41

ATOMIC STRUCTURE

41.1. **IDENTIFY** and **SET UP:** $L=\sqrt{l(l+1)}\hbar$. $L_z=m_l\hbar$. $l=0,\ 1,\ 2,...,\ n-1$. $m_l=0,\ \pm1,\ \pm2,...,\ \pm l$. $\cos\theta=L_z/L$.

EXECUTE: **(a)** $\underline{l=0}$: $L=0$, $L_z=0$. $\underline{l=1}$: $L=\sqrt{2}\hbar$, $L_z=\hbar,0,-\hbar$. $\underline{l=2}$: $L=\sqrt{6}\hbar$, $L_z=2\hbar,\hbar,0,-\hbar,-2\hbar$.

(b) In each case $\cos\theta=L_z/L$. $L=0$: θ not defined. $L=\sqrt{2}\hbar$: $45.0°,\ 90.0°,\ 135.0°$. $L=\sqrt{6}\hbar$: $35.3°,\ 65.9°,\ 90.0°,\ 114.1°,\ 144.7°$.

EVALUATE: There is no state where $\vec{L}$ is totally aligned along the z axis.

41.2. **IDENTIFY** and **SET UP:** $L=\sqrt{l(l+1)}\hbar$. $L_z=m_l\hbar$. $l=0,1,2,...,n-1$. $m_l=0,\pm1,\pm2,...,\pm l$. $\cos\theta=L_z/L$.

EXECUTE: **(a)** $\underline{l=0}$: $L=0$, $L_z=0$. $\underline{l=1}$: $L=\sqrt{2}\hbar$, $L_z=\hbar,0,-\hbar$. $\underline{l=2}$: $L=\sqrt{6}\hbar$, $L_z=2\hbar,\hbar,0,-\hbar,-2\hbar$. $\underline{l=3}$: $L=2\sqrt{3}\hbar$, $L_z=3\hbar,2\hbar,\hbar,0,-\hbar,-2\hbar,-3\hbar$. $\underline{l=4}$: $L=2\sqrt{5}\hbar$, $L_z=4\hbar,3\hbar,2\hbar,\hbar,0,-\hbar,-2\hbar,-3\hbar,-4\hbar$.

(b) $L=0$: θ not defined. $L=\sqrt{2}\hbar$: $45.0°,90.0°,135.0°$. $L=\sqrt{6}\hbar$: $35.3°,65.9°,90.0°,114.1°,144.7°$. $L=2\sqrt{3}\hbar$: $54.7°,73.2°,90.0°,106.8°,125.3°,150.0°$. $L=2\sqrt{5}\hbar$: $26.6°,47.9°,63.4°,77.1°,90.0°,102.9°,116.6°,132.1°,153.4°$.

(c) The minimum angle is $26.6°$ and occurs for $l=4$, $m_l=+4$. The maximum angle is $153.4°$ and occurs for $l=4$, $m_l=-4$.

41.3. **IDENTIFY** and **SET UP:** The magnitude of the orbital angular momentum L is related to the quantum number l by Eq.(41.4): $L=\sqrt{l(l+1)}\hbar,\ 1=0,\ 1,\ 2,\ldots$

EXECUTE: $l(l+1)=\left(\dfrac{L}{\hbar}\right)^2=\left(\dfrac{4.716\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}\right)=20$

And then $l(l+1)=20$ gives that $l=4$.

EVALUATE: l must be integer.

41.4. **(a)** $(m_l)_{\text{max}}=2$, so $(L_z)_{\text{max}}=2\hbar$.

(b) $\sqrt{l(l+1)}\hbar=\sqrt{6}\hbar=2.45\hbar$.

(c) The angle is arccos $\left(\dfrac{L_z}{L}\right)=\arccos\left(\dfrac{m_l}{\sqrt{6}}\right)$, and the angles are, for $m_l=-2$ to $m_l=2$, $144.7°$, $114.1°,\ 90.0°,\ 65.9°, 35.3°$. The angle corresponding to $m_l=l$ will always be larger for larger l.

41.5. **IDENTIFY** and **SET UP:** The angular momentum L is related to the quantum number l by Eq.(41.4), $L=\sqrt{l(l+1)}\hbar$. The maximum l, l_{max}, for a given n is $l_{\text{max}}=n-1$.

EXECUTE: For $n=2,\ l_{\text{max}}=1$ and $L=\sqrt{2}\hbar=1.414\hbar$.

For $n=20,\ l_{\text{max}}=19$ and $L=\sqrt{(19)(20)}\hbar=19.49\hbar$.

For $n=200,\ l_{\text{max}}=199$ and $L=\sqrt{(199)(200)}\hbar=199.5\hbar$.

EVALUATE: As n increases, the maximum L gets closer to the value $n\hbar$ postulated in the Bohr model.

41.6. The (l,m_l) combinations are $(0,\ 0)$, $(1,\ 0)$, $(1,\ \pm1)$, $(2,\ 0)$, $(2,\ \pm1)$, $(2,\ \pm2)$, $(3,\ 0)$, $(3,\ \pm1)$, $(3,\ \pm2)$, $(3,\ \pm3)$, $(4,\ 0)$, $(4,\ \pm1)$, $(4,\ \pm2)$, $(4,\ \pm3)$, and $(4,\ \pm4)$, a total of 25.

(b) Each state has the same energy (n is the same), $-\dfrac{13.60\ \text{eV}}{25}=-0.544$ eV.

41.7. $U=\dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r}=\dfrac{-1}{4\pi\epsilon_0}\dfrac{(1.60\times10^{-19}\ \text{C})^2}{1.0\times10^{-10}\ \text{m}}=-2.3\times10^{-18}\ \text{J}$

$$U=\frac{-2.3\times10^{-18}\ \text{J}}{1.60\times10^{-19}\ \text{J/eV}}=-14.4\ \text{eV}.$$

41.8. (**a**) As in Example 41.3, the probability is

$$P=\int_0^{a/2}|\psi_{1s}|^2\,4\pi r^2dr=\frac{4}{a^3}\left[\left(-\frac{ar^2}{2}-\frac{a^2r}{2}-\frac{a^3}{4}\right)e^{-2r/a}\right]_0^{a/2}=1-\frac{5e^{-1}}{2}=0.0803\,.$$

(**b**) The difference in the probabilities is $(1-5e^{-2})-(1-(5/2)e^{-1})=(5/2)(e^{-1}-2e^{-2})=0.243.$

41.9. (**a**) $|\psi|^2=\psi^*\psi=|R(r)|^2|\Theta(\theta)|^2\,(Ae^{-im_l\phi})(Ae^{+im_l\phi})=A^2|R(r)|^2|\Theta(\theta)|^2,$ which is independent of ϕ .

(**b**) $\int_0^{2\pi}|\Phi(\phi)|^2\,d\phi=A^2\int_0^{2\pi}d\phi=2\pi A^2=1\Rightarrow A=\dfrac{1}{\sqrt{2\pi}}.$

41.10. $E_n=-\dfrac{1}{(4\pi\epsilon_0)^2}\dfrac{m_re^4}{2n^2\hbar^2}\Delta E_{12}=E_2-E_1=\dfrac{E_1}{2^2}-E_1=-(0.75)E_1.$

(**a**) If $m_\mathrm{r}=m=9.11\times10^{-31}$ kg

$\dfrac{m_\mathrm{r}e^4}{(4\pi\epsilon_0)^2\hbar^2}=\dfrac{(9.109\times10^{-31}\text{ kg})(1.602\times10^{-19}\text{ C})^4}{2(1.055\times10^{-34}\text{ J}\cdot\text{s})^2}\left(8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}\right)^2=2.177\times10^{-18}\text{ J}=13.59\text{ eV}$

For $2\to1$ transition, the coefficient is $(0.75)(13.59\text{ eV})=10.19\text{ eV}.$

(**b**) If $m_\mathrm{r}=\dfrac{m}{2},$ using the result from part (a),

$$\frac{m_\mathrm{r}e^4}{(4\pi\epsilon_0)^2\hbar^2}=(13.59\text{ eV})\left(\frac{m/2}{m}\right)=\left(\frac{13.59\text{ eV}}{2}\right)=6.795\text{ eV}.$$

Similarly, the $2\to1$ transition, $\Rightarrow\left(\dfrac{10.19\text{ eV}}{2}\right)=5.095\text{ eV}.$

(**c**) If $m_\mathrm{r}=185.8m,$ using the result from part (a),

$$\frac{m_re^4}{(4\pi\epsilon_0)^2\hbar^2}=(13.59\text{ eV})\left(\frac{185.8m}{m}\right)=2525\text{ eV},$$

and the $2\to1$ transition gives $\Rightarrow$ (10.19 eV)(185.8) = 1893 eV.

