rolling friction, $f_r = \mu_r mg$.

$F - \mu_r mg = ma$. $a = \dfrac{F - \mu_r mg}{m} = 3.82\ \text{m/s}^2$.

EVALUATE: $f_k = \mu_k mg = 160$ N and $f_r = \mu_r mg = 4.36$ N, or, $\dfrac{f_r}{f_k} = \dfrac{\mu_r}{\mu_k}$. The rolling friction force is much less than the kinetic friction force.

5.40. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the truck. For constant speed, $a = 0$ and $F_{\text{horiz}} = f_r$.

SET UP: $f_r = \mu_r n = \mu_r mg$. Let $m_2 = 1.42 m_1$ and $\mu_{r2} = 0.81\mu_{r1}$.

EXECUTE: Since the speed is constant and we are neglecting air resistance, we can ignore the 2.4 m/s, and F_{net} in the horizontal direction must be zero. Therefore $f_r = \mu_r n = F_{\text{horiz}} = 200$ N before the weight and pressure changes are made. After the changes, $(0.81\mu_r)\,(1.42n) = F_{\text{horiz}}$, because the speed is still constant and $F_{\text{net}} = 0$. We can simply divide the two equations: $\dfrac{(0.81\mu_r)(1.42n)}{\mu_r n} = \dfrac{F_{\text{horiz}}}{200\ \text{N}}$ and $(0.81)\,(1.42)\,(200\ \text{N}) = F_{\text{horiz}} = 230$ N.

EVALUATE: The increase in weight increases the normal force and hence the friction force, whereas the decrease in μ_r reduces it. The percentage increase in the weight is larger, so the net effect is an increase in the friction force.

5.41. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. The target variables are the tension T in the cord and the acceleration a of the blocks. Then a can be used in a constant acceleration equation to find the speed of each block. The magnitude of the acceleration is the same for both blocks.

SET UP: The system is sketched in Figure 5.41a.

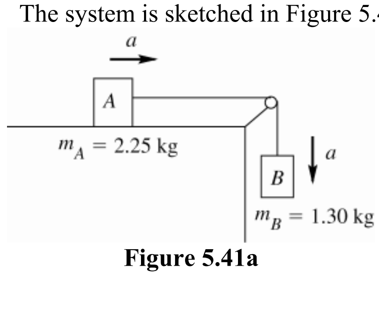


Figure 5.41a

For each block take a positive coordinate direction to be the direction of the block's acceleration.

block on the table: The free-body is sketched in Figure 5.41b.

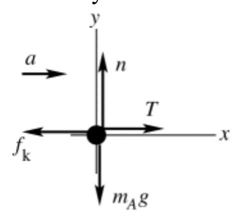


Figure 5.41b

EXECUTE:

$\sum F_y = ma_y$

$n - m_A g = 0$

$n = m_A g$

$f_k = \mu_k n = \mu_k m_A g$

$\sum F_x = ma_x$

$T - f_k = m_A a$

$T - \mu_k m_A g = m_A a$

SET UP: hanging block: The free-body is sketched in Figure 5.41c.

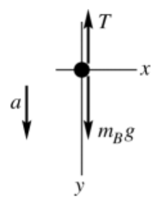


Figure 5.41c

EXECUTE:

$\sum F_y = ma_y$

$m_B g - T = m_B a$

$T = m_B g - m_B a$

(a) Use the second equation in the first

$$m_B g - m_B a - \mu_k m_A g = m_A a$$

$$(m_A + m_B)a = (m_B - \mu_k m_A)g$$

$$a = \frac{(m_B - \mu_k m_A)g}{m_A + m_B} = \frac{(1.30\text{ kg} - (0.45)(2.25\text{ kg}))(9.80\text{ m/s}^2)}{2.25\text{ kg} + 1.30\text{ kg}} = 0.7937\text{ m/s}^2$$

SET UP: Now use the constant acceleration equations to find the final speed. Note that the blocks have the same speeds. $x - x_0 = 0.0300$ m, $a_x = 0.7937\text{ m/s}^2$, $v_{0x} = 0$, $v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

EXECUTE: $v_x = \sqrt{2a_x(x - x_0)} = \sqrt{2(0.7937\text{ m/s}^2)(0.0300\text{ m})} = 0.218\text{ m/s} = 21.8\text{ cm/s}.$

(b) $T = m_B g - m_B a = m_B(g - a) = 1.30\text{ kg}(9.80\text{ m/s}^2 - 0.7937\text{ m/s}^2) = 11.7\text{ N}$

Or, to check, $T - \mu_k m_A g = m_A a$

$T = m_A(a + \mu_k g) = 2.25\text{ kg}(0.7937\text{ m/s}^2 + (0.45)(9.80\text{ m/s}^2)) = 11.7\text{ N}$, which checks.

EVALUATE: The force *T* exerted by the cord has the same value for each block. $T < m_B g$ since the hanging block accelerates downward. Also, $f_k = \mu_k m_A g = 9.92$ N. $T > f_k$ and the block on the table accelerates in the direction of *T*.

5.42. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the box. When the box is ready to slip the static friction force has its maximum possible value, $f_s = \mu_s n$.

SET UP: Use coordinates parallel and perpendicular to the ramp.

EXECUTE: **(a)** The normal force will be $w\cos\theta$ and the component of the gravitational force along the ramp is $w\sin\theta$. The box begins to slip when $w\sin\theta > \mu_s w\cos\theta$, or $\tan\theta > \mu_s = 0.35$, so slipping occurs at $\theta = \arctan(0.35) = 19.3°$.

(b) When moving, the friction force along the ramp is $\mu_k w\cos\theta$, the component of the gravitational force along the ramp is $w\sin\theta$, so the acceleration is

$$(w\sin\theta - w\mu_k\cos\theta)/m = g(\sin\theta - \mu_k\cos\theta) = 0.92\ \text{m/s}^2.$$

(c) Since $v_{0x} = 0$, $2ax = v^2$, so $v = (2ax)^{1/2}$, or $v = [(2)(0.92\text{ m/s}^2)(5\text{ m})]^{1/2} = 3\text{ m/s}$.

EVALUATE: When the box starts to move, friction changes from static to kinetic and the friction force becomes smaller.

5.43. **(a) IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the crate. Constant v implies $a = 0$. Crate moving says that the friction is kinetic friction. The target variable is the magnitude of the force applied by the woman.

SET UP: The free-body diagram for the crate is sketched in Figure 5.43.

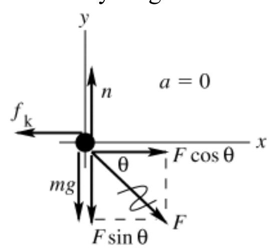


Figure 5.43

EXECUTE:

$\sum F_y = ma_y$

$n - mg - F\sin\theta = 0$

$n = mg + F\sin\theta$

$f_k = \mu_k n = \mu_k mg + \mu_k F\sin\theta$

$\sum F_x = ma_x$

$F\cos\theta - f_k = 0$

$F\cos\theta - \mu_k mg - \mu_k F\sin\theta = 0$

$F(\cos\theta - \mu_k \sin\theta) = \mu_k mg$

$F = \dfrac{\mu_k mg}{\cos\theta - \mu_k\sin\theta}$

(b) IDENTIFY and **SET UP:** "start the crate moving" means the same force diagram as in part (a), except that μ_k is replaced by μ_s. Thus $F = \dfrac{\mu_s mg}{\cos\theta - \mu_s\sin\theta}$.

EXECUTE: $F \to \infty$ if $\cos\theta - \mu_s\sin\theta = 0$. This gives $\mu_s = \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$.

EVALUATE: $\vec{F}$ has a downward component so $n > mg$. If $\theta = 0$ (woman pushes horizontally), $n = mg$ and $F = f_k = \mu_k mg$.

5.44. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the box.

SET UP: Let $+y$ be upward and $+x$ be horizontal, in the direction of the acceleration. Constant speed means $a = 0$.

EXECUTE: **(a)** There is no net force in the vertical direction, so $n + F\sin\theta - w = 0$, or $n = w - F\sin\theta = mg - F\sin\theta$. The friction force is $f_k = \mu_k n = \mu_k(mg - F\sin\theta)$. The net horizontal force is $F\cos\theta - f_k = F\cos\theta - \mu_k(mg - F\sin\theta)$, and so at constant speed,

$$F = \frac{\mu_k mg}{\cos\theta + \mu_k\sin\theta}$$

(b) Using the given values, $F = \dfrac{(0.35)(90\text{ kg})(9.80\text{ m/s}^2)}{(\cos 25° + (0.35)\sin 25°)} = 290\text{ N}$.

EVALUATE: If $\theta = 0°$, $F = \mu_k mg$.

5.45. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block.

SET UP: For block B use coordinates parallel and perpendicular to the incline. Since they are connected by ropes, blocks A and B also move with constant speed.

EXECUTE: **(a)** The free-body diagrams are sketched in Figure 5.45.

(b) The blocks move with constant speed, so there is no net force on block A; the tension in the rope connecting A and B must be equal to the frictional force on block A, $\mu_k = (0.35)\ (25.0\text{ N}) = 9\text{ N}$.

(c) The weight of block C will be the tension in the rope connecting B and C; this is found by considering the forces on block B. The components of force along the ramp are the tension in the first rope (9 N, from part (a)), the component of the weight along the ramp, the friction on block B and the tension in the second rope. Thus, the weight of block C is

$$w_C = 9\text{ N} + w_B(\sin 36.9° + \mu_k\cos 36.9°) = 9\text{ N} + (25.0\text{ N})(\sin 36.9° + (0.35)\cos 36.9°) = 31.0\text{ N}$$

The intermediate calculation of the first tension may be avoided to obtain the answer in terms of the common weight w of blocks A and B, $w_C = w(\mu_k + (\sin\theta + \mu_k\cos\theta))$, giving the same result.

(d) Applying Newton's Second Law to the remaining masses (B and C) gives:

$$a = g(w_C - \mu_k w_B\cos\theta - w_B\sin\theta)/(w_B + w_C) = 1.54\text{ m/s}^2.$$

EVALUATE: Before the rope between A and B is cut the net external force on the system is zero. When the rope is cut the friction force on A is removed from the system and there is a net force on the system of blocks B and C.

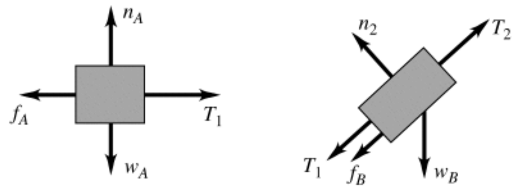


Figure 5.45

5.46. IDENTIFY and **SET UP:** The derivative of v_y gives a_y as a function of time, and the integral of v_y gives y as a function of time.

EXECUTE: Differentiating Eq. (5.10) with respect to time gives the acceleration $a = v_t\left(\frac{k}{m}\right)e^{-(k/m)t} = ge^{-(k/m)t}$, where Eq. (5.9), $v_t = mg/k$, has been used. Integrating Eq. (5.10) with respect to time with $y_0 = 0$ gives

$$y = \int_0^t v_t[1-e^{-(k/m)t}]\,dt = v_t\left[t + \left(\frac{m}{k}\right)e^{-(k/m)t}\right] - v_t\left(\frac{m}{k}\right) = v_t\left[t - \frac{m}{k}\left(1-e^{-(k/m)t}\right)\right].$$

EVALUATE: We can verify that $dy/dt = v_y$.

5.47. IDENTIFY and **SET UP:** Apply Eq.(5.13).

EXECUTE: **(a)** Solving for D in terms of v_t, $D = \frac{mg}{v_t^2} = \frac{(80\text{ kg})(9.80\text{ m/s}^2)}{(42\text{ m/s})^2} = 0.44\text{ kg/m}$.

(b) $v_t = \sqrt{\frac{mg}{D}} = \sqrt{\frac{(45\text{ kg})(9.80\text{ m/s}^2)}{(0.25\text{ kg/m})}} = 42\text{ m/s}$.

EVALUATE: v_t is less for the daughter since her mass is less.

5.48. IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to the ball. At the terminal speed, $f = mg$.

SET UP: The fluid resistance is directed opposite to the velocity of the object. At half the terminal speed, the magnitude of the frictional force is one-fourth the weight.

EXECUTE: **(a)** If the ball is moving up, the frictional force is down, so the magnitude of the net force is (5/4)w and the acceleration is (5/4)g, down.

(b) While moving down, the frictional force is up, and the magnitude of the net force is (3/4)w and the acceleration is (3/4)g, down.

EVALUATE: The frictional force is less than mg in each case and in each case the net force is downward and the acceleration is downward.

5.49. IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to one of the masses. The mass moves in a circular path, so has acceleration $a_{\text{rad}} = \frac{v^2}{R}$, directed toward the center of the path.

SET UP: In each case, $R = 0.200\text{ m}$. In part (a), let $+x$ be toward the center of the circle, so $a_x = a_{\text{rad}}$. In part (b) let $+y$ be toward the center of the circle, so $a_y = a_{\text{rad}}$. $+y$ is downward when the mass is at the top of the circle and $+y$ is upward when the mass is at the bottom of the circle. Since a_{rad} has its greatest possible value, $\vec{F}$ is in the direction of $\vec{a}_{\text{rad}}$ at both positions.

EXECUTE: **(a)** $\sum F_x = ma_x$ gives $F = ma_{\text{rad}} = m\frac{v^2}{R}$. $F = 75.0\text{ N}$ and $v = \sqrt{\frac{FR}{m}} = \sqrt{\frac{(75.0\text{ N})(0.200\text{ m})}{1.15\text{ kg}}} = 3.61\text{ m/s}$.

(b) The free-body diagrams for a mass at the top of the path and at the bottom of the path are given in figure 5.49. At the top, $\sum F_y = ma_y$ gives $F = ma_{\text{rad}} - mg$ and at the bottom it gives $F = mg + ma_{\text{rad}}$. For a given rotation rate and hence value of a_{rad}, the value of F required is larger at the bottom of the path.

(c) $F = mg + ma_{\text{rad}}$ so $\frac{v^2}{R} = \frac{F}{m} - g$ and

$$v = \sqrt{R\left(\frac{F}{m} - g\right)} = \sqrt{(0.200\text{ m})\left(\frac{75.0\text{ N}}{1.15\text{ kg}} - 9.80\text{ m/s}^2\right)} = 3.33\text{ m/s}$$

EVALUATE: The maximum speed is less for the vertical circle. At the bottom of the vertical path $\vec{F}$ and the weight are in opposite directions so F must exceed ma_{rad} by an amount equal to mg. At the top of the vertical path F and mg are in the same direction and together provide the required net force, so F must be larger at the bottom.

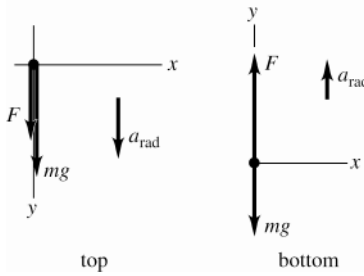


Figure 5.49

5.50. **IDENTIFY:** Since the car travels in an arc of a circle, it has acceleration $a_{\text{rad}} = v^2/R$, directed toward the center of the arc. The only horizontal force on the car is the static friction force exerted by the roadway. To calculate the minimum coefficient of friction that is required, set the static friction force equal to its maximum value, $f_s = \mu_s n$. Friction is static friction because the car is not sliding in the radial direction.

SET UP: The free-body diagram for the car is given in Figure 5.50. The diagram assumes the center of the curve is to the left of the car.

EXECUTE: **(a)** $\sum F_y = ma_y$ gives $n = mg$. $\sum F_x = ma_x$ gives $\mu_s n = m\dfrac{v^2}{R}$. $\mu_s mg = m\dfrac{v^2}{R}$ and

$$\mu_s = \frac{v^2}{gR} = \frac{(25.0\text{ m/s})^2}{(9.80\text{ m/s}^2)(220\text{ m})} = 0.290$$

(b) $\dfrac{v^2}{\mu_s} = Rg = \text{constant}$, so $\dfrac{v_1^2}{\mu_{s1}} = \dfrac{v_2^2}{\mu_{s2}}$. $v_2 = v_1\sqrt{\dfrac{\mu_{s2}}{\mu_{s1}}} = (25.0\text{ m/s})\sqrt{\dfrac{\mu_{s1}/3}{\mu_{s1}}} = 14.4\text{ m/s}$.

EVALUATE: A smaller coefficient of friction means a smaller maximum friction force, a smaller possible acceleration and therefore a smaller speed.

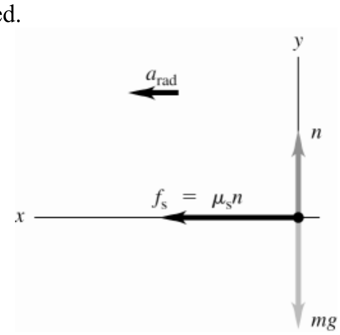


Figure 5.50

5.51. **IDENTIFY:** We can use the analysis done in Example 5.23. As in that example, we assume friction is negligible.

SET UP: From Example 5.23, the banking angle β is given by $\tan\beta = \dfrac{v^2}{gR}$. Also, $n = mg/\cos\beta$.

$65.0\text{ mi/h} = 29.1\text{ m/s}$.

EXECUTE: **(a)** $\tan\beta = \dfrac{(29.1\text{ m/s})^2}{(9.80\text{ m/s}^2)(225\text{ m})}$ and $\beta = 21.0°$. The expression for $\tan\beta$ does not involve the mass of the vehicle, so the truck and car should travel at the same speed.

(b) For the car, $n_{\text{car}} = \dfrac{(1125\text{ kg})(9.80\text{ m/s}^2)}{\cos 21.0°} = 1.18\times10^4\text{ N}$ and $n_{\text{truck}} = 2n_{\text{car}} = 2.36\times10^4\text{ N}$, since $m_{\text{truck}} = 2m_{\text{car}}$.

EVALUATE: The vertical component of the normal force must equal the weight of the vehicle, so the normal force is proportional to m.

