

MOTION ALONG A STRAIGHT LINE

- 2.1. **IDENTIFY:** The average velocity is $v_{\text{av-x}} = \frac{\Delta x}{\Delta t}$.

SET UP: Let $+x$ be upward.

EXECUTE: (a) $v_{\text{av-x}} = \frac{1000 \text{ m} - 63 \text{ m}}{4.75 \text{ s}} = 197 \text{ m/s}$

(b) $v_{\text{av-x}} = \frac{1000 \text{ m} - 0}{5.90 \text{ s}} = 169 \text{ m/s}$

EVALUATE: For the first 1.15 s of the flight, $v_{\text{av-x}} = \frac{63 \text{ m} - 0}{1.15 \text{ s}} = 54.8 \text{ m/s}$. When the velocity isn't constant the average velocity depends on the time interval chosen. In this motion the velocity is increasing.

- 2.2. **IDENTIFY:** $v_{\text{av-x}} = \frac{\Delta x}{\Delta t}$

SET UP: 13.5 days = $1.166 \times 10^5 \text{ s}$. At the release point, $x = +5.150 \times 10^6 \text{ m}$.

EXECUTE: (a) $v_{\text{av-x}} = \frac{x_2 - x_1}{\Delta t} = \frac{5.150 \times 10^6 \text{ m}}{1.166 \times 10^6 \text{ s}} = -4.42 \text{ m/s}$

(b) For the round trip, $x_2 = x_1$ and $\Delta x = 0$. The average velocity is zero.

EVALUATE: The average velocity for the trip from the nest to the release point is positive.

- 2.3. **IDENTIFY:** Target variable is the time Δt it takes to make the trip in heavy traffic. Use Eq.(2.2) that relates the average velocity to the displacement and average time.

SET UP: $v_{\text{av-x}} = \frac{\Delta x}{\Delta t}$ so $\Delta x = v_{\text{av-x}} \Delta t$ and $\Delta t = \frac{\Delta x}{v_{\text{av-x}}}$.

EXECUTE: Use the information given for normal driving conditions to calculate the distance between the two cities:

$$\Delta x = v_{\text{av-x}} \Delta t = (105 \text{ km/h})(1 \text{ h}/60 \text{ min})(140 \text{ min}) = 245 \text{ km}.$$

Now use $v_{\text{av-x}}$ for heavy traffic to calculate Δt ; Δx is the same as before:

$$\Delta t = \frac{\Delta x}{v_{\text{av-x}}} = \frac{245 \text{ km}}{70 \text{ km/h}} = 3.50 \text{ h} = 3 \text{ h and } 30 \text{ min}.$$

The trip takes an additional 1 hour and 10 minutes.

EVALUATE: The time is inversely proportional to the average speed, so the time in traffic is $(105/70)(140 \text{ min}) = 210 \text{ min}$.

- 2.4. **IDENTIFY:** The average velocity is $v_{\text{av-x}} = \frac{\Delta x}{\Delta t}$. Use the average speed for each segment to find the time traveled in that segment. The average speed is the distance traveled by the time.

SET UP: The post is 80 m west of the pillar. The total distance traveled is $200 \text{ m} + 280 \text{ m} = 480 \text{ m}$.

EXECUTE: (a) The eastward run takes time $\frac{200 \text{ m}}{5.0 \text{ m/s}} = 40.0 \text{ s}$ and the westward run takes $\frac{280 \text{ m}}{4.0 \text{ m/s}} = 70.0 \text{ s}$. The

average speed for the entire trip is $\frac{480 \text{ m}}{110.0 \text{ s}} = 4.4 \text{ m/s}$.

(b) $v_{\text{av-x}} = \frac{\Delta x}{\Delta t} = \frac{-80 \text{ m}}{110.0 \text{ s}} = -0.73 \text{ m/s}$. The average velocity is directed westward.

EVALUATE: The displacement is much less than the distance traveled and the magnitude of the average velocity is much less than the average speed. The average speed for the entire trip has a value that lies between the average speed for the two segments.

- 2.5. **IDENTIFY:** When they first meet the sum of the distances they have run is 200 m.

SET UP: Each runs with constant speed and continues around the track in the same direction, so the distance each runs is given by $d = vt$. Let the two runners be objects A and B .

EXECUTE: (a) $d_A + d_B = 200$ m, so $(6.20 \text{ m/s})t + (5.50 \text{ m/s})t = 200$ m and $t = \frac{200 \text{ m}}{11.70 \text{ m/s}} = 17.1$ s.

(b) $d_A = v_A t = (6.20 \text{ m/s})(17.1 \text{ s}) = 106$ m. $d_B = v_B t = (5.50 \text{ m/s})(17.1 \text{ s}) = 94$ m. The faster runner will be 106 m from the starting point and the slower runner will be 94 m from the starting point. These distances are measured around the circular track and are not straight-line distances.

EVALUATE: The faster runner runs farther.

- 2.6. **IDENTIFY:** To overtake the slower runner the first time the fast runner must run 200 m farther. To overtake the slower runner the second time the faster runner must run 400 m farther.

SET UP: t and x_0 are the same for the two runners.

EXECUTE: (a) Apply $x - x_0 = v_{0x}t$ to each runner: $(x - x_0)_f = (6.20 \text{ m/s})t$ and $(x - x_0)_s = (5.50 \text{ m/s})t$.

$(x - x_0)_f = (x - x_0)_s + 200$ m gives $(6.20 \text{ m/s})t = (5.50 \text{ m/s})t + 200$ m and $t = \frac{200 \text{ m}}{6.20 \text{ m/s} - 5.50 \text{ m/s}} = 286$ s.

$(x - x_0)_f = 1770$ m and $(x - x_0)_s = 1570$ m.

(b) Repeat the calculation but now $(x - x_0)_f = (x - x_0)_s + 400$ m. $t = 572$ s. The fast runner has traveled 3540 m. He has made 17 full laps for 3400 m and 140 m past the starting line in this 18th lap.

EVALUATE: In part (a) the fast runner will have run 8 laps for 1600 m and will be 170 m past the starting line in his 9th lap.

- 2.7. **IDENTIFY:** In time t_s the S-waves travel a distance $d = v_s t_s$ and in time t_p the P-waves travel a distance $d = v_p t_p$.

SET UP: $t_s = t_p + 33$ s

EXECUTE: $\frac{d}{v_s} = \frac{d}{v_p} + 33$ s. $d \left(\frac{1}{3.5 \text{ km/s}} - \frac{1}{6.5 \text{ km/s}} \right) = 33$ s and $d = 250$ km.

EVALUATE: The times of travel for each wave are $t_s = 71$ s and $t_p = 38$ s.

- 2.8. **IDENTIFY:** The average velocity is $v_{\text{av-x}} = \frac{\Delta x}{\Delta t}$. Use $x(t)$ to find x for each t .

SET UP: $x(0) = 0$, $x(2.00 \text{ s}) = 5.60$ m, and $x(4.00 \text{ s}) = 20.8$ m

EXECUTE: (a) $v_{\text{av-x}} = \frac{5.60 \text{ m} - 0}{2.00 \text{ s}} = +2.80$ m/s

(b) $v_{\text{av-x}} = \frac{20.8 \text{ m} - 0}{4.00 \text{ s}} = +5.20$ m/s

(c) $v_{\text{av-x}} = \frac{20.8 \text{ m} - 5.60 \text{ m}}{2.00 \text{ s}} = +7.60$ m/s

EVALUATE: The average velocity depends on the time interval being considered.

- 2.9. (a) **IDENTIFY:** Calculate the average velocity using Eq.(2.2).

SET UP: $v_{\text{av-x}} = \frac{\Delta x}{\Delta t}$ so use $x(t)$ to find the displacement Δx for this time interval.

EXECUTE: $t = 0$: $x = 0$

$t = 10.0$ s: $x = (2.40 \text{ m/s}^2)(10.0 \text{ s})^2 - (0.120 \text{ m/s}^3)(10.0 \text{ s})^3 = 240 \text{ m} - 120 \text{ m} = 120$ m.

Then $v_{\text{av-x}} = \frac{\Delta x}{\Delta t} = \frac{120 \text{ m}}{10.0 \text{ s}} = 12.0$ m/s.

(b) **IDENTIFY:** Use Eq.(2.3) to calculate $v_x(t)$ and evaluate this expression at each specified t .

SET UP: $v_x = \frac{dx}{dt} = 2bt - 3ct^2$.

EXECUTE: (i) $t = 0$: $v_x = 0$

(ii) $t = 5.0$ s: $v_x = 2(2.40 \text{ m/s}^2)(5.0 \text{ s}) - 3(0.120 \text{ m/s}^3)(5.0 \text{ s})^2 = 24.0 \text{ m/s} - 9.0 \text{ m/s} = 15.0$ m/s.

(iii) $t = 10.0$ s: $v_x = 2(2.40 \text{ m/s}^2)(10.0 \text{ s}) - 3(0.120 \text{ m/s}^3)(10.0 \text{ s})^2 = 48.0 \text{ m/s} - 36.0 \text{ m/s} = 12.0$ m/s.

(c) IDENTIFY: Find the value of t when $v_x(t)$ from part (b) is zero.

SET UP: $v_x = 2bt - 3ct^2$

$v_x = 0$ at $t = 0$.

$v_x = 0$ next when $2bt - 3ct^2 = 0$

EXECUTE: $2b = 3ct$ so $t = \frac{2b}{3c} = \frac{2(2.40 \text{ m/s}^2)}{30(.120 \text{ m/s}^3)} = 13.3 \text{ s}$

EVALUATE: $v_x(t)$ for this motion says the car starts from rest, speeds up, and then slows down again.

2.10. IDENTIFY and SET UP: The instantaneous velocity is the slope of the tangent to the x versus t graph.

EXECUTE: (a) The velocity is zero where the graph is horizontal; point IV.

(b) The velocity is constant and positive where the graph is a straight line with positive slope; point I.

(c) The velocity is constant and negative where the graph is a straight line with negative slope; point V.

(d) The slope is positive and increasing at point II.

(e) The slope is positive and decreasing at point III.

EVALUATE: The sign of the velocity indicates its direction.

2.11. IDENTIFY: The average velocity is given by $v_{\text{av-x}} = \frac{\Delta x}{\Delta t}$. We can find the displacement Δx for each constant velocity time interval. The average speed is the distance traveled divided by the time.

SET UP: For $t = 0$ to $t = 2.0 \text{ s}$, $v_x = 2.0 \text{ m/s}$. For $t = 2.0 \text{ s}$ to $t = 3.0 \text{ s}$, $v_x = 3.0 \text{ m/s}$. In part (b),

$v_x = -3.0 \text{ m/s}$ for $t = 2.0 \text{ s}$ to $t = 3.0 \text{ s}$. When the velocity is constant, $\Delta x = v_x \Delta t$.

EXECUTE: (a) For $t = 0$ to $t = 2.0 \text{ s}$, $\Delta x = (2.0 \text{ m/s})(2.0 \text{ s}) = 4.0 \text{ m}$. For $t = 2.0 \text{ s}$ to $t = 3.0 \text{ s}$,

$\Delta x = (3.0 \text{ m/s})(1.0 \text{ s}) = 3.0 \text{ m}$. For the first 3.0 s , $\Delta x = 4.0 \text{ m} + 3.0 \text{ m} = 7.0 \text{ m}$. The distance traveled is also 7.0 m .

The average velocity is $v_{\text{av-x}} = \frac{\Delta x}{\Delta t} = \frac{7.0 \text{ m}}{3.0 \text{ s}} = 2.33 \text{ m/s}$. The average speed is also 2.33 m/s .

(b) For $t = 2.0 \text{ s}$ to 3.0 s , $\Delta x = (-3.0 \text{ m/s})(1.0 \text{ s}) = -3.0 \text{ m}$. For the first 3.0 s , $\Delta x = 4.0 \text{ m} + (-3.0 \text{ m}) = +1.0 \text{ m}$.

The dog runs 4.0 m in the $+x$ -direction and then 3.0 m in the $-x$ -direction, so the distance traveled is still 7.0 m .

$v_{\text{av-x}} = \frac{\Delta x}{\Delta t} = \frac{1.0 \text{ m}}{3.0 \text{ s}} = 0.33 \text{ m/s}$. The average speed is $\frac{7.00 \text{ m}}{3.00 \text{ s}} = 2.33 \text{ m/s}$.

EVALUATE: When the motion is always in the same direction, the displacement and the distance traveled are equal and the average velocity has the same magnitude as the average speed. When the motion changes direction during the time interval, those quantities are different.

2.12. IDENTIFY and SET UP: $a_{\text{av-x}} = \frac{\Delta v_x}{\Delta t}$. The instantaneous acceleration is the slope of the tangent to the v_x versus t graph.

EXECUTE: (a) 0 s to 2 s : $a_{\text{av-x}} = 0$; 2 s to 4 s : $a_{\text{av-x}} = 1.0 \text{ m/s}^2$; 4 s to 6 s : $a_{\text{av-x}} = 1.5 \text{ m/s}^2$; 6 s to 8 s :

$a_{\text{av-x}} = 2.5 \text{ m/s}^2$; 8 s to 10 s : $a_{\text{av-x}} = 2.5 \text{ m/s}^2$; 10 s to 12 s : $a_{\text{av-x}} = 2.5 \text{ m/s}^2$; 12 s to 14 s : $a_{\text{av-x}} = 1.0 \text{ m/s}^2$; 14 s to

16 s : $a_{\text{av-x}} = 0$. The acceleration is not constant over the entire 16 s time interval. The acceleration is constant between 6 s and 12 s .

(b) The graph of v_x versus t is given in Fig. 2.12. $t = 9 \text{ s}$: $a_x = 2.5 \text{ m/s}^2$; $t = 13 \text{ s}$: $a_x = 1.0 \text{ m/s}^2$; $t = 15 \text{ s}$: $a_x = 0$.

EVALUATE: The acceleration is constant when the velocity changes at a constant rate. When the velocity is constant, the acceleration is zero.

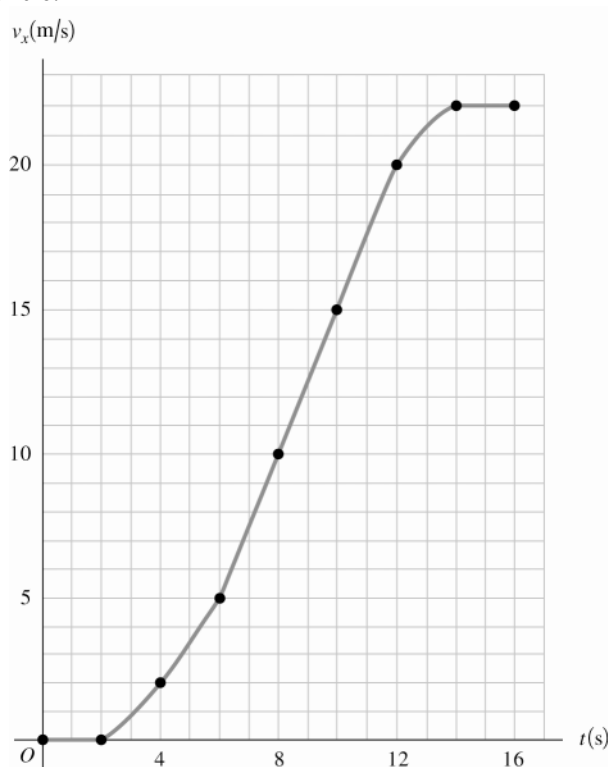


Figure 2.12

2.13. IDENTIFY: The average acceleration for a time interval Δt is given by $a_{\text{av-x}} = \frac{\Delta v_x}{\Delta t}$.

SET UP: Assume the car is moving in the $+x$ direction. $1 \text{ mi/h} = 0.447 \text{ m/s}$, so $60 \text{ mi/h} = 26.82 \text{ m/s}$, $200 \text{ mi/h} = 89.40 \text{ m/s}$ and $253 \text{ mi/h} = 113.1 \text{ m/s}$.

EXECUTE: (a) The graph of v_x versus t is sketched in Figure 2.13. The graph is not a straight line, so the acceleration is not constant.

(b) (i) $a_{\text{av-x}} = \frac{26.82 \text{ m/s} - 0}{2.1 \text{ s}} = 12.8 \text{ m/s}^2$ **(ii)** $a_{\text{av-x}} = \frac{89.40 \text{ m/s} - 26.82 \text{ m/s}}{20.0 \text{ s} - 2.1 \text{ s}} = 3.50 \text{ m/s}^2$ **(iii)**

$a_{\text{av-x}} = \frac{113.1 \text{ m/s} - 89.40 \text{ m/s}}{53 \text{ s} - 20.0 \text{ s}} = 0.718 \text{ m/s}^2$. The slope of the graph of v_x versus t decreases as t increases. This is consistent with an average acceleration that decreases in magnitude during each successive time interval.

EVALUATE: The average acceleration depends on the chosen time interval. For the interval between 0 and 53 s,

$a_{\text{av-x}} = \frac{113.1 \text{ m/s} - 0}{53 \text{ s}} = 2.13 \text{ m/s}^2$.

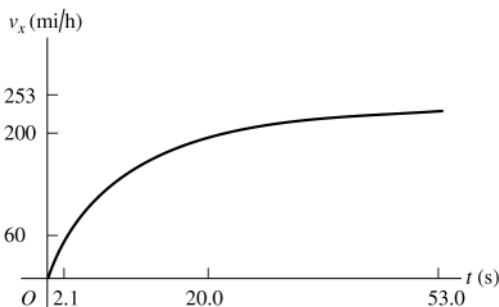


Figure 2.13

2.14. IDENTIFY: $a_{\text{av-x}} = \frac{\Delta v_x}{\Delta t}$. $a_x(t)$ is the slope of the v_x versus t graph.

SET UP: $60 \text{ km/h} = 16.7 \text{ m/s}$

EXECUTE: (a) (i) $a_{\text{av-x}} = \frac{16.7 \text{ m/s} - 0}{10 \text{ s}} = 1.7 \text{ m/s}^2$. (ii) $a_{\text{av-x}} = \frac{0 - 16.7 \text{ m/s}}{10 \text{ s}} = -1.7 \text{ m/s}^2$.

(iii) $\Delta v_x = 0$ and $a_{\text{av-x}} = 0$. (iv) $\Delta v_x = 0$ and $a_{\text{av-x}} = 0$.

(b) At $t = 20 \text{ s}$, v_x is constant and $a_x = 0$. At $t = 35 \text{ s}$, the graph of v_x versus t is a straight line and $a_x = a_{\text{av-x}} = -1.7 \text{ m/s}^2$.

EVALUATE: When $a_{\text{av-x}}$ and v_x have the same sign the speed is increasing. When they have opposite sign the speed is decreasing.

2.15. IDENTIFY and SET UP: Use $v_x = \frac{dx}{dt}$ and $a_x = \frac{dv_x}{dt}$ to calculate $v_x(t)$ and $a_x(t)$.

EXECUTE: $v_x = \frac{dx}{dt} = 2.00 \text{ cm/s} - (0.125 \text{ cm/s}^2)t$

$a_x = \frac{dv_x}{dt} = -0.125 \text{ cm/s}^2$

(a) At $t = 0$, $x = 50.0 \text{ cm}$, $v_x = 2.00 \text{ cm/s}$, $a_x = -0.125 \text{ cm/s}^2$.

(b) Set $v_x = 0$ and solve for t : $t = 16.0 \text{ s}$.

(c) Set $x = 50.0 \text{ cm}$ and solve for t . This gives $t = 0$ and $t = 32.0 \text{ s}$. The turtle returns to the starting point after 32.0 s .

(d) Turtle is 10.0 cm from starting point when $x = 60.0 \text{ cm}$ or $x = 40.0 \text{ cm}$.

Set $x = 60.0 \text{ cm}$ and solve for t : $t = 6.20 \text{ s}$ and $t = 25.8 \text{ s}$.

At $t = 6.20 \text{ s}$, $v_x = +1.23 \text{ cm/s}$.

At $t = 25.8 \text{ s}$, $v_x = -1.23 \text{ cm/s}$.

Set $x = 40.0 \text{ cm}$ and solve for t : $t = 36.4 \text{ s}$ (other root to the quadratic equation is negative and hence nonphysical).

At $t = 36.4 \text{ s}$, $v_x = -2.55 \text{ cm/s}$.

(e) The graphs are sketched in Figure 2.15.

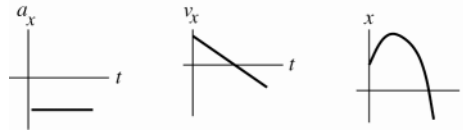


Figure 2.15

EVALUATE: The acceleration is constant and negative. v_x is linear in time. It is initially positive, decreases to zero, and then becomes negative with increasing magnitude. The turtle initially moves farther away from the origin but then stops and moves in the $-x$ -direction.

2.16. IDENTIFY: Use Eq.(2.4), with $\Delta t = 10 \text{ s}$ in all cases.

SET UP: v_x is negative if the motion is to the right.

EXECUTE: (a) $((5.0 \text{ m/s}) - (15.0 \text{ m/s})) / (10 \text{ s}) = -1.0 \text{ m/s}^2$

(b) $((-15.0 \text{ m/s}) - (-5.0 \text{ m/s})) / (10 \text{ s}) = -1.0 \text{ m/s}^2$

(c) $((-15.0 \text{ m/s}) - (+15.0 \text{ m/s})) / (10 \text{ s}) = -3.0 \text{ m/s}^2$

EVALUATE: In all cases, the negative acceleration indicates an acceleration to the left.

2.17. IDENTIFY: The average acceleration is $a_{\text{av-x}} = \frac{\Delta v_x}{\Delta t}$

SET UP: Assume the car goes from rest to 65 mi/h (29 m/s) in 10 s . In braking, assume the car goes from 65 mi/h to zero in 4.0 s . Let $+x$ be in the direction the car is traveling.

EXECUTE: (a) $a_{\text{av-x}} = \frac{29 \text{ m/s} - 0}{10 \text{ s}} = 2.9 \text{ m/s}^2$

(b) $a_{\text{av-x}} = \frac{0 - 29 \text{ m/s}}{4.0 \text{ s}} = -7.2 \text{ m/s}^2$

(c) In part (a) the speed increases so the acceleration is in the same direction as the velocity. If the velocity direction is positive, then the acceleration is positive. In part (b) the speed decreases so the acceleration is in the direction opposite to the direction of the velocity. If the velocity direction is positive then the acceleration is negative, and if the velocity direction is negative then the acceleration direction is positive.

EVALUATE: The sign of the velocity and of the acceleration indicate their direction.