41.11. **Identify** and **Set Up:** Eq.(41.8) gives $a=\dfrac{4\pi\epsilon_0\hbar^2}{m_\mathrm{r}e^2}=\dfrac{\epsilon_0h^2}{\pi m_\mathrm{r}e^2}.$

Execute: (**a**) $m_\mathrm{r}=m$

$a=\dfrac{\epsilon_0h^2}{\pi m_\mathrm{r}e^2}=\dfrac{(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{\pi(9.109\times10^{-31}\text{ kg})(1.602\times10^{-19}\text{ C})^2}=0.5293\times10^{-10}\text{ m}$

(**b**) $m_\mathrm{r}=m/2$

$a=2\left(\dfrac{\epsilon_0h^2}{\pi m_\mathrm{r}e^2}\right)=1.059\times10^{-10}\text{ m}$

(**c**) $m_\mathrm{r}=185.8m$

$a=\dfrac{1}{185.8}\left(\dfrac{\epsilon_0h^2}{\pi m_\mathrm{r}e^2}\right)=2.849\times10^{-13}\text{ m}$

Evaluate: a is the radius for the $n=1$ level in the Bohr model. When the reduced mass m_r increases, a decreases. For positronium and muonium the reduced mass effect is large.

41.12. $e^{im_l\phi}=\cos(m_l\phi)+i\sin(m_l\phi),$ and to be periodic with period $2\pi,$ $m_l2\pi$ must be an integer multiple of $2\pi,$ so m_l must be an integer.

41.13. $P(a)=\displaystyle\int_0^a|\psi_{1s}|2V=\int_0^a\frac{1}{\pi a^3}e^{-2r/a}(4\pi r^2dr)\,.$

$$P(a)=\frac{4}{a^3}\int_o^a r^2e^{-2r/a}dr=\frac{4}{a^3}\left[\left(\frac{-ar^2}{2}-\frac{a^2r}{2}-\frac{a^2}{4}\right)e^{-2r/a}\right]_0^a=\frac{4}{a^3}\left[\left(\frac{-a^3}{2}-\frac{a^3}{2}-\frac{a^3}{4}\right)e^{-2}+\frac{a^3}{4}e^0\right]$$

$\Rightarrow P(a)=1-5e^{-2}.$

41.14. (**a**) $\Delta E=\mu_\mathrm{B}B=(5.79\times10^{-5}\text{ eV/T})(0.400\text{ T})=2.32\times10^{-5}\text{ eV}$

(**b**) $m_l=-2$ the lowest possible value of m_l.

(c) The energy level diagram is sketched in Figure 41.14.

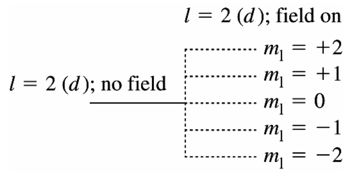

Figure 41.14

41.15. **IDENTIFY** and **SET UP:** The interaction energy between an external magnetic field and the orbital angular momentum of the atom is given by Eq.(41.18). The energy depends on m_l with the most negative m_l value having the lowest energy.

EXECUTE: **(a)** For the $5g$ level, $l = 4$ and there are $2l+1 = 9$ different m_l states. The $5g$ level is split into 9 levels by the magnetic field.

(b) Each m_l level is shifted in energy an amount given by $U = m_l \mu_B B$. Adjacent levels differ in m_l by one, so $\Delta U = \mu_B B$.

$$\mu_B = \frac{e\hbar}{2m} = \frac{(1.602\times10^{-19}\ \text{C})(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{2(9.109\times10^{-31}\ \text{kg})} = 9.277\times10^{-24}\ \text{A}\cdot\text{m}^2$$

$$\Delta U = \mu_B B = (9.277\times10^{-24}\ \text{A/m}^2)(0.600\ \text{T}) = 5.566\times10^{-24}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 3.47\times10^{-5}\ \text{eV}$$

(c) The level of highest energy is for the largest m_l, which is $m_l = l = 4$; $U_4 = 4\mu_B B$. The level of lowest energy is for the smallest m_l, which is $m_l = -l = -4$; $U_{-4} = -4\mu_B B$. The separation between these two levels is $U_4 - U_{-4} = 8\mu_B B = 8(3.47\times10^{-5}\ \text{eV}) = 2.78\times10^{-4}\ \text{eV}$.

EVALUATE: The energy separations are proportional to the magnetic field. The energy of the $n = 5$ level in the absence of the external magnetic field is $(-13.6\ \text{eV})/5^2 = -0.544\ \text{eV}$, so the interaction energy with the magnetic field is much less than the binding energy of the state.

41.16. **(a)** According to Figure 41.11 in the textbook there are three different transitions that are consistent with the selection rules. The initial m_l values are $0,\ \pm1$; and the final m_l value is 0.

(b) The transition from $m_l = 0$ to $m_l = 0$ produces the same wavelength (122 nm) that was seen without the magnetic field.

(c) The larger wavelength (smaller energy) is produced from the $m_l = -1$ to $m_l = 0$ transition.

(d) The shorter wavelength (greater energy) is produced from the $m_l = +1$ to $m_l = 0$ transition.

41.17. $3p \Rightarrow n = 3, l = 1, \Delta U = \mu_B B \Rightarrow B = \dfrac{U}{\mu_B} = \dfrac{(2.71\times10^{-5}\ \text{eV})}{(5.79\times10^{-5}\ \text{eV/T})} = 0.468\ \text{T}$

(b) Three: $m_l = 0, \pm1$.

41.18. **(a)** $U = +(2.00232)\left(\dfrac{e}{2m}\right)\left(\dfrac{-\hbar}{2}\right)B = -\dfrac{(2.00232)}{2}\mu_B B$

$$U = -\frac{(2.00232)}{2}(5.788\times10^{-5}\ \text{eV/T})(0.480\ \text{T}) = -2.78\times10^{-5}\ \text{eV}.$$

(b) Since $n = 1$, $l = 0$ so there is no orbital magnetic dipole interaction. But if $n \neq 0$ there could be since $l < n$ allows for $l \neq 0$.

41.19. **IDENTIFY** and **SET UP:** The interaction energy is $U = -\vec{\mu}\cdot\vec{B}$, with μ_z given by Eq.(41.22).

EXECUTE: $U = -\vec{\mu}\cdot\vec{B} = +\mu_z B$, since the magnetic field is in the negative z-direction.

$\mu_z = -(2.00232)\left(\dfrac{e}{2m}\right)S_z$, so $U = -(2.00232)\left(\dfrac{e}{2m}\right)S_z B$

$S_z = m_s\hbar$, so $U = -2.00232\left(\dfrac{e\hbar}{2m}\right)m_s B$

$\dfrac{e\hbar}{2m} = \mu_B = 5.788\times10^{-5}\ \text{eV/T}$

$U = -2.00232\mu_B m_s B$

The $m_s = +\dfrac{1}{2}$ level has lower energy.

$$\Delta U = U\left(m_s = -\frac{1}{2}\right) - U\left(m_s = +\frac{1}{2}\right) = -2.00232\mu_B B\left(-\frac{1}{2} - \left(+\frac{1}{2}\right)\right) = +2.00232\mu_B B$$

$$\Delta U = +2.00232(5.788\times10^{-5}\ \text{eV/T})(1.45\ \text{T}) = 1.68\times10^{-4}\ \text{eV}$$

EVALUATE: The interaction energy with the electron spin is the same order of magnitude as the interaction energy with the orbital angular momentum for states with $m_l \neq 0$. But a 1*s* state has $l = 0$ and $m_l = 0$, so there is no orbital magnetic interaction.

41.20. The allowed (l, j) combinations are $\left(0,\ \frac{1}{2}\right), \left(1,\ \frac{1}{2}\right), \left(1,\ \frac{3}{2}\right), \left(2,\ \frac{3}{2}\right)$ and $\left(2,\ \frac{5}{2}\right)$.

41.21. **IDENTIFY** and **SET UP:** *j* can have the values $l + 1/2$ and $l - 1/2$.
EXECUTE: If *j* takes the values 7/2 and 9/2 it must be that $l - 1/2 = 7/2$ and $l = 8/2 = 4$. The letter that labels this *l* is *g*.
EVALUATE: *l* must be an integer.

