Faculty
Civil Engineering and GeoSciences

Exam	**AESB1420-17**
	Electricity & Magnetism
Total number of pages	**5 pages, excl. cover page**
Date and time	**2-7-2018** from 9:00 till 12:00 AM
Responsible lecturer	**Herman Russchenberg**

Only the work / answers written on examination paper will be assessed, unless otherwise specified under 'Additional Information'.

Exam questions

Total number of questions: **16 questions (of which 8 open questions with sub-questions, 8 multiple-choice questions)**

Max. number of points to be granted: **10 points**

☐ **all questions have equal weight**
X **questions differ in weight** *(the weight is given in the sheets with questions)*

Use of tools and sources of information

- It is not allowed to use calculators. It is not allowed to use any electronic devices other than simple electronic watches and medical devices.
- It is not allowed to write exams with pencils.
- Not admitted during exams: tools and/or sources of information other than the formula sheet provided by the examiners.

Tools and sources of information that are admitted:

☐ **books** ☐ **notes** ☐ **dictionaries** ☐ **syllabus** ☐ **calculators**

X **formula sheets (see also above)**

Additional information

Give the answers to the multiple-choice questions on a separate sheet.

On each sheet write your name and study number,

On the first page the total number of sheets you submit, **AND** whether you are a first year's of higher year's student.

Last date exam is checked: 16-7-2018 (the marking period is 15 working days at most)

⚠ Every suspicion of fraud is mentioned to the Board of Examiners.

GSM OFF.

Electricity and magnetism, 02-07-2018

The exam consists of two parts. The first part is multiple-choice. You only have to give the answer. Every correct answer is worth 0.25 points, up to a total of 2 points. The second part consists of open questions. For these you have to motivate your answers. Answers without motivation are considered as wrong. The formulas on the formula sheet can be used as starting point. Everything else has to be derived. The total number of points for the open questions is 8. Every assignment is worth 1 point, uniformly distributed over the sub-questions.

1) Four equal negative point charges are located at the corners of a square, their positions in the *xy*-plane being (1, 1), (-1, 1), (-1, -1), (1, -1). The electric field on the *x*-axis at (1, 0) points in the same direction as

A) $\hat{j}$.

B) $\hat{i}$.

C) $-\hat{i}$

D) $\hat{k}$.

E) $-\hat{j}$.

2) At a distance *D* from a very long (essentially infinite) uniform line of charge, the electric field strength is 1000 N/C. At what distance from the line will the field strength to be 2000 N/C?

A) 2*D*

B) $\sqrt{2}D$

C) $D/\sqrt{2}$

D) *D*/2

E) *D*/4

3) A nonconducting sphere contains positive charge distributed uniformly throughout its volume. Which statement about the potential due to this sphere is true? All potentials are measured relative to infinity.

A) The potential is highest at the center of the sphere.

B) The potential at the center of the sphere is zero.

C) The potential at the center of the sphere is the same as the potential at the surface.

D) The potential at the surface is higher than the potential at the center.

E) The potential at the center is the same as the potential at infinity.

4) The charge on the square plates of a parallel-plate capacitor is *Q*. The potential across the plates is maintained with constant voltage by a battery as they are pulled apart to twice their original separation, which is small compared to the dimensions of the plates. The amount of charge on the plates is now equal to

A) 4*Q*.

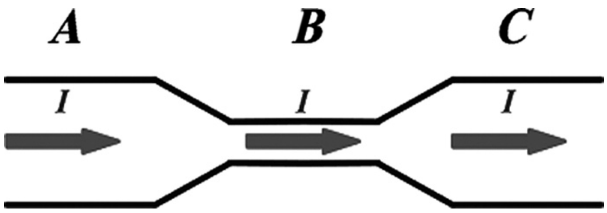
B) 2*Q*.

C) *Q*.

D) *Q*/2.

E) $Q/4$.

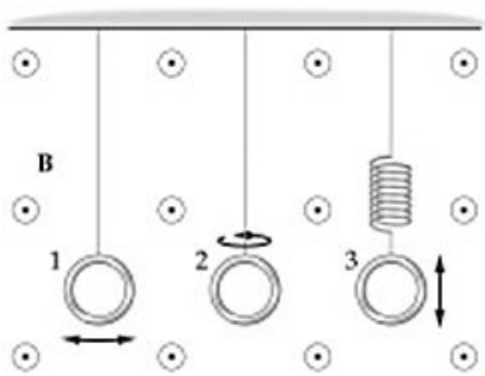
5) The figure shows a steady electric current passing through a wire with a narrow region. What happens to the drift velocity of the moving charges as they go from region *A* to region *B* and then to region *C*?