2.18. IDENTIFY: The average acceleration is $a_{\text{av-x}} = \frac{\Delta v_x}{\Delta t}$. Use $v_x(t)$ to find v_x at each t . The instantaneous acceleration

$$\text{is } a_x = \frac{dv_x}{dt}.$$

SET UP: $v_x(0) = 3.00 \text{ m/s}$ and $v_x(5.00 \text{ s}) = 5.50 \text{ m/s}$.

EXECUTE: (a) $a_{\text{av-x}} = \frac{\Delta v_x}{\Delta t} = \frac{5.50 \text{ m/s} - 3.00 \text{ m/s}}{5.00 \text{ s}} = 0.500 \text{ m/s}^2$

(b) $a_x = \frac{dv_x}{dt} = (0.100 \text{ m/s}^3)(2t) = (0.200 \text{ m/s}^3)t$. At $t = 0$, $a_x = 0$. At $t = 5.00 \text{ s}$, $a_x = 1.00 \text{ m/s}^2$.

(c) Graphs of $v_x(t)$ and $a_x(t)$ are given in Figure 2.18.

EVALUATE: $a_x(t)$ is the slope of $v_x(t)$ and increases as t increases. The average acceleration for $t = 0$ to $t = 5.00 \text{ s}$ equals the instantaneous acceleration at the midpoint of the time interval, $t = 2.50 \text{ s}$, since $a_x(t)$ is a linear function of t .

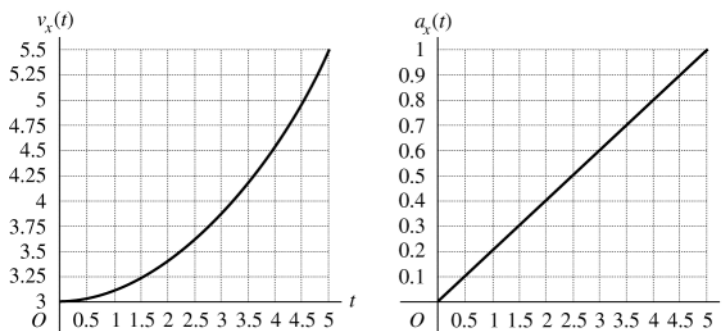


Figure 2.18

2.19. (a) IDENTIFY and SET UP: v_x is the slope of the x versus t curve and a_x is the slope of the v_x versus t curve.

EXECUTE: $t = 0$ to $t = 5 \text{ s}$: x versus t is a parabola so a_x is a constant. The curvature is positive so a_x is positive. v_x versus t is a straight line with positive slope. $v_{0x} = 0$.

$t = 5 \text{ s}$ to $t = 15 \text{ s}$: x versus t is a straight line so v_x is constant and $a_x = 0$. The slope of x versus t is positive so v_x is positive.

$t = 15 \text{ s}$ to $t = 25 \text{ s}$: x versus t is a parabola with negative curvature, so a_x is constant and negative. v_x versus t is a straight line with negative slope. The velocity is zero at 20 s , positive for 15 s to 20 s , and negative for 20 s to 25 s .

$t = 25 \text{ s}$ to $t = 35 \text{ s}$: x versus t is a straight line so v_x is constant and $a_x = 0$. The slope of x versus t is negative so v_x is negative.

$t = 35 \text{ s}$ to $t = 40 \text{ s}$: x versus t is a parabola with positive curvature, so a_x is constant and positive. v_x versus t is a straight line with positive slope. The velocity reaches zero at $t = 40 \text{ s}$.

The graphs of $v_x(t)$ and $a_x(t)$ are sketched in Figure 2.19a.

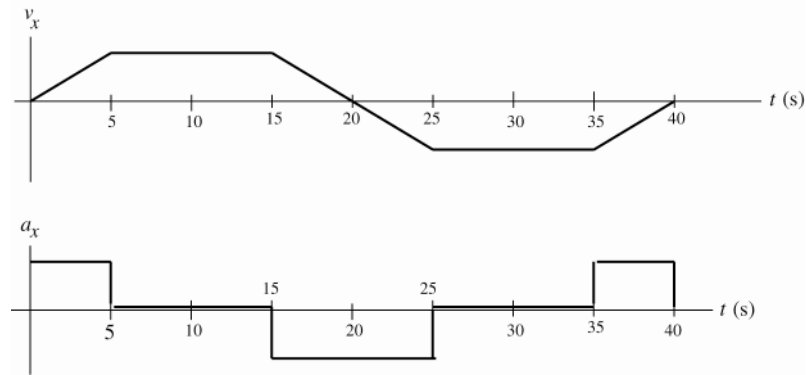


Figure 2.19a

(b) The motions diagrams are sketched in Figure 2.19b.

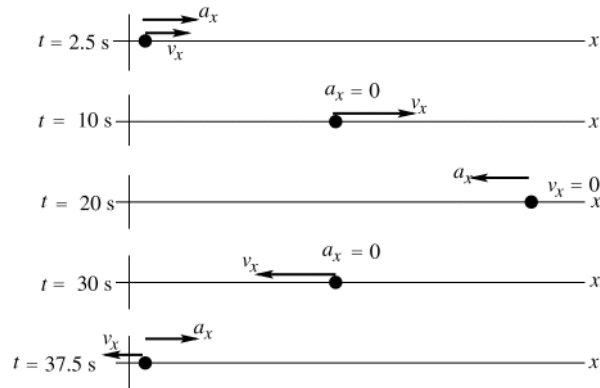


Figure 2.19b

EVALUATE: The spider speeds up for the first 5 s, since v_x and a_x are both positive. Starting at $t = 15$ s the spider starts to slow down, stops momentarily at $t = 20$ s, and then moves in the opposite direction. At $t = 35$ s the spider starts to slow down again and stops at $t = 40$ s.

2.20. IDENTIFY: $v_x(t) = \frac{dx}{dt}$ and $a_x(t) = \frac{dv_x}{dt}$

SET UP: $\frac{d}{dt}(t^n) = nt^{n-1}$ for $n \geq 1$.

EXECUTE: (a) $v_x(t) = (9.60 \text{ m/s}^2)t - (0.600 \text{ m/s}^6)t^5$ and $a_x(t) = 9.60 \text{ m/s}^2 - (3.00 \text{ m/s}^6)t^4$. Setting $v_x = 0$ gives $t = 0$ and $t = 2.00$ s. At $t = 0$, $x = 2.17$ m and $a_x = 9.60 \text{ m/s}^2$. At $t = 2.00$ s, $x = 15.0$ m and $a_x = -38.4 \text{ m/s}^2$.

(b) The graphs are given in Figure 2.20.

EVALUATE: For the entire time interval from $t = 0$ to $t = 2.00$ s, the velocity v_x is positive and x increases. While a_x is also positive the speed increases and while a_x is negative the speed decreases.

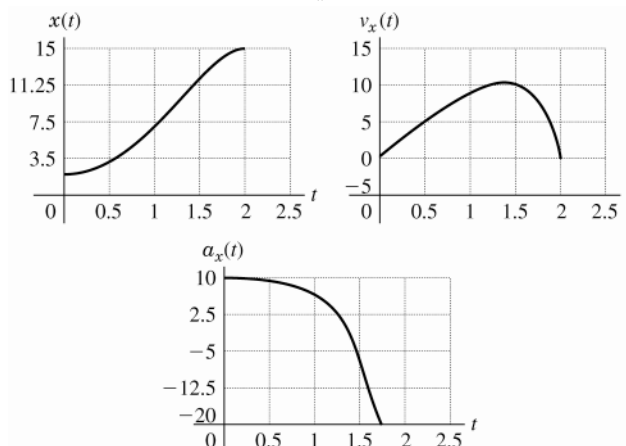


Figure 2.20

2.21. IDENTIFY: Use the constant acceleration equations to find v_{0x} and a_x .

(a) SET UP: The situation is sketched in Figure 2.21.

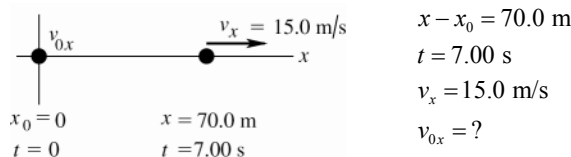


Figure 2.21

EXECUTE: Use $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$, so $v_{0x} = \frac{2(x - x_0)}{t} - v_x = \frac{2(70.0 \text{ m})}{7.00 \text{ s}} - 15.0 \text{ m/s} = 5.0 \text{ m/s}$.

(b) Use $v_x = v_{0x} + a_x t$, so $a_x = \frac{v_x - v_{0x}}{t} = \frac{15.0 \text{ m/s} - 5.0 \text{ m/s}}{7.00 \text{ s}} = 1.43 \text{ m/s}^2$.

EVALUATE: The average velocity is $(70.0 \text{ m})/(7.00 \text{ s}) = 10.0 \text{ m/s}$. The final velocity is larger than this, so the antelope must be speeding up during the time interval; $v_{0x} < v_x$ and $a_x > 0$.

2.22. IDENTIFY: Apply the constant acceleration kinematic equations.

SET UP: Let $+x$ be in the direction of the motion of the plane. $173 \text{ mi/h} = 77.33 \text{ m/s}$. $307 \text{ ft} = 93.57 \text{ m}$.

EXECUTE: (a) $v_{0x} = 0$, $v_x = 77.33 \text{ m/s}$ and $x - x_0 = 93.57 \text{ m}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(77.33 \text{ m/s})^2 - 0}{2(93.57 \text{ m})} = 32.0 \text{ m/s}^2.$$

(b) $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$ gives $t = \frac{2(x - x_0)}{v_{0x} + v_x} = \frac{2(93.57 \text{ m})}{0 + 77.33 \text{ m/s}} = 2.42 \text{ s}$

EVALUATE: Either $v_x = v_{0x} + a_x t$ or $x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$ could also be used to find t and would give the same result as in part (b).

2.23. IDENTIFY: For constant acceleration, Eqs. (2.8), (2.12), (2.13) and (2.14) apply.

SET UP: Assume the ball starts from rest and moves in the $+x$ -direction.

EXECUTE: (a) $x - x_0 = 1.50 \text{ m}$, $v_x = 45.0 \text{ m/s}$ and $v_{0x} = 0$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(45.0 \text{ m/s})^2}{2(1.50 \text{ m})} = 675 \text{ m/s}^2.$$

(b) $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$ gives $t = \frac{2(x - x_0)}{v_{0x} + v_x} = \frac{2(1.50 \text{ m})}{0 + 45.0 \text{ m/s}} = 0.0667 \text{ s}$

EVALUATE: We could also use $v_x = v_{0x} + a_x t$ to find $t = \frac{v_x}{a_x} = \frac{45.0 \text{ m/s}}{675 \text{ m/s}^2} = 0.0667 \text{ s}$ which agrees with our previous result. The acceleration of the ball is very large.

2.24. IDENTIFY: For constant acceleration, Eqs. (2.8), (2.12), (2.13) and (2.14) apply.

SET UP: Assume the ball moves in the $+x$ direction.

EXECUTE: (a) $v_x = 73.14 \text{ m/s}$, $v_{0x} = 0$ and $t = 30.0 \text{ ms}$. $v_x = v_{0x} + a_x t$ gives

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{73.14 \text{ m/s} - 0}{30.0 \times 10^{-3} \text{ s}} = 2440 \text{ m/s}^2.$$

$$(b) \quad x - x_0 = \left(\frac{v_{0x} + v_x}{2} \right) t = \left(\frac{0 + 73.14 \text{ m/s}}{2} \right) (30.0 \times 10^{-3} \text{ s}) = 1.10 \text{ m}$$

EVALUATE: We could also use $x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$ to calculate $x - x_0$:

$x - x_0 = \frac{1}{2} (2440 \text{ m/s}^2) (30.0 \times 10^{-3} \text{ s})^2 = 1.10 \text{ m}$, which agrees with our previous result. The acceleration of the ball is very large.

2.25. IDENTIFY: Assume that the acceleration is constant and apply the constant acceleration kinematic equations. Set $|a_x|$ equal to its maximum allowed value.

SET UP: Let $+x$ be the direction of the initial velocity of the car. $a_x = -250 \text{ m/s}^2$. $105 \text{ km/h} = 29.17 \text{ m/s}$.

EXECUTE: $v_{0x} = +29.17 \text{ m/s}$. $v_x = 0$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $x - x_0 = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - (29.17 \text{ m/s})^2}{2(-250 \text{ m/s}^2)} = 1.70 \text{ m}$.

EVALUATE: The car frame stops over a shorter distance and has a larger magnitude of acceleration. Part of your 1.70 m stopping distance is the stopping distance of the car and part is how far you move relative to the car while stopping.

2.26. IDENTIFY: Apply constant acceleration equations to the motion of the car.

SET UP: Let $+x$ be the direction the car is moving.

EXECUTE: (a) From Eq. (2.13), with $v_{0x} = 0$, $a_x = \frac{v_x^2}{2(x - x_0)} = \frac{(20 \text{ m/s})^2}{2(120 \text{ m})} = 1.67 \text{ m/s}^2$.

(b) Using Eq. (2.14), $t = 2(x - x_0)/v_x = 2(120 \text{ m})/(20 \text{ m/s}) = 12 \text{ s}$.

(c) $(12 \text{ s})(20 \text{ m/s}) = 240 \text{ m}$.

EVALUATE: The average velocity of the car is half the constant speed of the traffic, so the traffic travels twice as far.

2.27. IDENTIFY: The average acceleration is $a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t}$. For constant acceleration, Eqs. (2.8), (2.12), (2.13) and (2.14) apply.

SET UP: Assume the shuttle travels in the $+x$ direction. $161 \text{ km/h} = 44.72 \text{ m/s}$ and $1610 \text{ km/h} = 447.2 \text{ m/s}$. $1.00 \text{ min} = 60.0 \text{ s}$

EXECUTE: (a) (i) $a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{44.72 \text{ m/s} - 0}{8.00 \text{ s}} = 5.59 \text{ m/s}^2$

(ii) $a_{\text{av-}x} = \frac{447.2 \text{ m/s} - 44.72 \text{ m/s}}{60.0 \text{ s} - 8.00 \text{ s}} = 7.74 \text{ m/s}^2$

(b) (i) $t = 8.00 \text{ s}$, $v_{0x} = 0$, and $v_x = 44.72 \text{ m/s}$. $x - x_0 = \left(\frac{v_{0x} + v_x}{2} \right) t = \left(\frac{0 + 44.72 \text{ m/s}}{2} \right) (8.00 \text{ s}) = 179 \text{ m}$.

(ii) $\Delta t = 60.0 \text{ s} - 8.00 \text{ s} = 52.0 \text{ s}$, $v_{0x} = 44.72 \text{ m/s}$, and $v_x = 447.2 \text{ m/s}$.

$x - x_0 = \left(\frac{v_{0x} + v_x}{2} \right) t = \left(\frac{44.72 \text{ m/s} + 447.2 \text{ m/s}}{2} \right) (52.0 \text{ s}) = 1.28 \times 10^4 \text{ m}$.

EVALUATE: When the acceleration is constant the instantaneous acceleration throughout the time interval equals the average acceleration for that time interval. We could have calculated the distance in part (a) as

$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2 = \frac{1}{2} (5.59 \text{ m/s}^2) (8.00 \text{ s})^2 = 179 \text{ m}$, which agrees with our previous calculation.

2.28. IDENTIFY: Apply the constant acceleration kinematic equations to the motion of the car.

SET UP: $0.250 \text{ mi} = 1320 \text{ ft}$. $60.0 \text{ mph} = 88.0 \text{ ft/s}$. Let $+x$ be the direction the car is traveling.

EXECUTE: (a) braking: $v_{0x} = 88.0 \text{ ft/s}$, $x - x_0 = 146 \text{ ft}$, $v_x = 0$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{0 - (88.0 \text{ ft/s})^2}{2(146 \text{ ft})} = -26.5 \text{ ft/s}^2$$

Speeding up: $v_{0x} = 0$, $x - x_0 = 1320 \text{ ft}$, $t = 19.9 \text{ s}$. $x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$ gives

$$a_x = \frac{2(x - x_0)}{t^2} = \frac{2(1320 \text{ ft})}{(19.9 \text{ s})^2} = 6.67 \text{ ft/s}^2$$

(b) $v_x = v_{0x} + a_x t = 0 + (6.67 \text{ ft/s}^2)(19.9 \text{ s}) = 133 \text{ ft/s} = 90.5 \text{ mph}$

(c) $t = \frac{v_x - v_{0x}}{a_x} = \frac{0 - 88.0 \text{ ft/s}}{-26.5 \text{ ft/s}^2} = 3.32 \text{ s}$

EVALUATE: The magnitude of the acceleration while braking is much larger than when speeding up. That is why it takes much longer to go from 0 to 60 mph than to go from 60 mph to 0.

2.29. IDENTIFY: The acceleration a_x is the slope of the graph of v_x versus t .

SET UP: The signs of v_x and of a_x indicate their directions.

EXECUTE: (a) Reading from the graph, at $t = 4.0 \text{ s}$, $v_x = 2.7 \text{ cm/s}$, to the right and at $t = 7.0 \text{ s}$, $v_x = 1.3 \text{ cm/s}$, to the left.

(b) v_x versus t is a straight line with slope $-\frac{8.0 \text{ cm/s}}{6.0 \text{ s}} = -1.3 \text{ cm/s}^2$. The acceleration is constant and equal to

1.3 cm/s^2 , to the left. It has this value at all times.

(c) Since the acceleration is constant, $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$. For $t = 0$ to 4.5 s ,

$$x - x_0 = (8.0 \text{ cm/s})(4.5 \text{ s}) + \frac{1}{2}(-1.3 \text{ cm/s}^2)(4.5 \text{ s})^2 = 22.8 \text{ cm}.$$

$$x - x_0 = (8.0 \text{ cm/s})(7.5 \text{ s}) + \frac{1}{2}(-1.3 \text{ cm/s}^2)(7.5 \text{ s})^2 = 23.4 \text{ cm}$$

(d) The graphs of a_x and x versus t are given in Fig. 2.29.

EVALUATE: In part (c) we could have instead used $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$.

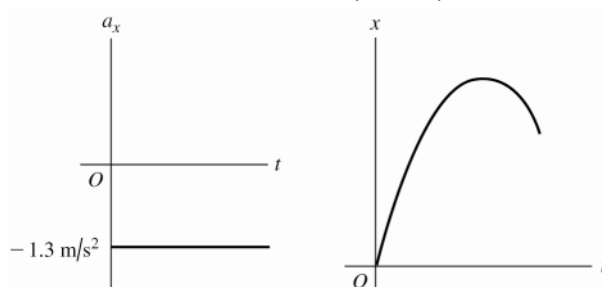


Figure 2.29

2.30. IDENTIFY: Use the constant acceleration equations to find x , v_{0x} , v_x and a_x for each constant-acceleration segment of the motion.

SET UP: Let $+x$ be the direction of motion of the car and let $x = 0$ at the first traffic light.

EXECUTE: (a) For $t = 0$ to $t = 8 \text{ s}$: $x = \left(\frac{v_{0x} + v_x}{2}\right)t = \left(\frac{0 + 20 \text{ m/s}}{2}\right)(8 \text{ s}) = 80 \text{ m}$.

$a_x = \frac{v_x - v_{0x}}{t} = \frac{20 \text{ m/s}}{8 \text{ s}} = +2.50 \text{ m/s}^2$. The car moves from $x = 0$ to $x = 80 \text{ m}$. The velocity v_x increases linearly

from zero to 20 m/s . The acceleration is a constant 2.50 m/s^2 .

Constant speed for 60 m : The car moves from $x = 80 \text{ m}$ to $x = 140 \text{ m}$. v_x is a constant 20 m/s . $a_x = 0$. This

interval starts at $t = 8 \text{ s}$ and continues until $t = \frac{60 \text{ m}}{20 \text{ m/s}} + 8 \text{ s} = 11 \text{ s}$.

Slowing from 20 m/s until stopped: The car moves from $x = 140 \text{ m}$ to $x = 180 \text{ m}$. The velocity decreases linearly

from 20 m/s to zero. $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$ gives $t = \frac{2(40 \text{ m})}{20 \text{ m/s} + 0} = 4 \text{ s}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$a_x = \frac{-(20.0 \text{ m/s})^2}{2(40 \text{ m})} = -5.00 \text{ m/s}^2$. This segment is from $t = 11 \text{ s}$ to $t = 15 \text{ s}$. The acceleration is a

constant -5.00 m/s^2 .

The graphs are drawn in Figure 2.30a.

(b) The motion diagram is sketched in Figure 2.30b.

EVALUATE: When \vec{a} and \vec{v} are in the same direction, the speed increases ($t = 0$ to $t = 8$ s). When \vec{a} and \vec{v} are in opposite directions, the speed decreases ($t = 11$ s to $t = 15$ s). When $a = 0$ the speed is constant $t = 8$ s to $t = 11$ s.

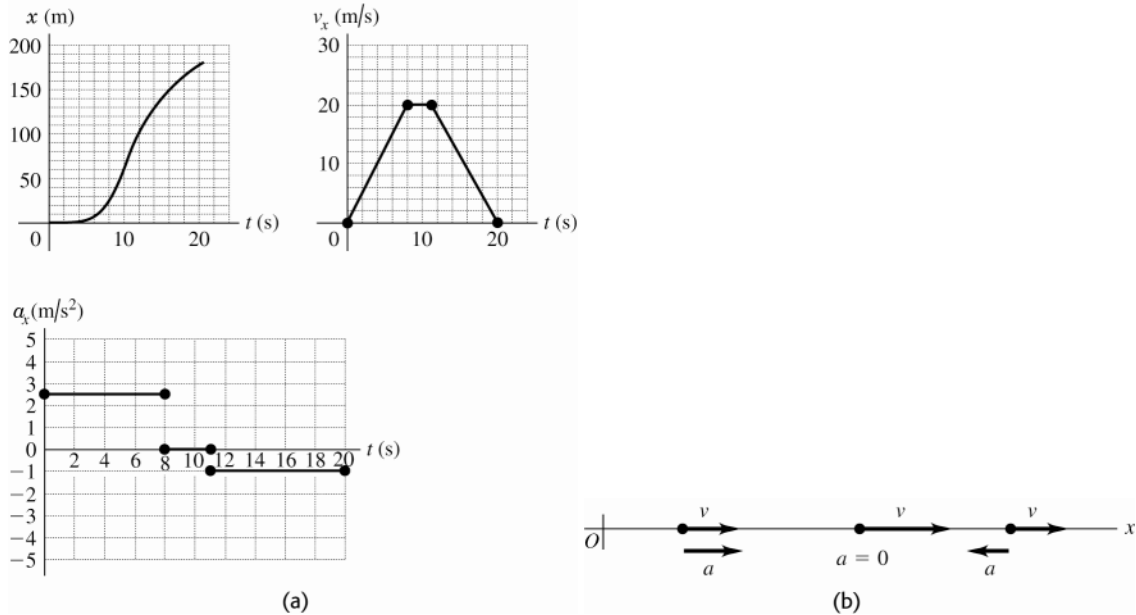


Figure 2.30a-b

2.31. (a) IDENTIFY and SET UP: The acceleration a_x at time t is the slope of the tangent to the v_x versus t curve at time t .