41.22. **(a)** $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(300\times10^{8}\text{ m/s})}{(5.9\times10^{-6}\text{ eV})} = 21\text{ cm},\ f = \dfrac{c}{\lambda} = \dfrac{(3.00\times10^{8}\text{ m/s})}{0.21\text{ m}} = 1.4\times10^{9}$ Hz, a short radio wave.
(b) As in Example 41.6, the effective field is $B \cong \Delta E/2\mu_{\text{B}} = 5.1\times10^{-2}$ T, for smaller than that found in the example.

41.23. **IDENTIFY** and **SET UP:** For a classical particle $L = I\omega$. For a uniform sphere with mass *m* and radius *R*, $I = \frac{2}{5}mR^2$, so $L = \left(\frac{2}{5}mR^2\right)\omega$. Solve for ω and then use $v = r\omega$ to solve for *v*.

EXECUTE: **(a)** $L = \sqrt{\frac{3}{4}}\hbar$ so $\frac{2}{5}mR^2\omega = \sqrt{\frac{3}{4}}\hbar$

$$\omega = \frac{5\sqrt{3/4}\hbar}{2mR^2} = \frac{5\sqrt{3/4}(1.055\times10^{-34}\text{ J}\cdot\text{s})}{2(9.109\times10^{-31}\text{ kg})(1.0\times10^{-17}\text{ m})^2} = 2.5\times10^{30}\text{ rad/s}$$

(b) $v = r\omega = (1.0\times10^{-17}\text{ m})(2.5\times10^{30}\text{ rad/s}) = 2.5\times10^{13}\text{ m/s}$.
EVALUATE: This is much greater than the speed of light *c*, so the model cannot be valid.

41.24. However the number of electrons is obtained, the results must be consistent with Table (41.3); adding two more electrons to the zinc configuration gives $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^2$.

41.25. The ten lowest energy levels for electrons are in the *n* = 1 and *n* = 2 shells.

$$n = 1, l = 0, m_l = 0, \qquad m_s = \pm\frac{1}{2}: 2\text{ states.}$$

$$n = 2, l = 0, m_l = 0, \qquad m_s = \pm\frac{1}{2}: 2\text{ states.}$$

$$n = 2, l = 1, m_l = 0, \pm 1, \quad m_s = \pm\frac{1}{2}: 6\text{ states.}$$

41.26. For the outer electrons, there are more inner electrons to screen the nucleus.

41.27. **IDENTIFY** and **SET UP:** The energy of an atomic level is given in terms of *n* and Z_{eff} by Eq.(41.27), $E_n = -\left(\dfrac{Z_{\text{eff}}^2}{n^2}\right)(13.6\text{ eV})$. The ionization energy for a level with energy $-E_n$ is $+E_n$.

EXECUTE: $n = 5$ and $Z_{\text{eff}} = 2.771$ gives $E_5 = -\dfrac{(2.771)^2}{5^2}(13.6\text{ eV}) = -4.18\text{ eV}$

The ionization energy is 4.18 eV.
EVALUATE: The energy of an atomic state is proportional to Z_{eff}^2.

41.28. For the $4s$ state, $E = -4.339$ eV and $Z_{\text{eff}} = 4\sqrt{(-4.339)/(-13.6)} = 2.26$. Similarly, $Z_{\text{eff}} = 1.79$ for the 4*p* state and 1.05 for the 4*d* state. The electrons in the states with higher *l* tend to be further away from the filled subshells and the screening is more complete.

41.29. **IDENTIFY** and **SET UP:** Use the exclusion principle to determine the ground-state electron configuration, as in Table 41.3. Estimate the energy by estimating Z_{eff}, taking into account the electron screening of the nucleus.
EXECUTE: **(a)** $Z = 7$ for nitrogen so a nitrogen atom has 7 electrons. N^{2+} has 5 electrons: $1s^2 2s^2 2p$.
(b) $Z_{\text{eff}} = 7 - 4 = 3$ for the 2*p* level.

$$E_n = -\left(\frac{Z_{\text{eff}}^2}{n^2}\right)(13.6\text{ eV}) = -\frac{3^2}{2^2}(13.6\text{ eV}) = -30.6\text{ eV}$$

(c) $Z = 15$ for phosphorus so a phosphorus atom has 15 electrons.
P^{2+} has 13 electrons: $1s^2 2s^2 2p^6 3s^2 3p$

(d) $Z_{\text{eff}} = 15 - 12 = 3$ for the 3p level.

$$E_n = -\left(\frac{Z_{\text{eff}}^2}{n^2}\right)(13.6\text{ eV}) = -\frac{3^2}{3^2}(13.6\text{ eV}) = -13.6\text{ eV}$$

EVALUATE: In these ions there is one electron outside filled subshells, so it is a reasonable approximation to assume full screening by these inner-subshell electrons.

41.30. **(a)** $E_2 = -\frac{13.6\text{ eV}}{4}Z_{\text{eff}}^2$, so $Z_{\text{eff}} = 1.26$.

(b) Similarly, $Z_{\text{eff}} = 2.26$.

(c) Z_{eff} becomes larger going down the columns in the periodic table.

41.31. **IDENTIFY** and **SET UP:** Estimate Z_{eff} by considering electron screening and use Eq.(41.27) to calculate the energy. Z_{eff} is calculated as in Example 41.8.

EXECUTE: **(a)** The element Be has nuclear charge $Z = 4$. The ion Be^+ has 3 electrons. The outermost electron sees the nuclear charge screened by the other two electrons so $Z_{\text{eff}} = 4 - 2 = 2$.

$$E_n = -\left(\frac{Z_{\text{eff}}^2}{n^2}\right)(13.6\text{ eV}) \text{ so } E_2 = -\frac{2^2}{2^2}(13.6\text{ eV}) = -13.6\text{ eV}$$

(b) The outermost electron in Ca^+ sees a $Z_{\text{eff}} = 2$. $E_4 = -\frac{2^2}{4^2}(13.6\text{ eV}) = -3.4\text{ eV}$

EVALUATE: For the electron in the highest l-state it is reasonable to assume full screening by the other electrons, as in Example 41.8. The highest l-states of Be^+, Mg^+, Ca^+, etc. all have a $Z_{\text{eff}} = 2$. But the energies are different because for each ion the outermost sublevel has a different n quantum number.

41.32. $E_{kx} \cong (Z-1)^2(10.2\text{ eV})$. $Z \approx 1 + \sqrt{\frac{7.46\times 10^3\text{ eV}}{10.2\text{ eV}}} = 28.0$, which corresponds to the element Nickel (Ni).

41.33. **(a)** $Z = 20: f = (2.48\times 10^{15}\text{ Hz})(20-1)^2 = 8.95\times 10^{17}\text{ Hz}$.

$E = hf = (4.14\times 10^{-15}\text{ eV}\cdot\text{s})(8.95\times 10^{17}\text{ Hz}) = 3.71\text{ keV}$. $\lambda = \frac{c}{f} = \frac{3.00\times 10^8\text{ m/s}}{8.95\times 10^{17}\text{ Hz}} = 3.35\times 10^{-10}\text{ m}$.

(b) $Z = 27$: $f = 1.68\times 10^{18}\text{ Hz}$. $E = 6.96\text{ keV}$. $\lambda = 1.79\times 10^{-10}\text{ m}$.

(c) $Z = 48: f = 5.48\times 10^{18}\text{ Hz}, E = 22.7\text{ keV}, \lambda = 5.47\times 10^{-11}\text{ m}$.

41.34. **IDENTIFY:** The orbital angular momentum is limited by the shell the electron is in.

SET UP: For an electron in the n shell, its orbital angular momentum quantum number l is limited by $0 \le l < n$, and its orbital angular momentum is given by $L = \sqrt{l(l+1)}\,\hbar$. The z-component of its angular momentum is $L_z = m_l\hbar$, where $m_l = 0, \pm 1, \ldots, \pm l$, and its spin angular momentum is $S = \sqrt{3/4}\,\hbar$ for all electrons. Its energy in the n^{th} shell is $E_n = -(13.6\text{eV})/n^2$.

EXECUTE: **(a)** $L = \sqrt{l(l+1)}\,\hbar = 12\hbar \Rightarrow l = 3$. Therefore the smallest that n can be is 4, so $E_n = -(13.6\text{ eV})/n^2 = -(13.6\text{ eV})/4^2 = -0.8500\text{ eV}$.

