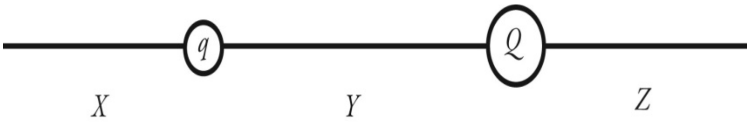
Electricity and magnetism, 3-7-2017

The exam consists of two parts. The first part is multiple-choice. You only have to give the answer. Every correct answer is worth 0.25 points, up to a total of 2 points. The second part consists of open questions. For these you have to motivate your answers. Answers without motivation are considered as wrong. The total number of points for the open questions is 8; 0.5 per sub-question.

<u>Multiple choice, 8 questions, 0.25 point per question</u>

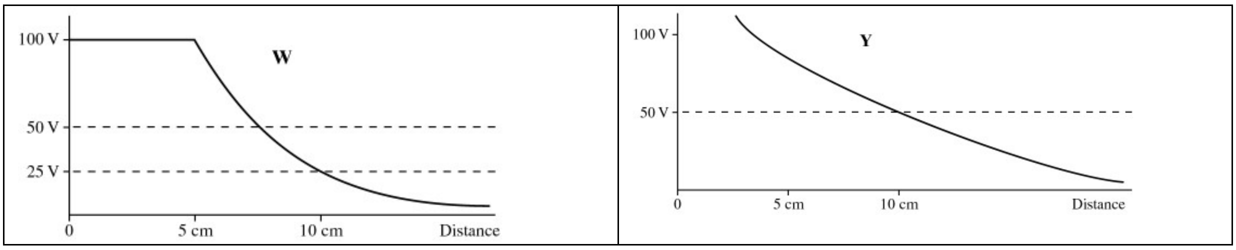
1) The figure shows two unequal point charges, q and Q, of opposite sign. Charge Q has greater magnitude than charge q. In which of the regions X, Y, Z will there be a point at which the net electric field due to these two charges is zero?

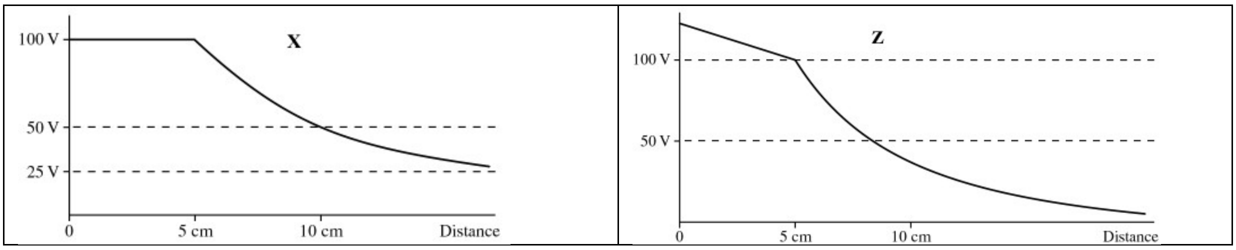


A) only regions X and Z
B) only region X
C) only region Y
D) only region Z
E) all three regions
Answer: B

2) Consider a spherical Gaussian surface of radius R centered at the origin. A charge Q is placed inside the sphere. To maximize the magnitude of the flux of the electric field through the Gaussian surface, the charge should be located
A) at $x = 0$, $y = 0$, $z = R/2$.
B) at the origin.
C) at $x = R/2$, $y = 0$, $z = 0$.
D) at $x = 0$, $y = R/2$, $z = 0$.
E) The charge can be located anywhere, since flux does not depend on the position of the charge as long as it is inside the sphere.
Answer: E

3) A metallic sphere of radius 5 cm is charged such that the potential of its surface is 100 V (relative to infinity). Which of the following plots correctly shows the potential as a function of distance from the center of the sphere?

