Essential University Physics

Richard Wolfson

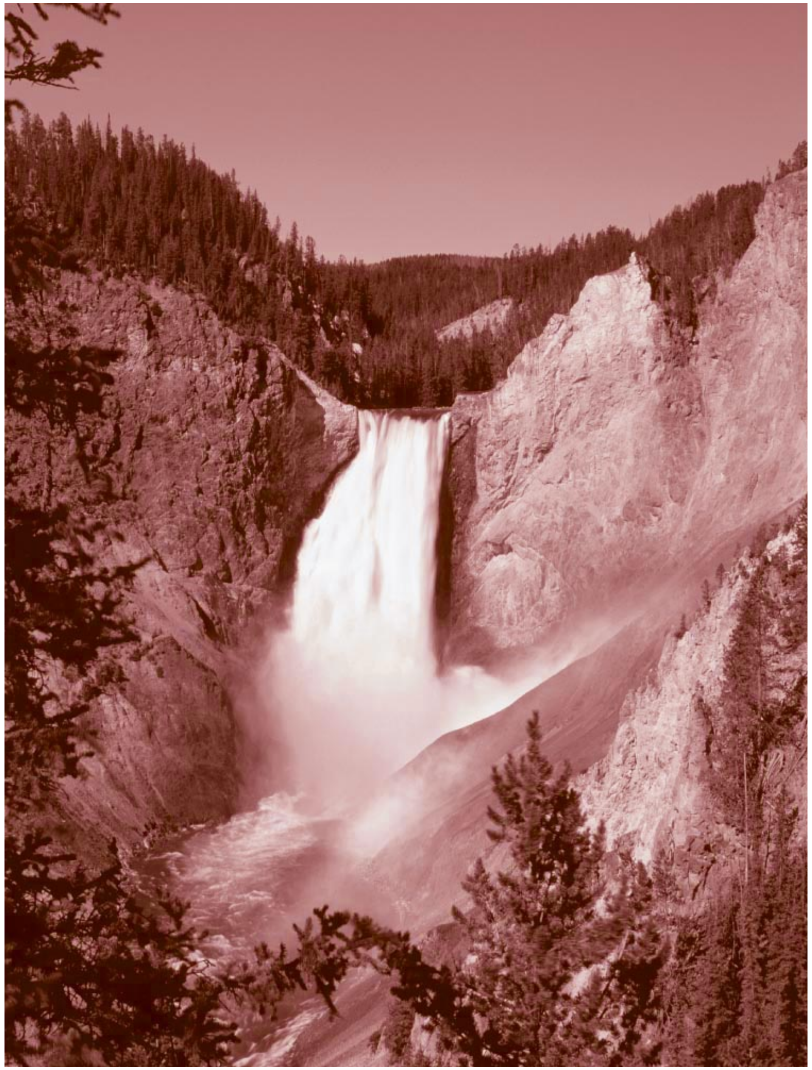
7

Conservation of Energy

PowerPoint® Lecture prepared by Richard Wolfson

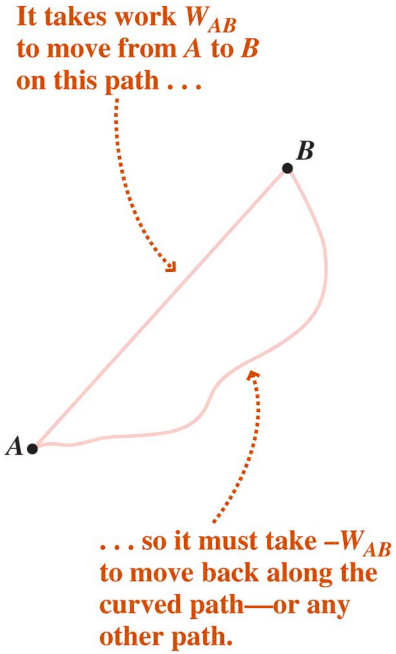
In this lecture you'll learn

- The difference between conservative and nonconservative forces
- The concept of potential energy
    - How to calculate potential energy
- Conservation of mechanical energy
    - A shortcut for solving mechanics problems
- Potential energy curves



Conservative and nonconservative forces

- A **conservative force** stores any work done against it, and can "give back" the stored work as kinetic energy.
- For a conservative force, the work done in moving between two points is independent of the path:



- Because the work done by a conservative force is path independent, the work done in going around any *closed path* is zero:

$$\oint \vec{F} \cdot d\vec{r} = 0$$

- A **nonconservative force** does not store work done against it, the work done may depend on path, and the work done going around a closed path need not be zero.

