

Exam	<u>AESB1420-17</u> <u>Electricity & Magnetism</u>
Total number of pages	4 pages, excl. cover page
Date and time	1-7-2019 from 9:00 till 12:00 AM
Responsible lecturer	Herman Russchenberg
<i>Only the work / answers written on examination paper will be assessed, unless otherwise specified under 'Additional Information'.</i>	
Exam questions	
Total number of questions:	16 questions (of which 8 open questions with sub-questions, 8 multiple-choice questions)
Max. number of points to be granted:	10 points
<input type="checkbox"/> all questions have equal weight	
<input checked="" type="checkbox"/> questions differ in weight (<i>the weight is given in the sheets with questions</i>)	
Use of tools and sources of information	
<ul style="list-style-type: none"> • It is <u>not</u> allowed to use calculators. It is <u>not</u> allowed to use any electronic devices other than simple electronic watches and medical devices. • It is <u>not</u> allowed to write exams with pencils. • <u>Not</u> admitted during exams: <u>tools and/or sources of information other than the formula sheet provided by the examiners.</u> 	
Tools and sources of information that are admitted:	
<input type="checkbox"/> books <input type="checkbox"/> notes <input type="checkbox"/> dictionaries <input type="checkbox"/> syllabus <input type="checkbox"/> calculators	
<input checked="" type="checkbox"/> formula sheets (see also above)	
Additional information	
Give the answers to the multiple-choice questions on a separate sheet.	
On each sheet write your name and study number,	
On the first page the total number of sheets you submit, AND whether you are a first year's of higher year's student.	
Last date exam is checked: 15-7-2019 (the marking period is 15 working days at most)	



Every suspicion of fraud is mentioned to the Board of Examiners.

GSM OFF.

The exam consists of two parts. The first part is multiple-choice. You only have to give the answer. Every correct answer is worth 0.25 points, up to a total of 2 points. The second part consists of open questions. For these you have to motivate your answers. **Answers without motivation are considered as wrong.** The total number of points for the open questions is 8, equally distributed over the sub-questions.

Multiple choice, 8 questions, 0.25 point per question

1) X and Y are two uncharged metal spheres on insulating stands, and are in contact with each other. A positively charged rod R is brought close to X as shown in Figure (a).

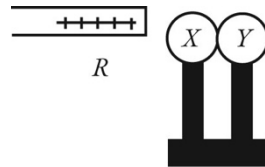


Figure (a)

Sphere Y is now moved away from X , as in Figure (b).

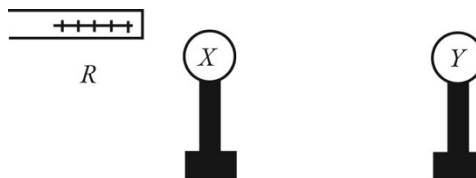


Figure (b)

What are the final charge states of X and Y ?

- A) Both X and Y are neutral.
- B) X is positive and Y is neutral.
- C) X is neutral and Y is positive.
- D) X is negative and Y is positive.**
- E) Both X and Y are negative.

2) Consider a spherical Gaussian surface of radius R centered at the origin. A charge Q is placed inside the sphere. To maximize the magnitude of the flux of the electric field through the Gaussian surface, the charge should be located

- A) at $x = 0, y = 0, z = R/2$.
- B) at the origin.
- C) at $x = R/2, y = 0, z = 0$.
- D) at $x = 0, y = R/2, z = 0$.
- E) The charge can be located anywhere, since flux does not depend on the position of the charge as long as it is inside the sphere.**

3) A negative charge is moved from point A to point B along an equipotential surface. Which of the following statements must be true for this case?

- A) The negative charge performs work in moving from point A to point B .
- B) Work is required to move the negative charge from point A to point B .
- C) No work is required to move the negative charge from point A to point B .**
- D) The work done on the charge depends on the distance between A and B .

E) Work is done in moving the negative charge from point A to point B .

4) The charge on the square plates of a parallel-plate capacitor is Q . The potential across the plates is maintained with constant voltage by a battery as they are pulled apart to twice their original separation, which is small compared to the dimensions of the plates. The amount of charge on the plates is now equal to

- A) $4Q$.
- B) $2Q$.
- C) Q .
- D) $Q/2$.**
- E) $Q/4$.

5) When electric current is flowing in a metal, the electrons are moving

- A) at nearly the speed of light.
- B) at the speed of light.
- C) at the speed of sound in the metal.
- D) at the speed of sound in air.
- E) at none of the above speeds.**

6) A horizontal wire carries a current straight toward you. From your point of view, the magnetic field at a point directly below the wire points

- A) directly away from you.
- B) to the left.
- C) to the right.**
- D) directly toward you.
- E) vertically upward.

