Essential University Physics

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8

Gravity

PowerPoint® Lecture prepared by Richard Wolfson

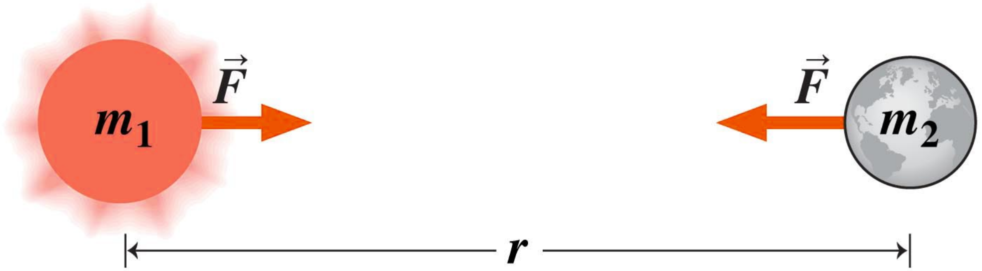
In this lecture you'll learn

- Newton's law of universal gravitation
- About motion in circular and other orbits
- How to calculate changes in gravitational potential energy
- The concept of escape speed
- The field concept

Universal gravitation

- Introduced by Isaac Newton, the **Law of Universal Gravitation** states that any two masses m_1 and m_2 attract with a force F that is proportional to the product of their distances and inversely proportional to the distance r between them:

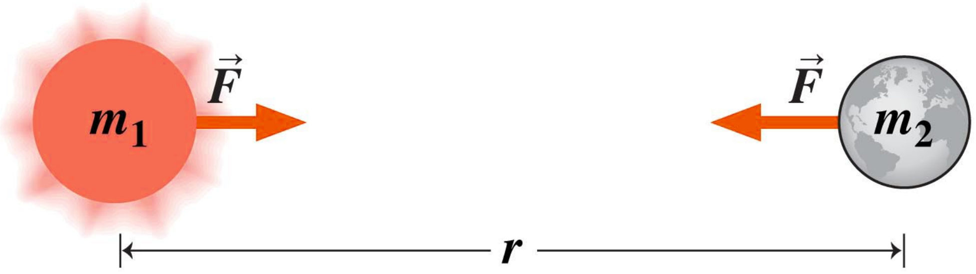
$$F = \frac{Gm_1 m_2}{r^2}$$



- Here $\boxed{G = 6.67 \cdot 10^{-11} \mathrm{N \cdot m^2/kg^2}}$ is the **constant of universal gravitation.**
- Strictly speaking, this law applies only to point masses. But Newton showed that it applies to spherical masses of any size, and it is a good approximation for any objects that are small compared with their separation.

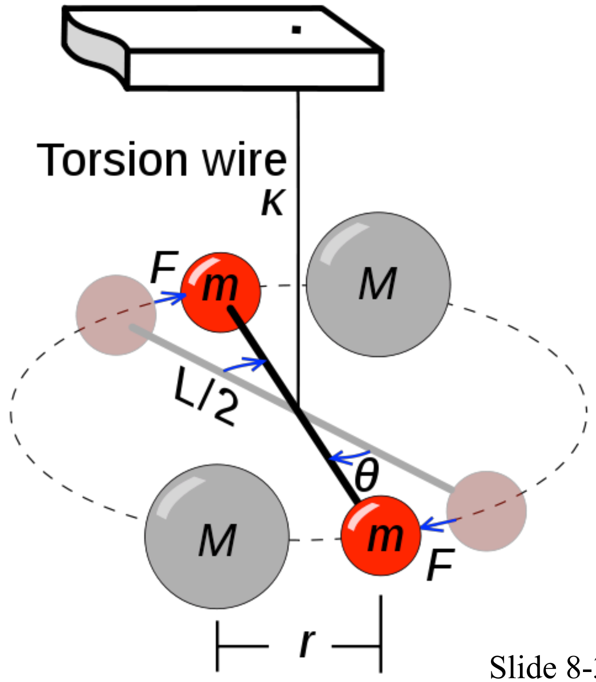
Universal gravitation

$$F = \frac{Gm_1 m_2}{r^2}$$



- Here $\boxed{G = 6.67 \cdot 10^{-11} \mathrm{N} \cdot \mathrm{m}^2/\mathrm{kg}^2}$ is the **constant of universal gravitation.**