(b) For $l = 3$, $m_l = \pm 3, \pm 2, \pm 1, 0$. Since $L_z = m_l\hbar$, the largest L_z can be is $3\hbar$ and the smallest it can be is $-3\hbar$.

(c) $S = \sqrt{3/4}\,\hbar$ for *all* electrons.

(d) In this case, $n = 3$, so $l = 2, 1, 0$. Therefore the maximum that L can be is $L_{\max} = \sqrt{2(2+1)}\,\hbar = \sqrt{6}\,\hbar$. The minimum L can be is zero when $l = 0$.

EVALUATE: At the quantum level, electrons in atoms can have only certain allowed values of their angular momentum.

41.35. **IDENTIFY:** The total energy determines what shell the electron is in, which limits its angular momentum.

SET UP: The electron's orbital angular momentum is given by $L = \sqrt{l(l+1)}\,\hbar$, and its total energy in the n^{th} shell is $E_n = -(13.6\text{ eV})/n^2$.

EXECUTE: **(a)** First find n: $E_n = -(13.6\text{eV})/n^2 = -0.5440\text{ eV}$ which gives $n = 5$, so $l = 4, 3, 2, 1, 0$. Therefore the possible values of L are given by $L = \sqrt{l(l+1)}\,\hbar$, giving $L = 0, \sqrt{2}\hbar, \sqrt{6}\hbar, \sqrt{12}\hbar, \sqrt{20}\hbar$.

(b) $E_6 = -(13.6\text{ eV})/6^2 = -0.3778\text{ eV}$. $\Delta E = E_6 - E_5 = -0.3778\text{ eV} - (-0.5440\text{ eV}) = +0.1662\text{ eV}$

This must be the energy of the photon, so $\Delta E = hc/\lambda$, which gives

$\lambda = hc/\Delta E = (4.136\times 10^{-15}\text{ eV}\cdot\text{s})(3.00\times 10^8\text{ m/s})/(0.1662\text{ eV}) = 7.47\times 10^{-6}\text{ m} = 7470\text{ nm}$, which is in the infrared and hence not visible.

EVALUATE: The electron can have any of the five possible values for its angular momentum, but it cannot have any others.

41.36. **IDENTIFY:** For the N shell, $n = 4$, which limits the values of the other quantum numbers.
SET UP: In the n^{th} shell, $0 \le l < n$, $m_l = 0, \pm 1, \ldots, \pm l$, and $m_s = \pm 1/2$. The orbital angular momentum of the electron is $L = \sqrt{l(l+1)}\,\hbar$ and its spin angular momentum is $S = \sqrt{3/4}\,\hbar$.
EXECUTE: **(a)** For $l = 3$ we can have $m_l = \pm 3, \pm 2\pm, \pm 1, 0$ and $m_s = \pm ½$; for $l = 2$ we can have $m_l = \pm 2, \pm 1, 0$ and $m_s = \pm ½$; for $l = 1$, we can have $m_l = \pm 1, 0$ and $m_s = \pm 1/2$; for $l = 0$, we can have $m_l = 0$ and $m_s = \pm 1/2$.
(b) For the N shell, $n = 4$, and for an f-electron, $l = 3$, giving $L = \sqrt{l(l+1)}\,\hbar = \sqrt{3(3+1)}\,\hbar = \sqrt{12}\hbar$. $L_z = m_l\hbar = \pm 3\hbar,\ \pm 2\hbar,\ \pm\hbar,\ 0,$ so the maximum value is $3\hbar$. $S = \sqrt{3/4}\,\hbar$ for all electrons.
(c) For a d-state electron, $l = 2$, giving $L = \sqrt{2(2+1)}\,\hbar = \sqrt{6}\hbar$. $L_z = m_l\hbar,$ and the maximum value of m_l is 2, so the maximum value of L_z is $2\hbar$. The smallest angle occurs when L_z is most closely aligned along the angular momentum vector, which is when L_z is greatest. Therefore $\cos\theta_{\text{min}} = \dfrac{L_z}{L} = \dfrac{2\hbar}{\sqrt{6}\hbar} = \dfrac{2}{\sqrt{6}}$ and $\theta_{\text{min}} = 35.3°$. The largest angle occurs when L_z is as far as possible from the L-vector, which is when L_z is most negative. Therefore $\cos\theta_{\text{max}} = \dfrac{-2\hbar}{\sqrt{6}\hbar} = -\dfrac{2}{\sqrt{6}}$ and $\theta_{\text{max}} = 144.7°$.
(d) This is not possible since $l = 3$ for an f-electron, but in the M shell the maximum value of l is 2.
EVALUATE: The fact that the angle in part (c) cannot be zero tells us that the orbital angular momentum of the electron cannot be totally aligned along any specified direction.

41.37. **IDENTIFY:** The inner electrons shield part of the nuclear charge from the outer electron.
SET UP: The electron's energy in the n^{th} shell, due to shielding, is $E_n = -\dfrac{Z_{\text{eff}}^2}{n^2}(13.6\text{eV})$, where $Z_{\text{eff}}e$ is the effective charge that the electron "sees" for the nucleus.
EXECUTE: **(a)** $E_n = -\dfrac{Z_{\text{eff}}^2}{n^2}(13.6\text{ eV})$ and $n = 4$ for the 4s state. Solving for Z_{eff} gives $Z_{\text{eff}} = \sqrt{-\dfrac{(4^2)(-1.947\text{ eV})}{13.6\text{ eV}}}$ = 1.51. The nucleus contains a charge of $+11e$, so the average number of electrons that screen this nucleus must be 11 – 1.51 = 9.49 electrons
(b) (i) The charge of the nucleus is $+19e$, but $17.2e$ is screened by the electrons, so the outer electron "sees" $19e - 17.2e = 1.8e$ and $Z_{\text{eff}} = 1.8$.
(ii) $E_n = -\dfrac{Z_{\text{eff}}^2}{n^2}(13.6\text{ eV}) = -\dfrac{(1.8)^2}{4^2}(13.6\text{ eV}) = -2.75\text{ eV}$
EVALUATE: Sodium has 11 protons, so the inner 10 electrons shield a large portion of this charge from the outer electron. But they don't shield 10 of the protons, since the inner electrons are not totally equivalent to a uniform spherical shell. (They are lumpy.)

41.38. See Example 41.3; $r^2|\psi|^2 = Cr^2e^{-2r/a}$, $\dfrac{d(r^2|\psi|^2)}{dr} = Ce^{-2r/a}(2r - (2r^2/a)),$ and for a maximum, $r = a,$ the distance of the electron from the nucleus in the Bohr model.

41.39. **(a) IDENTIFY** and **SET UP:** The energy is given by Eq.(38.18), which is identical to Eq.(41.3). The potential energy is given by Eq.(23.9), with $q = +Ze$ and $q_0 = -e$.

EXECUTE: $E_{1s} = -\dfrac{1}{(4\pi\epsilon_0)^2}\dfrac{me^4}{2\hbar^2}$; $U(r) = -\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{r}$

$E_{1s} = U(r)$ gives $-\dfrac{1}{(4\pi\epsilon_0)^2}\dfrac{me^4}{2\hbar^2} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{r}$

$$r = \frac{(4\pi\epsilon_0)2\hbar^2}{me^2} = 2a$$

EVALUATE: The turning point is twice the Bohr radius.
(b) IDENTIFY and **SET UP:** For the 1s state the probability that the electron is in the classically forbidden region is $P(r > 2a) = \int_{2a}^{\infty}|\psi_{1s}|^2 dV = 4\pi\int_{2a}^{\infty}|\psi_{1s}|^2 r^2 dr.$ The normalized wave function of the 1s state of hydrogen is given in Example 41.3: $\psi_{1s}(r) = \dfrac{1}{\sqrt{\pi a^3}}e^{-r/a}$. Evaluate the integral; the integrand is the same as in Example 41.3.

EXECUTE: $P(r > 2a) = 4\pi\left(\dfrac{1}{\pi a^3}\right)\int_{2a}^{\infty} r^2 e^{-2r/a}dr$

Use the integral formula $\int r^2 e^{-\alpha r}dr = -e^{-\alpha r}\left(\dfrac{r^2}{\alpha}+\dfrac{2r}{\alpha^2}+\dfrac{2}{\alpha^3}\right)$, with $\alpha = 2/a$.

$$P(r>2a) = -\frac{4}{a^3}\left[e^{-2r/a}\left(\frac{ar^2}{2}+\frac{a^2 r}{2}+\frac{a^3}{4}\right)\right]_{2a}^{\infty} = +\frac{4}{a^3}e^{-4}(2a^3+a^3+a^3/4)$$

$P(r>2a) = 4e^{-4}(13/4) = 13e^{-4} = 0.238.$

EVALUATE: These is a 23.8% probability of the electron being found in the classically forbidden region, where classically its kinetic energy would be negative.