Conservative and nonconservative forces

- Examples of conservative forces include
  - Gravity
  - The static electric force
  - The force of an ideal spring
- Nonconservative forces include
  - Friction
  - The electric force in the presence of changing magnetism

Clicker question

Suppose it takes the same amount of work to push a trunk across a rough floor as it does to lift a weight the same distance straight upward. How do the amounts of work compare if the trunk and the weight are moved instead on curved paths between the same starting and ending points?

A. The two amounts of work will remain equal to each other.

B. The amount of work to move the trunk will be greater.

C. The amount of work to move the weight will be greater.

Potential energy

- The "stored work" associated with a conservative force is called **potential energy.**
  - Potential energy is stored energy that can be released as kinetic energy.
- The change in potential energy is defined as the negative of the work done by a conservative force acting over any path between two points:

$$\Delta U_{AB} = -\int_A^B \vec{F} \cdot d\vec{r}$$

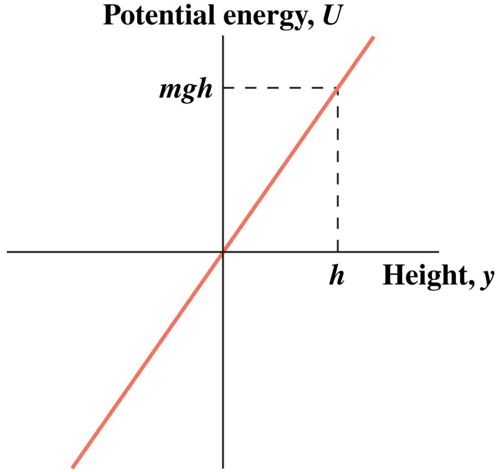
  - Potential energy change is independent of path.
  - Only *changes* in potential energy matter.
  - We're free to set the zero of potential energy at any convenient point.

Two common forms of potential energy

- **Gravitational potential energy** stores the work done against gravity:

$$\Delta U = mg \ \Delta y$$

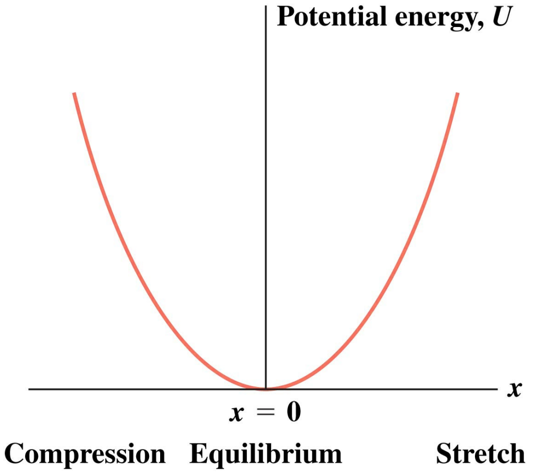
  - Gravitational potential energy increases linearly with height y.
  - This reflects the *constant* gravitational force near Earth's surface.



- **Elastic potential energy** stores the work done in stretching or compressing springs or spring-like systems:

$$U = \tfrac{1}{2}kx^2$$

  - Elastic potential energy increases *quadratically* with stretch or compression x.
  - This reflects the *linearly increasing* spring force.
  - Here the zero of potential energy is taken in the spring's equilibrium configuration.