7) A circular loop of wire lies in the plane of the paper. An increasing magnetic field points out of the paper. What is the direction of the induced current in the loop?

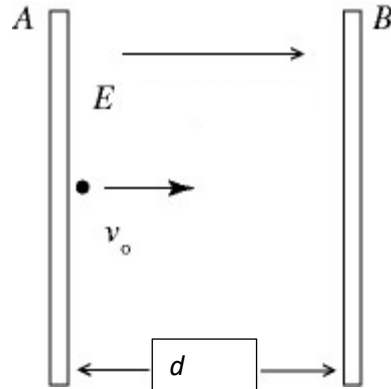
- A) counter-clockwise then clockwise
- B) clockwise then counter-clockwise
- C) clockwise**
- D) counter-clockwise
- E) There is no current induced in the loop.

8) In an electromagnetic wave, the electric and magnetic fields are oriented such that they are

- A) parallel to one another and perpendicular to the direction of wave propagation.
- B) parallel to one another and parallel to the direction of wave propagation.
- C) perpendicular to one another and perpendicular to the direction of wave propagation.**
- D) perpendicular to one another and parallel to the direction of wave propagation.

Open questions, 8 assignments

9) A pair of charged conducting plates produces a uniform field E , directed to the right, between the plates. The separation of the plates is d . An electron is projected from plate A , directly toward plate B , with an initial velocity of v_0 m/s, as shown in the figure. Calculate the distance of closest approach of the electron to plate B .



Answer:

The electric field exerts a force on the electron, directed to the left and slows down the electron. After a time t_0 the electron comes to a standstill. Calculate t_0 , and subsequently the travelled distance.

$$F = ma = -eE; a = -\frac{eE}{m}$$

$$v(t) = v_0 + at_0 = v_0 - \frac{eE}{m}t_0 = 0; t_0 = \frac{mv_0}{eE};$$

$$x(t_0) = v_0t_0 + \frac{1}{2}at_0^2 = \frac{mv_0^2}{eE} - \frac{1}{2}\frac{mv_0^2}{eE} = \frac{1}{2}\frac{mv_0^2}{eE};$$

$$\text{distance from B} = d - \frac{1}{2}\frac{mv_0^2}{eE};$$

10) A huge (essentially infinite) horizontal nonconducting sheet of thickness d has charge uniformly spread over both faces. The upper face carries $+\sigma_{up}$ C/m² while the lower face carries $-\sigma_{low}$ C/m².

a- Use Gauss law to derive the electric field of the upper face.

Answer:

$$E = \frac{\sigma_{up}}{2\epsilon_0}, \text{ directed away from the upper face (see book Example 21.6 for derivation)}$$

b - What is the magnitude of the electric field at a point within the sheet $0.25d$ below the upper face?

Answer

$$E_{upper} = \frac{\sigma_{up}}{2\epsilon_0}, \text{ directed towards the lower face}$$

$$E_{lower} = \frac{\sigma_{low}}{2\epsilon_0}, \text{ directed towards the lower face}$$

Total field is the sum of the two fields:

$$E_{lower} = \frac{\sigma_{up} + \sigma_{low}}{2\epsilon_0}, \text{ independent of the distance}$$

11) Four equal +Q C point charges are placed at the corners of a square L m on each side.

a – Derive the potential of a point charge, relative to infinity.

See book: chapter 22.2

b - What is the electric potential (relative to infinity) due to these four charges at the center of this square?

Superposition principle: total potential is the sum of the contributions of the individual contributions.

Potential in the centre due to one charge:

$$V = k \frac{Q}{r}; r = \frac{1}{2}\sqrt{L}$$

$$V_{tot} = 4k \frac{Q}{\frac{1}{2}\sqrt{L}} = 8k \frac{Q}{\sqrt{L}};$$

c - What is the magnitude of the electric field due to these charges at the center of the square?

The fields of the individual charges cancel each other out: $E=0$.

12) Two thin-walled concentric conducting spheres of radii R_{in} and R_{out} have a potential difference V between them.

a- What is the capacitance of this combination?