EXECUTE: At $t = 3$ s, the v_x versus t curve is a horizontal straight line, with zero slope. Thus $a_x = 0$.

At $t = 7$ s, the v_x versus t curve is a straight-line segment with slope $\frac{45 \text{ m/s} - 20 \text{ m/s}}{9 \text{ s} - 5 \text{ s}} = 6.3 \text{ m/s}^2$.

Thus $a_x = 6.3 \text{ m/s}^2$.

At $t = 11$ s the curve is again a straight-line segment, now with slope $\frac{-0 - 45 \text{ m/s}}{13 \text{ s} - 9 \text{ s}} = -11.2 \text{ m/s}^2$.

Thus $a_x = -11.2 \text{ m/s}^2$.

EVALUATE: $a_x = 0$ when v_x is constant, $a_x > 0$ when v_x is positive and the speed is increasing, and $a_x < 0$ when v_x is positive and the speed is decreasing.

(b) IDENTIFY: Calculate the displacement during the specified time interval.

SET UP: We can use the constant acceleration equations only for time intervals during which the acceleration is constant. If necessary, break the motion up into constant acceleration segments and apply the constant acceleration equations for each segment. For the time interval $t = 0$ to $t = 5$ s the acceleration is constant and equal to zero.

For the time interval $t = 5$ s to $t = 9$ s the acceleration is constant and equal to 6.25 m/s^2 . For the interval $t = 9$ s to $t = 13$ s the acceleration is constant and equal to -11.2 m/s^2 .

EXECUTE: During the first 5 seconds the acceleration is constant, so the constant acceleration kinematic formulas can be used.

$$v_{0x} = 20 \text{ m/s} \quad a_x = 0 \quad t = 5 \text{ s} \quad x - x_0 = ?$$

$$x - x_0 = v_{0x}t \quad (a_x = 0 \text{ so no } \frac{1}{2}a_x t^2 \text{ term})$$

$$x - x_0 = (20 \text{ m/s})(5 \text{ s}) = 100 \text{ m}; \text{ this is the distance the officer travels in the first 5 seconds.}$$

During the interval $t = 5$ s to 9 s the acceleration is again constant. The constant acceleration formulas can be applied to this 4 second interval. It is convenient to restart our clock so the interval starts at time $t = 0$ and ends at time $t = 5$ s. (Note that the acceleration is *not* constant over the entire $t = 0$ to $t = 9$ s interval.)

$$v_{0x} = 20 \text{ m/s} \quad a_x = 6.25 \text{ m/s}^2 \quad t = 4 \text{ s} \quad x_0 = 100 \text{ m} \quad x - x_0 = ?$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$x - x_0 = (20 \text{ m/s})(4 \text{ s}) + \frac{1}{2}(6.25 \text{ m/s}^2)(4 \text{ s})^2 = 80 \text{ m} + 50 \text{ m} = 130 \text{ m.}$$

$$\text{Thus } x - x_0 + 130 \text{ m} = 100 \text{ m} + 130 \text{ m} = 230 \text{ m.}$$

At $t = 9$ s the officer is at $x = 230$ m, so she has traveled 230 m in the first 9 seconds.

During the interval $t = 9$ s to $t = 13$ s the acceleration is again constant. The constant acceleration formulas can be applied for this 4 second interval but *not* for the whole $t = 0$ to $t = 13$ s interval. To use the equations restart our clock so this interval begins at time $t = 0$ and ends at time $t = 4$ s.

$$v_{0x} = 45 \text{ m/s (at the start of this time interval)}$$

$$a_x = -11.2 \text{ m/s}^2 \quad t = 4 \text{ s} \quad x_0 = 230 \text{ m} \quad x - x_0 = ?$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$x - x_0 = (45 \text{ m/s})(4 \text{ s}) + \frac{1}{2}(-11.2 \text{ m/s}^2)(4 \text{ s})^2 = 180 \text{ m} - 89.6 \text{ m} = 90.4 \text{ m.}$$

Thus $x = x_0 + 90.4 \text{ m} = 230 \text{ m} + 90.4 \text{ m} = 320 \text{ m}$.

At $t = 13$ s the officer is at $x = 320$ m, so she has traveled 320 m in the first 13 seconds.

EVALUATE: The velocity v_x is always positive so the displacement is always positive and displacement and distance traveled are the same. The average velocity for time interval Δt is $v_{\text{av-x}} = \Delta x / \Delta t$. For $t = 0$ to 5 s,

$$v_{\text{av-x}} = 20 \text{ m/s. For } t = 0 \text{ to } 9 \text{ s, } v_{\text{av-x}} = 26 \text{ m/s. For } t = 0 \text{ to } 13 \text{ s, } v_{\text{av-x}} = 25 \text{ m/s. These results are consistent with}$$

Fig. 2.33.

2.32. IDENTIFY: In each constant acceleration interval, the constant acceleration equations apply.

SET UP: When a_x is constant, the graph of v_x versus t is a straight line and the graph of x versus t is a parabola.

When $a_x = 0$, v_x is constant and x versus t is a straight line.

EXECUTE: The graphs are given in Figure 2.32.

EVALUATE: The slope of the x versus t graph is $v_x(t)$ and the slope of the v_x versus t graph is $a_x(t)$.

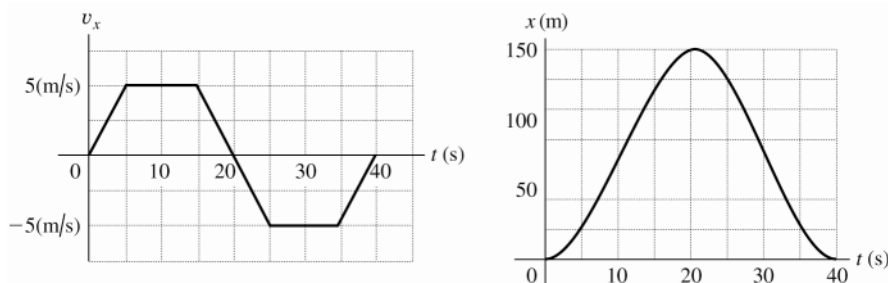


Figure 2.32

2.33. (a) IDENTIFY: The maximum speed occurs at the end of the initial acceleration period.

SET UP: $a_x = 20.0 \text{ m/s}^2$ $t = 15.0 \text{ min} = 900 \text{ s}$ $v_{0x} = 0$ $v_x = ?$

$$v_x = v_{0x} + a_x t$$

EXECUTE: $v_x = 0 + (20.0 \text{ m/s}^2)(900 \text{ s}) = 1.80 \times 10^4 \text{ m/s}$

(b) IDENTIFY: Use constant acceleration formulas to find the displacement Δx . The motion consists of three constant acceleration intervals. In the middle segment of the trip $a_x = 0$ and $v_x = 1.80 \times 10^4 \text{ m/s}$, but we can't directly find the distance traveled during this part of the trip because we don't know the time. Instead, find the distance traveled in the first part of the trip (where $a_x = +20.0 \text{ m/s}^2$) and in the last part of the trip (where

$a_x = -20.0 \text{ m/s}^2$). Subtract these two distances from the total distance of $3.84 \times 10^8 \text{ m}$ to find the distance traveled in the middle part of the trip (where $a_x = 0$).

first segment

SET UP: $x - x_0 = ?$ $t = 15.0 \text{ min} = 900 \text{ s}$ $a_x = +20.0 \text{ m/s}^2$ $v_{0x} = 0$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

EXECUTE: $x - x_0 = 0 + \frac{1}{2}(20.0 \text{ m/s}^2)(900 \text{ s})^2 = 8.10 \times 10^6 \text{ m} = 8.10 \times 10^3 \text{ km}$

second segment

SET UP: $x - x_0 = ?$ $t = 15.0 \text{ min} = 900 \text{ s}$ $a_x = -20.0 \text{ m/s}^2$

$$v_{0x} = 1.80 \times 10^4 \text{ m/s}$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

EXECUTE: $x - x_0 = (1.80 \times 10^4 \text{ m/s})(900 \text{ s}) + \frac{1}{2}(-20.0 \text{ m/s}^2)(900 \text{ s})^2 = 8.10 \times 10^6 \text{ m} = 8.10 \times 10^3 \text{ km}$ (The same distance as traveled as in the first segment.)

Therefore, the distance traveled at constant speed is

$$3.84 \times 10^8 \text{ m} - 8.10 \times 10^6 \text{ m} - 8.10 \times 10^6 \text{ m} = 3.678 \times 10^8 \text{ m} = 3.678 \times 10^5 \text{ km}.$$

The fraction this is of the total distance is $\frac{3.678 \times 10^8 \text{ m}}{3.84 \times 10^8 \text{ m}} = 0.958$.

(c) IDENTIFY: We know the time for each acceleration period, so find the time for the constant speed segment.

SET UP: $x - x_0 = 3.678 \times 10^8 \text{ m}$ $v_x = 1.80 \times 10^4 \text{ m/s}$ $a_x = 0$ $t = ?$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

EXECUTE: $t = \frac{x - x_0}{v_{0x}} = \frac{3.678 \times 10^8 \text{ m}}{1.80 \times 10^4 \text{ m/s}} = 2.043 \times 10^4 \text{ s} = 340.5 \text{ min}.$

The total time for the whole trip is thus $15.0 \text{ min} + 340.5 \text{ min} + 15.0 \text{ min} = 370 \text{ min}.$

EVALUATE: If the speed was a constant $1.80 \times 10^4 \text{ m/s}$ for the entire trip, the trip would take $(3.84 \times 10^8 \text{ m}) / (1.80 \times 10^4 \text{ m/s}) = 356 \text{ min}.$ The trip actually takes a bit longer than this since the average velocity is less than $1.80 \times 10^8 \text{ m/s}$ during the relatively brief acceleration phases.

2.34. IDENTIFY: Use constant acceleration equations to find $x - x_0$ for each segment of the motion.

SET UP: Let $+x$ be the direction the train is traveling.

EXECUTE: $t = 0$ to 14.0 s : $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(1.60 \text{ m/s}^2)(14.0 \text{ s})^2 = 157 \text{ m}.$

At $t = 14.0 \text{ s}$, the speed is $v_x = v_{0x} + a_x t = (1.60 \text{ m/s}^2)(14.0 \text{ s}) = 22.4 \text{ m/s}.$ In the next 70.0 s , $a_x = 0$ and

$$x - x_0 = v_{0x}t = (22.4 \text{ m/s})(70.0 \text{ s}) = 1568 \text{ m}.$$

For the interval during which the train is slowing down, $v_{0x} = 22.4 \text{ m/s}$, $a_x = -3.50 \text{ m/s}^2$ and $v_x = 0.$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \text{ gives } x - x_0 = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - (22.4 \text{ m/s})^2}{2(-3.50 \text{ m/s}^2)} = 72 \text{ m}.$$

The total distance traveled is $157 \text{ m} + 1568 \text{ m} + 72 \text{ m} = 1800 \text{ m}.$

EVALUATE: The acceleration is not constant for the entire motion but it does consist of constant acceleration segments and we can use constant acceleration equations for each segment.

2.35 IDENTIFY: $v_x(t)$ is the slope of the x versus t graph. Car B moves with constant speed and zero acceleration.

Car A moves with positive acceleration; assume the acceleration is constant.

SET UP: For car B , v_x is positive and $a_x = 0.$ For car A , a_x is positive and v_x increases with $t.$

EXECUTE: **(a)** The motion diagrams for the cars are given in Figure 2.35a.

(b) The two cars have the same position at times when their $x-t$ graphs cross. The figure in the problem shows this occurs at approximately $t = 1 \text{ s}$ and $t = 3 \text{ s}.$

(c) The graphs of v_x versus t for each car are sketched in Figure 2.35b.

(d) The cars have the same velocity when their $x-t$ graphs have the same slope. This occurs at approximately $t = 2 \text{ s}.$

(e) Car A passes car B when x_A moves above x_B in the $x-t$ graph. This happens at $t = 3 \text{ s}.$

(f) Car B passes car A when x_B moves above x_A in the $x-t$ graph. This happens at $t = 1 \text{ s}.$

EVALUATE: When $a_x = 0,$ the graph of v_x versus t is a horizontal line. When a_x is positive, the graph of v_x versus t is a straight line with positive slope.

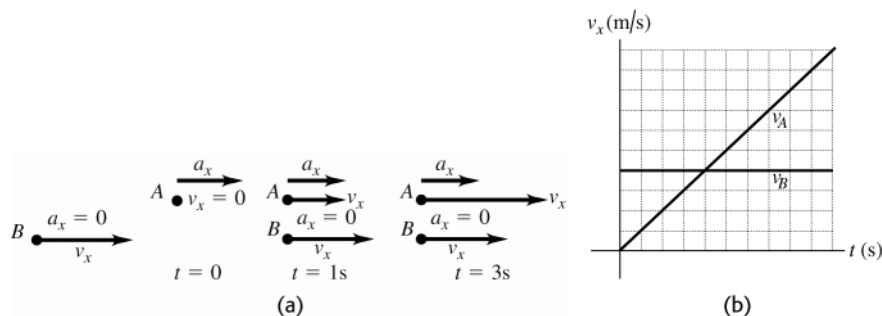


Figure 2.35a-b

2.36. IDENTIFY: Apply the constant acceleration equations to the motion of each vehicle. The truck passes the car when they are at the same x at the same $t > 0.$

SET UP: The truck has $a_x = 0$. The car has $v_{0x} = 0$. Let $+x$ be in the direction of motion of the vehicles. Both vehicles start at $x_0 = 0$. The car has $a_c = 3.20 \text{ m/s}^2$. The truck has $v_x = 20.0 \text{ m/s}$.

EXECUTE: (a) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x_T = v_{0T}t$ and $x_C = \frac{1}{2}a_c t^2$. Setting $x_T = x_C$ gives $t = 0$ and $v_{0T} = \frac{1}{2}a_c t$, so $t = \frac{2v_{0T}}{a_c} = \frac{2(20.0 \text{ m/s})}{3.20 \text{ m/s}^2} = 12.5 \text{ s}$. At this t , $x_T = (20.0 \text{ m/s})(12.5 \text{ s}) = 250 \text{ m}$ and $x = \frac{1}{2}(3.20 \text{ m/s}^2)(12.5 \text{ s})^2 = 250 \text{ m}$.

The car and truck have each traveled 250 m.

(b) At $t = 12.5 \text{ s}$, the car has $v_x = v_{0x} + a_x t = (3.20 \text{ m/s}^2)(12.5 \text{ s}) = 40 \text{ m/s}$.

(c) $x_T = v_{0T}t$ and $x_C = \frac{1}{2}a_c t^2$. The x - t graph of the motion for each vehicle is sketched in Figure 2.36a.

(d) $v_T = v_{0T}$. $v_C = a_c t$. The v_x - t graph for each vehicle is sketched in Figure 2.36b.

EVALUATE: When the car overtakes the truck its speed is twice that of the truck.

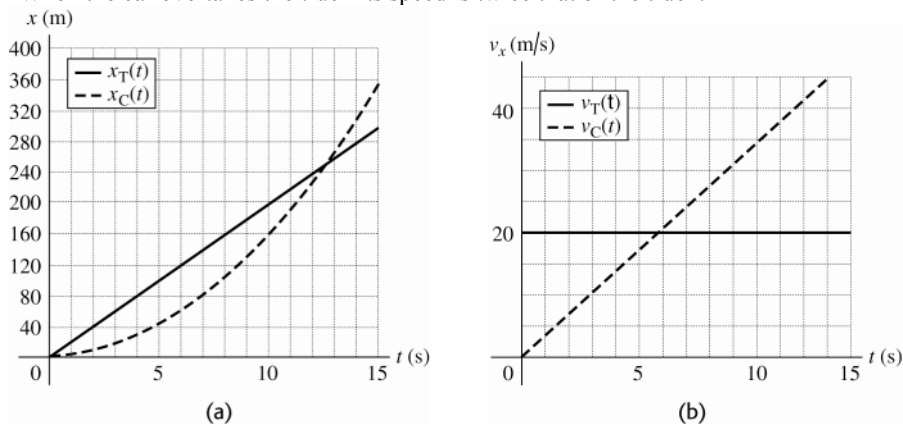


Figure 2.36a-b

2.37. IDENTIFY: For constant acceleration, Eqs. (2.8), (2.12), (2.13) and (2.14) apply.

SET UP: Take $+y$ to be downward, so the motion is in the $+y$ direction. $19,300 \text{ km/h} = 5361 \text{ m/s}$, $1600 \text{ km/h} = 444.4 \text{ m/s}$, and $321 \text{ km/h} = 89.2 \text{ m/s}$. $4.0 \text{ min} = 240 \text{ s}$.

EXECUTE: (a) Stage A: $t = 240 \text{ s}$, $v_{0y} = 5361 \text{ m/s}$, $v_y = 444.4 \text{ m/s}$. $v_y = v_{0y} + a_y t$ gives

$$a_y = \frac{v_y - v_{0y}}{t} = \frac{444.4 \text{ m/s} - 5361 \text{ m/s}}{240 \text{ s}} = -20.5 \text{ m/s}^2.$$

Stage B: $t = 94 \text{ s}$, $v_{0y} = 444.4 \text{ m/s}$, $v_y = 89.2 \text{ m/s}$. $v_y = v_{0y} + a_y t$ gives

$$a_y = \frac{v_y - v_{0y}}{t} = \frac{89.2 \text{ m/s} - 444.4 \text{ m/s}}{94 \text{ s}} = -3.8 \text{ m/s}^2.$$

Stage C: $y - y_0 = 75 \text{ m}$, $v_{0y} = 89.2 \text{ m/s}$, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{0 - (89.2 \text{ m/s})^2}{2(75 \text{ m})} = -53.0 \text{ m/s}^2. \text{ In each case the negative sign means that the acceleration is upward.}$$

$$(b) \text{ Stage A: } y - y_0 = \left(\frac{v_{0y} + v_y}{2} \right) t = \left(\frac{5361 \text{ m/s} + 444.4 \text{ m/s}}{2} \right) (240 \text{ s}) = 697 \text{ km}.$$

$$\text{Stage B: } y - y_0 = \left(\frac{444.4 \text{ m/s} + 89.2 \text{ m/s}}{2} \right) (94 \text{ s}) = 25 \text{ km}.$$

Stage C: The problem states that $y - y_0 = 75 \text{ m} = 0.075 \text{ km}$.

The total distance traveled during all three stages is $697 \text{ km} + 25 \text{ km} + 0.075 \text{ km} = 722 \text{ km}$.

EVALUATE: The upward acceleration produced by friction in stage A is calculated to be greater than the upward acceleration due to the parachute in stage B. The effects of air resistance increase with increasing speed and in reality the acceleration was probably not constant during stages A and B.

2.38. IDENTIFY: Assume an initial height of 200 m and a constant acceleration of 9.80 m/s^2 .

SET UP: Let $+y$ be downward. $1 \text{ km/h} = 0.2778 \text{ m/s}$ and $1 \text{ mi/h} = 0.4470 \text{ m/s}$.

EXECUTE: (a) $y - y_0 = 200 \text{ m}$, $a_y = 9.80 \text{ m/s}^2$, $v_{0y} = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$v_y = \sqrt{2(9.80 \text{ m/s}^2)(200 \text{ m})} = 60 \text{ m/s} = 200 \text{ km/h} = 140 \text{ mi/h}.$$

(b) Raindrops actually have a speed of about 1 m/s as they strike the ground.

(c) The actual speed at the ground is much less than the speed calculated assuming free-fall, so neglect of air resistance is a very poor approximation for falling raindrops.

EVALUATE: In the absence of air resistance raindrops would land with speeds that would make them very dangerous.

- 2.39. IDENTIFY:** Apply the constant acceleration equations to the motion of the flea. After the flea leaves the ground, $a_y = g$, downward. Take the origin at the ground and the positive direction to be upward.

(a) **SET UP:** At the maximum height $v_y = 0$.

$$v_y = 0 \quad y - y_0 = 0.440 \text{ m} \quad a_y = -9.80 \text{ m/s}^2 \quad v_{0y} = ?$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

EXECUTE: $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80 \text{ m/s}^2)(0.440 \text{ m})} = 2.94 \text{ m/s}$

(b) **SET UP:** When the flea has returned to the ground $y - y_0 = 0$.

$$y - y_0 = 0 \quad v_{0y} = +2.94 \text{ m/s} \quad a_y = -9.80 \text{ m/s}^2 \quad t = ?$$

$$y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$$

EXECUTE: With $y - y_0 = 0$ this gives $t = -\frac{2v_{0y}}{a_y} = -\frac{2(2.94 \text{ m/s})}{-9.80 \text{ m/s}^2} = 0.600 \text{ s}$.

EVALUATE: We can use $v_y = v_{0y} + a_yt$ to show that with $v_{0y} = 2.94 \text{ m/s}$, $v_y = 0$ after 0.300 s.

- 2.40. IDENTIFY:** Apply constant acceleration equations to the motion of the lander.

SET UP: Let $+y$ be positive. Since the lander is in free-fall, $a_y = +1.6 \text{ m/s}^2$.

EXECUTE: $v_{0y} = 0.8 \text{ m/s}$, $y - y_0 = 5.0 \text{ m}$, $a_y = +1.6 \text{ m/s}^2$ in $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = \sqrt{(0.8 \text{ m/s})^2 + 2(1.6 \text{ m/s}^2)(5.0 \text{ m})} = 4.1 \text{ m/s}.$$

EVALUATE: The same descent on earth would result in a final speed of 9.9 m/s, since the acceleration due to gravity on earth is much larger than on the moon.

- 2.41. IDENTIFY:** Apply constant acceleration equations to the motion of the meterstick. The time the meterstick falls is your reaction time.

SET UP: Let $+y$ be downward. The meter stick has $v_{0y} = 0$ and $a_y = 9.80 \text{ m/s}^2$. Let d be the distance the meterstick falls.

EXECUTE: (a) $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $d = (4.90 \text{ m/s}^2)t^2$ and $t = \sqrt{\frac{d}{4.90 \text{ m/s}^2}}$.

$$(b) t = \sqrt{\frac{0.176 \text{ m}}{4.90 \text{ m/s}^2}} = 0.190 \text{ s}$$

EVALUATE: The reaction time is proportional to the square of the distance the stick falls.

- 2.42. IDENTIFY:** Apply constant acceleration equations to the vertical motion of the brick.