Conservation of mechanical energy

- By the work-energy theorem, the change in an object's kinetic energy equals the net work done *on* the object: $\Delta K = W_{\text{net}}$
- When only conservative forces act, the net work is the negative of the potential-energy change: $W_{\text{net}} = -\Delta U$
- Therefore when only conservative forces act, any change in potential energy is compensated by an opposite change in kinetic energy:

$$\Delta K + \Delta U = 0$$

- Equivalently,

$$K + U = \text{constant} = K_0 + U_0$$

- Both these equations are statements of the law of **conservation of mechanical energy.**

Problem-solving with conservation of energy

- **Interpret** the problem to make sure all forces are conservative, so conservation of mechanical energy applies. Identify the quantity you're being asked to find, which may be an energy or some related quantity
- **Develop** your solution plan by drawing the object in a situation where you can determine both its kinetic and potential energy, then again in the situation where one quantity is unknown. Also draw bar graphs showing relative sizes of the various energies.
  - Set up the equation $K + U = K_0 + U_0$
- **Evaluate** to solve for the unknown quantity, which might be an energy, a spring stretch, a velocity, etc.
- **Assess** your solution to see that your answer makes sense, has the right physical units, and is consistent with your bar graphs.

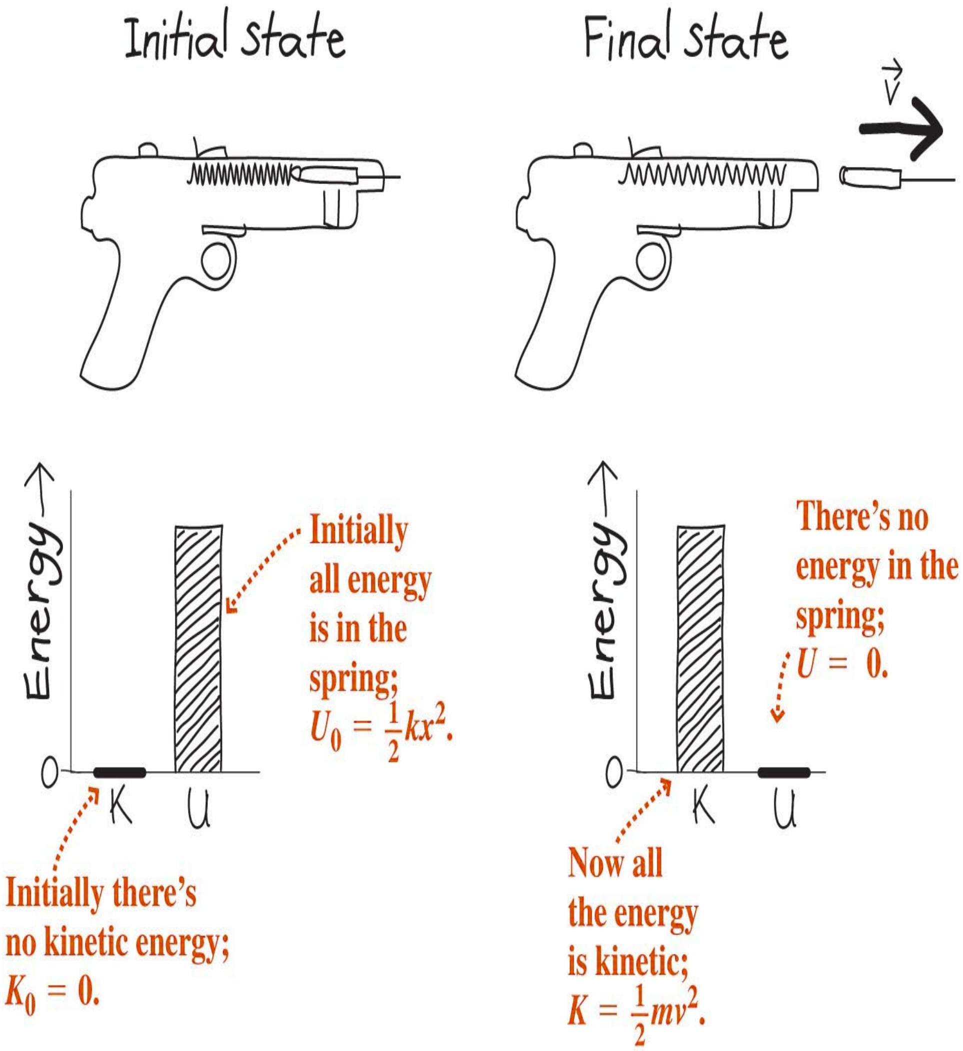
Examples

- A spring-loaded dart gun
  - What's the dart's speed?

