Electricity and magnetism, 18-8-2017

The exam consists of two parts. The first part is multiple-choice, with one correct answer per question. You only have to give the answer. Every correct answer is worth 0.25 points, up to a total of 2 points. The second part consists of open questions. For these you have to derive the requested answer from the information given. Answers without derivation are considered as wrong. The total number of points for the open questions is 8.

1) Two identical small charged spheres are a certain distance apart, and each one initially experiences an electrostatic force of magnitude F due to the other. With time, charge gradually leaks off of both spheres. When each of the spheres has lost half its initial charge, the magnitude of the electrostatic force will be
A) 1/16 *F.*
B) 1/8 *F.*
C) 1/4 *F.*
D) 1/2 *F.*
Answer: C

2 Under electrostatic conditions, the electric field just outside the surface of any charged conductor
A) is always parallel to the surface.
B) is always zero because the electric field is zero inside conductors.
C) is always perpendicular to the surface of the conductor.
D) is perpendicular to the surface of the conductor only if it is a sphere, a cylinder, or a flat sheet.
E) can have nonzero components perpendicular to and parallel to the surface of the conductor.
Answer: C

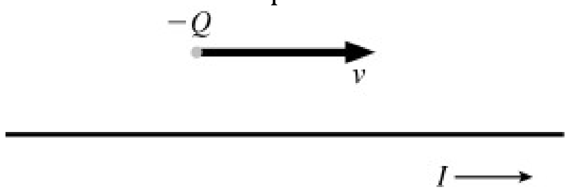
3) Suppose you have two point charges of opposite sign. As you move them farther and farther apart, the potential energy of this system relative to infinity
A) increases.
B) decreases.
C) stays the same.
Answer: A

4) When two or more capacitors are connected in parallel across a potential difference
A) the potential difference across each capacitor is the same.
B) each capacitor carries the same amount of charge.
C) the equivalent capacitance of the combination is less than the capacitance of any of the capacitors.
D) All of the above choices are correct.
E) None of the above choices are correct.
Answer: A

5) When a potential difference of 10 V is placed across a certain solid cylindrical resistor, the current through it is 2 A. If the diameter of this resistor is now tripled, the current will be

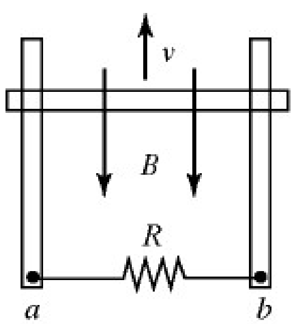
A) 2/9 A.
B) 2/3 A.
C) 2 A.
D) 3 A.
E) 18 A.
Answer: E

6) A negatively charged particle is moving to the right, directly above a wire having a current flowing to the right, as shown in the figure. In which direction is the magnetic force exerted on the particle?



A) into the page
B) out of the page
C) downward
D) upward
E) The magnetic force is zero since the velocity is parallel to the current.
Answer: D

7) In the figure, a copper bar is in contact with a pair of parallel metal rails and is in motion with velocity *v*. A uniform magnetic field is present pointing downward, as shown. The bar, the rails, and the resistor *R* are all in the same plane. The induced current through the resistor *R* is

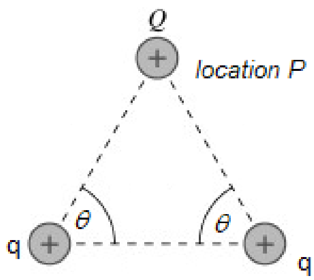


A) from *a* to *b*.
B) from *b* to *a*.
C) There is no induced current through the resistor.
Answer: C

8) The energy per unit volume in an electromagnetic wave is
A) equally divided between the electric and magnetic fields.
B) mostly in the electric field.
C) mostly in the magnetic field.
D) all in the electric field.
E) all in the magnetic field.
Answer: A

Open questions.

9) In the figure below a configuration of charges is given: two charges *q* at the baseline of a triangle and a charge *Q* at location *P.*



a) What is the magnitude and direction of the force on the charge *Q*? (0.25 points)

   We now add a charge at one of the sides of the triangle, such that it cancels the force of the two charges *q*.

b) What is the required charge and where should we put it? (0.25 points)

We remove the charge Q.

c) What is the electric potential (relative to infinity) at location P? (0.25 points)

d) Does the *potential energy* of a positive particle increase or decrease if it is brought from point P vertically downwards to the baseline of the triangle? (0.25 points)

Answers

a) The horizontal components of the force on Q due to two charges *q* work in opposite direction and cancel each other. Only the vertical components remain. All charges are positive, leading to a repelling force along the vertical.

$$Ftot = 2k\frac{qQ}{r^2}\sin(\theta)$$

b) The added charge should lead to a force -*Ftot* along the vertical. This can only be achieved by a negative charge in the middle of the horizontal base line of the triangle. The distance from there to the charge Q is $\sin(\theta)r$.

$$-Ftot = -2k\frac{qQ}{r^2}\sin(\theta) = k\frac{q^1Q}{(r\sin(\theta))^2} \rightarrow q^1{=}-2q(\sin(\theta))^3$$