Henry Cavendish experiment / 1798



Clicker question

Suppose the distance between two objects is cut in half. The gravitational force between them is …

A. doubled.

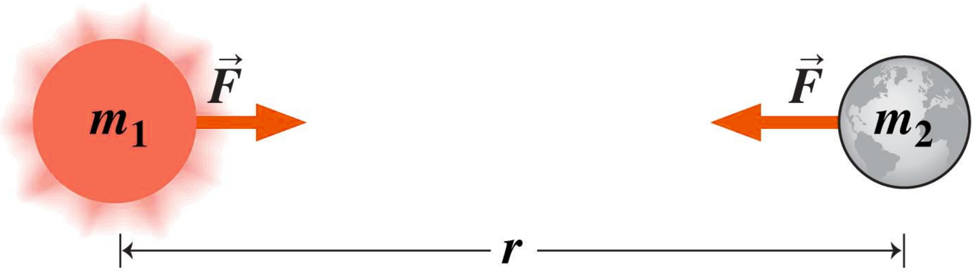
C. halved.

D. quadrupled.

E. quartered.

The acceleration of gravity: On Earth and in Space

$$F = \frac{Gm_1 m_2}{r^2}$$



Find the acceleration of gravity at:

1. Earth surface
2. International Space Station (ISS) (380 km)
3. Mars surface

$$m_1 = M, m_2 = m$$

$$m \cdot a = \frac{G \cdot m \cdot M}{r^2}$$

$$M_E = 5.97 \times 10^{24} \text{kg}$$
$$R_E = 6.37 \times 10^6 \text{ m}$$
$$R_{ISS} = 6.37 \times 10^6 + 380 \times 10^3 \text{ m}$$

$$a_E = 9.81 \text{ m/s}^2$$
$$a_{ISS} = 8.74 \text{ m/s}^2$$
$$a_M = 3.7 \text{ m/s}^2$$

Orbits

- Newton explained orbits using universal gravitation and his laws of motion:
  - Bound orbits are generally elliptical.
  - In the special case of a circular orbit, the orbiting object “falls” around a gravitating mass, always accelerating toward its center with the magnitude of its acceleration remaining constant.
  - Unbound orbits are hyperbolic or (borderline case) parabolic.

Clicker question

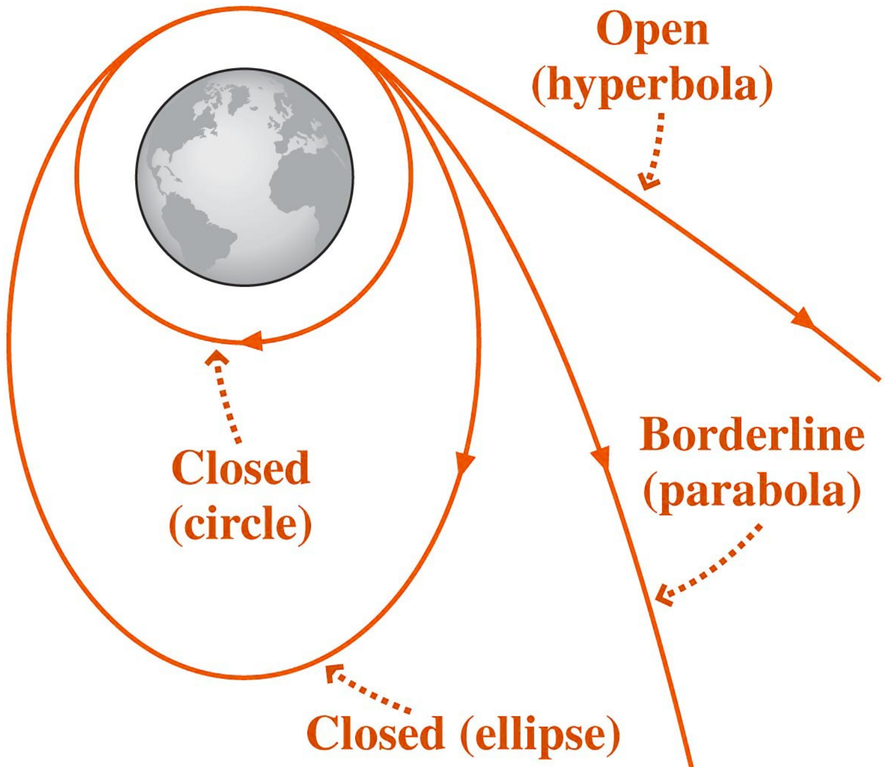
Suppose the paths in the figure are the paths of four projectiles. All four projectiles were launched from a common point at the top of the figure. Which projectile had the second-highest initial speed?