- $K + U = K_0 + U_0$ becomes

  $\tfrac{1}{2}mv^2 + 0 = 0 + \tfrac{1}{2}kx^2$

- So $v = \sqrt{k/m}\,x$
  where x is the initial spring compression.



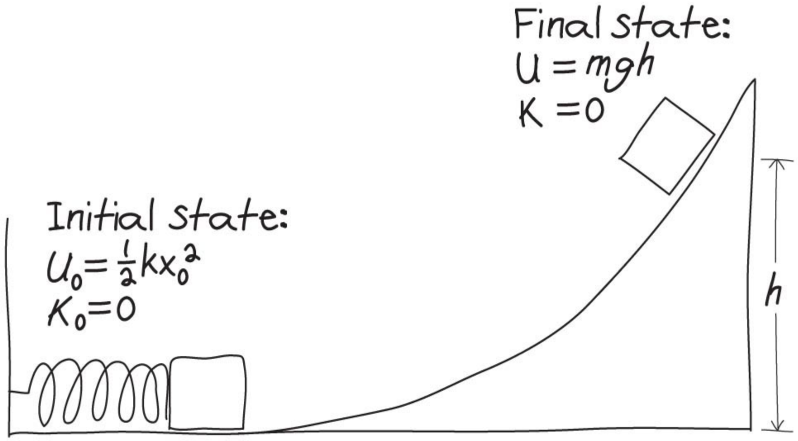
Initially all energy is in the spring; $U_0 = \frac{1}{2}kx^2$.

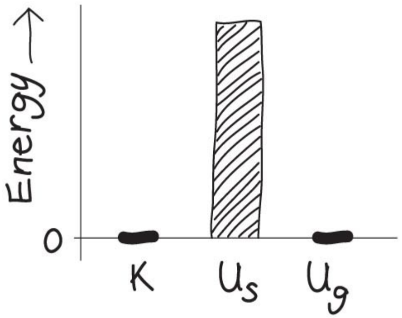
Initially there's no kinetic energy; $K_0 = 0$.

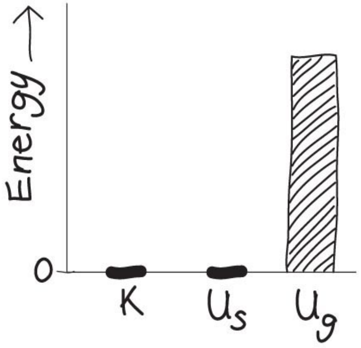
There's no energy in the spring; $U = 0$.

Now all the energy is kinetic; $K = \frac{1}{2}mv^2$.

Examples







- A spring and gravity
  - How high does the block go?
- $K + U = K_0 + U_0$ becomes

