43

NUCLEAR PHYSICS

43.1. **(a)** $^{28}_{14}\text{Si}$ has 14 protons and 14 neutrons.

(b) $^{85}_{37}\text{Rb}$ has 37 protons and 48 neutrons.

(c) $^{205}_{81}\text{Tl}$ has 81 protons and 124 neutrons.

43.2. **(a)** Using $R = (1.2\text{ fm})A^{1/3}$, the radii are roughly 3.6 fm, 5.3 fm, and 7.1 fm.

(b) Using $4\pi R^2$ for each of the radii in part (a), the areas are 163 fm^2, 353 fm^2 and 633 fm^2.

(c) $\frac{4}{3}\pi R^3$ gives 195 fm^3, 624 fm^3 and 1499 fm^3.

(d) The density is the same, since the volume and the mass are both proportional to A: $2.3\times10^{17}\text{ kg/m}^3$ (see Example 43.1).

(e) Dividing the result of part (d) by the mass of a nucleon, the number density is $0.14/\text{fm}^3 = 1.40\times10^{44}/\text{m}^3$.

43.3. **IDENTIFY:** Calculate the spin magnetic energy shift for each spin state of the 1*s* level. Calculate the energy splitting between these states and relate this to the frequency of the photons.

SET UP: When the spin component is parallel to the field the interaction energy is $U = -\mu_z B$. When the spin component is antiparallel to the field the interaction energy is $U = +\mu_z B$. The transition energy for a transition between these two states is $\Delta E = 2\mu_z B$, where $\mu_z = 2.7928\mu_\text{n}$. The transition energy is related to the photon frequency by $\Delta E = hf$, so $2\mu_z B = hf$.

EXECUTE: $B = \dfrac{hf}{2\mu_z} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(22.7\times10^6\text{ Hz})}{2(2.7928)(5.051\times10^{-27}\text{ J/T})} = 0.533\text{ T}$

EVALUATE: This magnetic field is easily achievable. Photons of this frequency have wavelength $\lambda = c/f = 13.2$ m. These are radio waves.

43.4. **(a)** As in Example 43.2, $\Delta E = 2(1.9130)(3.15245\times10^{-8}\text{ eV/T})(2.30\text{ T}) = 2.77\times10^{-7}\text{ eV}$. Since $\vec{\mu}$ and $\vec{S}$ are in opposite directions for a neutron, the antiparallel configuration is lower energy. This result is smaller than but comparable to that found in the example for protons.

(b) $f = \dfrac{\Delta E}{h} = 66.9\text{ MHz}$, $\lambda = \dfrac{c}{f} = 4.48\text{ m}$.

43.5. **IDENTIFY:** Calculate the spin magnetic energy shift for each spin component. Calculate the energy splitting between these states and relate this to the frequency of the photons.

(a) SET UP: From Example 43.2, when the *z*-component of $\vec{S}$ (and $\vec{\mu}$) is parallel to $\vec{B}$, $U = -|\mu_z|B = -2.7928\mu_\text{n}B$. When the *z*-component of $\vec{S}$ (and $\vec{\mu}$) is antiparallel to $\vec{B}$, $U = -|\mu_z|B = +2.7928\mu_\text{n}B$. The state with the proton spin component parallel to the field lies lower in energy. The energy difference between these two states is $\Delta E = 2(2.7928\mu_\text{n}B)$.

EXECUTE: $\Delta E = hf$ so $f = \dfrac{\Delta E}{h} = \dfrac{2(2.7928\mu_\text{n})B}{h} = \dfrac{2(2.7928)(5.051\times10^{-27}\text{ J/T})(1.65\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}}$

$f = 7.03\times10^7\text{ Hz} = 7.03\text{ MHz}$

And then $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^8\text{ m/s}}{7.03\times10^7\text{ Hz}} = 4.26\text{ m}$

EVALUATE: From Figure 32.4 in the textbook, these are radio waves.

(b) SET UP: From Eqs. (27.27) and (41.22) and Fig.41.14 in the textbook, the state with the z-component of $\vec{\mu}$ parallel to $\vec{B}$ has lower energy. But, since the charge of the electron is negative, this is the state with the electron spin component antiparallel to $\vec{B}$. That is, for the $m_s = -\frac{1}{2}$ state lies lower in energy.

EXECUTE: For the $m_s = +\frac{1}{2}$ state, $U = +(2.00232)\left(\frac{e}{2m}\right)\left(+\frac{\hbar}{2}\right)B = +\frac{1}{2}(2.00232)\left(\frac{e\hbar}{2m}\right)B = +\frac{1}{2}(2.00232)\mu_B B.$

For the $m_s = -\frac{1}{2}$ state, $U = -\frac{1}{2}(2.00232)\mu_B B$. The energy difference between these two states is $\Delta E = (2.00232)\mu_B B$.

$\Delta E = hf$ so $f = \frac{\Delta E}{h} = \frac{2.00232\mu_B B}{h} = \frac{(2.00232)(9.274\times10^{-24}\text{ J/T})(1.65\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = 4.62\times10^{10}\text{ Hz} = 46.2\text{ GHz}$. And

$\lambda = \frac{c}{f} = \frac{2.998\times10^{8}\text{ m/s}}{4.62\times10^{10}\text{ Hz}} = 6.49\times10^{-3}\text{ m} = 6.49\text{ mm}.$

EVALUATE: From Figure 32.4 in the textbook, these are microwaves. The interaction energy with the magnetic field is inversely proportional to the mass of the particle, so it is less for the proton than for the electron. The smaller transition energy for the proton produces a larger wavelength.

43.6. **(a)** $146m_n + 92m_H - m_U = 1.93\text{ u}$

(b) 1.80×10^3 MeV

(c) 7.56 MeV per nucleon (using 931.5 MeV/u and 238 nucleons).

43.7. **IDENTIFY** and **SET UP:** The text calculates that the binding energy of the deuteron is 2.224 MeV. A photon that breaks the deuteron up into a proton and a neutron must have at least this much energy.

$E = \frac{hc}{\lambda}$ so $\lambda = \frac{hc}{E}$

EXECUTE: $\lambda = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{2.224\times10^{6}\text{ eV}} = 5.575\times10^{-13}\text{ m} = 0.5575\text{ pm}.$

EVALUATE: This photon has gamma-ray wavelength.

43.8. **IDENTIFY:** The binding energy of the nucleus is the energy of its constituent particles minus the energy of the carbon-12 nucleus.

SET UP: In terms of the masses of the particles involved, the binding energy is

$$E_B = (6m_H + 6m_n - m_{C\text{-}12})c^2.$$

EXECUTE: **(a)** Using the values from Table 43.2, we get

$$E_B = [6(1.007825\text{ u}) + 6(1.008665\text{ u}) - 12.000000\text{ u})](931.5\text{ MeV/u}) = 92.16\text{ MeV}$$

(b) The binding energy per nucleon is (92.16 MeV)/(12 nucleons) = 7.680 MeV/nucleon

(c) The energy of the C-12 nucleus is (12.0000 u)(931.5 MeV/u) = 11178 MeV. Therefore the percent of the mass that is binding energy is $\frac{92.16\text{ MeV}}{11178\text{ MeV}} = 0.8245\%$.

EVALUATE: The binding energy of 92.16 MeV binds 12 nucleons. The binding energy per nucleon, rather than just the total binding energy, is a better indicator of the strength with which a nucleus is bound.

43.9. **IDENTIFY:** Conservation of energy tells us that the initial energy (photon plus deuteron) is equal to the energy after the split (kinetic energy plus energy of the proton and neutron). Therefore the kinetic energy released is equal to the energy of the photon minus the binding energy of the deuteron.

SET UP: The binding energy of a deuteron is 2.224 MeV and the energy of the photon is $E = hc/\lambda$. Kinetic energy is $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** The energy of the photon is

$$E_{ph} = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{3.50\times10^{-13}\text{ m}} = 5.68\times10^{-13}\text{ J}.$$

The binding of the deuteron is $E_B = 2.224\text{ MeV} = 3.56\times10^{-13}\text{ J}$. Therefore the kinetic energy is $K = (5.68 - 3.56)\times10^{-13}\text{ J} = 2.12\times10^{-13}\text{ J} = 1.32\text{ MeV}$.

(b) The particles share the energy equally, so each gets half. Solving the kinetic energy for v gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.06\times10^{-13}\text{ J})}{1.6605\times10^{-27}\text{ kg}}} = 1.13\times10^{7}\text{ m/s}$$

EVALUATE: Considerable energy has been released, because the particle speeds are in the vicinity of the speed of light.

43.10. **(a)** $7(m_n + m_H) - m_N = 0.112\text{ u}$, which is 105 MeV, or 7.48 MeV per nucleon.

(b) Similarly, $2(m_H + m_n) - m_{He} = 0.03038\text{ u} = 28.3\text{ MeV}$, or 7.07 MeV per nucleon, slightly lower (compare to Figure 43.2 in the textbook).

43.11. **(a) IDENTIFY:** Find the energy equivalent of the mass defect.

SET UP: A $^{11}_{5}\text{B}$ atom has 5 protons, $11-5=6$ neutrons, and 5 electrons. The mass defect therefore is $\Delta M = 5m_p + 6m_n + 5m_e - M(^{11}_{5}\text{B})$.

EXECUTE: $\Delta M = 5(1.0072765\text{ u}) + 6(1.0086649\text{ u}) + 5(0.0005485799\text{ u}) - 11.009305\text{ u} = 0.08181\text{ u}$. The energy equivalent is $E_B = (0.08181\text{ u})(931.5\text{ MeV/u}) = 76.21\text{ MeV}$.

(b) IDENTIFY and **SET UP:** Eq.(43.11): $E_B = C_1A - C_2A^{2/3} - C_3Z(Z-1)/A^{1/3} - C_4(A-2Z)^2/A$

The fifth term is zero since Z is odd but N is even. $A=11$ and $Z=5$.