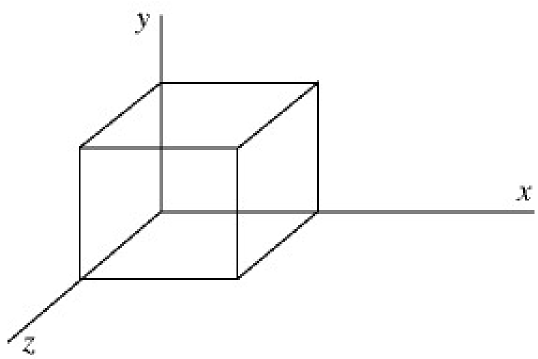
The required charge is -2q$sin^3(\theta)$

c) We have three charges at the baseline. Apply the superposition principle: the potential is the sum of the potential of the individual charges.

$$V_1 = V_2 = k\frac{q}{r}; V_3 = k\frac{-2q sin^3(\theta)}{r\sin(\theta)} = k\frac{-2q sin^2(\theta)}{r}; V_1 + V_2 + V_3 = 2k\frac{q}{r}cos^2(\theta)$$
$$= 2k\frac{q}{x}cos^3(\theta) \text{ with } x \text{ as the halflength of the baseline.}$$

d) The potential is energy is the product of charge and potential. The potential increases if we get closer to the baseline. The potential energy increases.

10) The cube of insulating material shown in the figure has one corner at the origin. Each side of the cube has length L. It is observed that there is an electric field $\vec{E} = \alpha y\hat{j}$ that is in the +*y* direction and whose magnitude depends only on *y*. Use Gauss's law to calculate the net charge enclosed by the cube. (1.5 points)



Answer

Gauss law: $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$

The electric is oriented along the vertical. It is perpendicular to the top and bottom sides of the cube and parallel to the other sides. Gauss law then simplifies to:

$$E_{y=0}A_{y=0} + E_{y=L}A_{y=L} = \frac{Q_{encl}}{\epsilon_0} \to \alpha L L^2 = \frac{Q_{encl}}{\epsilon_0} \to Q_{encl} = \alpha\epsilon_0 L^3$$

11) An air-filled capacitor is formed from two long conducting cylindrical shells with length *L* that are coaxial and have radii of R and 2R. The electric potential of the inner conductor with respect to the outer conductor is -Vcyl. Derive the energy of this capacitor in terms of L, R and Vcyl. (2 points)

Answer

The electric field due to the inner shell is given on the formula sheet.

$$\vec{E} = \frac{2k\lambda}{r}\vec{r};$$

The total energy is the given by the volume integral of the energy density:

$$U_{tot} = \int_V u_E dV = \tfrac{1}{2}\varepsilon_0 \int_V E^2 dV = \tfrac{1}{2}\varepsilon_0 \int_V \left(\frac{2k\lambda}{r}\right)^2 d\left(L\pi r^2\right) = \pi\varepsilon_0 4k^2\lambda^2 L \int_R^{2R} \frac{1}{r} dr = \pi\varepsilon_0 4k^2\lambda^2 L \ln(2)$$

Now we have to replace the charge density with the potential.

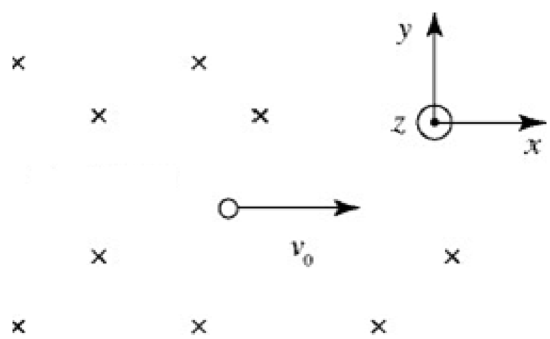
$$V_{R\to 2R} = -\int_R^{2R} \vec{E}\cdot d\vec{r} = -\int_R^{2R} \frac{2k\lambda}{r}\vec{r}\cdot\vec{r}dr = -2k\lambda \int_R^{2R} \frac{1}{r} dr = -2k\lambda \ln\left(\frac{2R}{R}\right) = -2k\lambda \ln(2)$$

$$-V_{cyl} = -2k\lambda \ln(2) \rightarrow \lambda = V_{cyl} / 2k\ln(2)$$

The total energy then becomes:

$$U_{tot} = \pi\varepsilon_0 4k^2 L \ln(2)\left(V_{cyl} / 2k\ln(2)\right)^2 = \frac{\pi\varepsilon_0 L V_{cyl}{}^2}{\ln(2)}$$

12) A uniform magnetic field of magnitude B in the negative *z*-direction is present in a region of space, as shown in the figure. A uniform electric field is also present. A stream of electrons that is projected with an initial velocity *v0* in the positive *x*-direction passes through the region without deflection.



a) What is the electric field vector in the region? (0.5 points)
b) The electric field is due to a charged conducting plate, located below the moving particle. The speed of the electrons suddenly doubles. What should we do with the charge density at the conducting plate to avoid deflection of the particle? (0.5 points)

