Electricity and magnetism, 14-8-2015

The exam consists of two parts. The first part is multiple-choice. You only have to give the answer. Every correct answer is worth 0.25 points, up to a total of 2 points. The second part consists of open questions. For these you have to motivate your answers. Answers without motivation are considered as wrong. The total number of points for the open questions is 8.

Multiple choice, 8 questions, 0.25 point per question

1) A positive point charge Q is fixed on a very large horizontal frictionless tabletop. A second positive point charge q is released from rest near the stationary charge and is free to move. Which statement best describes the motion of q after it is released?
A) Its speed will be greatest just after it is released.
B) Its acceleration is zero just after it is released.
C) As it moves farther and farther from Q, its acceleration will keep increasing.
D) As it moves farther and farther from Q, its speed will decrease.
E) As it moves farther and farther from Q, its speed will keep increasing.
Answer E.

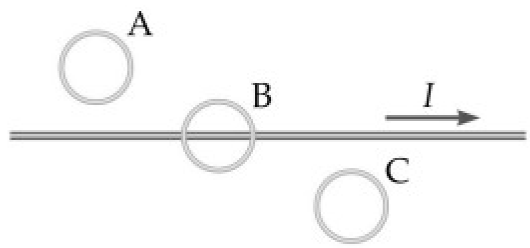
2) Two long straight parallel lines, #1 and #2, carry uniform positive linear charge densities. The charge density on line #2 is twice as great as the charge density on line #1. The locus of points where the electric field due to these lines is zero is
A) along a line between the lines closer to line #2 than line #1.
B) at a point midway between the lines.
C) along a line perpendicular to lines #1 and #2.
D) along a line between the lines closer to line #1 than line #2.
Answer: D

3) Suppose you have two point charges of opposite sign. As you move them farther and farther apart, the potential energy of this system relative to infinity
A) increases.
B) decreases.
C) stays the same.
Answer: A

4) An ideal air-filled parallel-plate capacitor has round plates and carries a fixed amount of equal but opposite charge on its plates. All the geometric parameters of the capacitor (plate diameter and plate separation) are now DOUBLED. If the original capacitance was C_0, what is the new capacitance?
A) $4C_0$
B) $2C_0$
C) C_0
D) $C_0/2$
E) $C_0/4$
Answer: B

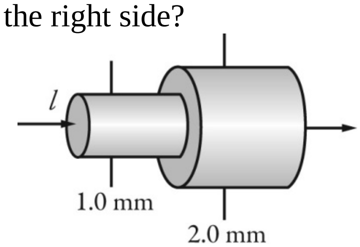
5) A horizontal wire carries a current straight toward you. From your point of view, the magnetic field at a point directly below the wire points
A) directly away from you.
B) to the left.
C) to the right.
D) directly toward you.
E) vertically upward.
Answer: C

6) The long straight wire in the figure carries a current *I* that is decreasing with time at a constant rate. The circular loops A, B, and C all lie in a plane containing the wire. The induced emf in each of the loops A, B, and C is such that



A) no emf is induced in any of the loops.
B) a counterclockwise emf is induced in all the loops.
C) loop A has a clockwise emf, loop B has no induced emf, and loop C has a counterclockwise emf.
D) loop A has a counter-clockwise emf, loop B has no induced emf, and loop C has a clockwise emf.
E) loop A has a counter-clockwise emf, loops B and C have clockwise emfs.
Answer: D

7) The figure shows two connected wires that are made of the same material. The current entering the wire on the left is 2.0 A and in that wire the electron drift speed is v_d. What is the electron drift speed in the wire on the right side?



A) $4v_d$
B) $2v_d$
C) v_d
D) $v_d/2$
E) $v_d/4$
Answer: E

8) The energy per unit volume in an electromagnetic wave is
A) equally divided between the electric and magnetic fields.

B) mostly in the electric field.
C) mostly in the magnetic field.
D) all in the electric field.
E) all in the magnetic field.
Answer: A

Open questions, 7 assignments

9) A dipole with a positive charge of q and a negative charge of $-q$ is centered at the origin and oriented along the x-axis with the positive charge located at a distance a to the right of the origin. Find the electric field due to this dipole at the point on the x-axis, far away from the dipole. Use the dipole moment in your answer.

