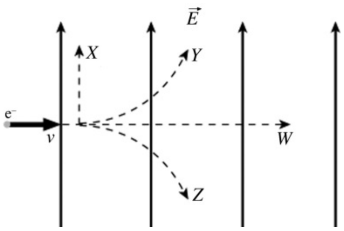
Electricity and magnetism, 29-6-2015

The exam consists of two parts. The first part is multiple-choice. You only have to give the answer. Every correct answer is worth 0.25 points, up to a total of 2 points. The second part consists of open questions. For these you have to motivate your answers. Answers without motivation are considered as wrong. The total number of points for the open questions is 8.

<u>Multiple choice, 8 questions, 0.25 point per question</u>

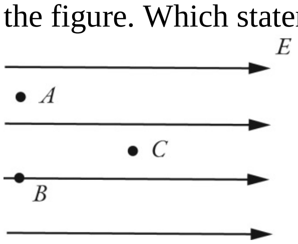
1) An electron is initially moving to the right when it enters a uniform electric field directed upwards. Which trajectory shown below will the electron follow?



A) trajectory *W*
B) trajectory *X*
C) trajectory *Y*
D) trajectory *Z*
Answer D

2) Consider a spherical Gaussian surface of radius *R* centered at the origin. A charge *Q* is placed inside the sphere. To maximize the magnitude of the flux of the electric field through the Gaussian surface, the charge should be located
A) at $x = 0$, $y = 0$, $z = R/2$.
B) at the origin.
C) at $x = R/2$, $y = 0$, $z = 0$.
D) at $x = 0$, $y = R/2$, $z = 0$.
E) The charge can be located anywhere, since flux does not depend on the position of the charge as long as it is inside the sphere.
Answer E

3) Suppose a region of space has a uniform electric field, directed towards the right, as shown in the figure. Which statement about the electric potential is true?



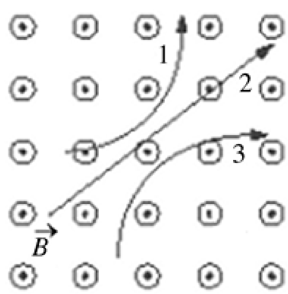
A) The potential at all three locations (*A*, *B*, *C*) is the same because the field is uniform.
B) The potential at points *A* and *B* are equal, and the potential at point *C* is higher than the

potential at point *A*.
C) The potential at points *A* and *B* are equal, and the potential at point *C* is lower than the potential at point *A*.
D) The potential at point *A* is the highest, the potential at point *B* is the second highest, and the potential at point *C* is the lowest.
Answer C

4) The electric field between square the plates of a parallel-plate capacitor has magnitude *E*. The potential across the plates is maintained with constant voltage by a battery as they are pulled apart to twice their original separation, which is small compared to the dimensions of the plates. The magnitude of the electric field between the plates is now equal to
A) 4*E*.
B) 2*E*.
C) *E*.
D) *E*/2.
E) *E*/4.
Answer D

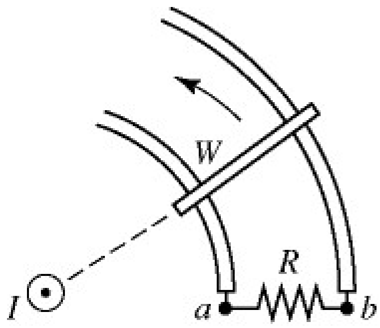
5) A wire of resistivity ρ must be replaced in a circuit by a wire of the same material but 4 times as long. If, however, the resistance of the new wire is to be the same as the resistance of the original wire, the diameter of the new wire must be
A) the same as the diameter of the original wire.
B) 1/2 the diameter of the original wire.
C) 1/4 the diameter of the original wire.
D) 2 times the diameter of the original wire.
E) 4 times the diameter of the original wire.
Answer D)

6) Three particles travel through a region of space where the magnetic field is out of the page, as shown in the figure. The electric charge of each of the three particles is, respectively,



A) 1 is neutral, 2 is negative, and 3 is positive.
B) 1 is neutral, 2 is positive, and 3 is negative.
C) 1 is positive, 2 is neutral, and 3 is negative.
D) 1 is positive, 2 is negative, and 3 is neutral.
E) 1 is negative, 2 is neutral, and 3 is positive.
Answer E

7) In the figure, a straight wire carries a steady current *I* perpendicular to the plane of the page. A bar is in contact with a pair of circular rails, and rotates about the straight wire. The direction of the induced current through the resistor *R* is

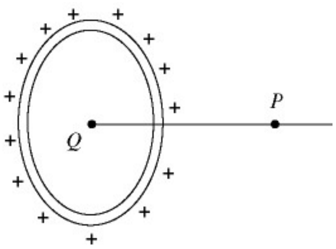


A) from *a* to *b*.
B) from *b* to *a*.
C) There is no induced current through the resistor.
Answer C

8) If the magnetic field of an electromagnetic wave is in the +*x*-direction and the electric field of the wave is in the +*y*-direction, the wave is traveling in the
A) *xy*-plane.
B) +*z*-direction.
C) -*z*-direction.
D) -*x*-direction.
E) -*y*-direction.
Answer C

Open questions, 6 assignments

9) In the figure, a ring with radius R carries a charge of Q_1 uniformly distributed over it.