5.52. **IDENTIFY:** The acceleration of the person is $a_{\text{rad}} = v^2/R$, directed horizontally to the left in the figure in the problem. The time for one revolution is the period $T = \dfrac{2\pi R}{v}$. Apply $\sum\vec{F} = m\vec{a}$ to the person.

SET UP: The person moves in a circle of radius $R = 3.00\text{ m} + (5.00\text{ m})\sin 30.0° = 5.50\text{ m}$. The free-body diagram is given in Figure 5.52. $\vec{F}$ is the force applied to the seat by the rod.

EXECUTE: **(a)** $\sum F_y = ma_y$ gives $F\cos 30.0° = mg$ and $F = \dfrac{mg}{\cos 30.0°}$. $\sum F_x = ma_x$ gives $F\sin 30.0° = m\dfrac{v^2}{R}$.

Combining these two equations gives $v = \sqrt{Rg\tan\theta} = \sqrt{(5.50\text{ m})(9.80\text{ m/s}^2)\tan 30.0°} = 5.58\text{ m/s}$. Then the period is $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(5.50\text{ m})}{5.58\text{ m/s}} = 6.19\text{ s}$.

(b) The net force is proportional to m so in $\sum \vec{F} = m\vec{a}$ the mass divides out and the angle for a given rate of rotation is independent of the mass of the passengers.

EVALUATE: The person moves in a horizontal circle so the acceleration is horizontal. The net inward force required for circular motion is produced by a component of the force exerted on the seat by the rod.

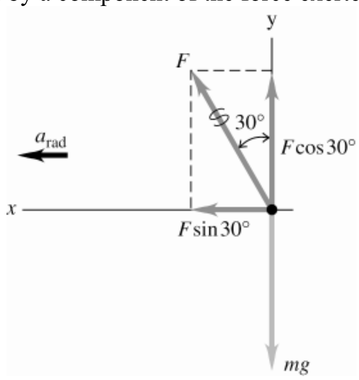


Figure 5.52

5.53. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the composite object of the person plus seat. This object moves in a horizontal circle and has acceleration a_{rad}, directed toward the center of the circle.

SET UP: The free-body diagram for the composite object is given in Figure 5.53. Let $+x$ be to the right, in the direction of $\vec{a}_{\text{rad}}$. Let $+y$ be upward. The radius of the circular path is $R = 7.50\text{ m}$. The total mass is $(255\text{ N} + 825\text{ N})/(9.80\text{ m/s}^2) = 110.2\text{ kg}$. Since the rotation rate is $32.0\text{ rev/min} = 0.5333\text{ rev/s}$, the period T is $\dfrac{1}{0.5333\text{ rev/s}} = 1.875\text{ s}$.

EXECUTE: $\sum F_y = ma_y$ gives $T_A\cos 40.0° - mg = 0$ and $T_A = \dfrac{mg}{\cos 40.0°} = \dfrac{255\text{ N} + 825\text{ N}}{\cos 40.0°} = 1410\text{ N}$.

$\sum F_x = ma_x$ gives $T_A\sin 40.0° + T_B = ma_{\text{rad}}$ and

$$T_B = m\frac{4\pi^2 R}{T^2} - T_A\sin 40.0° = (110.2\text{ kg})\frac{4\pi^2(7.50\text{ m})}{(1.875\text{ s})^2} - (1410\text{ N})\sin 40.0° = 8370\text{ N}.$$

The tension in the horizontal cable is 8370 N and the tension in the other cable is 1410 N.

EVALUATE: The weight of the composite object is 1080 N. The tension in cable A is larger than this since its vertical component must equal the weight. $ma_{\text{rad}} = 9280\text{ N}$. The tension in cable B is less than this because part of the required inward force comes from a component of the tension in cable A.

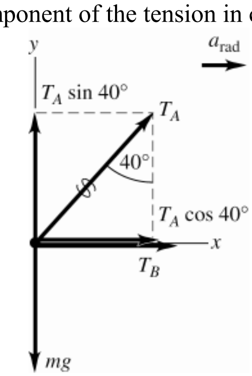


Figure 5.53

5.54. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the button. The button moves in a circle, so it has acceleration a_{rad}.

SET UP: The situation is equivalent to that of Example 5.22.

EXECUTE: **(a)** $\mu_s = \dfrac{v^2}{Rg}$. Expressing v in terms of the period T, $v = \dfrac{2\pi R}{T}$ so $\mu_s = \dfrac{4\pi^2 R}{T^2 g}$. A platform speed of 40.0 rev/min corresponds to a period of 1.50 s, so $\mu_s = \dfrac{4\pi^2(0.150\text{ m})}{(1.50\text{ s})^2(9.80\text{ m/s}^2)} = 0.269$.

(b) For the same coefficient of static friction, the maximum radius is proportional to the square of the period (longer periods mean slower speeds, so the button may be moved further out) and so is inversely proportional to the square of the speed. Thus, at the higher speed, the maximum radius is $(0.150\text{ m})\left(\dfrac{40.0}{60.0}\right)^2 = 0.067\text{ m}$.

EVALUATE: $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$. The maximum radial acceleration that friction can give is $\mu_s mg$. At the faster rotation rate T is smaller so R must be smaller to keep a_{rad} the same.

5.55. IDENTIFY: The acceleration due to circular motion is $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$.

SET UP: $R = 800\text{ m}$. $1/T$ is the number of revolutions per second.

EXECUTE: **(a)** Setting $a_{\text{rad}} = g$ and solving for the period T gives

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{400\text{ m}}{9.80\text{ m/s}^2}} = 40.1\text{ s},$$

so the number of revolutions per minute is $(60\text{ s/min})/(40.1\text{ s}) = 1.5\text{ rev/min}$.

(b) The lower acceleration corresponds to a longer period, and hence a lower rotation rate, by a factor of the square root of the ratio of the accelerations, $T' = (1.5\text{ rev/min}) \times \sqrt{3.70/9.8} = 0.92\text{ rev/min}$.

EVALUATE: In part (a) the tangential speed of a point at the rim is given by $a_{\text{rad}} = \dfrac{v^2}{R}$, so $v = \sqrt{Ra_{\text{rad}}} = \sqrt{Rg} = 62.6\text{ m/s}$; the space station is rotating rapidly.

5.56. IDENTIFY: $T = \dfrac{2\pi R}{v}$. The apparent weight of a person is the normal force exerted on him by the seat he is sitting on. His acceleration is $a_{\text{rad}} = v^2/R$, directed toward the center of the circle.

SET UP: The period is $T = 60.0\text{ s}$. The passenger has mass $m = w/g = 90.0\text{ kg}$.

EXECUTE: **(a)** $v = \dfrac{2\pi R}{T} = \dfrac{2\pi(50.0\text{ m})}{60.0\text{ s}} = 5.24\text{ m/s}$. Note that $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(5.24\text{ m/s})^2}{50.0\text{ m}} = 0.549\text{ m/s}^2$.

(b) The free-body diagram for the person at the top of his path is given in Figure 5.56a. The acceleration is downward, so take $+y$ downward. $\sum F_y = ma_y$ gives $mg - n = ma_{\text{rad}}$.

$n = m(g - a_{\text{rad}}) = (90.0\text{ kg})(9.80\text{ m/s}^2 - 0.549\text{ m/s}^2) = 833\text{ N}$.

The free-body diagram for the person at the bottom of his path is given in Figure 5.56b. The acceleration is upward, so take $+y$ upward. $\sum F_y = ma_y$ gives $n - mg = ma_{\text{rad}}$ and $n = m(g + a_{\text{rad}}) = 931\text{ N}$.

(c) Apparent weight $= 0$ means $n = 0$ and $mg = ma_{\text{rad}}$. $g = \dfrac{v^2}{R}$ and $v = \sqrt{gR} = 22.1\text{ m/s}$. The time for one revolution would be $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(50.0\text{ m})}{22.1\text{ m/s}} = 14.2\text{ s}$. Note that $a_{\text{rad}} = g$.

(d) $n = m(g + a_{\text{rad}}) = 2mg = 2(882\text{ N}) = 1760\text{ N}$, twice his true weight.

EVALUATE: At the top of his path his apparent weight is less than his true weight and at the bottom of his path his apparent weight is greater than his true weight.

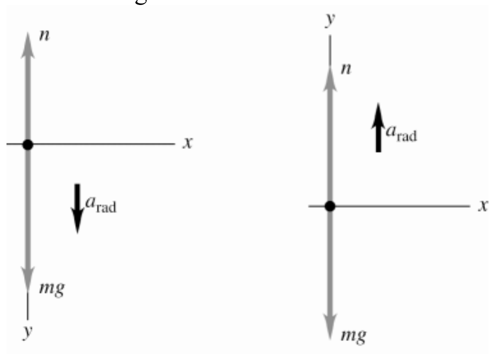


Figure 5.56a, b

5.57. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the motion of the pilot. The pilot moves in a vertical circle. The apparent weight is the normal force exerted on him. At each point $\vec{a}_{rad}$ is directed toward the center of the circular path.

(a) SET UP: "the pilot feels weightless" means that the vertical normal force *n* exerted on the pilot by the chair on which the pilot sits is zero. The force diagram for the pilot at the top of the path is given in Figure 5.57a.

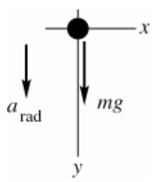


Figure 5.57a

EXECUTE:

$$\sum F_y = ma_y$$

$$mg = ma_{rad}$$

$$g = \frac{v^2}{R}$$

Thus $v = \sqrt{gR} = \sqrt{(9.80\text{ m/s}^2)(150\text{ m})} = 38.34\text{ m/s}$

$$v = (38.34\text{ m/s})\left(\frac{1\text{ km}}{10^3\text{ m}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = 138\text{ km/h}$$

(b) SET UP: The force diagram for the pilot at the bottom of the path is given in Figure 5.57b. Note that the vertical normal force exerted on the pilot by the chair on which the pilot sits is now upward.

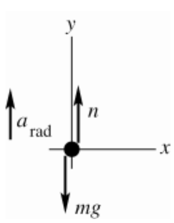


Figure 5.57b

EXECUTE:

$$\sum F_y = ma_y$$

$$n - mg = m\frac{v^2}{R}$$

$$n = mg + m\frac{v^2}{R}$$

This normal force is the pilot's apparent weight.

$w = 700\text{ N}$, so $m = \dfrac{w}{g} = 71.43\text{ kg}$

$$v = (280\text{ km/h})\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right) = 77.78\text{ m/s}$$

Thus $n = 700\text{ N} + 71.43\text{ kg}\dfrac{(77.78\text{ m/s})^2}{150\text{ m}} = 3580\text{ N}.$

EVALUATE: In part (b), $n > mg$ since the acceleration is upward. The pilot feels he is much heavier than when at rest. The speed is not constant, but it is still true that $a_{rad} = v^2/R$ at each point of the motion.

5.58. **IDENTIFY:** $a_{rad} = v^2/R$, directed toward the center of the circular path. At the bottom of the dive, $\vec{a}_{rad}$ is upward. The apparent weight of the pilot is the normal force exerted on her by the seat on which she is sitting.

SET UP: The free-body diagram for the pilot is given in Figure 5.58.

EXECUTE: **(a)** $a_{rad} = \dfrac{v^2}{R}$ gives $R = \dfrac{v^2}{a_{rad}} = \dfrac{(95.0\text{ m/s})^2}{4.00(9.80\text{ m/s}^2)} = 230\text{ m}$.

(b) $\sum F_y = ma_y$ gives $n - mg = ma_{rad}$.

$n = m(g + a_{rad}) = m(g + 4.00g) = 5.00mg = (5.00)(50.0\text{ kg})(9.80\text{ m/s}^2) = 2450\text{ N}$

EVALUATE: Her apparent weight is five times her true weight, the force of gravity the earth exerts on her.

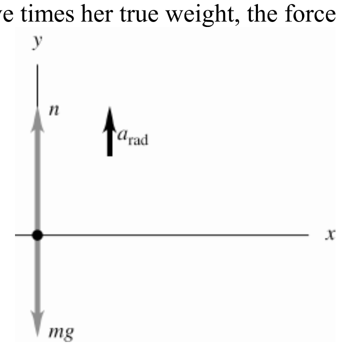


Figure 5.58

5.59. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the water. The water moves in a vertical circle. The target variable is the speed v; we will calculate a_{rad} and then get v from $a_{rad} = v^2/R$

SET UP: Consider the free-body diagram for the water when the pail is at the top of its circular path, as shown in Figures 5.59a and b.

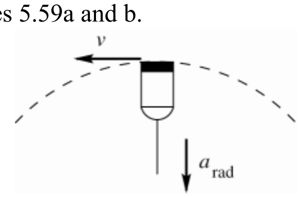


Figure 5.59a

The radial acceleration is in toward the center of the circle so at this point is downward. n is the downward normal force exerted on the water by the bottom of the pail.

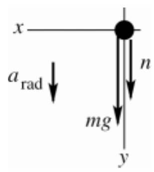


Figure 5.59b

EXECUTE:

$\sum F_y = ma_y$

$n + mg = m\dfrac{v^2}{R}$

At the minimum speed the water is just ready to lose contact with the bottom of the pail, so at this speed, $n \to 0$. (Note that the force n cannot be upward.)

With $n \to 0$ the equation becomes $mg = m\dfrac{v^2}{R}$. $v = \sqrt{gR} = \sqrt{(9.80\text{ m/s}^2)(0.600\text{ m})} = 2.42\text{ m/s}$.

EVALUATE: At the minimum speed $a_{rad} = g$. If v is less than this minimum speed, gravity pulls the water (and bucket) out of the circular path.

5.60. **IDENTIFY:** The ball has acceleration $a_{rad} = v^2/R$, directed toward the center of the circular path. When the ball is at the bottom of the swing, its acceleration is upward.

SET UP: Take $+y$ upward, in the direction of the acceleration. The bowling ball has mass $m = w/g = 7.27\text{ kg}$.

EXECUTE: **(a)** $a_{rad} = \dfrac{v^2}{R} = \dfrac{(4.20\text{ m/s})^2}{3.80\text{ m}} = 4.64\text{ m/s}$, upward.

(b) The free-body diagram is given in Figure 5.60. $\sum F_y = ma_y$ gives $T - mg = ma_{rad}$.

$T = m(g + a_{rad}) = (7.27\text{ kg})(9.80\text{ m/s}^2 + 4.64\text{ m/s}^2) = 105\text{ N}$

EVALUATE: The acceleration is upward, so the net force is upward and the tension is greater than the weight.

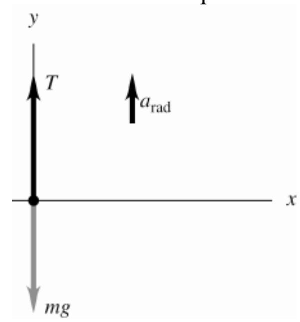


Figure 5.60

5.61. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the knot.

SET UP: $a = 0$. Use coordinates with axes that are horizontal and vertical.

EXECUTE: **(a)** The free-body diagram for the knot is sketched in Figure 5.61.

T_1 is more vertical so supports more of the weight and is larger. You can also see this from $\sum F_x = ma_x$:

$T_2 \cos 40° - T_1 \cos 60° = 0$. $T_2 \cos 40° - T_1 \cos 60° = 0$.

(b) T_1 is larger so set $T_1 = 5000$ N. Then $T_2 = T_1/1.532 = 3263.5$ N. $\sum F_y = ma_y$ gives

$T_1 \sin 60° + T_2 \sin 40° = w$ and $w = 6400$ N.

EVALUATE: The sum of the vertical components of the two tensions equals the weight of the suspended object. The sum of the tensions is greater than the weight.

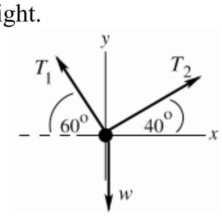


Figure 5.61

5.62. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to each object. Constant speed means $a = 0$.

SET UP: The free-body diagrams are sketched in Figure 5.62. T_1 is the tension in the lower chain, T_2 is the tension in the upper chain and $T = F$ is the tension in the rope.

EXECUTE: The tension in the lower chain balances the weight and so is equal to *w*. The lower pulley must have no net force on it, so twice the tension in the rope must be equal to *w* and the tension in the rope, which equals *F*, is $w/2$. Then, the downward force on the upper pulley due to the rope is also *w*, and so the upper chain exerts a force *w* on the upper pulley, and the tension in the upper chain is also *w*.

EVALUATE: The pulley combination allows the worker to lift a weight *w* by applying a force of only $w/2$.

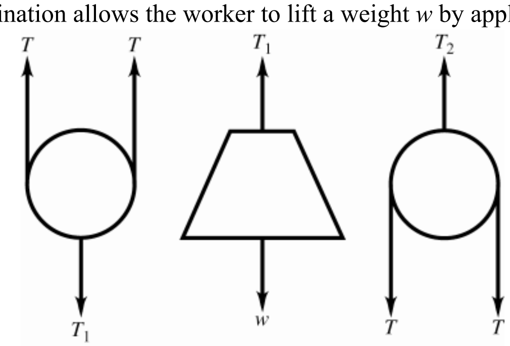


Figure 5.62

5.63. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the rope.

SET UP: The hooks exert forces on the ends of the rope. At each hook, the force that the hook exerts and the force due to the tension in the rope are an action-reaction pair.

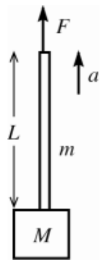
EXECUTE: **(a)** The vertical forces that the hooks exert must balance the weight of the rope, so each hook exerts an upward vertical force of $w/2$ on the rope. Therefore, the downward force that the rope exerts at each end is $T_{\text{end}} \sin\theta = w/2$, so $T_{\text{end}} = w/(2\sin\theta) = Mg/(2\sin\theta)$.