41.40. **(a)** For large values of n, the inner electrons will completely shield the nucleus, so $Z_{\text{eff}} = 1$ and the ionization energy would be $\dfrac{13.60\text{ eV}}{n^2}$.

(b) $\dfrac{13.60\text{ eV}}{350^2} = 1.11\times10^{-4}\text{ eV}$, $r_{350} = (350)^2 a_0 = (350)^2(0.529\times10^{-10}\text{ m}) = 6.48\times10^{-6}\text{ m}$.

(c) Similarly for n = 650, $\dfrac{13.60\text{ eV}}{(650)^2} = 3.22\times10^{-5}\text{ eV}$, $r_{650} = (650)^2(0.529\times10^{-10}\text{ m}) = 2.24\times10^{-5}\text{ m}$.

41.41. $\psi_{2s}(r) = \dfrac{1}{\sqrt{32\pi a^3}}\left(2-\dfrac{r}{a}\right)e^{-r/2a}$

(a) IDENTIFY and **SET UP:** Let $I = \int_0^\infty |\psi_{2s}|^2 dV = 4\pi\int_0^\infty |\psi_{2s}|^2 r^2 dr$. If ψ_{2s} is normalized then we will find that $I = 1$.

EXECUTE: $I = 4\pi\left(\dfrac{1}{32\pi a^3}\right)\displaystyle\int_0^\infty\left(2-\frac{r}{a}\right)^2 e^{-r/a}r^2 dr = \frac{1}{8a^3}\int_0^\infty\left(4r^2-\frac{4r^3}{a}+\frac{r^4}{a^2}\right)e^{-r/a}dr$

Use the integral formula $\int_0^\infty x^n e^{-\alpha x}dx = \dfrac{n!}{\alpha^{n+1}}$, with $\alpha = 1/a$

$I = \dfrac{1}{8a^3}\left(4(2!)(a^3) - \dfrac{4}{a}(3!)(a)^4 + \dfrac{1}{a^2}(4!)(a)^5\right) = \dfrac{1}{8}(8-24+24) = 1$; this ψ_{2s} is normalized.

(b) SET UP: For a spherically symmetric state such as the 2s, the probability that the electron will be found at $r < 4a$ is $P(r<4a) = \int_0^{4a}|\psi_{2s}|^2 dV = 4\pi\int_0^{4a}|\psi_{2s}|^2 r^2 dr.$

EXECUTE: $P(r<4a) = \dfrac{1}{8a^3}\displaystyle\int_0^{4a}\left(4r^2-\frac{4r^3}{a}+\frac{r^4}{a^2}\right)e^{-r/a}dr$

Let $P(r<4a) = \dfrac{1}{8a^3}(I_1+I_2+I_3)$.

$I_1 = 4\int_0^{4a} r^2 e^{-r/a}dr$

Use the integral formula $\int r^2 e^{-\alpha r}dr = -e^{-\alpha r}\left(\dfrac{r^2}{\alpha}+\dfrac{2r}{\alpha^2}+\dfrac{2}{\alpha^3}\right)$ with $\alpha = 1/a$.

$I_1 = -4[e^{-r/a}(r^2 a + 2ra^2 + 2a^3)]_0^{4a} = (-104e^{-4}+8)a^3.$

$I_2 = -\dfrac{4}{a}\displaystyle\int_0^{4a} r^3 e^{-r/a}dr$

Use the integral formula $\int r^3 e^{-\alpha r}dr = -e^{-\alpha r}\left(\dfrac{r^3}{\alpha}+\dfrac{3r^2}{\alpha^2}+\dfrac{6r}{a^3}+\dfrac{6}{\alpha^4}\right)$ with $\alpha = 1/a$.

$I_2 = \dfrac{4}{a}[e^{-r/a}(r^3 a + 3r^2 a^2 + 6ra^3 + 6a^4)]_0^{4a} = (568e^{-4}-24)a^3.$

$I_3 = \dfrac{1}{a^2}\displaystyle\int_0^{4a} r^4 e^{-r/a}dr$

Use the integral formula $\int r^4 e^{-\alpha r}dr = -e^{-\alpha r}\left(\dfrac{r^4}{\alpha}+\dfrac{4r^3}{\alpha^2}+\dfrac{12r^2}{\alpha^3}+\dfrac{24r}{a^4}+\dfrac{24}{a^5}\right)$ with $\alpha = 1/a$.

$I_3 = -\dfrac{1}{a^2}[e^{-r/a}(r^4 a + 4r^3 a^2 + 12r^2 a^3 + 24ra^4 + 24a^5)]_0^{4a} = (-824e^{-4}+24)a^3.$

Thus $P(r<4a)=\frac{1}{8a^3}(I_1+I_2+I_3)=\frac{1}{8a^3}a^3([8-24+24]+e^{-4}[-104+568-824])$

$P(r<4a)=\frac{1}{8}(8-360e^{-4})=1-45e^{-4}=0.176.$

EVALUATE: There is an 82.4% probability that the electron will be found at $r>4a$. In the Bohr model the electron is for certain at $r=4a$; this is a poor description of the radial probability distribution for this state.

41.42. **(a)** Since the given $\psi(r)$ is real, $r^2|\psi|^2=r^2\psi^2$. The probability density will be an extreme when $\frac{d}{dr}(r^2\psi^2)=2\left(r\psi^2+r^2\psi\frac{d\psi}{dr}\right)=2r\psi\left(\psi+r\frac{d\psi}{dr}\right)=0.$ This occurs at $r=0$, a minimum, and when $\psi=0$, also a minimum. A maximum must correspond to $\psi+r\frac{d\psi}{dr}=0.$ Within a multiplicative constant, $\psi(r)=(2-r/a)e^{-r/2a}$, $\frac{d\psi}{dr}=-\frac{1}{a}(2-r/2a)e^{-r/2a}$, and the condition for a maximum is $(2-r/a)=(r/a)(2-r/2a)$, or $r^2-6ra+4a^2=0.$ The solutions to the quadratic are $r=a(3\pm\sqrt{5})$. The ratio of the probability densities at these radii is 3.68, with the larger density at $r=a(3+\sqrt{5})$.

(b) $\psi=0$ at $r=2a$

Parts (a) and (b) are consistent with Figure 41.5 in the textbook; note the two relative maxima, one on each side of the minimum of zero at $r=2a$.

41.43. **IDENTIFY:** Use Figure 41.2 in the textbook to relate θ_L to L_z and L: $\cos\theta_L=\frac{L_z}{L}$ so $\theta_L=\arccos\left(\frac{L_z}{L}\right)$

(a) SET UP: The smallest angle $(\theta_L)_{\min}$ is for the state with the largest L and the largest L_z. This is the state with $l=n-1$ and $m_l=l=n-1$.

EXECUTE: $L_z=m_l\hbar=(n-1)\hbar$

$L=\sqrt{l(l+1)}\hbar=\sqrt{(n-1)n}\hbar$

$(\theta_L)_{\min}=\arccos\left(\frac{(n-1)\hbar}{\sqrt{(n-1)n}\hbar}\right)=\arccos\left(\frac{(n-1)}{\sqrt{(n-1)n}}\right)=\arccos\left(\sqrt{\frac{n-1}{n}}\right)=\arccos(\sqrt{1-1/n}).$

EVALUATE: Note that $(\theta_L)_{\min}$ approaches $0°$ as $n\to\infty$.

(b) SET UP: The largest angle $(\theta_L)_{\max}$ is for $l=n-1$ and $m_l=-l=-(n-1)$.

EXECUTE: A similar calculation to part (a) yields $(\theta_L)_{\max}=\arccos(-\sqrt{1-1/n})$

EVALUATE: Note that $(\theta_L)_{\max}$ approaches $180°$ as $n\to\infty$.

41.44. **(a)** $L_x^2+L_y^2=L^2-L_z^2=l(l+1)\hbar^2-m_l^2\hbar^2$ so $\sqrt{L_x^2+L_y^2}=\sqrt{l(l+1)-m_l^2}\hbar.$

(b) This is the magnitude of the component of angular momentum perpendicular to the z-axis.