A. The projectile with the closed path

B. The projectile with the hyperbolic path

C. The projectile with the parabolic path

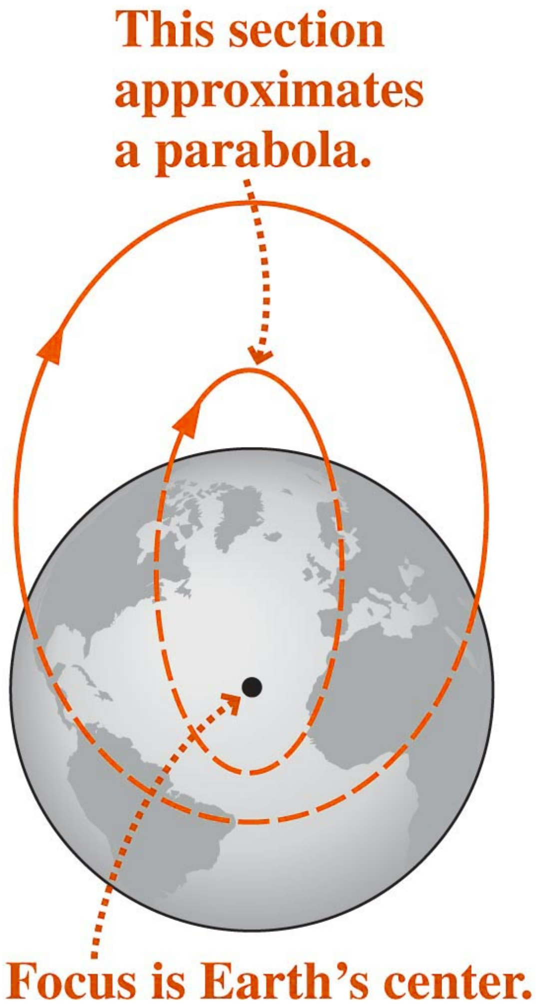
D. The projectile with the elliptical path.



Projectile motion and orbits

- The "parabolic" trajectories of projectiles near Earth's surface are actually sections of elliptical orbits that intersect Earth.

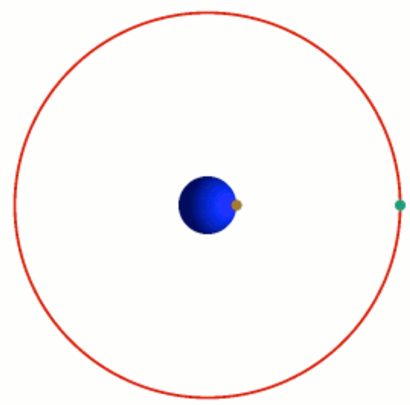
- The trajectories are parabolic only in the approximation that we can neglect Earth's curvature and the variation in gravity with distance from Earth's center.



Circular orbits

- In a circular orbit, gravity provides the force of magnitude mv^2/r needed to keep an object of mass m in its circular path about a much more massive object of mass M.
- Therefore

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

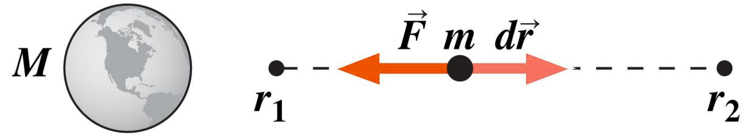


- This gives orbital speed $v = \sqrt{GM/r}$

  and orbital period $T^2 = \dfrac{4\pi^2 r^3}{GM}$.
  - For satellites in low-Earth orbit, the period is about 90 minutes.