(b) Each half of the rope is itself in equilibrium, so the tension in the middle must balance the horizontal force that each hook exerts, which is the same as the horizontal component of the force due to the tension at the end; $T_{\text{end}}\cos\theta = T_{\text{middle}}$, so $T_{\text{middle}} = Mg\cos\theta/(2\sin\theta) = Mg/(2\tan\theta)$.

(c) Mathematically speaking, $\theta \neq 0$ because this would cause a division by zero in the equation for T_{end} or T_{middle}. Physically speaking, we would need an infinite tension to keep a non-massless rope perfectly straight.

EVALUATE: The tension in the rope is not the same at all points along the rope.

5.64. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to the combined rope plus block to find a. Then apply $\sum\vec{F} = m\vec{a}$ to a section of the rope of length x. First note the limiting values of the tension. The system is sketched in Figure 5.64a.



At the top of the rope $T = F$
At the bottom of the rope $T = M(g + a)$

Figure 5.64a

SET UP: Consider the rope and block as one combined object, in order to calculate the acceleration: The free-body diagram is sketched in Figure 5.64b.

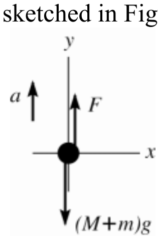


Figure 5.64b

EXECUTE:

$$\sum F_y = ma_y$$
$$F - (M + m)g = (M + m)a$$
$$a = \frac{F}{M + m} - g$$

SET UP: Now consider the forces on a section of the rope that extends a distance $x < L$ below the top. The tension at the bottom of this section is $T(x)$ and the mass of this section is $m(x/L)$. The free-body diagram is sketched in Figure 5.64c.

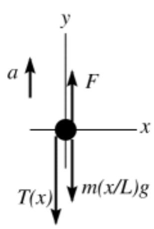


Figure 5.64c

EXECUTE:

$$\sum F_y = ma_y$$
$$F - T(x) - m(x/L)g = m(x/L)a$$
$$T(x) = F - m(x/L)g - m(x/L)a$$

Using our expression for a and simplifying gives

$$T(x) = F\left(1 - \frac{mx}{L(M + m)}\right)$$

EVALUATE: Important to check this result for the limiting cases:
$x = 0$: The expression gives the correct value of $T = F$.
$x = L$: The expression gives $T = F(M/(M + m))$. This should equal $T = M(g + a)$, and when we use the expression for a we see that it does.

5.65. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to each block.

SET UP: Constant speed means $a = 0$. When the blocks are moving, the friction force is f_k and when they are at rest, the friction force is f_s.

EXECUTE: **(a)** The tension in the cord must be $m_2 g$ in order that the hanging block move at constant speed. This tension must overcome friction and the component of the gravitational force along the incline, so $m_2 g = (m_1 g\sin\alpha + \mu_k m_1 g\cos\alpha)$ and $m_2 = m_1(\sin\alpha + \mu_k\cos\alpha)$.

(b) In this case, the friction force acts in the same direction as the tension on the block of mass m_1, so $m_2 g = (m_1 g\sin\alpha - \mu_k m_1 g\cos\alpha)$, or $m_2 = m_1(\sin\alpha - \mu_k\cos\alpha)$.

(c) Similar to the analysis of parts (a) and (b), the largest m_2 could be is $m_1(\sin\alpha + \mu_s \cos\alpha)$ and the smallest m_2 could be is $m_1(\sin\alpha - \mu_s \cos\alpha)$.

EVALUATE: In parts (a) and (b) the friction force changes direction when the direction of the motion of m_1 changes. In part (c), for the largest m_2 the static friction force on m_1 is directed down the incline and for the smallest m_2 the static friction force on m_1 is directed up the incline.

5.66. **IDENTIFY:** The system is in equilibrium. Apply Newton's 1st law to block *A*, to the hanging weight and to the knot where the cords meet. Target variables are the two forces.

(a) SET UP: The free-body diagram for the hanging block is given in Figure 5.66a.

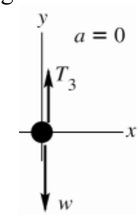


Figure 5.66a

EXECUTE:

$\sum F_y = ma_y$

$T_3 - w = 0$

$T_3 = 12.0\text{ N}$

SET UP: The free-body diagram for the knot is given in Figure 5.66b.

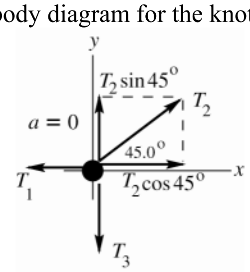


Figure 5.66b

EXECUTE:

$\sum F_y = ma_y$

$T_2 \sin 45.0° - T_3 = 0$

$T_2 = \dfrac{T_3}{\sin 45.0°} = \dfrac{12.0\text{ N}}{\sin 45.0°}$

$T_2 = 17.0\text{ N}$

$\sum F_x = ma_x$

$T_2 \cos 45.0° - T_1 = 0$

$T_1 = T_2 \cos 45.0° = 12.0\text{ N}$

SET UP: The free-body diagram for block *A* is given in Figure 5.66c.

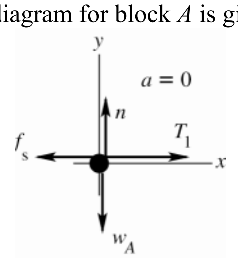


Figure 5.66c

EXECUTE:

$\sum F_x = ma_x$

$T_1 - f_s = 0$

$f_s = T_1 = 12.0\text{ N}$

EVALUATE: Also can apply $\sum F_y = ma_y$ to this block:

$$n - w_A = 0$$

$$n = w_A = 60.0\text{ N}$$

Then $\mu_s n = (0.25)(60.0\text{ N}) = 15.0\text{ N};$ this is the maximum possible value for the static friction force. We see that $f_s < \mu_s n;$ for this value of *w* the static friction force can hold the blocks in place.

(b) SET UP: We have all the same free-body diagrams and force equations as in part (a) but now the static friction force has its largest possible value, $f_s = \mu_s n = 15.0\text{ N}.$ Then $T_1 = f_s = 15.0\text{ N}.$

EXECUTE: From the equations for the forces on the knot

$T_2 \cos 45.0° - T_1 = 0$ implies $T_2 = T_1/\cos 45.0° = \dfrac{15.0\text{ N}}{\cos 45.0°} = 21.2\text{ N}$

$T_2 \sin 45.0° - T_3 = 0$ implies $T_3 = T_2 \sin 45.0° = (21.2\text{ N})\sin 45.0° = 15.0\text{ N}$

And finally $T_3 - w = 0$ implies $w = T_3 = 15.0\text{ N}.$

EVALUATE: Compared to part (a), the friction is larger in part (b) by a factor of (15.0/12.0) and *w* is larger by this same ratio.

5.67. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. Use Newton's 3rd law to relate forces on A and on B.

SET UP: Constant speed means $a = 0$.

EXECUTE: **(a)** Treat A and B as a single object of weight $w = w_A + w_B = 4.80\text{ N}$. The free-body diagram for this combined object is given in Figure 5.67a. $\sum F_y = ma_y$ gives $n = w = 4.80\text{ N}$. $f_k = \mu_k n = 1.44\text{ N}$. $\sum F_x = ma_x$ gives $F = f_k = 1.44\text{ N}$

(b) The free-body force diagrams for blocks A and B are given in Figure 5.67b. n and f_k are the normal and friction forces applied to block B by the tabletop and are the same as in part (a). f_{kB} is the friction force that A applies to B. It is to the right because the force from A opposes the motion of B. n_B is the downward force that A exerts on B. f_{kA} is the friction force that B applies to A. It is to the left because block B wants A to move with it. n_A is the normal force that block B exerts on A. By Newton's third law, $f_{kB} = f_{kA}$ and these forces are in opposite directions. Also, $n_A = n_B$ and these forces are in opposite directions.

$\sum F_y = ma_y$ for block A gives $n_A = w_A = 1.20\text{ N}$, so $n_B = 1.20\text{ N}$.

$f_{kA} = \mu_k n_A = (0.300)(1.20\text{ N}) = 0.36\text{ N}$, and $f_{kB} = 0.36\text{ N}$.

$\sum F_x = ma_x$ for block A gives $T = f_{kA} = 0.36\text{ N}$.

$\sum F_x = ma_x$ for block B gives $F = f_{kB} + f_k = 0.36\text{ N} + 1.44\text{ N} = 1.80\text{ N}$

EVALUATE: In part (a) block A is at rest with respect to B and it has zero acceleration. There is no horizontal force on A besides friction, and the friction force on A is zero. A larger force F is needed in part (b), because of the friction force between the two blocks.

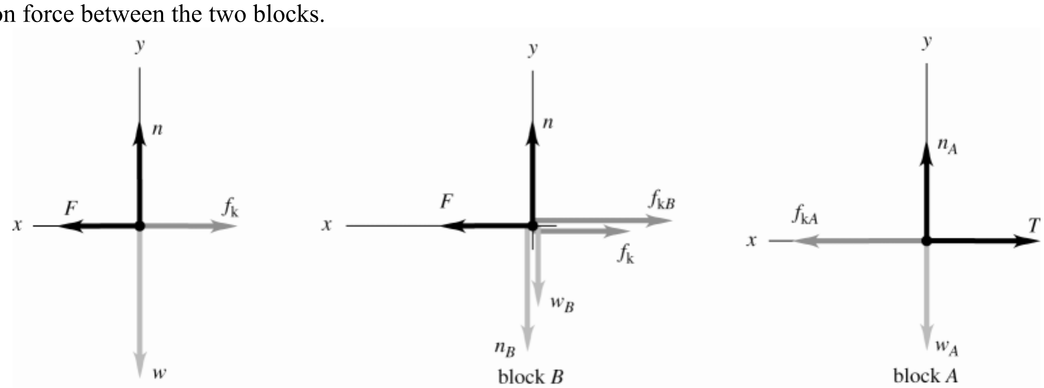


Figure 5.67a–c

5.68. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the brush. Constant speed means $a = 0$. Target variables are two of the forces on the brush.

SET UP: Note that the normal force exerted by the wall is horizontal, since it is perpendicular to the wall. The kinetic friction force exerted by the wall is parallel to the wall and opposes the motion, so it is vertically downward. The free-body diagram is given in Figure 5.68.

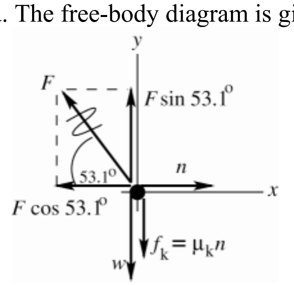


Figure 5.68

EXECUTE:

$\sum F_x = ma_x$

$n - F\cos 53.1° = 0$

$n = F\cos 53.1°$

$f_k = \mu_k n = \mu_k F\cos 53.1°$

$\sum F_y = ma_y$

$F\sin 53.1° - w - f_k = 0$

$F\sin 53.1° - w - \mu_k F\cos 53.1° = 0$

$F(\sin 53.1° - \mu_k \cos 53.1°) = w$

$$F = \frac{w}{\sin 53.1° - \mu_k \cos 53.1°}$$

(a) $F = \dfrac{w}{\sin 53.1° - \mu_k \cos 53.1°} = \dfrac{120\text{ N}}{\sin 53.1° - (0.15)\cos 53.1°} = 16.9\text{ N}$

(b) $n = F\cos 53.1° = (16.9\text{ N})\cos 53.1° = 10.1\text{ N}$

EVALUATE: In the absence of friction $w = F\sin 53.1°$, which agrees with our expression.

5.69. **IDENTIFY:** The net force at any time is $F_{\text{net}} = ma$.

SET UP: At $t = 0$, $a = 62g$. The maximum acceleration is 140*g* at $t = 1.2$ ms.

EXECUTE: **(a)** $F_{\text{net}} = ma = 62mg = 62(210\times10^{-9}\text{ kg})(9.80\text{ m/s}^2) = 1.3\times10^{-4}\text{ N}$. This force is 62 times the flea's weight.

(b) $F_{\text{net}} = 140mg = 2.9\times10^{-4}\text{ N}$.

(c) Since the initial speed is zero, the maximum speed is the area under the a_x-t graph. This gives 1.2 m/s.

EVALUATE: *a* is much larger than *g* and the net external force is much larger than the flea's weight.

5.70. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the instrument and calculate the acceleration. Then use constant acceleration equations to describe the motion.

SET UP: The free-body diagram for the instrument is given in Figure 5.70. The instrument has mass $m = w/g = 1.531\text{ kg}$.

EXECUTE: **(a)** For on the instrument, $\sum F_y = ma_y$ gives $T - mg = ma$ and $a = \dfrac{T - mg}{m} = 13.07\text{ m/s}^2$.

$v_{0y} = 0,\ v_y = 330\text{ m/s},\ a_y = 13.07\text{ m/s}^2,\ t = ?$ Then $v_y = v_{0y} + a_y t$ gives $t = 25.3\text{ s}$. Consider forces on the rocket; rocket has the same a_y. Let *F* be the thrust of the rocket engines. $F - mg = ma$ and

$F = m(g + a) = (25{,}000\text{ kg})(9.80\text{ m/s}^2 + 13.07\text{ m/s}^2) = 5.72\times10^5\text{ N}$.

(b) $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = 4170$ m.

EVALUATE: The rocket and instrument have the same acceleration. The tension in the wire is over twice the weight of the instrument and the upward acceleration is greater than *g*.

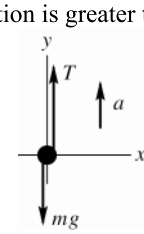


Figure 5.70

5.71. **IDENTIFY:** $a = dv/dt$. Apply $\sum \vec{F} = m\vec{a}$ to yourself.

SET UP: The reading of the scale is equal to the normal force the scale applies to you.

EXECUTE: The elevator's acceleration is

$$a = \frac{dv(t)}{dt} = 3.0\text{ m/s}^2 + 2(0.20\text{ m/s}^3)t = 3.0\text{ m/s}^2 + (0.40\text{ m/s}^3)t$$

At $t = 4.0$ s, $a = 3.0\text{ m/s}^2 + (0.40\text{ m/s}^3)(4.0\text{ s}) = 4.6\text{ m/s}^2$. From Newton's Second Law, the net force on you is $F_{\text{net}} = F_{\text{scale}} - w = ma$ and

$$F_{\text{scale}} = w + ma = (72\text{ kg})(9.8\text{ m/s}^2) + (72\text{ kg})(4.6\text{ m/s}^2) = 1040\text{ N}$$

EVALUATE: *a* increases with time, so the scale reading is increasing.

5.72. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the passenger to find the maximum allowed acceleration. Then use a constant acceleration equation to find the maximum speed.

SET UP: The free-body diagram for the passenger is given in Figure 5.72.

EXECUTE: $\sum F_y = ma_y$ gives $n - mg = ma$. $n = 1.6mg$, so $a = 0.60\,g = 5.88\text{ m/s}^2$.

$y - y_0 = 3.0\text{ m},\ a_y = 5.88\text{ m/s}^2,\ v_{0y} = 0$ so $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = 5.0\text{ m/s}$.

EVALUATE: A larger final speed would require a larger value of a_y, which would mean a larger normal force on the person.

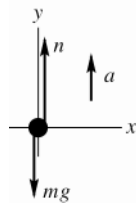


Figure 5.72

5.73. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the package. Calculate *a* and then use a constant acceleration equation to describe the motion.
SET UP: Let $+x$ be directed up the ramp.
EXECUTE: **(a)** $F_{\text{net}} = -mg\sin 37° - f_{\text{k}} = -mg\sin 37° - \mu_{\text{k}} mg\cos 37° = ma$ and

$$a = -(9.8\ \text{m/s}^2)(0.602 + (0.30)(0.799)) = -8.25\,\text{m/s}^2$$

Since we know the length of the slope, we can use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ with $x_0 = 0$ and $v_x = 0$ at the top.

$v_0^2 = -2ax = -2(-8.25\ \text{m/s}^2)(8.0\ \text{m}) = 132\ \text{m}^2/\text{s}^2$ and $v_0 = \sqrt{132\ \text{m}^2/\text{s}^2} = 11.5\ \text{m/s}$

(b) For the trip back down the slope, gravity and the friction force operate in opposite directions to each other.
$F_{\text{net}} = -mg\sin 37° + \mu_{\text{k}} mg\cos 37° = ma$ and

$a = g(-\sin 37° + 0.30\ \cos 37°) = (9.8\ \text{m/s}^2)((-0.602) + (0.30)(0.799)) = -3.55\ \text{m/s}^2$.

Now we have $v_0 = 0,\ x_0 = -8.0\ \text{m},\ x = 0$ and $v^2 = v_0^2 + 2a(x - x_0) = 0 + 2(-3.55\ \text{m/s}^2)(-8.0\ \text{m}) = 56.8\ \text{m}^2/\text{s}^2$, so

$v = \sqrt{56.8\ \text{m}^2/\text{s}^2} = 7.54\ \text{m/s}$.

EVALUATE: In both cases, moving up the incline and moving down the incline, the acceleration is directed down the incline. The magnitude of *a* is greater when the package is going up the incline, because $mg\sin 37°$ and f_{k} are in the same direction whereas when the package is going down these two forces are in opposite directions.

5.74. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the hammer. Since the hammer is at rest relative to the bus its acceleration equals that of the bus.
SET UP: The free-body diagram for the hammer is given in Figure 5.74.
EXECUTE: $\sum F_y = ma_y$ gives $T\sin 74° - mg = 0$ so $T\sin 74° = mg$. $\sum F_x = ma_x$ gives $T\cos 74° = ma$. Divide the second equation by the first: $\dfrac{a}{g} = \dfrac{1}{\tan 74°}$ and $a = 2.8\ \text{m/s}^2$.

EVALUATE: When the acceleration increases the angle between the rope and the ceiling of the bus decreases, and the angle the rope makes with the vertical increases.