A) The drift velocity decreases from A to B and increases from B to C.
B) The drift velocity increases all the time.
C) The drift velocity remains constant.
D) The drift velocity decreases all the time.
<u>E) The drift velocity increases from A to B and decreases from B to C.</u>

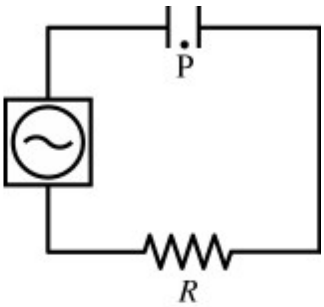
6) A charge is accelerated from rest through a potential difference V and then enters a uniform magnetic field oriented perpendicular to its path. The field deflects the particle into a circular arc of radius R. If the accelerating potential is tripled to $3V$, what will be the radius of the circular arc?
A) $9R$
B) $3R$
<u>C) $\sqrt{3}R$</u>
D) $R/\sqrt{3}$
E) $R/9$

7) The three loops of wire shown in the figure are all subject to the same uniform magnetic field $\vec{B}$ that does not vary with time. Loop 1 oscillates back and forth as the bob in a pendulum, loop 2 rotates about a vertical axis, and loop 3 oscillates up and down at the end of a spring. Which loop, or loops, will have an emf induced in them?



A) loop 1 only
<u>B) loop 2 only</u>
C) loop 3 only
D) loops 1 and 2
E) loops 2 and 3

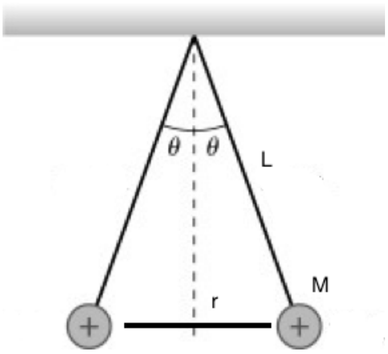
8) A capacitor is hooked up to a resistor and an AC voltage source as shown in the figure. The output of the source is given by $V(t) = V_0 \sin \omega t$. The plates of the capacitor are disks of radius R. Point P is directly between the two plates, equidistant from them and a distance $R/2$ from the center axis. At point P



A) there is no magnetic field because there is no charge moving between the plates.
B) there is a constant magnetic field.
C) there is a time-varying magnetic field.

Open assignments

9) The figure shows two tiny spheres with mass M suspended from two very thin long threads with length L. The spheres repel each other after being charged to a charge Q and hang at rest as shown.



Calculate the distance r between the spheres when equilibrium has been reached. You can assume that tanθ=sinθ.

Answer
Two forces are acting on the spheres: downwards gravity(Fm) and the horizontal, repelling electric force(Fq). In equilibrium the resultant force of Fm and Fq will be directed along the line, and the components of Fm and Fq perpendicular to the thread will cancel each other. Geometrical considerations then lead to:

$$tan\theta = \frac{Fq}{Fm} = \frac{kQ^2}{Mgr^2}; tan\theta = sin\theta = \frac{r}{2L}; r = \sqrt[3]{\frac{2kQ^2L}{Mg}}$$

10) Charge is distributed uniformly throughout a large insulating cylinder of radius R. The charge per unit length in the cylindrical volume is λ.

a) Use Gauss's law to find the magnitude of the electric field at a distance r from the central axis of the cylinder for $r < R$. Your answer should be in terms of r, R, λ, ε_0, and π.