Answer

The spacing between the + and – charges is $2a$. We will use $E = k\frac{q}{r^2}$ for each charge to find the total field at a point $x \gg a$.

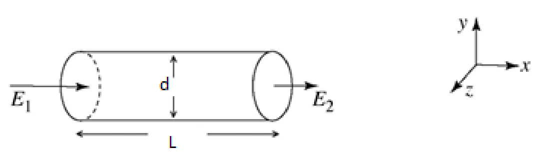
$$\vec{E} = k\frac{+q}{(x-a)^2}\hat{i} + k\frac{-q}{(x+a)^2}\hat{i} = kq[(x-a)^{-2} - (x+a)^{-2}]\hat{i}$$

$$\rightarrow \vec{E} = \frac{kq}{x^2}\hat{i}\left[\left(1-\frac{a}{x}\right)^{-2} - \left(1+\frac{a}{x}\right)^{-2}\right]$$

For $x \gg a$, $(1 \pm \frac{a}{x})^{-2} \approx 1 \mp 2\frac{a}{x}$, so $\vec{E} \approx \frac{kq}{x^2}\hat{i}[(1+2\frac{a}{x}) - (1-2\frac{a}{x})] = \frac{kq}{x^2}[4\frac{a}{x}] = 2\frac{k(2qa)}{x^3}\hat{i}$. But $p = qd = 2qa$, so $\vec{E} = \frac{2kp}{x^3}\hat{i}$.

(1 point)

10) A non-uniform electric field is directed along the x-axis at all points in space. This magnitude of the field varies with x, but not with respect to y or z. The axis of a cylindrical surface is aligned parallel to the x-axis, as shown in the figure. The electric fields E_1 and E_2, at the ends of the cylindrical surface are directed as shown. E1 is larger than E2. Calculate the average charge density in the cylinder.



Answer

Use Gauss law to relate the electric fluxes to the encapsulated charge. Because the charge does not have to be uniformly distributed in the cylinder, the charge density does not have the same everywhere. The *average* charge density is given by the total charge divided by the volume of the cylinder. The field lines are only passing through the outer ends of the cylinder. There is no flux in the y and z direction. The inward flux is larger than the outward flux, so the charge in the cylinder has to be negative.

$$\left(E_2 - E_1\right)A = \frac{Q}{\varepsilon_0} \rightarrow Q = \left(E_2 - E_1\right)A\varepsilon_0$$

$$\rho_{ave} = \frac{Q}{V} = \frac{\left(E_2 - E_1\right)A\varepsilon_0}{AL} = \frac{\left(E_2 - E_1\right)\varepsilon_0}{L}$$

(1 point)

11) In a certain region, the electric potential due to a charge distribution is given by the equation $V(x,y) = 2xy - x^2 - y$, where x and y are measured in meters and V is in volts. At which point is

the electric field equal to zero?

Answer
The electric field is given by the negative derivative of the potential.

$$\vec{E} = -\left(\tfrac{\partial}{\partial x}V(x,y)\vec{i} + \tfrac{\partial}{\partial y}V(x,y)\vec{j}\right)$$

$$2y - 2x = 0 \rightarrow y = x$$

$$2x - 1 = 0 \rightarrow x = \frac{1}{2} \rightarrow y = \frac{1}{2}$$

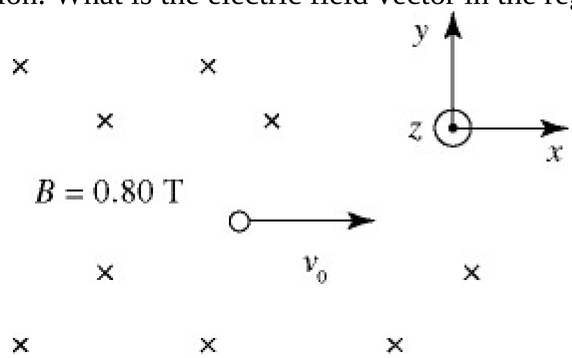
(1 point)

12) An air-filled capacitor is accidentally filled with water in such a way as not to discharge its plates. What happens to the capacitance? Explain your answer.

Answer
Water molecules are dipoles. They will be aligned by the electrical field in the capacitor, reducing the total field and potential. The charge remains constant, so the capacitance (C=Q/V) increases.