$$0 + mgh = 0 + \tfrac{1}{2}kx^2$$

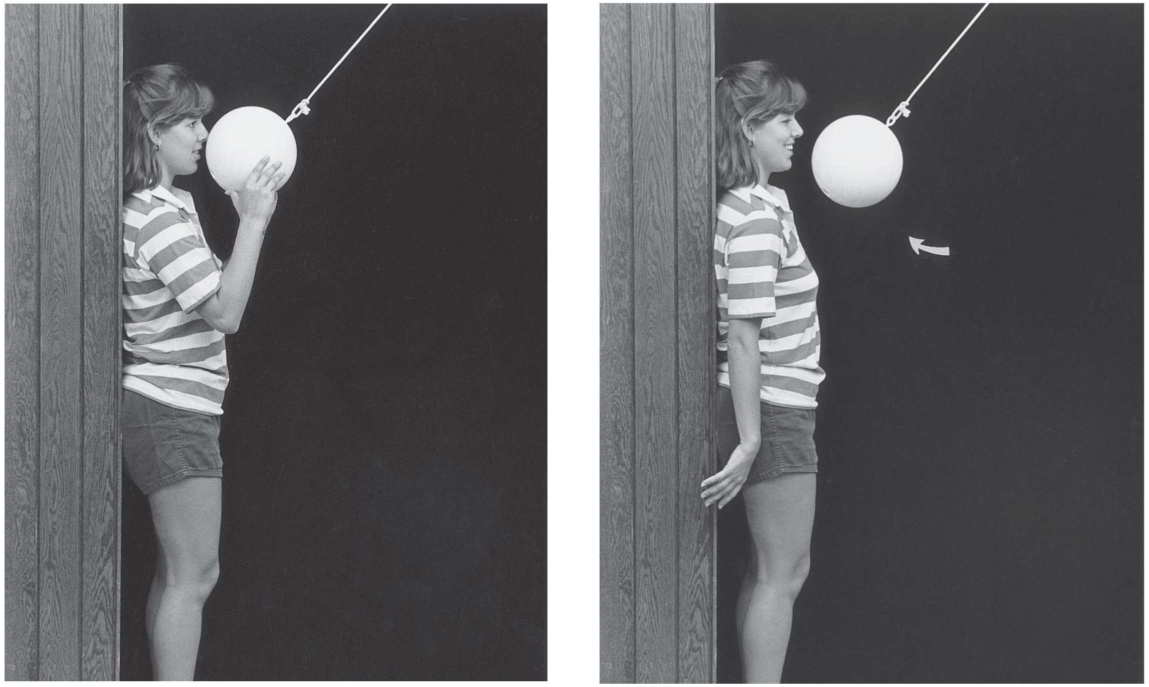
- So $h = \dfrac{kx^2}{2mg}$

Clicker question

A bowling ball is tied to the end of a long rope and suspended from the ceiling. A student stands at one side of the room, holds the ball to her nose, and then releases it from rest. Should she duck as it swings back?

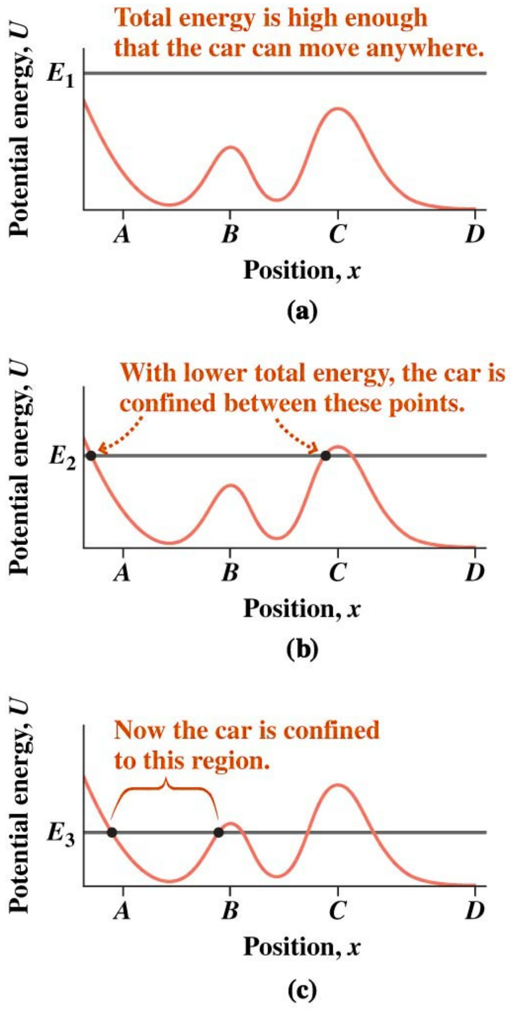
A. She should duck!

B. She does not need to duck.



Potential-energy curves

- Potential-energy curves depict a system's potential energy as a function of position or other quantities representing the system's configuration.
- An object with a given total energy can be "trapped" in a "potential well" established by points where its total energy equals its potential energy.
- These points are **turning points**, beyond which the object cannot move given its fixed total energy.
- Potential-energy curves for a roller-coaster car with three different total energies:


Total energy is high enough that the car can move anywhere.

(a)

With lower total energy, the car is confined between these points.