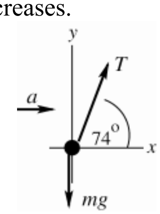


Figure 5.74

5.75. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the washer and to the crate. Since the washer is at rest relative to the crate, these two objects have the same acceleration.
SET UP: The free-body diagram for the washer is given in Figure 5.75.
EXECUTE: It's interesting to look at the string's angle measured from the perpendicular to the top of the crate. This angle is $\theta_{\text{string}} = 90° -$ angle measured from the top of the crate . The free-body diagram for the washer then leads to the following equations, using Newton's Second Law and taking the upslope direction as positive:

$$-m_{\text{w}} g\sin\theta_{\text{slope}} + T\sin\theta_{\text{string}} = m_{\text{w}} a \text{ and } T\sin\theta_{\text{string}} = m_{\text{w}}(a + g\ \sin\theta_{\text{slope}})$$

$$-m_{\text{w}} g\cos\theta_{\text{slope}} + T\cos\theta_{\text{string}} = 0 \text{ and } T\cos\theta_{\text{string}} = m_{\text{w}} g\cos\theta_{\text{slope}}$$

Dividing the two equations: $\tan\theta_{\text{string}} = \dfrac{a + g\sin\theta_{\text{slope}}}{g\cos\theta_{\text{slope}}}$

For the crate, the component of the weight along the slope is $-m_{\text{c}}g\sin\theta_{\text{slope}}$ and the normal force is $m_{\text{c}}g\cos\theta_{\text{slope}}$.

Using Newton's Second Law again: $-m_{\text{c}}g\sin\ \theta_{\text{slope}} + \mu_{\text{k}}m_{\text{c}}g\cos\theta_{\text{slope}} = m_{\text{c}}a$. $\mu_{\text{k}} = \dfrac{a + g\sin\theta_{\text{slope}}}{g\cos\theta_{\text{slope}}}$. This leads to the interesting observation that the string will hang at an angle whose tangent is equal to the coefficient of kinetic friction**:**

$$\mu_{\text{k}} = \tan\theta_{\text{string}} = \tan(90° - 68°) = \tan\ 22° = 0.40\ .$$

EVALUATE: In the limit that $\mu_{\text{k}} \to 0$, $\theta_{\text{string}} \to 0$ and the string is perpendicular to the top of the crate. As μ_{k} increases, θ_{string} increases.

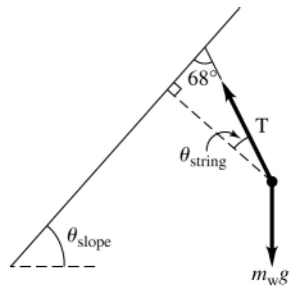


Figure 5.75

5.76. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to yourself and calculate a. Then use constant acceleration equations to describe the motion.

SET UP: The free-body diagram is given in Figure 5.76.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ gives $n = mg\cos\alpha$. $\Sigma F_x = ma_x$ gives $mg\sin\alpha - f_{\text{k}} = ma$. Combining these two equations, we have $a = g(\sin\alpha - \mu_{\text{k}}\cos\ \alpha) = -3.094\ \text{m/s}^2$. Find your stopping distance:

$v_x = 0,\ a_x = -3.094\ \text{m/s}^2,\ v_{0x} = 20\ \text{m/s}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $x - x_0 = 64.6$ m, which is greater than 40 m. You don't stop before you reach the hole, so you fall into it.

(b) $a_x = -3.094\ \text{m/s}^2,\ x - x_0 = 40\ \text{m},\ v_x = 0$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_{0x} = 16\ \text{m/s}$.

EVALUATE: Your stopping distance is proportional to the square of your initial speed, so your initial speed is proportional to the square root of your stopping distance. To stop in 40 m instead of 64.6 m your initial speed must be $(20\ \text{m/s})\sqrt{\dfrac{40\ \text{m}}{64.6\ \text{m}}} = 16\ \text{m/s}$.

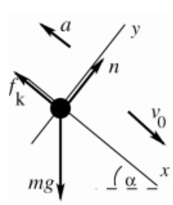


Figure 5.76

5.77. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to each block and to the rope. The key idea in solving this problem is to recognize that if the system is accelerating, the tension that block A exerts on the rope is different from the tension that block B exerts on the rope. (Otherwise the net force on the rope would be zero, and the rope couldn't accelerate.)

SET UP: Take a positive coordinate direction for each object to be in the direction of the acceleration of that object. All three objects have the same magnitude of acceleration.

EXECUTE: The Second Law equations for the three different parts of the system are**:**

Block A (The only horizontal forces on A are tension to the right, and friction to the left): $-\mu_{\text{k}}m_Ag + T_A = m_Aa.$

Block B (The only vertical forces on B are gravity down, and tension up): $m_Bg - T_B = m_Ba.$

Rope (The forces on the rope along the direction of its motion are the tensions at either end and the weight of the portion of the rope that hangs vertically): $m_R\left(\dfrac{d}{L}\right)g + T_B - T_A = m_Ra.$

To solve for a and eliminate the tensions, add the left hand sides and right hand sides of the three equations:

$-\mu_k m_A g + m_B g + m_R\left(\frac{d}{L}\right)g = (m_A + m_B + m_R)a$, or $a = g\frac{m_B + m_R(d/L) - \mu_k m_A}{(m_A + m_B + m_R)}$.

(a) When $\mu_k = 0$, $a = g\frac{m_B + m_R(d/L)}{(m_A + m_B + m_R)}$. As the system moves, d will increase, approaching L as a limit, and thus the acceleration will approach a maximum value of $a = g\frac{m_B + m_R}{(m_A + m_B + m_R)}$.

(b) For the blocks to just begin moving, $a > 0$, so solve $0 = [m_B + m_R(d/L) - \mu_s m_A]$ for d. Note that we must use static friction to find d for when the block will *begin* to move. Solving for d, $d = \frac{L}{m_R}(\mu_s m_A - m_B)$ or

$d = \frac{1.0\text{ m}}{0.160\text{ kg}}(0.25(2\text{ kg}) - 0.4\text{ kg}) = 0.63\text{ m}.$

(c) When $m_R = 0.04$ kg, $d = \frac{1.0\text{ m}}{0.04\text{ kg}}(0.25(2\text{ kg}) - 0.4\text{ kg}) = 2.50\text{ m}$. This is not a physically possible situation since $d > L$. The blocks won't move, no matter what portion of the rope hangs over the edge.

EVALUATE: For the blocks to move when released, the weight of B plus the weight of the rope that hangs vertically must be greater than the maximum static friction force on A, which is $\mu_s n = 4.9$ N.

5.78. **IDENTIFY:** Apply Newton's 1st law to the rope. Let m_1 be the mass of that part of the rope that is on the table, and let m_2 be the mass of that part of the rope that is hanging over the edge. ($m_1 + m_2 = m$, the total mass of the rope). Since the mass of the rope is not being neglected, the tension in the rope varies along the length of the rope. Let T be the tension in the rope at that point that is at the edge of the table.

SET UP: The free-body diagram for the hanging section of the rope is given in Figure 5.78a

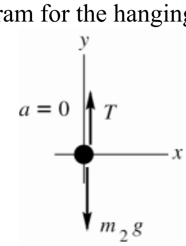


Figure 5.78a

EXECUTE:

$\sum F_y = ma_y$

$T - m_2 g = 0$

$T = m_2 g$

SET UP: The free-body diagram for that part of the rope that is on the table is given in Figure 5.78b.

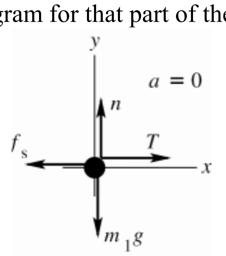


Figure 5.78b

EXECUTE:

$\sum F_y = ma_y$

$n - m_1 g = 0$

$n = m_1 g$

When the maximum amount of rope hangs over the edge the static friction has its maximum value:

$f_s = \mu_s n = \mu_s m_1 g$

$\sum F_x = ma_x$

$T - f_s = 0$

$T = \mu_s m_1 g$

Use the first equation to replace T:

$m_2 g = \mu_s m_1 g$

$m_2 = \mu_s m_1$

The fraction that hangs over is $\frac{m_2}{m} = \frac{\mu_s m_1}{m_1 + \mu_s m_1} = \frac{\mu_s}{1 + \mu_s}$.

EVALUATE: As $\mu_s \to 0$, the fraction goes to zero and as $\mu_s \to \infty$, the fraction goes to unity.

5.79. **IDENTIFY:** First calculate the maximum acceleration that the static friction force can give to the case. Apply $\sum \vec{F} = m\vec{a}$ to the case.

(a) SET UP: The static friction force is to the right in Figure 5.79a (northward) since it tries to make the case move with the truck. The maximum value it can have is $f_s = \mu_s N$.

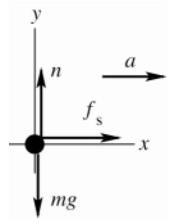


Figure 5.79a

EXECUTE:

$\sum F_y = ma_y$

$n - mg = 0$

$n = mg$

$f_s = \mu_s n = \mu_s mg$

$\sum F_x = ma_x$

$f_s = ma$

$\mu_s mg = ma$

$a = \mu_s g = (0.30)(9.80\text{ m/s}^2) = 2.94\text{ m/s}^2$

The truck's acceleration is less than this so the case doesn't slip relative to the truck; the case's acceleration is $a = 2.20\text{ m/s}^2$ (northward). Then $f_s = ma = (30.0\text{ kg})(2.20\text{ m/s}^2) = 66\text{ N}$, northward.

(b) IDENTIFY: Now the acceleration of the truck is greater than the acceleration that static friction can give the case. Therefore, the case slips relative to the truck and the friction is kinetic friction. The friction force still tries to keep the case moving with the truck, so the acceleration of the case and the friction force are both southward. The free-body diagram is sketched in Figure 5.79b.

SET UP:

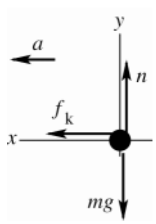


Figure 5.79b

EXECUTE:

$\sum F_y = ma_y$

$n - mg = 0$

$n = mg$

$f_k = \mu_k mg = (0.20)(30.0\text{ kg})(9.80\text{ m/s}^2)$

$f_k = 59\text{ N}$, southward

EVALUATE: $f_k = ma$ implies $a = \dfrac{f_k}{m} = \dfrac{59\text{ N}}{30.0\text{ kg}} = 2.0\text{ m/s}^2$. The magnitude of the acceleration of the case is less than that of the truck and the case slides toward the front of the truck. In both parts (a) and (b) the friction is in the direction of the motion and accelerates the case. Friction opposes *relative* motion between two surfaces in contact.

5.80. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the car to calculate its acceleration. Then use a constant acceleration equation to find the initial speed.

SET UP: Let $+x$ be in the direction of the car's initial velocity. The friction force f_k is then in the $-x$-direction. $192\text{ ft} = 58.52\text{ m}$.

EXECUTE: $n = mg$ and $f_k = \mu_k mg$. $\sum F_x = ma_x$ gives $-\mu_k mg = ma_x$ and $a_x = -\mu_k g = -(0.750)(9.80\text{ m/s}^2) = -7.35\text{ m/s}^2$. $v_x = 0$ (stops), $x - x_0 = 58.52\text{ m}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_{0x} = \sqrt{-2a_x(x - x_0)} = \sqrt{-2(-7.35\text{ m/s}^2)(58.52\text{ m})} = 29.3\text{ m/s} = 65.5\text{ mi/h}$. He was guilty.

EVALUATE: $x - x_0 = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = -\dfrac{v_{0x}^2}{2a_x}$. If his initial speed had been 45 mi/h he would have stopped in $\left(\dfrac{45\text{ mi/h}}{65.5\text{ mi/h}}\right)^2 (192\text{ ft}) = 91\text{ ft}$.

5.81. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the point where the three wires join and also to one of the balls. By symmetry the tension in each of the 35.0 cm wires is the same.

SET UP: The geometry of the situation is sketched in Figure 5.81a. The angle ϕ that each wire makes with the vertical is given by $\sin\phi = \dfrac{12.5\text{ cm}}{47.5\text{ cm}}$ and $\phi = 15.26°$. Let T_A be the tension in the vertical wire and let T_B be the tension in each of the other two wires. Neglect the weight of the wires. The free-body diagram for the left-hand ball is given in Figure 5.81b and for the point where the wires join in Figure 5.81c. n is the force one ball exerts on the other.

EXECUTE: **(a)** $\sum F_y = ma_y$ applied to the ball gives $T_B\cos\phi - mg = 0$.

$T_B = \dfrac{mg}{\cos\phi} = \dfrac{(15.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 15.26°} = 152\text{ N}$. Then $\sum F_y = ma_y$ applied in Figure 5.81c gives $T_A - 2T_B\cos\phi = 0$ and

$T_A = 2(152\text{ N})\cos\phi = 294\text{ N}$.

(b) $\sum F_x = ma_x$ applied to the ball gives $n - T_B\sin\phi = 0$ and $n = (152\text{ N})\sin 15.26° = 40.0\text{ N}$.

EVALUATE: T_A equals the total weight of the two balls.

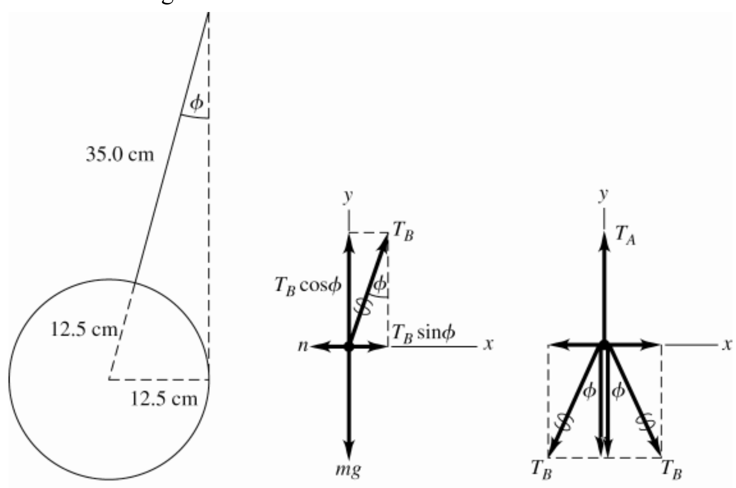


Figure 5.81a–c

5.82. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to the box. Compare the acceleration of the box to the acceleration of the truck and use constant acceleration equations to describe the motion.

SET UP: Both objects have acceleration in the same direction; take this to be the $+x$-direction.

EXECUTE: If the block were to remain at rest relative to the truck, the friction force would need to cause an acceleration of 2.20 m/s^2; however, the maximum acceleration possible due to static friction is $(0.19)(9.80\text{ m/s}^2) = 1.86\text{ m/s}^2$, and so the block will move relative to the truck; the acceleration of the box would be $\mu_k g = (0.15)(9.80\text{ m/s}^2) = 1.47\text{ m/s}^2$. The difference between the distance the truck moves and the distance the box moves (*i.e.*, the distance the box moves relative to the truck) will be 1.80 m after a time

$$t = \sqrt{\frac{2\Delta x}{a_{\text{truck}} - a_{\text{box}}}} = \sqrt{\frac{2(1.80\text{ m})}{(2.20\text{ m/s}^2 - 1.47\text{ m/s}^2)}} = 2.221\text{ s}.$$

In this time, the truck moves $\frac{1}{2}a_{\text{truck}}t^2 = \frac{1}{2}(2.20\text{ m/s}^2)\,(2.221\text{ s})^2 = 5.43\text{ m}$.

EVALUATE: To prevent the box from sliding off the truck the coefficient of static friction would have to be $\mu_s = (2.20\text{ m/s}^2)/g = 0.224$.

5.83. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to each block. Forces between the blocks are related by Newton's 3rd law. The target variable is the force *F*. Block *B* is pulled to the left at constant speed, so block *A* moves to the right at constant speed and $a = 0$ for each block.

SET UP: The free-body diagram for block *A* is given in Figure 5.83a. n_{BA} is the normal force that *B* exerts on *A*. $f_{BA} = \mu_k n_{BA}$ is the kinetic friction force that *B* exerts on *A*. Block *A* moves to the right relative to *B*, and f_{BA} opposes this motion, so f_{BA} is to the left.

Note also that F acts just on B, not on A.

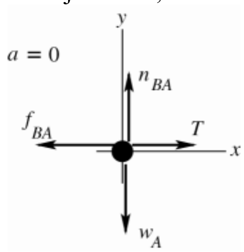


Figure 5.83a

EXECUTE:
$\sum F_y = ma_y$
$n_{BA} - w_A = 0$
$n_{BA} = 1.40\text{ N}$
$f_{BA} = \mu_k n_{BA} = (0.30)(1.40\text{ N}) = 0.420\text{ N}$

$\sum F_x = ma_x$
$T - f_{BA} = 0$
$T = f_{BA} = 0.420\text{ N}$

SET UP: The free-body diagram for block B is given in Figure 5.83b.

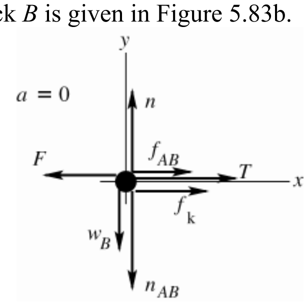


Figure 5.83b

EXECUTE: n_{AB} is the normal force that block A exerts on block B. By Newton's third law n_{AB} and n_{BA} are equal in magnitude and opposite in direction, so $n_{AB} = 1.40\text{ N}$. f_{AB} is the kinetic friction force that A exerts on B. Block B moves to the left relative to A and f_{AB} opposes this motion, so f_{AB} is to the right.
$f_{AB} = \mu_k n_{AB} = (0.30)(1.40\text{ N}) = 0.420\text{ N}.$
n and f_k are the normal and friction force exerted by the floor on block B; $f_k = \mu_k n.$ Note that block B moves to the left relative to the floor and f_k opposes this motion, so f_k is to the right.
$\sum F_y = ma_y$
$n - w_B - n_{AB} = 0$
$n = w_B + n_{AB} = 4.20\text{ N} + 1.40\text{ N} = 5.60\text{ N}$
Then $f_k = \mu_k n = (0.30)(5.60\text{ N}) = 1.68\text{ N}.$
$\sum F_x = ma_x$
$f_{AB} + T + f_k - F = 0$
$F = T + f_{AB} + f_k = 0.420\text{ N} + 0.420\text{ N} + 1.68\text{ N} = 2.52\text{ N}$

EVALUATE: Note that f_{AB} and f_{BA} are a third law action-reaction pair, so they must be equal in magnitude and opposite in direction and this is indeed what our calculation gives.