(c) The maximum value is $\sqrt{l(l+1)}\hbar=L$, when $m_l=0$. That is, if the electron is known to have no z-component of angular momentum, the angular momentum must be perpendicular to the z-axis. The minimum is $\sqrt{l}\hbar$ when $m_l=\pm l$.

41.45. $P(r)=\left(\frac{1}{24a^5}\right)r^4e^{-r/2a}$. $\frac{dP}{dr}=\left(\frac{1}{24a^5}\right)\left(4r^3-\frac{r^4}{a}\right)e^{-r/2a}$. $\frac{dP}{dr}=0$ when $4r^3-\frac{r^4}{a}=0$; $r=4a$. In the Bohr model, $r_n=n^2a$ so $r_2=4a$, which agrees.

41.46. The time required to transit the horizontal 50 cm region is $t=\frac{\Delta x}{v_x}=\frac{0.500\text{ m}}{525\text{ m/s}}=0.952$ ms. The force required to deflect each spin component by 0.50 mm is $F_z=ma_z=\pm m\frac{2\Delta z}{t^2}=\pm\left(\frac{0.1079\text{ kg/mol}}{6.022\times10^{23}\text{ atoms/mol}}\right)\frac{2(0.50\times10^{-3}\text{ m})}{(0.952\times10^{-3}\text{ s})^2}=$ $\pm1.98\times10^{-22}$ N. According to Eq.(41.22), the value of μ_z is $|\mu_z|=9.28\times10^{-24}\text{ A}\cdot\text{m}^2$. Thus, the required magnetic-field gradient is $\left|\frac{dB_z}{dz}\right|=\left|\frac{F_z}{\mu_z}\right|=\frac{1.98\times10^{-22}\text{ N}}{9.28\times10^{-24}\text{ J/T}}=21.3\text{ T/m}.$

41.47. Decay from a $3d$ to $2p$ state in hydrogen means that $n=3 \to n=2$ and $m_l = \pm 2, \pm 1, 0 \to m_l = \pm 1, 0.$ However selection rules limit the possibilities for decay. The emitted photon carries off one unit of angular momentum so l must change by 1 and hence m_l must change by 0 or $\pm 1.$ The shift in the transition energy from the zero field value is just $U = (m_{l_3} - m_{l_2})\mu_B B = \frac{e\hbar B}{2m}(m_{l_3} - m_{l_2}),$ where m_{l_3} is the $3d$ m_l value and m_{l_2} is the $2p$ m_l value. Thus there are only three different energy shifts. They and the transitions that have them, labeled by the m_l names, are:

$$\frac{e\hbar B}{2m}: 2 \to 1, \quad 1 \to 0, \quad 0 \to -1$$
$$0: 1 \to 1, \quad 0 \to 0, \quad -1 \to -1$$
$$-\frac{e\hbar B}{2m}: 0 \to 1, \quad -1 \to 0, \quad -2 \to -1$$

41.48. **IDENTIFY:** The presence of an external magnetic field shifts the energy levels up or down, depending upon the value of m_l.

SET UP: The selection rules tell us that for allowed transitions, $\Delta l = 1$ and $\Delta m_l = 0$ or ± 1.

EXECUTE: **(a)** $E = hc/\lambda = (4.136 \times 10^{-15}\ \text{eV}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})/(475.082\ \text{nm}) = 2.612\ \text{eV}$.

(b) For allowed transitions, $\Delta l = 1$ and $\Delta m_l = 0$ or ± 1. For the $3d$ state, $n = 3$, $l = 2$, and m_l can have the values 2, 1, 0, –1, –2. In the $2p$ state, $n = 2$, $l = 1$, and m_l can be 1, 0, –1. Therefore the 9 allowed transitions from the $3d$ state in the presence of a magnetic field are:

$l = 2$, $m_l = 2$ $\to$ $l = 1$, $m_l = 1$;
$l = 2$, $m_l = 1$ $\to$ $l = 1$, $m_l = 0$
$l = 2$, $m_l = 1$ $\to$ $l = 1$, $m_l = 1$
$l = 2$, $m_l = 0$ $\to$ $l = 1$, $m_l = 0$
$l = 2$, $m_l = 0$ $\to$ $l = 1$, $m_l = 1$
$l = 2$, $m_l = 0$ $\to$ $l = 1$, $m_l = -1$
$l = 2$, $m_l = -1$ $\to$ $l = 1$, $m_l = 0$
$l = 2$, $m_l = -1$ $\to$ $l = 1$, $m_l = -1$
$l = 2$, $m_l = -2$ $\to$ $l = 1$, $m_l = -1$

(c) $\Delta E = \mu_B B = (5.788 \times 10^{-5}\ \text{eV/T})(3.500\ \text{T}) = 0.000203\ \text{eV}$

So the energies of the new states are –8.50000 eV + 0 and –8.50000 eV ± 0.000203 eV, giving energies of: –8.50020 eV, –8.50000 eV, and –8.49980 eV

(d) The energy differences of the allowed transitions are equal to the energy differences if no magnetic field were present (2.61176 eV, from part (a)), and that value $\pm\Delta E$ (0.000203 eV, from part (c)). Therefore we get the following.

For $E = 2.61176$ eV: $\lambda = 475.082$ nm (which was given)

For $E = 2.61176\ \text{eV} + 0.000203\ \text{eV} = 2.611963\ \text{eV}$:

$$\lambda = hc/E = (4.136 \times 10^{-15}\ \text{eV}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})/(2.611963\ \text{eV}) = 475.045\ \text{nm}$$

For $E = 2.61176\ \text{eV} - 0.000203\ \text{eV} = 2.61156\ \text{eV}$:

$$\lambda = hc/E = (4.136 \times 10^{-15}\ \text{eV}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})/(2.61156\ \text{eV}) = 475.119\ \text{nm}$$

EVALUATE: Even a strong magnetic field produces small changes in the energy levels, and hence in the wavelengths of the emitted light.

41.49. **IDENTIFY:** The presence of an external magnetic field shifts the energy levels up or down, depending upon the value of m_l.

SET UP: The energy difference due to the magnetic field is $\Delta E = \mu_B B$ and the energy of a photon is $E = hc/\lambda$.

EXECUTE: For the p state, $m_l = 0$ or ± 1, and for the s state $m_l = 0$. Between any two adjacent lines, $\Delta E = \mu_B B$. Since the change in the wavelength ($\Delta\lambda$) is very small, the energy change (ΔE) is also very small, so we can use differentials. $E = hc/\lambda$. $|dE| = \frac{hc}{\lambda^2}d\lambda$ and $\Delta E = \frac{hc\Delta\lambda}{\lambda^2}$. Since $\Delta E = \mu_B B$, we get $\mu_B B = \frac{hc\Delta\lambda}{\lambda^2}$ and $B = \frac{hc\Delta\lambda}{\mu_B \lambda^2}$.

$B = (4.136 \times 10^{-15}\ \text{eV}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})(0.0462\ \text{nm})/(5.788 \times 10^{-5}\ \text{eV/T})(575.050\ \text{nm})^2 = 3.00\ \text{T}$

EVALUATE: Even a strong magnetic field produces small changes in the energy levels, and hence in the wavelengths of the emitted light.

41.50. **(a)** The energy shift from zero field is $\Delta U_0 = m_l \mu_B B$.

For $m_l = 2$, $\Delta U_0 = (2)(5.79 \times 10^{-5}\ \text{eV/T})(1.40\ \text{T}) = 1.62 \times 10^{-4}\ \text{eV}$.

For $m_l = 1$, $\Delta U_0 = (1)(5.79 \times 10^{-5}\ \text{eV/T})(1.40\ \text{T}) = 8.11 \times 10^{-5}\ \text{eV}$.

(b) $|\Delta\lambda| = \lambda_0 \frac{|\Delta E|}{E_0}$, where $E_0 = (13.6\ \text{eV})((1/4) - (1/9))$, $\lambda_0 = \left(\frac{36}{5}\right)\frac{1}{R} = 6.563 \times 10^{-7}\ \text{m}$

and $\Delta E = 1.62\times10^{-4}$ eV $- 8.11\times10^{-5}$ eV $= 8.09\times10^{-5}$ eV from part (a). Then, $|\Delta\lambda| = 2.81\times10^{-11}$ m $= 0.0281$ nm . The wavelength corresponds to a larger energy change, and so the wavelength is smaller.