Gravitational potential energy

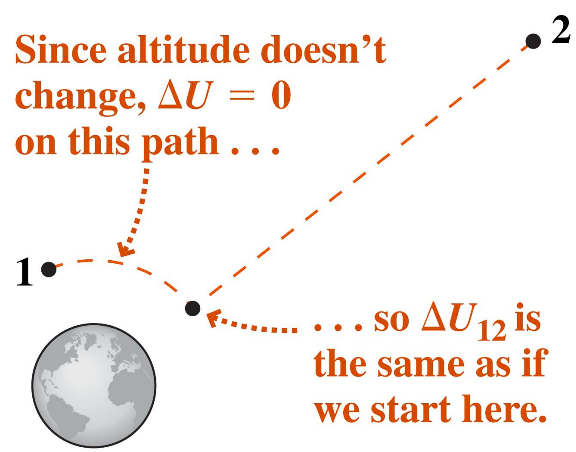
- Because the gravitational force changes with distance, it's necessary to integrate to calculate potential energy changes over large distances. This integration gives



$$\Delta U_{12} = \int_{r_1}^{r_2} \frac{GMm}{r^2}\,dr = GMm\int_{r_1}^{r_2} r^{-2}\,dr = GMm\frac{r^{-1}}{-1}\Bigg|_{r_1}^{r_2} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

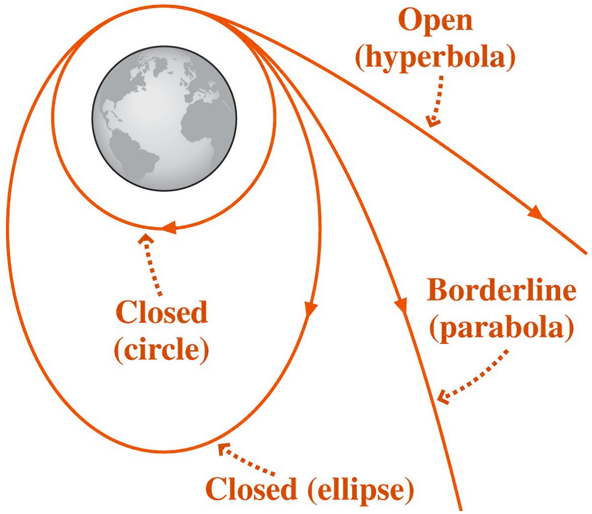
- This result holds regardless of whether the two points are on the same radial line.
- It's convenient to take the zero of gravitational potential energy at infinity. Then the gravitational potential energy becomes

$$U(r) = -\frac{GMm}{r}$$

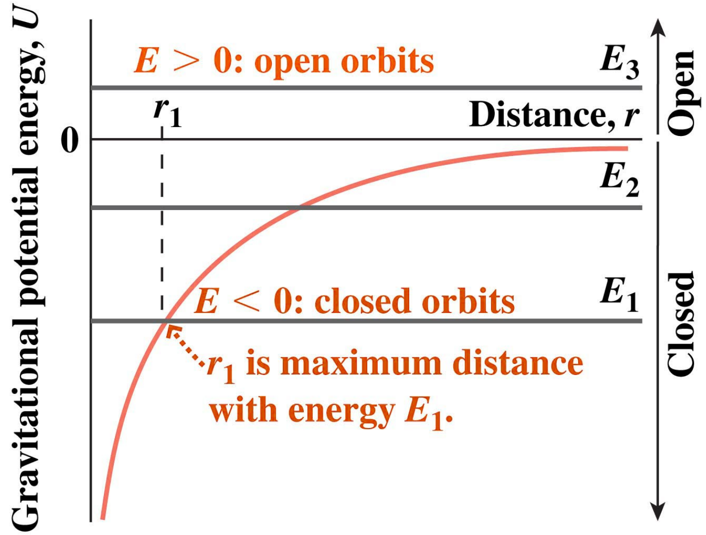


Energy and orbits

- The total energy E—the sum of kinetic energy K and potential energy U—determines the type of orbit an object follows:



  - For $E < 0$, the object is in a bound, elliptical orbit.
    - Special cases include circular orbits and the straight-line paths of falling objects.
  - For $E > 0$ the orbit is unbound and hyperbolic.
  - The borderline case $E = 0$ gives a parabolic orbit.



