3

MOTION IN TWO OR THREE DIMENSIONS

3.1. **IDENTIFY** and **SET UP:** Use Eq.(3.2), in component form.

EXECUTE: $(v_{\text{av}})_x = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{5.3\text{ m} - 1.1\text{ m}}{3.0\text{ s} - 0} = 1.4\text{ m/s}$

$(v_{\text{av}})_y = \dfrac{\Delta y}{\Delta t} = \dfrac{y_2 - y_1}{t_2 - t_1} = \dfrac{-0.5\text{ m} - 3.4\text{ m}}{3.0\text{ s} - 0} = -1.3\text{ m/s}$

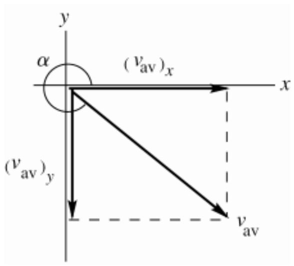


Figure 3.1

$\tan\alpha = \dfrac{(v_{\text{av}})_y}{(v_{\text{av}})_x} = \dfrac{-1.3\text{ m/s}}{1.4\text{ m/s}} = -0.9286$

$\alpha = 360° - 42.9° = 317°$

$v_{\text{av}} = \sqrt{(v_{\text{av}})_x^2 + (v_{\text{av}})_y^2}$

$v_{\text{av}} = \sqrt{(1.4\text{ m/s})^2 + (-1.3\text{ m/s})^2} = 1.9\text{ m/s}$

EVALUATE: Our calculation gives that $\vec{v}_{\text{av}}$ is in the 4th quadrant. This corresponds to increasing x and decreasing y.

3.2. **IDENTIFY:** Use Eq.(3.2), written in component form. The distance from the origin is the magnitude of $\vec{r}$.

SET UP: At time t_1, $x_1 = y_1 = 0$.

EXECUTE: **(a)** $x = (v_{\text{av-}x})\Delta t = (-3.8\text{ m/s})(12.0\text{ s}) = -45.6\text{ m}$ and $y = (v_{\text{av-}y})\Delta t = (4.9\text{ m/s})(12.0\text{ s}) = 58.8\text{ m}$.

(b) $r = \sqrt{x^2 + y^2} = \sqrt{(-45.6\text{ m})^2 + (58.8\text{ m})^2} = 74.4\text{ m}$.

EVALUATE: $\Delta\vec{r}$ is in the direction of $\vec{v}_{\text{av}}$. Therefore, Δx is negative since $v_{\text{av-}x}$ is negative and Δy is positive since $v_{\text{av-}y}$ is positive.

3.3. **(a) IDENTIFY** and **SET UP:** From $\vec{r}$ we can calculate x and y for any t.
Then use Eq.(3.2), in component form.

EXECUTE: $\vec{r} = \left[4.0\text{ cm} + (2.5\text{ cm/s}^2)t^2\right]\hat{i} + (5.0\text{ cm/s})t\hat{j}$

At $t = 0$, $\vec{r} = (4.0\text{ cm})\hat{i}$.

At $t = 2.0$ s, $\vec{r} = (14.0\text{ cm})\hat{i} + (10.0\text{ cm})\hat{j}$.

$(v_{\text{av}})_x = \dfrac{\Delta x}{\Delta t} = \dfrac{10.0\text{ cm}}{2.0\text{ s}} = 5.0\text{ cm/s}.$

$(v_{\text{av}})_y = \dfrac{\Delta y}{\Delta t} = \dfrac{10.0\text{ cm}}{2.0\text{ s}} = 5.0\text{ cm/s}.$

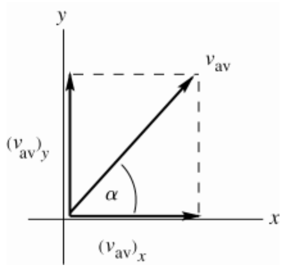


Figure 3.3a

$$v_{av} = \sqrt{(v_{av})_x^2 + (v_{av})_y^2} = 7.1 \text{ cm/s}$$

$$\tan\alpha = \frac{(v_{av})_y}{(v_{av})_x} = 1.00$$

$$\theta = 45°.$$

EVALUATE: Both x and y increase, so $\vec{v}_{av}$ is in the 1st quadrant.

(b) IDENTIFY and SET UP: Calculate $\vec{r}$ by taking the time derivative of $\vec{r}(t)$.

EXECUTE: $\vec{v} = \dfrac{d\vec{r}}{dt} = \left(\left[5.0 \text{ cm/s}^2\right]t\right)\hat{i} + (5.0 \text{ cm/s})\hat{j}$

$\underline{t=0}$: $v_x = 0$, $v_y = 5.0$ cm/s; $v = 5.0$ cm/s and $\theta = 90°$

$\underline{t = 1.0 \text{ s}}$: $v_x = 5.0$ cm/s, $v_y = 5.0$ cm/s; $v = 7.1$ cm/s and $\theta = 45°$

$\underline{t = 2.0 \text{ s}}$: $v_x = 10.0$ cm/s, $v_y = 5.0$ cm/s; $v = 11$ cm/s and $\theta = 27°$

(c) The trajectory is a graph of y versus x.

$x = 4.0 \text{ cm} + (2.5 \text{ cm/s}^2)t^2$, $y = (5.0 \text{ cm/s})t$

For values of t between 0 and 2.0 s, calculate x and y and plot y versus x.

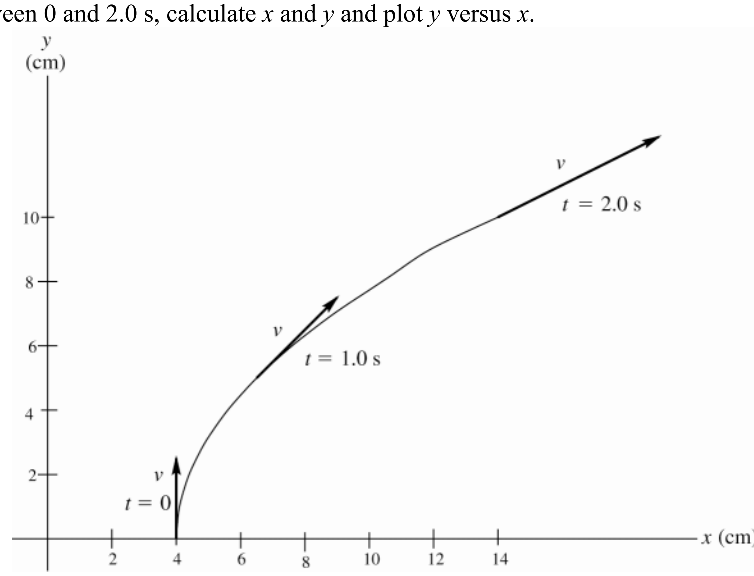


Figure 3.3b

EVALUATE: The sketch shows that the instantaneous velocity at any t is tangent to the trajectory.

3.4. **IDENTIFY:** $\vec{v} = d\vec{r}/dt$. This vector will make a 45°-angle with both axes when its x- and y-components are equal.

SET UP: $\dfrac{d(t^n)}{dt} = nt^{n-1}$.

EXECUTE: $\vec{v} = 2bt\hat{i} + 3ct^2\hat{j}$. $v_x = v_y$ gives $t = 2b/3c$.

EVALUATE: Both components of $\vec{v}$ change with t.

3.5. **IDENTIFY and SET UP:** Use Eq.(3.8) in component form to calculate $(a_{av})_x$ and $(a_{av})_y$.

EXECUTE: **(a)** The velocity vectors at $t_1 = 0$ and $t_2 = 30.0$ s are shown in Figure 3.5a.

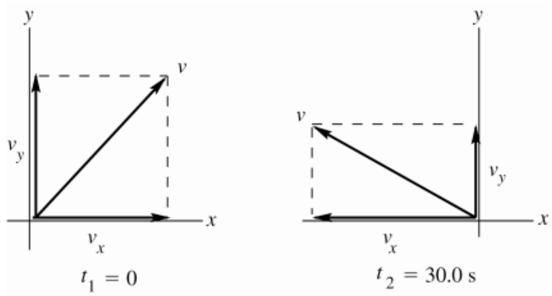


Figure 3.5a

(b) $(a_{\text{av}})_x = \dfrac{\Delta v_x}{\Delta t} = \dfrac{v_{2x} - v_{1x}}{t_2 - t_1} = \dfrac{-170 \text{ m/s} - 90 \text{ m/s}}{30.0 \text{ s}} = -8.67 \text{ m/s}^2$

$(a_{\text{av}})_y = \dfrac{\Delta v_y}{\Delta t} = \dfrac{v_{2y} - v_{1y}}{t_2 - t_1} = \dfrac{40 \text{ m/s} - 110 \text{ m/s}}{30.0 \text{ s}} = -2.33 \text{ m/s}^2$

(c)

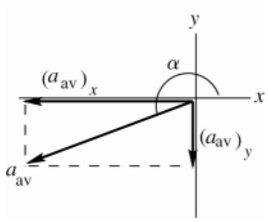


Figure 3.5b

$$a = \sqrt{(a_{\text{av}})_x^2 + (a_{\text{av}})_y^2} = 8.98 \text{ m/s}^2$$

$$\tan\alpha = \frac{(a_{\text{av}})_y}{(a_{\text{av}})_x} = \frac{-2.33 \text{ m/s}^2}{-8.67 \text{ m/s}^2} = 0.269$$

$$\alpha = 15° + 180° = 195°$$

EVALUATE: The changes in v_x and v_y are both in the negative x or y direction, so both components of $\vec{a}_{\text{av}}$ are in the 3rd quadrant.

3.6. **IDENTIFY:** Use Eq.(3.8), written in component form.

SET UP: $a_x = (0.45\,\text{m/s}^2)\cos 31.0° = 0.39\,\text{m/s}^2$, $a_y = (0.45\,\text{m/s}^2)\sin 31.0° = 0.23\,\text{m/s}^2$

EXECUTE: **(a)** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$ and $v_x = 2.6 \text{ m/s} + (0.39 \text{ m/s}^2)(10.0 \text{ s}) = 6.5 \text{ m/s}$. $a_{\text{av-}y} = \dfrac{\Delta v_y}{\Delta t}$ and

$v_y = -1.8 \text{ m/s} + (0.23 \text{ m/s}^2)(10.0 \text{ s}) = 0.52 \text{ m/s}$.

(b) $v = \sqrt{(6.5\,\text{m/s})^2 + (0.52\,\text{m/s})^2} = 6.48\,\text{m/s}$, at an angle of $\arctan\left(\dfrac{0.52}{6.5}\right) = 4.6°$ above the horizontal.

(c) The velocity vectors $\vec{v}_1$ and $\vec{v}_2$ are sketched in Figure 3.6. The two velocity vectors differ in magnitude and direction.

EVALUATE: $\vec{v}_1$ is at an angle of $35°$ below the $+x$-axis and has magnitude $v_1 = 3.2$ m/s, so $v_2 > v_1$ and the direction of $\vec{v}_2$ is rotated counterclockwise from the direction of $\vec{v}_1$.

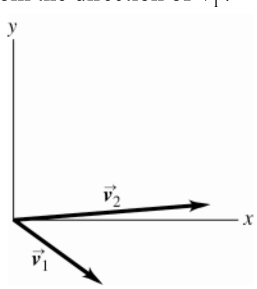


Figure 3.6

3.7. **IDENTIFY** and **SET UP:** Use Eqs.(3.4) and (3.12) to find v_x, v_y, a_x, and a_y as functions of time. The magnitude and direction of $\vec{r}$ and $\vec{a}$ can be found once we know their components.

EXECUTE: **(a)** Calculate x and y for t values in the range 0 to 2.0 s and plot y versus x. The results are given in Figure 3.7a.

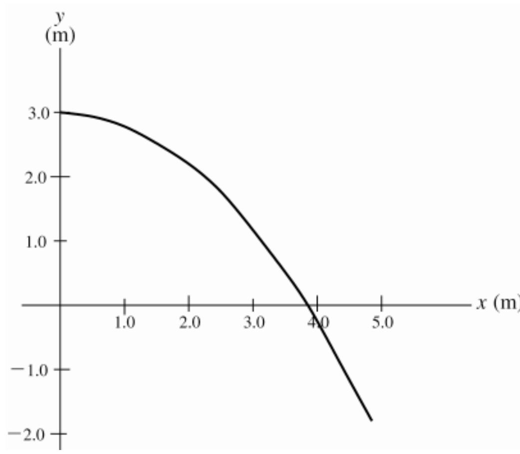


Figure 3.7a

(b) $v_x = \dfrac{dx}{dt} = \alpha \quad v_y = \dfrac{dy}{dt} = -2\beta t$

$a_y = \dfrac{dv_x}{dt} = 0 \quad a_y = \dfrac{dv_y}{dt} = -2\beta$

Thus $\vec{v} = a\hat{i} - 2\beta t\hat{j} \quad \vec{a} = -2\beta\hat{j}$

(c) <u>velocity</u>: At $t = 2.0$ s, $v_x = 2.4$ m/s, $v_y = -2(1.2 \text{ m/s}^2)(2.0 \text{ s}) = -4.8$ m/s

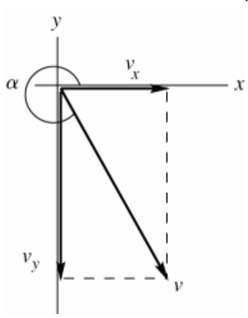


Figure 3.7b

$$v = \sqrt{v_x^2 + v_y^2} = 5.4 \text{ m/s}$$

$$\tan\alpha = \frac{v_y}{v_x} = \frac{-4.8 \text{ m/s}}{2.4 \text{ m/s}} = -2.00$$

$$\alpha = -63.4° + 360° = 297°$$

<u>acceleration</u>: At $t = 2.0$ s, $a_x = 0$, $a_y = -2(1.2 \text{ m/s}^2) = -2.4 \text{ m/s}^2$

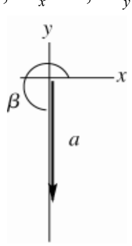


Figure 3.7c

$$a = \sqrt{a_x^2 + a_y^2} = 2.4 \text{ m/s}^2$$

$$\tan\beta = \frac{a_y}{a_x} = \frac{-2.4 \text{ m/s}^2}{0} = -\infty$$

$$\beta = 270°$$

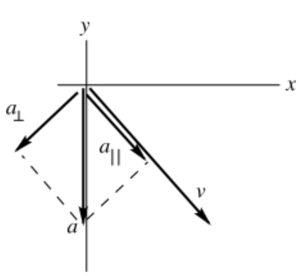


Figure 3.7d

EVALUATE: **(d)** $\vec{a}$ has a component $a_\parallel$ in the same direction as $\vec{v}$, so we know that v is increasing (the bird is speeding up.) $\vec{a}$ also has a component $a_\perp$ perpendicular to $\vec{v}$, so that the direction of $\vec{v}$ is changing; the bird is turning toward the $-y$-direction (toward the right)

$\vec{v}$ is always tangent to the path; $\vec{v}$ at $t = 2.0$ s shown in part (c) is tangent to the path at this t, conforming to this general rule. $\vec{a}$ is constant and in the $-y$-direction; the direction of $\vec{v}$ is turning toward the $-y$-direction.

3.8. **IDENTIFY:** The component $\vec{a}_\perp$ of $\vec{a}$ perpendicular to the path is related to the change in direction of $\vec{v}$ and the component $\vec{a}_\parallel$ of $\vec{a}$ parallel to the path is related to the change in the magnitude of $\vec{v}$.

SET UP: When the speed is increasing, $\vec{a}_\parallel$ is in the direction of $\vec{v}$ and when the speed is decreasing, $\vec{a}_\parallel$ is opposite to the direction of $\vec{v}$. When v is constant, $a_\parallel$ is zero and when the path is a straight line, $a_\perp$ is zero.

EXECUTE: The acceleration vectors in each case are sketched in Figure 3.8a-c.

EVALUATE: $\vec{a}_\perp$ is toward the center of curvature of the path.

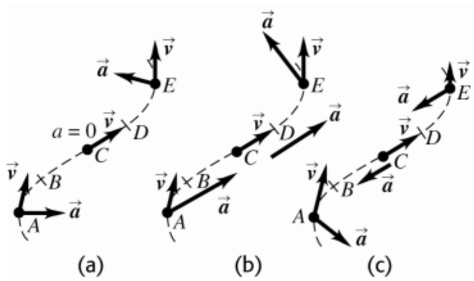


Figure 3.8a-c

3.9. **IDENTIFY:** The book moves in projectile motion once it leaves the table top. Its initial velocity is horizontal.

SET UP: Take the positive y-direction to be upward. Take the origin of coordinates at the initial position of the book, at the point where it leaves the table top.

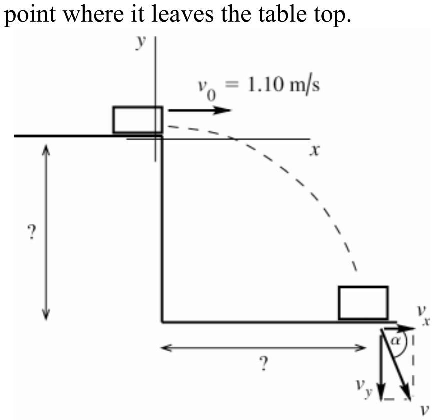


Figure 3.9a

x-component:
$a_x = 0, \;\; v_{0x} = 1.10$ m/s,
$t = 0.350$ s

y-component:
$a_y = -9.80\text{ m/s}^2$,
$v_{0y} = 0$,
$t = 0.350$ s

Use constant acceleration equations for the x and y components of the motion, with $a_x = 0$ and $a_y = -g$.

EXECUTE: **(a)** $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-9.80\text{ m/s}^2)(0.350\text{ s})^2 = -0.600\text{ m}$. The table top is 0.600 m above the floor.

(b) $x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (1.10\text{ m/s})(0.350\text{ s}) + 0 = 0.358\text{ m}$.

(c) $v_x = v_{0x} + a_x t = 1.10$ m/s (The x-component of the velocity is constant, since $a_x = 0$.)

$v_y = v_{0y} + a_y t = 0 + (-9.80\text{ m/s}^2)(0.350\text{ s}) = -3.43\text{ m/s}$

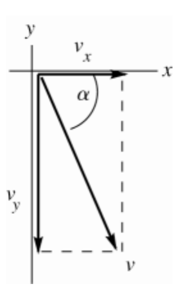


Figure 3.9b

$v = \sqrt{v_x^2 + v_y^2} = 3.60$ m/s

$\tan\alpha = \dfrac{v_y}{v_x} = \dfrac{-3.43\text{ m/s}}{1.10\text{ m/s}} = -3.118$

$\alpha = -72.2°$

Direction of $\vec{v}$ is $72.2°$ below the horizontal

(d) The graphs are given in Figure 3.9c

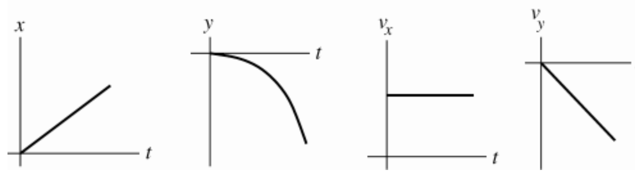


Figure 3.9c

EVALUATE: In the *x*-direction, $a_x = 0$ and v_x is constant. In the *y*-direction, $a_y = -9.80\ \text{m/s}^2$ and v_y is downward and increasing in magnitude since a_y and v_y are in the same directions. The *x* and *y* motions occur independently, connected only by the time. The time it takes the book to fall 0.600 m is the time it travels horizontally.

3.10. **IDENTIFY:** The bomb moves in projectile motion. Treat the horizontal and vertical components of the motion separately. The vertical motion determines the time in the air.

SET UP: The initial velocity of the bomb is the same as that of the helicopter. Take $+y$ downward, so $a_x = 0$, $a_y = +9.80\ \text{m/s}^2$, $v_{0x} = 60.0\ \text{m/s}$ and $v_{0y} = 0$.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 300\ \text{m}$ gives $t = \sqrt{\frac{2(y-y_0)}{a_y}} = \sqrt{\frac{2(300\ \text{m})}{9.80\ \text{m/s}^2}} = 7.82\ \text{s}$.

(b) The bomb travels a horizontal distance $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (60.0\ \text{m/s})(7.82\ \text{s}) = 470\ \text{m}$.

(c) $v_x = v_{0x} = 60.0\ \text{m/s}$. $v_y = v_{0y} + a_y t = (9.80\ \text{m/s}^2)(7.82\ \text{s}) = 76.6\ \text{m/s}$.

(d) The graphs are given in Figure 3.10.

(e) Because the airplane and the bomb always have the same *x*-component of velocity and position, the plane will be 300 m directly above the bomb at impact.

EVALUATE: The initial horizontal velocity of the bomb doesn't affect its vertical motion.

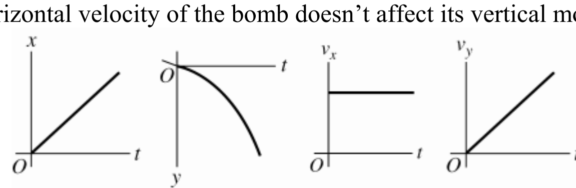


Figure 3.10

3.11. **IDENTIFY:** Each object moves in projectile motion.