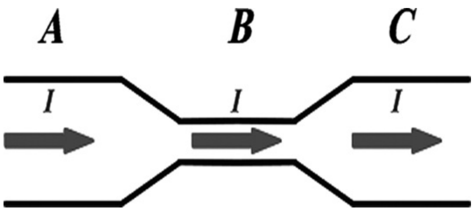




A) plot W
B) plot X
C) plot Y
D) plot Z
Answer: B

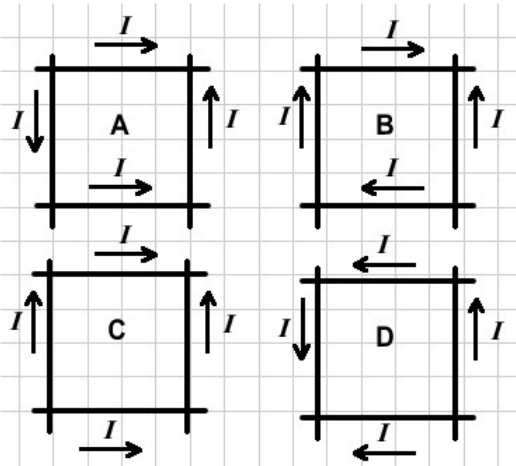
4) The charge on the square plates of a parallel-plate capacitor is *Q*. The potential across the plates is maintained with constant voltage by a battery as they are pulled apart to twice their original separation, which is small compared to the dimensions of the plates. The amount of charge on the plates is now equal to
A) 4*Q*.
B) 2*Q*.
C) *Q*.
D) *Q*/2.
E) *Q*/4.
Answer: D

5) The figure shows a steady electric current passing through a wire with a narrow region. What happens to the drift velocity of the moving charges as they go from region *A* to region *B* and then to region *C*?



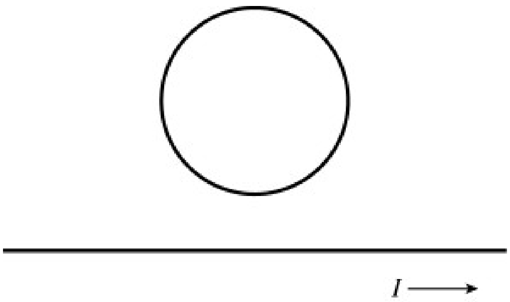
A) The drift velocity decreases from A to B and increases from B to C.
B) The drift velocity increases all the time.
C) The drift velocity remains constant.
D) The drift velocity decreases all the time.
E) The drift velocity increases from A to B and decreases from B to C.
Answer: E

6) The figure shows four different sets of insulated wires that cross each other at right angles without actually making electrical contact. The magnitude of the current is the same in all the wires, and the directions of current flow are as indicated. For which (if any) configuration will the magnetic field at the center of the square formed by the wires be equal to zero?



A) A
B) B
C) C
D) D
E) The field is not equal to zero in any of these cases.
Answer: C

7) A circular metal ring is situated above a long straight wire, as shown in the figure. The straight wire has a current flowing to the right, and the current is increasing in time at a constant rate. Which statement is true?

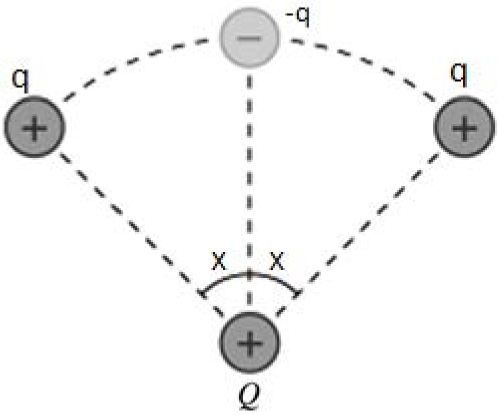


A) There is an induced current in the metal ring, flowing in a clockwise direction.
B) There is an induced current in the metal ring, flowing in a counter-clockwise direction.
C) There is no induced current in the metal ring because the current in the wire is changing at a constant rate.
Answer: A

8) In an electromagnetic wave, the electric and magnetic fields are oriented such that they are
A) parallel to one another and perpendicular to the direction of wave propagation.
B) parallel to one another and parallel to the direction of wave propagation.
C) perpendicular to one another and perpendicular to the direction of wave propagation.
D) perpendicular to one another and parallel to the direction of wave propagation.
Answer: C

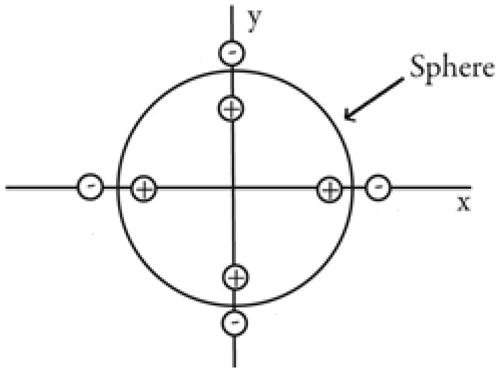
Open questions, 9 assignments

9) A point charge Q is located at the bottom of the figure, and the curve is a circular arc. At which angle X is there no net force on the charge *Q* due to the other point charges shown?