Escape speed

- An object with total energy E less than zero is in a bound orbit and can't escape from the gravitating center.
- With energy E greater than zero, the object is in an unbound orbit and can escape to infinitely far from the gravitating center.
- The minimum speed required to escape is given by

$$0 = K + U = \tfrac{1}{2}mv^2 - \frac{GMm}{r}$$

- Solving for v gives the escape speed:

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

  - Escape speed from Earth's surface is about 11 km/s.

Energy in circular orbits

- In the special case of a circular orbit, kinetic energy and potential energy are precisely related:

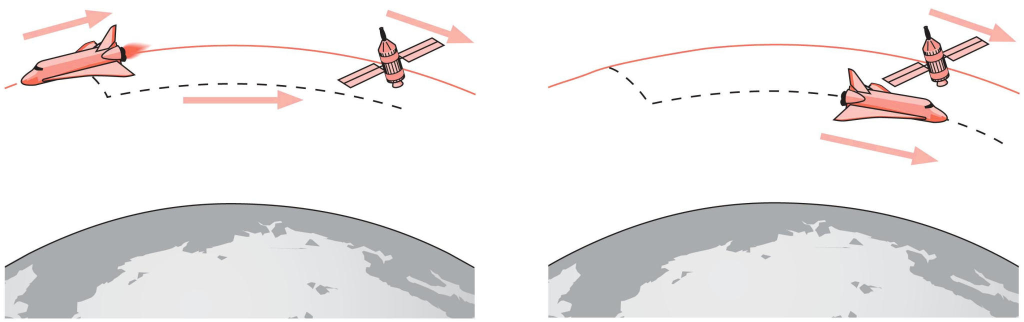
  $$U = -2K$$

  $$K = \frac{1}{2}mv^2 \quad \left(v = \sqrt{\frac{G \cdot M}{r}} \right) \longrightarrow K = \frac{1}{2}\frac{G \cdot M \cdot m}{r}$$

- Thus in a circular orbit the total energy is

  $$E = K + U = -K = \tfrac{1}{2}U = -\frac{GMm}{2r}$$

  - This negative energy shows that the orbit is bound.
  - The lower the orbit, the lower the total energy—but the faster the orbital speed.
    - This means an orbiting spacecraft needs to lose energy to gain speed.

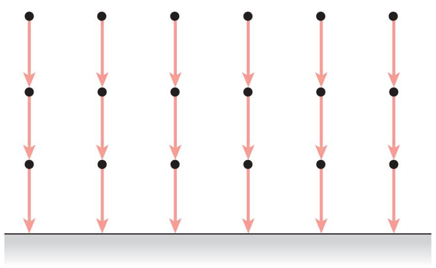


(a) (b)

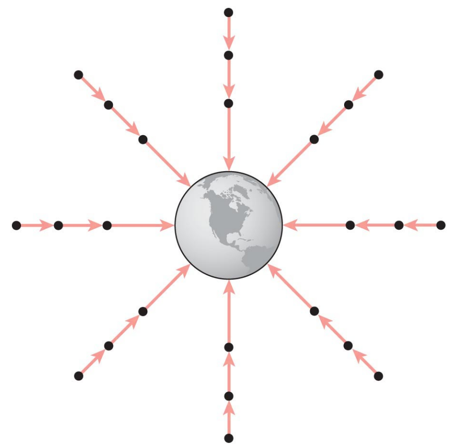
The gravitational field

- It's convenient to describe gravitation not in terms of "action at a distance" but rather in terms of a **gravitational field** that results from the presence of mass and that exists at all points in space.
    - A massive object creates a gravitational field in its vicinity, and other objects respond to the field *at their immediate locations*.
    - The gravitational field can be visualized with a set of vectors giving its strength (in N/kg; equivalently, m/s^2) and its direction.

Near Earth's surface:



On a larger scale:



Summary

- The **Law of Universal Gravitation** states that any two masses m_1 and m_2 attract with a force F that is proportional to the product of their distances and inversely proportional to the distance r between them:

$$F = \frac{Gm_1 m_2}{r^2}$$

- Motion under gravity includes
  - Bound elliptical and circular orbits when the orbiting object's total energy is less than zero
  - Open parabolic and hyperbolic orbits when the total energy is zero or greater
- The **gravitational field** describes the force of gravity in terms of a field at all points in space; an object then responds to the field in its immediate vicinity.