A) Calculate the electric field in point P on the axis at a distance x from the center.
(1 point)

See example 20.6 in the book. Argumentation:
- a small segment *dl* of the circle carrying charge *dq* creates a field in point P
- symmetry leads to cancellation of the vertical components of the fields creates by all segments along the circle; only the horizontal components need to be integrated

$$dE_P = k\frac{dq}{(x^2+R^2)}$$

$$dE_{P,x} = k\frac{dq}{(x^2+R^2)}\frac{x}{\sqrt{(x^2+R^2)}}$$

$$E_P = kQ_1\frac{x}{(x^2+R^2)^{3/2}}$$

A point charge Q is placed at the center of the ring, such that the electric field in P is zero.

B) Calculate the value of Q in terms of R, x and Q_1.
(1 point)

$$E_P^Q = kQ\frac{1}{x^2} = -E_{P,x} \rightarrow \quad kQ\frac{1}{x^2} = -kQ_1\frac{x}{(x^2+R^2)^{3/2}} \rightarrow Q = -Q_1\frac{x^3}{(x^2+R^2)^{3/2}}$$

10) A solid non-conducting sphere of radius R carries a uniform charge density throughout its volume. At which radial distance outside the sphere is the electric field strength equal to the field strength at a radial distance $r_1 = R/4$ from the center inside the sphere?
(1 point)

See example 21.1. Argumentation:
- apply Gauss law inside and outside the sphere

- Gaussian surface: a sphere
- The charge is uniformly distributed. Therefore, inside the sphere the encapsulated charge is given by the ratio of the volumes of the inner and outer sphere.
- Equate the fields inside and outside the sphere to calculate the distance

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

$$E_{inside} 4\pi r^2 = \frac{Q}{\varepsilon_0}\frac{r^3}{R^3} \to E_{inside} = \frac{Q}{4\pi\varepsilon_0}\frac{r}{R^3} = kQ\frac{r}{R^3}$$

$$E_{outside} 4\pi r^2 = \frac{Q}{\varepsilon_0} \to E_{inside} = \frac{Q}{4\pi\varepsilon_0 r^2} = kQ\frac{1}{r^2}$$

$$E_{outside} = E_{inside}(r1) \to \frac{1}{r^2} = \frac{r1}{R^3} \to r^2 = \frac{R^3}{r1} = 4\frac{R^3}{R} = 4R^2 \to r = 2R$$

11) Two parallel conducting flat plates are separated by a distance *d* and carry equal but opposite surface charge densities. If the potential difference between them is *V* what is the magnitude of the surface charge density on each plate?
(1 point)

See figure 21.27 and discussion. Argumentation:
- At a single conducting plate the charge is equally distributed on both sides of the plate. The field is given by the ratio of the surface charge density and permittivity.
- For two parallel conducting plates with opposite charges all charge will move to the inner sides of the plates. The field of each is given by half of the ratio of the surface charge density and permittivity.
-The total field between the plates is the sum of the fields of the separate plates.

$$E = \frac{V}{d} = \frac{\sigma}{\varepsilon_0} \to \sigma = \frac{\varepsilon_0 V}{d}$$

12) A cylindrical capacitor is made of two thin-walled concentric cylinders. The inner cylinder has radius *r1*, and the outer one a radius *r2*. The common length of the cylinders is *L*, much longer than the outer radius. What is the potential energy stored in this capacitor when a potential difference V is applied between the inner and outer cylinder?
(1 point)

See example 22.4 and problem 23.40. There are three strategies:
- Calculate the work needed to move charge from the inner to the outer cylinder. The work is the negative of the potential energy.
- Calculate the capacity first and from there calculate then energy.
- Calculate the energy density and integrate over the volume.
- The energy can be expressed in terms of charge or potential

$$U = \tfrac{1}{2}CV^2 \quad C = \frac{Q}{|V|} \quad V = -\int \vec{E}\cdot d\vec{r} \quad E = \frac{2k\lambda}{r} \quad Q = \lambda L$$

$$V = -\int \frac{2k\lambda}{r}\vec{r}\cdot\vec{r}dr = -2k\lambda\int_{r_1}^{r_2}\frac{1}{r}dr = -2k\lambda\ln\left(\frac{r_2}{r_1}\right) = -2k\frac{Q}{L}\ln\left(\frac{r_2}{r_1}\right)$$

$$approach\,1: U = -W = -\int_0^Q QVdQ = \frac{2k}{L}\ln\left(\frac{r_2}{r_1}\right)\int_0^Q QdQ = \frac{kQ^2}{L}\ln\left(\frac{r_2}{r_1}\right)$$

$$approach\,2: C = \frac{Q}{|V|} = \frac{1}{2k\frac{1}{L}\ln\left(\frac{r_2}{r_1}\right)} = \frac{L}{2k\ln\left(\frac{r_2}{r_1}\right)}$$