SET UP: Take $+y$ to be downward. For each cricket, $a_x = 0$ and $a_y = +9.80\ \text{m/s}^2$. For Chirpy, $v_{0x} = v_{0y} = 0$. For Milada, $v_{0x} = 0.950\ \text{m/s}$, $v_{0y} = 0$

EXECUTE: Milada's horizontal component of velocity has no effect on her vertical motion. She also reaches the ground in 3.50 s. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (0.950\ \text{m/s})(3.50\ \text{s}) = 3.32\ \text{m}$

EVALUATE: The *x* and *y* components of motion are totally separate and are connected only by the fact that the time is the same for both.

3.12. **IDENTIFY:** The person moves in projectile motion. She must travel 1.75 m horizontally during the time she falls 9.00 m vertically.

SET UP: Take $+y$ downward. $a_x = 0$, $a_y = +9.80\ \text{m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

EXECUTE: Time to fall 9.00 m: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y-y_0)}{a_y}} = \sqrt{\frac{2(9.00\ \text{m})}{9.80\ \text{m/s}^2}} = 1.36\ \text{s}$.

Speed needed to travel 1.75 m horizontally during this time: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$$v_0 = v_{0x} = \frac{x - x_0}{t} = \frac{1.75\ \text{m}}{1.36\ \text{s}} = 1.29\ \text{m/s}.$$

EVALUATE: If she increases her initial speed she still takes 1.36 s to reach the level of the ledge, but has traveled horizontally farther than 1.75 m.

3.13. **IDENTIFY:** The car moves in projectile motion. The car travels $21.3\ \text{m} - 1.80\ \text{m} = 19.5\ \text{m}$ downward during the time it travels 61.0 m horizontally.

SET UP: Take $+y$ to be downward. $a_x = 0$, $a_y = +9.80\ \text{m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

EXECUTE: Use the vertical motion to find the time in the air:

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(19.5\text{ m})}{9.80\text{ m/s}^2}} = 1.995\text{ s}$

Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $v_0 = v_{0x} = \frac{x - x_0}{t} = \frac{61.0\text{ m}}{1.995\text{ s}} = 30.6\text{ m/s}$.

(b) $v_x = 30.6\text{m/s}$ since $a_x = 0$. $v_y = v_{0y} + a_y t = -19.6\text{m/s}$. $v = \sqrt{v_x^2 + v_y^2} = 36.3\text{m/s}$.

EVALUATE: We calculate the final velocity by calculating its *x* and *y* components.

3.14. **IDENTIFY:** The marble moves with projectile motion, with initial velocity that is horizontal and has magnitude v_0. Treat the horizontal and vertical motions separately. If v_0 is too small the marble will land to the left of the hole and if v_0 is too large the marble will land to the right of the hole.

SET UP: Let $+x$ be horizontal to the right and let $+y$ be upward. $v_{0x} = v_0$, $v_{0y} = 0$, $a_x = 0$, $a_y = -9.80\text{ m/s}^2$

EXECUTE: Use the vertical motion to find the time it takes the marble to reach the height of the level ground; $y - y_0 = -2.75\text{ m}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(-2.75\text{ m})}{-9.80\text{ m/s}^2}} = 0.749\text{ s}$. The time does not depend on v_0.

Minimum v_0: $x - x_0 = 2.00\text{ m}$, $t = 0.749\text{ s}$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $v_0 = \frac{x - x_0}{t} = \frac{2.00\text{ m}}{0.749\text{ s}} = 2.67\text{ m/s}$.

Maximum v_0: $x - x_0 = 3.50\text{ m}$ and $v_0 = \frac{3.50\text{ m}}{0.749\text{ s}} = 4.67\text{ m/s}$.

EVALUATE: The horizontal and vertical motions are independent and are treated separately. Their only connection is that the time is the same for both.

3.15. **IDENTIFY:** The ball moves with projectile motion with an initial velocity that is horizontal and has magnitude v_0. The height *h* of the table and v_0 are the same; the acceleration due to gravity changes from $g_E = 9.80\text{ m/s}^2$ on earth to g_X on planet X.

SET UP: Let $+x$ be horizontal and in the direction of the initial velocity of the marble and let $+y$ be upward. $v_{0x} = v_0$, $v_{0y} = 0$, $a_x = 0$, $a_y = -g$, where *g* is either g_E or g_X.

EXECUTE: Use the vertical motion to find the time in the air: $y - y_0 = -h$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2h}{g}}$.

Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x - x_0 = v_{0x}t = v_0\sqrt{\frac{2h}{g}}$. $x - x_0 = D$ on earth and 2.76*D* on Planet X.

$(x - x_0)\sqrt{g} = v_0\sqrt{2h}$, which is constant, so $D\sqrt{g_E} = 2.76D\sqrt{g_X}$. $g_X = \frac{g_E}{(2.76)^2} = 0.131g_E = 1.28\text{ m/s}^2$.

EVALUATE: On Planet X the acceleration due to gravity is less, it takes the ball longer to reach the floor, and it travels farther horizontally.

3.16. **IDENTIFY:** The football moves in projectile motion.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -g$. At the highest point in the trajectory, $v_y = 0$.

EXECUTE: **(a)** $v_y = v_{0y} + a_y t$. The time *t* is $\frac{v_{0y}}{g} = \frac{16.0\text{m/s}}{9.80\text{m/s}^2} = 1.63\text{ s}$.

(b) Different constant acceleration equations give different expressions but the same numerical result:

$\frac{1}{2}gt^2 = \frac{1}{2}v_{y0}t = \frac{v_{0y}^2}{2g} = 13.1\text{ m}$.

(c) Regardless of how the algebra is done, the time will be twice that found in part (a), or 3.27 s

(d) $a_x = 0$, so $x - x_0 = v_{0x}t = (20.0\text{ m/s})(3.27\text{ s}) = 65.3\text{ m}$.

(e) The graphs are sketched in Figure 3.16.

EVALUATE: When the football returns to its original level, $v_x = 20.0$ m/s and $v_y = -16.0$ m/s .

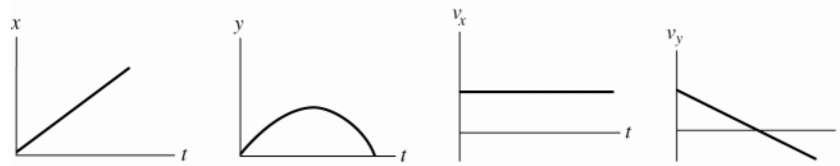


Figure 3.16

3.17. **IDENTIFY:** The shell moves in projectile motion.

SET UP: Let $+x$ be horizontal, along the direction of the shell's motion, and let $+y$ be upward. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$.

EXECUTE: **(a)** $v_{0x} = v_0 \cos\alpha_0 = (80.0\text{ m/s})\cos 60.0° = 40.0\text{ m/s}$, $v_{0y} = v_0 \sin\alpha_0 = (80.0\text{ m/s})\sin 60.0° = 69.3\text{ m/s}$.

(b) At the maximum height $v_y = 0$. $v_y = v_{0y} + a_y t$ gives $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{0 - 69.3\text{ m/s}}{-9.80\text{ m/s}^2} = 7.07\text{ s}$.

(c) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (69.3\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 245\text{ m}$.

(d) The total time in the air is twice the time to the maximum height, so $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (40.0\text{ m/s})(14.14\text{ s}) = 566\text{ m}$.

(e) At the maximum height, $v_x = v_{0x} = 40.0$ m/s and $v_y = 0$. At all points in the motion, $a_x = 0$ and $a_y = -9.80\text{ m/s}^2$.

EVALUATE: The equation for the horizontal range R derived in Example 3.8 is $R = \dfrac{v_0^2 \sin 2\alpha_0}{g}$. This gives $R = \dfrac{(80.0\text{ m/s})^2 \sin(120.0°)}{9.80\text{ m/s}^2} = 566\text{ m}$, which agrees with our result in part (d).

3.18. **IDENTIFY:** The flare moves with projectile motion. The equations derived in Example 3.8 can be used to find the maximum height h and range R.

SET UP: From Example 3.8, $h = \dfrac{v_0^2 \sin^2\alpha_0}{2g}$ and $R = \dfrac{v_0^2 \sin 2\alpha_0}{g}$.

EXECUTE: **(a)** $h = \dfrac{(125\text{ m/s})^2(\sin 55.0°)^2}{2(9.80\text{ m/s}^2)} = 535\text{ m}$. $R = \dfrac{(125\text{ m/s})^2(\sin 110.0°)}{9.80\text{ m/s}^2} = 1500\text{ m}$.

(b) h and R are proportional to $1/g$, so on the Moon, $h = \left(\dfrac{9.80\text{ m/s}^2}{1.67\text{ m/s}^2}\right)(535\text{ m}) = 3140\text{ m}$ and $R = \left(\dfrac{9.80\text{ m/s}^2}{1.67\text{ m/s}^2}\right)(1500\text{ m}) = 8800\text{ m}$.

EVALUATE: The projectile travels on a parabolic trajectory. It is incorrect to say that $h = (R/2)\tan\alpha_0$.

3.19. **IDENTIFY:** The baseball moves in projectile motion. In part (c) first calculate the components of the velocity at this point and then get the resultant velocity from its components.

SET UP: First find the x- and y-components of the initial velocity. Use coordinates where the $+y$-direction is upward, the $+x$-direction is to the right and the origin is at the point where the baseball leaves the bat.

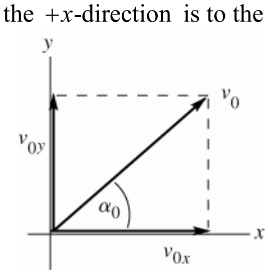


Figure 3.19a

$v_{0x} = v_0 \cos\alpha_0 = (30.0\text{ m/s})\cos 36.9° = 24.0\text{ m/s}$

$v_{0y} = v_0 \sin\alpha_0 = (30.0\text{ m/s})\sin 36.9° = 18.0\text{ m/s}$

Use constant acceleration equations for the x and y motions, with $a_x = 0$ and $a_y = -g$.

EXECUTE: **(a)** <u>y-component</u> (vertical motion):

$y - y_0 = +10.0$ m/s, $v_{0y} = 18.0$ m/s, $a_y = -9.80$ m/s^2, $t = ?$

$y - y_0 = v_{0y} + \frac{1}{2}a_y t^2$

$10.0 \text{ m} = (18.0 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$

$(4.90 \text{ m/s}^2)t^2 - (18.0 \text{ m/s})t + 10.0 \text{ m} = 0$

Apply the quadratic formula: $t = \frac{1}{9.80}\left[18.0 \pm \sqrt{(-18.0)^2 - 4(4.90)(10.0)}\right] \text{ s} = (1.837 \pm 1.154) \text{ s}$

The ball is at a height of 10.0 above the point where it left the bat at $t_1 = 0.683$ s and at $t_2 = 2.99$ s. At the earlier time the ball passes through a height of 10.0 m as its way up and at the later time it passes through 10.0 m on its way down.

(b) $v_x = v_{0x} = +24.0$ m/s, at all times since $a_x = 0$.

$v_y = v_{0y} + a_y t$

<u>$t_1 = 0.683$ s</u>: $v_y = +18.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(0.683 \text{ s}) = +11.3$ m/s. (v_y is positive means that the ball is traveling upward at this point.

<u>$t_2 = 2.99$ s</u>: $v_y = +18.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(2.99 \text{ s}) = -11.3$ m/s. (v_y is negative means that the ball is traveling downward at this point.)

(c) $v_x = v_{0x} = 24.0$ m/s

Solve for v_y:

$v_y = ?$, $y - y_0 = 0$ (when ball returns to height where motion started),

$a_y = -9.80$ m/s^2, $v_{0y} = +18.0$ m/s

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$v_y = -v_{0y} = -18.0$ m/s (negative, since the baseball must be traveling downward at this point)

Now that have the components can solve for the magnitude and direction of $\vec{v}$.

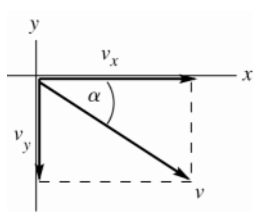


Figure 3.19b

$v = \sqrt{v_x^2 + v_y^2}$

$v = \sqrt{(24.0 \text{ m/s})^2 + (-18.0 \text{ m/s})^2} = 30.0$ m/s

$\tan\alpha = \frac{v_y}{v_x} = \frac{-18.0 \text{ m/s}}{24.0 \text{ m/s}}$

$\alpha = -36.9°$, $36.9°$ below the horizontal

The velocity of the ball when it returns to the level where it left the bat has magnitude 30.0 m/s and is directed at an angle of $36.9°$ below the horizontal.

EVALUATE: The discussion in parts (a) and (b) explains the significance of two values of t for which $y - y_0 = +10.0$ m. When the ball returns to its initial height, our results give that its speed is the same as its initial speed and the angle of its velocity below the horizontal is equal to the angle of its initial velocity above the horizontal; both of these are general results.

3.20. **IDENTIFY:** The shot moves in projectile motion.

SET UP: Let $+y$ be upward.

EXECUTE: **(a)** If air resistance is to be ignored, the components of acceleration are 0 horizontally and $-g = -9.80 \text{ m/s}^2$ vertically downward.

(b) The x-component of velocity is constant at $v_x = (12.0 \text{ m/s})\cos 51.0° = 7.55 \text{ m/s}$. The y-component is $v_{0y} = (12.0 \text{ m/s})\sin 51.0° = 9.32 \text{ m/s}$ at release and $v_y = v_{0y} - gt = (10.57 \text{ m/s}) - (9.80 \text{ m/s}^2)(2.08 \text{ s}) = -11.06 \text{ m/s}$ when the shot hits.

(c) $x - x_0 = v_{0x}t = (7.55 \text{ m/s})(2.08 \text{ s}) = 15.7$ m.

(d) The initial and final heights are not the same.

(e) With $y = 0$ and v_{0y} as found above, Eq.(3.18) gives $y_0 = 1.81$m.

(f) The graphs are sketched in Figure 3.20.

EVALUATE: When the shot returns to its initial height, $v_y = -9.32$ m/s . The shot continues to accelerate downward as it travels downward 1.81 m to the ground and the magnitude of v_y at the ground is larger than 9.32 m/s.

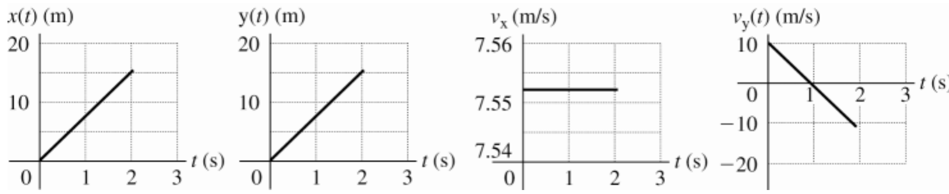
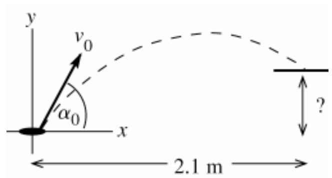


Figure 3.20

3.21. **IDENTIFY:** Take the origin of coordinates at the point where the quarter leaves your hand and take positive *y* to be upward. The quarter moves in projectile motion, with $a_x = 0$, and $a_y = -g$. It travels vertically for the time it takes it to travel horizontally 2.1 m.



$v_{0x} = v_0 \cos\alpha_0 = (6.4 \text{ m/s})\cos 60°$

$v_{0x} = 3.20$ m/s

$v_{0y} = v_0 \sin\alpha_0 = (6.4 \text{ m/s})\sin 60°$

$v_{0y} = 5.54$ m/s

Figure 3.21

(a) SET UP: Use the horizontal (*x*-component) of motion to solve for *t*, the time the quarter travels through the air:

$t = ?,\ \ x - x_0 = 2.1 \text{ m},\ \ v_{0x} = 3.2 \text{ m/s},\ \ a_x = 0$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 = v_{0x}t,$ since $a_x = 0$

EXECUTE: $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{2.1 \text{ m}}{3.2 \text{ m/s}} = 0.656 \text{ s}$

SET UP: Now find the vertical displacement of the quarter after this time:

$y - y_0 = ?,\ \ a_y = -9.80 \text{ m/s}^2,\ \ v_{0y} = +5.54 \text{ m/s},\ \ t = 0.656 \text{ s}$

$y - y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$

EXECUTE: $y - y_0 = (5.54 \text{ m/s})(0.656 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(0.656 \text{ s})^2 = 3.63 \text{ m} - 2.11 \text{ m} = 1.5 \text{ m}.$

(b) SET UP: $v_y = ?,\ \ t = 0.656 \text{ s},\ \ a_y = -9.80 \text{ m/s}^2,\ \ v_{0y} = +5.54 \text{ m/s}$

$v_y = v_{0y} + a_y t$

EXECUTE: $v_y = 5.54 \text{ m/s} + (-9.80 \text{ m/s}^2)(0.656 \text{ s}) = -0.89 \text{ m/s}.$

EVALUATE: The minus sign for v_y indicates that the *y*-component of $\vec{v}$ is downward. At this point the quarter has passed through the highest point in its path and is on its way down. The horizontal range if it returned to its original height (it doesn't!) would be 3.6 m. It reaches its maximum height after traveling horizontally 1.8 m, so at $x - x_0 = 2.1$ m it is on its way down.

3.22. **IDENTIFY:** Use the analysis of Example 3.10.

SET UP: From Example 3.10, $t = \dfrac{d}{v_0 \cos\alpha_0}$ and $y_{\text{dart}} = (v_0 \sin\alpha_0)t - \tfrac{1}{2}gt^2$.

EXECUTE: Substituting for *t* in terms of *d* in the expression for y_{dart} gives

$$y_{\text{dart}} = d\left(\tan\alpha_0 - \frac{gd}{2v_0^2\cos^2\alpha_0}\right).$$

Using the given values for *d* and α_0 to express this as a function of v_0 ,

$$y = (3.00 \text{ m})\left(0.90 - \frac{26.62 \text{ m}^2/\text{s}^2}{v_0^2}\right).$$

(a) $v_0 = 12.0$ m/s gives $y = 2.14$ m .

(b) $v_0 = 8.0$ m/s gives $y = 1.45$ m .

(c) $v_0 = 4.0$ m/s gives $y = -2.29$ m . In this case, the dart was fired with so slow a speed that it hit the ground before traveling the 3-meter horizontal distance.

EVALUATE: For (a) and (d) the trajectory of the dart has the shape shown in Figure 3.26 in the textbook. For (c) the dart moves in a parabola and returns to the ground before it reaches the *x*-coordinate of the monkey.

3.23. IDENTIFY: Take the origin of coordinates at the roof and let the $+y$-direction be upward. The rock moves in projectile motion, with $a_x = 0$ and $a_y = -g$. Apply constant acceleration equations for the *x* and *y* components of the motion.

SET UP:

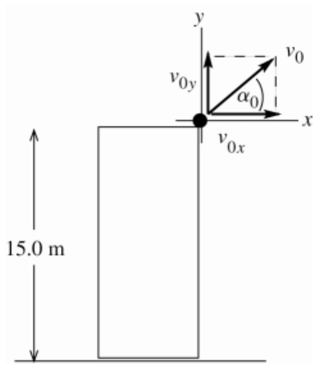


Figure 3.23a

$v_{0x} = v_0 \cos\alpha_0 = 25.2$ m/s

$v_{0y} = v_0 \sin\alpha_0 = 16.3$ m/s

(a) At the maximum height $v_y = 0$.

$a_y = -9.80 \text{ m/s}^2$, $v_y = 0$, $v_{0y} = +16.3$ m/s, $y - y_0 = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (16.3 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = +13.6 \text{ m}$

(b) SET UP: Find the velocity by solving for its *x* and *y* components.

$v_x = v_{0x} = 25.2$ m/s (since $a_x = 0$)

$v_y = ?$, $a_y = -9.80 \text{ m/s}^2$, $y - y_0 = -15.0$ m (negative because at the ground the rock is below its initial position),

$v_{0y} = 16.3$ m/s

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)}$ (v_y is negative because at the ground the rock is traveling downward.)

EXECUTE: $v_y = -\sqrt{(16.3 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-15.0 \text{ m})} = -23.7 \text{ m/s}$

Then $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(25.2 \text{ m/s})^2 + (-23.7 \text{ m/s})^2} = 34.6$ m/s.

