Electricity and magnetism, 27-6-2016

The exam consists of two parts. The first part is multiple-choice. You only have to give the answer. Every correct answer is worth 0.25 points, up to a total of 2 points. The second part consists of open questions. For these you have to motivate your answers. Answers without motivation are considered as wrong. The total number of points for the open questions is 8: 0.6 point per sub-question.

Multiple choice, 8 questions, 0.25 point per question

1) A point charge Q is located a short distance from a point charge $3Q$, and no other charges are present. If the electrical force on Q is F, what is the electrical force on $3Q$?
A) $F/3$
B) $F/\sqrt{3}$
C) F
D) $\sqrt{3}F$
E) $3F$
Answer: C

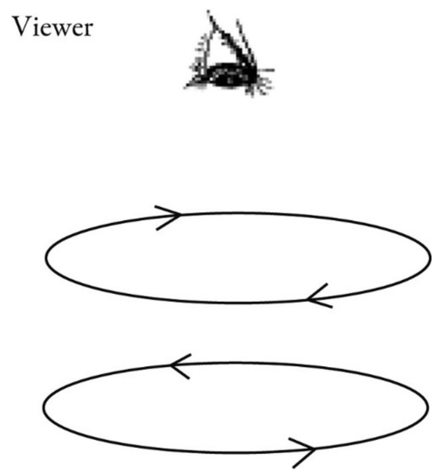
2) Suppose you have two negative point charges. As you move them farther and farther apart, the potential energy of this system relative to infinity
A) increases.
B) decreases.
C) stays the same.
Answer: B

3) An ideal parallel-plate capacitor consists of a set of two parallel plates of area A separated by a very small distance d. When this capacitor is connected to a battery that maintains a constant potential difference between the plates, the energy stored in the capacitor is U_0. If the separation between the plates is doubled, how much energy is stored in the capacitor?
A) $4U_0$
B) $2U_0$
C) U_0
D) $U_0/2$
E) $U_0/4$
Answer: D

4) After closing a switch a light bulb reacts almost instantaneously. This is because
A) the electrons in the wires move at nearly the speed of light.
B) the electric field builds up almost instantaneously.
C) the magnetic field builds up almost instantaneously.
D) the protons in the wires move at nearly the speed of light.
Answer: B

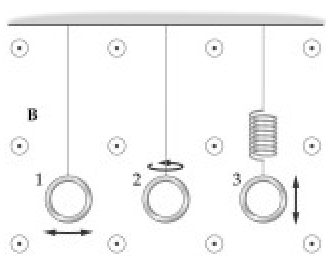
5) A ring with a clockwise current (as seen from above the ring) is situated with its center directly

above another ring, which has a counter-clockwise current, as shown in the figure. In what direction is the net magnetic force exerted on the top ring?



A) upward
B) downward
C) to the right
D) to the left
E) The net force is zero.
Answer: A

6) The three loops of wire shown in the figure are all subject to the same uniform magnetic field $\vec{B}$ that does not vary with time. Loop 1 oscillates back and forth as the bob in a pendulum, loop 2 rotates about a vertical axis, and loop 3 oscillates up and down at the end of a spring. Which loop, or loops, will have an emf induced in them?



A) loop 1 only
B) loop 2 only
C) loop 3 only
D) loops 1 and 2
E) loops 2 and 3
Answer: B

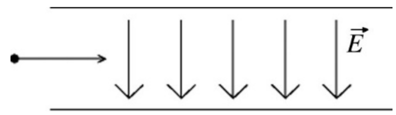
7) When an electromagnetic wave falls on a white, perfectly reflecting surface, it exerts a force F on that surface. If the surface is now painted a perfectly absorbing black, what will be the force that the same wave will exert on the surface?
A) 4F
B) 2F

C) *F*
D) *F*/2
E) *F*/4
Answer: D

8) Electromagnetic waves can propagate in vacuum due to:
A) induction
B) the electric field of free electrons
C) radiation pressure
D) the ε_0 and μ_0 of vacuum
Answer: A

Open questions, 5 assignments

9) In the figure, a proton (mass *m*, charge *e)* is projected horizontally midway between two parallel plates that are separated by *d* cm. The plates have a length *L*. The electrical field due to the plates has magnitude *E* between the plates away from the edges.



a) Find the minimum speed of the proton if it just misses the lower plate as it emerges from the field.

b) What happens to the potential difference between the plates if the speed of the proton is smaller than the speed of question *a*?

Answer

a) The proton enters the E-field midway between the plates with a given velocity. The electric field will push it to the lower plate. In the same time the proton has to travel over the full length of the plates

$$F = qE = ma; x = \frac{1}{2}at^2 = \frac{qE}{2m}t^2; x = \frac{d}{2};\ t = \sqrt{\left(\frac{md}{qE}\right)}; L = Vt \rightarrow V = \frac{L}{t} = \frac{L}{\sqrt{\left(\frac{md}{qE}\right)}} = L\sqrt{\left(\frac{qE}{md}\right)}$$

b) If the speed is smaller, the proton will hit the lower plate. This reduces the net negative charge of the plate. The field becomes weaker, and consequently the potential difference as well.