SET UP: Let $+y$ be downward. $a_y = 9.80 \text{ m/s}^2$

EXECUTE: (a) $v_{0y} = 0$, $t = 2.50 \text{ s}$, $a_y = 9.80 \text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 = \frac{1}{2}(9.80 \text{ m/s}^2)(2.50 \text{ s})^2 = 30.6 \text{ m}$. The building is 30.6 m tall.

$$(b) v_y = v_{0y} + a_yt = 0 + (9.80 \text{ m/s}^2)(2.50 \text{ s}) = 24.5 \text{ m/s}$$

(c) The graphs of a_y , v_y , and y versus t are given in Fig. 2.42. Take $y = 0$ at the ground.

EVALUATE: We could use either $y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t$ or $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to check our results.

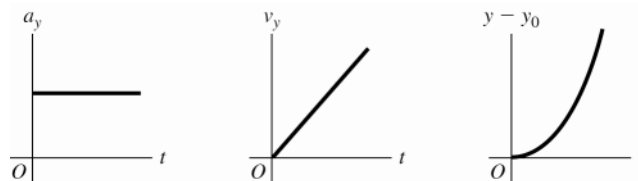


Figure 2.42

2.43. IDENTIFY: When the only force is gravity the acceleration is 9.80 m/s^2 , downward. There are two intervals of constant acceleration and the constant acceleration equations apply during each of these intervals.

SET UP: Let $+y$ be upward. Let $y = 0$ at the launch pad. The final velocity for the first phase of the motion is the initial velocity for the free-fall phase.

EXECUTE: (a) Find the velocity when the engines cut off. $y - y_0 = 525 \text{ m}$, $a_y = +2.25 \text{ m/s}^2$, $v_{0y} = 0$.

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \text{ gives } v_y = \sqrt{2(2.25 \text{ m/s}^2)(525 \text{ m})} = 48.6 \text{ m/s}.$$

Now consider the motion from engine cut off to maximum height: $y_0 = 525 \text{ m}$, $v_{0y} = +48.6 \text{ m/s}$, $v_y = 0$ (at the

maximum height), $a_y = -9.80 \text{ m/s}^2$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (48.6 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 121 \text{ m}$ and

$$y = 121 \text{ m} + 525 \text{ m} = 646 \text{ m}.$$

(b) Consider the motion from engine failure until just before the rocket strikes the ground: $y - y_0 = -525 \text{ m}$,

$a_y = -9.80 \text{ m/s}^2$, $v_{0y} = +48.6 \text{ m/s}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$v_y = -\sqrt{(48.6 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-525 \text{ m})} = -112 \text{ m/s}.$$

Then $v_y = v_{0y} + a_y t$ gives

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-112 \text{ m/s} - 48.6 \text{ m/s}}{-9.80 \text{ m/s}^2} = 16.4 \text{ s}.$$

(c) Find the time from blast-off until engine failure: $y - y_0 = 525 \text{ m}$, $v_{0y} = 0$, $a_y = +2.25 \text{ m/s}^2$.

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 \text{ gives } t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(525 \text{ m})}{2.25 \text{ m/s}^2}} = 21.6 \text{ s}.$$

The rocket strikes the launch pad

$21.6 \text{ s} + 16.4 \text{ s} = 38.0 \text{ s}$ after blast off. The acceleration a_y is $+2.25 \text{ m/s}^2$ from $t = 0$ to $t = 21.6 \text{ s}$. It is

-9.80 m/s^2 from $t = 21.6 \text{ s}$ to 38.0 s . $v_y = v_{0y} + a_y t$ applies during each constant acceleration segment, so the graph of v_y versus t is a straight line with positive slope of 2.25 m/s^2 during the blast-off phase and with negative slope of -9.80 m/s^2 after engine failure. During each phase $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. The sign of a_y determines the curvature of $y(t)$. At $t = 38.0 \text{ s}$ the rocket has returned to $y = 0$. The graphs are sketched in Figure 2.43.

EVALUATE: In part (b) we could have found the time from $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$, finding v_y first allows us to avoid solving for t from a quadratic equation.

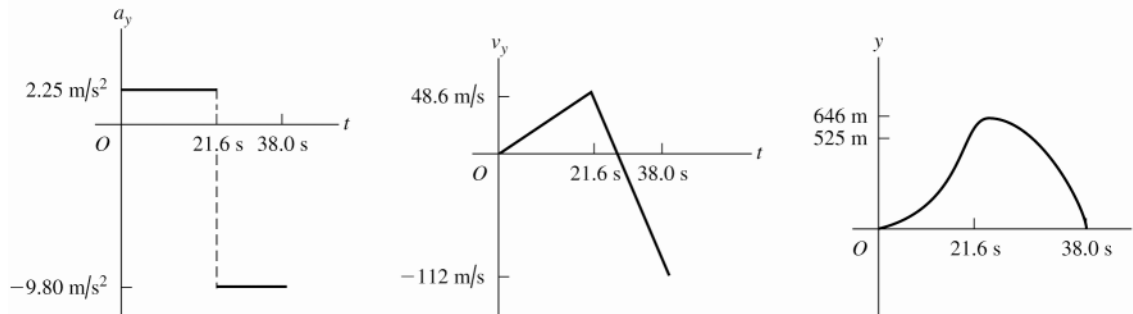


Figure 2.43

2.44. IDENTIFY: Apply constant acceleration equations to the vertical motion of the sandbag.

SET UP: Take $+y$ upward. $a_y = -9.80 \text{ m/s}^2$. The initial velocity of the sandbag equals the velocity of the balloon, so $v_{0y} = +5.00 \text{ m/s}$. When the balloon reaches the ground, $y - y_0 = -40.0 \text{ m}$. At its maximum height the sandbag has $v_y = 0$.

EXECUTE: (a) $t = 0.250 \text{ s}$: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (5.00 \text{ m/s})(0.250 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.250 \text{ s})^2 = 0.94 \text{ m}$. The sandbag is 40.9 m above the ground. $v_y = v_{0y} + a_y t = +5.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(0.250 \text{ s}) = 2.55 \text{ m/s}$.

$t = 1.00 \text{ s}$: $y - y_0 = (5.00 \text{ m/s})(1.00 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(1.00 \text{ s})^2 = 0.10 \text{ m}$. The sandbag is 40.1 m above the ground. $v_y = v_{0y} + a_y t = +5.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(1.00 \text{ s}) = -4.80 \text{ m/s}$.

(b) $y - y_0 = -40.0 \text{ m}$, $v_{0y} = 5.00 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-40.0 \text{ m} = (5.00 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$. $(4.90 \text{ m/s}^2)t^2 - (5.00 \text{ m/s})t - 40.0 \text{ m} = 0$ and

$$t = \frac{1}{9.80} \left(5.00 \pm \sqrt{(-5.00)^2 - 4(4.90)(-40.0)} \right) \text{ s} = (0.51 \pm 2.90) \text{ s}. \quad t \text{ must be positive, so } t = 3.41 \text{ s}.$$

(c) $v_y = v_{0y} + a_y t = +5.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(3.41 \text{ s}) = -28.4 \text{ m/s}$

(d) $v_{0y} = 5.00 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (5.00 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 1.28 \text{ m}. \quad \text{The maximum height is 41.3 m above the ground.}$$

(e) The graphs of a_y , v_y , and y versus t are given in Fig. 2.44. Take $y = 0$ at the ground.

EVALUATE: The sandbag initially travels upward with decreasing velocity and then moves downward with increasing speed.

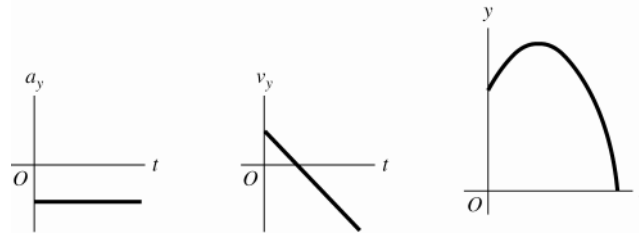


Figure 2.44

2.45. IDENTIFY: The balloon has constant acceleration $a_y = g$, downward.

(a) **SET UP:** Take the $+y$ direction to be upward.

$$t = 2.00 \text{ s}, \quad v_{0y} = -6.00 \text{ m/s}, \quad a_y = -9.80 \text{ m/s}^2, \quad v_y = ?$$

EXECUTE: $v_y = v_{0y} + a_y t = -6.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(2.00 \text{ s}) = -25.5 \text{ m/s}$

(b) **SET UP:** $y - y_0 = ?$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (-6.00 \text{ m/s})(2.00 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(2.00 \text{ s})^2 = -31.6 \text{ m}$

(c) **SET UP:** $y - y_0 = -10.0 \text{ m}$, $v_{0y} = -6.00 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$, $v_y = ?$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

EXECUTE: $v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)} = -\sqrt{(-6.00 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-10.0 \text{ m})} = -15.2 \text{ m/s}$

(d) The graphs are sketched in Figure 2.45.

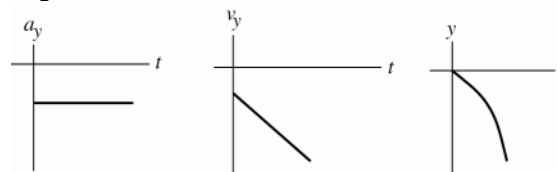


Figure 2.45

EVALUATE: The speed of the balloon increases steadily since the acceleration and velocity are in the same direction. $|v_y| = 25.5 \text{ m/s}$ when $|y - y_0| = 31.6 \text{ m}$, so $|v_y|$ is less than this (15.2 m/s) when $|y - y_0|$ is less (10.0 m).

2.46. IDENTIFY: Since air resistance is ignored, the egg is in free-fall and has a constant downward acceleration of magnitude 9.80 m/s^2 . Apply the constant acceleration equations to the motion of the egg.

SET UP: Take $+y$ to be upward. At the maximum height, $v_y = 0$.

EXECUTE: (a) $y - y_0 = -50.0 \text{ m}$, $t = 5.00 \text{ s}$, $a_y = -9.80 \text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$v_{0y} = \frac{y - y_0}{t} - \frac{1}{2}a_y t = \frac{-50.0 \text{ m}}{5.00 \text{ s}} - \frac{1}{2}(-9.80 \text{ m/s}^2)(5.00 \text{ s}) = +14.5 \text{ m/s}.$$

(b) $v_{0y} = +14.5 \text{ m/s}$, $v_y = 0$ (at the maximum height), $a_y = -9.80 \text{ m/s}^2$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (14.5 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 10.7 \text{ m}.$$

(c) At the maximum height $v_y = 0$.

(d) The acceleration is constant and equal to 9.80 m/s^2 , downward, at all points in the motion, including at the maximum height.

(e) The graphs are sketched in Figure 2.46.

EVALUATE: The time for the egg to reach its maximum height is $t = \frac{v_y - v_{0y}}{a_y} = \frac{-14.5 \text{ m/s}}{-9.8 \text{ m/s}^2} = 1.48 \text{ s}$. The egg has returned to the level of the cornice after 2.96 s and after 5.00 s it has traveled downward from the cornice for 2.04 s .

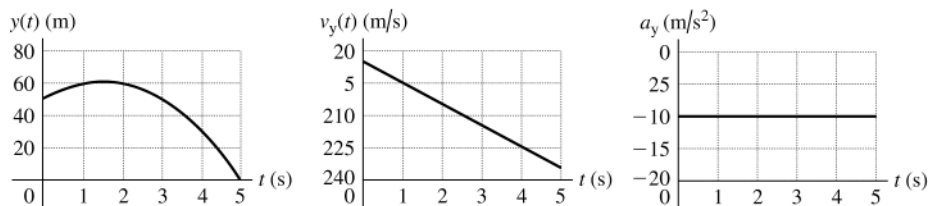


Figure 2.46

2.47. IDENTIFY: Use the constant acceleration equations to calculate a_x and $x - x_0$.

(a) **SET UP:** $v_x = 224 \text{ m/s}$, $v_{0x} = 0$, $t = 0.900 \text{ s}$, $a_x = ?$

$$v_x = v_{0x} + a_x t$$

EXECUTE: $a_x = \frac{v_x - v_{0x}}{t} = \frac{224 \text{ m/s} - 0}{0.900 \text{ s}} = 249 \text{ m/s}^2$

(b) $a_x / g = (249 \text{ m/s}^2) / (9.80 \text{ m/s}^2) = 25.4$

(c) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(249 \text{ m/s}^2)(0.900 \text{ s})^2 = 101 \text{ m}$

(d) **SET UP:** Calculate the acceleration, assuming it is constant:

$t = 1.40 \text{ s}$, $v_{0x} = 283 \text{ m/s}$, $v_x = 0$ (stops), $a_x = ?$

$$v_x = v_{0x} + a_x t$$

EXECUTE: $a_x = \frac{v_x - v_{0x}}{t} = \frac{0 - 283 \text{ m/s}}{1.40 \text{ s}} = -202 \text{ m/s}^2$

$$a_x / g = (-202 \text{ m/s}^2) / (9.80 \text{ m/s}^2) = -20.6; \quad a_x = -20.6g$$

If the acceleration while the sled is stopping is constant then the magnitude of the acceleration is only $20.6g$. But if the acceleration is not constant it is certainly possible that at some point the instantaneous acceleration could be as large as $40g$.

EVALUATE: It is reasonable that for this motion the acceleration is much larger than g .

2.48. IDENTIFY: Since air resistance is ignored, the boulder is in free-fall and has a constant downward acceleration of magnitude 9.80 m/s^2 . Apply the constant acceleration equations to the motion of the boulder.

SET UP: Take $+y$ to be upward.

EXECUTE: (a) $v_{0y} = +40.0 \text{ m/s}$, $v_y = +20.0 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$. $v_y = v_{0y} + a_y t$ gives

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{20.0 \text{ m/s} - 40.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = +2.04 \text{ s}.$$

$$(b) v_y = -20.0 \text{ m/s} . t = \frac{v_y - v_{0y}}{a_y} = \frac{-20.0 \text{ m/s} - 40.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = +6.12 \text{ s} .$$

$$(c) y - y_0 = 0 , v_{0y} = +40.0 \text{ m/s} , a_y = -9.80 \text{ m/s}^2 . y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 \text{ gives } t = 0 \text{ and } t = -\frac{2v_{0y}}{a_y} = -\frac{2(40.0 \text{ m/s})}{-9.80 \text{ m/s}^2} = +8.16 \text{ s} .$$

$$(d) v_y = 0 , v_{0y} = +40.0 \text{ m/s} , a_y = -9.80 \text{ m/s}^2 . v_y = v_{0y} + a_y t \text{ gives } t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - 40.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = 4.08 \text{ s} .$$

(e) The acceleration is 9.80 m/s^2 , downward, at all points in the motion.

(f) The graphs are sketched in Figure 2.48.

EVALUATE: $v_y = 0$ at the maximum height. The time to reach the maximum height is half the total time in the air, so the answer in part (d) is half the answer in part (c). Also note that $2.04 \text{ s} < 4.08 \text{ s} < 6.12 \text{ s}$. The boulder is going upward until it reaches its maximum height and after the maximum height it is traveling downward.

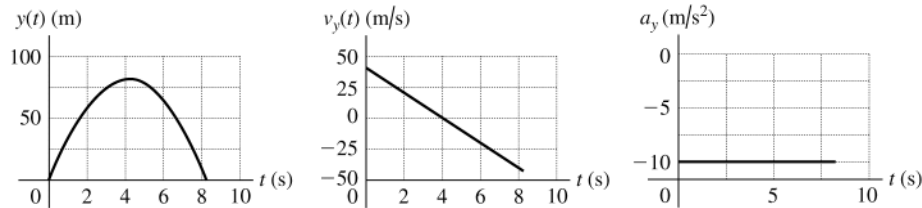


Figure 2.48

2.49. IDENTIFY: We can avoid solving for the common height by considering the relation between height, time of fall and acceleration due to gravity and setting up a ratio involving time of fall and acceleration due to gravity.

SET UP: Let g_{En} be the acceleration due to gravity on Enceladus and let g be this quantity on earth. Let h be the common height from which the object is dropped. Let $+y$ be downward, so $y - y_0 = h$. $v_{0y} = 0$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $h = \frac{1}{2}g t_E^2$ and $h = \frac{1}{2}g_{\text{En}} t_{\text{En}}^2$. Combining these two equations gives

$$g t_E^2 = g_{\text{En}} t_{\text{En}}^2 \text{ and } g_{\text{En}} = g \left(\frac{t_E}{t_{\text{En}}} \right)^2 = (9.80 \text{ m/s}^2) \left(\frac{1.75 \text{ s}}{18.6 \text{ s}} \right)^2 = 0.0868 \text{ m/s}^2 .$$

EVALUATE: The acceleration due to gravity is inversely proportional to the square of the time of fall.

2.50. IDENTIFY: The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use Eqs.(2.17) and (2.18). Use the values of v_x and of x at $t = 1.0 \text{ s}$ to evaluate v_{0x} and x_0 .

$$\text{SET UP: } \int t^n dt = \frac{1}{n+1} t^{n+1}, \text{ for } n \geq 0 .$$

EXECUTE: (a) $v_x = v_{0x} + \int_0^t \alpha dt = v_{0x} + \frac{1}{2} \alpha t^2 = v_{0x} + (0.60 \text{ m/s}^3) t^2$. $v_x = 5.0 \text{ m/s}$ when $t = 1.0 \text{ s}$ gives

$$v_{0x} = 4.4 \text{ m/s} . \text{ Then, at } t = 2.0 \text{ s} , v_x = 4.4 \text{ m/s} + (0.60 \text{ m/s}^3)(2.0 \text{ s})^2 = 6.8 \text{ m/s} .$$

(b) $x = x_0 + \int_0^t (v_{0x} + \frac{1}{2} \alpha t^2) dt = x_0 + v_{0x} t + \frac{1}{6} \alpha t^3$. $x = 6.0 \text{ m}$ at $t = 1.0 \text{ s}$ gives $x_0 = 1.4 \text{ m}$. Then, at $t = 2.0 \text{ s}$,

$$x = 1.4 \text{ m} + (4.4 \text{ m/s})(2.0 \text{ s}) + \frac{1}{6}(1.24 \text{ m/s}^3)(2.0 \text{ s})^3 = 11.8 \text{ m} .$$

(c) $x(t) = 1.4 \text{ m} + (4.4 \text{ m/s})t + (0.20 \text{ m/s}^3)t^3$. $v_x(t) = 4.4 \text{ m/s} + (0.60 \text{ m/s}^3)t^2$. $a_x(t) = (1.20 \text{ m/s}^3)t$. The graphs are sketched in Figure 2.50.

EVALUATE: We can verify that $a_x = \frac{dv_x}{dt}$ and $v_x = \frac{dx}{dt}$.

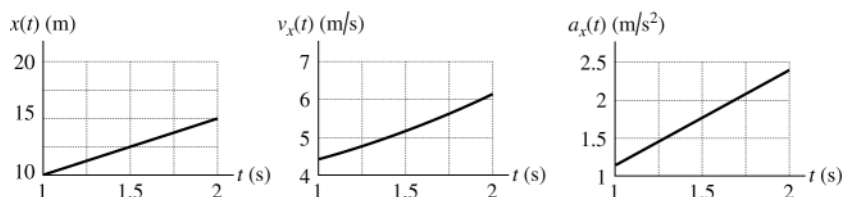


Figure 2.50

2.51. $a_x = At - Bt^2$ with $A = 1.50 \text{ m/s}^3$ and $B = 0.120 \text{ m/s}^4$

(a) **IDENTIFY:** Integrate $a_x(t)$ to find $v_x(t)$ and then integrate $v_x(t)$ to find $x(t)$.

SET UP: $v_x = v_{0x} + \int_0^t a_x dt$

EXECUTE: $v_x = v_{0x} + \int_0^t (At - Bt^2) dt = v_{0x} + \frac{1}{2}At^2 - \frac{1}{3}Bt^3$

At rest at $t = 0$ says that $v_{0x} = 0$, so

$$v_x = \frac{1}{2}At^2 - \frac{1}{3}Bt^3 = \frac{1}{2}(1.50 \text{ m/s}^3)t^2 - \frac{1}{3}(0.120 \text{ m/s}^4)t^3$$

$$v_x = (0.75 \text{ m/s}^3)t^2 - (0.040 \text{ m/s}^4)t^3$$

SET UP: $x - x_0 + \int_0^t v_x dt$

EXECUTE: $x = x_0 + \int_0^t \left(\frac{1}{2}At^2 - \frac{1}{3}Bt^3\right) dt = x_0 + \frac{1}{6}At^3 - \frac{1}{12}Bt^4$

At the origin at $t = 0$ says that $x_0 = 0$, so

$$x = \frac{1}{6}At^3 - \frac{1}{12}Bt^4 = \frac{1}{6}(1.50 \text{ m/s}^3)t^3 - \frac{1}{12}(0.120 \text{ m/s}^4)t^4$$

$$x = (0.25 \text{ m/s}^3)t^3 - (0.010 \text{ m/s}^4)t^4$$

EVALUATE: We can check our results by using them to verify that $v_x(t) = \frac{dx}{dt}$ and $a_x(t) = \frac{dv_x}{dt}$.

(b) **IDENTIFY and SET UP:** At time t , when v_x is a maximum, $\frac{dv_x}{dt} = 0$. (Since $a_x = \frac{dv_x}{dt}$, the maximum velocity is when $a_x = 0$. For earlier times a_x is positive so v_x is still increasing. For later times a_x is negative and v_x is decreasing.)

EXECUTE: $a_x = \frac{dv_x}{dt} = 0$ so $At - Bt^2 = 0$

One root is $t = 0$, but at this time $v_x = 0$ and not a maximum.

The other root is $t = \frac{A}{B} = \frac{1.50 \text{ m/s}^3}{0.120 \text{ m/s}^4} = 12.5 \text{ s}$

At this time $v_x = (0.75 \text{ m/s}^3)t^2 - (0.040 \text{ m/s}^4)t^3$ gives

$$v_x = (0.75 \text{ m/s}^3)(12.5 \text{ s})^2 - (0.040 \text{ m/s}^4)(12.5 \text{ s})^3 = 117.2 \text{ m/s} - 78.1 \text{ m/s} = 39.1 \text{ m/s}.$$

EVALUATE: For $t < 12.5 \text{ s}$, $a_x > 0$ and v_x is increasing. For $t > 12.5 \text{ s}$, $a_x < 0$ and v_x is decreasing.

2.52. **IDENTIFY:** $a(t)$ is the slope of the v versus t graph and the distance traveled is the area under the v versus t graph.

SET UP: The v versus t graph can be approximated by the graph sketched in Figure 2.52.

EXECUTE: (a) Slope $= a = 0$ for $t \geq 1.3 \text{ ms}$.

