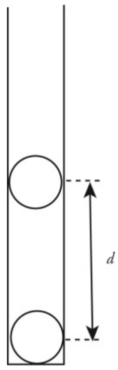
Electricity and magnetism, 29-1-2018

The exam consists of two parts. The first part is multiple-choice. You only have to give the answer. Every correct answer is worth 0.25 points, up to a total of 2 points. The second part consists of open questions. For these you have to motivate your answers. Answers without motivation are considered as wrong. The total number of points for the open questions is 8.

<u>Multiple choice, 8 questions, 0.25 point per question</u>

1) One very small uniformly charged plastic ball is located directly above another such charge in a test tube as shown in the figure. The balls are in equilibrium a distance *d* apart. If the charge on each ball is doubled, the distance between the balls in the test tube would become



A) $\sqrt{2}\,d$.
B) 2*d*.
C) 4*d*.
D) 8*d*.
Answer: B

2) Two long straight parallel lines, #1 and #2, carry uniform positive linear charge densities. The charge density on line #2 is twice as great as the charge density on line #1. The locus of points where the electric field due to these lines is zero is
A) along a line between the lines closer to line #2 than line #1.
B) at a point midway between the lines.
C) along a line perpendicular to lines #1 and #2.
D) along a line between the lines closer to line #1 than line #2.
Answer: D

3) A negative charge, if free, will tend to move
A) from high potential to low potential.
B) from low potential to high potential.
C) toward infinity.
D) away from infinity.
E) in the direction of the electric field.
Answer: B

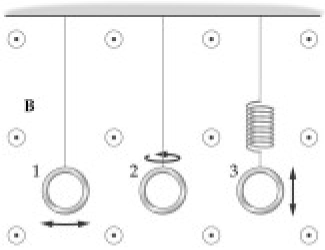
4) The charge on the square plates of a parallel-plate capacitor is *Q*. The potential across the plates is maintained with constant voltage by a battery as they are pulled apart to twice their

original separation, which is small compared to the dimensions of the plates. The amount of charge on the plates is now equal to
A) 4*Q*.
B) 2*Q*.
C) *Q*.
D) *Q*/2.
E) *Q*/4.
Answer: D

5) A narrow copper wire of length *L* and radius *b* is attached to a wide copper wire of length *L* and radius 2*b*, forming one long wire of length 2*L*. This long wire is attached to a battery, and a current is flowing through it. If the electric field in the narrow wire is *E*, the electric field in the wide wire is
A) *E*.
B) 2*E*.
C) 4*E*.
D) *E*/2.
E) *E*/4.
Answer: E

6) A charge is accelerated from rest through a potential difference *V* and then enters a uniform magnetic field oriented perpendicular to its path. The field deflects the particle into a circular arc of radius *R*. If the accelerating potential is tripled to 3*V*, what will be the radius of the circular arc?
A) 9*R*
B) 3*R*
C) $\sqrt{3}\,R$
D) $R/\sqrt{3}$
E) *R*/9
Answer: C

7) The three loops of wire shown in the figure are all subject to the same uniform magnetic field $\vec{B}$ that does not vary with time. Loop 1 oscillates back and forth as the bob in a pendulum, loop 2 rotates about a vertical axis, and loop 3 oscillates up and down at the end of a spring. Which loop, or loops, will have an emf induced in them?

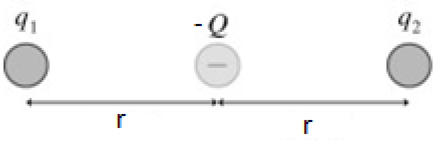


A) loop 1 only
B) loop 2 only
C) loop 3 only
D) loops 1 and 2
E) loops 2 and 3
Answer: B

8) The energy per unit volume in an electromagnetic wave is
A) equally divided between the electric and magnetic fields.
B) mostly in the electric field.
C) mostly in the magnetic field.
D) all in the electric field.
E) all in the magnetic field.
Answer: A

Open questions, 6 assignments

9) In the figure, all the charges are point charges and the charge in the middle is -Q. For what charge q_1 will charge q_2 be in static equilibrium?