5.84. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the person to find the acceleration the PAPS unit produces. Apply constant acceleration equations to her free-fall motion and to her motion after the PAPS fires.

SET UP: We take the upward direction as positive.

EXECUTE: The explorer's vertical acceleration is -3.7 m/s^2 for the first 20 s. Thus at the end of that time her vertical velocity will be $v_y = a_y t = (-3.7\text{ m/s}^2)(20\text{ s}) = -74\text{ m/s}.$ She will have fallen a distance $d = v_{av}t = \left(\frac{-74\text{ m/s}}{2}\right)(20\text{ s}) = -740\text{ m}$ and will thus be $1200\text{ m} - 740\text{ m} = 460\text{ m}$ above the surface. Her vertical velocity must reach zero as she touches the ground; therefore, taking the ignition point of the PAPS as

$y_0 = 0,\ v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y = \dfrac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \dfrac{0 - (-74\ \text{m/s})^2}{-460\ \text{m}} = 5.95\ \text{m/s}^2$, which is the vertical acceleration that must be provided by the PAPS. The time it takes to reach the ground is given by

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - (-74\ \text{m/s})}{5.95\ \text{m/s}^2} = 12.4\ \text{s}$$

Using Newton's Second Law for the vertical direction $F_{\text{PAPSv}} + mg = ma$. This gives

$$F_{\text{PAPSv}} = ma - mg = m(a + g) = (150\ \text{kg})(5.95 - (-3.7))\ \text{m/s}^2 = 1450\ \text{N}\ ,$$

which is the vertical component of the PAPS force. The vehicle must also be brought to a stop horizontally in 12.4 seconds; the acceleration needed to do this is

$$a_y = \frac{v_y - v_{0y}}{t} = \frac{0 - 33\ \text{m/s}^2}{12.4\ \text{s}} = 2.66\ \text{m/s}^2$$

and the force needed is $F_{\text{PAPSh}} = ma = (150\ \text{kg})(2.66\ \text{m/s}^2) = 400\ \text{N}$, since there are no other horizontal forces.

EVALUATE: The acceleration produced by the PAPS must bring to zero both her horizontal and vertical components of velocity.

5.85. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. Parts (a) and (b) will be done together.

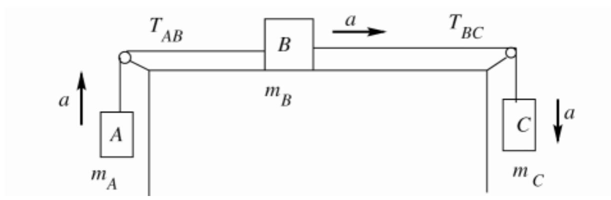


Figure 5.85a

Note that each block has the same magnitude of acceleration, but in different directions. For each block let the direction of $\vec{a}$ be a positive coordinate direction.

SET UP: The free-body diagram for block A is given in Figure 5.85b.

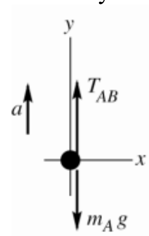


Figure 5.85b

EXECUTE:

$\sum F_y = ma_y$

$T_{AB} - m_A g = m_A a$

$T_{AB} = m_A(a + g)$

$T_{AB} = 4.00\ \text{kg}(2.00\ \text{m/s}^2 + 9.80\ \text{m/s}^2) = 47.2\ \text{N}$

SET UP: The free-body diagram for block B is given in Figure 5.85b.

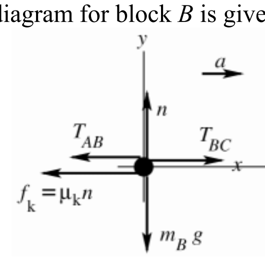


Figure 5.85c

EXECUTE:

$\sum F_y = ma_y$

$n - m_B g = 0$

$n = m_B g$

$f_k = \mu_k n = \mu_k m_B g = (0.25)(12.0\ \text{kg})(9.80\ \text{m/s}^2) = 29.4\ \text{N}$

$\sum F_x = ma_x$

$T_{BC} - T_{AB} - f_k = m_B a$

$T_{BC} = T_{AB} + f_k + m_B a = 47.2\ \text{N} + 29.4\ \text{N} + (12.0\ \text{kg})(2.00\ \text{m/s}^2)$

$T_{BC} = 100.6\ \text{N}$

SET UP: The free-body diagram for block C is sketched in Figure 5.85d.

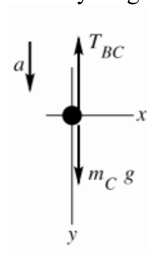


Figure 5.85d

EXECUTE:

$\sum F_y = ma_y$

$m_C g - T_{BC} = m_C a$

$m_C(g-a) = T_{BC}$

$$m_C = \frac{T_{BC}}{g-a} = \frac{100.6\text{ N}}{9.80\text{ m/s}^2 - 2.00\text{ m/s}^2} = 12.9\text{ kg}$$

EVALUATE: If all three blocks are considered together as a single object and $\sum \vec{F} = m\vec{a}$ is applied to this combined object, $m_C g - m_A g - \mu_k m_B g = (m_A + m_B + m_C)a$. Using the values for μ_k, m_A and m_B given in the problem and the mass m_C we calculated, this equation gives $a = 2.00\text{ m/s}^2$, which checks.

5.86. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. They have the same magnitude of acceleration, a.

SET UP: Consider positive accelerations to be to the right (up and to the right for the left-hand block, down and to the right for the right-hand block).

EXECUTE: **(a)** The forces along the inclines and the accelerations are related by $T - (100\text{ kg})g\sin 30° = (100\text{ kg})a$ and $(50\text{ kg})g\sin 53° - T = (50\text{ kg})a$, where T is the tension in the cord and a the mutual magnitude of acceleration. Adding these relations, $(50\text{ kg}\sin 53° - 100\text{ kg}\sin 30°)g = (50\text{ kg} + 100\text{ kg})a$, or $a = -0.067g$. Since a comes out negative, the blocks will slide to the left; the 100-kg block will slide down. Of course, if coordinates had been chosen so that positive accelerations were to the left, a would be $+0.067g$.

(b) $a = 0.067(9.80\text{ m/s}^2) = 0.658\text{ m/s}^2$.

(c) Substituting the value of a (including the proper sign, depending on choice of coordinates) into either of the above relations involving T yields 424 N.

EVALUATE: For part (a) we could have compared $mg\sin\theta$ for each block to determine which direction the system would move.

5.87. **IDENTIFY:** Let the tensions in the ropes be T_1 and T_2.

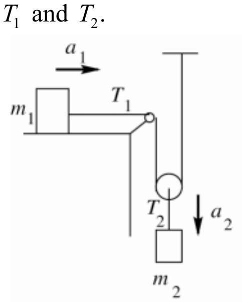


Figure 5.87a

Consider the forces on each block. In each case take a positive coordinate direction in the direction of the acceleration of that block.

SET UP: The free-body diagram for m_1 is given in Figure 5.87b.

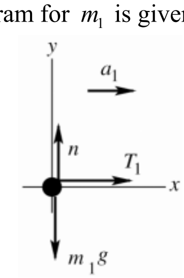


Figure 5.87b

EXECUTE:

$\sum F_x = ma_x$

$T_1 = m_1 a_1$

SET UP: The free-body diagram for m_2 is given in Figure 5.87c.

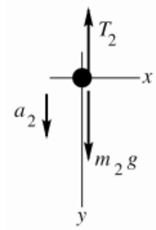


Figure 5.87c

EXECUTE:
$\sum F_y = ma_y$
$m_2 g - T_2 = m_2 a_2$

This gives us two equations, but there are 4 unknowns (T_1, T_2, a_1, and a_2) so two more equations are required.

SET UP: The free-body diagram for the moveable pulley (mass *m*) is given in Figure 5.87d.

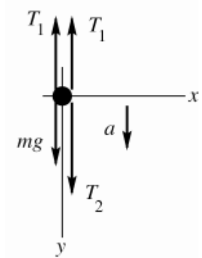


Figure 5.87d

EXECUTE:
$\sum F_y = ma_y$
$mg + T_2 - 2T_1 = ma$

But our pulleys have negligible mass, so $mg = ma = 0$ and $T_2 = 2T_1$. Combine these three equations to eliminate T_1 and T_2: $m_2 g - T_2 = m_2 a_2$ gives $m_2 g - 2T_1 = m_2 a_2$. And then with $T_1 = m_1 a_1$ we have $m_2 g - 2m_1 a_1 = m_2 a_2$.

SET UP: There are still two unknowns, a_1 and a_2. But the accelerations a_1 and a_2 are related. In any time interval, if m_1 moves to the right a distance *d*, then in the same time m_2 moves downward a distance $d/2$. One of the constant acceleration kinematic equations says $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$, so if m_2 moves half the distance it must have half the acceleration of m_1: $a_2 = a_1/2$, or $a_1 = 2a_2$.

EXECUTE: This is the additional equation we need. Use it in the previous equation and get
$m_2 g - 2m_1(2a_2) = m_2 a_2$.
$a_2(4m_1 + m_2) = m_2 g$
$a_2 = \dfrac{m_2 g}{4m_1 + m_2}$ and $a_1 = 2a_2 = \dfrac{2m_2 g}{4m_1 + m_2}$.

EVALUATE: If $m_2 \to 0$ or $m_1 \to \infty$, $a_1 = a_2 = 0$. If $m_2 >> m_1$, $a_2 = g$ and $a_1 = 2g$.

5.88. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to block *B*, to block *A* and *B* as a composite object and to block *C*. If *A* and *B* slide together all three blocks have the same magnitude of acceleration.

SET UP: If *A* and *B* don't slip the friction between them is static. The free-body diagrams for block *B*, for blocks *A* and *B*, and for *C* are given in Figures 5.88a-c. Block *C* accelerates downward and *A* and *B* accelerate to the right. In each case take a positive coordinate direction to be in the direction of the acceleration. Since block *A* moves to the right, the friction force f_s on block *B* is to the right, to prevent relative motion between the two blocks. When *C* has its largest mass, f_s has its largest value: $f_s = \mu_s n$.

EXECUTE: $\sum F_x = ma_x$ applied to the block *B* gives $f_s = m_B a$. $n = m_B g$ and $f_s = \mu_s m_B g$. $\mu_s m_B g = m_B a$ and $a = \mu_s g$. $\sum F_x = ma_x$ applied to blocks $A + B$ gives $T = m_{AB} a = m_{AB}\mu_s g$. $\sum F_y = ma_y$ applied to block *C* gives $m_C g - T = m_C a$. $m_C g - m_{AB}\mu_s g = m_C \mu_s g$. $m_C = \dfrac{m_{AB}\mu_s}{1-\mu_s} = (5.00\text{ kg} + 8.00\text{ kg})\left(\dfrac{0.750}{1-0.750}\right) = 39.0\text{ kg}$.

EVALUATE: With no friction from the tabletop, the system accelerates no matter how small the mass of C is. If m_C is less than 39.0 kg, the friction force that A exerts on B is less than $\mu_s n$. If m_C is greater than 39.0 kg, blocks C and A have a larger acceleration than friction can give to block B and A accelerates out from under B.

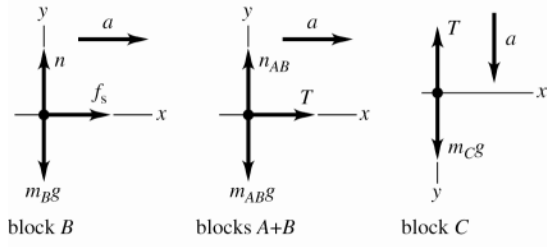


Figure 5.88

5.89. **IDENTIFY:** Apply the method of Exercise 5.19 to calculate the acceleration of each object. Then apply constant acceleration equations to the motion of the 2.00 kg object.

SET UP: After the 5.00 kg object reaches the floor, the 2.00 kg object is in free-fall, with downward acceleration g.

EXECUTE: The 2.00-kg object will accelerate upward at $g\dfrac{5.00\text{ kg}-2.00\text{ kg}}{5.00\text{ kg}+2.00\text{ kg}}=3g/7,$ and the 5.00-kg object will accelerate downward at $3g/7$. Let the initial height above the ground be h_0. When the large object hits the ground, the small object will be at a height $2h_0$, and moving upward with a speed given by $v_0^2=2ah_0=6gh_0/7$. The small object will continue to rise a distance $v_0^2/2g=3h_0/7,$ and so the maximum height reached will be $2h_0+3h_0/7=17h_0/7=1.46\text{ m}$ above the floor , which is 0.860 m above its initial height.

EVALUATE: The small object is 1.20 m above the floor when the large object strikes the floor, and it rises an additional 0.26 m after that.

5.90. **IDENTIFY:** Apply $\sum\vec{F}=m\vec{a}$ to the box.

SET UP: The box has an upward acceleration of $a=1.90\text{ m/s}^2$.

EXECUTE: The floor exerts an upward force n on the box, obtained from $n-mg=ma,$ or $n=m(a+g).$ The friction force that needs to be balanced is

$$\mu_k n=\mu_k m(a+g)\ =(0.32)(28.0\text{ kg})(1.90\ \text{m/s}^2+9.80\ \text{m/s}^2)=105\text{ N}.$$

EVALUATE: If the elevator wasn't accelerating the normal force would be $n=mg$ and the friction force that would have to be overcome would be 87.8 N. The upward acceleration increases the normal force and that increases the friction force.

5.91. **IDENTIFY:** Apply $\sum\vec{F}=m\vec{a}$ to the block. The cart and the block have the same acceleration. The normal force exerted by the cart on the block is perpendicular to the front of the cart, so is horizontal and to the right. The friction force on the block is directed so as to hold the block up against the downward pull of gravity. We want to calculate the minimum a required, so take static friction to have its maximum value, $f_s=\mu_s n$.

SET UP: The free-body diagram for the block is given in Figure 5.91.

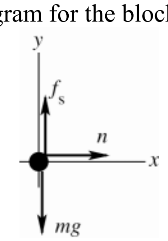


Figure 5.91

EXECUTE:

$\sum F_x=ma_x$

$n=ma$

$f_s=\mu_s n=\mu_s ma$

$\sum F_y=ma_y$

$f_s-mg=0$

$\mu_s ma=mg$

$a=g/\mu_s$

EVALUATE: An observer on the cart sees the block pinned there, with no reason for a horizontal force on it because the block is at rest relative to the cart. Therefore, such an observer concludes that $n=0$ and thus $f_s=0,$ and he doesn't understand what holds the block up against the downward force of gravity. The reason for this difficulty is that $\sum \vec{F} = m\vec{a}$ does not apply in a coordinate frame attached to the cart. This reference frame is accelerated, and hence not inertial. The smaller μ_s is, the larger a must be to keep the block pinned against the front of the cart.

5.92. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block.

SET UP: Use coordinates where $+x$ is directed down the incline.

EXECUTE: **(a)** Since the larger block (the trailing block) has the larger coefficient of friction, it will need to be pulled down the plane; *i.e.*, the larger block will not move faster than the smaller block, and the blocks will have the same acceleration. For the smaller block, $(4.00\text{ kg})g(\sin 30° - (0.25)\cos\ 30°) - T = (4.00\text{ kg})a$, or $11.11\text{ N} - T = (4.00\text{ kg})a$, and similarly for the larger, $15.44\text{ N} + T = (8.00\text{ kg})a$. Adding these two relations, $26.55\text{ N} = (12.00\text{ kg})a$, $a = 2.21\ \text{m/s}^2$.

(b) Substitution into either of the above relations gives $T = 2.27\ \text{N}$.

(c) The string will be slack. The 4.00-kg block will have $a = 2.78\ \text{m/s}^2$ and the 8.00-kg block will have $a = 1.93\ \text{m/s}^2$, until the 4.00-kg block overtakes the 8.00-kg block and collides with it.

EVALUATE: If the string is cut the acceleration of each block will be independent of the mass of that block and will depend only on the slope angle and the coefficient of kinetic friction. The 8.00-kg block would have a smaller acceleration even though it has a larger mass, since it has a larger μ_k.

5.93. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the block and to the plank.

SET UP: Both objects have $a = 0$.

EXECUTE: Let n_B be the normal force between the plank and the block and n_A be the normal force between the block and the incline. Then, $n_B = w\cos\theta$ and $n_A = n_B + 3w\cos\theta = 4w\cos\theta$. The net frictional force on the block is $\mu_k(n_A + n_B) = \mu_k 5w\cos\theta$. To move at constant speed, this must balance the component of the block's weight along the incline, so $3w\sin\theta = \mu_k 5w\cos\theta$, and $\mu_k = \frac{3}{5}\tan\theta = \frac{3}{5}\tan 37° = 0.452.$

EVALUATE: In the absence of the plank the block slides down at constant speed when the slope angle and coefficient of friction are related by $\tan\theta = \mu_k$. For $\theta = 36.9°$, $\mu_k = 0.75$. A smaller μ_k is needed when the plank is present because the plank provides an additional friction force.

5.94. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the ball, to m_1 and to m_2

SET UP: The free-body diagrams for the ball, m_1 and m_2 are given in Figures 5.94a-c. All three objects have the same magnitude of acceleration. In each case take the direction of $\vec{a}$ to be a positive coordinate direction.

EXECUTE: **(a)** $\sum F_y = ma_y$ applied to the ball gives $T\cos\theta = mg$. $\sum F_x = ma_x$ applied to the ball gives $T\sin\theta = ma$. Combining these two equations to eliminate T gives $\tan\theta = a/g$.