(b)

$$h_{\max} = \text{Area under } v\text{-}t \text{ graph} \approx A_{\text{Triangle}} + A_{\text{Rectangle}} \approx \frac{1}{2}(1.3 \text{ ms})(133 \text{ cm/s}) + (2.5 \text{ ms} - 1.3 \text{ ms})(133 \text{ cm/s}) \approx 0.25 \text{ cm}$$

(c) $a = \text{slope of } v\text{-}t \text{ graph. } a(0.5 \text{ ms}) \approx a(1.0 \text{ ms}) \approx \frac{133 \text{ cm/s}}{1.3 \text{ ms}} = 1.0 \times 10^5 \text{ cm/s}^2$.

$a(1.5 \text{ ms}) = 0$ because the slope is zero.

(d) $h = \text{area under } v\text{-}t \text{ graph. } h(0.5 \text{ ms}) \approx A_{\text{Triangle}} = \frac{1}{2}(0.5 \text{ ms})(33 \text{ cm/s}) = 8.3 \times 10^{-3} \text{ cm} .$

$$h(1.0 \text{ ms}) \approx A_{\text{Triangle}} = \frac{1}{2}(1.0 \text{ ms})(100 \text{ cm/s}) = 5.0 \times 10^{-2} \text{ cm} .$$

$$h(1.5 \text{ ms}) \approx A_{\text{Triangle}} + A_{\text{Rectangle}} = \frac{1}{2}(1.3 \text{ ms})(133 \text{ cm/s}) + (0.2 \text{ ms})(133) = 0.11 \text{ cm}$$

EVALUATE: The acceleration is constant until $t = 1.3 \text{ ms}$, and then it is zero. $g = 980 \text{ cm/s}^2$. The acceleration during the first 1.3 ms is much larger than this and gravity can be neglected for the portion of the jump that we are considering.

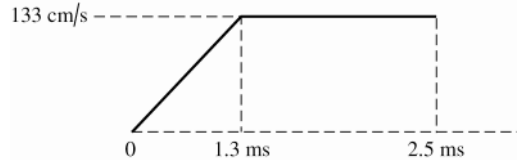


Figure 2.52

2.53. (a) IDENTIFY and SET UP: The change in speed is the area under the a_x versus t curve between vertical lines at $t = 2.5 \text{ s}$ and $t = 7.5 \text{ s}$.

EXECUTE: This area is $\frac{1}{2}(4.00 \text{ cm/s}^2 + 8.00 \text{ cm/s}^2)(7.5 \text{ s} - 2.5 \text{ s}) = 30.0 \text{ cm/s}$

This acceleration is positive so the change in velocity is positive.

(b) Slope of v_x versus t is positive and increasing with t . The graph is sketched in Figure 2.53.

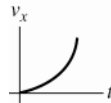


Figure 2.53

EVALUATE: The calculation in part (a) is equivalent to $\Delta v_x = (a_{\text{av-x}})\Delta t$. Since a_x is linear in t ,

$$a_{\text{av-x}} = (a_{0x} + a_x)/2. \text{ Thus } a_{\text{av-x}} = \frac{1}{2}(4.00 \text{ cm/s}^2 + 8.00 \text{ cm/s}^2) \text{ for the time interval } t = 2.5 \text{ s} \text{ to } t = 7.5 \text{ s}.$$

2.54. IDENTIFY: The average speed is the total distance traveled divided by the total time. The elapsed time is the distance traveled divided by the average speed.

SET UP: The total distance traveled is 20 mi. With an average speed of 8 mi/h for 10 mi, the time for that first 10 miles is $\frac{10 \text{ mi}}{8 \text{ mi/h}} = 1.25 \text{ h}$.

EXECUTE: (a) An average speed of 4 mi/h for 20 mi gives a total time of $\frac{20 \text{ mi}}{4 \text{ mi/h}} = 5.0 \text{ h}$. The second 10 mi must be covered in $5.0 \text{ h} - 1.25 \text{ h} = 3.75 \text{ h}$. This corresponds to an average speed of $\frac{10 \text{ mi}}{3.75 \text{ h}} = 2.7 \text{ mi/h}$.

(b) An average speed of 12 mi/h for 20 mi gives a total time of $\frac{20 \text{ mi}}{12 \text{ mi/h}} = 1.67 \text{ h}$. The second 10 mi must be covered in $1.67 \text{ h} - 1.25 \text{ h} = 0.42 \text{ h}$. This corresponds to an average speed of $\frac{10 \text{ mi}}{0.42 \text{ h}} = 24 \text{ mi/h}$.

(c) An average speed of 16 mi/h for 20 mi gives a total time of $\frac{20 \text{ mi}}{16 \text{ mi/h}} = 1.25 \text{ h}$. But 1.25 h was already spent during the first 10 miles and the second 10 miles would have to be covered in zero time. This is not possible and an average speed of 16 mi/h for the 20-mile ride is not possible.

EVALUATE: The average speed for the total trip is not the average of the average speeds for each 10-mile segment. The rider spends a different amount of time traveling at each of the two average speeds.

2.55. IDENTIFY: $v_x(t) = \frac{dx}{dt}$ and $a_x = \frac{dv_x}{dt}$.

SET UP: $\frac{d}{dt}(t^n) = nt^{n-1}$, for $n \geq 1$.

EXECUTE: (a) $v_x(t) = (9.00 \text{ m/s}^3)t^2 - (20.0 \text{ m/s}^2)t + 9.00 \text{ m/s}$. $a_x(t) = (18.0 \text{ m/s}^3)t - 20.0 \text{ m/s}^2$. The graphs are sketched in Figure 2.55.

(b) The particle is instantaneously at rest when $v_x(t) = 0$. $v_{0x} = 0$ and the quadratic formula gives

$t = \frac{1}{18.0}(20.0 \pm \sqrt{(20.0)^2 - 4(9.00)(9.00)})$ s = 1.11 s \pm 0.48 s. $t = 0.63$ s and $t = 1.59$ s. These results agree with the v_x - t graphs in part (a).

(c) For $t = 0.63$ s, $a_x = (18.0 \text{ m/s}^3)(0.63 \text{ s}) - 20.0 \text{ m/s}^2 = -8.7 \text{ m/s}^2$. For $t = 1.59$ s, $a_x = +8.6 \text{ m/s}^2$. At $t = 0.63$ s the slope of the v_x - t graph is negative and at $t = 1.59$ s it is positive, so the same answer is deduced from the $v_x(t)$ graph as from the expression for $a_x(t)$.

(d) $v_x(t)$ is instantaneously not changing when $a_x = 0$. This occurs at $t = \frac{20.0 \text{ m/s}^2}{18.0 \text{ m/s}^3} = 1.11$ s.

(e) When the particle is at its greatest distance from the origin, $v_x = 0$ and $a_x < 0$ (so the particle is starting to move back toward the origin). This is the case for $t = 0.63$ s, which agrees with the x - t graph in part (a). At $t = 0.63$ s, $x = 2.45$ m.

(f) The particle's speed is changing at its greatest rate when a_x has its maximum magnitude. The a_x - t graph in part (a) shows this occurs at $t = 0$ and at $t = 2.00$ s. Since v_x is always positive in this time interval, the particle is speeding up at its greatest rate when a_x is positive, and this is for $t = 2.00$ s.

The particle is slowing down at its greatest rate when a_x is negative and this is for $t = 0$.

EVALUATE: Since $a_x(t)$ is linear in t , $v_x(t)$ is a parabola and is symmetric around the point where $|v_x(t)|$ has its minimum value ($t = 1.11$ s). For this reason, the answer to part (d) is midway between the two times in part (c).

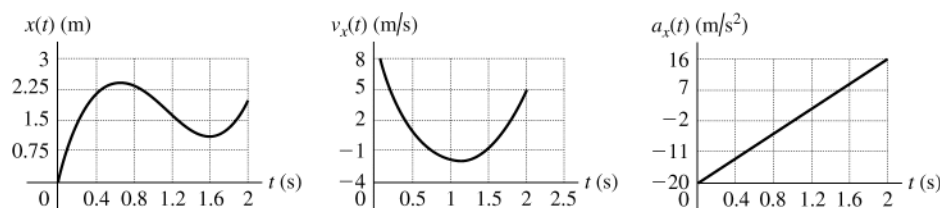


Figure 2.55

2.56. IDENTIFY: The average velocity is $v_{\text{av-x}} = \frac{\Delta x}{\Delta t}$. The average speed is the distance traveled divided by the elapsed time.

SET UP: Let $+x$ be in the direction of the first leg of the race. For the round trip, $\Delta x \geq 0$ and the total distance traveled is 50.0 m. For each leg of the race both the magnitude of the displacement and the distance traveled are 25.0 m.

EXECUTE: (a) $|v_{\text{av-x}}| = \frac{|\Delta x|}{\Delta t} = \frac{25.0 \text{ m}}{20.0 \text{ s}} = 1.25 \text{ m/s}$. This is the same as the average speed for this leg of the race.

(b) $|v_{\text{av-x}}| = \frac{|\Delta x|}{\Delta t} = \frac{25.0 \text{ m}}{15.0 \text{ s}} = 1.67 \text{ m/s}$. This is the same as the average speed for this leg of the race.

(c) $\Delta x = 0$ so $v_{\text{av-x}} = 0$.

(d) The average speed is $\frac{50.0 \text{ m}}{35.0 \text{ s}} = 1.43 \text{ m/s}$.

EVALUATE: Note that the average speed for the round trip is not equal to the arithmetic average of the average speeds for each leg.

2.57. IDENTIFY: Use information about displacement and time to calculate average speed and average velocity. Take the origin to be at Seward and the positive direction to be west.

(a) **SET UP:** average speed = $\frac{\text{distance traveled}}{\text{time}}$

EXECUTE: The distance traveled (different from the net displacement $(x - x_0)$) is 76 km + 34 km = 110 km.

Find the total elapsed time by using $v_{\text{av-x}} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t}$ to find t for each leg of the journey.

Seward to Auora: $t = \frac{x - x_0}{v_{\text{av-x}}} = \frac{76 \text{ km}}{88 \text{ km/h}} = 0.8636 \text{ h}$

$$\text{Auora to York: } t = \frac{x - x_0}{v_{\text{av-x}}} = \frac{-34 \text{ km}}{-72 \text{ km/h}} = 0.4722 \text{ h}$$

$$\text{Total } t = 0.8636 \text{ h} + 0.4722 \text{ h} = 1.336 \text{ h}.$$

$$\text{Then average speed} = \frac{110 \text{ km}}{1.336 \text{ h}} = 82 \text{ km/h}.$$

(b) SET UP: $v_{\text{av-x}} = \frac{\Delta x}{\Delta t}$, where Δx is the displacement, not the total distance traveled.

$$\text{For the whole trip he ends up } 76 \text{ km} - 34 \text{ km} = 42 \text{ km} \text{ west of his starting point. } v_{\text{av-x}} = \frac{42 \text{ km}}{1.336 \text{ h}} = 31 \text{ km/h}.$$

EVALUATE: The motion is not uniformly in the same direction so the displacement is less than the distance traveled and the magnitude of the average velocity is less than the average speed.

- 2.58. IDENTIFY:** The vehicles are assumed to move at constant speed. The speed (mi/h) divided by the frequency with which vehicles pass a given point (vehicles/h) is the total space per vehicle (the length of the vehicle plus space to the next vehicle).

$$\text{SET UP: } 96 \text{ km/h} = 96 \times 10^3 \text{ m/h}$$

EXECUTE: (a) The total space per vehicle is $\frac{96 \times 10^3 \text{ m/h}}{2400 \text{ vehicles/h}} = 40 \text{ m/vehicle}$. Since the average length of a vehicle is 4.6 m, the average space between vehicles is $40 \text{ m} - 4.6 \text{ m} = 35 \text{ m}$.

(b) The frequency of vehicles (vehicles/h) is $\frac{96 \times 10^3 \text{ m/h}}{(4.6 + 9.2) \text{ m/vehicle}} = 7000 \text{ vehicles/h}$.

EVALUATE: The traffic flow rate per lane would nearly triple. Note that the traffic flow rate is directly proportional to the traffic speed.

- 2.59. (a) IDENTIFY:** Calculate the average acceleration using $a_{\text{av-x}} = \frac{\Delta v_x}{\Delta t} = \frac{v_x - v_{0x}}{t}$. Use the information about the time and total distance to find his maximum speed.

SET UP: $v_{0x} = 0$ since the runner starts from rest.

$t = 4.0 \text{ s}$, but we need to calculate v_x , the speed of the runner at the end of the acceleration period.

EXECUTE: For the last $9.1 \text{ s} - 4.0 \text{ s} = 5.1 \text{ s}$ the acceleration is zero and the runner travels a distance of $d_1 = (5.1 \text{ s})v_x$ (obtained using $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$)

During the acceleration phase of 4.0 s , where the velocity goes from 0 to v_x , the runner travels a distance

$$d_2 = \left(\frac{v_{0x} + v_x}{2} \right) t = \frac{v_x}{2} (4.0 \text{ s}) = (2.0 \text{ s})v_x$$

The total distance traveled is 100 m , so $d_1 + d_2 = 100 \text{ m}$. This gives $(5.1 \text{ s})v_x + (2.0 \text{ s})v_x = 100 \text{ m}$.

$$v_x = \frac{100 \text{ m}}{7.1 \text{ s}} = 14.08 \text{ m/s}.$$

Now we can calculate $a_{\text{av-x}}$: $a_{\text{av-x}} = \frac{v_x - v_{0x}}{t} = \frac{14.08 \text{ s} - 0}{4.0 \text{ s}} = 3.5 \text{ m/s}^2$.

(b) For this time interval the velocity is constant, so $a_{\text{av-x}} = 0$.

EVALUATE: Now that we have v_x we can calculate $d_1 = (5.1 \text{ s})(14.08 \text{ m/s}) = 71.9 \text{ m}$ and $d_2 = (2.0 \text{ s})(14.08 \text{ m/s}) = 28.2 \text{ m}$. So, $d_1 + d_2 = 100 \text{ m}$, which checks.

(c) IDENTIFY and SET UP: $a_{\text{av-x}} = \frac{v_x - v_{0x}}{t}$, where now the time interval is the full 9.1 s of the race.

We have calculated the final speed to be 14.08 m/s , so

$$a_{\text{av-x}} = \frac{14.08 \text{ m/s}}{9.1 \text{ s}} = 1.5 \text{ m/s}^2.$$

EVALUATE: The acceleration is zero for the last 5.1 s , so it makes sense for the answer in part (c) to be less than half the answer in part (a).

(d) The runner spends different times moving with the average accelerations of parts (a) and (b).

- 2.60. IDENTIFY:** Apply the constant acceleration equations to the motion of the sled. The average velocity for a time interval Δt is $v_{\text{av-x}} = \frac{\Delta x}{\Delta t}$.

SET UP: Let $+x$ be parallel to the incline and directed down the incline. The problem doesn't state how much time it takes the sled to go from the top to 14.4 m from the top.

EXECUTE: (a) 14.4 m to 25.6 m: $v_{\text{av-}x} = \frac{25.6 \text{ m} - 14.4 \text{ m}}{2.00 \text{ s}} = 5.60 \text{ m/s}$. 25.6 to 40.0 m:

$$v_{\text{av-}x} = \frac{40.0 \text{ m} - 25.6 \text{ m}}{2.00 \text{ s}} = 7.20 \text{ m/s}. \quad 40.0 \text{ m to } 57.6 \text{ m: } v_{\text{av-}x} = \frac{57.6 \text{ m} - 40.0 \text{ m}}{2.00 \text{ s}} = 8.80 \text{ m/s}.$$

(b) For each segment we know $x - x_0$ and t but we don't know v_{0x} or v_x . Let $x_1 = 14.4 \text{ m}$ and $x_2 = 25.6 \text{ m}$. For

this interval $\left(\frac{v_1 + v_2}{2}\right) = \frac{x_2 - x_1}{t}$ and $at = v_2 - v_1$. Solving for v_2 gives $v_2 = \frac{1}{2}at + \frac{x_2 - x_1}{t}$. Let $x_2 = 25.6 \text{ m}$ and

$x_3 = 40.0 \text{ m}$. For this second interval, $\left(\frac{v_2 + v_3}{2}\right) = \frac{x_3 - x_2}{t}$ and $at = v_3 - v_2$. Solving for v_2 gives

$$v_2 = -\frac{1}{2}at + \frac{x_3 - x_2}{t}. \quad \text{Setting these two expressions for } v_2 \text{ equal to each other and solving for } a \text{ gives}$$

$$a = \frac{1}{t^2}[(x_3 - x_2) - (x_2 - x_1)] = \frac{1}{(2.00 \text{ s})^2}[(40.0 \text{ m} - 25.6 \text{ m}) - (25.6 \text{ m} - 14.4 \text{ m})] = 0.80 \text{ m/s}^2.$$

Note that this expression for a says $a = \frac{v_{\text{av-}23} - v_{\text{av-}12}}{t}$, where $v_{\text{av-}12}$ and $v_{\text{av-}23}$ are the average speeds for successive 2.00 s intervals.

(c) For the motion from $x = 14.4 \text{ m}$ to $x = 25.6 \text{ m}$, $x - x_0 = 11.2 \text{ m}$, $a_x = 0.80 \text{ m/s}^2$ and $t = 2.00 \text{ s}$.

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \text{ gives } v_{0x} = \frac{x - x_0}{t} - \frac{1}{2}a_x t = \frac{11.2 \text{ m}}{2.00 \text{ s}} - \frac{1}{2}(0.80 \text{ m/s}^2)(2.00 \text{ s}) = 4.80 \text{ m/s}.$$

(d) For the motion from $x = 0$ to $x = 14.4 \text{ m}$, $x - x_0 = 14.4 \text{ m}$, $v_{0x} = 0$, and $v_x = 4.8 \text{ m/s}$.

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t \text{ gives } t = \frac{2(x - x_0)}{v_{0x} + v_x} = \frac{2(14.4 \text{ m})}{4.8 \text{ m/s}} = 6.0 \text{ s}.$$

(e) For this 1.00 s time interval, $t = 1.00 \text{ s}$, $v_{0x} = 4.8 \text{ m/s}$, $a_x = 0.80 \text{ m/s}^2$.

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (4.8 \text{ m/s})(1.00 \text{ s}) + \frac{1}{2}(0.80 \text{ m/s}^2)(1.00 \text{ s})^2 = 5.2 \text{ m}.$$

EVALUATE: With $x = 0$ at the top of the hill, $x(t) = v_{0x}t + \frac{1}{2}a_x t^2 = (0.40 \text{ m/s}^2)t^2$. We can verify that $t = 6.0 \text{ s}$ gives $x = 14.4 \text{ m}$, $t = 8.0 \text{ s}$ gives 25.6 m , $t = 10.0 \text{ s}$ gives 40.0 m , and $t = 12.0 \text{ s}$ gives 57.6 m .

2.61. IDENTIFY: When the graph of v_x versus t is a straight line the acceleration is constant, so this motion consists of two constant acceleration segments and the constant acceleration equations can be used for each segment. Since v_x is always positive the motion is always in the $+x$ direction and the total distance moved equals the magnitude of the displacement. The acceleration a_x is the slope of the v_x versus t graph.

SET UP: For the $t = 0$ to $t = 10.0 \text{ s}$ segment, $v_{0x} = 4.00 \text{ m/s}$ and $v_x = 12.0 \text{ m/s}$. For the $t = 10.0 \text{ s}$ to 12.0 s segment, $v_{0x} = 12.0 \text{ m/s}$ and $v_x = 0$.

EXECUTE: (a) For $t = 0$ to $t = 10.0 \text{ s}$, $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = \left(\frac{4.00 \text{ m/s} + 12.0 \text{ m/s}}{2}\right)(10.0 \text{ s}) = 80.0 \text{ m}$. For

$t = 10.0 \text{ s}$ to $t = 12.0 \text{ s}$, $x - x_0 = \left(\frac{12.0 \text{ m/s} + 0}{2}\right)(2.00 \text{ s}) = 12.0 \text{ m}$. The total distance traveled is 92.0 m .

(b) $x - x_0 = 80.0 \text{ m} + 12.0 \text{ m} = 92.0 \text{ m}$

(c) For $t = 0$ to 10.0 s , $a_x = \frac{12.0 \text{ m/s} - 4.00 \text{ m/s}}{10.0 \text{ s}} = 0.800 \text{ m/s}^2$. For $t = 10.0 \text{ s}$ to 12.2 s ,

$$a_x = \frac{0 - 12.0 \text{ m/s}}{2.00 \text{ s}} = -6.00 \text{ m/s}^2. \quad \text{The graph of } a_x \text{ versus } t \text{ is given in Figure 2.61.}$$

EVALUATE: When v_x and a_x are both positive, the speed increases. When v_x is positive and a_x is negative, the speed decreases.

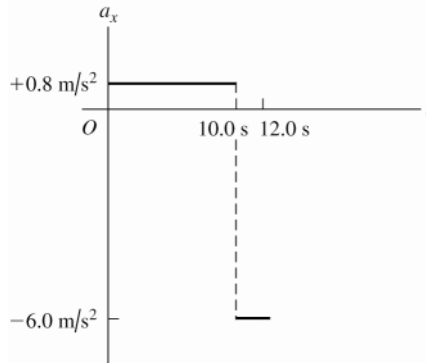


Figure 2.61

2.62. IDENTIFY: Since light travels at constant speed, $d = ct$

SET UP: The distance from the earth to the sun is 1.50×10^{11} m. The distance from the earth to the moon is 3.84×10^8 m. $c = 186,000$ mi/s.

EXECUTE: (a) $d = ct = (3.0 \times 10^8 \text{ m/s})(1 \text{ y}) \left(\frac{365 \frac{1}{4} \text{ d}}{1 \text{ y}} \right) \left(\frac{24 \text{ h}}{1 \text{ d}} \right) \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) = 9.5 \times 10^{15} \text{ m}$

(b) $d = ct = (3.0 \times 10^8 \text{ m/s})(10^{-9} \text{ s}) = 0.30 \text{ m}$

(c) $t = \frac{d}{c} = \frac{1.5 \times 10^{11} \text{ m}}{3.0 \times 10^8 \text{ m/s}} = 500 \text{ s} = 8.33 \text{ min}$

(d) $t = \frac{d}{c} = \frac{2(3.84 \times 10^8 \text{ m})}{3.0 \times 10^8 \text{ m/s}} = 2.6 \text{ s}$

(e) $t = \frac{d}{c} = \frac{3 \times 10^9 \text{ mi}}{186,000 \text{ mi/s}} = 16,100 \text{ s} = 4.5 \text{ h}$

EVALUATE: The speed of light is very large but it still takes light a measurable length of time to travel a large distance.

2.63. IDENTIFY: Speed is distance d divided by time t . The distance around a circular path is $d = 2\pi R$, where R is the radius of the circular path.