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\frac{L}{2k\ln\left(\frac{r_2}{r_1}\right)}\left(2k\frac{Q}{L}\ln\left(\frac{r_2}{r_1}\right)\right)^2 = \frac{kQ^2}{L}\ln\left(\frac{r_2}{r_1}\right)$$

$$approach\,3: \quad u = \tfrac{1}{2}\varepsilon_0E^2 \quad U = \int_V udV = \tfrac{1}{2}\varepsilon_0\int_{r_1}^{r_2}\left(\frac{2k\lambda}{r}\right)^2 2\pi rLdr = 2\pi\varepsilon_0k^2\lambda^2L\int_{r_1}^{r_2}\left(\frac{1}{r}\right)dr$$

$$U = 4\pi\varepsilon_0k^2\lambda^2L\ln\left(\frac{r_2}{r_1}\right) = \frac{kQ^2}{L}\ln\left(\frac{r_2}{r_1}\right)$$

$Finally:$

$$|V| = 2k\frac{Q}{L}\ln\left(\frac{r_2}{r_1}\right) \rightarrow Q = \frac{|V|L}{2k\ln\left(\frac{r_2}{r_1}\right)}$$

$$U = \frac{|V|^2L}{4k\ln\left(\frac{r_2}{r_1}\right)}$$

13) A conducting bar moves along frictionless conducting rails connected to a 4.00-Ω resistor as shown in the figure. The length of the bar is 1.60 m and a uniform magnetic field of 2 T is applied perpendicular to the paper pointing outward, as shown.

(a) What is the applied external force required to move the bar to the right with a constant speed of 1.00 m/s?

(1 point)

$$\vec{F} = I\vec{l} \times \vec{B} \quad I = \frac{|\varepsilon|}{R} \quad \varepsilon = -\frac{\partial \Phi_B}{\partial t}$$

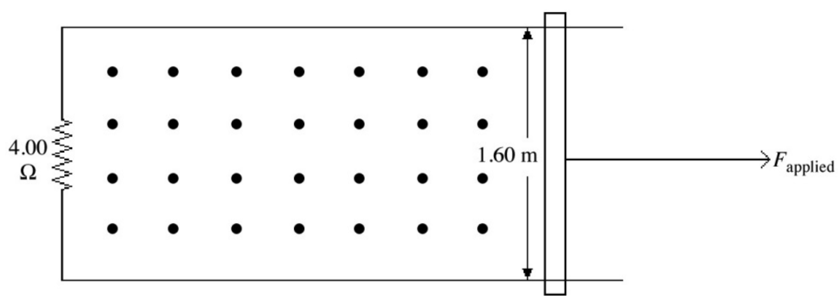
$$\varepsilon = -\frac{\partial \Phi_B}{\partial t} = -\frac{\partial BA}{\partial t} = -Bl\frac{\partial x}{\partial t} = -Blv \to I = \frac{Blv}{R}$$

$$F = IlB = \frac{B^2 l^2 v}{R}$$

$$R = 4 \; B = 2 \; v = 6 \;\; l = 1.6 \to F = 2.56N$$

(b) Does the magnetic force pull the bar back to its original position when external force suddenly stops?
(1 point)

No change in flux anymore, no current, no force, bar remains where it is



Magnetic field is outward (●)

14) Unpolarized light passes through three polarizing filters. The first one is oriented with an arbitrary transmission axis, the second filter has its transmission axis 45° from the horizontal, and the third one has a vertical transmission axis. What percent of the light gets through this combination of filters?
(1 point)

Argumentation:
--unpolarized light > first polarizer filters 50%
arbitrary transmission axis > define polarization of light after first filter
-second polarizer: Malus Law with difference between transmission of axis first and second polarizer; polarization of remaining light is 45°
-third polarizer: Malus Law with difference between transmission of axis second and third polarizer; polarization of remaining light is 90°

$$General: \quad I = I_0 \cos^2 \theta$$

$$I_{1st} = \tfrac{1}{2} I_0 \; (polarization\, angle : \theta_{1st})$$

$$I_{2nd} = \tfrac{1}{2} I_0 \cos^2 (\theta_{1st} - 45)$$

$$I_{3rd} = \tfrac{1}{2} I_0 \cos^2 (\theta_{1st} - 45) \cos^2 45$$