(1 point)

13) A uniform magnetic field of magnitude 0.80 T in the negative *z*-direction is present in a region of space, as shown in the figure. A uniform electric field is also present. An electron that is projected with an initial velocity $v_0 = 9 \times 10^4$ m/s in the positive *x*-direction passes through the region without deflection. What is the electric field vector in the region?

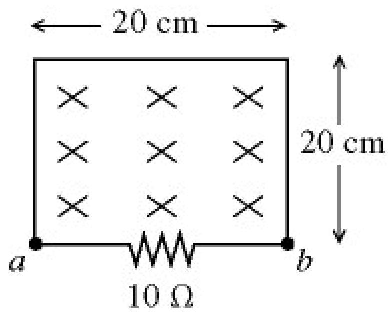


Answer
No deflection implies that the magnetic force is balanced by the electrical force.
Applying the right-hand rule shows that the magnetic force is directed along the *negative* y-axis.
The electrical force is therefore directed along the *positive* y-axis.
The particle is an electron. The electric field is directed along the *negative* y-axis.

$$\vec{F}_B == -q\vec{v}\times\vec{B} = -qvB\vec{y}$$
$$\vec{F}_E == -q\vec{E}$$
$$\vec{F}_E = -\vec{F}_B \rightarrow -q\vec{E} = qvB\vec{y} \rightarrow \vec{E} = -vB\vec{y}$$
$$v = 9\cdot 10^4 m/s$$
$$B = 0.8T$$
$$\left|E\right| = 7,2\cdot 10^4 Tm/s = 72kV/m$$

(1 point)

14) As shown in the figure, a wire and a 10-Ω resistor are used to form a circuit in the shape of a square, 20 cm by 20 cm. A uniform but nonsteady magnetic field is directed into the plane of the circuit. The magnitude of the magnetic field is decreased from 1.50 T to 0.50 T in a time interval of 0,2 ms. Calculate the induced current.



Answer
The changing magnetic field induces an emf according to Lenz' law. The current is calculate with Ohm's law. The external magnetic field decreases. The induced magnetic field will compensate this and is therefore directed into the plane of the circuit. The corresponding current has to flow from *b* to *a*.

$$\varepsilon = -\frac{\partial \Phi_B}{\partial t} = -\frac{\partial BA}{\partial t} = -A\frac{\partial B}{\partial t}$$
$$I = \varepsilon / R = -\frac{1}{R}A\frac{\partial B}{\partial t}$$
$$\frac{\partial B}{\partial t} = \frac{-1T}{0.2ms}$$
$$I = -\frac{1}{10}\cdot(0,2)^2\frac{-1}{0,2\cdot 10^{-3}} = \frac{0,2}{10^{-2}} = 20A$$

(1 point)

15) A sinusoidal electromagnetic wave is propagating in vacuum. At a given point *P* and at a particular time, the electric field is in the +*x* direction and the magnetic field is in the -*y* direction.

(a) What is the direction of propagation of the wave? Use the coordination system of assignment

13.

Answer
The direction is given by the vector product of E and B: -*z* direction
(0.25 point)

(b) Express the *average* intensity of the wave in terms of electrical field.

Answer
The energy flow is given by the Poynting vector S. The average intensity is half the peak intensity.

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} \to S = \frac{EB}{\mu_0}$$

$$\overline{S} = \frac{1}{2}\frac{E_p B_p}{\mu_0}$$

$$E_p = cB_p \to S = \frac{1}{2}\frac{E_p^2}{c\mu_0}$$

(0.75 point)

(c) A transmitter at location *x= 0* sends a wave with power *P*. At a distance *x=R* the wave is reflected by a small reflector with area *A*. What is the intensity of the reflected wave at *x=0*?

Answer
The power of the waves at a distance R is distributed equally over a sphere with radius R. The power intercepted by the reflector is the product of intensity and area.

$$from\ x = 0\ to\ x = R:$$

$$S_{x=R} = \frac{P}{4\pi R^2}$$

$$P_{x=R} = S_{x=R} \cdot A = \frac{PA}{4\pi R^2}$$

$$from\ x = R\ to\ x = 0:$$

$$S_{x=0} = \frac{P_{x=R}}{4\pi R^2} = \frac{1}{4\pi R^2}\frac{PA}{4\pi R^2} = \frac{PA}{\left(4\pi R^2\right)^2}$$

(1 point)