(c) SET UP: Use the vertical motion (*y*-component) to find the time the rock is in the air:

$t = ?$, $v_y = -23.7$ m/s (from part (b)), $a_y = -9.80 \text{ m/s}^2$, $v_{0y} = +16.3$ m/s

EXECUTE: $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-23.7 \text{ m/s} - 16.3 \text{ m/s}}{-9.80 \text{ m/s}^2} = +4.08 \text{ s}$

SET UP: Can use this *t* to calculate the horizontal range:

$t = 4.08$ s, $v_{0x} = 25.2$ m/s, $a_x = 0$, $x - x_0 = ?$

EXECUTE: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (25.2 \text{ m/s})(4.08 \text{ s}) + 0 = 103 \text{ m}$

(d) Graphs of *x* versus *t*, *y* versus *t*, v_x versus *t*, and v_y versus *t*:

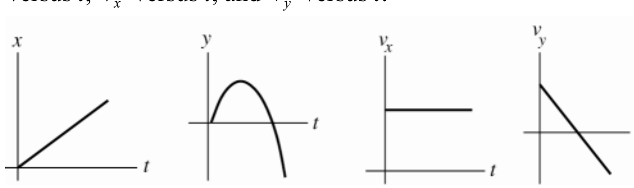


Figure 3.23b

EVALUATE: The time it takes the rock to travel vertically to the ground is the time it has to travel horizontally. With $v_{0y} = +16.3$ m/s the time it takes the rock to return to the level of the roof $(y = 0)$ is $t = 2v_{0y}/g = 3.33$ s. The time in the air is greater than this because the rock travels an additional 15.0 m to the ground.

3.24. **IDENTIFY:** Consider the horizontal and vertical components of the projectile motion. The water travels 45.0 m horizontally in 3.00 s.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. $v_{0x} = v_0\cos\theta_0$, $v_{0y} = v_0\sin\theta_0$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$ gives $x - x_0 = v_0(\cos\theta_0)t$ and $\cos\theta_0 = \dfrac{45.0\text{ m}}{(25.0\text{ m/s})(3.00\text{ s})} = 0.600$; $\theta_0 = 53.1°$

(b) At the highest point $v_x = v_{0x} = (25.0\text{ m/s})\cos 53.1° = 15.0$ m/s, $v_y = 0$ and $v = \sqrt{v_x^2 + v_y^2} = 15.0$ m/s. At all points in the motion, $a = 9.80\text{ m/s}^2$ downward.

(c) Find $y - y_0$ when $t = 3.00$ s:

$y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 = (25.0\text{ m/s})(\sin 53.1°)(3.00\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(3.00\text{ s})^2 = 15.9\text{ m}$

$v_x = v_{0x} = 15.0$ m/s, $v_y = v_{0y} + a_yt = (25.0\text{ m/s})(\sin 53.1°) - (9.80\text{m/s}^2)(3.00\text{ s}) = -9.41$ m/s, and

$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{(15.0\text{ m/s})^2 + (-9.41\text{ m/s})^2} = 17.7\text{ m/s}$

EVALUATE: The acceleration is the same at all points of the motion. It takes the water $t = -\dfrac{v_{0y}}{a_y} = -\dfrac{20.0\text{ m/s}}{-9.80\text{ m/s}^2} = 2.04$ s to reach its maximum height. When the water reaches the building it has passed its maximum height and its vertical component of velocity is downward.

3.25. **IDENTIFY** and **SET UP:** The stone moves in projectile motion. Its initial velocity is the same as that of the balloon. Use constant acceleration equations for the *x* and *y* components of its motion. Take $+y$ to be upward.

EXECUTE: **(a)** Use the vertical motion of the rock to find the initial height.

$t = 6.00$ s, $v_{0y} = +20.0$ m/s, $a_y = +9.80\text{ m/s}^2$, $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $y - y_0 = 296$ m

(b) In 6.00 s the balloon travels downward a distance $y - y_0 = (20.0\text{ s})(6.00\text{ s}) = 120$ m. So, its height above ground when the rock hits is $296\text{ m} - 120\text{ m} = 176$ m.

(c) The horizontal distance the rock travels in 6.00 s is 90.0 m. The vertical component of the distance between the rock and the basket is 176 m, so the rock is $\sqrt{(176\text{ m})^2 + (90\text{ m})^2} = 198$ m from the basket when it hits the ground.

(d) (i) The basket has no horizontal velocity, so the rock has horizontal velocity 15.0 m/s relative to the basket. Just before the rock hits the ground, its vertical component of velocity is $v_y = v_{0y} + a_yt =$ $20.0\text{ m/s} + (9.80\text{ m/s}^2)(6.00\text{ s}) = 78.8$ m/s, downward, relative to the ground. The basket is moving downward at 20.0 m/s, so relative to the basket the rock has downward component of velocity 58.8 m/s.

(e) horizontal: 15.0 m/s; vertical: 78.8 m/s

EVALUATE: The rock has a constant horizontal velocity and accelerates downward

3.26. **IDENTIFY:** The shell moves as a projectile. To just clear the top of the cliff, the shell must have $y - y_0 = 25.0$ m when it has $x - x_0 = 60.0$ m.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -g$. $v_{0x} = v_0\cos 43°$, $v_{0y} = v_0\sin 43°$.

EXECUTE: **(a)** horizontal motion: $x - x_0 = v_{0x}t$ so $t = \dfrac{60.0\text{ m}}{(v_0\cos 43°)t}$.

vertical motion: $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $25.0\text{m} = (v_0\sin 43.0°)t + \frac{1}{2}(-9.80\text{m/s}^2)t^2$.

Solving these two simultaneous equations for v_0 and *t* gives $v_0 = 3.26$ m/s and $t = 2.51$ s.

(b) v_y when shell reaches cliff:

$$v_y = v_{0y} + a_yt = (32.6\text{ m/s})\sin 43.0° - (9.80\text{ m/s}^2)(2.51\text{ s}) = -2.4\text{ m/s}$$

The shell is traveling downward when it reaches the cliff, so it lands right at the edge of the cliff.

EVALUATE: The shell reaches its maximum height at $t = -\dfrac{v_{0y}}{a_y} = 2.27$ s, which confirms that at $t = 2.51$ s it has passed its maximum height and is on its way down when it strikes the edge of the cliff.

3.27. **IDENTIFY:** The suitcase moves in projectile motion. The initial velocity of the suitcase equals the velocity of the airplane.

SET UP: Take $+y$ to be upward. $a_x = 0$, $a_y = -g$.

EXECUTE: Use the vertical motion to find the time it takes the suitcase to reach the ground:
$v_{0y} = v_0 \sin 23°$, $a_y = -9.80 \text{ m/s}^2$, $y - y_0 = -114 \text{ m}$, $t = ?$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 9.60 \text{ s}$.
The distance the suitcase travels horizontally is $x - x_0 = v_{0x} = (v_0 \cos 23.0°)t = 795 \text{ m}$.

EVALUATE: An object released from rest at a height of 114 m strikes the ground at $t = \sqrt{\dfrac{2(y - y_0)}{-g}} = 4.82 \text{ s}$. The suitcase is in the air much longer than this since it initially has an upward component of velocity.

3.28. IDENTIFY: Determine how a_{rad} depends on the rotational period T.

SET UP: $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$.

EXECUTE: For any item in the washer, the centripetal acceleration will be inversely proportional to the square of the rotational period; tripling the centripetal acceleration involves decreasing the period by a factor of $\sqrt{3}$, so that the new period T' is given in terms of the previous period T by $T' = T/\sqrt{3}$.
EVALUATE: The rotational period must be decreased in order to increase the rate of rotation and therefore increase the centripetal acceleration.

3.29. IDENTIFY: Apply Eq. (3.30).
SET UP: $T = 24 \text{ h}$.

EXECUTE: **(a)** $a_{\text{rad}} = \dfrac{4\pi^2(6.38\times10^6 \text{ m})}{((24 \text{ h})(3600 \text{ s/h}))^2} = 0.034 \text{ m/s}^2 = 3.4\times10^{-3} g.$

(b) Solving Eq. (3.30) for the period T with $a_{\text{rad}} = g$, $T = \sqrt{\dfrac{4\pi^2(6.38\times10^6 \text{m})}{9.80 \text{ m/s}^2}} = 5070 \text{ s} = 1.4 \text{ h}.$

EVALUATE: a_{rad} is proportional to $1/T^2$, so to increase a_{rad} by a factor of $\dfrac{1}{3.4\times10^{-3}} = 294$ requires that T be multiplied by a factor of $\dfrac{1}{\sqrt{294}}$. $\dfrac{24 \text{ h}}{\sqrt{294}} = 1.4 \text{ h}$.

3.30. IDENTIFY: Each blade tip moves in a circle of radius $R = 3.40 \text{ m}$ and therefore has radial acceleration $a_{\text{rad}} = v^2/R$.

SET UP: 550 rev/min = 9.17 rev/s, corresponding to a period of $T = \dfrac{1}{9.17 \text{ rev/s}} = 0.109 \text{ s}$.

EXECUTE: **(a)** $v = \dfrac{2\pi R}{T} = 196 \text{ m/s}$.

(b) $a_{\text{rad}} = \dfrac{v^2}{R} = 1.13\times10^4 \text{ m/s}^2 = 1.15\times10^3 g$.

EVALUATE: $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$ gives the same results for a_{rad} as in part (b).

3.31. IDENTIFY: Apply Eq.(3.30).
SET UP: $R = 7.0 \text{ m}$. $g = 9.80 \text{ m/s}^2$.
EXECUTE: **(a)** Solving Eq. (3.30) for T in terms of R and a_{rad},
$T = \sqrt{4\pi^2 R/a_{\text{rad}}} = \sqrt{4\pi^2(7.0 \text{ m})/(3.0)(9.80 \text{ m/s}^2)} = 3.07 \text{ s}$.
(b) $a_{\text{rad}} = 10g$ gives $T = 1.68 \text{ s}$.
EVALUATE: When a_{rad} increases, T decreases.

3.32. IDENTIFY: Each planet moves in a circular orbit and therefore has acceleration $a_{\text{rad}} = v^2/R$.
SET UP: The radius of the earth's orbit is $r = 1.50\times10^{11} \text{ m}$ and its orbital period is $T = 365 \text{ days} = 3.16\times10^7 \text{ s}$.
For Mercury, $r = 5.79\times10^{10} \text{ m}$ and $T = 88.0 \text{ days} = 7.60\times10^6 \text{ s}$.

EXECUTE: **(a)** $v = \dfrac{2\pi r}{T} = 2.98\times10^4 \text{ m/s}$

(b) $a_{\text{rad}} = \dfrac{v^2}{r} = 5.91\times10^{-3} \text{ m/s}^2$.

(c) $v = 4.79\times10^4 \text{ m/s}$, and $a_{\text{rad}} = 3.96\times10^{-2} \text{ m/s}^2$.

EVALUATE: Mercury has a larger orbital velocity and a larger radial acceleration than earth.

3.33. **IDENTIFY:** Uniform circular motion.

SET UP: Since the magnitude of $\vec{v}$ is constant. $v_{\tan} = \dfrac{d|\vec{v}|}{dt} = 0$ and the resultant acceleration is equal to the radial component. At each point in the motion the radial component of the acceleration is directed in toward the center of the circular path and its magnitude is given by v^2/R.

EXECUTE: **(a)** $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(7.00\ \text{m/s})^2}{14.0\ \text{m}} = 3.50\ \text{m/s}^2$, upward.

(b) The radial acceleration has the same magnitude as in part (a), but now the direction toward the center of the circle is downward. The acceleration at this point in the motion is $3.50\ \text{m/s}^2$, downward.

(c) SET UP: The time to make one rotation is the period T, and the speed v is the distance for one revolution divided by T.

EXECUTE: $v = \dfrac{2\pi R}{T}$ so $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(14.0\ \text{m})}{7.00\ \text{m/s}} = 12.6\ \text{s}$

EVALUATE: The radial acceleration is constant in magnitude since v is constant and is at every point in the motion directed toward the center of the circular path. The acceleration is perpendicular to $\vec{v}$ and is nonzero because the direction of $\vec{v}$ changes.

3.34. **IDENTIFY:** The acceleration is the vector sum of the two perpendicular components, a_{rad} and $a_{\tan}$.

SET UP: $a_{\tan}$ is parallel to $\vec{v}$ and hence is associated with the change in speed; $a_{\tan} = 0.500\ \text{m/s}^2$.

EXECUTE: **(a)** $a_{\text{rad}} = v^2/R = (3\ \text{m/s})^2/(14\ \text{m}) = 0.643\ \text{m/s}^2$.

$a = ((0.643\ \text{m/s}^2)^2 + (0.5\ \text{m/s}^2)^2)^{1/2} = 0.814\ \text{m/s}^2$, $37.9°$ to the right of vertical.

(b) The sketch is given in Figure 3.34.

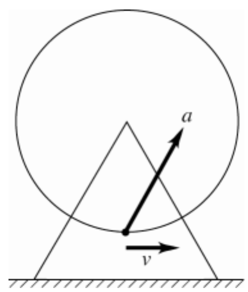


Figure 3.34

3.35. **IDENTIFY:** Each part of his body moves in uniform circular motion, with $a_{\text{rad}} = \dfrac{v^2}{R}$. The speed in rev/s is $1/T$, where T is the period in seconds (time for 1 revolution). The speed v increases with R along the length of his body but all of him rotates with the same period T.

SET UP: For his head $R = 8.84\ \text{m}$ and for his feet $R = 6.84\ \text{m}$.

EXECUTE: **(a)** $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(8.84\ \text{m})(12.5)(9.80\ \text{m/s}^2)} = 32.9\ \text{m/s}$

(b) Use $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$. Since his head has $a_{\text{rad}} = 12.5g$ and $R = 8.84\ \text{m}$,

$T = 2\pi\sqrt{\dfrac{R}{a_{\text{rad}}}} = 2\pi\sqrt{\dfrac{8.84\ \text{m}}{12.5(9.80\ \text{m/s}^2)}} = 1.688\ \text{s}$. Then his feet have $a_{\text{rad}} = \dfrac{R}{T^2} = \dfrac{4\pi^2(6.84\ \text{m})}{(1.688\ \text{s})^2} = 94.8\ \text{m/s}^2 = 9.67g$.

The difference between the acceleration of his head and his feet is $12.5g - 9.67g = 2.83g = 27.7\ \text{m/s}^2$.

(c) $\dfrac{1}{T} = \dfrac{1}{1.69\ \text{s}} = 0.592\ \text{rev/s} = 35.5\ \text{rpm}$

EVALUATE: His feet have speed $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(6.84\ \text{m})(94.8\ \text{m/s}^2)} = 25.5\ \text{m/s}$

3.36. **IDENTIFY:** The relative velocities are $\vec{v}_{\text{S/F}}$, the velocity of the scooter relative to the flatcar, $\vec{v}_{\text{S/G}}$, the scooter relative to the ground and $\vec{v}_{\text{F/G}}$, the flatcar relative to the ground. $\vec{v}_{\text{S/G}} = \vec{v}_{\text{S/F}} + \vec{v}_{\text{F/G}}$. Carry out the vector addition by drawing a vector addition diagram.

SET UP: $\vec{v}_{\text{S/F}} = \vec{v}_{\text{S/G}} - \vec{v}_{\text{F/G}}$. $\vec{v}_{\text{F/G}}$ is to the right, so $-\vec{v}_{\text{F/G}}$ is to the left.

EXECUTE: In each case the vector addition diagram gives

(a) 5.0 m/s to the right
(b) 16.0 m/s to the left
(c) 13.0 m/s to the left.

EVALUATE: The scooter has the largest speed relative to the ground when it is moving to the right relative to the flatcar, since in that case the two velocities $\vec{v}_{S/F}$ and $\vec{v}_{F/G}$ are in the same direction and their magnitudes add.

3.37. IDENTIFY: Relative velocity problem. The time to walk the length of the moving sidewalk is the length divided by the velocity of the woman relative to the ground.

SET UP: Let W stand for the woman, G for the ground, and S for the sidewalk. Take the positive direction to be the direction in which the sidewalk is moving.

The velocities are $v_{W/G}$ (woman relative to the ground), $v_{W/S}$ (woman relative to the sidewalk), and $v_{S/G}$ (sidewalk relative to the ground).

Eq.(3.33) becomes $v_{W/G} = v_{W/S} + v_{S/G}$.

The time to reach the other end is given by $t = \dfrac{\text{distance traveled relative to ground}}{v_{W/G}}$

EXECUTE: **(a)** $v_{S/G} = 1.0$ m/s

$v_{W/S} = +1.5$ m/s

$v_{W/G} = v_{W/S} + v_{S/G} = 1.5 \text{ m/s} + 1.0 \text{ m/s} = 2.5 \text{ m/s}.$

$$t = \frac{35.0 \text{ m}}{v_{W/G}} = \frac{35.0 \text{ m}}{2.5 \text{ m/s}} = 14 \text{ s}.$$

(b) $v_{S/G} = 1.0$ m/s

$v_{W/S} = -1.5$ m/s

$v_{W/G} = v_{W/S} + v_{S/G} = -1.5 \text{ m/s} + 1.0 \text{ m/s} = -0.5 \text{ m/s}.$ (Since $v_{W/G}$ now is negative, she must get on the moving sidewalk at the opposite end from in part (a).)

$$t = \frac{-35.0 \text{ m}}{v_{W/G}} = \frac{-35.0 \text{ m}}{-0.5 \text{ m/s}} = 70 \text{ s}.$$

EVALUATE: Her speed relative to the ground is much greater in part (a) when she walks with the motion of the sidewalk.

3.38. IDENTIFY: Calculate the rower's speed relative to the shore for each segment of the round trip.

SET UP: The boat's speed relative to the shore is 6.8 km/h downstream and 1.2 km/h upstream.

EXECUTE: The walker moves a total distance of 3.0 km at a speed of 4.0 km/h, and takes a time of three fourths of an hour (45.0 min).

The total time the rower takes is $\dfrac{1.5 \text{ km}}{6.8 \text{ km/h}} + \dfrac{1.5 \text{ km}}{1.2 \text{ km/h}} = 1.47 \text{ h} = 88.2 \text{ min}.$

EVALUATE: It takes the rower longer, even though for half the distance his speed is greater than 4.0 km/h. The rower spends more time at the slower speed.

3.39. IDENTIFY: Apply the relative velocity relation.

SET UP: The relative velocities are $\vec{v}_{C/E}$, the canoe relative to the earth, $\vec{v}_{R/E}$, the velocity of the river relative to the earth and $\vec{v}_{C/R}$, the velocity of the canoe relative to the river.

EXECUTE: $\vec{v}_{C/E} = \vec{v}_{C/R} + \vec{v}_{R/E}$ and therefore $\vec{v}_{C/R} = \vec{v}_{C/E} - \vec{v}_{R/E}$. The velocity components of $\vec{v}_{C/R}$ are $-0.50 \text{ m/s} + (0.40 \text{ m/s})/\sqrt{2}$, east and $(0.40 \text{ m/s})/\sqrt{2}$, south, for a velocity relative to the river of 0.36 m/s, at $52.5°$ south of west.

EVALUATE: The velocity of the canoe relative to the river has a smaller magnitude than the velocity of the canoe relative to the earth.

3.40. IDENTIFY: Use the relation that relates the relative velocities.

SET UP: The relative velocities are the velocity of the plane relative to the ground, $\vec{v}_{P/G}$, the velocity of the plane relative to the air, $\vec{v}_{P/A}$, and the velocity of the air relative to the ground, $\vec{v}_{A/G}$. $\vec{v}_{P/G}$ must due west and $\vec{v}_{A/G}$ must be south. $v_{A/G} = 80$ km/h and $v_{P/A} = 320$ km/h. $\vec{v}_{P/G} = \vec{v}_{P/A} + \vec{v}_{A/G}$. The relative velocity addition diagram is given in Figure 3.40.

EXECUTE: **(a)** $\sin\theta = \dfrac{v_{A/G}}{v_{P/A}} = \dfrac{80 \text{ km/h}}{320 \text{ km/h}}$ and $\theta = 14°$, north of west.

(b) $v_{P/G} = \sqrt{v_{P/A}^2 - v_{A/G}^2} = \sqrt{(320 \text{ km/h})^2 - (80.0 \text{ km/h})^2} = 310 \text{ km/h}$.

EVALUATE: To travel due west the velocity of the plane relative to the air must have a westward component and also a component that is northward, opposite to the wind direction.

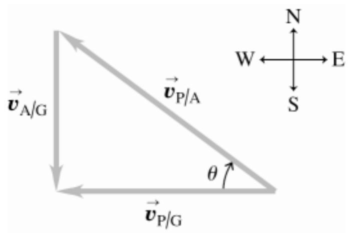


Figure 3.40

3.41. **IDENTIFY:** Relative velocity problem in two dimensions. His motion relative to the earth (time displacement) depends on his velocity relative to the earth so we must solve for this velocity.

(a) SET UP: View the motion from above.

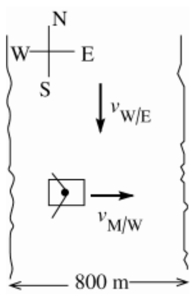


Figure 3.41a

The velocity vectors in the problem are:

$\vec{v}_{M/E}$, the velocity of the man relative to the earth

$\vec{v}_{W/E}$, the velocity of the water relative to the earth

$\vec{v}_{M/W}$, the velocity of the man relative to the water

The rule for adding these velocities is

$\vec{v}_{M/E} = \vec{v}_{M/W} + \vec{v}_{W/E}$

The problem tells us that $\vec{v}_{W/E}$ has magnitude 2.0 m/s and direction due south. It also tells us that $\vec{v}_{M/W}$ has magnitude 4.2 m/s and direction due east. The vector addition diagram is then as shown in Figure 3.41b

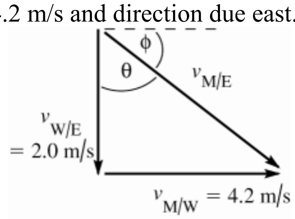


Figure 3.41b

This diagram shows the vector addition $\vec{v}_{M/E} = \vec{v}_{M/W} + \vec{v}_{W/E}$ and also has $\vec{v}_{M/W}$ and $\vec{v}_{W/E}$ in their specified directions. Note that the vector diagram forms a right triangle.