10) A nonconducting spherical shell of inner radius R_1 and outer radius R_2 contains a uniform volume charge density ρ throughout the shell. Derive he magnitude of the electric field at the following radial distances *r* from the center of the sphere:

a) $r < R_1$
b) $R_1 < r < R_2$
c) $r > R_2$

Your answers should be in terms of ρ, R_1, R_2, r, ε_0 , and π.

d) We now replace the shell with conducting material. All dimensions remain the same. How does the field change in case *a*, *b* and *c*?
e) We cover the shell with dielectric material of a certain thickness and dielectric constant κ. What is the field strength in the dielectric?

Answer: Apply Gauss law for the three cases.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} = E4\pi r^2; E = \frac{Q_{encl}}{4\pi r^2 \epsilon_0};\ Q_{encl} = \rho \frac{4}{3}\pi(r^3 - R_1^3)\ for\ (R_1 < r < R_2)$$

(a) $E = 0$; there is no enclosed charge

(b) $E = \frac{\rho}{3\varepsilon_0 r^2}\left(r^3 - R_1{}^3\right)$

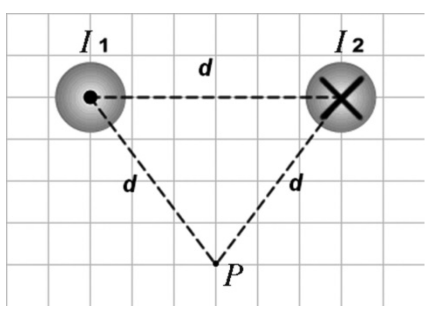
(c) $E = \frac{\rho}{3\varepsilon_0 r^2}\left(R_2{}^3 - R_1{}^3\right)$

(d) There is no field inside a conductor. All charge is on the outside of the shell. And so the field for r < R2 is zero.

(e) The field is weakened by a factor 1/ κ.

11) The figure shows two long, parallel current-carrying wires. The wires carry equal currents *I* in the directions indicated and are located a distance *d* apart.

(a) Use Ampere's law to derive the magnetic field of wire 1.
(b) Calculate the magnitude and direction of the magnetic field at the point *P* that is located an equal distance *d* from each wire.
(c) Will the wires attract or repel each other? Explain.



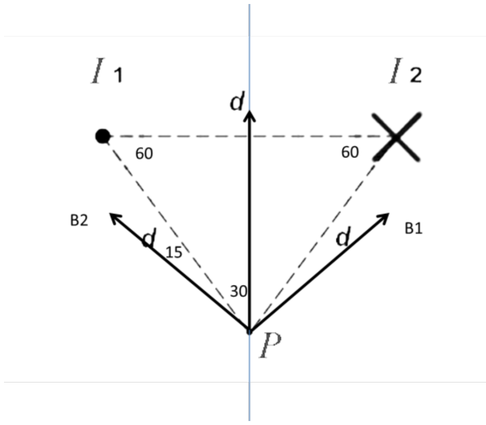
Answer
(a)
Circular field lines around the wire. Choose Amperian loop such that coincides with the field lines: dot product in integral becomes *Bdr* .

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I \ \rightarrow B_1 2\pi r = \mu_0 I_1 \ \rightarrow B_1 = \frac{\mu_0 I_1}{2\pi r};$$

right hand rule: field lines counterclockwise

(b) Field lines of each wires are circles. Field vector is perpendicular to the lines connecting P

and the wires. Due to symmetry we can neglect the horizontal components of the B-vectors. The angle between each B-vector and the vertical line is 45 degrees. Each B-vector contributes with a component *cos(45)B* to the total. The total field is therefore given as $B = \frac{\mu_0 I}{2\pi d}$ in the upward direction.



c) The magnetic force on a current is given by $\vec{F} = I\vec{l} \times \vec{B}$. The magnetic field vector B2 at the location of wire 1 is oriented upwards, and vice versa for B1 at I2. Using the right hand rule we see that the magnetic forces are outward pointing. The wires repel each other.

12) A circular coil having *N* turns and radius *r* is rotating in a uniform magnetic field *B* with a frequency *f*. The coil is rotating with a uniform angular speed. The rotation axis is perpendicular to the direction of the field. At t=0 the opening of the coil is perpendicular to the B-field.

(a) Derive the *emf*.
(b) We connect a resistor *R* between the ends of the coil. What is the peak power in the resistor?
(c) What happens to the *emf* if we change the direction of rotation?

Answer

(a) The *emf* is given by

$$emf = -\frac{d\varphi_B}{dt} = -\frac{dBA}{dt} = -NB\pi r^2 \frac{dcos(2\pi ft)}{dt} = 2NB\pi^2 r^2 f sin(2\pi ft)$$

(b) $the\ emf\ can\ be\ regarded\ as\ a\ potential\ difference\ U\ overthe\ resistor$:

$$P = UI = \frac{U^2}{R}$$

(c) Nothing. The change of flux is the same in both directions.

13) The following are positioned in sequence: A source of a beam of natural light of intensity I_0;

three ideal polarizers *A*, *B*, and *C*; and an observer. Polarizer axis angles are measured clockwise from the vertical, from the perspective of the observer. The axis angle of polarizer *A* is set at 0° (vertical), and the axis angle of polarizer *C* is set at 50°. Polarizer *B* is set so that the beam intensity is zero at the observer.

(a) Give the definition of polarization.
(b) Give two possible angles of polarizer *B*.

Answer:

(a) The direction of the E-field with respect to the propagation direction of the wave.
(b) The middle polarizer should be perpendicular to the first one (90 degrees) or the third one (140 degrees).