(b) $\sum F_x = ma_x$ applied to m_2 gives $T = m_2 a$. $\sum F_y = ma_y$ applied to m_1 gives $m_1 g - T = m_1 a$. Combining these two equations gives $a = \left(\dfrac{m_1}{m_1 + m_2}\right)g$. Then $\tan\theta = \dfrac{m_1}{m_1 + m_2} = \dfrac{250\text{ kg}}{1500\text{ kg}}$ and $\theta = 9.46°$.

(c) As m_1 becomes much larger than m_2, $a \to g$ and $\tan\theta \to 1$, so $\theta \to 45°$.

EVALUATE: The device requires that the ball is at rest relative to the platform; any motion swinging back and forth must be damped out. When $m_1 << m_2$ the system still accelerates, but with small a and $\theta \to 0°$.

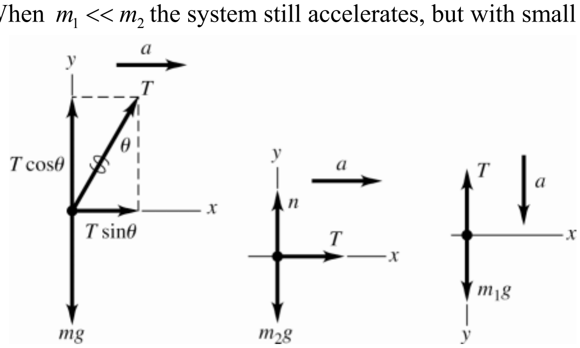


Figure 5.94a–c

5.95. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the automobile.

SET UP: The "correct" banking angle is for zero friction and is given by $\tan\beta = \dfrac{v_0^2}{gR}$, as derived in Example 5.23. Use coordinates that are vertical and horizontal, since the acceleration is horizontal.

EXECUTE: For speeds larger than v_0, a frictional force is needed to keep the car from skidding. In this case, the inward force will consist of a part due to the normal force n and the friction force f; $n\sin\beta + f\cos\beta = ma_{\text{rad}}$. The normal and friction forces both have vertical components; since there is no vertical acceleration, $n\cos\beta - f\sin\beta = mg$. Using $f = \mu_s n$ and $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(1.5v_0)^2}{R} = 2.25\,g\tan\beta$, these two relations become $n\sin\beta + \mu_s n\cos\beta = 2.25\,mg\tan\beta$ and $n\cos\beta - \mu_s n\sin\beta = mg$. Dividing to cancel n gives $\dfrac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta} = 2.25\tan\beta$. Solving for μ_s and simplifying yields $\mu_s = \dfrac{1.25\sin\beta\cos\beta}{1 + 1.25\sin^2\beta}$. Using $\beta = \arctan\left(\dfrac{(20\text{ m/s})^2}{(9.80\text{ m/s}^2)(120\text{ m})}\right) = 18.79°$ gives $\mu_s = 0.34$.

EVALUATE: If μ_s is insufficient, the car skids away from the center of curvature of the roadway, so the friction in inward.

5.96. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the car. The car moves in the arc of a horizontal circle, so $\vec{a} = \vec{a}_{\text{rad}},$ directed toward the center of curvature of the roadway. The target variable is the speed of the car. a_{rad} will be calculated from the forces and then v will be calculated from $a_{\text{rad}} = v^2/R$.

(a) To keep the car from sliding up the banking the static friction force is directed down the incline. At maximum speed the static friction force has its maximum value $f_s = \mu_s n$.

SET UP: The free-body diagram for the car is sketched in Figure 5.96a.

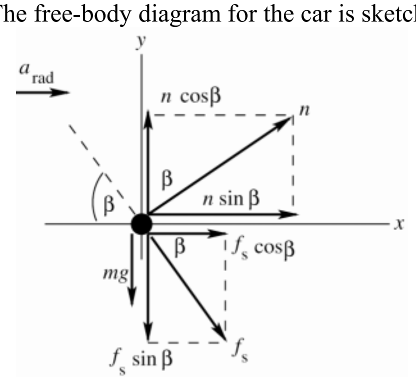


Figure 5.96a

EXECUTE:

$\sum F_y = ma_y$

$n\cos\beta - f_s\sin\beta - mg = 0$

But $f_s = \mu_s n$, so

$n\cos\beta - \mu_s n\sin\beta - mg = 0$

$$n = \frac{mg}{\cos\beta - \mu_s\sin\beta}$$

$\sum F_x = ma_x$

$n\sin\beta + \mu_s n\cos\beta = ma_{\text{rad}}$

$n(\sin\beta + \mu_s\cos\beta) = ma_{\text{rad}}$

Use the $\sum F_y$ equation to replace n:

$$\left(\frac{mg}{\cos\beta - \mu_s\sin\beta}\right)(\sin\beta + \mu_s\cos\beta) = ma_{\text{rad}}$$

$$a_{\text{rad}} = \left(\frac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta}\right)g = \left(\frac{\sin 25° + (0.30)\cos 25°}{\cos 25° - (0.30)\sin 25°}\right)(9.80\text{ m/s}^2) = 8.73\text{ m/s}^2$$

$a_{\text{rad}} = v^2/R$ implies $v = \sqrt{a_{\text{rad}}R} = \sqrt{(8.73\text{ m/s}^2)(50\text{ m})} = 21\text{ m/s}$.

(b) IDENTIFY: To keep the car from sliding *down* the banking the static friction force is directed up the incline. At the minimum speed the static friction force has its maximum value $f_s = \mu_s n$.

SET UP: The free-body diagram for the car is sketched in Figure 5.96b.

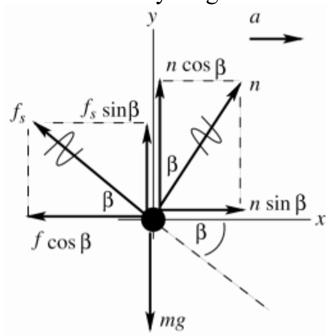


Figure 5.96b

The free-body diagram is identical to that in part (a) except that now the components of f_s have opposite directions. The force equations are all the same except for the opposite sign for terms containing μ_s.

EXECUTE: $a_{rad} = \left(\dfrac{\sin\beta - \mu_s\cos\beta}{\cos\beta + \mu_s\sin\beta}\right)g = \left(\dfrac{\sin 25° - (0.30)\cos 25°}{\cos 25° + (0.30)\sin 25°}\right)(9.80\text{ m/s}^2) = 1.43\text{ m/s}^2$

$v = \sqrt{a_{rad}R} = \sqrt{(1.43\text{ m/s}^2)(50\text{ m})} = 8.5\text{ m/s}.$

EVALUATE: For v between these maximum and minimum values, the car is held on the road at a constant height by a static friction force that is less than $\mu_s n$. When $\mu_s \to 0$, $a_{rad} = g\tan\beta$. Our analysis agrees with the result of Example 5.23 in this special case.

5.97. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the car.

SET UP: $1\text{ mi/h} = 0.447\text{ m/s}$. The acceleration of the car is $a_{rad} = v^2/r$, directed toward the center of curvature of the roadway.

EXECUTE: **(a)** $80\text{ mi/h} = 35.7\text{ m/s}$. The centripetal force needed to keep the car on the road is provided by friction; thus $\mu_s mg = \dfrac{mv^2}{r}$ and $r = \dfrac{v^2}{\mu_s g} = \dfrac{(35.7\text{ m/s})^2}{(0.76)(9.8\text{ m/s}^2)} = 171\text{ m}$.

(b) If $\mu_s = 0.20$,

$$v = \sqrt{r\mu_s g} = \sqrt{(171\text{ m})\,(0.20)\,(9.8\text{ m/s}^2)} = 18.3\text{ m/s or about } 41\text{ mi/h}.$$

(c) If $\mu_s = 0.37$,

$$v = \sqrt{(171\text{ m})\,(0.37)\,(9.8\text{ m/s}^2)} = 24.9\text{ m/s or about } 56\text{ mi/h}$$

The speed limit is evidently designed for these conditions.

EVALUATE: The maximum safe speed is proportional to $\sqrt{\mu_s}$. $\sqrt{0.20/0.76} = 0.51$, so the maximum safe speed for wet-ice conditions is about half what it is for a dry road.

5.98. **IDENTIFY:** The analysis of this problem is the same as that of Example 5.21.

SET UP: From Example 5.21, $\tan\beta = \dfrac{a_{rad}}{g} = \dfrac{v^2}{rg}$.

EXECUTE: Solving for v in terms of β and R, $v = \sqrt{gR\tan\beta} = \sqrt{(9.80\text{ m/s}^2)\,(50.0)\tan 30.0°} = 16.8\text{ m/s}$, about 60.6 km/h.

EVALUATE: The greater the speed of the bus the larger will be the angle β, so T will have a larger horizontal, inward component.

5.99. **IDENTIFY** and **SET UP:** The monkey and bananas have the same mass and the tension in the rope has the same upward value at the bananas and at the monkey. Therefore, the monkey and bananas will have the same net force and hence the same acceleration, in both magnitude and direction.

EXECUTE: **(a)** For the monkey to move up, $T > mg$. The bananas also move up.

(b) The bananas and monkey move with the same acceleration and the distance between them remains constant.

(c) Both the monkey and bananas are in free fall. They have the same initial velocity and as they fall the distance between them doesn't change.

(d) The bananas will slow down at the same rate as the monkey. If the monkey comes to a stop, so will the bananas.

EVALUATE: None of these actions bring the monkey any closer to the bananas.

5.100. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$, with $f = kv$.

SET UP: Follow the analysis that leads to Eq.(5.10), except now the initial speed is $v_{0y} = 3mg/k = 3v_{\text{t}}$ rather than zero.

EXECUTE: The separated equation of motion has a lower limit of $3v_{\text{t}}$ instead of 0; specifically,

$$\int_{3v_{\text{t}}}^{v} \frac{dv}{v - v_{\text{t}}} = \ln\frac{v_{\text{t}} - v}{-2v_{\text{t}}} = \ln\left(\frac{v}{2v_{\text{t}}} - \frac{1}{2}\right) = -\frac{k}{m}t, \text{ or } v = 2v_{\text{t}}\left[\frac{1}{2} + e^{-(k/m)t}\right].$$

EVALUATE: As $t \to \infty$ the speed approaches v_{t}. The speed is always greater than v_{t} and this limit is approached from above.

5.101. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the rock.

SET UP: Equations 5.9 through 5.13 apply, but with a_0 rather than g as the initial acceleration.

EXECUTE: **(a)** The rock is released from rest, and so there is initially no resistive force and $a_0 = (18.0\text{ N})/(3.00\text{ kg}) = 6.00\text{ m/s}^2$.

(b) $(18.0\text{ N} - (2.20\text{ N}\cdot\text{s/m})\ (3.00\text{ m/s}))/(3.00\text{ kg}) = 3.80\text{ m/s}^2$.

(c) The net force must be 1.80 N, so $kv = 16.2\text{ N}$ and $v = (16.2\text{ N})/(2.20\text{ N}\cdot\text{s/m}) = 7.36\text{ m/s}$.

(d) When the net force is equal to zero, and hence the acceleration is zero, $kv_{\text{t}} = 18.0\text{ N}$ and $v_{\text{t}} = (18.0\text{ N})/(2.20\text{ N}\cdot\text{s/m}) = 8.18\text{ m/s}$.

(e) From Eq.(5.12),

$$y = (8.18\text{ m/s})\left[(2.00\text{ s}) - \frac{3.00\text{ kg}}{2.20\text{ N}\cdot\text{s/m}}\left(1 - e^{-((2.20\text{ N}\cdot\text{s/m})/(3.00\text{ kg}))(2.00\text{ s})}\right)\right] = +7.78\text{ m}.$$

From Eq. (5.10), $v = (8.18\text{ m/s})[1 - e^{-((2.20\text{ N}\cdot\text{s/m})/(3.00\text{ kg}))(2.00\text{ s})}] = 6.29\text{ m/s}$.

From Eq.(5.11), but with a_0 instead of g, $a = (6.00\text{ m/s}^2)e^{-((2.20\text{ N}\cdot\text{s/m})/(3.00\text{ kg}))(2.00\text{ s})} = 1.38\text{ m/s}^2$.

(f) $1 - \dfrac{v}{v_{\text{t}}} = 0.1 = e^{-(k/m)t}$ and $t = \dfrac{m}{k}\ln\ (10) = 3.14\text{ s}$.

EVALUATE: The acceleration decreases with time until it becomes zero when $v = v_{\text{t}}$. The speed increases with time and approaches v_{t} as $t \to \infty$.

5.102. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the rock. $a = \dfrac{dv}{dt}$ and $v = \dfrac{dx}{dt}$ yield differential equations that can be integrated to give $v(t)$ and $x(t)$.

SET UP: The retarding force of the surface is the only horizontal force acting.

EXECUTE: **(a)** Thus $a = \dfrac{F_{\text{net}}}{m} = \dfrac{F_R}{m} = \dfrac{-kv^{1/2}}{m} = \dfrac{dv}{dt}$ and $\dfrac{dv}{v^{1/2}} = -\dfrac{k}{m}dt$. Integrating gives $\int_{v_0}^{v}\dfrac{dv}{v^{1/2}} = -\dfrac{k}{m}\int_0^t dt$ and $2v^{1/2}\Big|_{v_0}^{v} = -\dfrac{kt}{m}$. This gives $v = v_0 - \dfrac{v_0^{1/2}kt}{m} + \dfrac{k^2t^2}{4m^2}$.

For the rock's position: $\dfrac{dx}{dt} = v_0 - \dfrac{v_0^{1/2}kt}{m} + \dfrac{k^2t^2}{4m^2}$ and $dx = v_0dt - \dfrac{v_0^{1/2}ktdt}{m} + \dfrac{k^2t^2dt}{4m^2}$.

Integrating gives $x = v_0t - \dfrac{v_0^{1/2}kt^2}{2m} + \dfrac{k^2t^3}{12m^2}$.

(b) $v = 0 = v_0 - \dfrac{v_0^{1/2}kt}{m} + \dfrac{k^2t^2}{2m^2}$. This is a quadratic equation in t; from the quadratic formula we can find the single solution $t = \dfrac{2mv_0^{1/2}}{k}$.

(c) Substituting the expression for t into the equation for x:

$$x = v_0 \cdot \frac{2mv_0^{1/2}}{k} - \frac{v_0^{1/2}k}{2m}\cdot\frac{4m^2v_0}{k^2} + \frac{k^2}{12m^2}\cdot\frac{8m^3v_0^{3/2}}{k^3} = \frac{2mv_0^{3/2}}{3k}$$

EVALUATE: The magnitude of the average acceleration is $a_{\text{av}} = \left|\dfrac{\Delta v}{\Delta t}\right| - \dfrac{v_0}{(2mv_0^{1/2}/k)} = \dfrac{1}{2}\dfrac{kv_0^{1/2}}{m}$. The average force is $F_{\text{av}} = ma_{\text{av}} = \frac{1}{2}kv_0^{1/2}$, which is $\frac{1}{2}$ times the initial value of the force.

5.103. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the object, with and without including the buoyancy force.

SET UP: At the terminal speed v_t, $a = 0$.

EXECUTE: Without buoyancy, $kv_t = mg$, so $k = \dfrac{mg}{v_t} = \dfrac{mg}{0.36\text{ s}}$. With buoyancy included there is the additional upward buoyancy force B, so $B + kv_t = mg$. $B = mg - kv_t = mg\left(1 - \dfrac{0.24\text{ m/s}}{0.36\text{ m/s}}\right) = mg/3$.

EVALUATE: At the terminal speed, B and $f = kv$ together equal mg. The presence of B reduces the value of f required, so the presence of B reduces the terminal speed.

5.104. **IDENTIFY:** The block has acceleration $a_{rad} = v^2/r$, directed to the left in the figure in the problem. Apply $\sum \vec{F} = m\vec{a}$ to the block.

SET UP: The block moves in a horizontal circle of radius $r = \sqrt{(1.25\text{ m})^2 - (1.00\text{ m})^2} = 0.75\text{ m}$. Each string makes an angle θ with the vertical. $\cos\theta = \dfrac{1.00\text{ m}}{1.25\text{ m}}$, so $\theta = 36.9°$. The free-body diagram for the block is given in Figure 5.104. Let $+x$ be to the left and let $+y$ be upward.

EXECUTE: **(a)** $\sum F_y = ma_y$ gives $T_u\cos\theta - T_l\cos\theta - mg = 0$.

$$T_l = T_u - \frac{mg}{\cos\theta} = 80.0\text{ N} - \frac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{\cos 36.9°} = 31.0\text{ N}.$$

(b) $\sum F_x = ma_x$ gives $(T_u + T_l)\sin\theta = m\dfrac{v^2}{r}$.

$v = \sqrt{\dfrac{r(T_u + T_l)\sin\theta}{m}} = \sqrt{\dfrac{(0.75\text{ m})(80.0\text{ N} + 31.0\text{ N})\sin 36.9°}{4.00\text{ kg}}} = 3.53\text{ m/s}$. The number of revolutions per second is

$$\frac{v}{2\pi r} = \frac{3.53\text{ m/s}}{2\pi(0.75\text{ m})} = 0.749\text{ rev/s} = 44.9\text{ rev/min}.$$

(c) If $T_l \to 0$, $T_u\cos\theta = mg$ and $T_u = \dfrac{mg}{\cos\theta} = \dfrac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{\cos 36.9°} = 49.0\text{ N}$. $T_u\sin\theta = m\dfrac{v^2}{r}$.

$v = \sqrt{\dfrac{rT_u\sin\theta}{m}} = \sqrt{\dfrac{(0.75\text{ m})(49.0\text{ N})\sin 36.9°}{4.00\text{ kg}}} = 2.35\text{ m/s}$. The number of revolutions per minute is

$$(44.9\text{ rev/min})\left(\frac{2.35\text{ m/s}}{3.53\text{ m/s}}\right) = 29.9\text{ rev/min}$$

EVALUATE: The tension in the upper string must be greater than the tension in the lower string so that together they produce an upward component of force that balances the weight of the block.