41.51. **IDENTIFY:** The ratio according to the Boltzmann distribution is given by Eq.(38.21): $\frac{n_1}{n_0} = e^{-(E_1-E_0)/kT}$, where 1 is the higher energy state and 0 is the lower energy state.

SET UP: The interaction energy with the magnetic field is $U = -\mu_z B = 2.00232\left(\frac{e\hbar}{2m}\right)m_s B$ (Example 41.5.). The energy of the $m_s = +\frac{1}{2}$ level is increased and the energy of the $m_s = -\frac{1}{2}$ level is decreased.

$$\frac{n_{1/2}}{n_{-1/2}} = e^{-(U_{1/2}-U_{-1/2})/kT}$$

EXECUTE: $U_{1/2} - U_{-1/2} = 2.00232\left(\frac{e\hbar}{2m}\right)B\left(\frac{1}{2}-\left(-\frac{1}{2}\right)\right) = 2.00232\left(\frac{e\hbar}{2m}\right)B = 2.00232\mu_B B$

$$\frac{n_{1/2}}{n_{-1/2}} = e^{-(2.00232)\mu_B B/kT}$$

(a) $B = 5.00\times10^{-5}$ T

$$\frac{n_{1/2}}{n_{-1/2}} = e^{-2.00232(9.274\times10^{-24}\ \text{A/m}^2)(5.00\times10^{-5}\ \text{T})/([1.381\times10^{-23}\ \text{J/K}][300\ \text{K}])}$$

$$\frac{n_{1/2}}{n_{-1/2}} = e^{-2.24\times10^{-7}} = 0.99999978 = 1 - 2.2\times10^{-7}$$

(b) $B = 5.00\times10^{-5}$ T, $\frac{n_{1/2}}{n_{-1/2}} = e^{-2.24\times10^{-3}} = 0.9978$

(c) $B = 5.00\times10^{-5}$ T, $\frac{n_{1/2}}{n_{-1/2}} = e^{-2.24\times10^{-2}} = 0.978$

EVALUATE: For small fields the energy separation between the two spin states is much less than kT for $T = 300$ K and the states are equally populated. For $B = 5.00$ T the energy spacing is large enough for there to be a small excess of atoms in the lower state.

41.52. Using Eq.(41.4), $L = mvr = \sqrt{l(l+1)}\hbar$, and the Bohr radius from Eq.(38.15), we obtain the following value for v:

$v = \frac{\sqrt{l(l+1)}\hbar}{m(n^2a_0)} = \frac{\sqrt{2}(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi(9.11\times10^{-31}\ \text{kg})\,(4)\,(5.29\times10^{-11}\ \text{m})} = 7.74\times10^5$ m/s. The magnetic field generated by the "moving" proton at the electrons position can be calculated from Eq.(28.1):

$$B = \frac{\mu_0}{4\pi}\frac{|q|v\sin\phi}{r^2} = (10^{-7}\ \text{T}\cdot\text{m/A})\frac{(1.60\times10^{-19}\text{C})\,(7.74\times10^5\ \text{m/s})\sin(90°)}{(4)^2(5.29\times10^{-11}\ \text{m})^2} = 0.277\ \text{T}.$$

41.53. **IDENTIFY** and **SET UP:** m_s can take on 4 different values: $m_s = -\frac{3}{2},\ -\frac{1}{2},\ +\frac{1}{2},\ +\frac{3}{2}$. Each nlm_l state can have 4 electrons, each with one of the four different m_s values. Apply the exclusion principle to determine the electron configurations.

EXECUTE: **(a)** For a filled $n = 1$ shell, the electron configuration would be $1s^4$; four electrons and $Z = 4$. For a filled $n = 2$ shell, the electron configuration would be $1s^4 2s^4 2p^{12}$; twenty electrons and $Z = 20$.

(b) Sodium has $Z = 11$; 11 electrons. The ground-state electron configuration would be $1s^4 2s^4 2p^3$.

EVALUATE: The chemical properties of each element would be very different.

41.54. **(a)** $Z^2\,(-13.6\ \text{eV}) = (7)^2\,(-13.6\ \text{eV}) = -666$ eV.

(b) The negative of the result of part (a), 666 eV.

(c) The radius of the ground state orbit is inversely proportional to the nuclear charge, and

$\frac{a}{Z} = (0.529\times10^{-10}\ \text{m})/7 = 7.56\times10^{-12}$ m.

(d) $\lambda = \frac{hc}{\Delta E} = \frac{hc}{E_0\left(\frac{1}{1^2}-\frac{1}{2^2}\right)}$, where E_0 is the energy found in part (b), and $\lambda = 2.49$ nm.

41.55. **(a) IDENTIFY** and **SET UP:** The energy of the photon equals the transition energy of the atom: $\Delta E = hc/\lambda.$ The energies of the states are given by Eq.(41.3).

EXECUTE: $E_n = -\dfrac{13.60\text{ eV}}{n^2}$ so $E_2 = -\dfrac{13.60\text{ eV}}{4}$ and $E_1 = -\dfrac{13.60\text{ eV}}{1}$

$$\Delta E = E_2 - E_1 = 13.60\text{ eV}\left(-\frac{1}{4}+1\right) = \frac{3}{4}(13.60\text{ eV}) = 10.20\text{ eV} = (10.20\text{ eV})(1.602\times10^{-19}\text{ J/eV}) = 1.634\times10^{-18}\text{ J}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{1.634\times10^{-18}\text{ J}} = 1.22\times10^{-7}\text{ m} = 122\text{ nm}$$

(b) IDENTIFY and **SET UP:** Calculate the change in ΔE due to the orbital magnetic interaction energy, Eq.(41.17), and relate this to the shift $\Delta\lambda$ in the photon wavelength.

EXECUTE: The shift of a level due to the energy of interaction with the magnetic field in the *z*-direction is $U = m_l\mu_B B.$ The ground state has $m_l = 0$ so is unaffected by the magnetic field. The $n = 2$ initial state has $m_l = -1$ so its energy is shifted downward an amount $U = m_l\mu_B B = (-1)(9.274\times10^{-24}\text{ A/m}^2)(2.20\text{ T}) =$ $(-2.040\times10^{-23}\text{ J})(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 1.273\times10^{-4}\text{ eV}$

Note that the shift in energy due to the magnetic field is a very small fraction of the 10.2 eV transition energy. Problem 39.56c shows that in this situation $|\Delta\lambda/\lambda| = |\Delta E/E|.$ This gives

$$|\Delta\lambda| = \lambda|\Delta E/E| = 122\text{ nm}\left(\frac{1.273\times10^{-4}\text{ eV}}{10.2\text{ eV}}\right) = 1.52\times10^{-3}\text{ nm} = 1.52\text{ pm}.$$

EVALUATE: The upper level in the transition is lowered in energy so the transition energy is decreased. A smaller ΔE means a larger λ; the magnetic field increases the wavelength. The fractional shift in wavelength, $\Delta\lambda/\lambda$ is small, only 1.2×10^{-5}.

41.56. The effective field is that which gives rise to the observed difference in the energy level transition, $B = \dfrac{\Delta E}{\mu_B} = \dfrac{hc}{\mu_B}\left(\dfrac{\lambda_1-\lambda_2}{\lambda_1\lambda_2}\right) = \dfrac{2\pi mc}{e}\left(\dfrac{\lambda_1-\lambda_2}{\lambda_1\lambda_2}\right).$ Substitution of numerical values gives $B = 3.64\times10^{-3}$ T, much smaller than that for sodium.

41.57. **IDENTIFY:** Estimate the atomic transition energy and use Eq.(38.6) to relate this to the photon wavelength.