(b)

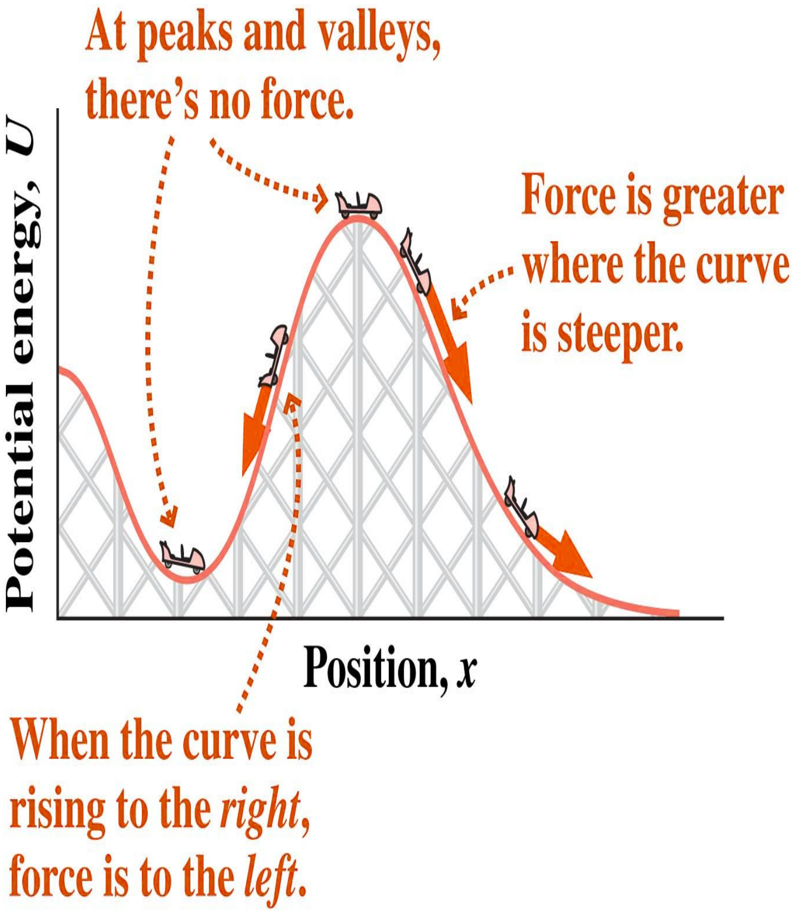
Now the car is confined to this region.

(c)

Force and potential energy

- Force is greatest where potential energy increases most rapidly.
- Mathematically, the component of force in a given direction is the negative derivative of the potential energy with respect to position in that direction:

$$F_x = -\frac{dU}{dx}$$



At peaks and valleys, there's no force.

Force is greater where the curve is steeper.

When the curve is rising to the *right*, force is to the *left*.

Clicker question

This figure shows the potential energy for an electron in a microelectronic device. From among the labeled points, find the point **where the force on the electron is greatest**.