The Pythagorean theorem applied to the vector addition diagram gives $v_{M/E}^2 = v_{M/W}^2 + v_{W/E}^2$.

EXECUTE: $v_{M/E} = \sqrt{v_{M/W}^2 + v_{W/E}^2} = \sqrt{(4.2 \text{ m/s})^2 + (2.0 \text{ m/s})^2} = 4.7 \text{ m/s}$ $\tan\theta = \dfrac{v_{M/W}}{v_{W/E}} = \dfrac{4.2 \text{ m/s}}{2.0 \text{ m/s}} = 2.10;$ $\theta = 65°;$ or $\phi = 90° - \theta = 25°$. The velocity of the man relative to the earth has magnitude 4.7 m/s and direction 25° S of E.

(b) This requires careful thought. To cross the river the man must travel 800 m due east relative to the earth. The man's velocity relative to the earth is $\vec{v}_{M/E}$. But, from the vector addition diagram the eastward component of $v_{M/E}$ equals $v_{M/W} = 4.2$ m/s.

Thus $t = \dfrac{x - x_0}{v_x} = \dfrac{800 \text{ m}}{4.2 \text{ m/s}} = 190$ s.

(c) The southward component of $\vec{v}_{M/E}$ equals $v_{W/E} = 2.0$ m/s. Therefore, in the 190 s it takes him to cross the river the distance south the man travels relative to the earth is

$$y - y_0 = v_y t = (2.0 \text{ m/s})(190 \text{ s}) = 380 \text{ m}.$$

EVALUATE: If there were no current he would cross in the same time, $(800 \text{ m})/(4.2 \text{ m/s}) = 190$ s. The current carries him downstream but doesn't affect his motion in the perpendicular direction, from bank to bank.

3.42. **IDENTIFY:** Use the relation that relates the relative velocities.

SET UP: The relative velocities are the water relative to the earth, $\vec{v}_{W/E}$, the boat relative to the water, $\vec{v}_{B/W}$, and the boat relative to the earth, $\vec{v}_{B/E}$. $\vec{v}_{B/E}$ is due east, $\vec{v}_{W/E}$ is due south and has magnitude 2.0 m/s. $v_{B/W} = 4.2$ m/s. $\vec{v}_{B/E} = \vec{v}_{B/W} + \vec{v}_{W/E}$. The velocity addition diagram is given in Figure 3.42.

EXECUTE: **(a)** Find the direction of $\vec{v}_{\text{B/W}}$. $\sin\theta = \frac{v_{\text{W/E}}}{v_{\text{B/W}}} = \frac{2.0\text{ m/s}}{4.2\text{ m/s}}$. $\theta = 28.4°$, north of east.

(b) $v_{\text{B/E}} = \sqrt{v_{\text{B/W}}^2 - v_{\text{W/E}}^2} = \sqrt{(4.2\text{ m/s})^2 - (2.0\text{ m/s})^2} = 3.7\text{ m/s}$

(c) $t = \frac{800\text{ m}}{v_{\text{B/E}}} = \frac{800\text{ m}}{3.7\text{ m/s}} = 216\text{ s}$.

EVALUATE: It takes longer to cross the river in this problem than it did in Problem 3.41. In the direction straight across the river (east) the component of his velocity relative to the earth is lass than 4.2 m/s.

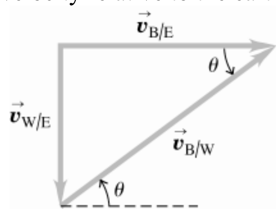


Figure 3.42

3.43. **IDENTIFY:** Relative velocity problem in two dimensions.

(a) SET UP: $\vec{v}_{\text{P/A}}$ is the velocity of the plane relative to the air. The problem states that $\vec{v}_{\text{P/A}}$ has magnitude 35 m/s and direction south.

$\vec{v}_{\text{A/E}}$ is the velocity of the air relative to the earth. The problem states that $\vec{v}_{\text{A/E}}$ is to the southwest (45° S of W) and has magnitude 10 m/s.

The relative velocity equation is $\vec{v}_{\text{P/E}} = \vec{v}_{\text{P/A}} + \vec{v}_{\text{A/E}}$.

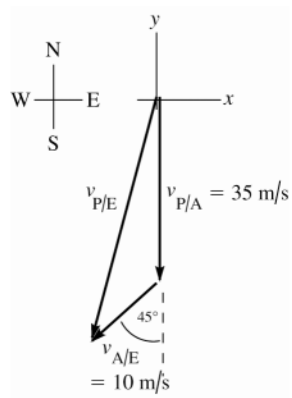


Figure 3.43a

EXECUTE: **(b)** $(v_{\text{P/A}})_x = 0$, $(v_{\text{P/A}})_y = -35\text{ m/s}$

$(v_{\text{A/E}})_x = -(10\text{ m/s})\cos 45° = -7.07\text{ m/s}$,

$(v_{\text{A/E}})_y = -(10\text{ m/s})\sin 45° = -7.07\text{ m/s}$

$(v_{\text{P/E}})_x = (v_{\text{P/A}})_x + (v_{\text{A/E}})_x = 0 - 7.07\text{ m/s} = -7.1\text{ m/s}$

$(v_{\text{P/E}})_y = (v_{\text{P/A}})_y + (v_{\text{A/E}})_y = -35\text{ m/s} - 7.07\text{ m/s} = -42\text{ m/s}$

(c)

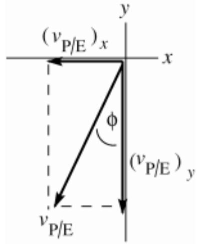


Figure 3.43b

$v_{\text{P/E}} = \sqrt{(v_{\text{P/E}})_x^2 + (v_{\text{P/E}})_y^2}$

$v_{\text{P/E}} = \sqrt{(-7.1\text{ m/s})^2 + (-42\text{ m/s})^2} = 43\text{ m/s}$

$\tan\phi = \frac{(v_{\text{P/E}})_x}{(v_{\text{P/E}})_y} = \frac{-7.1}{-42} = 0.169$

$\phi = 9.6°$; (9.6° west of south)

EVALUATE: The relative velocity addition diagram does not form a right triangle so the vector addition must be done using components. The wind adds both southward and westward components to the velocity of the plane relative to the ground.

3.44. IDENTIFY: Use Eqs.(2.17) and (2.18).

SET UP: At the maximum height $v_y = 0$.

EXECUTE: **(a)** $v_x = v_{0x} + \frac{\alpha}{3}t^3$, $v_y = v_{0y} + \beta t - \frac{\gamma}{2}t^2$, and $x = v_{0x}t + \frac{\alpha}{12}t^4$, $y = v_{0y}t + \frac{\beta}{2}t^2 - \frac{\gamma}{6}t^3$.

(b) Setting $v_y = 0$ yields a quadratic in t, $0 = v_{0y} + \beta t - \frac{\gamma}{2}t^2$, which has as the positive solution

$t = \frac{1}{\gamma}\left[\beta + \sqrt{\beta^2 + 2v_0\gamma}\right] = 13.59\ \text{s}$. Using this time in the expression for $y(t)$ gives a maximum height of 341 m.

(c) The path of the rocket is sketched in Figure 3.44.

(d) $y = 0$ gives $0 = v_{0y}t + \frac{\beta}{2}t^2 - \frac{\gamma}{6}t^3$ and $\frac{\gamma}{6}t^2 - \frac{\beta}{2}t - v_{0y} = 0$. The positive solution is $t = 20.73\ \text{s}$. For this t,

$x = 3.85 \times 10^4\ \text{m}$.

EVALUATE: The graph in part (c) shows the path is not symmetric about the highest point and the time to return to the ground is less than twice the time to the maximum height.

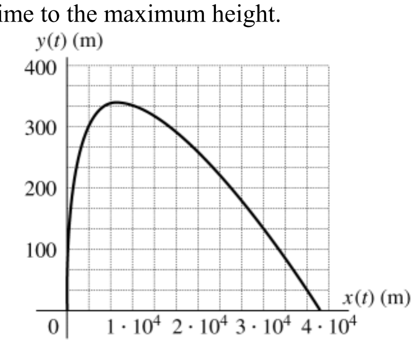


Figure 3.44

3.45. IDENTIFY: $\vec{v} = \frac{d\vec{r}}{dt}$ and $\vec{a} = \frac{d\vec{v}}{dt}$

SET UP: $\frac{d}{dt}(t^n) = nt^{n-1}$. At $t = 1.00\ \text{s}$, $a_x = 4.00\ \text{m/s}^2$ and $a_y = 3.00\ \text{m/s}^2$. At $t = 0$, $x = 0$ and $y = 50.0\ \text{m}$.

EXECUTE: **(a)** $v_x = \frac{dx}{dt} = 2Bt$. $a_x = \frac{dv_x}{dt} = 2B$, which is independent of t. $a_x = 4.00\ \text{m/s}^2$ gives $B = 2.00\ \text{m/s}^2$.

$v_y = \frac{dy}{dt} = 3Dt^2$. $a_y = \frac{dv_y}{dt} = 6Dt$. $a_y = 3.00\ \text{m/s}^2$ gives $D = 0.500\ \text{m/s}^2$. $x = 0$ at $t = 0$ gives $A = 0$. $y = 50.0$ m at $t = 0$ gives $C = 50.0\ \text{m}$.

(b) At $t = 0$, $v_x = 0$ and $v_y = 0$, so $\vec{v} = 0$. At $t = 0$, $a_x = 2B = 4.00\ \text{m/s}^2$ and $a_y = 0$, so $\vec{a} = (4.00\ \text{m/s}^2)\hat{i}$.

(c) At $t = 10.0\ \text{s}$, $v_x = 2(2.00\ \text{m/s}^2)(10.0\ \text{s}) = 40.0\ \text{m/s}$ and $v_y = 3(0.500\ \text{m/s}^3)(10.0\ \text{s})^2 = 150\ \text{m/s}$.

$v = \sqrt{v_x^2 + v_y^2} = 155\ \text{m/s}$.

(d) $x = (2.00\ \text{m/s}^2)(10.0\ \text{s})^2 = 200\ \text{m}$, $y = 50.0\ \text{m} + (0.500\ \text{m/s}^3)(10.0\ \text{s})^3 = 550\ \text{m}$. $\vec{r} = (200\ \text{m})\hat{i} + (550\ \text{m})\hat{j}$.

EVALUATE: The velocity and acceleration vectors as functions of time are

$\vec{v}(t) = (2Bt)\hat{i} + (3Dt^2)\hat{j}$ and $\vec{a}(t) = (2B)\hat{i} + (6Dt)\hat{j}$. The acceleration is not constant.

3.46. IDENTIFY: $\vec{r} = \vec{r}_0 + \int_0^t \vec{v}(t)dt$ and $a = \frac{d\vec{v}}{dt}$.

SET UP: At $t = 0$, $x_0 = 0$ and $y_0 = 0$.

EXECUTE: **(a)** Integrating, $\vec{r} = (\alpha t - \frac{\beta}{3}t^3)\hat{i} + (\frac{\gamma}{2}t^2)\hat{j}$. Differentiating, $\vec{a} = (-2\beta t)\hat{i} + \gamma\hat{j}$.

(b) The positive time at which $x = 0$ is given by $t^2 = 3\alpha/\beta$. At this time, the y-coordinate is

$$y = \frac{\gamma}{2}t^2 = \frac{3\alpha\gamma}{2\beta} = \frac{3(2.4\ \text{m/s})(4.0\ \text{m/s}^2)}{2(1.6\ \text{m/s}^3)} = 9.0\ \text{m}.$$

EVALUATE: The acceleration is not constant.

3.47. **IDENTIFY:** Once the rocket leaves the incline it moves in projectile motion. The acceleration along the incline determines the initial velocity and initial position for the projectile motion.

SET UP: For motion along the incline let $+x$ be directed up the incline. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = \sqrt{2(1.25\text{ m/s}^2)(200\text{ m})} = 22.36\text{ m/s}$. When the projectile motion begins the rocket has $v_0 = 22.36\text{ m/s}$ at $35.0°$ above the horizontal and is at a vertical height of $(200.0\text{ m})\sin 35.0° = 114.7\text{ m}$. For the projectile motion let $+x$ be horizontal to the right and let $+y$ be upward. Let $y = 0$ at the ground. Then $y_0 = 114.7\text{ m}$, $v_{0x} = v_0 \cos 35.0° = 18.32\text{ m/s}$, $v_{0y} = v_0 \sin 35.0° = 12.83\text{ m/s}$, $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. Let $x = 0$ at point *A*, so $x_0 = (200.0\text{ m})\cos 35.0° = 163.8\text{ m}$.

EXECUTE: **(a)** At the maximum height $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (12.83\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 8.40\text{ m}$ and $y = 114.7\text{ m} + 8.40\text{ m} = 123\text{ m}$. The maximum height above ground is 123 m.

(b) The time in the air can be calculated from the vertical component of the projectile motion: $y - y_0 = -114.7\text{ m}$, $v_{0y} = 12.83\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $(4.90\text{ m/s}^2)t^2 - (12.83\text{ m/s})t - 114.7\text{ m}$. The quadratic formula gives $t = \dfrac{1}{9.80}\left(12.83 \pm \sqrt{(12.83)^2 + 4(4.90)(114.7)}\right)\text{ s}$. The positive root is $t = 6.32\text{ s}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (18.32\text{ m/s})(6.32\text{ s}) = 115.8\text{ m}$ and $x = 163.8\text{ m} + 115.8\text{ m} = 280\text{ m}$. The horizontal range of the rocket is 280 m.

EVALUATE: The expressions for *h* and *R* derived in Example 3.8 do not apply here. They are only for a projectile fired on level ground.

3.48. **IDENTIFY:** The person moves in projectile motion. Use the results in Example 3.8 to determine how *T*, *h* and *D* depend on *g* and set up a ratio.

SET UP: From Example 3.8, the time in the air is $t = \dfrac{2v_0 \sin\alpha_0}{g}$, the maximum height is $h = \dfrac{v_0^2 \sin^2\alpha_0}{2g}$ and the horizontal range (called *D* in the problem) is $D = \dfrac{v_0^2 \sin 2\alpha_0}{g}$. The person has the same v_0 and α_0 on Mars as on the earth.

EXECUTE: $tg = 2v_0 \sin\alpha_0$, which is constant, so $t_E g_E = t_M g_M$. $t_M = \left(\dfrac{g_E}{g_M}\right)t_E = \left(\dfrac{g_E}{0.379 g_E}\right)t_E = 2.64 t_E$.

$hg = \dfrac{v_0^2 \sin^2\alpha_0}{2}$, which is constant, so $h_E g_E = h_M g_M$. $h_M = \left(\dfrac{g_E}{g_M}\right)h_E = 2.64 h_E$. $Dg = v_0^2 \sin 2\alpha_0$, which is constant, so $D_E g_E = D_M g_M$. $D_M = \left(\dfrac{g_E}{g_M}\right)D_E = 2.64 D_E$.

EVALUATE: All three quantities are proportional to $1/g$ so all increase by the same factor of $g_E/g_M = 2.64$.

3.49. **IDENTIFY:** The range for a projectile that lands at the same height from which it was launched is $R = \dfrac{v_0^2 \sin 2\alpha}{g}$.

SET UP: The maximum range is for $\alpha = 45°$.

EXECUTE: Assuming $\alpha = 45°$, and $R = 50\text{ m}$, $v_0 = \sqrt{gR} = 22\text{ m/s}$.

EVALUATE: We have assumed that debris was launched at all angles, including the angle of $45°$ that gives maximum range.

3.50. **IDENTIFY:** The velocity has a horizontal tangential component and a vertical component. The vertical component of acceleration is zero and the horizontal component is $a_{\text{rad}} = \dfrac{v_x^2}{R}$

SET UP: Let $+y$ be upward and $+x$ be in the direction of the tangential velocity at the instant we are considering.

EXECUTE: **(a)** The bird's tangential velocity can be found from

$$v_x = \frac{\text{circumference}}{\text{time of rotation}} = \frac{2\pi(8.00\text{ m})}{5.00\text{ s}} = \frac{50.27\text{ m}}{5.00\text{ s}} = 10.05\text{ m/s}$$

Thus its velocity consists of the components $v_x = 10.05$ m/s and $v_y = 3.00$ m/s. The speed relative to the ground is then $v = \sqrt{v_x^2 + v_y^2} = 10.5$ m/s.

(b) The bird's speed is constant, so its acceleration is strictly centripetal–entirely in the horizontal direction, toward the center of its spiral path–and has magnitude $a_{\text{rad}} = \frac{v_x^2}{r} = \frac{(10.05\text{ m/s})^2}{8.00\text{ m}} = 12.6\text{ m/s}^2$.

(c) Using the vertical and horizontal velocity components $\theta = \tan^{-1}\frac{3.00\text{ m/s}}{10.05\text{ m/s}} = 16.6°$.

EVALUATE: The angle between the bird's velocity and the horizontal remains constant as the bird rises.

3.51. **IDENTIFY:** Take $+y$ to be downward. Both objects have the same vertical motion, with v_{0y} and $a_y = +g$. Use constant acceleration equations for the x and y components of the motion.

SET UP: Use the vertical motion to find the time in the air:

$v_{0y} = 0,\ a_y = 9.80\text{ m/s}^2,\ y - y_0 = 25\text{ m},\ t = ?$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 2.259$ s

During this time the dart must travel 90 m, so the horizontal component of its velocity must be

$$v_{0x} = \frac{x - x_0}{t} = \frac{90\text{ m}}{2.25\text{ s}} = 40\text{ m/s}$$

EVALUATE: Both objects hit the ground at the same time. The dart hits the monkey for any muzzle velocity greater than 40 m/s.

3.52. **IDENTIFY:** The person moves in projectile motion. Her vertical motion determines her time in the air.

SET UP: Take $+y$ upward. $v_{0x} = 15.0$ m/s, $v_{0y} = +10.0$ m/s, $a_x = 0$, $a_y = -9.80\text{ m/s}^2$.

EXECUTE: **(a)** Use the vertical motion to find the time in the air: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = -30.0$ m gives $-30.0\text{ m} = (10.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2$. The quadratic formula gives

$t = \frac{1}{2(4.9)}\left(+10.0 \pm \sqrt{(-10.0)^2 - 4(4.9)(-30)}\right)$ s. The positive solution is $t = 3.70$ s. During this time she travels a horizontal distance $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (15.0\text{ m/s})(3.70\text{ s}) = 55.5$ m. She will land 55.5 m south of the point where she drops from the helicopter and this is where the mats should have been placed.

(b) The x-t, y-t, v_x-t and v_y-t graphs are sketched in Figure 3.52.

EVALUATE: If she had dropped from rest at a height of 30.0 m it would have taken her $t = \sqrt{\frac{2(30.0\text{ m})}{9.80\text{ m/s}^2}} = 2.47$ s. She is in the air longer than this because she has an initial vertical component of velocity that is upward.

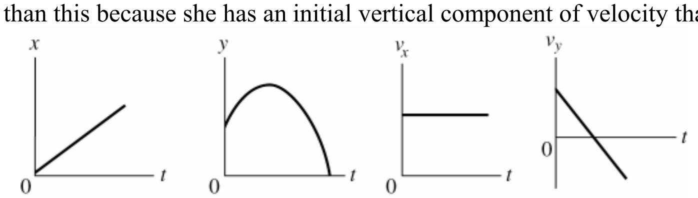


Figure 3.52

3.53. **IDENTIFY:** The cannister moves in projectile motion. Its initial velocity is horizontal. Apply constant acceleration equations for the x and y components of motion.

SET UP:

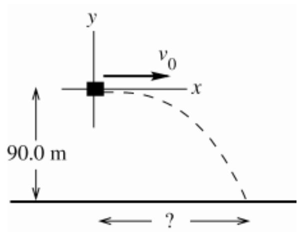


Figure 3.53

Take the origin of coordinates at the point where the canister is released. Take $+y$ to be upward. The initial velocity of the canister is the velocity of the plane, 64.0 m/s in the $+x$-direction.

Use the vertical motion to find the time of fall:
$t = ?,\ v_{0y} = 0,\ a_y = -9.80\text{ m/s}^2,\ y - y_0 = -90.0\text{ m}$ (When the canister reaches the ground it is 90.0 m below the origin.)

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$$

EXECUTE: Since $v_{0y} = 0,\ t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(-90.0\text{ m})}{-9.80\text{ m/s}^2}} = 4.286\text{ s}.$

SET UP: Then use the horizontal component of the motion to calculate how far the canister falls in this time:
$x - x_0 = ?,\ a_x - 0,\ v_{0x} = 64.0\text{ m/s},$

EXECUTE: $x - x_0 = v_0 t + \tfrac{1}{2}at^2 = (64.0\text{ m/s})(4.286\text{ s}) + 0 = 274\text{ m}.$

EVALUATE: The time it takes the cannister to fall 90.0 m, starting from rest, is the time it travels horizontally at constant speed.