EXECUTE: $E_B = (15.75\text{ MeV})(11) - (17.80\text{ MeV})(11)^{2/3} - (0.7100\text{ MeV})5(4)/11^{1/3} - (23.69\text{ MeV})(11-10)^2/11$.

$E_B = +173.25\text{ MeV} - 88.04\text{ MeV} - 6.38\text{ MeV} - 2.15\text{ MeV} = 76.68\text{ MeV}$

The percentage difference between the calculated and measured E_B is $\dfrac{76.68\text{ MeV} - 76.21\text{ MeV}}{76.21\text{ MeV}} = 0.6\%$

EVALUATE: Eq.(43.11) has a greater percentage accuracy for ^{62}Ni. The semi-empirical mass formula is more accurate for heavier nuclei.

43.12. **(a)** $34m_n + 29m_H - m_{Cu} = 34(1.008665)\text{ u} + 29(1.007825)\text{ u} - 62.929601\text{ u} = 0.592\text{ u}$, which is 551 MeV, or 8.75 MeV per nucleon (using 931.5 MeV/u and 63 nucleons).

(b) In Eq.(43.11), $Z = 29$ and $N = 34$, so the fifth term is zero. The predicted binding energy is

$$E_B = (15.75\text{ MeV})(63) - (17.80\text{ MeV})(63)^{2/3} - (0.7100\text{ MeV})\frac{(29)(28)}{(63)^{1/3}} - (23.69\text{ MeV})\frac{(5)^2}{(63)}.$$

$E_B = 556\text{ MeV}$. The fifth term is zero since the number of neutrons is even while the number of protons is odd, making the pairing term zero. This result differs from the binding energy found from the mass deficit by 0.86%, a very good agreement comparable to that found in Example 43.4.

43.13. **IDENTIFY** In each case determine how the decay changes A and Z of the nucleus. The β^+ and β^- particles have charge but their nucleon number is $A=0$.

(a) SET UP: α-decay: Z increases by 2, $A = N + Z$ decreases by 4 (an α particle is a $^{4}_{2}\text{He}$ nucleus)

EXECUTE: $^{239}_{94}\text{Pu} \rightarrow {}^{4}_{2}\text{He} + {}^{235}_{92}\text{U}$

(b) SET UP: β^- decay: Z increases by 1, $A = N + Z$ remains the same (a β^- particle is an electron, $^{0}_{-1}\text{e}$)

EXECUTE: $^{24}_{11}\text{Na} \rightarrow {}^{0}_{-1}\text{e} + {}^{24}_{12}\text{Mg}$

(c) SET UP β^+ decay: Z decreases by 1, $A = N + Z$ remains the same (a β^+ particle is a positron, $^{0}_{+1}\text{e}$)

EXECUTE: $^{15}_{8}\text{O} \rightarrow {}^{0}_{+1}\text{e} + {}^{15}_{7}\text{N}$

EVALUATE: In each case the total charge and total number of nucleons for the decay products equals the charge and number of nucleons for the parent nucleus; these two quantities are conserved in the decay.

43.14. **(a)** The energy released is the energy equivalent of $m_n - m_p - m_e = 8.40\times10^{-4}\text{ u}$, or 783 keV.

(b) $m_n > m_p$, and the decay is not possible.

43.15. **IDENTIFY:** The energy of the photon must be equal to the difference in energy of the two nuclear energy levels.

SET UP: The energy difference is $\Delta E = hc/\lambda$.

EXECUTE: $$\Delta E = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{0.0248\times10^{-9}\text{ J}} = 8.015\times10^{-15}\text{ J} = 0.0501\text{ MeV}$$

EVALUATE: Since the wavelength of this photon is *much* shorter than the wavelengths of visible light, its energy is much greater than visible-light photons which are frequently emitted during electron transitions in atoms. This tells us that the energy difference between the nuclear shells is *much* greater than the energy difference between electron shells in atoms, meaning that nuclear energies are *much* greater than the energies of orbiting electrons.

43.16. **IDENTIFY:** The energy released is equal to the mass defect of the initial and final nuclei.

SET UP: The mass defect is equal to the difference between the initial and final masses of the constituent particles.

EXECUTE: **(a)** The mass defect is 238.050788 u – 234.043601 u – 4.002603 u = 0.004584 u. The energy released is (0.004584 u)(931.5 MeV/u) = 4.270 MeV.

(b) Take the ratio of the two kinetic energies, using the fact that $K = p^2/2m$:

$$\frac{K_{Th}}{K_\alpha} = \frac{\dfrac{p_{Th}^2}{2m_{Th}}}{\dfrac{p_\alpha^2}{2m_\alpha}} = \frac{m_\alpha}{m_{Th}} = \frac{4}{234}.$$

The kinetic energy of the Th is

$$K_{\text{Th}} = \frac{4}{234+4}K_{\text{Total}} = \frac{4}{238}(4.270\,\text{MeV}) = 0.07176\ \text{MeV} = 1.148\ \times 10^{-14}\ \text{J}$$

Solving for v in the kinetic energy gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.148\times 10^{-14}\ \text{J})}{(234.043601)\left(1.6605\times 10^{-27}\ \text{kg}\right)}} = 2.431\times 10^{5}\ \text{m/s}$$

EVALUATE: As we can see by the ratio of kinetic energies in part (b) , the alpha particle will have a much higher kinetic energy than the thorium.

43.17. If β^- decay of ^{14}C is possible, then we are considering the decay $^{14}_{6}\text{C} \to {}^{14}_{7}\text{N} + \beta^-$.

$\Delta m = M(^{14}_{6}\text{C}) - M(^{14}_{7}\text{N}) - m_{\text{e}}$

$\Delta m = (14.003242\ \text{u} - 6(0.000549\ \text{u})) - (14.003074\ \text{u} - 7(0.000549\ \text{u})) - 0.0005491\ \text{u}$

$\Delta m = +1.68\times 10^{-4}$ u. So $E = (1.68\times 10^{-4}\ \text{u})(931.5\ \text{MeV/u}) = 0.156\ \text{MeV} = 156\ \text{keV}$

43.18. **(a)** A proton changes to a neutron, so the emitted particle is a positron (β^+).

(b) The number of nucleons in the nucleus decreases by 4 and the number of protons by 2, so the emitted particle is an alpha-particle.

(c) A neutron changes to a proton, so the emitted particle is an electron (β^-).

43.19. **(a)** As in the example, $(0.000898\ \text{u})(931.5\ \text{MeV/u}) = 0.836\ \text{MeV}$.

(b) $0.836\ \text{MeV} - 0.122\ \text{MeV} - 0.014\ \text{MeV} = 0.700\ \text{MeV}$.

43.20. **(a)** $^{90}_{39}\text{Sr} \to \beta^- + {}^{90}_{39}\text{X}$. X has 39 protons and 90 protons plus neutrons, so it must be ^{90}Y.

(b) Use base 2 because we know the half life. $A = A_0 2^{-t/T_{1/2}}$ and $0.01A_0 = A_0 2^{-t/T_{1/2}}$.

$t = -\dfrac{T_{1/2}\log 0.01}{\log 2} = -\dfrac{(28\ \text{yr})\log 0.01}{\log 2} = 190\,\text{yr}$.

43.21. **IDENTIFY** and **SET UP:** $T_{1/2} = \dfrac{\ln 2}{\lambda}$ The mass of a single nucleus is $124m_{\text{p}} = 2.07\times 10^{-25}$ kg .

$|\Delta N/\Delta t| = 0.350\ \text{Ci} = 1.30\times 10^{10}\ \text{Bq}$; $|\Delta N/\Delta t| = \lambda N$

EXECUTE: $N = \dfrac{6.13\times 10^{-3}\ \text{kg}}{2.07\times 10^{-25}\ \text{kg}} = 2.96\times 10^{22}$; $\lambda = \dfrac{\Delta N/\Delta t}{N} = \dfrac{1.30\times 10^{10}\ \text{Bq}}{2.96\times 10^{22}} = 4.39\times 10^{-13}\ \text{s}^{-1}$

$T_{1/2} = \dfrac{\ln 2}{\lambda} = 1.58\times 10^{12}\ \text{s} = 5.01\times 10^{4}\ \text{yr}$

43.22. Note that Eq.(43.17) can be written as follows: $N = N_0 2^{-t/T_{1/2}}$. The amount of elapsed time since the source was created is roughly 2.5 years. Thus, we expect the current activity to be $N = (5000\ \text{Ci})2^{-(2.5\ \text{yr})/(5.271\ \text{yr})} = 3600\,\text{Ci}$. The source is barely usable. Alternatively, we could calculate $\lambda = \dfrac{\ln(2)}{T_{1/2}} = 0.132(\text{years})^{-1}$ and use the Eq. 43.17 directly to obtain the same answer.

43.23. **IDENTIFY** and **SET UP:** As discussed in Section 43.4, the activity $A = |dN/dt|$ obeys the same decay equation as Eq. (43.17): $A = A_0 e^{-\lambda t}$. For ^{14}C, $T_{1/2} = 5730$ y and $\lambda = \ln 2/T_{1/2}$ so $A = A_0 e^{-(\ln 2)t/T_{1/2}}$; Calculate A at each t; $A_0 = 180.0$ decays/min.

EXECUTE: **(a)** $t = 1000$ y, $A = 159$ decays/min

(b) $t = 50{,}000$ y, $A = 0.43$ decays/min

EVALUATE: The time in part (b) is 8.73 half-lives, so the decay rate has decreased by a factor or $(\frac{1}{2})^{8.73}$.

43.24. **IDENTIFY and SET UP:** The decay rate decreases by a factor of 2 in a time of one half-life.

EXECUTE: **(a)** 24 d is $3T_{1/2}$ so the activity is $(375\ \text{Bq})/(2^3) = 46.9\ \text{Bq}$

(b) The activity is proportional to the number of radioactive nuclei, so the percent is $\dfrac{17.0\ \text{Bq}}{46.9\ \text{Bq}} = 36.2\%$

(c) $^{131}_{53}\text{I} \to {}^{0}_{-1}\text{e} + {}^{131}_{54}\text{Xe}$ The nucleus $^{131}_{54}\text{Xe}$ is produced.