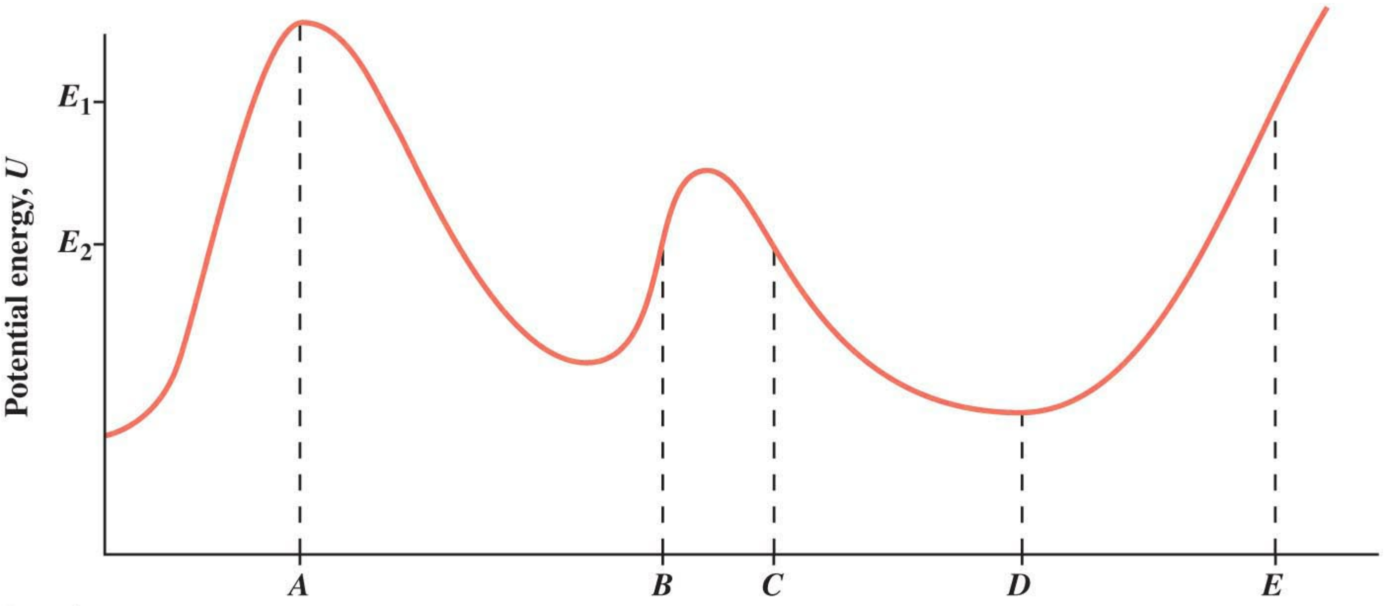
A. Point *A*

C. Point *B*

D. Point C

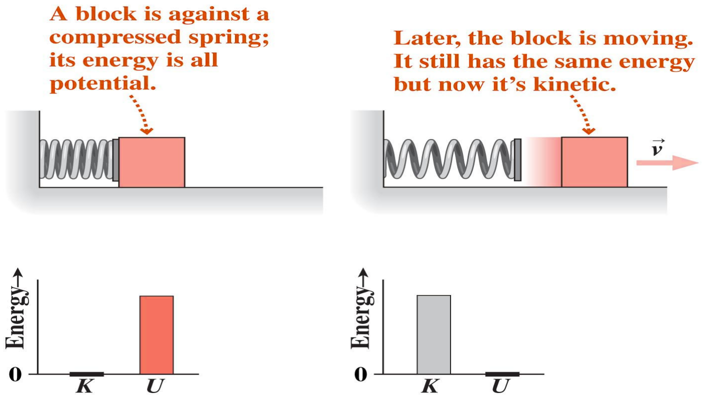
E. Point *D*

F. Point *E*



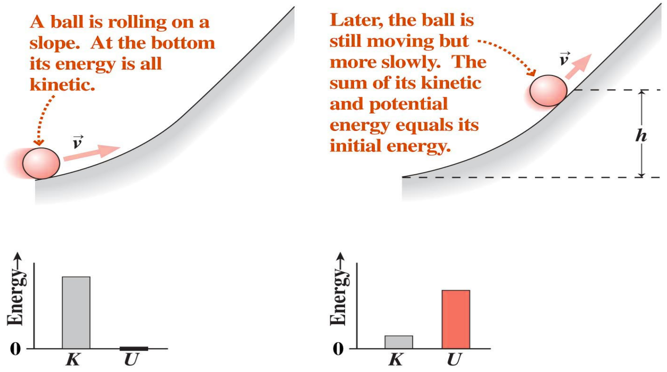
Summary

- **Potential energy** is stored energy that can be converted to kinetic energy.
- The change in potential energy is the negative of the work done by a conservative force as an object is moved on any path between two points: $\Delta U = -\int_A^B \vec{F} \cdot d\vec{r}$
- When only conservative forces act, the total mechanical energy $K + U$ is conserved:



A block is against a compressed spring; its energy is all potential.

Later, the block is moving. It still has the same energy but now it's kinetic.



A ball is rolling on a slope. At the bottom its energy is all kinetic.

Later, the ball is still moving but more slowly. The sum of its kinetic and potential energy equals its initial energy.

- **Potential-energy curves** describe potential energy as a function of position or configuration.
- **Force** is the negative of derivative of potential energy: $F_x = -dU/dx$.

Clicker question

This figure shows the potential energy for an electron in a microelectronic device. From among the labeled points, find the rightmost point where the force on the electron points to the left.

A. Point *A*

C. Point *B*

D. Point C

E. Point *D*

F. Point *E*