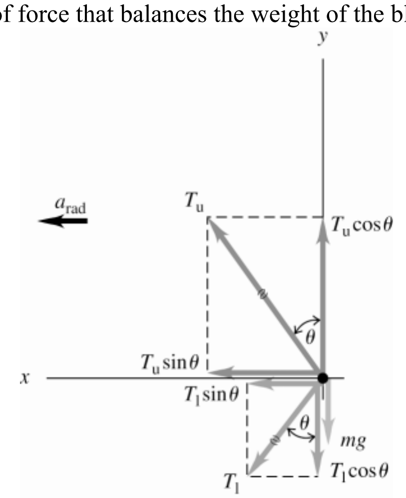


Figure 5.104

5.105. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the falling object.

SET UP: Follow the steps that lead to Eq.(5.10), except now $v_{0y} = v_0$ and is not zero.

EXECUTE: **(a)** Newton's 2nd law gives $m\dfrac{dv_y}{dt} = mg - kv_y$, where $\dfrac{mg}{k} = v_t$. $\displaystyle\int_{v_0}^{v_y} \frac{dv_y}{v_y - v_t} = -\frac{k}{m}\int_0^t dt$. This is the same expression used in the derivation of Eq. (5.10), except the lower limit in the velocity integral is the initial speed v_0 instead of zero. Evaluating the integrals and rearranging gives $v = v_0 e^{-kt/m} + v_t(1 - e^{-kt/m})$. Note that at $t = 0$ this expression says $v_y = v_0$ and at $t \to \alpha$ it says $v_y \to v_t$.

(b) The downward gravity force is larger than the upward fluid resistance force so the acceleration is downward, until the fluid resistance force equals gravity when the terminal speed is reached. The object speeds up until $v_y = v_t$. Take $+y$ to be downward. The graph is sketched in Figure 5.105a.

(c) The upward resistance force is larger than the downward gravity force so the acceleration is upward and the object slows down, until the fluid resistance force equals gravity when the terminal speed is reached. Take $+y$ to be downward. The graph is sketched in Figure 5.105b.

(d) When $v_0 = v_t$ the acceleration at $t = 0$ is zero and remains zero; the velocity is constant and equal to the terminal velocity.

EVALUATE: In all cases the speed becomes v_t as $t \to \infty$.

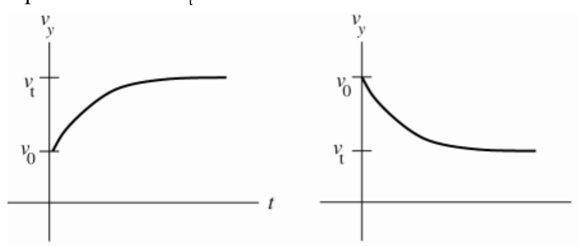


Figure 5.105a, b

5.106. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the rock.

SET UP: At the maximum height, $v_y = 0$. Let $+y$ be upward. Suppress the *y* subscripts on *v* and *a*.

EXECUTE: **(a)** To find the maximum height and time to the top without fluid resistance: $v^2 = v_0^2 + 2a(y - y_0)$ and

$$y - y_0 = \frac{v^2 - v_0^2}{2a} = \frac{0 - (6.0 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = 1.84 \text{ m}. \quad t = \frac{v - v_0}{a} = \frac{0 - 6.0 \text{ m/s}}{-9.8 \text{ m/s}^2} = 0.61 \text{ s}.$$

(b) Starting from Newton's Second Law for this situation $m\dfrac{dv}{dt} = mg - kv$. We rearrange and integrate, taking downward as positive as in the text and noting that the velocity at the top of the rock's flight is zero:

$$\int_v^0 \frac{dv}{v - v_t} = -\frac{k}{m}t. \quad \ln(v - v_t)\Big|_v^0 = \ln\frac{-v_t}{v - v_t} = \ln\frac{-2.0 \text{ m/s}}{-6.0 \text{ m/s} - 2.0 \text{ m/s}} = \ln(0.25) = -1.386$$

From Eq.(5.9), $m/k = v_t/g = (2.0 \text{ m/s}^2)/(9.8 \text{ m/s}^2) = 0.204$ s, and $t = -\frac{m}{k}(-1.386) = (0.204 \text{ s})(1.386) = 0.283$ s to the top. Equation 5.10 in the text gives us $\dfrac{dx}{dt} = v_t(1 - e^{-(k/m)t}) = v_t - v_t e^{-(k/m)t}$.

$$x = \int_0^x dx = \int_0^t v_t dt - \int_0^t v_t e^{-(k/m)t} dt = v_t t + \frac{v_t m}{k}(e^{-(k/m)t} - 1).$$

$$x = (2.0 \text{ m/s})(0.283 \text{ s}) + (2.0 \text{ m/s})(0.204 \text{ s})(e^{-1.387} - 1) = 0.26 \text{ m}.$$

EVALUATE: With fluid resistance present the maximum height is much less and the time to reach it is less.

5.107. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the car.

SET UP: The forces on the car are the air drag force $f_D = Dv^2$ and the rolling friction force $\mu_r mg$. Take the velocity to be in the $+x$-direction. The forces are opposite in direction to the velocity.

EXECUTE: **(a)** $\sum F_x = ma_x$ gives $-Dv^2 - \mu_r mg = ma$. We can write this equation twice, once with $v = 32$ m/s and $a = -0.42 \text{ m/s}^2$ and once with $v = 24$ m/s and $a = -0.30 \text{ m/s}^2$. Solving these two simultaneous equations in the unknowns *D* and μ_r gives $\mu_r = 0.015$ and $D = 0.36 \text{ N}\cdot\text{s}^2/\text{m}^2$.

(b) $n = mg\cos\beta$ and the component of gravity parallel to the incline is $mg\sin\beta$, where $\beta = 2.2°$. For constant speed, $mg\sin 2.2° - \mu_r mg\cos 2.2° - Dv^2 = 0$. Solving for v gives $v = 29$ m/s.

(c) For angle β, $mg\sin\beta - \mu_r mg\cos\beta - Dv^2 = 0$ and $v = \sqrt{\dfrac{mg(\sin\beta - \mu_r\cos\beta)}{D}}$. The terminal speed for a falling object is derived from $Dv_t^2 - mg = 0$, so $v_t = \sqrt{mg/D}$. $v/v_t = \sqrt{\sin\beta - \mu_r\cos\beta}$. And since $\mu_r = 0.015$, $v/v_t = \sqrt{\sin\beta - (0.015)\cos\beta}$.

EVALUATE: In part (c), $v \to v_t$ as $\beta \to 90°$, since in that limit the incline becomes vertical.

5.108. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the person and to the cart.

SET UP: The apparent weight, w_{app}, which is the same as the upward force on the person exerted by the car seat.

EXECUTE: **(a)** The apparent weight is the actual weight of the person minus the centripetal force needed to keep him moving in its circular path**:**

$$w_{app} = mg - \frac{mv^2}{R} = (70\text{ kg})\left[(9.8\text{ m/s}^2) - \frac{(12\text{ m/s})^2}{40\text{ m}}\right] = 434\text{ N}.$$

(b) The cart will lose contact with the surface when its apparent weight is zero; i.e., when the road no longer has to exert any upward force on it**:** $mg - \dfrac{mv^2}{R} = 0$. $v = \sqrt{Rg} = \sqrt{(40\text{ m})(9.8\text{ m/s}^2)} = 19.8\text{ m/s}$**.** The answer doesn't depend on the cart's mass, because the centripetal force needed to hold it on the road is proportional to its mass and so to its weight, which provides the centripetal force in this situation.

EVALUATE: At the speed calculated in part (b), the downward force needed for circular motion is provided by gravity. For speeds greater than this more, downward force is needed and there is no source for it and the cart leaves the circular path. For speeds less than this, less downward force than gravity is needed, so the roadway must exert an upward vertical force.

5.109. **(a) IDENTIFY:** Use the information given about Jena to find the time t for one revolution of the merry-go-round. Her acceleration is a_{rad}, directed in toward the axis. Let $\vec{F}_1$ be the horizontal force that keeps her from sliding off. Let her speed be v_1 and let R_1 be her distance from the axis. Apply $\sum \vec{F} = m\vec{a}$ to Jena, who moves in uniform circular motion.

SET UP: The free-body diagram for Jena is sketched in Figure 5.109a

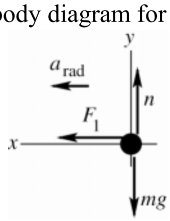


Figure 5.109a

EXECUTE:

$\sum F_x = ma_x$

$F_1 = ma_{rad}$

$F_1 = m\dfrac{v_1^2}{R_1}$, $v_1 = \sqrt{\dfrac{R_1F_1}{m}} = 1.90\text{ m/s}$

The time for one revolution is $t = \dfrac{2\pi R_1}{v_1} = 2\pi R_1\sqrt{\dfrac{m}{R_1F_1}}$. Jackie goes around once in the same time but her speed (v_2) and the radius of her circular path (R_2) are different.

$$v_2 = \frac{2\pi R_2}{t} = 2\pi R_2\left(\frac{1}{2\pi R_1}\right)\sqrt{\frac{R_1F_1}{m}} = \frac{R_2}{R_1}\sqrt{\frac{R_1F_1}{m}}.$$

IDENTIFY: Now apply $\sum \vec{F} = m\vec{a}$ to Jackie. She also moves in uniform circular motion.

SET UP: The free-body diagram for Jackie is sketched in Figure 5.109b.

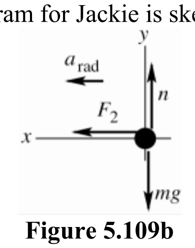


Figure 5.109b

EXECUTE:

$\sum F_x = ma_x$

$F_2 = ma_{rad}$

$$F_2 = m\frac{v_2^2}{R_2} = \left(\frac{m}{R_2}\right)\left(\frac{R_2^2}{R_1^2}\right)\left(\frac{R_1F_1}{m}\right) = \left(\frac{R_2}{R_1}\right)F_1 = \left(\frac{3.60\text{ m}}{1.80\text{ m}}\right)(60.0\text{ N}) = 120.0\text{ N}$$

(b) $F_2 = m\dfrac{v_2^2}{R_2}$, so $v_2 = \sqrt{\dfrac{F_2R_2}{m}} = \sqrt{\dfrac{(120.0\text{ N})(3.60\text{ m})}{30.0\text{ kg}}} = 3.79\text{ m/s}$

EVALUATE: Both girls rotate together so have the same period T. By Eq.(5.16), a_{rad} is larger for Jackie so the force on her is larger. Eq.(5.15) says $R_1/v_1 = R_2/v_2$ so $v_2 = v_1(R_2/R_1)$; this agrees with our result in (a).

5.110. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the passenger. The passenger has acceleration a_{rad}, directed inward toward the center of the circular path.

SET UP: The passenger's velocity is $v = 2\pi R/t = 8.80$ m/s. The vertical component of the seat's force must balance the passenger's weight and the horizontal component must provide the centripetal force.

EXECUTE: **(a)** $F_{seat}\sin\theta = mg = 833\text{ N}$ and $F_{seat}\cos\theta = \dfrac{mv^2}{R} = 188\text{ N}$. Therefore

$\tan\theta = (833\text{ N})/(188\text{ N}) = 4.43$; $\theta = 77.3°$ above the horizontal. The magnitude of the net force exerted by the seat (note that this is not the net force on the passenger) is

$$F_{seat} = \sqrt{(833\text{ N})^2 + (188\text{ N})^2} = 854\text{ N}$$

(b) The magnitude of the force is the same, but the horizontal component is reversed.

EVALUATE: At the highest point in the motion, $F_{seat} = mg - m\dfrac{v^2}{R} = 645\text{ N}$. At the lowest point in the motion, $F_{seat} = mg + m\dfrac{v^2}{R} = 1021\text{ N}$. The result in parts (a) and (b) lies between these extreme values.

5.111. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the person. The person moves in a horizontal circle so his acceleration is $a_{rad} = v^2/R$, directed toward the center of the circle. The target variable is the coefficient of static friction between the person and the surface of the cylinder. $v = (0.60\text{ rev/s})\left(\dfrac{2\pi R}{1\text{ rev}}\right) = (0.60\text{ rev/s})\left(\dfrac{2\pi(2.5\text{ m})}{1\text{ rev}}\right) = 9.425\text{ m/s}$

(a) SET UP: The problem situation is sketched in Figure 5.111a.

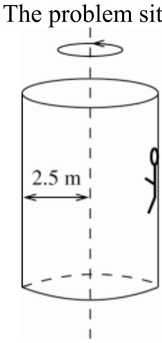


Figure 5.111a

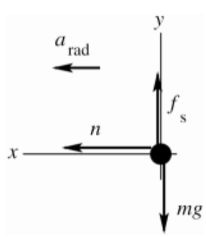


Figure 5.111b

The free-body diagram for the person is sketched in Figure 5.111b.
The person is held up against gravity by the static friction force exerted on him by the wall. The acceleration of the person is a_{rad}, directed in towards the axis of rotation.

(b) EXECUTE: To calculate the minimum μ_s required, take f_s to have its maximum value, $f_s = \mu_s n$.

$\sum F_y = ma_y$

$f_s - mg = 0$

$\mu_s n = mg$

$\sum F_x = ma_x$

$n = mv^2/R$

Combine these two equations to eliminate n:

$\mu_s mv^2/R = mg$

$$\mu_s = \frac{Rg}{v^2} = \frac{(2.5\text{ m})(9.80\text{ m/s}^2)}{(9.425\text{ m/s})^2} = 0.28$$

(c) EVALUATE: No, the mass of the person divided out of the equation for μ_s. Also, the smaller μ_s is, the larger v must be to keep the person from sliding down. For smaller μ_s the cylinder must rotate faster to make n larger enough.

5.112. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the combined object of motorcycle plus rider.

SET UP: The object has acceleration $a_{rad} = v^2/r$, directed toward the center of the circular path.

EXECUTE: **(a)** For the tires not to lose contact, there must be a downward force on the tires. Thus, the (downward) acceleration at the top of the sphere must exceed mg, so $m\frac{v^2}{R} > mg$, and

$v > \sqrt{gR} = \sqrt{(9.80\ \text{m/s}^2)\ (13.0\ \text{m})} = 11.3\ \text{m/s}.$

(b) The (upward) acceleration will then be $4g$, so the upward normal force must be $5mg = 5(110\ \text{kg})\ (9.80\ \text{m/s}^2) = 5390\ \ \text{N}.$

EVALUATE: At any nonzero speed the normal force at the bottom of the path exceeds the weight of the object.

5.113. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to your friend. Your friend moves in the arc of a circle as the car turns.

(a) Turn to the right. The situation is sketched in Figure 5.113a.

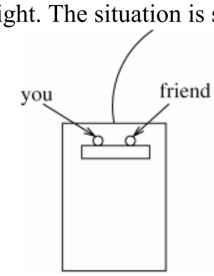


Figure 5.113a

As viewed in an inertial frame, in the absence of sufficient friction your friend doesn't make the turn completely and you move to the right toward your friend.

(b) The maximum radius of the turn is the one that makes a_{rad} just equal to the maximum acceleration that static friction can give to your friend, and for this situation f_s has its maximum value $f_s = \mu_s n.$

SET UP: The free-body diagram for your friend, as viewed by someone standing behind the car, is sketched in Figure 5.113b.

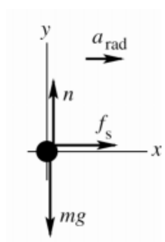


Figure 5.113b

EXECUTE:

$\sum F_y = ma_y$

$n - mg = 0$

$n = mg$

$\sum F_x = ma_x$

$f_s = ma_{rad}$

$\mu_s n = mv^2/R$

$\mu_s mg = mv^2/R$

$$R = \frac{v^2}{\mu_s g} = \frac{(20\ \text{m/s})^2}{(0.35)(9.80\ \text{m/s}^2)} = 120\ \text{m}$$

EVALUATE: The larger μ_s is, the smaller the radius R must be.

5.114. **IDENTIFY:** The tension F in the string must be the same as the weight of the hanging block, and must also provide the resultant force necessary to keep the block on the table in uniform circular motion.

SET UP: The acceleration of the block is $a_{rad} = v^2/r$, directed toward the hole.

EXECUTE: $Mg = F = m\frac{v^2}{r}$, so $v = \sqrt{grM/m}.$

EVALUATE: The larger M is the greater must be the speed v, if r remains the same.

5.115. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the circular motion of the bead. Also use Eq.(5.16) to relate a_{rad} to the period of rotation T.

SET UP: The bead and hoop are sketched in Figure 5.115a.

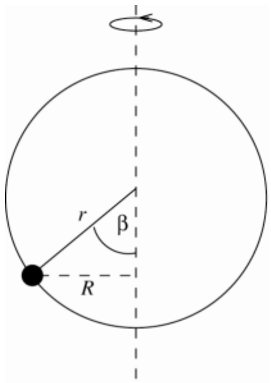


Figure 5.115a

The bead moves in a circle of radius $R = r\sin\beta$.
The normal force exerted on the bead by the hoop is radially inward.

The free-body diagram for the bead is sketched in Figure 5.115b.

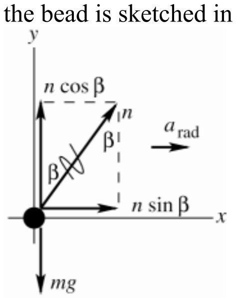


Figure 5.115b

EXECUTE:

$\sum F_y = ma_y$

$n\cos\beta - mg = 0$

$n = mg/\cos\beta$

$\sum F_x = ma_x$

$n\sin\beta = ma_{\text{rad}}$

Combine these two equations to eliminate n:

$$\left(\frac{mg}{\cos\beta}\right)\sin\beta = ma_{\text{rad}}$$

$$\frac{\sin\beta}{\cos\beta} = \frac{a_{\text{rad}}}{g}$$

$a_{\text{rad}} = v^2/R$ and $v = 2\pi R/T$, so $a_{\text{rad}} = 4\pi^2 R/T^2$, where T is the time for one revolution.