Answer

$$k\left|\frac{-Qq_2}{r^2}\right| = k\frac{q_1q_2}{4r^2} \rightarrow Q = \frac{q_1}{4} \rightarrow q_1 = 4Q$$

10) A solid nonconducting sphere of radius *R* carries a charge *Q* distributed uniformly throughout its volume. At a certain distance r_1 ($r_1 < R$) from the center of the sphere, the electric field has magnitude *E*. If the same charge *Q* were distributed uniformly throughout a sphere of radius 2*R*,what would then be the magnitude of the electric field at the same distance r_1 from the center ?

Answer

$$EA = \frac{Q}{\varepsilon_0} \rightarrow E4\pi r_1^2 = \frac{Q}{\varepsilon_0}\frac{r_1^3}{R^3} \rightarrow E = \frac{Q}{4\pi\varepsilon_0}\frac{r_1}{R^3}; R \rightarrow 2R; E \rightarrow E/8$$

11) Two point charges, *Q* and -3*Q*, are located on the *x*-axis a distance *d* apart, with -3*Q* to the right of *Q*. Find the location of ALL the points on the *x*-axis (not counting infinity) at which the potential (relative to infinity) due to this pair of charges is equal to zero.

Answer

- To the right of particle -3Q the potential will not go 0, because the potential of -3Q will always 'outweigh' the potential of particle Q which is at a larger distance.
- In between the two particles the potential can go to 0 for a point closer to particle Q than particle -3Q:

$$V_Q = k\frac{Q}{r}; V_{-3Q} = \frac{-3Q}{d-r} \rightarrow \frac{1}{r} + \frac{-3}{d-r} = 0 \rightarrow r = \frac{d}{4}$$

To the left of particle Q the potential can become also 0:

$$V_Q = k\frac{Q}{-r}; V_{-3Q} = \frac{-3Q}{d-r} \rightarrow \frac{1}{-r} + \frac{-3}{d-r} = 0 \rightarrow r = \frac{d}{2}\ to\, the\, left$$

12) A cylindrical capacitor is made of two thin-walled concentric cylinders. The inner cylinder has radius *r1*, and the outer one a radius *r2*. The common length of the cylinders is *L*, much longer than the outer radius. What is the potential energy stored in this capacitor when a potential difference *V* is applied between the inner and outer cylinder? Express your answer in terms of the potential difference, *r1, r2,* and *L*.
(1 point)

See example 22.4 and problem 23.40. There are three strategies:

- Calculate the work needed to move charge from the inner to the outer cylinder. The work is the negative of the potential energy.
- Calculate the capacity first and from there calculate then energy.
- Calculate the energy density and integrate over the volume.

$$U=\tfrac{1}{2}CV^2 \quad C=\frac{Q}{|V|} \quad V=-\int \vec{E}\cdot d\vec{r} \quad E=\frac{2k\lambda}{r} \quad Q=\lambda L$$

$$V=-\int \frac{2k\lambda}{r}\vec{r}\cdot\vec{r}dr=-2k\lambda\int_{r_1}^{r_2}\frac{1}{r}dr=-2k\lambda\ln\left(\frac{r_2}{r_1}\right)=-2k\frac{Q}{L}\ln\left(\frac{r_2}{r_1}\right)$$

$$approach\,1: U=-W=-\int_0^Q qVdq=\frac{2k}{L}\ln\left(\frac{r_2}{r_1}\right)\int_0^Q qdq=\frac{kQ^2}{L}\ln\left(\frac{r_2}{r_1}\right)$$

$$approach\,2: C=\frac{Q}{|V|}=\frac{1}{2k\frac{1}{L}\ln\left(\frac{r_2}{r_1}\right)}=\frac{L}{2k\ln\left(\frac{r_2}{r_1}\right)}$$