EVALUATE: Both the activity and the number of radioactive nuclei present decrease by a factor of 2 in one half-life.

43.25. **(a)** ${}^{3}_{1}\text{H} \to {}^{0}_{-1}\text{e} + {}^{3}_{2}\text{He}$

(b) $N = N_0 e^{-\lambda t}, N = 0.100 N_0$ and $\lambda = (\ln 2)/T_{1/2}$

$$0.100 = e^{-t(\ln 2)/T_{1/2}};\quad -t(\ln 2)/T_{1/2} = \ln(0.100);\quad t = \frac{-\ln(0.100)T_{1/2}}{\ln 2} = 40.9 \text{ y}$$

43.26. **(a)** $\dfrac{dN}{dt} = 500\ \mu\text{Ci} = (500\times10^{-6})(3.70\times10^{10}\ \text{s}^{-1}) = 1.85\times10^{7}$ decays/s

$T_{1/2} = \dfrac{\ln 2}{\lambda} \to \lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{12\ \text{d}(86{,}400\,\text{s/d})} = 6.69\times10^{-7}$ s .

$\dfrac{dN}{dt} = \lambda N \Rightarrow N = \dfrac{dN/dt}{\lambda} = \dfrac{1.85\times10^{7}\ \text{decays / s}}{6.69\times10^{-7}\,\text{s}^{-1}} = 2.77\times10^{13}$ nuclei . The mass of this many ^{131}Ba nuclei is

$m = 2.77\times10^{13}\,\text{nuclei}\times(131\times1.66\times10^{-27}\ \text{kg/nucleus}) = 6.0\times10^{-12}\,\text{kg} = 6.0\times10^{-9}\ \text{g} = 6.0\ \text{ng}$

(b) $A = A_0 e^{-\lambda t}$. $1\ \mu\text{Ci} = (500\ \mu\text{Ci})\, e^{-\lambda t}$. $\ln(1/500) = -\lambda t$.

$$t = -\frac{\ln(1/500)}{\lambda} = -\frac{\ln(1/500)}{6.69\times10^{-7}\,\text{s}^{-1}} = 9.29\times10^{6}\ \text{s}\left(\frac{1\ \text{d}}{86{,}400\ \text{s}}\right) = 108\ \text{days}$$

43.27. $A = A_0 e^{-\lambda t} = A_0 e^{-t(\ln 2)/T_{1/2}}$. $-\dfrac{(\ln 2)t}{T_{½}} = \ln(A/A_0)$.

$$T_{½} = -\frac{(\ln 2)t}{\ln(A/A_0)} = -\frac{(\ln 2)(4.00\ \text{days})}{\ln(3091/8318)} = 2.80\ \text{days}$$

43.28. $\dfrac{dN}{dt} = \lambda N$. $\lambda = \dfrac{\ln 2}{T_{½}} = \dfrac{\ln 2}{1620\ \text{yr}\left(3.15\times10^{7}\ \text{s/yr}\right)} = 1.36\times10^{-11}\ \text{s}^{-1}$.

$$N = 1\ \text{g}\left(\frac{6.022\times10^{23}\,\text{atoms}}{226\ \text{g}}\right) = 2.665\times10^{25}\ \text{atoms} .$$

$$\frac{dN}{dt} = \lambda N = (2.665\times10^{25})(1.36\times10^{-11}\text{s}^{-1}) = 3.62\times10^{10}\ \text{decays/s} = 3.62\times10^{10}\,\text{Bq}$$

Convert to Ci: $3.62\times10^{10}\ \text{Bq}\left(\dfrac{1\,\text{Ci}}{3.70\times10^{10}\ \text{Bq}}\right) = 0.98\,\text{Ci}$

43.29. **IDENTIFY** and **SET UP:** Calculate the number N of ^{14}C atoms in the sample and then use Eq. (43.17) to find the decay constant λ. Eq. (43.18) then gives $T_{1/2}$.

EXECUTE: Find the total number of carbon atoms in the sample.

$n = m/M$;

$N_{\text{tot}} = nN_{\text{A}} = mN_{\text{A}}/M = (12.0\times10^{-3}\ \text{kg})(6.022\times10^{23}\ \text{atoms/mol})/(12.011\times10^{-3}\ \text{kg/mol})$

$\text{N}_{\text{tot}} = 6.016\times10^{23}$ atoms, so $(1.3\times10^{-12})(6.016\times10^{23}) = 7.82\times10^{11}$ carbon-14 atoms

$\Delta N/\Delta t = -180$ decays/min $= -3.00$ decays/s

$\Delta N/\Delta t = -\lambda N;\quad \lambda = \dfrac{-\Delta N/\Delta t}{N} = 3.836\times10^{-12}\ \text{s}^{-1}$

$T_{1/2} = (\ln 2)/\lambda = 1.807\times10^{11}\ \text{s} = 5730\ \text{y}$

EVALUATE: The value we calculated agrees with the value given in Section 43.4.

43.30. $\dfrac{360\times10^{6}\,\text{decays}}{86{,}400\ \text{s}} = 4.17\times10^{3}\ \text{Bq} = 1.13\times10^{-7}\ \text{Ci} = 0.113\ \mu\text{Ci}.$

43.31. **(a)** $\left|\dfrac{dN}{dt}\right| = 7.56\times10^{11}\ \text{Bq} = 7.56\times10^{11}$ decays/s . $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{(30.8\ \text{min})(60\ \text{s/min})} = 3.75\times10^{-4}\ \text{s}^{-1}.$

$$N_0 = \frac{1}{\lambda}\left|\frac{dN}{dt}\right| = \frac{7.56\times10^{11}\ \text{decays/s}}{3.75\times10^{-4}\ \text{s}^{-1}} = 2.02\times10^{15}\ \text{nuclei.}$$

(b) The number of nuclei left after one half-life is $\frac{N_0}{2}=1.01\times10^{15}$ nuclei, and the activity is half:

$\left|\frac{dN}{dt}\right|=3.78\times10^{11}\text{ decays/s}.$

(c) After three half lives (92.4 minutes) there is an eighth of the original amount, so $N=2.53\times10^{14}$ nuclei, and an eighth of the activity: $\left(\frac{dN}{dt}\right)=9.45\times10^{10}\text{ decays/s}.$

43.32. The activity of the sample is $\frac{3070\text{ decays/min}}{(60\text{ sec/min})\,(0.500\text{ kg})}=102\text{ Bq/kg}$, while the activity of atmospheric carbon is 255 Bq/kg (see Example 43.9). The age of the sample is then $t=-\frac{\ln(102/255)}{\lambda}=-\frac{\ln(102/255)}{1.21\times10^{-4}/\text{y}}=7573\text{ y}.$

43.33. **IDENTIFY** and **SET UP:** Find λ from the half-life and the number N of nuclei from the mass of one nucleus and the mass of the sample. Then use Eq.(43.16) to calculate $|dN/dt|$, the number of decays per second.

EXECUTE: **(a)** $|dN/dt|=\lambda N$

$\lambda=\frac{0.693}{T_{1/2}}=\frac{0.693}{(1.28\times10^{9}\text{ y})(3.156\times10^{7}\text{ s/1 y})}=1.715\times10^{-17}\text{ s}^{-1}$

The mass of ^{40}K atom is approximately 40 u, so the number of ^{40}K nuclei in the sample is

$N=\frac{1.63\times10^{-9}\text{ kg}}{40\text{ u}}=\frac{1.63\times10^{-9}\text{ kg}}{40(1.66054\times10^{-27}\text{ kg})}=2.454\times10^{16}.$

Then $|dN/dt|=\lambda N=(1.715\times10^{-17}\text{ s}^{-1})(2.454\times10^{16})=0.421\text{ decays/s}$

(b) $|dN/dt|=(0.421\text{ decays/s})(1\text{ Ci}/(3.70\times10^{10}\text{ decays/s}))=1.14\times10^{-11}\text{ Ci}$

EVALUATE: The very small sample still contains a very large number of nuclei. But the half life is very large, so the decay rate is small.

43.34. **(a)** rem = rad $\times$ RBE. 200 = x(10) and x = 20 rad.

(b) 1 rad deposits 0.010 J/kg, so 20 rad deposit 0.20 J/kg. This radiation affects 25 g (0.025 kg) of tissue, so the total energy is $(0.025\text{ kg})(0.20\text{ J/kg})=5.0\times10^{-3}\text{ J}=5.0\text{ mJ}$

(c) Since RBE = 1 for β-rays, so rem = rad. Therefore 20 rad = 20 rem.

43.35. 1 rad = 10^{-2} Gy, so 1 Gy = 100 rad and the dose was 500 rad.

rem = (rad)(RBE) = (500 rad)(4.0) = 2000 rem. $1\text{ Gy}=1\text{ J/kg}$, so 5.0 J/kg.

43.36. **IDENTIFY** and **SET UP:** For x rays $\text{RBE}=1$ so the equivalent dose in Sv is the same as the absorbed dose in J/kg.

EXECUTE: One whole-body scan delivers $(75\text{ kg})(12\times10^{-3}\text{ J/kg})=0.90\text{ J}$. One chest x ray delivers $(5.0\text{ kg})(0.20\times10^{-3}\text{ J/kg})=1.0\times10^{-3}\text{ J}$. It takes $\frac{0.90\text{ J}}{1.0\times10^{-3}\text{ J}}=900$ chest x rays to deliver the same total energy.

43.37. **IDENTIFY** and **SET UP:** For x rays $\text{RBE}=1$ and the equivalent dose equals the absorbed dose.

EXECUTE: **(a)** $175\text{ krad}=175\text{ krem}=1.75\text{ kGy}=1.75\text{ kSv}$

$(1.75\times10^{3}\text{ J/kg})(0.150\text{ kg})=2.62\times10^{2}\text{ J}$

(b) $175\text{ krad}=1.75\text{ kGy}$; $(1.50)(175\text{ krad})=262\text{ krem}=2.62\text{ kSv}$

The energy deposited would be $2.62\times10^{2}\text{ J}$, the same as in (a).