Answers

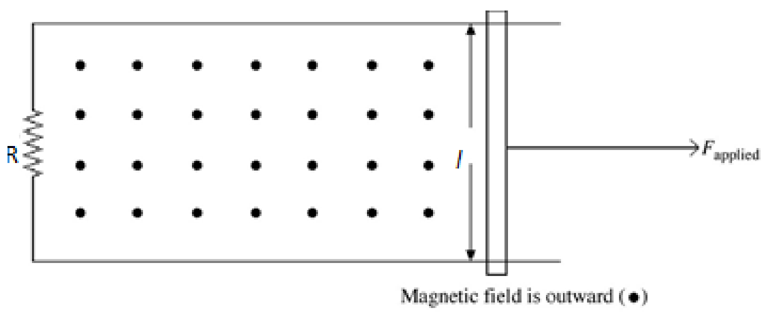
a)
No deflection implies that the magnetic force is balanced by the electrical force.

Applying the right-hand rule shows that the magnetic force is directed along the *negative* y-axis.
The electrical force is therefore directed along the *positive* y-axis.
The particle is an electron. The electric field is directed along the *negative* y-axis.

$$\vec{F}_B == -e\vec{v_0} \times \vec{B} = -ev_0 B\vec{y}$$
$$\vec{F}_E == -e\vec{E}$$
$$\vec{F}_E = -\vec{F}_B \to -e\vec{E} = ev_0 B\vec{y} \to \vec{E} = -v_0 B\vec{y}$$

b) The electric field of a conducting plate is given by $E = \frac{\sigma}{\varepsilon_0} = -v_0 B$. If the speed doubles, then the charge density also has to double.

13) A conducting bar moves along frictionless conducting rails connected to a resistor *R* as shown in the figure. The length of the bar is *l* and a uniform magnetic field *B* is applied perpendicular to the paper pointing outward, as shown.



a) What is the applied force required to move the bar to the right with a constant speed v? (0.5 points)

b) At what rate is energy dissipated in the resistor? (0.5 points)

We now put a closed conducting loop with a certain resistance inside the rectangular frame.

c) Describe qualitatively what will happens. (0.5 points)

Answers

a) The bar moves with a constant speed, which implies that there is no net force working on it. The bar moves to the right, thereby increasing the magnetic flux through the area of the frame. This leads to an *emf*, a current and magnetic force. The latter has to be balanced by the applied force to ensure a constant velocity.

$$\vec{F} = I\vec{l} \times \vec{B} \quad I = \frac{|\varepsilon|}{R} \quad \varepsilon = -\frac{\partial \Phi_B}{\partial t}$$

$$\varepsilon = -\frac{\partial \Phi_B}{\partial t} = -\frac{\partial BA}{\partial t} = -Bl\frac{\partial x}{\partial t} = -Blv \rightarrow I = \frac{Blv}{R}$$

$$F = IlB = \frac{B^2 l^2 v}{R}$$

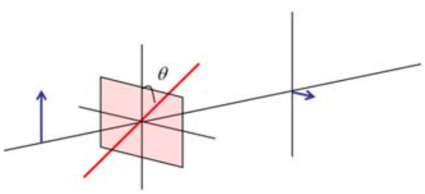
b)

$$P = I^2 R = \frac{(Blv)^2}{R}$$

c)

- The moving bar creates an increasing flux (out of the paper) through the rectangle, but not through the circular loop, because that area has not changed.
- The rectangle 'responds' by inducing an opposing flux (into the paper), also through the loop.
- The loop induces a flux to counter act this change (out of the paper).

14) Given two polarization filters, each with a *different* transmission angle. One of the filters is given in the figure below. We want to rotate the polarization of incoming vertically polarized light to horizontal by putting the second filter behind the first one. At which transmission angle θ do we get the largest intensity of horizontally polarized light? (1 point) WRONG QUESTION! REPLACED BY a) AND b).



a) What does the transmission angle of the second filter has to be? (0.25 points)
b) At which transmission angle θ do we get the largest intensity of horizontally polarized light? (0.75 points)

Answer

a) The second filter has to have a transmission angle of 90 degrees with respect to the vertical to get horizontal polarization.

b) The first filter rotates the polarization with an angle θ. The intensity is $I_0 \cos^2(\theta)$, (assuming the initial intensity is I_0). The angle of the second filter is equivalent to (90- θ), relative to the incoming light. The intensity of the light then becomes:

$$I = I_0 \cos^2(\theta)\cos^2(90-\theta) = I_0 \cos^2(\theta)\sin^2(\theta) = I_0 \frac{1}{4}\sin^2(2\theta)$$

To find its maximum we differentiate and set to 0:

$$\frac{dI}{d\theta} = I_0 \sin(2\theta)\cos(2\theta) = I_0 \frac{1}{2}\sin(4\theta) = 0 \rightarrow \theta = 0 \; or \; \theta = 45$$

At $\theta = 0$ the intensity is at its minimum 0. At $\theta = 45$ the intensity is at its maximum $\frac{I_0}{4}$.