Answer Apply superposition principle. The force $F_{(-q)}$ of $-q$ on Q is attractive and directed upwards. The forces $F_{(q)}$ of the two positive charges q on Q are repelling. The horizontal components of these two forces cancel each other. Only the vertical components cos(X) $F_{(q)}$ of these forces remain. The total force acting on Q then becomes 2cos(X) $F_{(q)} + F_{(-q)} = F_{(q)}$ (2cos(X) -1). This becomes zero when cos(X) = 0.5, and X=60 degrees.

10) Four dipoles, each consisting of a +q charge and a -q charge separated by a distance *d*, are located in the *xy*-plane with their centers at a distance R from the origin, as shown. A sphere with radius R passes through the dipoles, as shown in the figure.



a) What is the electric flux through the sphere due to these dipoles?
b) What is the electric field strength at a distance R from the origin?
c) What is the electrical force on a charge Q at a distance 2R from the origin?
d) What is the potential at the origin?

Answer

a) Gauss law: electrical flux equals $Q_{enclosed} / \varepsilon_0 = 4q / \varepsilon_0$
b) $\int \vec{E} \cdot d\vec{A} = \frac{4q}{\varepsilon_0} \rightarrow E = \frac{4q}{4\pi\varepsilon_0 R^2} = \frac{q}{\pi\varepsilon_0 R^2}$
c) If we draw a Gaussian surface of radius 2R, then the enclosed charge, and consequently the electric field, becomes 0. There is no force on Q.

d) Calculate the potential of a dipole (see example 22.5 of the book), and then apply the superposition principle:

$$V_{one\ dipole} = \frac{kp\cos(\theta)}{R^2};\ \theta = 0,\ with\ p = qd \rightarrow total\ V = 4\frac{kp}{R^2}$$

11) An electron is released from rest at a distance *L* from a proton. If the proton is held in place, how fast will the electron be moving when it is L/2 from the proton?
Answer The electron is attracted by the proton. When moving towards the proton, the electron will gain kinetic energy and loose potential energy to the same amount. The electron starts from the rest position (no kinetic energy).

$$\triangle\left(E_{kin}\right) = \tfrac{1}{2}mv^2 = eV_{\triangle L};\quad V(r) = k\frac{e}{r};\quad V(L/2) - V(L) = k\frac{e}{L};\quad \triangle U_{pot} = k\frac{e^2}{L};$$

$$\tfrac{1}{2}mv^2 = k\frac{e^2}{L};\quad v = \sqrt{\frac{2ke^2}{mL}} = e\sqrt{\frac{2k}{mL}}$$

12) An air-filled capacitor is formed from two long conducting cylindrical shells that are coaxial and have radii of *R* and 2*R*. The electric potential of the inner conductor with respect to the outer conductor is $-V_{cyl}$. The length of the capacitor is *L*.

a) What is the charge density on the inner shell?
b) How much energy is stored in the capacitor?
c) What is the capacity?
d) If we double the radius of the outer shell and keep the potential difference constant, how much becomes the energy stored in the capacitor then?

Answer

a) The electric field due to the inner shell is given on the formula sheet.

$$\vec{E} = \frac{2k\lambda}{r}\vec{r};\quad V_{R\rightarrow 2R} = -\int_R^{2R}\vec{E}\cdot d\vec{r} = -\int_R^{2R}\frac{2k\lambda}{r}\vec{r}\cdot\vec{r}dr = -2k\lambda\int_R^{2R}\frac{1}{r}dr = -2k\lambda\ln\left(\frac{2R}{R}\right) = -2k\lambda\ln(2)$$