EVALUATE: The energy required to raise the temperature of 0.150 kg of water 1 C° is 628 J, and $2.62\times10^{2}\text{ J}$ is less than this. The energy deposited corresponds to a very small amount of heating.

43.38. **(a)** $5.4\text{ Sv}\,(100\text{ rem/Sv})=540\text{ rem}.$

(b) The RBE of 1 gives an absorbed dose of 540 rad.

(c) The absorbed dose is 5.4 Gy, so the total energy absorbed is $(5.4\text{ Gy})\,(65\text{ kg})=351\text{ J}$. The energy required to raise the temperature of 65 kg by $0.010°\text{ C}$ is $(65\text{ kg})\,(4190\text{ J/kg}\cdot\text{K})\,(0.01\text{C}^\circ)=3\text{ kJ}.$

43.39. **(a)** We need to know how many decays per second occur.

$$\lambda=\frac{0.693}{T_{1/2}}=\frac{0.693}{(12.3\text{ y})\,(3.156\times10^{7}\text{ s/y})}=1.79\times10^{-9}\text{s}^{-1}.$$

The number of tritium atoms is $N_0=\frac{1}{\lambda}\left|\frac{dN}{dt}\right|=\frac{(0.35\text{ Ci})\,(3.70\times10^{10}\text{ Bq/Ci})}{1.79\times10^{-9}\text{ s}^{-1}}=7.2540\times10^{18}$ nuclei .

The number of remaining nuclei after one week is

$N = N_0 e^{-\lambda t} = (7.25\times10^{18})e^{-(1.79\times10^{-9}\text{ s}^{-1})(7)(24)(3600\text{s})} = 7.2462\times10^{18}$ nuclei. $\Delta N = N_0 - N = 7.8\times10^{15}$ decays. So the energy absorbed is $E_{\text{total}} = \Delta N\, E_\gamma = (7.8\times10^{15})\,(5000\text{ eV})\,(1.60\times10^{-19}\text{ J/eV}) = 6.24\text{ J}$. The absorbed dose is $\dfrac{(6.24\text{ J})}{(50\text{ kg})} = 0.125\text{ J/kg} = 12.5\text{ rad}$. Since RBE = 1, then the equivalent dose is 12.5 rem.

(b) In the decay, antinetrinos are also emitted. These are not absorbed by the body, and so some of the energy of the decay is lost (about 12 keV).

43.40. $(0.72\times10^{-6}\text{ Ci})\,(3.7\times10^{10}\text{ Bq/Ci})\,(3.156\times10^{7}\text{ s}) = 8.41\times10^{11}\ \alpha$ particles. The absorbed dose is

$\dfrac{(8.41\times10^{11})\,(4.0\times10^{6}\text{ eV})\,(1.602\times10^{-19}\text{ J/eV})}{(0.50\text{ kg})} = 1.08\text{ Gy} = 108\text{ rad}$. The equivalent dose is $(20)\,(108\text{ rad}) = 2160\text{ rem}$.

43.41. **(a) IDENTIFY** and **SET UP:** Determine X by balancing the charge and nucleon number on the two sides of the reaction equation.

EXECUTE: X must have $A = 2+14-10 = 6$ and $Z = 1+7-5 = 3$. Thus X is ${}^{6}_{3}\text{Li}$ and the reaction is

${}^{2}_{1}\text{H} + {}^{14}_{7}\text{N} \rightarrow {}^{6}_{3}\text{Li} + {}^{10}_{5}\text{B}$

(b) IDENTIFY and **SET UP:** Calculate the mass decrease and find its energy equivalent.

EXECUTE: The neutral atoms on each side of the reaction equation have a total of 8 electrons, so the electron masses cancel when neutral atom masses are used. The neutral atom masses are found in Table 43.2.

mass of ${}^{2}_{1}\text{H} + {}^{14}_{7}\text{N}$ is $2.014102\text{ u} + 14.003074\text{ u} = 16.017176\text{ u}$

mass of ${}^{6}_{3}\text{Li} + {}^{10}_{5}\text{B}$ is $6.015121\text{ u} + 10.012937\text{ u} = 16.028058\text{ u}$

The mass increases, so energy is absorbed by the reaction. The Q value is

$(16.017176\text{ u} - 16.028058\text{ u})(931.5\text{ MeV/u}) = -10.14\text{ MeV}$

(c) IDENTIFY and **SET UP:** The available energy in the collision, the kinetic energy K_{cm} in the center of mass reference frame, is related to the kinetic energy K of the bombarding particle by Eq. (43.24).

EXECUTE: The kinetic energy that must be available to cause the reaction is 10.14 MeV. Thus $K_{\text{cm}} = 10.14\text{ MeV}$. The mass M of the stationary target $({}^{14}_{7}\text{N})$ is $M = 14\text{ u}$. The mass m of the colliding particle $({}^{2}_{1}\text{H})$ is 2 u. Then by Eq. (43.24) the minimum kinetic energy K that the ${}^{2}_{1}\text{H}$ must have is

$$K = \left(\frac{M+m}{M}\right) K_{\text{cm}} = \left(\frac{14\text{ u} + 2\text{ u}}{14\text{ u}}\right)(10.14\text{ MeV}) = 11.59\text{ MeV}$$

EVALUATE: The projectile $\left({}^{2}_{1}\text{H}\right)$ is much lighter than the target $\left({}^{14}_{7}\text{N}\right)$ so K is not much larger than K_{cm}. The K we have calculated is what is required to allow the mass increase. We would also need to check to see if at this energy the projectile can overcome the Coulomb repulsion to get sufficiently close to the target nucleus for the reaction to occur.

43.42. $m_{{}^{3}_{2}\text{He}} + m_{{}^{2}_{1}\text{H}} - m_{{}^{4}_{2}\text{He}} - m_{{}^{1}_{1}\text{H}} = 1.97\times10^{-2}\text{u}$, so the energy released is 18.4 MeV.

43.43. **IDENTIFY** and **SET UP:** Determine X by balancing the charge and the nucleon number on the two sides of the reaction equation.

EXECUTE: X must have $A = +2+9-4 = 7$ and $Z = +1+4-2 = 3$. Thus X is ${}^{7}_{3}\text{Li}$ and the reaction is

${}^{2}_{1}\text{H} + {}^{9}_{4}\text{Be} = {}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He}$

(b) IDENTIFY and **SET UP:** Calculate the mass decrease and find its energy equivalent.

EXECUTE: If we use the neutral atom masses then there are the same number of electrons (five) in the reactants as in the products. Their masses cancel, so we get the same mass defect whether we use nuclear masses or neutral atom masses. The neutral atoms masses are given in Table 43.2.

${}^{2}_{1}\text{H} + {}^{9}_{4}\text{Be}$ has mass $2.014102\text{ u} + 9.012182\text{ u} = 11.26284\text{ u}$

${}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He}$ has mass $7.016003\text{ u} + 4.002603\text{ u} = 11.018606\text{ u}$

The mass decrease is $11.026284\text{ u} - 11.018606\text{ u} = 0.007678\text{ u}$.

This corresponds to an energy release of $0.007678\text{ u}(931.5\text{ MeV/1 u}) = 7.152\text{ MeV}$.

(c) IDENTIFY and **SET UP:** Estimate the threshold energy by calculating the Coulomb potential energy when the ${}^{2}_{1}\text{H}$ and ${}^{9}_{4}\text{Be}$ nuclei just touch. Obtain the nuclear radii from Eq. (43.1).

EXECUTE: The radius R_{Be} of the ${}^{9}_{4}\text{Be}$ nucleus is $R_{\text{Be}} = (1.2\times10^{-15}\text{ m})(9)^{1/3} = 2.5\times10^{-15}\text{ m}$.

The radius R_{H} of the ${}^{2}_{1}\text{H}$ nucleus is $R_{\text{H}} = (1.2\times10^{-15}\text{ m})(2)^{1/3} = 1.5\times10^{-15}\text{ m}$.

The nuclei touch when their center-to-center separation is

$R = R_{\text{Be}} + R_{\text{H}} = 4.0\times10^{-15}\text{ m}$.

The Coulomb potential energy of the two reactant nuclei at this separation is

$$U=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}=\frac{1}{4\pi\epsilon_0}\frac{e(4e)}{r}$$

$$U=(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4(1.602\times10^{-19}\ \text{C})^2}{(4.0\times10^{-15}\ \text{m})(1.602\times10^{-19}\ \text{J/eV})}=1.4\ \text{MeV}$$

This is an estimate of the threshold energy for this reaction.

EVALUATE: The reaction releases energy but the total initial kinetic energy of the reactants must be 1.4 MeV in order for the reacting nuclei to get close enough to each other for the reaction to occur. The nuclear force is strong but is very short-range.

43.44. IDENTIFY and SET UP: 0.7% of naturally occurring uranium is the isotope ^{235}U. The mass of one ^{235}U nucleus is about $235m_\text{p}$.

EXECUTE: **(a)** The number of fissions needed is $\dfrac{1.0\times10^{19}\ \text{J}}{(200\times10^6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}=3.13\times10^{29}$. The mass of ^{235}U required is $(3.13\times10^{29})(235m_\text{p})=1.23\times10^5\ \text{kg}$.

(b) $\dfrac{1.23\times10^5\ \text{kg}}{0.7\times10^{-2}}=1.76\times10^7\ \text{kg}$

EVALUATE: The calculation assumes 100% conversion of fission energy to electrical energy.

43.45. IDENTIFY and **SET UP:** The energy released is the energy equivalent of the mass decrease. 1 u is equivalent to 931.5 MeV. The mass of one ^{235}U nucleus is $235m_\text{p}$.