SET UP: The radius of the earth is $R_E = 6.38 \times 10^6$ m. The earth rotates once in 1 day = 86,400 s. The radius of the earth's orbit around the sun is 1.50×10^{11} m and the earth completes this orbit in 1 year = 3.156×10^7 s. The speed of light in vacuum is $c = 3.00 \times 10^8$ m/s.

EXECUTE: (a) $v = \frac{d}{t} = \frac{2\pi R_E}{t} = \frac{2\pi(6.38 \times 10^6 \text{ m})}{86,400 \text{ s}} = 464 \text{ m/s}$.

(b) $v = \frac{2\pi R}{t} = \frac{2\pi(1.50 \times 10^{11} \text{ m})}{3.156 \times 10^7 \text{ s}} = 2.99 \times 10^4 \text{ m/s}$.

(c) The time for light to go around once is $t = \frac{d}{c} = \frac{2\pi R_E}{c} = \frac{2\pi(6.38 \times 10^6 \text{ m})}{3.00 \times 10^8 \text{ m/s}} = 0.1336 \text{ s}$. In 1.00 s light would go around the earth $\frac{1.00 \text{ s}}{0.1336 \text{ s}} = 7.49$ times.

EVALUATE: All these speeds are large compared to speeds of objects in our everyday experience.

2.64. IDENTIFY: When the graph of v_x versus t is a straight line the acceleration is constant, so this motion consists of two constant acceleration segments and the constant acceleration equations can be used for each segment. For $t = 0$ to 5.0 s, v_x is positive and the ball moves in the $+x$ direction. For $t = 5.0$ s to 20.0 s, v_x is negative and the ball moves in the $-x$ direction. The acceleration a_x is the slope of the v_x versus t graph.

SET UP: For the $t = 0$ to $t = 5.0$ s segment, $v_{0x} = 0$ and $v_x = 30.0$ m/s. For the $t = 5.0$ s to $t = 20.0$ s segment, $v_{0x} = -20.0$ m/s and $v_x = 0$.

EXECUTE: (a) For $t = 0$ to 5.0 s, $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = \left(\frac{0 + 30.0 \text{ m/s}}{2}\right)(5.0 \text{ m/s}) = 75.0 \text{ m}$. The ball travels a distance of 75.0 m. For $t = 5.0$ s to 20.0 s, $x - x_0 = \left(\frac{-20.0 \text{ m/s} + 0}{2}\right)(15.0 \text{ m/s}) = -150.0 \text{ m}$. The total distance traveled is $75.0 \text{ m} + 150.0 \text{ m} = 225.0 \text{ m}$.

(b) The total displacement is $x - x_0 = 75.0 \text{ m} + (-150.0 \text{ m}) = -75.0 \text{ m}$. The ball ends up 75.0 m in the negative x -direction from where it started.

(c) For $t = 0$ to 5.0 s, $a_x = \frac{30.0 \text{ m/s} - 0}{5.0 \text{ s}} = 6.00 \text{ m/s}^2$. For $t = 5.0$ s to 20.0 s, $a_x = \frac{0 - (-20.0 \text{ m/s})}{15.0 \text{ s}} = +1.33 \text{ m/s}^2$.

The graph of a_x versus t is given in Figure 2.64.

(d) The ball is in contact with the floor for a small but nonzero period of time and the direction of the velocity doesn't change instantaneously. So, no, the actual graph of $v_x(t)$ is not really vertical at 5.00 s.

EVALUATE: For $t = 0$ to 5.0 s, both v_x and a_x are positive and the speed increases. For $t = 5.0$ s to 20.0 s, v_x is negative and a_x is positive and the speed decreases. Since the direction of motion is not the same throughout, the displacement is not equal to the distance traveled.

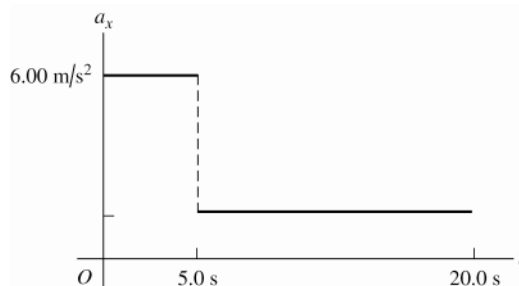


Figure 2.64

2.65. IDENTIFY and SET UP: Apply constant acceleration equations.

Find the velocity at the start of the second 5.0 s; this is the velocity at the end of the first 5.0 s. Then find $x - x_0$ for the first 5.0 s.

EXECUTE: For the first 5.0 s of the motion, $v_{0x} = 0$, $t = 5.0$ s.

$$v_x = v_{0x} + a_x t \text{ gives } v_x = a_x(5.0 \text{ s}).$$

This is the initial speed for the second 5.0 s of the motion. For the second 5.0 s:

$$v_{0x} = a_x(5.0 \text{ s}), \quad t = 5.0 \text{ s}, \quad x - x_0 = 150 \text{ m}.$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \text{ gives } 150 \text{ m} = (25 \text{ s}^2)a_x + (12.5 \text{ s}^2)a_x \text{ and } a_x = 4.0 \text{ m/s}^2$$

Use this a_x and consider the first 5.0 s of the motion:

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(4.0 \text{ m/s}^2)(5.0 \text{ s})^2 = 50.0 \text{ m}.$$

EVALUATE: The ball is speeding up so it travels farther in the second 5.0 s interval than in the first. In fact, $x - x_0$ is proportional to t^2 since it starts from rest. If it goes 50.0 m in 5.0 s, in twice the time (10.0 s) it should go four times as far. In 10.0 s we calculated it went $50 \text{ m} + 150 \text{ m} = 200 \text{ m}$, which is four times 50 m.

2.66. IDENTIFY: Apply $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to the motion of each train. A collision means the front of the passenger train is at the same location as the caboose of the freight train at some common time.

SET UP: Let P be the passenger train and F be the freight train. For the front of the passenger train $x_0 = 0$ and for the caboose of the freight train $x_0 = 200 \text{ m}$. For the freight train $v_F = 15.0 \text{ m/s}$ and $a_F = 0$. For the passenger train $v_P = 25.0 \text{ m/s}$ and $a_P = -0.100 \text{ m/s}^2$.

EXECUTE: (a) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ for each object gives $x_P = v_P t + \frac{1}{2}a_P t^2$ and $x_F = 200 \text{ m} + v_F t$. Setting

$$x_P = x_F \text{ gives } v_P t + \frac{1}{2}a_P t^2 = 200 \text{ m} + v_F t. \quad (0.0500 \text{ m/s}^2)t^2 - (10.0 \text{ m/s})t + 200 \text{ m} = 0. \text{ The}$$

quadratic formula gives $t = \frac{1}{0.100} \left(+10.0 \pm \sqrt{(10.0)^2 - 4(0.0500)(200)} \right) \text{ s} = (100 \pm 77.5) \text{ s}$. The collision occurs at $t = 100 \text{ s} - 77.5 \text{ s} = 22.5 \text{ s}$. The equations that specify a collision have a physical solution (real, positive t), so a collision does occur.

(b) $x_p = (25.0 \text{ m/s})(22.5 \text{ s}) + \frac{1}{2}(-0.100 \text{ m/s}^2)(22.5 \text{ s})^2 = 537 \text{ m}$. The passenger train moves 537 m before the collision. The freight train moves $(15.0 \text{ m/s})(22.5 \text{ s}) = 337 \text{ m}$.

(c) The graphs of x_F and x_p versus t are sketched in Figure 2.66.

EVALUATE: The second root for the equation for t , $t = 177.5 \text{ s}$ is the time the trains would meet again if they were on parallel tracks and continued their motion after the first meeting.

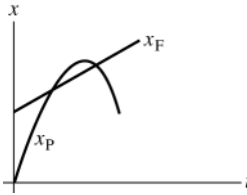


Figure 2.66

2.67. IDENTIFY: Apply constant acceleration equations to the motion of the two objects, you and the cockroach. You catch up with the roach when both objects are at the same place at the same time. Let T be the time when you catch up with the cockroach.

SET UP: Take $x = 0$ to be at the $t = 0$ location of the roach and positive x to be in the direction of motion of the two objects.

roach:

$$v_{0x} = 1.50 \text{ m/s}, \quad a_x = 0, \quad x_0 = 0, \quad x = 1.20 \text{ m}, \quad t = T$$

you:

$$v_{0x} = 0.80 \text{ m/s}, \quad x_0 = -0.90 \text{ m}, \quad x = 1.20 \text{ m}, \quad t = T, \quad a_x = ?$$

Apply $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to both objects:

EXECUTE: **roach:** $1.20 \text{ m} = (1.50 \text{ m/s})T$, so $T = 0.800 \text{ s}$.

you: $1.20 \text{ m} - (-0.90 \text{ m}) = (0.80 \text{ m/s})T + \frac{1}{2}a_x T^2$

$$2.10 \text{ m} = (0.80 \text{ m/s})(0.800 \text{ s}) + \frac{1}{2}a_x(0.800 \text{ s})^2$$

$$2.10 \text{ m} = 0.64 \text{ m} + (0.320 \text{ s}^2)a_x$$

$$a_x = 4.6 \text{ m/s}^2.$$

EVALUATE: Your final velocity is $v_x = v_{0x} + a_x t = 4.48 \text{ m/s}$. Then $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = 2.10 \text{ m}$, which checks.

You have to accelerate to a speed greater than that of the roach so you will travel the extra 0.90 m you are initially behind.

2.68. IDENTIFY: The insect has constant speed 15 m/s during the time it takes the cars to come together.

SET UP: Each car has moved 100 m when they hit.

EXECUTE: The time until the cars hit is $\frac{100 \text{ m}}{10 \text{ m/s}} = 10 \text{ s}$. During this time the grasshopper travels a distance of

$$(15 \text{ m/s})(10 \text{ s}) = 150 \text{ m}.$$

EVALUATE: The grasshopper ends up 100 m from where it started, so the magnitude of his final displacement is 100 m. This is less than the total distance he travels since he spends part of the time moving in the opposite direction.

2.69. IDENTIFY: Apply constant acceleration equations to each object.

Take the origin of coordinates to be at the initial position of the truck, as shown in Figure 2.69a

Let d be the distance that the auto initially is behind the truck, so $x_0(\text{auto}) = -d$ and $x_0(\text{truck}) = 0$. Let T be the

time it takes the auto to catch the truck. Thus at time T the truck has undergone a displacement $x - x_0 = 40.0 \text{ m}$, so is at $x = x_0 + 40.0 \text{ m} = 40.0 \text{ m}$. The auto has caught the truck so at time T is also at $x = 40.0 \text{ m}$.

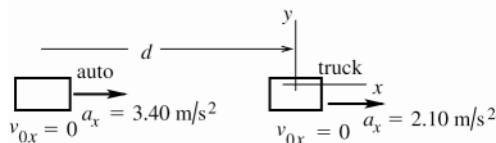


Figure 2.69a

(a) **SET UP:** Use the motion of the truck to calculate T :

$$x - x_0 = 40.0 \text{ m}, \quad v_{0x} = 0 \quad (\text{starts from rest}), \quad a_x = 2.10 \text{ m/s}^2, \quad t = T$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$\text{Since } v_{0x} = 0, \text{ this gives } t = \sqrt{\frac{2(x - x_0)}{a_x}}$$

$$\text{EXECUTE: } T = \sqrt{\frac{2(40.0 \text{ m})}{2.10 \text{ m/s}^2}} = 6.17 \text{ s}$$

(b) **SET UP:** Use the motion of the auto to calculate d :

$$x - x_0 = 40.0 \text{ m} + d, \quad v_{0x} = 0, \quad a_x = 3.40 \text{ m/s}^2, \quad t = 6.17 \text{ s}$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$\text{EXECUTE: } d + 40.0 \text{ m} = \frac{1}{2}(3.40 \text{ m/s}^2)(6.17 \text{ s})^2$$

$$d = 64.8 \text{ m} - 40.0 \text{ m} = 24.8 \text{ m}$$

$$\text{(c) auto: } v_x = v_{0x} + a_x t = 0 + (3.40 \text{ m/s}^2)(6.17 \text{ s}) = 21.0 \text{ m/s}$$

$$\text{truck: } v_x = v_{0x} + a_x t = 0 + (2.10 \text{ m/s}^2)(6.17 \text{ s}) = 13.0 \text{ m/s}$$

(d) The graph is sketched in Figure 2.69b.

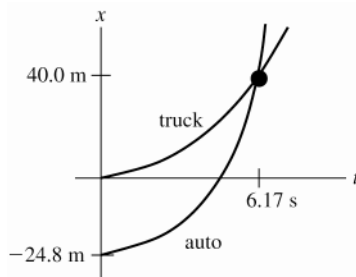


Figure 2.69b

EVALUATE: In part (c) we found that the auto was traveling faster than the truck when they come abreast. The graph in part (d) agrees with this: at the intersection of the two curves the slope of the x - t curve for the auto is greater than that of the truck. The auto must have an average velocity greater than that of the truck since it must travel farther in the same time interval.

2.70. IDENTIFY: Apply the constant acceleration equations to the motion of each car. The collision occurs when the cars are at the same place at the same time.

SET UP: Let $+x$ be to the right. Let $x = 0$ at the initial location of car 1, so $x_{01} = 0$ and $x_{02} = D$. The cars collide when $x_1 = x_2$. $v_{0x1} = 0$, $a_{x1} = a_x$, $v_{0x2} = -v_0$ and $a_{x2} = 0$.

EXECUTE: (a) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x_1 = \frac{1}{2}a_x t^2$ and $x_2 = D - v_0 t$. $x_1 = x_2$ gives $\frac{1}{2}a_x t^2 = D - v_0 t$.

$\frac{1}{2}a_x t^2 + v_0 t - D = 0$. The quadratic formula gives $t = \frac{1}{a_x} \left(-v_0 \pm \sqrt{v_0^2 + 2a_x D} \right)$. Only the positive root is physical,

$$\text{so } t = \frac{1}{a_x} \left(-v_0 + \sqrt{v_0^2 + 2a_x D} \right).$$

$$\text{(b) } v_1 = a_x t = \sqrt{v_0^2 + 2a_x D} - v_0$$

(c) The x - t and v_x - t graphs for the two cars are sketched in Figure 2.70.

EVALUATE: In the limit that $a_x = 0$, $D - v_0 t = 0$ and $t = D/v_0$, the time it takes car 2 to travel distance D . In the limit that $v_0 = 0$, $t = \sqrt{\frac{2D}{a_x}}$, the time it takes car 1 to travel distance D .

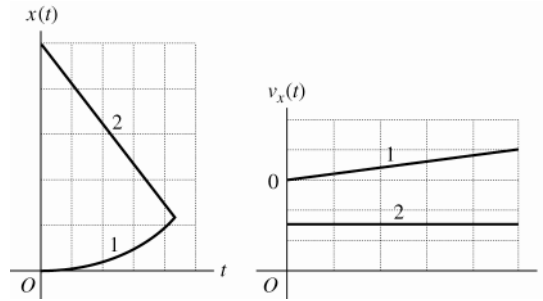


Figure 2.70

2.71. **IDENTIFY:** The average speed is the distance traveled divided by the time. The average velocity is $v_{\text{av-x}} = \frac{\Delta x}{\Delta t}$.

SET UP: The distance the ball travels is half the circumference of a circle of diameter 50.0 cm so is $\frac{1}{2}\pi d = \frac{1}{2}\pi(50.0 \text{ cm}) = 78.5 \text{ cm}$. Let $+x$ be horizontally from the starting point toward the ending point, so Δx equals the diameter of the bowl.

EXECUTE: (a) The average speed is $\frac{\frac{1}{2}\pi d}{t} = \frac{78.5 \text{ cm}}{10.0 \text{ s}} = 7.85 \text{ cm/s}$.

(b) The average velocity is $v_{\text{av-x}} = \frac{\Delta x}{\Delta t} = \frac{50.0 \text{ cm}}{10.0 \text{ s}} = 5.00 \text{ cm/s}$.

EVALUATE: The average speed is greater than the magnitude of the average velocity, since the distance traveled is greater than the magnitude of the displacement.

2.72. **IDENTIFY:** a_x is the slope of the v_x versus t graph. x is the area under the v_x versus t graph.

SET UP: The slope of v_x is positive and decreasing in magnitude. As v_x increases, the displacement in a given amount of time increases.

EXECUTE: The a_x - t and x - t graphs are sketched in Figure 2.72.

EVALUATE: v_x is the slope of the x versus t graph. The $x(t)$ graph we sketch has zero slope at $t = 0$, the slope is always positive, and the slope initially increases and then approaches a constant. This behavior agrees with the $v_x(t)$ that is given in the graph in the problem.

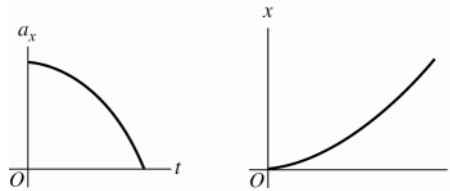


Figure 2.72

2.73. **IDENTIFY:** Apply constant acceleration equations to each vehicle.

SET UP: (a) It is very convenient to work in coordinates attached to the truck.

Note that these coordinates move at constant velocity relative to the earth. In these coordinates the truck is at rest, and the initial velocity of the car is $v_{0x} = 0$. Also, the car's acceleration in these coordinates is the same as in coordinates fixed to the earth.

EXECUTE: First, let's calculate how far the car must travel relative to the truck: The situation is sketched in Figure 2.73.

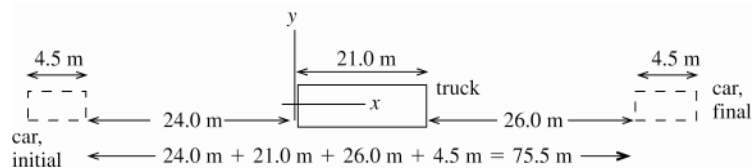


Figure 2.73

The car goes from $x_0 = -24.0$ m to $x = 51.5$ m. So $x - x_0 = 75.5$ m for the car.

Calculate the time it takes the car to travel this distance:

$$a_x = 0.600 \text{ m/s}^2, \quad v_{0x} = 0, \quad x - x_0 = 75.5 \text{ m}, \quad t = ?$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$t = \sqrt{\frac{2(x - x_0)}{a_x}} = \sqrt{\frac{2(75.5 \text{ m})}{0.600 \text{ m/s}^2}} = 15.86 \text{ s}$$

It takes the car 15.9 s to pass the truck.

(b) Need how far the car travels relative to the earth, so go now to coordinates fixed to the earth. In these coordinates $v_{0x} = 20.0$ m/s for the car. Take the origin to be at the initial position of the car.

$$v_{0x} = 20.0 \text{ m/s}, \quad a_x = 0.600 \text{ m/s}^2, \quad t = 15.86 \text{ s}, \quad x - x_0 = ?$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (20.0 \text{ m/s})(15.86 \text{ s}) + \frac{1}{2}(0.600 \text{ m/s}^2)(15.86 \text{ s})^2$$

$$x - x_0 = 317.2 \text{ m} + 75.5 \text{ m} = 393 \text{ m}.$$

(c) In coordinates fixed to the earth:

$$v_x = v_{0x} + a_x t = 20.0 \text{ m/s} + (0.600 \text{ m/s}^2)(15.86 \text{ s}) = 29.5 \text{ m/s}$$

EVALUATE: In 15.9 s the truck travels $x - x_0 = (20.0 \text{ m/s})(15.86 \text{ s}) = 317.2$ m. The car travels

$392.7 \text{ m} - 317.2 \text{ m} = 75 \text{ m}$ farther than the truck, which checks with part (a). In coordinates attached to the truck,

for the car $v_{0x} = 0$, $v_x = 9.5$ m/s and in 15.86 s the car travels $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = 75 \text{ m}$, which checks with

part (a).

2.74. IDENTIFY: The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use

$$a_x(t) = \frac{dv_x}{dt} \text{ and } x = x_0 + \int_0^t v_x(t) dt.$$

$$\text{SET UP: } \int t^n dt = \frac{1}{n+1} t^{n+1} \text{ for } n \geq 0.$$

EXECUTE: (a) $x(t) = x_0 + \int_0^t [\alpha - \beta t^2] dt = x_0 + \alpha t - \frac{1}{3} \beta t^3$. $x = 0$ at $t = 0$ gives $x_0 = 0$ and

$$x(t) = \alpha t - \frac{1}{3} \beta t^3 = (4.00 \text{ m/s})t - (0.667 \text{ m/s}^3)t^3. \quad a_x(t) = \frac{dv_x}{dt} = -2\beta t = -(4.00 \text{ m/s}^3)t.$$

(b) The maximum positive x is when $v_x = 0$ and $a_x < 0$. $v_x = 0$ gives $\alpha - \beta t^2 = 0$ and

$$t = \sqrt{\frac{\alpha}{\beta}} = \sqrt{\frac{4.00 \text{ m/s}}{2.00 \text{ m/s}^3}} = 1.41 \text{ s}. \text{ At this } t, a_x \text{ is negative. For } t = 1.41 \text{ s,}$$

$$x = (4.00 \text{ m/s})(1.41 \text{ s}) - (0.667 \text{ m/s}^3)(1.41 \text{ s})^3 = 3.77 \text{ m}.$$

EVALUATE: After $t = 1.41$ s the object starts to move in the $-x$ direction and goes to $x = -\infty$ as $t \rightarrow \infty$.

2.75. a(t) = \alpha + \beta t, with $\alpha = -2.00$ m/s² and $\beta = 3.00$ m/s³

(a) IDENTIFY and SET UP: Integrate $a_x(t)$ to find $v_x(t)$ and then integrate $v_x(t)$ to find $x(t)$.

$$\text{EXECUTE: } v_x = v_{0x} + \int_0^t a_x dt = v_{0x} + \int_0^t (\alpha + \beta t) dt = v_{0x} + \alpha t + \frac{1}{2} \beta t^2$$

$$x = x_0 + \int_0^t v_x dt = x_0 + \int_0^t (v_{0x} + \alpha t + \frac{1}{2} \beta t^2) dt = x_0 + v_{0x}t + \frac{1}{2} \alpha t^2 + \frac{1}{6} \beta t^3$$

At $t = 0$, $x = x_0$.