3.54. **IDENTIFY:** The equipment moves in projectile motion. The distance D is the horizontal range of the equipment plus the distance the ship moves while the equipment is in the air.

SET UP: For the motion of the equipment take $+x$ to be to the right and $+y$ to be upwards. Then $a_x = 0$, $a_y = -9.80\text{ m/s}^2$, $v_{0x} = v_0\cos\alpha_0 = 7.50\text{ m/s}$ and $v_{0y} = v_0\sin\alpha_0 = 13.0\text{ m/s}$. When the equipment lands in the front of the ship, $y - y_0 = -8.75\text{ m}$.

EXECUTE: Use the vertical motion of the equipment to find its time in the air: $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives

$t = \dfrac{1}{9.80}\left(13.0 \pm \sqrt{(-13.0)^2 + 4(4.90)(8.75)}\right)\text{ s}$. The positive root is $t = 3.21\text{ s}$. The horizontal range of the equipment is $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 = (7.50\text{ m/s})(3.21\text{ s}) = 24.1\text{ m}$. In 3.21 s the ship moves a horizontal distance $(0.450\text{ m/s})(3.21\text{ s}) = 1.44\text{ m}$, so $D = 24.1\text{ m} + 1.44\text{ m} = 25.5\text{ m}$.

EVALUATE: The equation $R = \dfrac{v_0^2\sin 2\alpha_0}{g}$ from Example 3.8 can't be used because the starting and ending points of the projectile motion are at different heights.

3.55. **IDENTIFY:** Projectile motion problem.

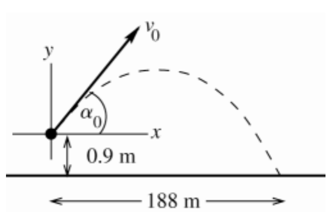


Figure 3.55

Take the origin of coordinates at the point where the ball leaves the bat, and take $+y$ to be upward.
$v_{0x} = v_0\cos\alpha_0$
$v_{0y} = v_0\sin\alpha_0,$
but we don't know v_0.

Write down the equation for the horizontal displacement when the ball hits the ground and the corresponding equation for the vertical displacement. The time t is the same for both components, so this will give us two equations in two unknowns (v_0 and t).

(a) SET UP: <u>y-component</u>:

$a_y = -9.80\ \text{m/s}^2$, $y - y_0 = -0.9\ \text{m}$, $v_{0y} = v_0 \sin 45°$

$y - y_0 = v_{0y} + t\tfrac{1}{2}a_y t^2$

EXECUTE: $-0.9\ \text{m} = (v_0 \sin 45°)t + \tfrac{1}{2}(-9.80\ \text{m/s}^2)t^2$

SET UP: <u>x-component</u>:

$a_x = 0$, $x - x_0 = 188\ \text{m}$, $v_{0x} = v_0 \cos 45°$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$

EXECUTE: $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{188\ \text{m}}{v_0 \cos 45°}$

Put the expression for t from the x-component motion into the y-component equation and solve for v_0. (Note that $\sin 45° = \cos 45°$.)

$$-0.9\ \text{m} = (v_0 \sin 45°)\left(\frac{188\ \text{m}}{v_0 \cos 45°}\right) - (4.90\ \text{m/s}^2)\left(\frac{188\ \text{m}}{v_0 \cos 45°}\right)^2$$

$$4.90\ \text{m/s}^2\left(\frac{188\ \text{m}}{v_0 \cos 45°}\right)^2 = 188\ \text{m} + 0.9\ \text{m} = 188.9\ \text{m}$$

$$\left(\frac{v_0 \cos 45°}{188\ \text{m}}\right)^2 = \frac{4.90\ \text{m/s}^2}{188.9\ \text{m}},\quad v_0 = \left(\frac{188\ \text{m}}{\cos 45°}\right)\sqrt{\frac{4.90\ \text{m/s}^2}{188.9\ \text{m}}} = 42.8\ \text{m/s}$$

(b) Use the horizontal motion to find the time it takes the ball to reach the fence:

SET UP: <u>x-component</u>:

$x - x_0 = 116\ \text{m}$, $a_x = 0$, $v_{0x} = v_0 \cos 45° = (42.8\ \text{m/s})\cos 45° = 30.3\ \text{m/s}$, $t = ?$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$

EXECUTE: $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{116\ \text{m}}{30.3\ \text{m/s}} = 3.83\ \text{s}$

SET UP: Find the vertical displacement of the ball at this t:

<u>y-component</u>:

$y - y_0 = ?$, $a_y = -9.80\ \text{m/s}^2$, $v_{0y} = v_0 \sin 45° = 30.3\ \text{m/s}$, $t = 3.83\ \text{s}$

$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$

EXECUTE: $y - y_0 = (30.3\ \text{s})(3.83\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(3.83\ \text{s})^2$

$y - y_0 = 116.0\ \text{m} - 71.9\ \text{m} = +44.1\ \text{m}$, above the point where the ball was hit. The height of the ball above the ground is $44.1\ \text{m} + 0.90\ \text{m} = 45.0\ \text{m}$. It's height then above the top of the fence is $45.0\ \text{m} - 3.0\ \text{m} = 42.0\ \text{m}$.

EVALUATE: With $v_0 = 42.8\ \text{m/s}$, $v_{0y} = 30.3\ \text{m/s}$ and it takes the ball 6.18 s to return to the height where it was hit and only slightly longer to reach a point 0.9 m below this height. $t = (188\ \text{m})/(v_0 \cos 45°)$ gives $t = 6.21\ \text{s}$, which agrees with this estimate. The ball reaches its maximum height approximately $(188\ \text{m})/2 = 94\ \text{m}$ from home plate, so at the fence the ball is not far past its maximum height of 47.6 m, so a height of 45.0 m at the fence is reasonable.

3.56. IDENTIFY: The water moves in projectile motion.

SET UP: Let $x_0 = y_0 = 0$ and take $+y$ to be positive. $a_x = 0$, $a_y = -g$.

EXECUTE: The equations of motions are $y = (v_0\ \sin\alpha)t - \tfrac{1}{2}gt^2$ and $x = (v_0\ \cos\alpha)t$. When the water goes in the tank for the *minimum* velocity, $y = 2D$ and $x = 6D$. When the water goes in the tank for the *maximum* velocity, $y = 2D$ and $x = 7D$. In both cases, $\sin\alpha = \cos\alpha = \sqrt{2}/2$.

To reach the *minimum* distance: $6D = \dfrac{\sqrt{2}}{2}v_0 t$, and $2D = \dfrac{\sqrt{2}}{2}v_0 t - \tfrac{1}{2}gt^2$. Solving the first equation for t gives $t = \dfrac{6D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 6D - \tfrac{1}{2}g\left(\dfrac{6D\sqrt{2}}{v_0}\right)^2$. Solving this for v_0 gives $v_0 = 3\sqrt{gD}$.

To reach the *maximum* distance: $7D = \frac{\sqrt{2}}{2} v_0 t$, and $2D = \frac{\sqrt{2}}{2} v_0 t - \frac{1}{2} g t^2$. Solving the first equation for *t* gives $t = \frac{7D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 7D - \frac{1}{2} g \left(\frac{7D\sqrt{2}}{v_0} \right)^2$. Solving this for v_0 gives $v_0 = \sqrt{49gD/5} = 3.13\sqrt{gD}$, which, as expected, is larger than the previous result.

EVALUATE: A launch speed of $v_0 = \sqrt{6}\sqrt{gD} = 2.45\sqrt{gD}$ is required for a horizontal range of 6*D*. The minimum speed required is greater than this, because the water must be at a height of at least 2*D* when it reaches the front of the tank.

3.57. **IDENTIFY:** The equations for *h* and *R* from Example 3.8 can be used.

SET UP: $h = \frac{v_0^2 \sin^2 \alpha_0}{2g}$ and $R = \frac{v_0^2 \sin 2\alpha_0}{g}$. If the projectile is launched straight up, $\alpha_0 = 90°$.

EXECUTE: **(a)** $h = \frac{v_0^2}{2g}$ and $v_0 = \sqrt{2gh}$.

(b) Calculate α_0 that gives a maximum height of *h* when $v_0 = 2\sqrt{2gh}$. $h = \frac{8gh \sin^2 \alpha_0}{2g} = 4h \sin^2 \alpha_0$. $\sin \alpha_0 = \frac{1}{2}$ and $\alpha_0 = 30.0°$.

(c) $R = \frac{\left(2\sqrt{2gh}\right)^2 \sin 60.0°}{g} = 6.93h$.

EVALUATE: $\frac{v_0^2}{g} = \frac{2h}{\sin^2 \alpha_0}$ so $R = \frac{2h \sin(2\alpha_0)}{\sin^2 \alpha_0}$. For a given α_0 , *R* increases when *h* increases. For $\alpha_0 = 90°$, $R = 0$ and for $\alpha_0 = 0°$, $h = 0$ and $R = 0$. For $\alpha_0 = 45°$, $R = 4h$.

3.58. **IDENTIFY:** To clear the bar the ball must have a height of 10.0 ft when it has a horizontal displacement of 36.0 ft. The ball moves as a projectile. When v_0 is very large, the ball reaches the goal posts in a very short time and the acceleration due to gravity causes negligible downward displacement.

SET UP: $36.0 \text{ ft} = 10.97 \text{ m}$; $10.0 \text{ ft} = 3.048 \text{ m}$. Let $+x$ be to the right and $+y$ be upward, so $a_x = 0$, $a_y = -g$, $v_{0x} = v_0 \cos \alpha_0$ and $v_{0y} = v_0 \sin \alpha_0$

EXECUTE: **(a)** The ball cannot be aimed lower than directly at the bar. $\tan \alpha_0 = \frac{10.0 \text{ ft}}{36.0 \text{ ft}}$ and $\alpha_0 = 15.5°$.

(b) $x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$ gives $t = \frac{x - x_0}{v_{0x}} = \frac{x - x_0}{v_0 \cos \alpha_0}$. Then $y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$ gives

$$y - y_0 = (v_0 \sin \alpha_0) \left(\frac{x - x_0}{v_0 \cos \alpha_0} \right) - \frac{1}{2} g \frac{(x - x_0)^2}{v_0^2 \cos^2 \alpha_0} = (x - x_0) \tan \alpha_0 - \frac{1}{2} g \frac{(x - x_0)^2}{v_0^2 \cos^2 \alpha_0} .$$

$$v_0 = \frac{(x - x_0)}{\cos \alpha_0} \sqrt{\frac{g}{2[(x - x_0)\tan \alpha_0 - (y - y_0)]}} = \frac{10.97 \text{ m}}{\cos 45.0°} \sqrt{\frac{9.80 \text{ m/s}^2}{2[10.97 \text{ m} - 3.048 \text{ m}]}} = 12.2 \text{ m/s}$$

EVALUATE: With the v_0 in part (b) the horizontal range of the ball is $R = \frac{v_0^2 \sin 2\alpha_0}{g} = 15.2 \text{ m} = 49.9 \text{ ft}$. The ball reaches the highest point in its trajectory when $x - x_0 = R/2$, so when it reaches the goal posts it is on its way down.

3.59. **IDENTIFY:** Apply Eq.(3.27) and solve for *x*.

SET UP: The change in height is $y = -h$.

EXECUTE: **(a)** We get a quadratic equation in *x*, the solution to which is

$$x = \frac{v_0^2 \cos \alpha_0}{g} \left[\tan^2 \alpha_0 + \frac{2gh}{v_0^2 \cos \alpha_0} \right] = \frac{v_0 \cos \alpha_0}{g} \left[v_0 \sin \alpha_0 + \sqrt{v_0^2 \sin^2 \alpha_0 + 2gh} \right] .$$

If $h = 0$, the square root reduces to $v_0 \sin \alpha_0$, and $x = R$.

(b) The expression for x becomes $x = (10.2\text{ m})\cos\alpha_0 + [\sin^2\alpha_0 \; + \sqrt{\sin^2\alpha_0 + 0.98}]$. The graph of x as a function of α_0 is sketched in Figure 3.59. The angle $\alpha_0 = 90°$ corresponds to the projectile being launched straight up, and there is no horizontal motion. If $\alpha_0 = 0$, the projectile moves horizontally until it has fallen the distance h.

(d) The graph shows that the maximum horizontal distance is for an angle less than $45°$.

EVALUATE: For $\alpha_0 = 45°$ the x and y components of the initial velocity are equal. For $\alpha_0 < 45°$ the x component of the initial velocity is less than the y component. Height comes from the initial position and less vertical component of initial velocity is needed for the maximum range.

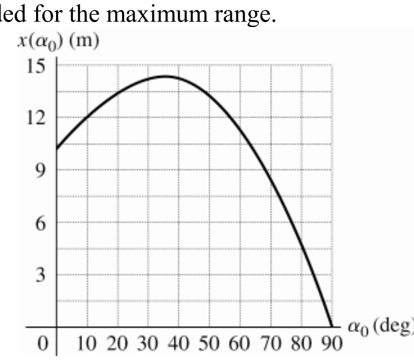


Figure 3.59

3.60. **IDENTIFY:** The snowball moves in projectile motion. In part (a) the vertical motion determines the time in the air. In part (c), find the height of the snowball above the ground after it has traveled horizontally 4.0 m.

SET UP: Let $+y$ be downward. $a_x = 0$, $a_y = +9.80\text{ m/s}^2$. $v_{0x} = v_0\cos\theta_0 = 5.36\text{ m/s}$, $v_{0y} = v_0\sin\theta_0 = 4.50\text{ m/s}$.

EXECUTE: **(a)** Use the vertical motion to find the time in the air: $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ with $y - y_0 = 14.0\text{ m}$ gives $14.0\text{ m} = (4.50\text{ m/s})t + (4.9\text{ m/s}^2)t^2$. The quadratic formula gives $t = \frac{1}{2(4.9)}\left(-4.50 \pm \sqrt{(4.50)^2 - 4(4.9)(-14.0)}\right)\text{ s}$. The positive root is $t = 1.29\text{ s}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2 = (5.36\text{ m/s})(1.29\text{ s}) = 6.91\text{ m}$.

(b) The x-t, y-t, v_x-t and v_y-t graphs are sketched in Figure 3.60.

(c) $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$ gives $t = \frac{x - x_0}{v_{0x}} = \frac{4.0\text{ m}}{5.36\text{ m/s}} = 0.746\text{ s}$. In this time the snowball travels downward a distance $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 = 6.08\text{ m}$ and is therefore $14.0\text{ m} - 6.08\text{ m} = 7.9\text{ m}$ above the ground. The snowball passes well above the man and doesn't hit him.

EVALUATE: If the snowball had been released from rest at a height of 14.0 m it would have reached the ground in $t = \sqrt{\frac{2(14.0\text{ m})}{9.80\text{ m/s}^2}} = 1.69\text{ s}$. The snowball reaches the ground in a shorter time than this because of its initial downward component of velocity.

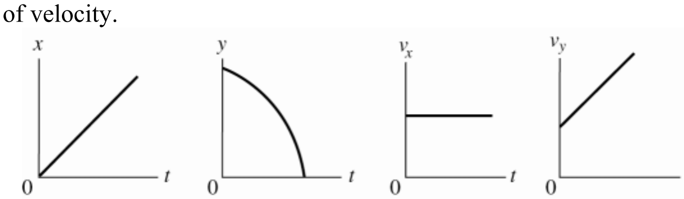


Figure 3.60

3.61. **(a) IDENTIFY** and **SET UP:** Use the equation derived in Example 3.8:

$$R = (v_0\cos\alpha_0)\left(\frac{2v_0\sin\alpha_0}{g}\right)$$

Call the range R_1 when the angle is α_0 and R_2 when the angle is $90° - \alpha$.

$$R_1 = (v_0 \cos\alpha_0)\left(\frac{2v_0 \sin\alpha_0}{g}\right)$$

$$R_2 = (v_0 \cos(90° - \alpha_0))\left(\frac{2v_0 \sin(90° - \alpha_0)}{g}\right)$$

The problem asks us to show that $R_1 = R_2$.

Execute: We can use the trig identities in Appendix B to show:

$\cos(90° - \alpha_0) = \cos(\alpha_0 - 90°) = \sin\alpha_0$

$\sin(90° - \alpha_0) = -\sin(\alpha_0 - 90°) = -(-\cos\alpha_0) = +\cos\alpha_0$

Thus $R_2 = (v_0 \sin\alpha_0)\left(\frac{2v_0 \cos\alpha_0}{g}\right) = (v_0 \cos\alpha_0)\left(\frac{2v_0 \sin\alpha_0}{g}\right) = R_1$.

(b) $R = \frac{v_0^2 \sin 2\alpha_0}{g}$ so $\sin 2\alpha_0 = \frac{Rg}{v_0^2} = \frac{(0.25\text{ m})(9.80\text{ m/s}^2)}{(2.2\text{ m/s})^2}$.

This gives $\alpha = 15°$ or $75°$.

Evaluate: $R = (v_0^2 \sin 2\alpha_0)/g$, so the result in part (a) requires that $\sin^2(2\alpha_0) = \sin^2(180° - 2\alpha_0)$, which is true. (Try some values of α_0 and see!)

3.62. **Identify:** Mary Belle moves in projectile motion.

Set Up: Let $+y$ be upward. $a_x = 0$, $a_y = -g$.

Execute: **(a)** Eq.(3.27) with $x = 8.2$ m, $y = 6.1$ m and $\alpha_0 = 53°$ gives $v_0 = 13.8$ m/s.

(b) When she reached Joe Bob, $t = \frac{8.2\text{ m}}{v_0 \cos 53°} = 0.9874$ s. $v_x = v_{0x} = 8.31$ m/s and $v_y = v_{0y} + a_y t = +1.34$ m/s.

$v = 8.4$ m/s, at an angle of $9.16°$.

(c) The graph of $v_x(t)$ is a horizontal line. The other graphs are sketched in Figure 3.62.

(d) Use Eq. (3.27), which becomes $y = (1.327)x - (0.071115\text{ m}^{-1})x^2$. Setting $y = -8.6$ m gives $x = 23.8$ m as the positive solution.

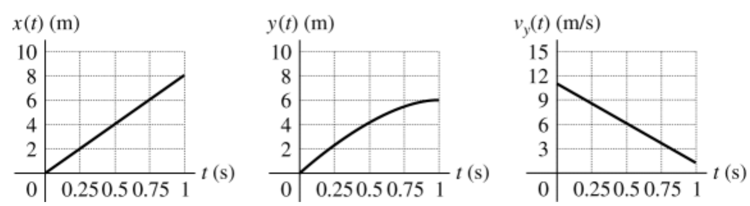


Figure 3.62

3.63. **(a) Identify:** Projectile motion.

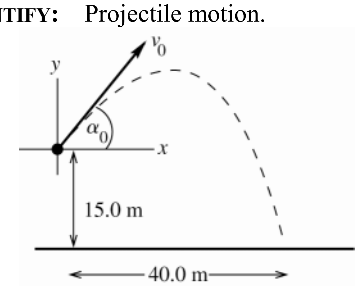


Figure 3.63

Take the origin of coordinates at the top of the ramp and take $+y$ to be upward. The problem specifies that the object is displaced 40.0 m to the right when it is 15.0 m below the origin.

We don't know *t*, the time in the air, and we don't know v_0. Write down the equations for the horizontal and vertical displacements. Combine these two equations to eliminate one unknown.

Set Up: <u>*y*-component</u>:

$y - y_0 = -15.0$ m, $a_y = -9.80$ m/s^2, $v_{0y} = v_0 \sin 53.0°$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

Execute: $-15.0\text{ m} = (v_0 \sin 53.0°)t - (4.90\text{ m/s}^2)t^2$

SET UP: <u>x-component</u>:

$x - x_0 = 40.0\text{ m},\ \ a_x = 0,\ \ v_{0x} = v_0\cos 53.0°$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$

EXECUTE: $40.0\text{ m} = (v_0 t)\cos 53.0°$

The second equation says $v_0 t = \dfrac{40.0\text{ m}}{\cos 53.0°} = 66.47\text{ m}.$

Use this to replace $v_0 t$ in the first equation:

$$-15.0\text{ m} = (66.47\text{ m})\sin 53° - (4.90\text{ m/s}^2)t^2$$

$$t = \sqrt{\frac{(66.46\text{ m})\sin 53° + 15.0\text{ m}}{4.90\text{ m/s}^2}} = \sqrt{\frac{68.08\text{ m}}{4.90\text{ m/s}^2}} = 3.727\text{ s}.$$

Now that we have t we can use the x-component equation to solve for v_0:

$$v_0 = \frac{40.0\text{ m}}{t\cos 53.0°} = \frac{40.0\text{ m}}{(3.727\text{ s})\cos 53.0°} = 17.8\text{ m/s}.$$

EVALUATE: Using these values of v_0 and t in the $y = y_0 = v_{0y} + \tfrac{1}{2}a_y t^2$ equation verifies that $y - y_0 = -15.0\text{ m}$.