Answer

$$C = \frac{Q}{V};$$

$$V = - \int \vec{E} \cdot \vec{dr} = - \int_{R_{in}}^{R_{out}} E dr = - \int_{R_{in}}^{R_{out}} \frac{Q}{4\pi\epsilon_0 r^2} dr = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{R_{out}} - \frac{1}{R_{in}} \right]$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\epsilon_0} \left[\frac{1}{R_{out}} - \frac{1}{R_{in}} \right]} = \frac{1}{\frac{1}{4\pi\epsilon_0} \left[\frac{1}{R_{out}} - \frac{1}{R_{in}} \right]} = \frac{4\pi\epsilon_0}{\left[\frac{1}{R_{out}} - \frac{1}{R_{in}} \right]} = \frac{4\pi\epsilon_0(R_{in} - R_{out})}{R_{in}R_{out}};$$

b- What is the charge carried by each sphere?

Answer

$$Q = CV = \frac{4\pi\epsilon_0 V (R_{in} - R_{out})}{R_{in}R_{out}}$$

13) A proton beam that carries a total current of I A. The current density in the proton beam increases linearly with distance from the center. This is expressed mathematically as $J(r) = J_0 (r/R)$, where R is the radius of the beam and J_0 is the current density at the edge. Determine the value of J_0 .

Answer

$$I = \int \vec{j} \cdot d\vec{A}; J \text{ is perpendicular to } A; I = \int_0^R J dA$$

$$I = \int_0^R J dA = \frac{J_0}{R} \int_0^R r 2\pi r dr = 2\pi \frac{J_0}{R} \int_0^R r^2 dr = \frac{2\pi}{3} J_0 R^2$$

$$J_0 = \frac{3}{2\pi R^2} I$$

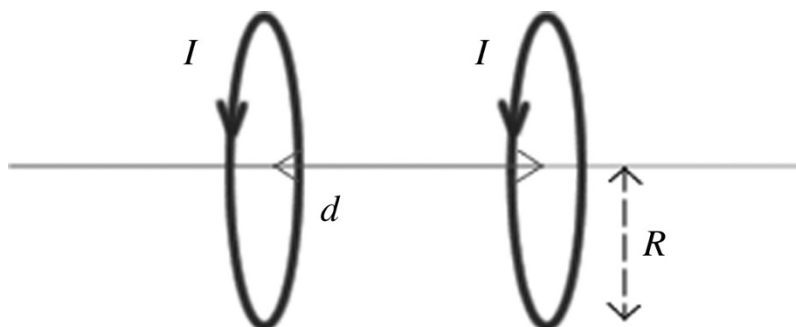
14) Two coaxial circular coils of radius R , each carrying a current I in the same direction, are positioned a distance d apart, as shown in the figure.

a- Use Biot-Savart to derive the magnetic field due to one coil at a given point on the line connecting their centers.

Answer

See book, Example 26.3; $B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$ with x as the coordinate along the line

b- Calculate the magnitude of the magnetic field halfway between the coils along the line connecting their centers.



Answer

The fields of the two loops have the same strength and are pointing in the same direction.

$$B = (B_1 + B_2) = \frac{\mu_0 I R^2}{\left(\left(\frac{d}{2}\right)^2 + R^2\right)^{3/2}}$$

15) A rectangular coil having N turns and measuring X by Y m is rotating in a uniform magnetic field B with a frequency f . The rotation axis is perpendicular to the direction of the field.

a- If the coil develops a sinusoidal emf of maximum value V , what is the value of N ?

Answer

The magnetic flux varies over time, as so does the emf:

$$\epsilon = - \frac{dBXY \sin(2\pi ft)}{dt} = NBXY 2\pi f \cos(2\pi ft)$$

$$\epsilon_{max} = V = NBXY 2\pi f$$

$$N = \frac{V}{BXY 2\pi f}$$

b- How does the emf change when the rotation axis is normal to the surface of coil?

Answer

The flux does not change over time: no emf.

16) For a short time dT , a plate with a mass M and surface area A is bombarded by light of average intensity S_0 W/m², normal to the surface. All light is absorbed by the plate. You can assume that the rate of change of momentum per unit of area of the incoming wave equals S_0/c .

a- How much does the velocity of the mirror change during that time?

Answer

The radiation pressure (force per unit of area) sets the plate into motion and the plate gains momentum p . When all energy is absorbed, the momentum of the wave is transferred to the plate.

$$Prad = \frac{F}{A} = \frac{1}{A} \frac{dp}{dt} = \frac{Ma}{A} = \frac{So}{c}$$
$$a = \frac{A So}{M c}; v(dT) = adT = \frac{A So}{M c} dT$$

b- How does this change when the plate reflects all light?

Answer

When all light is reflected, the plate gains twice as much momentum. The speed doubles.