EXECUTE: **(a)** $^{235}_{92}\text{U}+^1_0\text{n}\rightarrow{}^{144}_{56}\text{Ba}+^{89}_{36}\text{Kr}+3^1_0\text{n}$

We can use atomic masses since the same number of electrons are included on each side of the reaction equation and the electron masses cancel. The mass decrease is

$$\Delta M=m\left(^{235}_{92}\text{U}\right)+m\left(^1_0\text{n}\right)-\left[m\left(^{144}_{56}\text{Ba}\right)+m\left(^{89}_{36}\text{Kr}\right)+3m\left(^1_0\text{n}\right)\right]$$

$$\Delta M=235.043930\ \text{u}+1.0086649\ \text{u}-143.922953\ \text{u}-88.917630\ \text{u}-3(1.0086649\ \text{u})$$

$\Delta M=0.1860\ \text{u}$. The energy released is $(0.1860\ \text{u})(931.5\ \text{MeV/u})=173.3\ \text{MeV}$.

(b) The number of ^{235}U nuclei in 1.00 g is $\dfrac{1.00\times10^{-3}\ \text{kg}}{235m_\text{p}}=2.55\times10^{21}$. The energy released per gram is $(173.3\ \text{MeV/nucleus})(2.55\times10^{21}\ \text{nuclei/g})=4.42\times10^{23}\ \text{MeV/g}$.

43.46. **(a)** $^{28}_{14}\text{Si}+\gamma\Rightarrow{}^{24}_{12}\text{Mg}+^A_Z\text{X}$. $A+24=28$ so $A=4$. $Z+12=14$ so $Z=2$. X is an α particle.

(b) $E_\gamma=-\Delta mc^2=(23.985042\ \text{u}+4.002603\ \text{u}-27.976927\ \text{u})\ (931.5\ \text{MeV/u})=9.984\ \text{MeV}$

43.47. The energy liberated will be

$M(^3_2\text{He})+M(^4_2\text{He})-M(^7_4\text{Be})=(3.016029\ \text{u}+4.002603\ \text{u}-7.016929\ \text{u})(931.5\ \text{MeV/u})=1.586\ \text{MeV}.$

43.48. **(a)** $Z=3+2-0=5$ and $A=4+7-1=10$.

(b) The nuclide is a boron nucleus, and $m_\text{He}+m_\text{Li}-m_\text{n}-m_\text{B}=-3.00\times10^{-3}\ \text{u}$, and so 2.79 MeV of energy is absorbed.

43.49. Nuclei: $^A_Z\text{X}^{Z+}\rightarrow{}^{A-4}_{Z-2}\text{Y}^{(Z-2)+}+^4_2\text{He}^{2+}$. Add the mass of Z electrons to each side and we find:

$\Delta m=M(^A_Z\text{X})-M(^{A-4}_{Z-2}\text{Y})-M(^4_2\text{He})$, where now we have the mass of the neutral atoms. So as long as the mass of the original neutral atom is greater than the sum of the neutral products masses, the decay can happen.

43.50. Denote the reaction as $^A_Z\text{X}\rightarrow{}_{Z+1}{}^A\text{Y}+\text{e}^-$. The mass defect is related to the change in the neutral atomic masses by

$$[m_\text{X}-Zm_\text{e}]-[m_\text{Y}-(Z+1)m_\text{e}]-m_\text{e}=(m_\text{X}-m_\text{Y}),$$

where m_X and m_Y are the masses as tabulated in, for instance, Table (43.2).

43.51. $^A_Z\text{X}^{Z+}\rightarrow{}_{Z-1}{}^A\text{Y}^{(Z-1)+}+\beta^+$. Adding $(Z-1)$ electrons to both sides yields $^A_Z\text{X}^+\rightarrow{}_{Z-1}{}^A\text{Y}+\beta^+$. So in terms of masses:

$\Delta m=M\left(^A_Z\text{X}^+\right)-M\left(_{Z-1}{}^A\text{Y}\right)-m_\text{e}=\left(M\left(^A_Z\text{X}\right)-m_\text{e}\right)-M\left(_{Z-1}{}^A\text{Y}\right)-m_\text{e}=M\left(^A_Z\text{X}\right)-M\left(_{Z-1}{}^A\text{Y}\right)-2m_\text{e}$. So the decay will occur as long as the original neutral mass is greater than the sum of the neutral product mass and two electron masses.

43.52. IDENTIFY and SET UP: $m=\rho V$. $1\ \text{gal}=3.788\ \text{L}=3.788\times10^{-3}\ \text{m}^3$. The mass of a ^{235}U nucleus is $235m_\text{p}$. $1\ \text{MeV}=1.60\times10^{-13}\ \text{J}$

EXECUTE: **(a)** For 1 gallon, $m=\rho V=(737\ \text{kg/m}^3)(3.788\times10^{-3}\ \text{m}^3)=2.79\ \text{kg}=2.79\times10^3\ \text{g}$

$$\frac{1.3\times10^8\ \text{J/gal}}{2.79\times10^3\ \text{g/gal}}=4.7\times10^4\ \text{J/g}$$

(b) 1 g contains $\dfrac{1.00\times10^{-3}\text{ kg}}{235m_\text{p}}=2.55\times10^{21}$ nuclei

$(200\text{ MeV/nucleus})(1.60\times10^{-13}\text{ J/MeV})(2.55\times10^{21}\text{ nuclei})=8.2\times10^{10}\text{ J/g}$

(c) A mass of $6m_\text{p}$ produces 26.7 MeV.

$$\frac{(26.7\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{6m_\text{p}}=4.26\times10^{14}\text{ J/kg}=4.26\times10^{11}\text{ J/g}$$

(d) The total energy available would be $(1.99\times10^{30}\text{ kg})(4.7\times10^{7}\text{ J/kg})=9.4\times10^{37}\text{ J}$

$$\text{power}=\frac{\text{energy}}{t}\text{ so }t=\frac{\text{energy}}{\text{power}}=\frac{9.4\times10^{37}\text{ J}}{3.86\times10^{26}\text{ W}}=2.4\times10^{11}\text{ s}=7600\text{ yr}$$

EVALUATE: If the mass of the sun were all proton fuel, it would contain enough fuel to last $(7600\text{ yr})\left(\dfrac{4.3\times10^{11}\text{ J/g}}{4.7\times10^{4}\text{ J/g}}\right)=7.0\times10^{10}\text{ yr}$.

43.53. Using Eq: (43.12): ${}^{A}_{Z}M=ZM_\text{H}+Nm_\text{n}-E_\text{B}/c^2\Rightarrow M({}^{24}_{11}\text{Na})=11M_\text{H}+13m_\text{n}-E_\text{B}/c^2$.

But $E_\text{B}=(15.75\text{ MeV})(24)-(17.80\text{ MeV})(24)^{2/3}-(0.7100\text{ MeV})\dfrac{(11)(10)}{(24)^{1/3}}-$

$(23.69\text{ MeV})\dfrac{(24-2(11))^2}{24}-(39\text{ MeV})(24)^{-4/3}=198.31\text{ MeV}.$

$$\Rightarrow M({}^{24}_{11}\text{Na})=11(1.007825\text{ u})+13(1.008665\text{ u})-\frac{(198.31\text{ MeV})}{931.5\text{ MeV/u}}=23.9858\text{ u}$$

$$\%\text{ error}=\frac{23.990963-23.9858}{23.990963}\times100=0.022\%.$$

If the binding energy term is neglected, $M({}^{24}_{11}\text{Na})=24.1987\text{ u}$ and the percentage error would be

$$\frac{24.1987-23.990963}{23.990963}\times100=0.87\%.$$

43.54. The α-particle will have $\dfrac{226}{230}$ of the released energy (see Example 43.5). $\dfrac{226}{230}(m_\text{Th}-m_\text{Ra}-m_\alpha)=$ 5.032×10^{-3} u or 4.69 MeV.

43.55. **(a) IDENTIFY and SET UP:** The heavier nucleus will decay into the lighter one.

EXECUTE: ${}^{25}_{13}\text{Al}$ will decay into ${}^{25}_{12}\text{Mg}$.

(b) IDENTIFY and SET UP: Determine the emitted particle by balancing A and Z in the decay reaction.

EXECUTE: This gives ${}^{25}_{13}\text{Al}\rightarrow{}^{25}_{12}\text{Mg}+{}^{0}_{+1}\text{e}$. The emitted particle must have charge $+e$ and its nucleon number must be zero. Therefore, it is a β^+ particle, a positron.

(c) IDENTIFY and SET UP: Calculate the energy defect ΔM for the reaction and find the energy equivalent of ΔM. Use the nuclear masses for ${}^{25}_{13}\text{Al}$ and ${}^{25}_{12}\text{Mg}$, to avoid confusion in including the correct number of electrons if neutral atom masses are used.

EXECUTE: The nuclear mass for ${}^{25}_{13}\text{Al}$ is $M_\text{nuc}({}^{25}_{13}\text{Al})=24.990429\text{ u}-13(0.000548580\text{ u})=24.983297\text{ u}.$

The nuclear mass for ${}^{25}_{12}\text{Mg}$ is $M_\text{nuc}({}^{25}_{12}\text{Mg})=24.985837\text{ u}-12(0.000548580\text{ u})=24.979254\text{ u}.$

The mass defect for the reaction is

$\Delta M=M_\text{nuc}({}^{25}_{13}\text{Al})-M_\text{nuc}({}^{25}_{12}\text{Mg})-M({}^{0}_{+1}\text{e})=24.983297\text{ u}-24.979254\text{ u}-0.00054858\text{ u}=0.003494\text{ u}$

$Q=(\Delta M)c^2=0.003494\text{ u}(931.5\text{ MeV/1 u})=3.255\text{ MeV}$

EVALUATE: The mass decreases in the decay and energy is released. Note: ${}^{25}_{13}\text{Al}$ can also decay into ${}^{25}_{12}\text{Mg}$ by the electron capture.