$$-V_{cyl} = -2k\lambda\ln(2) \rightarrow \lambda = V_{cyl}/2k\ln(2)$$

b) $$U_{tot} = \int_V u_E dV = \tfrac{1}{2}\varepsilon_0\int_V E^2 dV = \tfrac{1}{2}\varepsilon_0\int_V\left(\frac{2k\lambda}{r}\right)^2 d\left(L\pi r^2\right) = \pi\varepsilon_0 4k^2\lambda^2 L\int_R^{2R}\frac{1}{r}dr = \pi\varepsilon_0 4k^2\lambda^2 L\ln(2)$$

$$\lambda = V_{cyl}/2k\ln(2)$$

$$U_{tot} = \pi\varepsilon_0 4k^2 L\ln(2)\left(V_{cyl}/2k\ln(2)\right)^2 = \frac{\pi\varepsilon_0 L V_{cyl}{}^2}{\ln(2)}$$

$$c)\ U_{tot} = \tfrac{1}{2} C V_{cyl}{}^2 \rightarrow C = \frac{2U_{tot}}{V_{cyl}{}^2} = \frac{2\dfrac{\pi\varepsilon_0 L V_{cyl}{}^2}{\ln(2)}}{V_{cyl}{}^2} = \frac{2\pi\varepsilon_0 L}{\ln(2)}$$

$d)\ U_{tot} == \dfrac{\pi\varepsilon_0 L V_{cyl}}{\ln(4)}$, because now we deal with ln(4R/R) instead of ln(2R/R), (see answer a).

13) A cylindrical wire of diameter *d* carries a current *I*. The potential difference between points on the wire that are at a distance L apart is V_L.
(a) What is the electric field in the wire?
(b) What is the resistivity of the material of which the wire is made?

Answer

a) $E = \dfrac{V_L}{L}$

b) $R = \rho \dfrac{L}{A} \rightarrow \rho = \dfrac{A}{L} R = \dfrac{A}{L}\dfrac{V_L}{I} = \dfrac{\pi d^2}{4L}\dfrac{V_L}{I}$

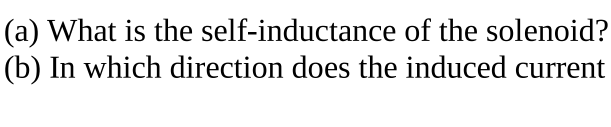
14) A wire carrying a current is shaped in the form of a circular loop of radius *R*. If the magnetic field strength that this current produces at the center of the loop is *B*, what is the magnitude of the current that flows through the wire?

Answer: Apply Biot-Savart

$d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\vec{dl}\times\vec{r}}{r^2} \rightarrow dB = \dfrac{\mu_0}{4\pi}\dfrac{Idl}{r^2}$ (the unit vector is directed towards the center and perpendicular to dl)

$$B = \int \frac{\mu_0}{4\pi}\frac{Idl}{r^2} = \frac{\mu_0 I}{4\pi r^2}\int dl = \frac{\mu_0 I 2\pi r}{4\pi r^2} = \frac{\mu_0 I}{2r} \rightarrow I = \frac{2Br}{\mu_0}$$

15) In the figure, the current in a solenoid having no appreciable resistance is flowing from *b* to *a* and is decreasing at a rate of I_0 A/s. The self-induced emf in the solenoid is found to be V_0. The self-inductance is defined as the ratio of magnetic flux and current.



(a) What is the self-inductance of the solenoid?
(b) In which direction does the induced current flow?

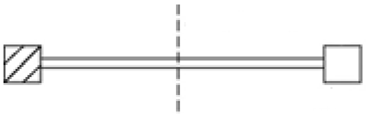
Answer

a)

$$L = \frac{\Phi_B}{I}; emf = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt} \rightarrow L = \frac{-emf}{\frac{dI}{dt}} = \frac{-V_0}{-I_0} = \frac{V_0}{I_0}$$

b)The current from *b* to *a* is decreasing. The induced current compensates for this and flows from *b* to *a*.

16) A radiometer has two square vanes, attached to a light horizontal cross arm, and pivoted about a vertical axis through the center, as shown in the figure. One vane is silvered and it reflects all radiant energy incident upon it. The other vane is blackened and it absorbs all incident radiant energy. Electromagnetic waves, coming from all directions, are incident upon the vanes. The device is placed in vacuum. Describe what will happen.



Answer The incident wave contains energy and therefore momentum. The momentum transferred to the reflecting silver vane is twice as much as the momentum given to the absorbing black vane. The system will start to rotate around the vertical axis.