(b) IDENTIFY: $v_0 = (17.8\text{ m/s})/2 = 8.9\text{ m/s}$

This is less than the speed required to make it to the other side, so he lands in the river.
Use the vertical motion to find the time it takes him to reach the water:

SET UP: $y - y_0 = -100\text{ m};\ \ v_{0y} = +v_0\sin 53.0° = 7.11\text{ m/s};\ \ a_y = -9.80\text{ m/s}^2$

$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $-100 = 7.11t - 4.90t^2$

EXECUTE: $4.90t^2 - 7.11t - 100 = 0$ and $t = \tfrac{1}{9.80}\left(7.11 \pm \sqrt{(7.11)^2 - 4(4.90)(-100)}\right)$

$t = 0.726\text{ s} \pm 4.57\text{ s}$ so $t = 5.30\text{ s}.$

The horizontal distance he travels in this time is

$$x - x_0 = v_{0x}t = (v_0\cos 53.0°)t = (5.36\text{ m/s})(5.30\text{ s}) = 28.4\text{ m}.$$

He lands in the river a horizontal distance of 28.4 m from his launch point.

EVALUATE: He has half the minimum speed and makes it only about halfway across.

3.64. IDENTIFY: The rock moves in projectile motion.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -g$. Eqs.(3.22) and (3.23) give v_x and v_y.

EXECUTE: Combining equations 3.25, 3.22 and 3.23 gives

$v^2 = v_0^2\cos^2\alpha_0 + (v_0\sin\alpha_0 - gt)^2 = v_0^2(\sin^2\alpha_0 + \cos^2\alpha_0) - 2v_0\sin\alpha_0 gt + (gt)^2$.

$v^2 = v_0^2 - 2g(v_0\sin\alpha_0 t - \dfrac{1}{2}gt^2) = v_0^2 - 2gy$, where Eq.(3.21) has been used to eliminate t in favor of y. For the case of a rock thrown from the roof of a building of height h, the speed at the ground is found by substituting $y = -h$ into the above expression, yielding $v = \sqrt{v_0^2 + 2gh}$, which is independent of α_0.

EVALUATE: This result, as will be seen in the chapter dealing with conservation of energy (Chapter 7), is valid for any y, positive, negative or zero, as long as $v_0^2 - 2gy > 0$.

3.65. IDENTIFY and SET UP: Take $+y$ to be upward. The rocket moves with projectile motion, with $v_{0y} = +40.0\text{ m/s}$ and $v_{0x} = 30.0\text{ m/s}$ relative to the ground. The vertical motion of the rocket is unaffected by its horizontal velocity.

EXECUTE: **(a)** $v_y = 0$ (at maximum height), $v_{0y} = +40.0\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$, $y - y_0 = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = 81.6\text{ m}$

(b) Both the cart and the rocket have the same constant horizontal velocity, so both travel the same horizontal distance while the rocket is in the air and the rocket lands in the cart.

(c) Use the vertical motion of the rocket to find the time it is in the air.

$v_{0y} = 40\text{ m/s},\ \ a_y = -9.80\text{ m/s}^2,\ \ v_y = -40\text{ m/s},\ \ t = ?$

$v_y = v_{0y} + a_y t$ gives $t = 8.164\text{ s}$

Then $x - x_0 = v_{0x}t = (30.0\text{ m/s})(8.164\text{ s}) = 245\text{ m}.$

(d) Relative to the ground the rocket has initial velocity components $v_{0x} = 30.0$ m/s and $v_{0y} = 40.0$ m/s, so it is traveling at $53.1°$ above the horizontal.
(e) (i)

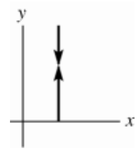


Figure 3.65a

Relative to the cart, the rocket travels straight up and then straight down
(ii)

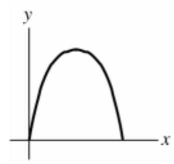


Figure 3.65b

Relative to the ground the rocket travels in a parabola.
EVALUATE: Both the cart and rocket have the same constant horizontal velocity. The rocket lands in the cart.

3.66. **IDENTIFY:** The ball moves in projectile motion.
SET UP: The woman and ball travel for the same time and must travel the same horizontal distance, so for the ball $v_{0x} = 6.00\text{ m/s}$.

EXECUTE: **(a)** $v_{0x} = v_0 \cos\theta_0$. $\cos\theta_0 = \dfrac{v_{0x}}{v_0} = \dfrac{6.00\text{ m/s}}{20.0\text{ m/s}}$ and $\theta_0 = 72.5°$.

(b) Relative to the ground the ball moves in a parabola. The ball and the runner have the same horizontal component of velocity, so relative to the runner the ball has only vertical motion. The trajectories as seen by each observer are sketched in Figure 3.66.
EVALUATE: The ball could be thrown with a different speed, so long as the angle at which it was thrown was adjusted to keep $v_{0x} = 6.00\text{ m/s}$.

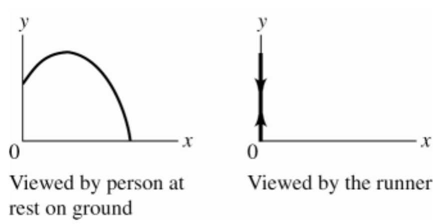


Figure 3.66

3.67. **IDENTIFY:** The boulder moves in projectile motion.
SET UP: Take $+y$ downward. $v_{0x} = v_0$, $v_{0y} = 0$. $a_x = 0$, $a_y = +9.80\text{ m/s}^2$.
EXECUTE: **(a)** Use the vertical motion to find the time for the boulder to reach the level of the lake:
$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ with $y - y_0 = +20$ m gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(20\text{ m})}{9.80\text{ m/s}^2}} = 2.02$ s. The rock must travel horizontally 100 m during this time. $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$ gives $v_0 = v_{0x} = \dfrac{x-x_0}{t} = \dfrac{100\text{ m}}{2.02\text{ s}} = 49.5\text{ m/s}$

(b) In going from the edge of the cliff to the plain, the boulder travels downward a distance of $y - y_0 = 45$ m.
$t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(45\text{ m})}{9.80\text{ m/s}^2}} = 3.03$ s and $x - x_0 = v_{0x}t = (49.5\text{ m/s})(3.03\text{ s}) = 150$ m. The rock lands $150\text{ m} - 100\text{ m} = 50$ m beyond the foot of the dam.
EVALUATE: The boulder passes over the dam 2.02 s after it leaves the cliff and then travels an additional 1.01 s before landing on the plain. If the boulder has an initial speed that is less than 49 m/s, then it lands in the lake.

3.68. **IDENTIFY:** The bagels move in projectile motion. Find Henrietta's location when the bagels reach the ground, and require the bagels to have this horizontal range.

Set Up: Let $+y$ be downward and let $x_0 = y_0 = 0$. $a_x = 0$, $a_y = +g$. When the bagels reach the ground, $y = 43.9\text{ m}$.

Execute: **(a)** When she catches the bagels, Henrietta has been jogging for 9.00 s plus the time for the bagels to fall 43.9 m from rest. Get the time to fall: $y = \frac{1}{2}gt^2$, $43.9\text{ m} = \frac{1}{2}(9.80\text{ m/s}^2)t^2$ and $t = 2.99\text{ s}$. So, she has been jogging for $9.00\text{ s} + 2.99\text{ s} = 12.0\text{ s}$. During this time she has gone $x = vt = (3.05\text{ m/s})(12.0\text{ s}) = 36.6\text{ m}$. Bruce must throw the bagels so they travel 36.6 m horizontally in 2.99 s. This gives $x = vt$. $36.6\text{ m} = v(2.99\text{ s})$ and $v = 12.2\text{ m/s}$.

(b) 36.6 m from the building.

Evaluate: If $v > 12.2\text{ m/s}$ the bagels land in front of her and if $v < 12.2\text{ m/s}$ they land behind her. There is a range of velocities greater than 12.2 m/s for which she would catch the bagels in the air, at some height above the sidewalk.

3.69. Identify: The shell moves in projectile motion. To find the horizontal distance between the tanks we must find the horizontal velocity of one tank relative to the other. Take $+y$ to be upward.

(a) Set Up: The vertical motion of the shell is unaffected by the horizontal motion of the tank. Use the vertical motion of the shell to find the time the shell is in the air:

$v_{0y} = v_0 \sin\alpha = 43.4\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$, $y - y_0 = 0$ (returns to initial height), $t = ?$

Execute: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 8.86\text{ s}$

Set Up: Consider the motion of one tank relative to the other.

Execute: Relative to tank #1 the shell has a constant horizontal velocity $v_0 \cos\alpha = 246.2\text{ m/s}$. Relative to the ground the horizontal velocity component is $246.2\text{ m/s} + 15.0\text{ m/s} = 261.2\text{ m/s}$. Relative to tank #2 the shell has horizontal velocity component $261.2\text{ m/s} - 35.0\text{ m/s} = 226.2\text{ m/s}$. The distance between the tanks when the shell was fired is the $(226.2\text{ m/s})(8.86\text{ s}) = 2000\text{ m}$ that the shell travels relative to tank #2 during the 8.86 s that the shell is in the air.

(b) The tanks are initially 2000 m apart. In 8.86 s tank #1 travels 133 m and tank #2 travels 310 m, in the same direction. Therefore, their separation increases by $310\text{ m} - 133\text{ m} = 177\text{ m}$. So, the separation becomes 2180 m (rounding to 3 significant figures).

Evaluate: The retreating tank has greater speed than the approaching tank, so they move farther apart while the shell is in the air. We can also calculate the separation in part (b) as the relative speed of the tanks times the time the shell is in the air: $(35.0\text{ m/s} - 15.0\text{ m/s})(8.86\text{ s}) = 177\text{ m}$.

3.70. Identify: The object moves with constant acceleration in both the horizontal and vertical directions.

Set Up: Let $+y$ be downward and let $+x$ be the direction in which the firecracker is thrown.

Execute: The firecracker's falling time can be found from the vertical motion: $t = \sqrt{\dfrac{2h}{g}}$.

The firecracker's horizontal position at any time t (taking the student's position as $x = 0$) is $x = vt - \frac{1}{2}at^2$. $x = 0$ when cracker hits the ground, so $t = 2v/a$. Combining this with the expression for the falling time gives $\dfrac{2v}{a} = \sqrt{\dfrac{2h}{g}}$ and $h = \dfrac{2v^2 g}{a^2}$.

Evaluate: When h is smaller, the time in the air is smaller and either v must be smaller or a must be larger.

3.71. Identify: The velocity $\vec{v}_{T/G}$ of the tank relative to the ground is related to the velocity $\vec{v}_{R/G}$ of the rocket relative to the ground and the velocity $\vec{v}_{T/R}$ of the tank relative to the rocket by $\vec{v}_{T/G} = \vec{v}_{T/R} + \vec{v}_{R/G}$.

Set Up: Let $+y$ be upward and take $y = 0$ at the ground. Let $+x$ be in the direction of the horizontal component of the tank's motion. Once the tank is released it has $a_x = 0$, $a_y = -9.80\text{ m/s}^2$, relative to the ground.

Execute: **(a)** For the rocket $v_y = v_{0y} + a_y t = (1.75\text{m/s}^2)(22.0\text{ s}) = 38.5\text{ m/s}$ and $v_x = 0$. The rocket has speed 38.5 m/s at the instant when the fuel tank is released.

(b) (i) The rocket's path is vertical, so relative to the crew member $v_{T/R\text{-}x} = +25.0\text{ m/s}$ and $v_{T/R\text{-}y} = 0$. (ii) $\vec{v}_{R/G}$ is vertical and $\vec{v}_{T/R}$ is horizontal, so $v_{T/G\text{-}x} = +25.0\text{ m/s}$ and $v_{T/G-y} = +38.5\text{ m/s}$.

(c) (i) The tank initially moves horizontally, at an angle of zero. (ii) $\tan\alpha_0 = \dfrac{v_{T/G\text{-}y}}{v_{T/G\text{-}x}} = \dfrac{38.5\text{ m/s}}{25.0\text{ m/s}}$ and $\alpha_0 = 57.0°$.

(d) Consider the motion of the tank, in the reference frame of the technician on the ground. At the instant the tank is released the rocket at a height $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2=\frac{1}{2}(1.75\text{ m/s}^2)(22.0\text{ s})^2=423.5\text{ m}$. So, for the tank $y_0=423.5\text{ m}$, $v_{0y}=38.5\text{ m/s}$ and $a_y=-9.80\text{ m/s}^2$. $v_y=0$ at the maximum height. $v_y^2=v_{0y}^2+2a_y(y-y_0)$ gives $y-y_0=\dfrac{v_y^2-v_{0y}^2}{2a_y}=\dfrac{0-(38.5\text{ m/s})^2}{2(9.80\text{ m/s}^2)}=75.6\text{ m}$. $y=423.5\text{ m}+75.6\text{ m}=499\text{ m}$. The tank reaches a height of 499 m above the launch pad.

EVALUATE: Relative to the crew member in the rocket the jettisoned tank has an acceleration of $1.75\text{ m/s}^2+9.80\text{ m/s}^2=11.5\text{ m/s}^2$, downward. Relative to the rocket the tank follows a parabolic path, but with zero initial vertical velocity and with a downward acceleration that has magnitude greater than *g*.

3.72. **IDENTIFY:** The velocity $\vec{v}_{R/G}$ of the rocket relative to the ground is related to the velocity $\vec{v}_{S/G}$ of the secondary rocket relative to the ground and the velocity $\vec{v}_{S/R}$ of the secondary rocket relative to the rocket by $\vec{v}_{S/G}=\vec{v}_{S/R}+\vec{v}_{R/G}$.

SET UP: Let $+y$ be upward and let $y=0$ at the ground. Let $+x$ be in the direction of the horizontal component of the secondary rocket's motion. After it is launched the secondary rocket has $a_x=0$ and $a_y=-9.80\text{ m/s}^2$, relative to the ground.

EXECUTE: **(a)** (i) $v_{S/R\text{-}x}=(12.0\text{ m/s})\cos 53.0°=7.22\text{ m/s}$ and $v_{S/R\text{-}y}=(12.0\text{ m/s})\sin 53.0°=9.58\text{ m/s}$.

(ii) $v_{R/G\text{-}x}=0$ and $v_{R/G\text{-}y}=8.50\text{ m/s}$. $v_{S/G\text{-}x}=v_{S/R\text{-}x}+v_{R/G\text{-}x}=7.22\text{ m/s}$ and $v_{S/G\text{-}y}=v_{S/R\text{-}y}+v_{R/G\text{-}y}=$ $9.58\text{ m/s}+8.50\text{ m/s}=18.1\text{ m/s}$.

(b) $v_{S/G}=\sqrt{(v_{S/G\text{-}x})^2+(v_{S/G\text{-}y})^2}=19.5\text{ m/s}$. $\tan\alpha_0=\dfrac{v_{S/G\text{-}y}}{v_{S/G\text{-}x}}=\dfrac{18.1\text{ m/s}}{7.22\text{ m/s}}$ and $\alpha_0=68.3°$.

(c) Relative to the ground the secondary rocket has $y_0=145\text{ m}$, $v_{0y}=+18.1\text{ m/s}$, $a_y=-9.80\text{ m/s}^2$ and $v_y=0$ (at the maximum height). $v_y^2=v_{0y}^2+2a_y(y-y_0)$ gives $y-y_0=\dfrac{v_y^2-v_{0y}^2}{2a_y}=\dfrac{0-(18.1\text{ m/s})^2}{2(-9.80\text{ m/s}^2)}=16.7\text{ m}$.

$y=145\text{ m}+16.7\text{ m}=162\text{ m}$.

EVALUATE: The secondary rocket reaches its maximum height in time $t=\dfrac{v_y-v_{0y}}{a_y}=\dfrac{-18.1\text{ m/s}}{-9.80\text{ m/s}^2}=1.85\text{ s}$ after it is launched. At this time the primary rocket has height $145\text{ m}+(8.50\text{ m/s})(1.85\text{ s})=161\text{ m}$, so is at nearly the same height as the secondary rocket. The secondary rocket first moves upward from the primary rocket but then loses vertical velocity due to the acceleration of gravity.

3.73. **IDENTIFY:** The original firecracker moves as a projectile. At its maximum height it's velocity is horizontal. The velocity $\vec{v}_{A/G}$ of fragment *A* relative to the ground is related to the velocity $\vec{v}_{F/G}$ of the original firecracker relative to the ground and the velocity $\vec{v}_{A/F}$ of the fragment relative to the original firecracker by $\vec{v}_{A/G}=\vec{v}_{A/F}+\vec{v}_{F/G}$. Fragment *B* obeys a similar equation.

SET UP: Let $+x$ be along the direction of the horizontal motion of the firecracker before it explodes and let $+y$ be upward. Fragment *A* moves at $53.0°$ above the $+x$ direction and fragment *B* moves at $53.0°$ below the $+x$ direction. Before it explodes the firecracker has $a_x=0$ and $a_y=-9.80\text{ m/s}^2$

EXECUTE: The horizontal component of the firecracker's velocity relative to the ground is constant (since $a_x=0$), so $v_{F/G\text{-}x}=(25.0\text{ m/s})\cos 30.0°=21.65\text{ m/s}$. At the time of the explosion, $v_{F/G\text{-}y}=0$. For fragment *A*, $v_{A/F\text{-}x}=(20.0\text{ m/s})\cos 53.0°=12.0\text{ m/s}$ and $v_{A/F\text{-}y}=(20.0\text{ m/s})\sin 53.0°=16.0\text{ m/s}$.

$v_{A/G\text{-}x}=v_{A/F\text{-}x}+v_{F/G\text{-}x}=12.0\text{ m/s}+21.65\text{ m/s}=33.7\text{ m/s}$. $v_{A/G\text{-}y}=v_{A/F\text{-}y}+v_{F/G\text{-}y}=16.0\text{ m/s}$.

$\tan\alpha_0=\dfrac{v_{A/G\text{-}y}}{v_{A/G\text{-}x}}=\dfrac{16.0\text{ m/s}}{33.7\text{ m/s}}$ and $\alpha_0=25.4°$. The calculation for fragment *B* is the same, except $v_{A/F\text{-}y}=-16.0\text{ m/s}$.

The fragments move at $25.4°$ above and $25.4°$ below the horizontal.

EVALUATE: As the initial velocity of the firecracker increases the angle with the horizontal for the fragments, as measured from the ground, decreases.

3.74. **IDENTIFY:** The grenade moves in projectile motion. $110\text{ km/h}=30.6\text{ m/s}$. The horizontal range *R* of the grenade must be 15.8 m plus the distance *d* that the enemy's car travels while the grenade is in the air.

SET UP: For the grenade take $+y$ upward, so $a_x = 0$, $a_y = -g$. Let v_0 be the magnitude of the velocity of the grenade relative to the hero. $v_{0x} = v_0 \cos 45°$, $v_{0y} = v_0 \sin 45°$. 90 km/h = 25 m/s ; The enemy's car is traveling away from the hero's car with a relative velocity of $v_{\rm rel} = 30.6\ \text{m/s} - 25\ \text{m/s} = 5.6\ \text{m/s}$.

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 0$ gives $t = -\dfrac{2v_{0y}}{a_y} = \dfrac{2v_0 \sin 45°}{g}$. $d = v_{\rm rel}t = \dfrac{\sqrt{2}v_0 v_{\rm rel}}{g}$.

$R = v_{0x}t = v_0(\cos 45°)t = \dfrac{2v_0^2 \sin 45° \cos 45°}{g} = \dfrac{v_0^2}{g}$. $R = d + 15.8\ \text{m}$ gives that $\dfrac{v_0^2}{g} = \dfrac{\sqrt{2}v_{\rm rel}}{g}v_0 + 15.8\ \text{m}$.

$v_0^2 - \sqrt{2}v_{\rm rel}v_0 - (15.8\ \text{m})g = 0$. $v_0^2 - 7.92v_0 - 154.8 = 0$. The quadratic formula gives $v_0 = 17.0\ \text{m/s} = 61.2\ \text{km/h}$. The grenade has velocity of magnitude 61.2 km/h relative to the hero. Relative to the hero the velocity of the grenade has components $v_{0x} = v_0 \cos 45° = 43.3\ \text{km/h}$ and $v_{0y} = v_0 \sin 45° = 43.3\ \text{km/h}$. Relative to the earth the velocity of the grenade has components $v_{\rm Ex} = 43.3\ \text{km/h} + 90\ \text{km/h} = 133.3\ \text{km/h}$ and $v_{\rm Ey} = 43.3\ \text{km/h}$. The magnitude of the velocity relative to the earth is $v_{\rm E} = \sqrt{v_{\rm Ex}^2 + v_{\rm Ey}^2} = 140\ \text{km/h}$.

EVALUATE: The time the grenade is in the air is $t = \dfrac{2v_0 \sin 45°}{g} = \dfrac{2(17.0\ \text{m/s})\sin 45°}{9.80\ \text{m/s}^2} = 2.45\ \text{s}$. During this time the grenade travels a horizontal distance $x - x_0 = (133.3\ \text{km/h})(2.45\ \text{s})(1\ \text{h}/3600\ \text{s}) = 90.7\ \text{m}$, relative to the earth, and the enemy's car travels a horizontal distance $x - x_0 = (110\ \text{km/h})(2.45\ \text{s})(1\ \text{h}/3600\ \text{s}) = 74.9\ \text{m}$, relative to the earth. The grenade has traveled 15.8 m farther.