To have $x = x_0$ at $t_1 = 4.00$ s requires that $v_{0x}t_1 + \frac{1}{2} \alpha t_1^2 + \frac{1}{6} \beta t_1^3 = 0$.

$$\text{Thus } v_{0x} = -\frac{1}{6} \beta t_1^2 - \frac{1}{2} \alpha t_1 = -\frac{1}{6} (3.00 \text{ m/s}^3)(4.00 \text{ s})^2 - \frac{1}{2} (-2.00 \text{ m/s}^2)(4.00 \text{ s}) = -4.00 \text{ m/s}.$$

(b) With v_{0x} as calculated in part (a) and $t = 4.00$ s,

$$v_0 = v_{0x} + \alpha t + \frac{1}{2} \beta t^2 = -4.00 \text{ m/s} + (-2.00 \text{ m/s}^2)(4.00 \text{ s}) + \frac{1}{2} (3.00 \text{ m/s}^3)(4.00 \text{ s})^2 = +12.0 \text{ m/s}.$$

EVALUATE: $a_x = 0$ at $t = 0.67$ s. For $t > 0.67$ s, $a_x > 0$. At $t = 0$, the particle is moving in the $-x$ -direction and is speeding up. After $t = 0.67$ s, when the acceleration is positive, the object slows down and then starts to move in the $+x$ -direction with increasing speed.

- 2.76. IDENTIFY:** Find the distance the professor walks during the time t it takes the egg to fall to the height of his head.
SET UP: Let $+y$ be downward. The egg has $v_{0y} = 0$ and $a_y = 9.80 \text{ m/s}^2$. At the height of the professor's head, the egg has $y - y_0 = 44.2 \text{ m}$.

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(44.2 \text{ m})}{9.80 \text{ m/s}^2}} = 3.00 \text{ s}$. The professor walks a distance

$x - x_0 = v_{0x}t = (1.20 \text{ m/s})(3.00 \text{ s}) = 3.60 \text{ m}$. Release the egg when your professor is 3.60 m from the point directly below you.

EVALUATE: Just before the egg lands its speed is $(9.80 \text{ m/s}^2)(3.00 \text{ s}) = 29.4 \text{ m/s}$. It is traveling much faster than the professor.

- 2.77. IDENTIFY:** Use the constant acceleration equations to establish a relationship between maximum height and acceleration due to gravity and between time in the air and acceleration due to gravity.

SET UP: Let $+y$ be upward. At the maximum height, $v_y = 0$. When the rock returns to the surface, $y - y_0 = 0$.

EXECUTE: (a) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y H = -\frac{1}{2}v_{0y}^2$, which is constant, so $a_E H_E = a_M H_M$.

$$H_M = H_E \left(\frac{a_E}{a_M} \right) = H \left(\frac{9.80 \text{ m/s}^2}{3.71 \text{ m/s}^2} \right) = 2.64H.$$

(b) $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 0$ gives $a_y t = -2v_{0y}$, which is constant, so $a_E T_E = a_M T_M$.

$$T_M = T_E \left[\frac{a_E}{a_M} \right] = 2.64T.$$

EVALUATE: On Mars, where the acceleration due to gravity is smaller, the rocks reach a greater height and are in the air for a longer time.

- 2.78. IDENTIFY:** Calculate the time it takes her to run to the table and return. This is the time in the air for the thrown ball. The thrown ball is in free-fall after it is thrown. Assume air resistance can be neglected.

SET UP: For the thrown ball, let $+y$ be upward. $a_y = -9.80 \text{ m/s}^2$. $y - y_0 = 0$ when the ball returns to its original position.

EXECUTE: (a) It takes her $\frac{5.50 \text{ m}}{2.50 \text{ m/s}} = 2.20 \text{ s}$ to reach the table and an equal time to return. For the ball,

$$y - y_0 = 0, \quad t = 4.40 \text{ s} \text{ and } a_y = -9.80 \text{ m/s}^2. \quad y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 \text{ gives}$$

$$v_{0y} = -\frac{1}{2}a_y t = -\frac{1}{2}(-9.80 \text{ m/s}^2)(4.40 \text{ s}) = 21.6 \text{ m/s}.$$

(b) Find $y - y_0$ when $t = 2.20 \text{ s}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (21.6 \text{ m/s})(2.20 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(2.20 \text{ s})^2 = 23.8 \text{ m}$

EVALUATE: It takes the ball the same amount of time to reach its maximum height as to return from its maximum height, so when she is at the table the ball is at its maximum height. Note that this large maximum height requires that the act either be done outdoors, or in a building with a very high ceiling.

- 2.79. (a) IDENTIFY:** Use constant acceleration equations, with $a_y = g$, downward, to calculate the speed of the diver when she reaches the water.

SET UP: Take the origin of coordinates to be at the platform, and take the $+y$ -direction to be downward.

$$y - y_0 = +21.3 \text{ m}, \quad a_y = +9.80 \text{ m/s}^2, \quad v_{0y} = 0 \text{ (since diver just steps off)}, \quad v_y = ?$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$\text{EXECUTE: } v_y = +\sqrt{2a_y(y - y_0)} = +\sqrt{2(9.80 \text{ m/s}^2)(31.3 \text{ m})} = +20.4 \text{ m/s}.$$

We know that v_y is positive because the diver is traveling downward when she reaches the water.

The announcer has exaggerated the speed of the diver.

EVALUATE: We could also use $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ to find $t = 2.085 \text{ s}$. The diver gains 9.80 m/s of speed each second, so has $v_y = (9.80 \text{ m/s}^2)(2.085 \text{ s}) = 20.4 \text{ m/s}$ when she reaches the water, which checks.

(b) IDENTIFY: Calculate the initial upward velocity needed to give the diver a speed of 25.0 m/s when she reaches the water. Use the same coordinates as in part (a).

SET UP: $v_{0y} = ?$, $v_y = +25.0 \text{ m/s}$, $a_y = +9.80 \text{ m/s}^2$, $y - y_0 = +21.3 \text{ m}$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

EXECUTE: $v_{0y} = -\sqrt{v_y^2 - 2a_y(y - y_0)} = -\sqrt{(25.0 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(21.3 \text{ m})} = -14.4 \text{ m/s}$

(v_{0y} is negative since the direction of the initial velocity is upward.)

EVALUATE: One way to decide if this speed is reasonable is to calculate the maximum height above the platform it would produce:

$$v_{0y} = -14.4 \text{ m/s}, \quad v_y = 0 \text{ (at maximum height)}, \quad a_y = +9.80 \text{ m/s}^2, \quad y - y_0 = ?$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (-14.4 \text{ s})^2}{2(+9.80 \text{ m/s}^2)} = -10.6 \text{ m}$$

This is not physically attainable; a vertical leap of 10.6 m upward is not possible.

- 2.80. IDENTIFY:** The flowerpot is in free-fall. Apply the constant acceleration equations. Use the motion past the window to find the speed of the flowerpot as it reaches the top of the window. Then consider the motion from the windowsill to the top of the window.

SET UP: Let $+y$ be downward. Throughout the motion $a_y = +9.80 \text{ m/s}^2$.

EXECUTE: Motion past the window: $y - y_0 = 1.90 \text{ m}$, $t = 0.420 \text{ s}$, $a_y = +9.80 \text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$v_{0y} = \frac{y - y_0}{t} - \frac{1}{2}a_y t = \frac{1.90 \text{ m}}{0.420 \text{ s}} - \frac{1}{2}(9.80 \text{ m/s}^2)(0.420 \text{ s}) = 2.466 \text{ m/s} . \text{ This is the velocity of the flowerpot when it is at the top of the window.}$$

Motion from the windowsill to the top of the window: $v_{0y} = 0$, $v_y = 2.466 \text{ m/s}$, $a_y = +9.80 \text{ m/s}^2$.

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \text{ gives } y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{(2.466 \text{ m/s})^2 - 0}{2(9.80 \text{ m/s}^2)} = 0.310 \text{ m} . \text{ The top of the window is } 0.310 \text{ m}$$

below the windowsill.

EVALUATE: It takes the flowerpot $t = \frac{v_y - v_{0y}}{a_y} = \frac{2.466 \text{ m/s}}{9.80 \text{ m/s}^2} = 0.252 \text{ s}$ to fall from the sill to the top of the

window. Our result says that from the windowsill the pot falls $0.310 \text{ m} + 1.90 \text{ m} = 2.21 \text{ m}$ in

$$0.252 \text{ s} + 0.420 \text{ s} = 0.672 \text{ s} . \quad y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80 \text{ m/s}^2)(0.672 \text{ s})^2 = 2.21 \text{ m} , \text{ which checks.}$$

- 2.81. IDENTIFY:** For parts (a) and (b) apply the constant acceleration equations to the motion of the bullet. In part (c) neglect air resistance, so the bullet is free-fall. Use the constant acceleration equations to establish a relation between initial speed v_0 and maximum height H .

SET UP: For parts (a) and (b) let $+x$ be in the direction of motion of the bullet. For part (c) let $+y$ be upward, so $a_y = -g$. At the maximum height, $v_y = 0$.

EXECUTE: (a) $x - x_0 = 0.700 \text{ m}$, $v_{0x} = 0$, $v_x = 965 \text{ m/s}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(965 \text{ m/s})^2 - 0}{2(0.700 \text{ m})} = 6.65 \times 10^5 \text{ m/s}^2 . \quad \frac{a_x}{g} = 6.79 \times 10^4 , \text{ so } a_x = (6.79 \times 10^4)g .$$

(b) $x - x_0 = \left(\frac{v_{0x} + v_x}{2} \right) t$ gives $t = \frac{2(x - x_0)}{v_{0x} + v_x} = \frac{2(0.700 \text{ m})}{0 + 965 \text{ m/s}} = 1.45 \text{ ms} .$

(c) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $v_y = 0$ gives $\frac{v_{0y}^2}{y - y_0} = -2a_y$, which is constant. $\frac{v_{01}^2}{H_1} = \frac{v_{02}^2}{H_2}$.

$$H_2 = H_1 \left(\frac{v_{02}^2}{v_{01}^2} \right) = H \left(\frac{\frac{1}{2}v_{01}}{v_{01}} \right)^2 = H/4 .$$

EVALUATE: $H = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{-(965 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 47.5 \text{ km} .$ Rifle bullets fired vertically don't actually reach such a

large height; it is not an accurate approximation to ignore air resistance.

- 2.82. IDENTIFY:** Assume the firing of the second stage lasts a very short time, so the rocket is in free-fall after 25.0 s. The motion consists of two constant acceleration segments.

SET UP: Let $+y$ be upward. After $t = 25.0 \text{ s}$, $a_y = -9.80 \text{ m/s}^2$.

EXECUTE: (a) Find the height of the rocket at $t = 25.0 \text{ s}$: $v_{0y} = 0$, $a_y = +3.50 \text{ m/s}^2$, $t = 25.0 \text{ s}$.

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(3.50 \text{ m/s}^2)(25.0 \text{ s})^2 = 1.0938 \times 10^3 \text{ m} . \text{ Find the displacement of the rocket from firing of the}$$

second stage until the maximum height is reached: $v_{0y} = 132.5 \text{ m/s}$, $v_y = 0$ (at maximum height), $a_y = -9.80 \text{ m/s}^2$.

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \text{ gives } y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (132.5 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 896 \text{ m} . \text{ The total height is}$$

$$1094 \text{ m} + 896 \text{ m} = 1990 \text{ m} .$$

(b) $v_{0y} = +132.5 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$, $y - y_0 = -1094 \text{ m}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$-1093.8 \text{ m} = (132.5 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$. The quadratic formula gives $t = 33.7 \text{ s}$ as the positive root. The rocket returns to the launch pad 33.7 s after the second stage fires.

(c) $v_y = v_{0y} + a_y t = +132.5 \text{ m/s} + (-9.80 \text{ m/s}^2)(33.7 \text{ s}) = -198 \text{ m/s}$. The rocket has speed 198 m/s as it reaches the launch pad.

EVALUATE: The speed when the rocket returns to the launch pad is greater than 132.5 m/s. When the rocket returns to the height where the second stage fired, its velocity is 132.5 m/s downward and it continues to speed up during the rest of the descent.

2.83. Take positive y to be upward.

(a) IDENTIFY: Consider the motion from when he applies the acceleration to when the shot leaves his hand.

SET UP: $v_{0y} = 0$, $v_y = ?$, $a_y = 45.0 \text{ m/s}^2$, $y - y_0 = 0.640 \text{ m}$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

EXECUTE: $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(45.0 \text{ m/s}^2)(0.640 \text{ m})} = 7.59 \text{ m/s}$

(b) IDENTIFY: Consider the motion of the shot from the point where he releases it to its maximum height, where $v = 0$. Take $y = 0$ at the ground.

SET UP: $y_0 = 2.20 \text{ m}$, $y = ?$, $a_y = -9.80 \text{ m/s}^2$ (free fall), $v_{0y} = 7.59 \text{ m/s}$

(from part (a), $v_y = 0$ at maximum height)

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

EXECUTE: $y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (7.59 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 2.94 \text{ m}$

$$y = 2.20 \text{ m} + 2.94 \text{ m} = 5.14 \text{ m} .$$

(c) IDENTIFY: Consider the motion of the shot from the point where he releases it to when it returns to the height of his head. Take $y = 0$ at the ground.

SET UP: $y_0 = 2.20 \text{ m}$, $y = 1.83 \text{ m}$, $a_y = -9.80 \text{ m/s}^2$, $v_{0y} = +7.59 \text{ m/s}$, $t = ?$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: $1.83 \text{ m} - 2.20 \text{ m} = (7.59 \text{ m/s})t + \frac{1}{2}(-9.80 \text{ m/s}^2)t^2$
 $= (7.59 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$

$$4.90t^2 - 7.59t - 0.37 = 0, \text{ with } t \text{ in seconds} .$$

Use the quadratic formula to solve for t :

$$t = \frac{1}{9.80} \left(7.59 \pm \sqrt{(7.59)^2 - 4(4.90)(-0.37)} \right) = 0.774 \pm 0.822$$

t must be positive, so $t = 0.774 \text{ s} + 0.822 \text{ s} = 1.60 \text{ s}$

EVALUATE: Calculate the time to the maximum height: $v_y = v_{0y} + a_y t$, so

$t = (v_y - v_{0y})/a_y = -(7.59 \text{ m/s})/(-9.80 \text{ m/s}^2) = 0.77 \text{ s}$. It also takes 0.77 s to return to 2.2 m above the ground, for a total time of 1.54 s. His head is a little lower than 2.20 m, so it is reasonable for the shot to reach the level of his head a little later than 1.54 s after being thrown; the answer of 1.60 s in part (c) makes sense.

2.84. IDENTIFY: The teacher is in free-fall and falls with constant acceleration 9.80 m/s^2 , downward. The sound from her shout travels at constant speed. The sound travels from the top of the cliff, reflects from the ground and then travels upward to her present location. If the height of the cliff is h and she falls a distance y in 3.0 s, the sound must travel a distance $h + (h - y)$ in 3.0 s.

SET UP: Let $+y$ be downward, so for the teacher $a_y = 9.80 \text{ m/s}^2$ and $v_{0y} = 0$. Let $y = 0$ at the top of the cliff.

EXECUTE: **(a)** For the teacher, $y = \frac{1}{2}(9.80 \text{ m/s}^2)(3.0 \text{ s})^2 = 44.1 \text{ m}$. For the sound, $h + (h - y) = v_s t$.

$$h = \frac{1}{2}(v_s t + y) = \frac{1}{2}([340 \text{ m/s}][3.0 \text{ s}] + 44.1 \text{ m}) = 532 \text{ m} , \text{ which rounds to } 530 \text{ m} .$$

(b) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(9.80 \text{ m/s}^2)(532 \text{ m})} = 102 \text{ m/s}$.

EVALUATE: She is in the air for $t = \frac{v_y - v_{0y}}{a_y} = \frac{102 \text{ m/s}}{9.80 \text{ m/s}^2} = 10.4 \text{ s}$ and strikes the ground at high speed.

- 2.85. IDENTIFY and SET UP:** Let $+y$ be upward. Each ball moves with constant acceleration $a_y = -9.80 \text{ m/s}^2$. In parts (c) and (d) require that the two balls be at the same height at the same time.

EXECUTE: (a) At ceiling, $v_y = 0$, $y - y_0 = 3.0 \text{ m}$, $a_y = -9.80 \text{ m/s}^2$. Solve for v_{0y} .

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \text{ gives } v_{0y} = 7.7 \text{ m/s.}$$

(b) $v_y = v_{0y} + a_y t$ with the information from part (a) gives $t = 0.78 \text{ s}$.

(c) Let the first ball travel downward a distance d in time t . It starts from its maximum height, so $v_{0y} = 0$.

$$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 \text{ gives } d = (4.9 \text{ m/s}^2) t^2$$

The second ball has $v_{0y} = \frac{2}{3}(7.7 \text{ m/s}) = 5.1 \text{ m/s}$. In time t it must travel upward $3.0 \text{ m} - d$ to be at the same place as the first ball.

$$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 \text{ gives } 3.0 \text{ m} - d = (5.1 \text{ m/s}) t - (4.9 \text{ m/s}^2) t^2.$$

We have two equations in two unknowns, d and t . Solving gives $t = 0.59 \text{ s}$ and $d = 1.7 \text{ m}$.

(d) $3.0 \text{ m} - d = 1.3 \text{ m}$

EVALUATE: In 0.59 s the first ball falls $d = (4.9 \text{ m/s}^2)(0.59 \text{ s})^2 = 1.7 \text{ m}$, so is at the same height as the second ball.

- 2.86. IDENTIFY:** The helicopter has two segments of motion with constant acceleration: upward acceleration for 10.0 s and then free-fall until it returns to the ground. Powers has three segments of motion with constant acceleration: upward acceleration for 10.0 s , free-fall for 7.0 s and then downward acceleration of 2.0 m/s^2 .

SET UP: Let $+y$ be upward. Let $y = 0$ at the ground.

EXECUTE: (a) When the engine shuts off both objects have upward velocity

$v_y = v_{0y} + a_y t = (5.0 \text{ m/s}^2)(10.0 \text{ s}) = 50.0 \text{ m/s}$ and are at $y = v_{0y} t + \frac{1}{2} a_y t^2 = \frac{1}{2}(5.0 \text{ m/s}^2)(10.0 \text{ s})^2 = 250 \text{ m}$. For the helicopter, $v_y = 0$ (at the maximum height), $v_{0y} = +50.0 \text{ m/s}$, $y_0 = 250 \text{ m}$, and $a_y = -9.80 \text{ m/s}^2$.

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \text{ gives } y = \frac{v_y^2 - v_{0y}^2}{2a_y} + y_0 = \frac{0 - (50.0 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} + 250 \text{ m} = 378 \text{ m}, \text{ which rounds to } 380 \text{ m.}$$

(b) The time for the helicopter to crash from the height of 250 m where the engines shut off can be found using $v_{0y} = +50.0 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$, and $y - y_0 = -250 \text{ m}$. $y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$ gives

$$-250 \text{ m} = (50.0 \text{ m/s}) t - (4.90 \text{ m/s}^2) t^2. \quad (4.90 \text{ m/s}^2) t^2 - (50.0 \text{ m/s}) t - 250 \text{ m} = 0. \text{ The quadratic formula gives}$$

$$t = \frac{1}{9.80} \left(50.0 \pm \sqrt{(50.0)^2 + 4(4.90)(250)} \right) \text{ s. Only the positive solution is physical, so } t = 13.9 \text{ s. Powers therefore}$$

has free-fall for 7.0 s and then downward acceleration of 2.0 m/s^2 for $13.9 \text{ s} - 7.0 \text{ s} = 6.9 \text{ s}$. After 7.0 s of free-fall he is at $y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 = 250 \text{ m} + (50.0 \text{ m/s})(7.0 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(7.0 \text{ s})^2 = 360 \text{ m}$ and has velocity

$$v_x = v_{0x} + a_x t = 50.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(7.0 \text{ s}) = -18.6 \text{ m/s}. \text{ After the next } 6.9 \text{ s he is at}$$

$$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 = 360 \text{ m} + (-18.6 \text{ m/s})(6.9 \text{ s}) + \frac{1}{2}(-2.00 \text{ m/s}^2)(6.9 \text{ s})^2 = 184 \text{ m}. \text{ Powers is } 184 \text{ m above the ground when the helicopter crashes.}$$

EVALUATE: When Powers steps out of the helicopter he retains the initial velocity he had in the helicopter but his acceleration changes abruptly from 5.0 m/s^2 upward to 9.80 m/s^2 downward. Without the jet pack he would have crashed into the ground at the same time as the helicopter. The jet pack slows his descent so he is above the ground when the helicopter crashes.

- 2.87. IDENTIFY:** Apply the constant acceleration equations to his motion. Consider two segments of the motion: the last 1.0 s and the motion prior to that. The final velocity for the first segment is the initial velocity for the second segment.

SET UP: Let $+y$ be downward, so $a_y = +9.80 \text{ m/s}^2$.

EXECUTE: Motion from the roof to a height of $h/4$ above ground: $y - y_0 = 3h/4$, $a_y = +9.80 \text{ m/s}^2$, $v_{0y} = 0$.

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \text{ gives } v_y = \sqrt{2a_y(y - y_0)} = 3.834\sqrt{h} \sqrt{\text{m}}/\text{s}. \text{ Motion from height of } h/4 \text{ to the ground:}$$

$$y - y_0 = h/4, \quad a_y = +9.80 \text{ m/s}^2, \quad v_{0y} = 3.834\sqrt{h} \sqrt{\text{m}}/\text{s}, \quad t = 1.00 \text{ s}. \quad y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 \text{ gives}$$

$$\frac{h}{4} = 3.834\sqrt{h} \sqrt{m} + 4.90 \text{ m}. \text{ Let } h = u^2 \text{ and solve for } u. \frac{1}{4}u^2 - 3.834u \sqrt{m} - 4.90 \text{ m} = 0.$$

$u = 2\left(3.834 \pm \sqrt{(-3.834)^2 + 4.90}\right) \sqrt{m}$. Only the positive root is physical, so $u = 16.52 \sqrt{m}$ and $h = u^2 = 273 \text{ m}$, which rounds to 270 m. The building is 270 m tall.

EVALUATE: With $h = 273 \text{ m}$ the total time of fall is $t = \sqrt{\frac{2h}{a_y}} = 7.46 \text{ s}$. In $7.47 \text{ s} - 1.00 \text{ s} = 6.46 \text{ s}$ Spider-Man

falls a distance $y - y_0 = \frac{1}{2}(9.80 \text{ m/s}^2)(6.46 \text{ s})^2 = 204 \text{ m}$. This leaves 69 m for the last 1.0 s of fall, which is $h/4$.

2.88. IDENTIFY: Apply constant acceleration equations to the motion of the rock. Sound travels at constant speed.

SET UP: Let t_{fall} be the time for the rock to fall to the ground and let t_s be the time it takes the sound to travel from the impact point back to you. $t_{\text{fall}} + t_s = 10.0 \text{ s}$. Both the rock and sound travel a distance d that is equal to the height of the cliff. Take $+y$ downward for the motion of the rock. The rock has $v_{0y} = 0$ and $a_y = 9.80 \text{ m/s}^2$.

EXECUTE: (a) For the rock, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t_{\text{fall}} = \sqrt{\frac{2d}{9.80 \text{ m/s}^2}}$.