${}^{25}_{13}\text{Al}+{}^{0}_{-1}\text{e}\rightarrow{}^{25}_{12}\text{Mg}$

The ${}^{0}_{-1}$ electron in the reaction is an orbital electron in the neutral ${}^{25}_{13}\text{Al}$ atom. The mass defect can be calculated using the nuclear masses:

$\Delta M=M_\text{nuc}\left({}^{25}_{13}\text{Al}\right)+M({}^{0}_{-1}\text{e})-M_\text{nuc}\left({}^{25}_{12}\text{Mg}\right)=24.983287\text{ u}+0.00054858\text{ u}-24.979254\text{ u}=0.004592\text{ u}.$

$Q=\left(\Delta M\right)^2c^2=(0.004592\text{ u})(931.5\text{ MeV/1 u})=4.277\text{ MeV}$

The mass decreases in the decay and energy is released.

43.56. **(a)** $m_{^{210}_{84}\text{Po}} - m_{^{206}_{82}\text{Pb}} - m_{^{4}_{2}\text{He}} = 5.81\times10^{-3}$ u, or $Q = 5.41$ MeV. The energy of the alpha particle is $(206/210)$ times this, or 5.30 MeV (see Example 43.5).

(b) $m_{^{210}_{84}\text{Po}} - m_{^{209}_{83}\text{Bi}} - m_{^{1}_{1}\text{H}} = -5.35\times10^{-3}\,\text{u} < 0,$ so the decay is not possible.

(c) $m_{^{210}_{84}\text{Po}} - m_{^{209}_{84}\text{Po}} - m_{\text{n}} = -8.22\times10^{-3}\,\text{u} < 0,$ so the decay is not possible.

(d) $m_{^{210}_{85}\text{At}} > m_{^{210}_{84}\text{Po}},$ so the decay is not possible (see Problem (43.50)).

(e) $m_{^{210}_{83}\text{Bi}} + 2m_{\text{e}} > m_{^{210}_{84}\text{Po}},$ so the decay is not possible (see Problem (43.51)).

43.57. **IDENTIFY and SET UP:** The amount of kinetic energy released is the energy equivalent of the mass change in the decay. $m_{\text{e}} = 0.0005486$ u and the atomic mass of $^{14}_{7}\text{N}$ is 14.003074 u. The energy equivalent of 1 u is 931.5 MeV. ^{14}C has a half-life of $T_{1/2} = 5730\text{ yr} = 1.81\times10^{11}$ s. The RBE for an electron is 1.0.

EXECUTE: **(a)** $^{14}_{6}\text{C} \to \text{e}^- + {}^{14}_{7}\text{N} + \overline{\nu}_{\text{e}}$

(b) The mass decrease is $\Delta M = m\left(^{14}_{6}\text{C}\right) - \left[m_{\text{e}} + m\left(^{14}_{7}\text{N}\right)\right]$. Use nuclear masses, to avoid difficulty in accounting for atomic electrons. The nuclear mass of $^{14}_{6}\text{C}$ is $14.003242\text{ u} - 6m_{\text{e}} = 13.999950$ u.

The nuclear mass of $^{14}_{7}\text{N}$ is $14.003074\text{ u} - 7m_{\text{e}} = 13.999234$ u.

$\Delta M = 13.999950\text{ u} - 13.999234\text{ u} - 0.000549\text{ u} = 1.67\times10^{-4}$ u. The energy equivalent of ΔM is 0.156 MeV.

(c) The mass of carbon is $(0.18)(75\text{ kg}) = 13.5$ kg. From Example 43.9, the activity due to 1 g of carbon in a living organism is 0.255 Bq. The number of decay/s due to 13.5 kg of carbon is $(13.5\times10^3)(0.255\text{ Bq/g}) =$ 3.4×10^3 decays/s.

(d) Each decay releases 0.156 MeV so 3.4×10^3 decays/s releases $530\text{ MeV/s} = 8.5\times10^{-11}$ J/s.

(e) The total energy absorbed in 1 yr is $(8.5\times10^{-11}\text{ J/s})(3.156\times10^7\text{ s}) = 2.7\times10^{-3}$ J. The absorbed dose is $\dfrac{2.7\times10^{-3}\text{ J}}{75\text{ kg}} = 3.6\times10^{-5}\text{ J/kg} = 36\ \mu\text{Gy} = 3.6\text{ mrad}$. With RBE = 1.0, the equivalent dose is $36\ \mu\text{Sv} = 3.6$ mrem.

43.58. **IDENTIFY** and **SET UP:** $m_\pi = 264m_{\text{e}} = 2.40\times10^{-28}$ kg. The total energy of the two photons equals the rest mass energy $m_\pi c^2$ of the pion.

EXECUTE: **(a)** $E_{\text{ph}} = \frac{1}{2}m_\pi c^2 = \frac{1}{2}(2.40\times10^{-28}\text{ kg})(3.00\times10^8\text{ m/s})^2 = 1.08\times10^{-11}\text{ J} = 67.5\text{ MeV}$

$$E_{\text{ph}} = \frac{hc}{\lambda} \text{ so } \lambda = \frac{hc}{E_{\text{ph}}} = \frac{1.24\times10^{-6}\text{ eV}\cdot\text{m}}{67.5\times10^{6}\text{ eV}} = 1.84\times10^{-14}\text{ m} = 18.4\text{ fm}$$

These are gamma ray photons, so they have RBE = 1.0.

(b) Each pion delivers $2(1.08\times10^{-11}\text{ J}) = 2.16\times10^{-11}$ J.

The absorbed dose is $200\text{ rad} = 2.00\text{ Gy} = 2.00$ J/kg.

The energy deposited is $(25\times10^{-3}\text{ kg})(2.00\text{ J/kg}) = 0.050$ J.

The number of π^0 mesons needed is $\dfrac{0.050\text{ J}}{2.16\times10^{-11}\text{ J/meson}} = 2.3\times10^9$ mesons.

EVALUATE: Note that charge is conserved in the decay since the pion is neutral. If the pion is initially at rest the photons must have equal momenta in opposite directions so the two photons have the same λ and are emitted in opposite directions. The photons also have equal energies since they have the same momentum and $E = pc$.

43.59. **IDENTIFY** and **SET UP:** Find the energy equivalent of the mass decrease. Part of the released energy appears as the emitted photon and the rest as kinetic energy of the electron.

EXECUTE: $^{198}_{79}\text{Au} \to {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e}$

The mass change is $197.968225\text{ u} - 197.966752\text{ u} = 1.473\times10^{-3}$ u

(The neutral atom masses include 79 electrons before the decay and 80 electrons after the decay. This one additional electron in the products accounts correctly for the electron emitted by the nucleus.) The total energy released in the decay is $(1.473\times10^{-3}\text{ u})(931.5\text{ MeV/u}) = 1.372$ MeV. This energy is divided between the energy of the emitted photon and the kinetic energy of the β^- particle. Thus the β^- particle has kinetic energy equal to $1.372\text{ MeV} - 0.412\text{ MeV} = 0.960$ MeV.

EVALUATE: The emitted electron is much lighter than the $^{198}_{80}\text{Hg}$ nucleus, so the electron has almost all the final kinetic energy. The final kinetic energy of the ^{198}Hg nucleus is very small.

43.60. (See Problem (43.51)) $m_{^{11}_{6}\text{C}} - m_{^{11}_{5}\text{B}} - 2m_{\text{e}} = 1.03\times10^{-3}$ u. Decay is energetically possible.

43.61. **IDENTIFY** and **SET UP:** The decay is energetically possible if the total mass decreases. Determine the nucleus produced by the decay by balancing A and Z on both sides of the equation. ${}^{13}_{7}\text{N} \to {}^{0}_{+1}\text{e} + {}^{13}_{6}\text{C}$. To avoid confusion in including the correct number of electrons with neutral atom masses, use nuclear masses, obtained by subtracting the mass of the atomic electrons from the neutral atom masses.

EXECUTE: The nuclear mass for ${}^{13}_{7}\text{N}$ is $M_{\text{nuc}}\left({}^{13}_{7}\text{N}\right) = 13.005739\text{ u} - 7(0.00054858\text{ u}) = 13.001899\text{ u}.$

The nuclear mass for ${}^{13}_{6}\text{C}$ is $M_{\text{nuc}}\left({}^{13}_{6}\text{C}\right) = 13.003355\text{ u} - 6(0.00054858\text{ u}) = 13.000064\text{ u}.$

The mass defect for the reaction is

$\Delta M = M_{\text{nuc}}\left({}^{13}_{7}\text{N}\right) - M_{\text{nuc}}\left({}^{13}_{6}\text{C}\right) - M\left({}^{0}_{+1}\text{e}\right)$. $\Delta M = 13.001899\text{ u} - 13.000064\text{ u} - 0.00054858\text{ u} = 0.001286\text{ u}.$

EVALUATE: The mass decreases in the decay, so energy is released. This decay is energetically possible.

43.62. **(a)** A least-squares fit to log of the activity *vs.* time gives a slope of $\lambda = 0.5995\text{ h}^{-1}$, for a half-life of $\dfrac{\ln 2}{\lambda} = 1.16\text{ h}.$

(b) The initial activity is $N_0\lambda$, and this gives $N_0 = \dfrac{(2.00\times10^4\text{ Bq})}{(0.5995\text{ hr}^{-1})(1\text{ hr}/3600\text{ s})} = 1.20\times10^8.$

(c) $N_0 e^{-\lambda t} = 1.81\times10^6.$

43.63. The activity $A(t) \equiv \dfrac{dN(t)}{dt}$ but $\dfrac{dN(t)}{dt} = -\lambda N(t)$ so $-\lambda N_0 = A_0$. Taking the derivative of

$N(t) = N_0 e^{-\lambda t} \Rightarrow \dfrac{dN(t)}{dt} = -\lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$, or $A(t) = A_0 e^{-\lambda t}$.

43.64. From Eq.43.17 $N(t) = N_0 e^{-\lambda t}$ but $N_0 e^{-\lambda t} = N_0 e^{-(\ln 2)\left(t/T_{1/2}\right)}$

$= N_0\left[e^{-(\ln 2)}\right]^{\left(t/T_{1/2}\right)} = N_0\left[e^{\ln(1/2)}\right]^{\left(t/T_{1/2}\right)}$. So $N(t) = N_0\left(\dfrac{1}{2}\right)^n$ where $n = \dfrac{t}{T_{1/2}}$.