$$U=\tfrac{1}{2}CV^2=\tfrac{1}{2}\frac{L}{2k\ln\left(\frac{r_2}{r_1}\right)}\left(2k\frac{Q}{L}\ln\left(\frac{r_2}{r_1}\right)\right)^2=\frac{kQ^2}{L}\ln\left(\frac{r_2}{r_1}\right)$$

$$approach\,3:\quad u=\tfrac{1}{2}\varepsilon_0E^2 \quad U=\int_V udV=\tfrac{1}{2}\varepsilon_0\int_{r_1}^{r_2}\left(\frac{2k\lambda}{r}\right)^2 2\pi rLdr=2\pi\varepsilon_0k^2\lambda^2L\int_{r_1}^{r_2}\left(\frac{1}{r}\right)dr$$

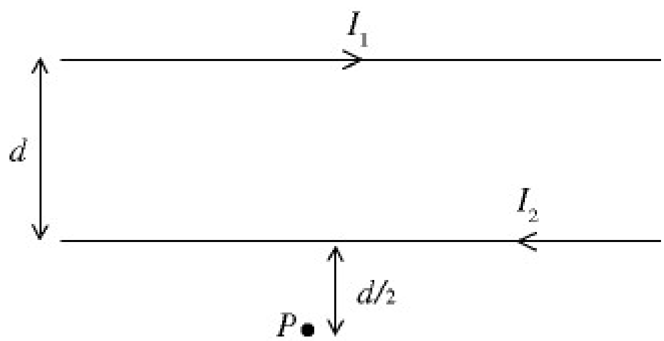
$$U=4\pi\varepsilon_0k^2\lambda^2L\ln\left(\frac{r_2}{r_1}\right)=\frac{kQ^2}{L}\ln\left(\frac{r_2}{r_1}\right)$$

Finally:

$$|V|=2k\frac{Q}{L}\ln\left(\frac{r_2}{r_1}\right)\rightarrow Q=\frac{|V|L}{2k\ln\left(\frac{r_2}{r_1}\right)}$$

$$U=\frac{|V|^2L}{4k\ln\left(\frac{r_2}{r_1}\right)}$$

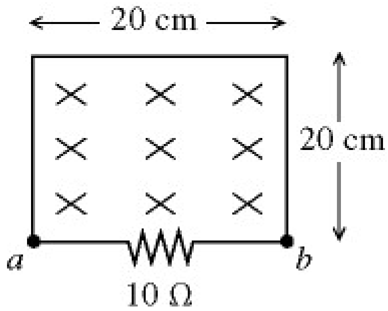
13) As shown in the figure, two long straight wires are separated by a distance of *d*. The currents are I_1 to the right in the upper wire and I_2 to the left in the lower wire. What are the magnitude and direction of the magnetic field at point *P*, which is a distance d/2 below the lower wire?



Answer
The magnetic field of the upper wire points into paper at point P.
The magnetic field of the lower wire points out of the paper at point P.

$$B_{upper}(P) = \mu_0 \frac{I_1}{2\pi \frac{3}{2} d}; B_{lower}(P) = \mu_0 \frac{I_2}{2\pi \frac{1}{2} d}; B_{upper} - B_{lower} = \left[\frac{2/3I_1 - 2I_2}{d}\right]$$

14) As shown in the figure, a wire and a 10-Ω resistor are used to form a circuit in the shape of a square, 20 cm by 20 cm. A uniform but nonsteady magnetic field is directed into the plane of the circuit. The magnitude of the magnetic field is decreased from 1.50 T to 0.50 T in a time interval of 40 ms. What is the average induced current and its direction through the resistor, in this time interval?



$$\left|emf\right| = \frac{dBA}{dt} = IR \rightarrow \frac{1T \cdot 0.04m2}{40ms} = 1 = 10I \rightarrow I = 0.1A$$

The magnetic into the paper decreases. This will be compensated by the induced current, which then flows from b to a.