$R = r\sin\beta$, so $a_{\text{rad}} = \dfrac{4\pi^2 r\sin\beta}{T^2}$

Use this in the above equation: $\dfrac{\sin\beta}{\cos\beta} = \dfrac{4\pi^2 r\sin\beta}{T^2 g}$

This equation is satisfied by $\sin\beta = 0$, so $\beta = 0$, or by

$\dfrac{1}{\cos\beta} = \dfrac{4\pi^2 r}{T^2 g}$, which gives $\cos\beta = \dfrac{T^2 g}{4\pi^2 r}$

(a) 4.00 rev/s implies $T = (1/4.00)\ \text{s} = 0.250\ \text{s}$

Then $\cos\beta = \dfrac{(0.250\ \text{s})^2(9.80\ \text{m/s}^2)}{4\pi^2(0.100\ \text{m})}$ and $\beta = 81.1°$.

(b) This would mean $\beta = 90°$. But $\cos 90° = 0$, so this requires $T \to 0$. So β approaches $90°$ as the hoop rotates very fast, but $\beta = 90°$ is not possible.

(c) 1.00 rev/s implies $T = 1.00\ \text{s}$

The $\cos\beta = \dfrac{T^2 g}{4\pi^2 r}$ equation then says $\cos\beta = \dfrac{(1.00\ \text{s})^2(9.80\ \text{m/s}^2)}{4\pi^2(0.100\ \text{m})} = 2.48$, which is not possible. The only way to have the $\sum \vec{F} = m\vec{a}$ equations satisfied is for $\sin\beta = 0$. This means $\beta = 0$; the bead sits at the bottom of the hoop.

EVALUATE: $\beta \to 90°$ as $T \to 0$ (hoop moves faster). The largest value T can have is given by $T^2 g/(4\pi^2 r) = 1$ so $T = 2\pi\sqrt{r/g} = 0.635$ s. This corresponds to a rotation rate of $(1/0.635)$ rev/s $= 1.58$ rev/s. For a rotation rate less than 1.58 rev/s, $\beta = 0$ is the only solution and the bead sits at the bottom of the hoop. Part (c) is an example of this.

5.116. **IDENTIFY:** $a_x = \dfrac{d^2x}{dt^2}$ and $a_y = \dfrac{d^2y}{dt^2}$. Then apply $\sum \vec{F} = m\vec{a}$ to calculate the components of the net force.

SET UP: The components of $\vec{F}$ determine its magnitude and direction.

EXECUTE: **(a)** Differentiating twice, $a_x = -6\beta t$ and $a_y = -2\delta$, so

$F_x = ma_x = (2.20\text{ kg})(-0.72\text{ N/s})t = -(1.58\text{ N/s})t$ and $F_y = ma_y = (2.20\text{ kg})(-2.00\text{ m/s}^2) = -4.40\text{ N}$.

(b) The graph is given in Figure 5.116.

(c) At $t = 3.00$ s, $F_x = -4.75$ N and $F_y = -4.40$ N, so $F = \sqrt{(-4.75\text{ N})^2 + (-4.40\text{ N})^2} = 6.48$ N at an angle of $\arctan\left(\dfrac{-4.40}{-4.75}\right) = 223°$.

EVALUATE: F_y is constant and negative. F_x is zero at $t = 0$ and becomes increasingly more negative as t increases.

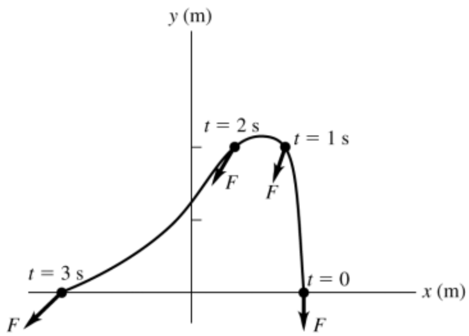


Figure 5.116

5.117. **IDENTIFY:** The velocity is tangent to the path. The acceleration has a tangential component when the speed is changing and a radial component when the path is curving.

SET UP: $\vec{a}_{\text{rad}}$ is toward the center of curvature of the path. $\vec{a}_{\tan}$ is parallel to $\vec{v}$ when the speed is increasing and antiparallel to $\vec{v}$ when the speed is decreasing. The net force $\vec{F}$ is proportional to $\vec{a}$.

EXECUTE: The diagram is sketched in Figure 5.117.

EVALUATE: $\vec{v}$, $\vec{a}$, and $\vec{F}$ all change during the motion.

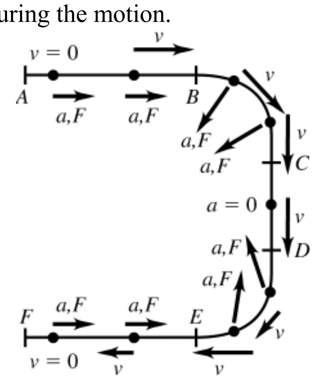


Figure 5.117

5.118. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the car. It has acceleration $\vec{a}_{\text{rad}}$, directed toward the center of the circular path.

SET UP: The analysis is the same as in Example 5.24.

EXECUTE: **(a)** $F_A = m\left(g + \dfrac{v^2}{R}\right) = (1.60\text{ kg})\left(9.80\text{ m/s}^2 + \dfrac{(12.0\text{ m/s})^2}{5.00\text{ m}}\right) = 61.8\text{ N}.$

(b) $F_B = m\left(g - \dfrac{v^2}{R}\right) = (1.60\text{ kg})\left(9.80\text{ m/s}^2 - \dfrac{(12.0\text{ m/s})^2}{5.00\text{ m}}\right) = -30.4\text{ N}.$, where the minus sign indicates that the track pushes *down* on the car. The magnitude of this force is 30.4 N.

EVALUATE: $|F_A| > |F_B|$. $|F_A| - 2mg$.

5.119. **IDENTIFY:** The analysis is the same as for Problem 5.96.

SET UP: The speed is related to the period by $v = 2\pi R/T = 2\pi h(\tan\beta)/T$, or $T = 2\pi h(\tan\beta)/v$.

EXECUTE: The maximum and minimum speeds are the same as those found in Problem 5.96,

$$v_{\text{max}} = \sqrt{gh\tan\beta\frac{\cos\beta + \mu_s\sin\beta}{\sin\beta - \mu_s\cos\beta}} \text{ and } v_{\text{min}} = \sqrt{gh\tan\beta\frac{\cos\beta - \mu_s\sin\beta}{\sin\beta + \mu_s\cos\beta}}.$$

The minimum and maximum values of the period T are then

$$T_{\text{min}} = 2\pi\sqrt{\frac{h\tan\beta}{g}\frac{\sin\beta - \mu_s\cos\beta}{\cos\beta + \mu_s\sin\beta}} \text{ and } T_{\text{max}} = 2\pi\sqrt{\frac{h\tan\beta}{g}\frac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta}}.$$

EVALUATE: For $\mu_s = 0$ the results for the speeds reduce to $v_{\text{min}} = v_{\text{max}} = \sqrt{gh}$. $h = \dfrac{R}{\tan\beta}$. The result for v then agrees with the result in Example 5.23, if we take into account that in this problem β is measured from the vertical whereas in Example 5.23 it is measured relative to the horizontal.

5.120. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to the block and to the wedge.

SET UP: For both parts, take the x-direction to be horizontal and positive to the right, and the y-direction to be vertical and positive upward. The normal force between the block and the wedge is n; the normal force between the wedge and the horizontal surface will not enter, as the wedge is presumed to have zero vertical acceleration. The horizontal acceleration of the wedge is A, and the components of acceleration of the block are a_x and a_y.

EXECUTE: **(a)** The equations of motion are then $MA = -n\sin\alpha$, $ma_x = n\sin\alpha$ and $ma_y = n\cos\alpha - mg$. Note that the normal force gives the wedge a negative acceleration; the wedge is expected to move to the left. These are three equations in four unknowns, A, a_x, a_y and n. Solution is possible with the imposition of the relation between A, a_x and a_y. An observer on the wedge is not in an inertial frame, and should not apply Newton's laws, but the kinematic relation between the components of acceleration are not so restricted. To such an observer, the vertical acceleration of the block is a_y, but the horizontal acceleration of the block is $a_x - A$. To this observer, the block descends at an angle α, so the relation needed is $\dfrac{a_y}{a_x - A} = -\tan\alpha$. At this point, algebra is unavoidable. A possible approach is to eliminate a_x by noting that $a_x = -\dfrac{M}{m}A$, using this in the kinematic constraint to eliminate a_y and then eliminating n. The results are:

$$A = \frac{-gm}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_x = \frac{gM}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_y = \frac{-g(M+\text{m})\tan\alpha}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

(b) When $M >> m, A \to 0$, as expected (the large block won't move). Also,

$a_x \to \dfrac{g}{\tan\alpha + (1/\tan\alpha)} = g\dfrac{\tan\alpha}{\tan^2\alpha + 1} = g\sin\alpha\cos\alpha$ which is the acceleration of the block ($g\sin\alpha$ in this case), with the factor of $\cos\alpha$ giving the horizontal component. Similarly, $a_y \to -g\sin^2\alpha$.

(c) The trajectory is a spiral.

EVALUATE: If $m >> M$, our general results give $a_x = 0$ and $a_y = -g$. The massive block accelerates straight downward, as if it were in free-fall.

5.121. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to the block and to the wedge.

SET UP: From Problem 5.120, $ma_x = n\sin\alpha$ and $ma_y = n\cos\alpha - mg$ for the block. $a_y = 0$ gives $a_x = g\tan\alpha$.

EXECUTE: If the block is not to move vertically, both the block and the wedge have this horizontal acceleration and the applied force must be $F = (M+m)a = (M+m)g\tan\alpha$.

EVALUATE: $F \to 0$ as $\alpha \to 0$ and $F \to \infty$ as $\alpha \to \infty$.

5.122. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$.

SET UP: Let $+x$ be directed up the ramp.

EXECUTE: The normal force that the ramp exerts on the box will be $n = w\cos\alpha - T\sin\alpha$. The rope provides a force of $T\cos\theta$ up the ramp, and the component of the weight down the ramp is $w\sin\alpha$. Thus, the net force up the ramp is

$$F = T\cos\theta - w\sin\alpha - \mu_k(w\cos\alpha - T\sin\theta) = T(\cos\theta + \mu_k\sin\theta) - w(\sin\alpha + \mu_k\cos\alpha)$$

The acceleration will be the greatest when the first term in parentheses is greatest and this occurs when $\tan\theta = \mu_k$.

EVALUATE: Small θ means F is more nearly in the direction of the motion. But $\theta \to 90°$ means F is directed to reduce the normal force and thereby reduce friction. The optimum value of θ is somewhere in between and depends on μ_k. When $\mu_k = 0$, the optimum value of θ is $\theta = 0°$.

5.123. **IDENTIFY:** Use the results of Problem 5.44.

SET UP: $f(x)$ is a minimum when $\dfrac{df}{dx} = 0$ and $\dfrac{d^2 f}{dx^2} > 0$.

EXECUTE: **(a)** $F = \mu_k w/(\cos\theta + \mu_k\sin\theta)$

(b) The graph of F versus θ is given in Figure 5.123.

(c) F is minimized at $\tan\theta = \mu_k$. For $\mu_k = 0.25$, $\theta = 14.0°$.

EVALUATE: Small θ means F is more nearly in the direction of the motion. But $\theta \to 90°$ means F is directed to reduce the normal force and thereby reduce friction. The optimum value of θ is somewhere in between and depends on μ_k.

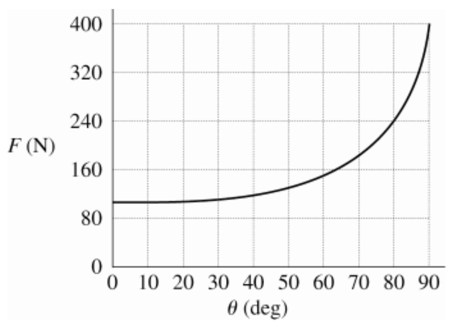


Figure 5.123

5.124. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the ball. At the terminal speed, $a = 0$.

SET UP: For convenience, take the positive direction to be down, so that for the baseball released from rest, the acceleration and velocity will be positive, and the speed of the baseball is the same as its positive component of velocity. Then the resisting force, directed against the velocity, is upward and hence negative.

EXECUTE: **(a)** The free-body diagram for the falling ball is sketched in Figure 5.124.

(b) Newton's Second Law is then $ma = mg - Dv^2$. Initially, when $v = 0$, the acceleration is g, and the speed increases. As the speed increases, the resistive force increases and hence the acceleration decreases. This continues as the speed approaches the terminal speed.

(c) At terminal velocity, $a = 0$, so $v_t = \sqrt{\dfrac{mg}{D}}$ in agreement with Eq. (5.13).

(d) The equation of motion may be rewritten as $\dfrac{dv}{dt} = \dfrac{g}{v_t^2}(v_t^2 - v^2)$. This is a separable equation and may be expressed as $\displaystyle\int \frac{dv}{v_t^2 - v^2} = \frac{g}{v_t^2}\int dt$ or $\dfrac{1}{v_t}\,\text{arctanh}\left(\dfrac{v}{v_t}\right) = \dfrac{gt}{v_t^2}$. $v = v_t\,\tanh(gt/v_t)$.

EVALUATE: $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$. At $t \to 0$, $\tanh(gt/v_t) \to 0$ and $v \to 0$. At $t \to \infty$, $\tanh(gt/v_t) \to \infty$ and $v \to v_t$.

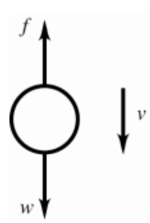


Figure 5.124

5.125. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each of the three masses and to the pulley B.

SET UP: Take all accelerations to be positive downward. The equations of motion are straightforward, but the kinematic relations between the accelerations, and the resultant algebra, are not immediately obvious. If the acceleration of pulley B is a_B, then $a_B = -a_3$, and a_B is the average of the accelerations of masses 1 and 2, or $a_1 + a_2 = 2a_B = -2a_3$.

EXECUTE: **(a)** There can be no net force on the massless pulley B, so $T_C = 2T_A$. The five equations to be solved are then $m_1 g - T_A = m_1 a_1$, $m_2 g - T_A = m_2 a_2$, $m_3 g - T_C = m_3 a_3$, $a_1 + a_2 + 2a_3 = 0$ and $2T_A - T_C = 0$. These are five equations in five unknowns, and may be solved by standard means.

The accelerations a_1 and a_2 may be eliminated by using $2a_3 = -(a_1 + a_2) = -(2g - T_A((1/m_1) + (1/m_2)))$.

The tension T_A may be eliminated by using $T_A = (1/2)T_C = (1/2)m_3(g - a_3)$.

Combining and solving for a_3 gives $a_3 = g\dfrac{-4m_1m_2 + m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$.

(b) The acceleration of the pulley B has the same magnitude as a_3 and is in the opposite direction.

(c) $a_1 = g - \dfrac{T_A}{m_1} = g - \dfrac{T_C}{2m_1} = g - \dfrac{m_3}{2m_1}(g - a_3)$. Substituting the above expression for a_3 gives

$a_1 = g\dfrac{4m_1m_2 - 3m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$.

(d) A similar analysis (or, interchanging the labels 1 and 2) gives $a_2 = g\dfrac{4m_1m_2 - 3m_1m_3 + m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$.

(e), **(f)** Once the accelerations are known, the tensions may be found by substitution into the appropriate equation of motion, giving $T_A = g\dfrac{4m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$, $T_C = g\dfrac{8m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$.

(g) If $m_1 = m_2 = m$ and $m_3 = 2m$, all of the accelerations are zero, $T_C = 2mg$ and $T_A = mg$. All masses and pulleys are in equilibrium, and the tensions are equal to the weights they support, which is what is expected.

EVALUATE: It is useful to consider special cases. For example, when $m_1 = m_2 >> m_3$ our general result gives $a_1 = a_2 = +g$ and $a_3 = -g$.

5.126. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. The tension in the string is the same at both ends. If $T < w$ for a block, that block remains at rest.

SET UP: In all cases, the tension in the string will be half of F.

EXECUTE: **(a)** $F/2 = 62$ N, which is insufficient to raise either block; $a_1 = a_2 = 0$.

(b) $F/2 = 62$ N. The larger block (of weight 196 N) will not move, so $a_1 = 0$, but the smaller block, of weight 98 N, has a net upward force of 49 N applied to it, and so will accelerate upwards with $a_2 = \dfrac{49 \text{ N}}{10.0 \text{ kg}} = 4.9 \text{ m/s}^2$.

(c) $F/2 = 212$ N, so the net upward force on block A is 16 N and that on block B is 114 N, so

$a_1 = \dfrac{16 \text{ N}}{20.0 \text{ kg}} = 0.8 \text{ m/s}^2$ and $a_2 = \dfrac{114 \text{ N}}{10.0 \text{ kg}} = 11.4 \text{ m/s}^2$.

EVALUATE: The two blocks need not have accelerations with the same magnitudes.

5.127. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the ball at each position.

SET UP: When the ball is at rest, $a = 0$. When the ball is swinging in an arc it has acceleration component $a_{\text{rad}} = \dfrac{v^2}{R}$, directed inward.

EXECUTE: Before the horizontal string is cut, the ball is in equilibrium, and the vertical component of the tension force must balance the weight, so $T_A \cos\beta = w$ or $T_A = w/\cos\beta$. At point B, the ball is not in equilibrium; its speed is instantaneously 0, so there is no radial acceleration, and the tension force must balance the radial component of the weight, so $T_B = w\cos\beta$ and the ratio $(T_B/T_A) = \cos^2\beta$.

EVALUATE: At point B the net force on the ball is not zero; the ball has a tangential acceleration.