(a) SET UP: vanadium, $Z = 23$

minimum wavelength; corresponds to largest transition energy

EXECUTE: The highest occupied shell is the *N* shell $(n = 4).$ The highest energy transition is $N \to K,$ with transition energy $\Delta E = E_N - E_K.$ Since the shell energies scale like $1/n^2$ neglect E_N relative to $E_K,$ so $\Delta E = E_K = (Z-1)^2(13.6\text{ eV}) = (23-1)^2(13.6\text{ eV}) = 6.582\times10^3\text{ eV} = 1.055\times10^{-15}\text{ J}.$ The energy of the emitted photon equals this transition energy, so the photon's wavelength is given by $\Delta E = hc/\lambda$ so $\lambda = hc/\Delta E.$

$$\lambda = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{1.055\times10^{-15}\text{ J}} = 1.88\times10^{-10}\text{ m} = 0.188\text{ nm}.$$

SET UP: maximum wavelength; corresponds to smallest transition energy, so for the K_α transition

EXECUTE: The frequency of the photon emitted in this transition is given by Moseley's law (Eq.41.29):

$$f = (2.48\times10^{15}\text{ Hz})(Z-1)^2 = (2.48\times10^{15}\text{ Hz})(23-1)^2 = 1.200\times10^{18}\text{ Hz}$$

$$\lambda = \frac{c}{f} = \frac{2.998\times10^{8}\text{ m/s}}{1.200\times10^{18}\text{ Hz}} = 2.50\times10^{-10}\text{ m} = 0.250\text{ nm}$$

(b) rhenium, $Z = 45$

Apply the analysis of part (a), just with this different value of *Z*.

minimum wavelength

$$\Delta E = E_K = (Z-1)^2(13.6\text{ eV}) = (45-1)^2(13.6\text{ eV}) = 2.633\times10^4\text{ eV} = 4.218\times10^{-15}\text{ J}.$$

$$\lambda = hc/\Delta E = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{4.218\times10^{-15}\text{ J}} = 4.71\times10^{-11}\text{ m} = 0.0471\text{ nm}.$$

maximum wavelength

$$f = (2.48\times10^{15}\text{ Hz})(Z-1)^2 = (2.48\times10^{15}\text{ Hz})(45-1)^2 = 4.801\times10^{18}\text{ Hz}$$

$$\lambda = \frac{c}{f} = \frac{2.998\times10^{8}\text{ m/s}}{4.801\times10^{18}\text{ Hz}} = 6.24\times10^{-11}\text{ m} = 0.0624\text{ nm}$$

EVALUATE: Our calculated wavelengths have values corresponding to x rays. The transition energies increase when *Z* increases and the photon wavelengths decrease.

41.58. **(a)** $\Delta E = (2.00232)\frac{e}{2m}B\Delta S_z \approx \frac{e\hbar}{m}B = \frac{hc}{\lambda} \Rightarrow B = \frac{2\pi mc}{\lambda e}.$

(b) $B = \frac{2\pi(9.11\times10^{-31}\text{ kg})(3.00\times10^{8}\text{ m/s})}{(0.0350\text{ m})(1.60\times10^{-19}\text{ C})} = 0.307\text{ T}.$

41.59. **(a)** To calculate the total number of states for the n^{th} principal quantum number shell we must multiply all the possibilities. The spin states multiply everything by 2. The maximum l value is (n –1), and each l value has $(2l+1)m_l$ values. So the total number of states is

$$N = 2\sum_{l=0}^{n-1}(2l+1) = 2\sum_{l=0}^{n-1}1 + 4\sum_{l=0}^{n-1}l = 2n + \frac{4(n-1)(n)}{2} = 2n + 2n^2 - 2n = 2n^2.$$

(b) The n = 5 shell (O-shell) has 50 states.

41.60. **IDENTIFY:** We treat the Earth as an electron.

SET UP: The intrinsic spin angular momentum of an electron is $S = \sqrt{\frac{3}{4}}\hbar$, and the angular momentum of the spinning Earth is $S = I\omega$, where I = 2/5 mR^2.

EXECUTE: **(a)** Using $S = I\omega = \sqrt{\frac{3}{4}}\hbar$ and solving for ω gives

$$\omega = \frac{\sqrt{\frac{3}{4}}\hbar}{\frac{2}{5}mR^2} = \frac{\sqrt{\frac{3}{4}}\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)}{\frac{2}{5}\left(5.97\times10^{24}\text{ kg}\right)\left(6.38\times10^{6}\text{ m}\right)^2} = 9.40\times10^{-73}\text{ rad/s}$$

(b) We could not use this approach on the electron because in quantum physics we do not view it in the classical sense as a spinning ball.

EVALUATE: The angular velocity we have just calculated for the Earth would certainly be masked by its present angular spin of one revolution per day.

41.61. The potential $U(x) = \frac{1}{2}k'x^2$ is that of a simple harmonic oscillator. Treated quantum mechanically (see Section 40.4) each *energy* state has energy $E_n = \hbar\omega\,(n+\frac{1}{2})$. Since electrons obey the exclusion principle, this allows us to put *two* electrons (one for each $m_s = \pm\frac{1}{2}$) for every value of n—each quantum state is then defined by the ordered pair of quantum numbers (n, m_s). By placing two electrons in each energy level the lowest energy is then

$$2\left(\sum_{n=0}^{N-1}E_n\right) = 2\left(\sum_{n=0}^{N-1}\hbar\omega\left(n+\frac{1}{2}\right)\right) = 2\hbar\omega\left[\sum_{n=0}^{N-1}n + \sum_{n=0}^{N-1}\frac{1}{2}\right] = 2\hbar\omega\left[\frac{(N-1)(N)}{2} + \frac{N}{2}\right] =$$

$$\hbar\omega[N^2 - N + N] = \hbar\omega N^2 = \hbar N^2\sqrt{\frac{k'}{m}}.$$

Here we used the hint from Problem 41.59 to do the first sum, realizing that the first value of n is zero and the last value of n is N – 1, giving us a total of N energy levels filled.

41.62. **(a)** Apply Coulomb's law to the orbiting electron and set it equal to the centripetal force. There is an attractive force with charge +2e a distance r away and a repulsive force a distance 2r away. So, $\frac{(+2e)(-e)}{4\pi\epsilon_0 r^2} + \frac{(-e)(-e)}{4\pi\epsilon_0(2r)^2} = \frac{-mv^2}{r}$. But, from the quantization of angular momentum in the first Bohr orbit, $L = mvr = \hbar \Rightarrow v = \frac{\hbar}{mr}$.

So $\frac{-2e^2}{4\pi\epsilon_0 r^2} + \frac{e^2}{4\pi\epsilon_0(4r)^2} = \frac{-mv^2}{r} = \frac{-m\left(\frac{\hbar}{mr}\right)^2}{r} = -\frac{\hbar^2}{mr^3} \Rightarrow \frac{-7}{4}\frac{e^2}{r^2} = -\frac{4\pi\epsilon_0\hbar^2}{mr^3}$.

$r = \frac{4}{7}\left(\frac{4\pi\epsilon_0\hbar^2}{me^2}\right) = \frac{4}{7}a_0 = \frac{4}{7}(0.529\times10^{-10}\text{ m}) = 3.02\times10^{-11}\text{ m}.$

And $v = \frac{-\hbar}{mr} = \frac{7}{4}\frac{\hbar}{ma_0} = \frac{7}{4}\frac{(1.054\times10^{-34}\text{ J}\cdot\text{s})}{(9.11\times10^{-31}\text{ kg})(0.529\times10^{-10}\text{ m})} = 3.83\times10^{6}\text{ m/s}.$

(b) $K = 2\left(\frac{1}{2}mv^2\right) = 9.11\times10^{-31}\text{ kg }(3.83\times10^{6}\text{ m/s})^2 = 1.34\times10^{-17}\text{ J} = 83.5\text{ eV}.$

(c) $U = 2\left(\frac{-2e^2}{4\pi\epsilon_0 r}\right) + \frac{e^2}{4\pi\epsilon_0(2r)} = \frac{-4e^2}{4\pi\epsilon_0 r} + \frac{e^2}{4\pi E_0(2r)} = \frac{-7}{2}\left(\frac{e^2}{4\pi\epsilon_0 r}\right) = -2.67\times10^{-17}\text{ J} = -166.9\text{ eV}$

(d) $E_\infty = -[-166.9\text{ eV} + 83.5\text{ eV}] = 83.4\text{ eV},$ which is only off by about 5% from the real value of 79.0 eV.

41.63. **(a)** The radius is inversely proportional to *Z*, so the classical turning radius is $2a/Z$.

(b) The normalized wave function is $\psi_{1s}(r) = \frac{1}{\sqrt{\pi a^3/Z^3}}e^{-Zr/a}$ and the probability of the electron being found outside the classical turning point is $P = \int_{2a/Z}^{\infty} |\psi_{1s}|^2 4\pi r^2 dr = \frac{4}{a^3/Z^3}\int_{2a/Z}^{\infty} e^{-2Zr/a} r^2 dr.$ Making the change of variable $u = Zr/a,\ dr = (a/Z)du$ changes the integral to $P = 4\int_2^{\infty} e^{-2u}u^2 du$, which is independent of *Z*. The probability is that found in Problem 41.39, 0.238, independent of *Z*.