3.75. **IDENTIFY** and **SET UP:** Use Eqs. (3.4) and (3.12) to get the velocity and acceleration components from the position components.

EXECUTE: $x = R\cos\omega t,\ \ y = R\sin\omega t$

(a) $r = \sqrt{x^2 + y^2} = \sqrt{R^2\cos^2\omega t + R^2\sin^2\omega t} = \sqrt{R^2(\sin^2\omega t + \cos^2\omega t)} = \sqrt{R^2} = R,$

since $\sin^2\omega t + \cos^2\omega t = 1.$

(b) $v_x = \dfrac{dx}{dt} = -R\omega\sin\omega t,\ \ v_y = \dfrac{dy}{dt} = R\omega\cos\omega t$

$\vec{v}\cdot\vec{r} = v_x x + v_y y = (-R\omega\sin\omega t)(R\cos\omega t) + (R\omega\cos\omega t)(R\sin\omega t)$

$\vec{v}\cdot\vec{r} = R^2\omega(-\sin\omega t\cos\omega t + \sin\omega t\cos\omega t) = 0,$ so $\vec{v}$ is perpendicular to $\vec{r}$.

(c) $a_x = \dfrac{dv_x}{dt} = -R\omega^2\cos\omega t = -\omega^2 x$

$a_y = \dfrac{dv_y}{dt} = -R\omega^2\sin\omega t = -\omega^2 y$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\omega^4x^2 + \omega^4y^2} = \omega^2\sqrt{x^2 + y^2} = R\omega^2.$

$\vec{a} = a_x\hat{i} + a_y\hat{j} = -\omega^2(x\hat{i} + y\hat{j}) = -\omega^2\vec{r}.$

Since ω^2 is positive this means that the direction of $\vec{a}$ is opposite to the direction of $\vec{r}$.

(d) $v = \sqrt{v_x^2 + v_y^2} = \sqrt{R^2\omega^2\sin^2\omega t + R^2\omega^2\cos^2\omega t} = \sqrt{R^2\omega^2(\sin^2\omega t + \cos^2\omega t)}.$ $v = \sqrt{R^2\omega^2} = R\omega.$

(e) $a = R\omega^2,\ \ \omega = v/R,$ so $a = R(v^2/R^2) = v^2/R.$

EVALUATE: The rock moves in uniform circular motion. The position vector is radial, the velocity is tangential, and the acceleration is radially inward.

3.76. **IDENTIFY:** All velocities are constant, so the distance traveled is $d = v_{\rm B/E}t$, where $v_{\rm B/E}$ is the magnitude of the velocity of the boat relative to the earth. The relative velocities $\vec{v}_{\rm B/E}$, $\vec{v}_{\rm S/W}$ (boat relative to the water) and $\vec{v}_{\rm W/E}$ (water relative to the earth) are related by $\vec{v}_{\rm B/E} = \vec{v}_{\rm B/W} + \vec{v}_{\rm W/E}$.

SET UP: Let $+x$ be east and let $+y$ be north. $v_{\text{W/E-}x} = +30.0\ \text{m/min}$ and $v_{\text{W/E-}y} = 0$. $v_{\rm B/W} = 100.0\ \text{m/min}$. The direction of $\vec{v}_{\rm B/W}$ is the direction in which the boat is pointed or aimed.

EXECUTE: **(a)** $v_{\text{B/W-}y} = +100.0\ \text{m/min}$ and $v_{\text{B/W-}x} = 0$. $v_{\text{B/E-}x} = v_{\text{B/W-}x} + v_{\text{W/E-}x} = 30.0\ \text{m/min}$ and $v_{\text{B/E-}y} = v_{\text{B/W-}y} + v_{\text{W/E-}y} = 100.0\ \text{m/min}$. The time to cross the river is $t = \dfrac{y - y_0}{v_{\text{B/E-}y}} = \dfrac{400.0\ \text{m}}{100.0\ \text{m/min}} = 4.00\ \text{min}$.

$x - x_0 = (30.0\text{ m/min})(4.00\text{ min}) = 120.0\text{ m}$. You will land 120.0 m east of point *B*, which is 45.0 m east of point *C*. The distance you will have traveled is $\sqrt{(400.0\text{ m})^2 + (120.0\text{ m})^2} = 418\text{ m}$.

(b) $\vec{v}_{\text{B/W}}$ is directed at angle ϕ east of north, where $\tan\phi = \dfrac{75.0\text{ m}}{400.0\text{ m}}$ and $\phi = 10.6°$.

$v_{\text{B/W-}x} = (100.0\text{ m/min})\sin 10.6° = 18.4\text{ m/min}$ and $v_{\text{B/W-}y} = (100.0\text{ m/min})\cos 10.6° = 98.3\text{ m/min}$.

$v_{\text{B/E-}x} = v_{\text{B/W-}x} + v_{\text{W/E-}x} = 18.4\text{ m/min} + 30.0\text{ m/min} = 48.4\text{ m/min}$. $v_{\text{B/E-}y} = v_{\text{B/W-}y} + v_{\text{W/E-}y} = 98.3\text{ m/min}$.

$t = \dfrac{y - y_0}{v_{\text{B/E-}y}} = \dfrac{400.0\text{ m}}{98.3\text{ m/min}} = 4.07\text{ min}$. $x - x_0 = (48.4\text{ m/min})(4.07\text{ min}) = 197\text{ m}$. You will land 197 m downstream from *B*, so 122 m downstream from *C*.

(c) (i) If you reach point *C*, then $\vec{v}_{\text{B/E}}$ is directed at $10.6°$ east of north, which is $79.4°$ north of east. We don't know the magnitude of $\vec{v}_{\text{B/E}}$ and the direction of $\vec{v}_{\text{B/W}}$. In part (a) we found that if we aim the boat due north we will land east of *C*, so to land at *C* we must aim the boat west of north. Let $\vec{v}_{\text{B/W}}$ be at an angle ϕ of north of west. The relative velocity addition diagram is sketched in Figure 3.76. The law of sines says $\dfrac{\sin\theta}{v_{\text{W/E}}} = \dfrac{\sin 79.4°}{v_{\text{B/W}}}$.

$\sin\theta = \left(\dfrac{30.0\text{ m/min}}{100.0\text{ m/min}}\right)\sin 79.4°$ and $\theta = 17.15°$. Then $\phi = 180° - 79.4° - 17.15° = 83.5°$. The boat will head $83.5°$ north of west, so $6.5°$ west of north.

$v_{\text{B/E-}x} = v_{\text{B/W-}x} + v_{\text{W/E-}x} = -(100.0\text{ m/min})\cos 83.5° + 30.0\text{ m/min} = 18.7\text{ m/min}$.

$v_{\text{B/E-}y} = v_{\text{B/W-}y} + v_{\text{W/E-}y} = -(100.0\text{ m/min})\sin 83.5° = 99.4\text{ m/min}$. Note that these two components do give the direction of $\vec{v}_{\text{B/E}}$ to be $79.4°$ north of east, as required. (ii) The time to cross the river is

$t = \dfrac{y - y_0}{v_{\text{B/E-}y}} = \dfrac{400.0\text{ m}}{99.4\text{ m/min}} = 4.02\text{ min}$. (iii) You travel from *A* to *C*, a distance of $\sqrt{(400.0\text{ m})^2 + (75.0\text{ m})^2} = 407\text{ m}$.

(iv) $v_{\text{B/E}} = \sqrt{(v_{\text{B/E-}x})^2 + (v_{\text{B/E-}y})^2} = 101\text{ m/min}$. Note that $v_{\text{B/E}}t = 406\text{ m}$, the distance traveled (apart from a small difference due to rounding).

EVALUATE: You cross the river in the shortest time when you head toward point *B*, as in part (a), even though you travel farther than in part (c).

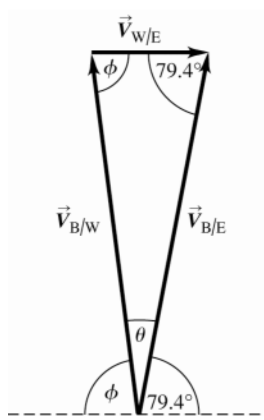


Figure 3.76

3.77. IDENTIFY: $v_x = dx/dt$, $v_y = dy/dt$, $a_x = dv_x/dt$ and $a_y = dv_y/dt$.

SET UP: $\dfrac{d(\sin\omega t)}{dt} = \omega\cos(\omega t)$ and $\dfrac{d(\cos\omega t)}{dt} = -\omega\sin(\omega t)$.

EXECUTE: **(a)** The path is sketched in Figure 3.77.

(b) To find the velocity components, take the derivative of *x* and *y* with respect to time: $v_x = R\omega(1 - \cos\omega t)$, and $v_y = R\omega\sin\omega t$. To find the acceleration components, take the derivative of v_x and v_y with respect to time: $a_x = R\omega^2\sin\omega t$, and $a_y = R\omega^2\cos\omega t$.

(c) The particle is at rest $(v_y = v_x = 0)$ every period, namely at $t = 0,\ 2\pi/\omega,\ 4\pi/\omega, \ldots$. At that time, $x = 0,\ 2\pi R,\ 4\pi R, \ldots$; and $y = 0$. The acceleration is $a = R\omega^2$ in the $+y$-direction.

(d) No, since $a = \left[\left(R\omega^2 \sin \omega t \right)^2 + \left(R\omega^2 \cos \omega t \right)^2 \right]^{1/2} = R\omega^2$. The magnitude of the acceleration is the same as for uniform circular motion.

EVALUATE: The velocity is tangent to the path. v_{0x} is always positive; v_y changes sign during the motion.

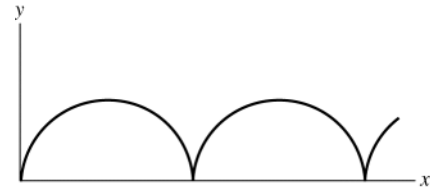


Figure 3.77

3.78. **IDENTIFY:** At the highest point in the trajectory the velocity of the projectile relative to the earth is horizontal. The velocity $\vec{v}_{\mathrm{P/E}}$ of the projectile relative to the earth, the velocity $\vec{v}_{\mathrm{F/P}}$ of a fragment relative to the projectile, and the velocity $\vec{v}_{\mathrm{F/E}}$ of a fragment relative to the earth are related by $\vec{v}_{\mathrm{F/E}} = \vec{v}_{\mathrm{F/P}} + \vec{v}_{\mathrm{P/E}}$.

SET UP: Let $+x$ be along the horizontal component of the projectile motion. Let the speed of each fragment relative to the projectile be v. Call the fragments 1 and 2, where fragment 1 travels in the $+x$ direction and fragment 2 is in the $-x$-direction , and let the speeds just after the explosion of the two fragments relative to the earth be v_1 and v_2. Let v_{p} be the speed of the projectile just before the explosion.

EXECUTE: $v_{\mathrm{F/E}\text{-}x} = v_{\mathrm{F/P}\text{-}x} + v_{\mathrm{P/E}\text{-}x}$ gives $v_1 = v_{\mathrm{p}} + v$ and $-v_2 = v_{\mathrm{p}} - v$. Both fragments start from the same height with zero vertical component of velocity relative to the earth, so they both fall for the same time t, and this is also the same time as it took for the projectile to travel a horizontal distance D, so $v_{\mathrm{p}}t = D$. Since fragment 2 lands at A it travels a horizontal distance D as it falls and $v_2 t = D$. $-v_2 = +v_{\mathrm{p}} - v$ gives $v = v_{\mathrm{p}} + v_2$ and $vt = v_{\mathrm{p}}t + v_2 t = 2D$. Then $v_1 t = v_{\mathrm{p}}t + vt = 3D$. This fragment lands a horizontal distance 3D from the point of explosion and hence 4D from A.

EVALUATE: Fragment 1, that is ejected in the direction of the motion of the projectile travels with greater speed relative to the earth than the fragment that travels in the opposite direction.

3.79. **IDENTIFY:** $a_{\mathrm{rad}} = \dfrac{v^2}{R} = \dfrac{4\pi^2 R}{T^2}$. All points on the centrifuge have the same period T.

SET UP: The period T in seconds is related to n, the number of revolutions per minute, by $n = \dfrac{60\text{ s/min}}{T}$.

EXECUTE: **(a)** $\dfrac{a_{\mathrm{rad}}}{R} = \dfrac{4\pi^2}{T^2}$, which is constant. $\dfrac{a_{\mathrm{rad},1}}{R_1} = \dfrac{a_{\mathrm{rad},2}}{R_2}$. Let $R_1 = R$, so $a_{\mathrm{rad},1} = 5.00g$ and let $R_2 = R/2$.

$a_{\mathrm{rad},2} = a_{\mathrm{rad},1}\left(\dfrac{R_2}{R_1}\right) = (5.00g)(1/2) = 2.50g$.

(b) $T = \left(\dfrac{60\text{ s/min}}{n}\right)$ and $a_{\mathrm{rad}} = \dfrac{4\pi^2 R}{T^2}$ gives $a_{\mathrm{rad}} = 4\pi^2 R n^2/(60\text{ s/min})^2$. $\dfrac{a_{\mathrm{rad}}}{n^2} = \dfrac{4\pi^2 R}{(60\text{ s/min})^2}$, which is constant.

$\dfrac{a_{\mathrm{rad},1}}{n_1^2} = \dfrac{a_{\mathrm{rad},2}}{n_2^2}$. Let $a_{\mathrm{rad},1} = 5.00g$, so $n_1 = n$ and $a_{\mathrm{rad},2} = 5g_{\mathrm{Mercury}} = 5(0.378)g$. Then

$n_2 = n_1\sqrt{\dfrac{a_{\mathrm{rad},2}}{a_{\mathrm{rad},1}}} = n\sqrt{\dfrac{5(0.378)g}{5.00g}} = 0.615n$.

EVALUATE: The radial acceleration is less for points closer to the rotation axis. Since $g_{\mathrm{Mercury}} < g$, a smaller rotation rate is required to produce $5g_{\mathrm{Mercury}}$ than to produce $5g$.

3.80. **IDENTIFY:** Use the relation that relates the relative velocities.

SET UP: The relative velocities are the raindrop relative to the earth, $\vec{v}_{\mathrm{R/E}}$, the raindrop relative to the train, $\vec{v}_{\mathrm{R/T}}$, and the train relative to the earth, $\vec{v}_{\mathrm{T/E}}$. $\vec{v}_{\mathrm{R/E}} = \vec{v}_{\mathrm{R/T}} + \vec{v}_{\mathrm{T/E}}$. $\vec{v}_{\mathrm{T/E}}$ is due east and has magnitude 12.0 m/s. $\vec{v}_{\mathrm{R/T}}$ is 30.0° west of vertical. $\vec{v}_{\mathrm{R/E}}$ is vertical. The relative velocity addition diagram is given in Figure 3.80.

EXECUTE: **(a)** $\vec{v}_{\mathrm{R/E}}$ is vertical and has zero horizontal component. The horizontal component of $\vec{v}_{\mathrm{R/T}}$ is $-\vec{v}_{\mathrm{T/E}}$, so is 12.0 m/s westward.

(b) $v_{\mathrm{R/E}} = \dfrac{v_{\mathrm{T/E}}}{\tan 30.0^\circ} = \dfrac{12.0\text{ m/s}}{\tan 30.0^\circ} = 20.8\text{ m/s}$. $v_{\mathrm{R/T}} = \dfrac{v_{\mathrm{T/E}}}{\sin 30.0^\circ} = \dfrac{12.0\text{ m/s}}{\sin 30.0^\circ} = 24.0\text{ m/s}$.

EVALUATE: The speed of the raindrop relative to the train is greater than its speed relative to the earth, because of the motion of the train.

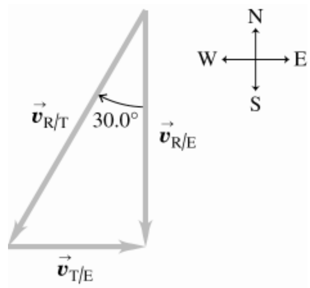


Figure 3.80

3.81. **IDENTIFY:** Relative velocity problem. The plane's motion relative to the earth is determined by its velocity relative to the earth.

SET UP: Select a coordinate system where $+y$ is north and $+x$ is east.

The velocity vectors in the problem are:

$\vec{v}_{P/E}$, the velocity of the plane relative to the earth.

$\vec{v}_{P/A}$, the velocity of the plane relative to the air (the magnitude $v_{P/A}$ is the air speed of the plane and the direction of $\vec{v}_{P/A}$ is the compass course set by the pilot).

$\vec{v}_{A/E}$, the velocity of the air relative to the earth (the wind velocity).

The rule for combining relative velocities gives $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$.

(a) We are given the following information about the relative velocities:

$\vec{v}_{P/A}$ has magnitude 220 km/h and its direction is west. In our coordinates is has components $(v_{P/A})_x = -220$ km/h and $(v_{P/A})_y = 0$.

From the displacement of the plane relative to the earth after 0.500 h, we find that $\vec{v}_{P/E}$ has components in our coordinate system of

$$(v_{P/E})_x = -\frac{120\text{ km}}{0.500\text{ h}} = -240\text{ km/h (west)}$$

$$(v_{P/E})_y = -\frac{20\text{ km}}{0.500\text{ h}} = -40\text{ km/h (south)}$$

With this information the diagram corresponding to the velocity addition equation is shown in Figure 3.81a.

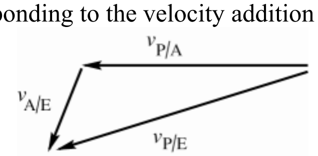


Figure 3.81a

We are asked to find $\vec{v}_{A/E}$, so solve for this vector:

$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$ gives $\vec{v}_{A/E} = \vec{v}_{P/E} - \vec{v}_{P/A}$.

EXECUTE: The x-component of this equation gives

$(v_{A/E})_x = (v_{P/E})_x - (v_{P/A})_x = -240\text{ km/h} - (-220\text{ km/h}) = -20\text{ km/h}$.

The y-component of this equation gives

$(v_{A/E})_y = (v_{P/E})_y - (v_{P/A})_y = -40\text{ km/h}$.

Now that we have the components of $\vec{v}_{A/E}$ we can find its magnitude and direction.

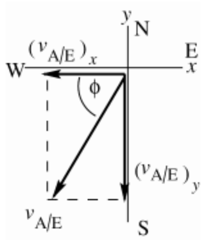


Figure 3.81b

$$v_{A/E} = \sqrt{(v_{A/E})_x^2 + (v_{A/E})_y^2}$$

$$v_{A/E} = \sqrt{(-20\text{ km/h})^2 + (-40\text{ km/h})^2} = 44.7\text{ km/h}$$

$$\tan\phi = \frac{40\text{ km/h}}{20\text{ km/h}} = 2.00;\quad \phi = 63.4°$$

The direction of the wind velocity is $63.4°$ S of W, or $26.6°$ W of S.

EVALUATE: The plane heads west. It goes farther west than it would without wind and also travels south, so the wind velocity has components west and south.

(b) SET UP: The rule for combining the relative velocities is still $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$, but some of these velocities have different values than in part (a).

$\vec{v}_{P/A}$ has magnitude 220 km/h but its direction is to be found.

$\vec{v}_{A/E}$ has magnitude 40 km/h and its direction is due south.

The direction of $\vec{v}_{P/E}$ is west; its magnitude is not given.

The vector diagram for $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$ and the specified directions for the vectors is shown in Figure 3.81c.

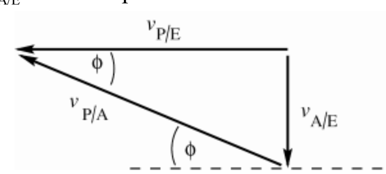


Figure 3.81c

The vector addition diagram forms a right triangle.

EXECUTE: $\sin\phi = \dfrac{v_{A/E}}{v_{P/A}} = \dfrac{40\text{ km/h}}{220\text{ km/h}} = 0.1818;\quad \phi = 10.5°.$

The pilot should set her course $10.5°$ north of west.

EVALUATE: The velocity of the plane relative to the air must have a northward component to counteract the wind and a westward component in order to travel west.

3.82. IDENTIFY: Both the bolt and the elevator move vertically with constant acceleration.

SET UP: Let $+y$ be upward and let $y = 0$ at the initial position of the floor of the elevator, so y_0 for the bolt is 3.00 m.

EXECUTE: **(a)** The position of the bolt is $3.00\text{ m} + (2.50\text{ m/s})t - (1/2)(9.80\text{ m/s}^2)t^2$ and the position of the floor is (2.50 m/s)t. Equating the two, $3.00\text{ m} = (4.90\text{ m/s}^2)t^2$. Therefore, $t = 0.782\text{ s}$.

(b) The velocity of the bolt is $2.50\text{ m/s} - (9.80\text{ m/s}^2)(0.782\text{ s}) = -5.17\text{ m/s}$ relative to Earth, therefore, relative to an observer in the elevator $v = -5.17\text{ m/s} - 2.50\text{ m/s} = -7.67\text{ m/s}$.