Answer:

Gauss law: $\int \vec{E} \cdot \overrightarrow{dA} = \frac{Qencl}{\epsilon_0}$. De electric field is radiating outwards, parallel to the normal vector of the cylinder. We can then simplify Gauss law to $EA = E2\pi rL = \frac{Qencl}{\epsilon_0}; E = \frac{Qencl}{2\pi\epsilon_0 rL}$

From the center of the cylinder the enclosed charge will increase as $Q = \frac{\pi r^2 L}{\pi R^2 L} Qencl = \frac{\pi r^2}{\pi R^2} \lambda L$

The electric field now becomes: $E = \frac{Qencl}{2\pi\epsilon_0 rL} = \frac{\lambda r}{2\pi\epsilon_0 R^2}$

b) We now drill a hole into the cylinder such that is becomes a hollow pipe with inner radius R/2 and outer radius R. What is the magnitude of electric field for r<R?

Answer:

By drilling the hole we have removed charged, and consequently the line charge density has changed. The inner core of radius R/2 occupies ¼ of the total volume, so ¼ of the charge is removed. The total charge in the remainder of the cylinder is ¾ of the original charge, and consequently the line charge density if ¾ of the original line charge density.

For r<R/2 there is no enclosed charge, so the electric field is 0.

For r>R/2, $E = \frac{3/4\lambda r}{2\pi\epsilon_0 R^2}$

11) A very small object with mass M carrying -Q of charge is attracted to a large, well-anchored, positively charged conducting plate with a surface charge density σ.

a) How much time does it take for the negatively charged object to travel through a potential difference of V0? The object starts with a velocity of 0 m/s.

Answer

While the particle travels through the potential difference V0, it will gain kinetic energy. The conducting plate exerts a force on the particle: $F = QE = Ma$. The conducting plate has a uniform field, so the potential difference is given by $V0 = -E\Delta r$. The distance the particle travels is given by $\Delta r = 1/2at^2$. The electric field of a conducting plate is given by $E = \frac{\sigma}{\varepsilon_0}$. Combining it all:

$$t^2 = \frac{2\Delta r}{a} = \frac{2V0}{aE} = \frac{2MV0}{QE^2} = \frac{\varepsilon_0{}^2 2MV0}{Q\sigma^2}; t = \frac{\varepsilon_0}{\sigma}\sqrt{\frac{2MV0}{Q}}$$

b) How much time will it take if we double the charge on the conducting plate?

Answer: following from question a: doubling the charge leads the half the time.

12) A long piece of coaxial cable of length L has a wire with a radius of R1 and a concentric conductor with inner radius R2. The length of the cable is much longer than R1 and R2. The line charge density is λ.

a) Calculate the potential difference.

Answer The field inside the cable is radially symmetric. Gauss law then becomes:

$$EA = \frac{Q}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}; E = \frac{\lambda}{2\pi\epsilon_0 r}$$

The potential difference is given by:

$$V = -\int_{R1}^{R2} \vec{E} \cdot \overrightarrow{dr} = -\int_{R1}^{R2} Edr = -\frac{\lambda}{2\pi\epsilon_0}\int_{R1}^{R2} \frac{1}{r} dr = -\frac{\lambda}{2\pi\epsilon_0} ln\frac{R2}{R1}$$

We now fill the space between the cable and the conductor with dielectric material. The dielectric constant equals 4.

b) How much does the potential difference change?

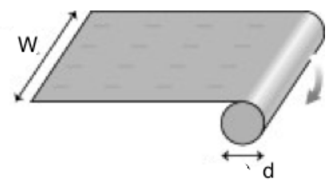
Answer
The dielectric material weakens the electric field with a factor 4, and consequently also the potential difference with the same factor.

c) Calculate the capacity.

Answer
The charge has not changed:

$$C = \frac{Q}{V}; Cnew = \frac{Q}{V/4} = \frac{4Q}{\frac{\lambda}{2\pi\epsilon_0} ln\frac{R2}{R1}} = \frac{4\lambda L}{\frac{\lambda}{2\pi\epsilon_0} ln\frac{R2}{R1}} = \frac{8\pi\epsilon_0 L}{ln\frac{R2}{R1}}$$

13) The figure shows a diameter roller with diameter *d* that turns at R rpm. A wide plastic film with a width W is being wrapped onto the roller, and this plastic carries an excess electric charge having a uniform surface charge density of σ. What is the current of the moving film?