(We have used that $a\ln x = \ln(x^a)$, $e^{ax} = (e^x)^a$, and $e^{\ln x} = x$.)

43.65. **IDENTIFY and SET UP:** One-half of the sample decays in a time of $T_{1/2}$.

EXECUTE: **(a)** $\dfrac{10\times10^9\text{ yr}}{200{,}000\text{ yr}} = 5.0\times10^4$

(b) $(\tfrac{1}{2})^{5.0\times10^4}$. This exponent is too large for most hand-held calculators. But $(\tfrac{1}{2}) = 10^{-0.301}$ so

$(\tfrac{1}{2})^{5.0\times10^4} = (10^{-0.301})^{5.0\times10^4} = 10^{-15{,}000}$

43.66. **IDENTIFY and SET UP:** $T_{1/2} = \dfrac{\ln 2}{\lambda}$. The mass of a single nucleus is $149m_{\text{p}} = 2.49\times10^{-25}$ kg. $\Delta N/\Delta t = -\lambda N$.

EXECUTE: $N = \dfrac{12.0\times10^{-3}\text{ kg}}{2.49\times10^{-25}\text{ kg}} = 4.82\times10^{22}$. $\Delta N/\Delta t = -2.65$ decays/s

$\lambda = -\dfrac{\Delta N/\Delta t}{N} = \dfrac{2.65\text{ decays/s}}{4.82\times10^{22}} = 5.50\times10^{-23}\text{ s}^{-1}$; $T_{1/2} = \dfrac{\ln 2}{\lambda} = 1.26\times10^{22}\text{ s} = 3.99\times10^{14}\text{ yr}$

43.67. **IDENTIFY:** Use Eq. (43.17) to relate the initial number of radioactive nuclei, N_0, to the number , N, left after time t.

SET UP: We have to be careful; after ${}^{87}\text{Rb}$ has undergone radioactive decay it is no longer a rubidium atom. Let N_{85} be the number of ${}^{85}\text{Rb}$ atoms; this number doesn't change. Let N_0 be the number of ${}^{87}\text{Rb}$ atoms on earth when the solar system was formed. Let N be the present number of ${}^{87}\text{Rb}$ atoms.

EXECUTE: The present measurements say that $0.2783 = N/(N + N_{85})$.

$(N + N_{85})(0.2783) = N$, so $N = 0.3856N_{85}$. The percentage we are asked to calculate is $N_0/(N_0 + N_{85})$.

N and N_0 are related by $N = N_0 e^{-\lambda t}$ so $N_0 = e^{+\lambda t}N$.

Thus $\dfrac{N_0}{N_0 + N_{85}} = \dfrac{Ne^{\lambda t}}{Ne^{\lambda t} + N_{85}} = \dfrac{(0.3855e^{\lambda t})N_{85}}{(0.3856e^{\lambda t})N_{85} + N_{85}} = \dfrac{0.3856e^{\lambda t}}{0.3856e^{\lambda t} + 1}.$

$t = 4.6\times10^9\text{ y}$; $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{4.75\times10^{10}\text{ y}} = 1.459\times10^{-11}\text{ y}^{-1}$

$e^{\lambda t} = e^{(1.459\times10^{-11}\text{ y}^{-1})(4.6\times10^9\text{ y})} = e^{0.16711} = 1.0694$

Thus $\dfrac{N_0}{N_0 + N_{85}} = \dfrac{(0.3856)(1.0694)}{(0.3856)(1.0694)+1} = 29.2\%.$

EVALUATE: The half-life for ^{87}Rb is a factor of 10 larger than the age of the solar system, so only a small fraction of the ^{87}Rb nuclei initially present have decayed; the percentage of rubidium atoms that are radioactive is only a bit less now than it was when the solar system was formed.

43.68. **(a)** $(6.25\times10^{12})(4.77\times10^{6}\ \text{MeV})(1.602\times10^{-19}\ \text{J/eV})/(70.0\ \text{kg}) = 0.0682\ \text{Gy} = 0.682\ \text{rad}$

(b) (20)(6.82 rad) = 136 rem

(c) $N\lambda = \dfrac{m}{Am_\text{p}}\dfrac{\ln(2)}{T_{1/2}} = 1.17\times10^{9}\ \text{Bq} = 31.6\ \text{mCi}$.

(d) $\dfrac{6.25\times10^{12}}{1.17\times10^{9}\ \text{Bq}} = 5.34\times10^{3}$ s, about an hour and a half. Note that this time is so small in comparison with the half-life that the decrease in activity of the source may be neglected.

43.69. **IDENTIFY** and **SET UP:** Find the energy emitted and the energy absorbed each second. Convert the absorbed energy to absorbed dose and to equivalent dose.

EXECUTE: **(a)** First find the number of decays each second:

$$2.6\times10^{-4}\ \text{Ci}\left(\frac{3.70\times10^{10}\ \text{decays/s}}{1\ \text{Ci}}\right) = 9.6\times10^{6}\ \text{decays/s}$$

The average energy per decay is 1.25 MeV, and one-half of this energy is deposited in the tumor. The energy delivered to the tumor per second then is

$\frac{1}{2}(9.6\times10^{6}\ \text{decays/s})(1.25\times10^{6}\ \text{eV/decay})(1.602\times10^{-19}\ \text{J/eV}) = 9.6\times10^{-7}\ \text{J/s}.$

(b) The absorbed dose is the energy absorbed divided by the mass of the tissue:

$$\frac{9.6\times10^{-7}\ \text{J/s}}{0.500\ \text{kg}} = (1.9\times10^{-6}\ \text{J/kg}\cdot\text{s})(1\ \text{rad/(0.01 J/kg)}) = 1.9\times10^{-4}\ \text{rad/s}$$

(c) equivalent dose (REM) = RBE × absorbed dose (rad)

In one second the equivalent dose is $0.70(1.9\times10^{-4}\ \text{rad}) = 1.3\times10^{-4}$ rem.

(d) $(200\ \text{rem}/1.3\times10^{-4}\ \text{rem/s}) = 1/5\times10^{6}\ \text{s}(1\ \text{h}/3600\ \text{s}) = 420\ \text{h} = 17$ days.

EVALUATE: The activity of the source is small so that absorbed energy per second is small and it takes several days for an equivalent dose of 200 rem to be absorbed by the tumor. A 200 rem dose equals 2.00 Sv and this is large enough to damage the tissue of the tumor.

43.70. **(a)** After 4.0 min = 240 s, the ratio of the number of nuclei is $\dfrac{2^{-240/122.2}}{2^{-240/26.9}} = 2^{(240)\left(\frac{1}{26.9}-\frac{1}{122.2}\right)} = 124.$

(b) After 15.0 min = 900 s, the ratio is 7.15×10^{7}.

43.71. **IDENTIFY** and **SET UP:** The number of radioactive nuclei left after time *t* is given by $N = N_0e^{-\lambda t}$. The problem says $N/N_0 = 0.21$; solve for *t*.

EXECUTE: $0.21 = e^{-\lambda t}$ so $\ln(0.21) = -\lambda t$ and $t = -\ln(0.21)/\lambda$

Example 43.9 gives $\lambda = 1.209\times10^{-4}\ \text{y}^{-1}$ for ^{14}C. Thus $t = \dfrac{-\ln(0.21)}{1.209\times10^{-4}\ \text{y}} = 1.3\times10^{4}$ y.

EVALUATE: The half-life of ^{14}C is 5730 y, so our calculated *t* is more than two half-lives, so the fraction remaining is less than $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

43.72. **IDENTIFY:** The tritium (H-3) decays to He-3. The ratio of the number of He-3 atoms to H-3 atoms allows us to calculate the time since the decay began, which is when the H-3 was formed by the nuclear explosion. The H-3 decay is exponential.

SET UP: The number of tritium (H-3) nuclei decreases exponentially as $N_\text{H} = N_{0,\text{H}}e^{-\lambda t}$, with a half-life of 12.3 years. The amount of He-3 present after a time *t* is equal to the original amount of tritium minus the number of tritium nuclei that are still undecayed after time *t*.

EXECUTE: The number of He-3 nuclei after time *t* is

$$N_\text{He} = N_{0,\text{H}} - N_\text{H} = N_{0,\text{H}} - N_{0,\text{H}}e^{-\lambda t} = N_{0,\text{H}}\left(1-e^{-\lambda t}\right).$$

Taking the ratio of the number of He-3 atoms to the number of tritium (H-3) atoms gives

$$\frac{N_\text{He}}{N_\text{H}} = \frac{N_{0,\text{H}}\left(1-e^{-\lambda t}\right)}{N_{0,\text{H}}e^{-\lambda t}} = \frac{1-e^{-\lambda t}}{e^{-\lambda t}} = e^{\lambda t} - 1.$$

Solving for t gives $t = \frac{\ln(1 + N_{\text{He}}/N_{\text{H}})}{\lambda}$. Using the given numbers and $T_{1/2} = \frac{\ln 2}{\lambda}$, we have

$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{12.3\text{ y}} = 0.0563/\text{y}$ and $t = \frac{\ln(1+4.3)}{0.0563/\text{y}}$ = 30 years.

EVALUATE: One limitation on this method would be that after many years the ratio of H to He would be too small to measure accurately.

43.73. **(a) IDENTIFY** and **SET UP:** Use Eq.(43.1) to calculate the radius R of a ${}^2_1\text{H}$ nucleus. Calculate the Coulomb potential energy (Eq.23.9) of the two nuclei when they just touch.

EXECUTE: The radius of ${}^2_1\text{H}$ is $R = (1.2\times10^{-15}\text{ m})(2)^{1/3} = 1.51\times10^{-15}$ m. The barrier energy is the Coulomb potential energy of two ${}^2_1\text{H}$ nuclei with their centers separated by twice this distance:

$$U = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(1.602\times10^{-19}\text{ C})^2}{2(1.51\times10^{-15}\text{ m})} = 7.64\times10^{-14}\text{ J} = 0.48\text{ MeV}$$

(b) IDENTIFY and **SET UP:** Find the energy equivalent of the mass decrease.