For the sound, $t_s = \frac{d}{330 \text{ m/s}} = 10.0 \text{ s}$. Let $\alpha^2 = d$. $0.00303\alpha^2 + 0.4518\alpha - 10.0 = 0$. $\alpha = 19.6$ and $d = 384 \text{ m}$.

(b) You would have calculated $d = \frac{1}{2}(9.80 \text{ m/s}^2)(10.0 \text{ s})^2 = 490 \text{ m}$. You would have overestimated the height of the cliff. It actually takes the rock less time than 10.0 s to fall to the ground.

EVALUATE: Once we know d we can calculate that $t_{\text{fall}} = 8.8 \text{ s}$ and $t_s = 1.2 \text{ s}$. The time for the sound of impact to travel back to you is 12% of the total time and cannot be neglected. The rock has speed 86 m/s just before it strikes the ground.

2.89. (a) IDENTIFY: Let $+y$ be upward. The can has constant acceleration $a_y = -g$. The initial upward velocity of the can equals the upward velocity of the scaffolding; first find this speed.

SET UP: $y - y_0 = -15.0 \text{ m}$, $t = 3.25 \text{ s}$, $a_y = -9.80 \text{ m/s}^2$, $v_{0y} = ?$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $v_{0y} = 11.31 \text{ m/s}$

Use this v_{0y} in $v_y = v_{0y} + a_y t$ to solve for v_y : $v_y = -20.5 \text{ m/s}$

(b) **IDENTIFY:** Find the maximum height of the can, above the point where it falls from the scaffolding:

SET UP: $v_y = 0$, $v_{0y} = +11.31 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$, $y - y_0 = ?$

EXECUTE: $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = 6.53 \text{ m}$

The can will pass the location of the other painter. Yes, he gets a chance.

EVALUATE: Relative to the ground the can is initially traveling upward, so it moves upward before stopping momentarily and starting to fall back down.

2.90. IDENTIFY: Both objects are in free-fall. Apply the constant acceleration equations to the motion of each person.

SET UP: Let $+y$ be downward, so $a_y = +9.80 \text{ m/s}^2$ for each object.

EXECUTE: (a) Find the time it takes the student to reach the ground: $y - y_0 = 180 \text{ m}$, $v_{0y} = 0$, $a_y = 9.80 \text{ m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(180 \text{ m})}{9.80 \text{ m/s}^2}} = 6.06 \text{ s}$. Superman must reach the ground in

$6.06 \text{ s} - 5.00 \text{ s} = 1.06 \text{ s}$: $t = 1.06 \text{ s}$, $y - y_0 = 180 \text{ m}$, $a_y = +9.80 \text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$v_{0y} = \frac{y - y_0}{t} - \frac{1}{2}a_y t = \frac{180 \text{ m}}{1.06 \text{ s}} - \frac{1}{2}(9.80 \text{ m/s}^2)(1.06 \text{ s}) = 165 \text{ m/s}$. Superman must have initial speed $v_0 = 165 \text{ m/s}$.

(b) The graphs of $y-t$ for Superman and for the student are sketched in Figure 2.90.

(c) The minimum height of the building is the height for which the student reaches the ground in 5.00 s, before Superman jumps. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80 \text{ m/s}^2)(5.00 \text{ s})^2 = 122 \text{ m}$. The skyscraper must be at least 122 m high.

EVALUATE: $165 \text{ m/s} = 369 \text{ mi/h}$, so only Superman could jump downward with this initial speed.

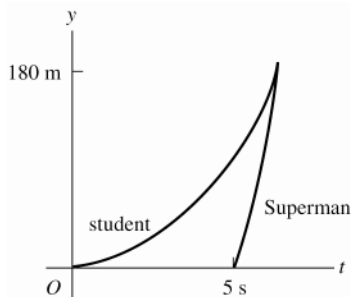


Figure 2.90

- 2.91. IDENTIFY:** Apply constant acceleration equations to the motion of the rocket and to the motion of the canister after it is released. Find the time it takes the canister to reach the ground after it is released and find the height of the rocket after this time has elapsed. The canister travels up to its maximum height and then returns to the ground.
SET UP: Let $+y$ be upward. At the instant that the canister is released, it has the same velocity as the rocket.

After it is released, the canister has $a_y = -9.80 \text{ m/s}^2$. At its maximum height the canister has $v_y = 0$.

EXECUTE: (a) Find the speed of the rocket when the canister is released: $v_{0y} = 0$, $a_y = 3.30 \text{ m/s}^2$,

$y - y_0 = 235 \text{ m}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(3.30 \text{ m/s}^2)(235 \text{ m})} = 39.4 \text{ m/s}$. For the motion of the canister after it is released, $v_{0y} = +39.4 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$, $y - y_0 = -235 \text{ m}$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-235 \text{ m} = (39.4 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$. The quadratic formula gives $t = 12.0 \text{ s}$ as the positive solution. Then for the motion of the rocket during this 12.0 s ,

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 235 \text{ m} + (39.4 \text{ m/s})(12.0 \text{ s}) + \frac{1}{2}(3.30 \text{ m/s}^2)(12.0 \text{ s})^2 = 945 \text{ m}$.

(b) Find the maximum height of the canister above its release point: $v_{0y} = +39.4 \text{ m/s}$, $v_y = 0$, $a_y = -9.80 \text{ m/s}^2$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (39.4 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 79.2 \text{ m}$. After its release the canister travels

upward 79.2 m to its maximum height and then back down $79.2 \text{ m} + 235 \text{ m}$ to the ground. The total distance it travels is 393 m .

EVALUATE: The speed of the rocket at the instant that the canister returns to the launch pad is

$v_y = v_{0y} + a_y t = 39.4 \text{ m/s} + (3.30 \text{ m/s}^2)(12.0 \text{ s}) = 79.0 \text{ m/s}$. We can calculate its height at this instant by

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ with $v_{0y} = 0$ and $v_y = 79.0 \text{ m/s}$. $y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{(79.0 \text{ m/s})^2}{2(3.30 \text{ m/s}^2)} = 946 \text{ m}$, which agrees

with our previous calculation.

- 2.92. IDENTIFY:** Both objects are in free-fall and move with constant acceleration 9.80 m/s^2 , downward. The two balls collide when they are at the same height at the same time.

SET UP: Let $+y$ be upward, so $a_y = -9.80 \text{ m/s}^2$ for each ball. Let $y = 0$ at the ground. Let ball A be the one thrown straight up and ball B be the one dropped from rest at height H . $y_{0A} = 0$, $y_{0B} = H$.

EXECUTE: (a) $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ applied to each ball gives $y_A = v_0 t - \frac{1}{2}gt^2$ and $y_B = H - \frac{1}{2}gt^2$. $y_A = y_B$ gives

$$v_0 t - \frac{1}{2}gt^2 = H - \frac{1}{2}gt^2 \text{ and } t = \frac{H}{v_0}.$$

(b) For ball A at its highest point, $v_{yA} = 0$ and $v_y = v_{0y} + a_y t$ gives $t = \frac{v_0}{g}$. Setting this equal to the time in

part (a) gives $\frac{H}{v_0} = \frac{v_0}{g}$ and $H = \frac{v_0^2}{g}$.

EVALUATE: In part (a), using $t = \frac{H}{v_0}$ in the expressions for y_A and y_B gives $y_A = y_B = H \left(1 - \frac{gH}{2v_0^2}\right)$. H must be

less than $\frac{2v_0^2}{g}$ in order for the balls to collide before ball A returns to the ground. This is because it takes ball A

time $t = \frac{2v_0}{g}$ to return to the ground and ball B falls a distance $\frac{1}{2}gt^2 = \frac{2v_0^2}{g}$ during this time. When $H = \frac{2v_0^2}{g}$ the two balls collide just as ball A reaches the ground and for H greater than this ball A reaches the ground before they collide.

2.93. IDENTIFY and SET UP: Use $v_x = dx/dt$ and $a_x = dv_x/dt$ to calculate $v_x(t)$ and $a_x(t)$ for each car. Use these equations to answer the questions about the motion.

EXECUTE: $x_A = \alpha t + \beta t^2$, $v_{Ax} = \frac{dx_A}{dt} = \alpha + 2\beta t$, $a_{Ax} = \frac{dv_{Ax}}{dt} = 2\beta$

$x_B = \gamma t^2 - \delta t^3$, $v_{Bx} = \frac{dx_B}{dt} = 2\gamma t - 3\delta t^2$, $a_{Bx} = \frac{dv_{Bx}}{dt} = 2\gamma - 6\delta t$

(a) IDENTIFY and SET UP: The car that initially moves ahead is the one that has the larger v_{0x} .

EXECUTE: At $t=0$, $v_{Ax} = \alpha$ and $v_{Bx} = 0$. So initially car A moves ahead.

(b) IDENTIFY and SET UP: Cars at the same point implies $x_A = x_B$.

$\alpha t + \beta t^2 = \gamma t^2 - \delta t^3$

EXECUTE: One solution is $t=0$, which says that they start from the same point. To find the other solutions, divide by t : $\alpha + \beta t = \gamma t - \delta t^2$

$\delta t^2 + (\beta - \gamma)t + \alpha = 0$

$t = \frac{1}{2\delta} \left(-(\beta - \gamma) \pm \sqrt{(\beta - \gamma)^2 - 4\delta\alpha} \right) = \frac{1}{0.40} \left(+1.60 \pm \sqrt{(1.60)^2 - 4(0.20)(2.60)} \right) = 4.00 \text{ s} \pm 1.73 \text{ s}$

So $x_A = x_B$ for $t=0$, $t=2.27 \text{ s}$ and $t=5.73 \text{ s}$.

EVALUATE: Car A has constant, positive a_x . Its v_x is positive and increasing. Car B has $v_{0x} = 0$ and a_x that is initially positive but then becomes negative. Car B initially moves in the $+x$ -direction but then slows down and finally reverses direction. At $t=2.27 \text{ s}$ car B has overtaken car A and then passes it. At $t=5.73 \text{ s}$, car B is moving in the $-x$ -direction as it passes car A again.

(c) IDENTIFY: The distance from A to B is $x_B - x_A$. The rate of change of this distance is $\frac{d(x_B - x_A)}{dt}$. If this

distance is not changing, $\frac{d(x_B - x_A)}{dt} = 0$. But this says $v_{Bx} - v_{Ax} = 0$. (The distance between A and B is neither decreasing nor increasing at the instant when they have the same velocity.)

SET UP: $v_{Ax} = v_{Bx}$ requires $\alpha + 2\beta t = 2\gamma t - 3\delta t^2$

EXECUTE: $3\delta t^2 + 2(\beta - \gamma)t + \alpha = 0$

$t = \frac{1}{6\delta} \left(-2(\beta - \gamma) \pm \sqrt{4(\beta - \gamma)^2 - 12\delta\alpha} \right) = \frac{1}{1.20} \left(3.20 \pm \sqrt{4(-1.60)^2 - 12(0.20)(2.60)} \right)$

$t = 2.667 \text{ s} \pm 1.667 \text{ s}$, so $v_{Ax} = v_{Bx}$ for $t=1.00 \text{ s}$ and $t=4.33 \text{ s}$.

EVALUATE: At $t=1.00 \text{ s}$, $v_{Ax} = v_{Bx} = 5.00 \text{ m/s}$. At $t=4.33 \text{ s}$, $v_{Ax} = v_{Bx} = 13.0 \text{ m/s}$. Now car B is slowing down while A continues to speed up, so their velocities aren't ever equal again.

(d) IDENTIFY and SET UP: $a_{Ax} = a_{Bx}$ requires $2\beta = 2\gamma - 6\delta t$

EXECUTE: $t = \frac{\gamma - \beta}{3\delta} = \frac{2.80 \text{ m/s}^2 - 1.20 \text{ m/s}^2}{3(0.20 \text{ m/s}^3)} = 2.67 \text{ s}$.

EVALUATE: At $t=0$, $a_{Bx} > a_{Ax}$, but a_{Bx} is decreasing while a_{Ax} is constant. They are equal at $t=2.67 \text{ s}$ but for all times after that $a_{Bx} < a_{Ax}$.

2.94. IDENTIFY: The apple has two segments of motion with constant acceleration. For the motion from the tree to the top of the grass the acceleration is g , downward and the apple falls a distance $H - h$. For the motion from the top of the grass to the ground the acceleration is a , upward, the apple travels downward a distance h , and the final speed is zero.

SET UP: Let $+y$ be upward and let $y=0$ at the ground. The apple is initially a height $H + h$ above the ground.

EXECUTE: (a) Motion from $y_0 = H + h$ to $y = H$: $y - y_0 = -H$, $v_{0y} = 0$, $a_y = -g$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = -\sqrt{2gH}$. The speed of the apple is $\sqrt{2gH}$ as it enters the grass.

(b) Motion from $y_0 = h$ to $y = 0$: $y - y_0 = -h$, $v_{0y} = -\sqrt{2gH}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{-2gH}{2(-h)} = \frac{gH}{h}. \text{ The acceleration of the apple while it is in the grass is } gH/h, \text{ upward.}$$

(c) Graphs of $y-t$, v_y-t and a_y-t are sketched in Figure 2.94.

EVALUATE: The acceleration a produced by the grass increases when H increases and decreases when h increases.

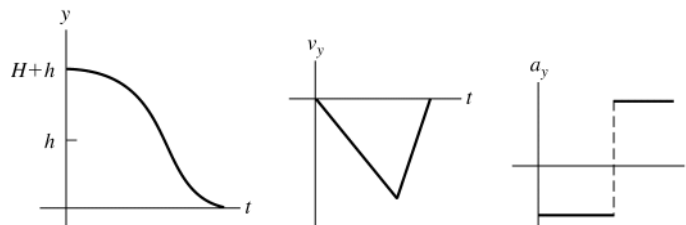


Figure 2.94

2.95. IDENTIFY: Apply constant acceleration equations to the motion of the two objects, the student and the bus.

SET UP: For convenience, let the student's (constant) speed be v_0 and the bus's initial position be x_0 . Note that these quantities are for separate objects, the student and the bus. The initial position of the student is taken to be zero, and the initial velocity of the bus is taken to be zero. The positions of the student x_1 and the bus x_2 as functions of time are then $x_1 = v_0 t$ and $x_2 = x_0 + (1/2)at^2$.

EXECUTE: (a) Setting $x_1 = x_2$ and solving for the times t gives $t = \frac{1}{a} \left(v_0 \pm \sqrt{v_0^2 - 2ax_0} \right)$.

$$t = \frac{1}{(0.170 \text{ m/s}^2)} \left((5.0 \text{ m/s}) \pm \sqrt{(5.0 \text{ m/s})^2 - 2(0.170 \text{ m/s}^2)(40.0 \text{ m})} \right) = 9.55 \text{ s and } 49.3 \text{ s}.$$

The student will be likely to hop on the bus the first time she passes it (see part (d) for a discussion of the later time). During this time, the student has run a distance $v_0 t = (5 \text{ m/s})(9.55 \text{ s}) = 47.8 \text{ m}$.

(b) The speed of the bus is $(0.170 \text{ m/s}^2)(9.55 \text{ s}) = 1.62 \text{ m/s}$.

(c) The results can be verified by noting that the x lines for the student and the bus intersect at two points, as shown in Figure 2.95a.

(d) At the later time, the student has passed the bus, maintaining her constant speed, but the accelerating bus then catches up to her. At this later time the bus's velocity is $(0.170 \text{ m/s}^2)(49.3 \text{ s}) = 8.38 \text{ m/s}$.

(e) No; $v_0^2 < 2ax_0$, and the roots of the quadratic are imaginary. When the student runs at 3.5 m/s , Figure 2.95b shows that the two lines do *not* intersect:

(f) For the student to catch the bus, $v_0^2 > 2ax_0$, and so the minimum speed is

$$\sqrt{2(0.170 \text{ m/s}^2)(40 \text{ m})} = 3.688 \text{ m/s. She would be running for a time } \frac{3.69 \text{ m/s}}{0.170 \text{ m/s}^2} = 21.7 \text{ s, and covers a distance } (3.688 \text{ m/s})(21.7 \text{ s}) = 80.0 \text{ m}.$$

However, when the student runs at 3.688 m/s , the lines intersect at *one* point, at $x = 80 \text{ m}$, as shown in Figure 2.95c.

EVALUATE: The graph in part (c) shows that the student is traveling faster than the bus the first time they meet but at the second time they meet the bus is traveling faster.

$$t_2 = t_{\text{tot}} - t_1$$

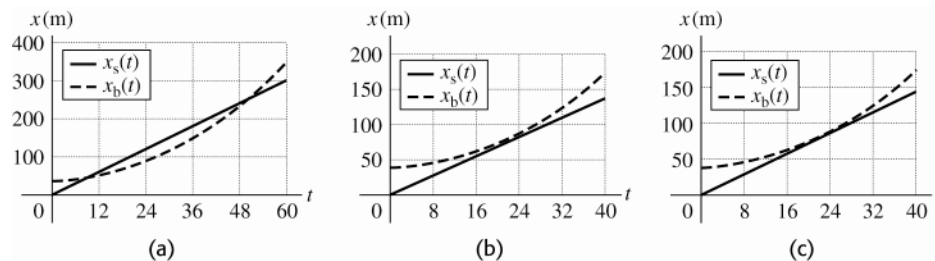


Figure 2.95

2.96. IDENTIFY: Apply $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ to the motion from the maximum height, where $v_{0y} = 0$. The time spent above $y_{\max}/2$ on the way down equals the time spent above $y_{\max}/2$ on the way up.

SET UP: Let $+y$ be downward. $a_y = g$. $y - y_0 = y_{\max}/2$ when he is a distance $y_{\max}/2$ above the floor.

EXECUTE: The time from the maximum height to $y_{\max}/2$ above the floor is given by $y_{\max}/2 = \frac{1}{2}gt_1^2$. The time from the maximum height to the floor is given by $y_{\max} = \frac{1}{2}gt_{\text{tot}}^2$ and the time from a height of $y_{\max}/2$ to the floor is .

$$\frac{t_1}{t_2} = \frac{\sqrt{y_{\max}/2}}{\sqrt{y_{\max} - y_{\max}/2}} = \frac{1}{\sqrt{2}-1} = 2.4 .$$

EVALUATE: The person spends over twice as long above $y_{\max}/2$ as below $y_{\max}/2$. His average speed is less above $y_{\max}/2$ than it is when he is below this height.

2.97. IDENTIFY: Apply constant acceleration equations to both objects.

SET UP: Let $+y$ be upward, so each ball has $a_y = -g$. For the purpose of doing all four parts with the least

repetition of algebra, quantities will be denoted symbolically. That is, let $y_1 = h + v_0t - \frac{1}{2}gt^2$, $y_2 = h - \frac{1}{2}g(t - t_0)^2$.

In this case, $t_0 = 1.00$ s .

EXECUTE: (a) Setting $y_1 = y_2 = 0$, expanding the binomial $(t - t_0)^2$ and eliminating the common term

$$\frac{1}{2}gt^2 \text{ yields } v_0t = gt_0t - \frac{1}{2}gt_0^2 . \text{ Solving for } t: t = \frac{\frac{1}{2}gt_0^2}{gt_0 - v_0} = \frac{t_0}{2} \left(\frac{1}{1 - v_0/(gt_0)} \right) .$$

Substitution of this into the expression for y_1 and setting $y_1 = 0$ and solving for h as a function of v_0 yields, after

some algebra, $h = \frac{1}{2}gt_0^2 \frac{(\frac{1}{2}gt_0 - v_0)^2}{(gt_0 - v_0)^2}$. Using the given value $t_0 = 1.00$ s and $g = 9.80$ m/s²,

$$h = 20.0 \text{ m} = (4.9 \text{ m}) \left(\frac{4.9 \text{ m/s} - v_0}{9.8 \text{ m/s} - v_0} \right)^2 .$$

This has two solutions, one of which is unphysical (the first ball is still going up when the second is released; see part (c)). The physical solution involves taking the negative square root before solving for v_0 , and yields 8.2 m/s.

The graph of y versus t for each ball is given in Figure 2.97.

(b) The above expression gives for (i), 0.411 m and for (ii) 1.15 km.

(c) As v_0 approaches 9.8 m/s, the height h becomes infinite, corresponding to a relative velocity at the time the second ball is thrown that approaches zero. If $v_0 > 9.8$ m/s, the first ball can never catch the second ball.

(d) As v_0 approaches 4.9 m/s, the height approaches zero. This corresponds to the first ball being closer and closer (on its way down) to the top of the roof when the second ball is released. If $v_0 < 4.9$ m/s, the first ball will already have passed the roof on the way down before the second ball is released, and the second ball can never catch up.

EVALUATE: Note that the values of v_0 in parts (a) and (b) are all greater than v_{\min} and less than v_{\max} .

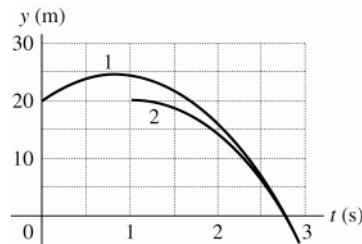


Figure 2.97

2.98. IDENTIFY: Apply constant acceleration equations to the motion of the boulder.

SET UP: Let $+y$ be downward, so $a_y = +g$.

EXECUTE: (a) Let the height be h and denote the 1.30-s interval as Δt ; the simultaneous equations

$h = \frac{1}{2}gt^2$, $\frac{2}{3}h = \frac{1}{2}g(t - \Delta t)^2$ can be solved for t . Eliminating h and taking the square root, $\frac{t}{t - \Delta t} = \sqrt{\frac{3}{2}}$, and

$t = \frac{\Delta t}{1 - \sqrt{2/3}}$, and substitution into $h = \frac{1}{2}gt^2$ gives $h = 246$ m.

(b) The above method assumed that $t > 0$ when the square root was taken. The negative root (with $\Delta t = 0$) gives an answer of 2.51 m, clearly not a “cliff”. This would correspond to an object that was initially near the bottom of this “cliff” being thrown upward and taking 1.30 s to rise to the top and fall to the bottom. Although physically possible, the conditions of the problem preclude this answer.

EVALUATE: For the first two-thirds of the distance, $y - y_0 = 164$ m, $v_{0y} = 0$, and $a_y = 9.80$ m/s².

$v_y = \sqrt{2a_y(y - y_0)} = 56.7$ m/s. Then for the last third of the distance, $y - y_0 = 82.0$ m, $v_{0y} = 56.7$ m/s and

$a_y = 9.80$ m/s². $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $(4.90 \text{ m/s}^2)t^2 + (56.7 \text{ m/s})t - 82.0 \text{ m} = 0$.

$t = \frac{1}{9.8} \left(-56.7 + \sqrt{(56.7)^2 + 4(4.9)(82.0)} \right) \text{ s} = 1.30 \text{ s}$, as required.