(c) As calculated in part (b), the speed relative to Earth is 5.17 m/s.

(d) Relative to Earth, the distance the bolt traveled is

$(2.50\text{ m/s})t - (1/2)(9.80\text{ m/s}^2)t^2 = (2.50\text{ m/s})(0.782\text{ s}) - (4.90\text{ m/s}^2)(0.782\text{ s})^2 = -1.04\text{ m}$.

EVALUATE: As viewed by an observer in the elevator, the bolt has $v_{0y} = 0$ and $a_y = -9.80\text{ m/s}^2$, so in 0.782 s it falls $-\frac{1}{2}(9.80\text{ m/s}^2)(0.782\text{ s})^2 = -3.00\text{ m}$.

3.83. IDENTIFY: In an earth frame the elevator accelerates upward at 4.00 m/s^2 and the bolt accelerates downward at 9.80 m/s^2. Relative to the elevator the bolt has a downward acceleration of $4.00\text{ m/s}^2 + 9.80\text{ m/s}^2 = 13.80\text{ m/s}^2$. In either frame, that of the earth or that of the elevator, the bolt has constant acceleration and the constant acceleration equations can be used.

SET UP: Let $+y$ be upward. The bolt travels 3.00 m downward relative to the elevator.

EXECUTE: **(a)** In the frame of the elevator, $v_{0y} = 0$, $y - y_0 = -3.00\text{ m}$, $a_y = -13.8\text{ m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(-3.00\text{ m})}{-13.8\text{ m/s}^2}} = 0.659\text{ s}$.

(b) $v_y = v_{0y} + a_y t$. $v_{0y} = 0$ and $t = 0.659\ \text{s}$. (i) $a_y = -13.8\ \text{m/s}^2$ and $v_y = -9.09\ \text{m/s}$. The bolt has speed 9.09 m/s when it reaches the floor of the elevator. (ii) $a_y = -9.80\ \text{m/s}^2$ and $v_y = -6.46\ \text{m/s}$. In this frame the bolt has speed 6.46 m/s when it reaches the floor of the elevator.

(c) $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. $v_{0y} = 0$ and $t = 0.659\ \text{s}$. (i) $a_y = -13.8\ \text{m/s}^2$ and $y - y_0 = \frac{1}{2}(-13.8\ \text{m/s}^2)(0.659\ \text{s})^2 = -3.00\ \text{m}$. The bolt falls 3.00 m, which is correctly the distance between the floor and roof of the elevator. (ii) $a_y = -9.80\ \text{m/s}^2$ and $y - y_0 = \frac{1}{2}(-9.80\ \text{m/s}^2)(0.659\ \text{s})^2 = -2.13\ \text{m}$. The bolt falls 2.13 m.

EVALUATE: In the earth's frame the bolt falls 2.13 m and the elevator rises $\frac{1}{2}(4.00\ \text{m/s}^2)(0.659\ \text{s})^2 = 0.87\ \text{m}$ during the time that the bolt travels from the ceiling to the floor of the elevator.

3.84. **IDENTIFY:** The velocity $\vec{v}_{\text{P/E}}$ of the plane relative to the earth is related to the velocity $\vec{v}_{\text{P/A}}$ of the plane relative to the air and the velocity $\vec{v}_{\text{A/E}}$ of the air relative to the earth (the wind velocity) by $\vec{v}_{\text{P/E}} = \vec{v}_{\text{P/A}} + \vec{v}_{\text{A/E}}$.

SET UP: Let $+x$ be to the east. With no wind $v_{\text{P/A}} = v_{\text{P/E}} = \dfrac{5550\ \text{km}}{6.60\ \text{h}} = 840.9\ \text{km/h}$. $v_{\text{A/E-}x} = +225\ \text{km/h}$. The distance between A and B is 2775 km.

EXECUTE: $v_{\text{P/E-}x} = v_{\text{P/A-}x} + v_{\text{A/E-}x}$. For the trip A to B, $v_{\text{P/A-}x} = +840.9\ \text{km/h}$ and $v_{\text{P/E-}x} = 840.9\ \text{km/h} + 225\ \text{km/h} = 1065.9\ \text{km/h}$ and the travel time is $t_{AB} = \dfrac{2775\ \text{km}}{1065.9\ \text{km/h}} = 2.60\ \text{h}$. For the trip B to A, $v_{\text{P/A-}x} = -840.9\ \text{km/h}$ and $v_{\text{P/E-}x} = -840.9\ \text{km/h} + 225\ \text{km/h} = -615.9\ \text{km/h}$ and the travel time is $t_{BA} = \dfrac{-2775\ \text{km}}{-615.9\ \text{km/h}} = 4.51\ \text{h}$. The total time for the round trip will be $t = t_{AB} + t_{BA} = 7.11\ \text{h}$.

EVALUATE: The round trip takes longer when the wind blows, even though the plane travels with the wind for one leg of the trip. The arithmetic average of the speeds for each leg is $\dfrac{1065.9\ \text{km/h} + 615.9\ \text{km/h}}{2} = 840.9\ \text{km/h}$, the same speed when there is no wind. But the plane spends more time traveling at the slower speed relative to the ground and the average speed is less than the arithmetic average of the speeds for each half of the trip.

3.85. **IDENTIFY:** Relative velocity problem.

SET UP: The three relative velocities are:

$\vec{v}_{\text{J/G}}$, Juan relative to the ground. This velocity is due north and has magnitude $v_{\text{J/G}} = 8.00$ m/s.

$\vec{v}_{\text{B/G}}$, the ball relative to the ground. This vector is $37.0°$ east of north and has magnitude $v_{\text{B/G}} = 12.00$ m/s.

$\vec{v}_{\text{B/J}}$, the ball relative to Juan. We are asked to find the magnitude and direction of this vector.

The relative velocity addition equation is $\vec{v}_{\text{B/G}} = \vec{v}_{\text{B/J}} + \vec{v}_{\text{J/G}}$, so $\vec{v}_{\text{B/J}} = \vec{v}_{\text{B/G}} - \vec{v}_{\text{J/G}}$.

The relative velocity addition diagram does not form a right triangle so we must do the vector addition using components.

Take $+y$ to be north and $+x$ to be east.

EXECUTE: $v_{\text{B/J}x} = +v_{\text{B/G}} \sin 37.0° = 7.222\ \text{m/s}$

$v_{\text{B/J}y} = +v_{\text{B/G}} \cos 37.0° - v_{\text{J/G}} = 1.584\ \text{m/s}$

These two components give $v_{\text{B/J}} = 7.39\ \text{m/s}$ at $12.4°$ north of east.

EVALUATE: Since Juan is running due north, the ball's eastward component of velocity relative to him is the same as its eastward component relative to the earth. The northward component of velocity for Juan and the ball are in the same direction, so the component for the ball relative to Juan is the difference in their components of velocity relative to the ground.

3.86. **IDENTIFY:** **(a)** The ball moves in projectile motion. When it is moving horizontally, $v_y = 0$.

SET UP: Let $+x$ be to the right and let $+y$ be upward. $a_x = 0$, $a_y = -g$.

EXECUTE: **(a)** $v_{0y} = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(4.90\ \text{m})} = 9.80\ \text{m/s}$.

(b) $v_{0y}/g = 1.00\ \text{s}$.

(c) The horizontal component of the velocity of the ball relative to the man is $\sqrt{(10.8\ \text{m/s})^2 - (9.80\ \text{m/s})^2} = 4.54\ \text{m/s}$, the horizontal component of the velocity relative to the hoop is $4.54\ \text{m/s} + 9.10\ \text{m/s} = 13.6\ \text{m/s}$, and the man must be 13.6 m in front of the hoop at release.

(d) Relative to the flat car, the ball is projected at an angle $\theta = \tan^{-1}\left(\frac{9.80 \text{ m/s}}{4.54 \text{ m/s}}\right) = 65°.$ Relative to the ground the angle is $\theta = \tan^{-1}\left(\frac{9.80 \text{ m/s}}{4.54 \text{ m/s} + 9.10 \text{ m/s}}\right) = 35.7°$.

EVALUATE: In both frames of reference the ball moves in a parabolic path with $a_x = 0$ and $a_y = -g$. The only difference between the description of the motion in the two frames is the horizontal component of the ball's velocity.

3.87. **IDENTIFY:** The pellets move in projectile motion. The vertical motion determines their time in the air.

SET UP: $v_{0x} = v_0 \cos 1.0°$, $v_{0y} = v_0 \sin 1.0°$.

EXECUTE: **(a)** $t = \frac{2v_{0y}}{g}$. $x - x_0 = v_{0x}t$ gives $x - x_0 = (v_0 \cos 1.0°)\left(\frac{2v_0 \sin 1.0°}{g}\right) = 80 \text{ m}$.

(b) The probability is 1000 times the ratio of the area of the top of the person's head to the area of the circle in which the pellets land. $(1000)\left(\frac{\pi(10\times10^{-2} \text{ m})^2}{\pi(80 \text{ m})^2}\right) = 1.6\times10^{-3}.$

(c) The slower rise will tend to reduce the time in the air and hence reduce the radius. The slower horizontal velocity will also reduce the radius. The lower speed would tend to increase the time of descent, hence increasing the radius. As the bullets fall, the friction effect is smaller than when they were rising, and the overall effect is to decrease the radius.

EVALUATE: The small angle of deviation from the vertical still causes the pellets to spread over a large area because their time in the air is large.

3.88. **IDENTIFY:** Write an expression for the square of the distance (D^2) from the origin to the particle, expressed as a function of time. Then take the derivative of D^2 with respect to *t*, and solve for the value of *t* when this derivative is zero. If the discriminant is zero or negative, the distance *D* will never decrease.

SET UP: $D^2 = x^2 + y^2$, with $x(t)$ and $y(t)$ given by Eqs.(3.20) and (3.21).

EXECUTE: Following this process, $\sin^{-1}\sqrt{8/9} = 70.5°.$

EVALUATE: We know that if the object is thrown straight up it moves away from *P* and then returns, so we are not surprised that the projectile angle must be less than some maximum value for the distance to always increase with time.

3.89. **IDENTIFY:** The baseball moves in projectile motion.

SET UP: Use coordinates where the *x*-axis is horizontal and the *y*-axis is vertical.

EXECUTE: **(a)** The trajectory of the projectile is given by Eq. (3.27), with $\alpha_0 = \theta + \varphi,$ and the equation describing the incline is $y = x\tan\theta.$ Setting these equal and factoring out the $x = 0$ root (where the projectile is on the incline) gives a value for $x_0;$ the range measured along the incline is

$$x/\cos\theta = \left[\frac{2v_0^2}{g}\right][\tan(\theta+\varphi) - \tan\theta]\left[\frac{\cos^2(\theta+\varphi)}{\cos\theta}\right].$$

(b) Of the many ways to approach this problem, a convenient way is to use the same sort of substitution, involving double angles, as was used to derive the expression for the range along a horizontal incline. Specifically, write the above in terms of $\alpha = \theta + \varphi,$ as

$$R = \left[\frac{2v_0^2}{g\cos^2\theta}\right][\sin\alpha\cos\alpha\cos\theta - \cos^2\alpha\sin\theta].$$

The dependence on α and hence φ is in the second term. Using the identities $\sin\alpha\cos\alpha = (1/2)\sin 2\alpha$ and $\cos^2\alpha = (1/2)(1+\cos 2\alpha),$ this term becomes

$$(1/2)[\cos\theta\sin 2\alpha - \sin\theta\cos 2\alpha - \sin\theta] = (1/2)[\sin(2\alpha-\theta) - \sin\theta].$$

This will be a maximum when $\sin(2\alpha - \theta)$ is a maximum, at $2\alpha - \theta = 2\varphi + \theta = 90°,$ or $\varphi = 45° - \theta/2.$

EVALUATE: Note that the result reduces to the expected forms when $\theta = 0$ (a flat incline, $\varphi = 45°$ and when $\theta = -90°$ (a vertical cliff), when a horizontal launch gives the greatest distance).

3.90. **IDENTIFY:** The arrow moves in projectile motion.

SET UP: Use coordinates that for which the axes are horizontal and vertical. Let θ be the angle of the slope and let ϕ be the angle of projection relative to the sloping ground.

EXECUTE: The horizontal distance x in terms of the angles is

$$\tan\theta = \tan(\theta+\phi) - \left(\frac{gx}{2v_0^2}\right)\frac{1}{\cos^2(\theta+\phi)}.$$

Denote the dimensionless quantity $gx/2v_0^2$ by β; in this case

$$\beta = \frac{(9.80 \text{ m/s}^2)(60.0 \text{ m})\cos 30.0°}{2(32.0 \text{ m/s})^2} = 0.2486.$$

The above relation can then be written, on multiplying both sides by the product $\cos\theta\cos(\theta+\phi)$,

$$\sin\theta\cos(\theta+\phi) = \sin(\theta+\phi)\cos\theta - \frac{\beta\cos\theta}{\cos(\theta+\phi)},$$

and so $\sin(\theta+\phi)\cos\theta - \cos(\theta+\phi)\sin\theta = \dfrac{\beta\cos\theta}{\cos(\theta+\phi)}$. The term on the left is $\sin((\theta+\phi)-\theta) = \sin\phi$, so the result of this combination is $\sin\phi\cos(\theta+\phi) = \beta\cos\theta.$

Although this can be done numerically (by iteration, trial-and-error, or other methods), the expansion $\sin a\cos b = \frac{1}{2}(\sin(a+b)+\sin(a-b))$ allows the angle ϕ to be isolated; specifically, then

$\dfrac{1}{2}(\sin(2\phi+\theta)+\sin(-\theta)) = \beta\cos\theta$, with the net result that $\sin(2\phi+\theta) = 2\beta\cos\theta + \sin\theta.$

(a) For $\theta = 30°$, and β as found above, $\phi = 19.3°$ and the angle above the horizontal is $\theta+\phi = 49.3°$. For level ground, using $\beta = 0.2871$, gives $\phi = 17.5°$.

(b) For $\theta = -30°$, the same β as with $\theta = 30°$ may be used $(\cos 30° = \cos(-30°))$, giving $\phi = 13.0°$ and $\phi+\theta = -17.0°$.

EVALUATE: For $\theta = 0$ the result becomes $\sin(2\phi) = 2\beta = gx/v_0^2$. This is equivalent to the expression $R = \dfrac{v_0^2\sin(2\alpha_0)}{g}$ derived in Example 3.8.

3.91. **IDENTIFY:** Find $\Delta\vec{v}$ and use this to calculate the magnitude and direction of the average acceleration.

SET UP: In a time Δt, the velocity vector has moved through an angle (in radians) $\Delta\phi = \dfrac{v\Delta t}{R}$ (see Figure 3.28 in the textbook). By considering the isosceles triangle formed by the two velocity vectors, the magnitude $|\Delta\vec{\mathbf{v}}|$ is seen to be $2v\sin(\phi/2)$.

EXECUTE: $|\vec{a}_{av}| = \dfrac{|\Delta\vec{v}|}{\Delta t} = 2\dfrac{v}{\Delta t}\sin\left(\dfrac{v\Delta t}{2R}\right) = \dfrac{10 \text{ m/s}}{\Delta t}\sin([1.0/\text{s}]\Delta t)$

Using the given values gives magnitudes of $9.59 \text{ m/s}^2, 9.98 \text{ m/s}^2$ and 10.0 m/s^2. The changes in direction of the velocity vectors are given by $\Delta\theta = \dfrac{v\Delta t}{R}$ and are, respectively, 1.0 rad, 0.2 rad, and 0.1 rad. Therefore, the angle of the average acceleration vector with the original velocity vector is $\dfrac{\pi+\Delta\theta}{2} = \pi/2 + 1/2 \text{ rad(or } 118.6°)$, $\pi/2 + 0.1 \text{ rad(or } 95.7°)$, and $\pi/2 + 0.05 \text{ rad(or } 92.9°)$.

EVALUATE: The instantaneous acceleration magnitude, $v^2/R = (5.00 \text{ m/s})^2/(2.50 \text{ m}) = 10.0 \text{ m/s}^2$ is indeed approached in the limit at $\Delta t \to 0$. Also, the direction of $\vec{a}_{av}$ approaches the radially inward direction as $\Delta t \to 0$.

3.92. **IDENTIFY:** The rocket has two periods of constant acceleration motion.

SET UP: Let $+y$ be upward. During the free-fall phase, $a_x = 0$ and $a_y = -g$. After the engines turn on, $a_x = (3.00g)\cos 30.0°$ and $a_y = (3.00g)\sin 30.0°$. Let t be the total time since the rocket was dropped and let T be the time the rocket falls before the engine starts.

EXECUTE: (i) The diagram is given in Figure 3.92a.

(ii) The x-position of the plane is $(236 \text{ m/s})t$ and the x-position of the rocket is $(236 \text{ m/s})t + (1/2)(3.00)(9.80 \text{ m/s}^2)\cos 30°(t-T)^2$. The graphs of these two equations are sketched in Figure 3.92b.

(iii) If we take $y = 0$ to be the altitude of the airliner, then

$y(t) = -1/2gT^2 - gT(t-T) + 1/2(3.00)(9.80\text{ m/s}^2)(\sin 30°)(t-T)^2$ for the rocket. The airliner has constant y. The graphs are sketched in Figure 3.92b.

In each of the Figures 3.92a-c, the rocket is dropped at $t = 0$ and the time T when the motor is turned on is indicated.

By setting $y = 0$ for the rocket, we can solve for t in terms of T:

$0 = -(4.90\text{ m/s}^2)T^2 - (9.80\text{ m/s}^2)T(t-T) + (7.35\text{ m/s}^2)(t-T)^2$. Using the quadratic formula for the variable $x = t - T$ we find $x = t - T = \dfrac{(9.80\text{ m/s}^2)T + \sqrt{(9.80\text{ m/s}^2 T)^2 + (4)(7.35\text{ m/s}^2)(4.9)T^2}}{2(7.35\text{ m/s}^2)}$, or $t = 2.72\ T$. Now, using the condition that $x_{\text{rocket}} - x_{\text{plane}} = 1000\text{ m}$, we find $(236\text{ m/s})t + (12.7\text{ m/s}^2)(t-T)^2 - (236\text{ m/s})t = 1000\text{ m}$, or $(1.72T)^2 = 78.6\text{ s}^2$. Therefore $T = 5.15\text{ s}$.

EVALUATE: During the free-fall phase the rocket and airliner have the same x coordinate but the rocket moves downward from the airliner. After the engines fire, the rocket starts to move upward and its horizontal component of velocity starts to exceed that of the airliner.

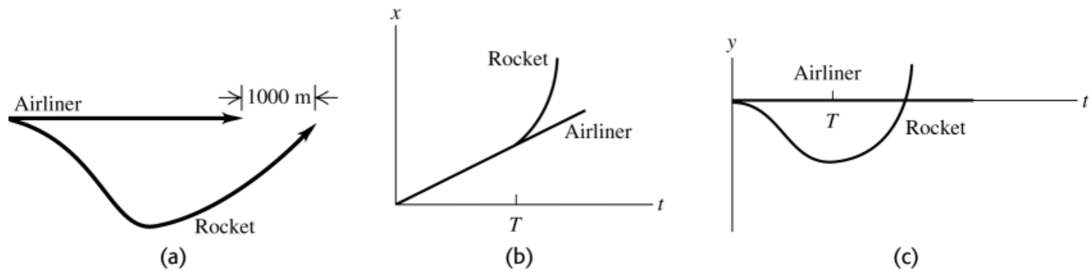


Figure 3.92

3.93. IDENTIFY: Apply the relative velocity relation.

SET UP: Let $v_{\text{C/W}}$ be the speed of the canoe relative to water and $v_{\text{W/G}}$ be the speed of the water relative to the ground.

EXECUTE: **(a)** Taking all units to be in km and h, we have three equations. We know that heading upstream $v_{\text{C/W}} - v_{\text{W/G}} = 2$. We know that heading downstream for a time t, $(v_{\text{C/W}} + v_{\text{W/G}})t = 5$. We also know that for the bottle $v_{\text{W/G}}(t+1) = 3$. Solving these three equations for $v_{\text{W/G}} = x$, $v_{\text{C/W}} = 2 + x$, therefore $(2 + x + x)t = 5$ or $(2 + 2x)t = 5$. Also $t = 3/x - 1$, so $(2+2x)\left(\dfrac{3}{x} - 1\right) = 5$ or $2x^2 + x - 6 = 0$. The positive solution is $x = v_{\text{W/G}} = 1.5\text{ km/h}$.

(b) $v_{\text{C/W}} = 2\text{ km/h} + v_{\text{W/G}} = 3.5\text{ km/h}$.

EVALUATE: When they head upstream, their speed relative to the ground is $3.5\text{ km/h} - 1.5\text{ km/h} = 2.0\text{ km/h}$. When they head downstream, their speed relative to the ground is $3.5\text{ km/h} + 1.5\text{ km/h} = 5.0\text{ km/h}$. The bottle is moving downstream at 1.5 km/s relative to the earth, so they are able to overtake it.