EXECUTE: ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + {}^1_0\text{n}$

If we use neutral atom masses there are two electrons on each side of the reaction equation, so their masses cancel. The neutral atom masses are given in Table 43.2.

${}^2_1\text{H} + {}^2_1\text{H}$ has mass $2(2.014102\text{ u}) = 4.028204\text{ u}$

${}^3_2\text{He} + {}^1_0\text{n}$ has mass $3.016029\text{ u} + 1.008665\text{ u} = 4.024694\text{ u}$

The mass decrease is $4.028204\text{ u} - 4.024694\text{ u} = 3.510\times10^{-3}$ u. This corresponds to a liberated energy of $(3.510\times10^{-3}\text{ u})(931.5\text{ MeV/u}) = 3.270\text{ MeV}$, or $(3.270\times10^6\text{ eV})(1.602\times10^{-19}\text{ J/eV}) = 5.239\times10^{-13}$ J.

(c) IDENTIFY and **SET UP:** We know the energy released when two ${}^2_1\text{H}$ nuclei fuse. Find the number of reactions obtained with one mole of ${}^2_1\text{H}$.

EXECUTE: Each reaction takes two ${}^2_1\text{H}$ nuclei. Each mole of D_2 has 6.022×10^{23} molecules, so 6.022×10^{23} pairs of atoms. The energy liberated when one mole of deuterium undergoes fusion is $(6.022\times10^{23})(5.239\times10^{-13}\text{ J}) = 3.155\times10^{11}$ J/mol.

EVALUATE: The energy liberated per mole is more than a million times larger than from chemical combustion of one mole of hydrogen gas.

43.74. In terms of the number N of cesium atoms that decay in one week and the mass $m = 1.0$ kg, the equivalent dose is

$3.5\text{ Sv} = \frac{N}{m}((\text{RBE})_\gamma \text{E}_\gamma + (\text{RBE})_\text{e}\text{E}_\text{e}) = \frac{N}{m}((1)(0.66\text{ MeV}) + (1.5)(0.51\text{ MeV})) = \frac{N}{m}(2.283\times10^{-13}\text{ J})$, so

$N = \frac{(1.0\text{ kg})(3.5\text{ Sv})}{(2.283\times10^{-13}\text{ J})} = 1.535\times10^{13}$. The number N_0 of atoms present is related to

N by $N_0 = Ne^{\lambda t}$. $\lambda = \frac{\ln 2}{Ty_2}\frac{0.693}{(30.07\text{ yr})(3.156\times10^7\text{ sec/yr})} = 7.30\times10^{-10}\text{ sec}^{-1}$.

Then $N_0 = Ne^{\lambda t} = (1.535\times10^{13})e^{(7.30\times10^{-10}\text{ s}^{-1})(7\text{ days})(8.64\times10^4\text{ s/day})} = 1.536\times10^{13}$.

43.75. **(a)** $v_{\text{cm}} = v\frac{m}{m+M}$. $v'_m = v - v\frac{m}{m+M} = \left(\frac{M}{m+M}\right)v$. $v'_M = \frac{vm}{m+M}$.

$$K' = \frac{1}{2}mv'^2_m + \frac{1}{2}Mv'^2_M = \frac{1}{2}\frac{mM^2}{(m+M)^2}v^2 + \frac{1}{2}\frac{Mm^2}{(m+M)^2}v^2 = \frac{1}{2}\frac{M}{(m+M)}\left(\frac{mM}{m+M} + \frac{m^2}{m+M}\right)v^2.$$

$$K' = \frac{M}{m+M}\left(\frac{1}{2}mv^2\right) \Rightarrow K' = \frac{M}{m+M}K \equiv K_{\text{cm}}.$$

(b) For an endoergic reaction $K_{\text{cm}} = -Q\,(Q < 0)$ at threshold. Putting this into part (a) gives

$$-Q = \frac{M}{M+m}K_{\text{th}} \Rightarrow K_{\text{th}} = \frac{-(M+m)}{M}Q$$

43.76. $K = \dfrac{M_\alpha}{M_\alpha + m} K_\infty$, where K_∞ is the energy that the α-particle would have if the nucleus were infinitely massive.

Then, $M = M_{\text{Os}} - M_\alpha - K_\infty = M_{\text{Os}} - M_\alpha - \dfrac{186}{182}\left(2.76\ \text{MeV}/c^2\right) = 181.94821\ \text{u}$.

43.77. $\Delta m = M\left({}^{235}_{92}\text{U}\right) - M\left({}^{140}_{54}\text{Xe}\right) - M\left({}^{94}_{38}\text{Sr}\right) - m_{\text{n}}$

$\Delta m = 235.043923\ \text{u} - 139.921636\ \text{u} - 93.915360\ \text{u} - 1.008665\ \text{u} = 0.1983\ \text{u}$

$\Rightarrow E = (\Delta m)c^2 = (0.1983\ \text{u})(931.5\ \text{MeV/u}) = 185\ \text{MeV}$.

43.78. **(a)** A least-squares fit of the log of the activity *vs.* time for the times later than 4.0 h gives a fit with correlation $-\left(1 - 2\times10^{-6}\right)$ and decay constant of $0.361\ \text{h}^{-1}$, corresponding to a half-life of 1.92 h. Extrapolating this back to time 0 gives a contribution to the rate of about 2500/s for this longer-lived species. A least-squares fit of the log of the activity *vs*. time for times earlier than 2.0 h gives a fit with correlation = 0.994, indicating the presence of only two species.

(b) By trial and error, the data is fit by a decay rate modeled by $R = (5000\ \text{Bq})e^{-t(1.733/\text{h})} + (2500\ \text{Bq})e^{-t(0.361/\text{h})}$. This would correspond to half-lives of 0.400 h and 1.92 h.

(c) In this model, there are 1.04×10^7 of the shorter-lived species and 2.49×10^7 of the longer-lived species.

(d) After 5.0 h, there would be 1.80×10^3 of the shorter-lived species and 4.10×10^6 of the longer-lived species.

43.79. **(a)** There are two processes occurring: the creation of ${}^{128}\text{I}$ by the neutron irradiation, and the decay of the newly produced ${}^{128}\text{I}$. So $\dfrac{dN}{dt} = K - \lambda N$ where K is the rate of production by the neutron irradiation. Then

$\int_0^N \dfrac{dN'}{K - \lambda N'} = \int_0^t dt$. $\left[\ln\left(K - \lambda N'\right)\right]_0^N = -\lambda t$. $\ln(K - \lambda N) = \ln K - \lambda t$. $N(t) = \dfrac{K\left(1 - e^{-\lambda t}\right)}{\lambda}$. The graph is given in Figure 43.79.

(b) The activity of the sample is $\lambda N(t) = K\left(1 - e^{-\lambda t}\right) = \left(1.5\times10^6\ \text{decays/s}\right)\times\left(1 - e^{-\left(\frac{0.693}{25\ \text{min}}\right)t}\right)$. So the activity is

$\left(1.5\times10^6\ \text{decays/s}\right)\left(1 - e^{-0.02772\,t}\right)$, with *t* in minutes. So the activity $\left(\dfrac{-dN'}{dt}\right)$ at various times is:

$$\frac{-dN'}{dt}(t = 1\ \text{min}) = 4.1\times10^4\ \text{Bq};\ \frac{-dN'}{dt}(t = 10\ \text{min}) = 3.6\times10^5\ \text{Bq};$$
$$\frac{-dN'}{dt}(t = 25\ \text{min}) = 7.5\times10^5\ \text{Bq};\ \frac{-dN'}{dt}(t = 50\ \text{min}) = 1.1\times10^6\ \text{Bq};$$
$$\frac{-dN'}{dt}(t = 75\ \text{min}) = 1.3\times10^6\ \text{Bq};\ \frac{-dN'}{dt}(t = 180\ \text{min}) = 1.5\times10^6\ \text{Bq};$$

(c) $N_{\text{max}} = \dfrac{K}{\lambda} = \dfrac{(1.5\times10^6)\,(60)}{(0.02772)} = 3.2\times10^9$ atoms.

(d) The maximum activity is at saturation, when the rate being produced equals that decaying and so it equals 1.5×10^6 decays/s.

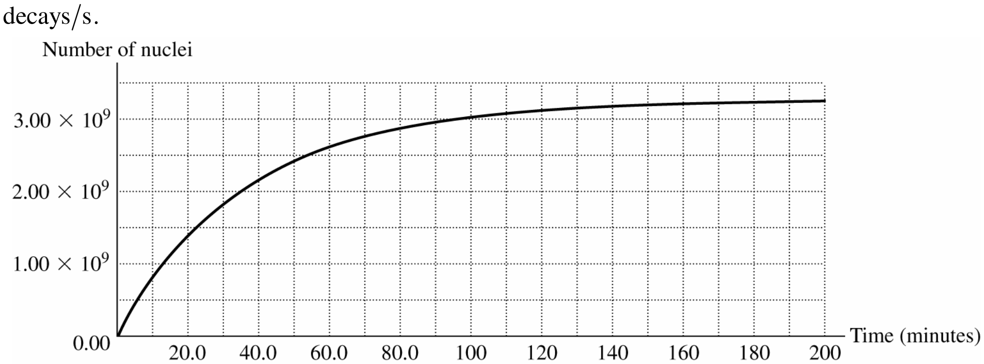

Figure 43.79

43.80. The activity of the original iron, after 1000 hours of operation, would be $(9.4\times10^{-6}\,\text{Ci})\,(3.7\times10^{10}\ \text{Bq/Ci})2^{-(1000\ \text{h})/(45\ \text{d}\times24\ \text{h/d})} = 1.8306\times10^5\ \text{Bq}$. The activity of the oil is 84 Bq, or 4.5886×10^{-4} of the total iron activity, and this must be the fraction of the mass worn, or mass of $4.59\times10^{-2}\ \text{g}$. The rate at which the piston rings lost their mass is then $4.59\times10^{-5}\ \text{g/h}$.