Answer
Per rotation the amount of charge that is transported equals $\sigma A = \sigma\pi dW$. The total charge transported during R rotations in a minute becomes $\sigma\pi dWR$. The current then becomes:

$$I = \frac{\Delta Q}{\Delta t} = \frac{1}{60}\sigma\pi dWR \text{ C/s}$$

14) As shown in the figure, a small particle of charge -*Q* and mass M has velocity Vo as it enters a region of uniform magnetic field. The particle is observed to travel in the semicircular path shown, with radius *R*.

a) Calculate the magnitude and direction of the magnetic field in the region.

Answer
The magnetic force points towards the centre of the semi-circle. Applying the right-hand rule for a *negative* particle then leads to a magnetic field into the paper.

$$F = -QVoB = \frac{MVo^2}{R}; |B| = \frac{MV_0}{RQ}$$

b) Show that the time it takes to the leave the region of the magnetic field does not depend or R and V0.

Answer

$T = \frac{\pi R}{Vo} = \frac{\pi MVo}{VoBQ} = \frac{\pi M}{BQ}$

c) What is its speed when it leaves the region of the magnetic field?

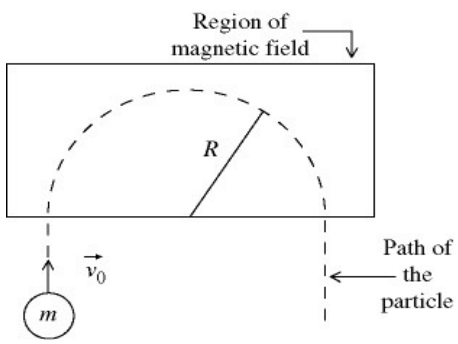
Answer
Vo: The force is perpendicular to the direction: the speed does not change.

d) Another particle is entering the region parallel to the magnetic field. Describe its trajectory.

Answer
The motion is parallel to the magnetic field: no magnetic force. The particle follows a straight line.

e) A third particle is entering region, making an angle between 0 and 90 degrees with the magnetic field. Describe its trajectory.

Answer
There now is parallel and perpendicular component of the velocity vector to the magnetic field. The perpendicular component leads to a circular path. The parallel component leads to a straight line. Combined this leads to a spiral form of the path.



15) A conducting loop with area A and resistance R lies in the *x-y* plane. A spatially uniform magnetic field points in the *z*-direction. The field varies with time *t* according to B= at^2-b where a and b are positive constants.
a) Find the loop current as function of time

Answer
The uniform magnetic field is perpendicular to the loop. The flux through the loop is given by BA. The magnetic field changes as function of time. The emf is then given by:

$$emf = -\varepsilon_0 \frac{d\psi}{dt} = -A\varepsilon_0 \frac{dB}{dt} = -2\varepsilon_0 aAt$$

The current then is:

$$I = \frac{emf}{R} = \frac{-2\varepsilon_0 aAt}{R}$$

b) Find the loop current when B=0
Answer
B=0: B= at^2-b=0

$$I = \frac{-2\varepsilon_0 aA}{R}\sqrt{\frac{b}{a}}$$

c) Suppose we can also change the area of the loop. How should we change it as function of time to prevent the induction of the loop current?
Answer

$$emf = -\varepsilon_0 \frac{d\psi}{dt} = -\varepsilon_0 \frac{dBA}{dt} = 0{:}\, BA = constant; A = \frac{constant}{B} = \frac{constant}{\mathrm{at}^2 - \mathrm{b}}$$

16) A sinusoidal electromagnetic wave is propagating in vacuum. At a given point and at a particular time, the electric field is in the +*x* direction and the magnetic field is in the -*y* direction.

a) What is the direction of propagation of the wave?

Answer
The direction of the wave is given by the Poynting vector: $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$. With the right hand rule we find that the wave is propagating in the negative z-direction.

b) If the intensity of the wave is Po W/m2 what is the electric field amplitude at that point?

Answer

$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$; $S = \frac{EB}{\mu_0}$; $E = cB$; $S = \frac{E^2}{c\mu_0}$: $E = \sqrt{Sc\mu_0}$ with S=Po.

c) The wave passes through a polarizer with its transmission axis at 30 degrees from the vertical. What is the intensity of wave after passing through the polarizer?

Answer

The electric field is in the x-direction: the wave is horizontally polarized. The angle between polarization and the transmission is 90-30=60 degrees. Malus law:

$S1 = S0\, cos^2(\theta) : S1 = S0\, cos^2(90 - 30)$=1/4S0