## Geometric Optics

34.1. Identify and Set Up: Plane mirror: $s=-s^{\prime}$ (Eq.34.1) and $m=y^{\prime} / y=-s^{\prime} / s=+1$ (Eq.34.2). We are given $s$ and $y$ and are asked to find $s^{\prime}$ and $y^{\prime}$.
Execute: The object and image are shown in Figure 34.1.


$$
\begin{aligned}
& s^{\prime}=-s=-39.2 \mathrm{~cm} \\
& \left|y^{\prime}\right|=|m||y|=(+1)(4.85 \mathrm{~cm}) \\
& \left|y^{\prime}\right|=4.85 \mathrm{~cm}
\end{aligned}
$$

Figure 34.1
The image is 39.2 cm to the right of the mirror and is 4.85 cm tall.
Evaluate: For a plane mirror the image is always the same distance behind the mirror as the object is in front of the mirror. The image always has the same height as the object.
34.2. IDENTIFY: Similar triangles say $\frac{h_{\text {tree }}}{h_{\text {mirror }}}=\frac{d_{\text {tree }}}{d_{\text {mirror }}}$.

SET UP: $\quad d_{\text {mirror }}=0.350 \mathrm{~m}, ~ h_{\text {mirror }}=0.0400 \mathrm{~m}$ and $d_{\text {tree }}=28.0 \mathrm{~m}+0.350 \mathrm{~m}$.
EXECUTE: $\quad h_{\text {tree }}=h_{\text {mirror }} \frac{d_{\text {tree }}}{d_{\text {mirror }}}=0.040 \mathrm{~m} \frac{28.0 \mathrm{~m}+0.350 \mathrm{~m}}{0.350 \mathrm{~m}}=3.24 \mathrm{~m}$.
Evaluate: The image of the tree formed by the mirror is 28.0 m behind the mirror and is 3.24 m tall.
34.3. Identify: Apply the law of reflection.

SET UP: If up is the $+y$-direction and right is the $+x$-direction, then the object is at $\left(-x_{0},-y_{0}\right)$ and $P_{2}^{\prime}$ is at $\left(x_{0},-y_{0}\right)$.
Execute: Mirror 1 flips the $y$-values, so the image is at $\left(x_{0}, y_{0}\right)$ which is $P_{3}^{\prime}$.
Evaluate: Mirror 2 uses $P_{1}^{\prime}$ as an object and forms an image at $P_{3}^{\prime}$.
34.4. Identify: $f=R / 2$

SET UP: For a concave mirror $R>0$.
EXECUTE: (a) $f=\frac{R}{2}=\frac{34.0 \mathrm{~cm}}{2}=17.0 \mathrm{~cm}$
Evaluate: (b) The image formation by the mirror is determined by the law of reflection and that is unaffected by the medium in which the light is traveling. The focal length remains 17.0 cm .
34.5. Identify and Set Up: Use Eq.(34.6) to calculate $s^{\prime}$ and use Eq.(34.7) to calculate $y^{\prime}$. The image is real if $s^{\prime}$ is positive and is erect if $m>0$. Concave means $R$ and $f$ are positive, $R=+22.0 \mathrm{~cm} ; f=R / 2=+11.0 \mathrm{~cm}$.
Execute: (a)


Three principal rays, numbered as in Sect. 34.2, are shown in Figure 34.5. The principal ray diagram shows that the image is real, inverted, and enlarged.

Figure 34.5
(b) $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$
$\frac{1}{s^{\prime}}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{s f}$ so $s^{\prime}=\frac{s f}{s-f}=\frac{(16.5 \mathrm{~cm})(11.0 \mathrm{~cm})}{16.5 \mathrm{~cm}-11.0 \mathrm{~cm}}=+33.0 \mathrm{~cm}$
$s^{\prime}>0$ so real image, 33.0 cm to left of mirror vertex
$m=-\frac{s^{\prime}}{s}=-\frac{33.0 \mathrm{~cm}}{16.5 \mathrm{~cm}}=-2.00\left(m<0\right.$ means inverted image) $\left|y^{\prime}\right|=|m||y|=2.00(0.600 \mathrm{~cm})=1.20 \mathrm{~cm}$
Evaluate: The image is 33.0 cm to the left of the mirror vertex. It is real, inverted, and is 1.20 cm tall (enlarged). The calculation agrees with the image characterization from the principal ray diagram. A concave mirror used alone always forms a real, inverted image if $s>f$ and the image is enlarged if $f<s<2 f$.
34.6. Identify: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and $m=-\frac{s^{\prime}}{s}$.

SET UP: For a convex mirror, $R<0 . R=-22.0 \mathrm{~cm}$ and $f=\frac{R}{2}=-11.0 \mathrm{~cm}$.
Execute: (a) The principal-ray diagram is sketched in Figure 34.6.
(b) $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} . \quad s^{\prime}=\frac{s f}{s-f}=\frac{(16.5 \mathrm{~cm})(-11.0 \mathrm{~cm})}{16.5 \mathrm{~cm}-(-11.0 \mathrm{~cm})}=-6.6 \mathrm{~cm} . \quad m=-\frac{s^{\prime}}{s}=-\frac{-6.6 \mathrm{~cm}}{16.5 \mathrm{~cm}}=+0.400$.
$\left|y^{\prime}\right|=|m| y=(0.400)(0.600 \mathrm{~cm})=0.240 \mathrm{~cm}$. The image is 6.6 cm to the right of the mirror. It is 0.240 cm tall. $s^{\prime}<0$, so the image is virtual. $m>0$, so the image is erect.
Evaluate: The calculated image properties agree with the image characterization from the principal-ray diagram.


Figure 34.6
34.7. IDENTIFY: $\quad \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} . m=-\frac{s^{\prime}}{s} .|m|=\frac{\left|y^{\prime}\right|}{y}$. Find $m$ and calculate $y^{\prime}$.

SET UP: $\quad f=+1.75 \mathrm{~m}$.
EXECUTE: $\quad s \gg f$ so $s^{\prime}=f=1.75 \mathrm{~m}$.
$m=-\frac{s^{\prime}}{s}=-\frac{1.75 \mathrm{~m}}{5.58 \times 10^{10} \mathrm{~m}}=-3.14 \times 10^{-11} .\left|y^{\prime}\right|=|m| y=\left(3.14 \times 10^{-11}\right)\left(6.794 \times 10^{6} \mathrm{~m}\right)=2.13 \times 10^{-4} \mathrm{~m}=0.213 \mathrm{~mm}$.
Evaluate: The image is real and is 1.75 m in front of the mirror.
34.8. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and $m=-\frac{s^{\prime}}{s}$.

SET Up: The mirror surface is convex so $R=-3.00 \mathrm{~cm} . \quad s=24.0 \mathrm{~cm}-3.00 \mathrm{~cm}=21.0 \mathrm{~cm}$.
EXECUTE: $\quad f=\frac{R}{2}=-1.50 \mathrm{~cm} \cdot \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \cdot s^{\prime}=\frac{s f}{s-f}=\frac{(21.0 \mathrm{~cm})(-1.50 \mathrm{~cm})}{21.0 \mathrm{~cm}-(-1.50 \mathrm{~cm})}=-1.40 \mathrm{~cm}$. The image is
1.40 cm behind the surface so it is $3.00 \mathrm{~cm}-1.40 \mathrm{~cm}=1.60 \mathrm{~cm}$ from the center of the ornament, on the same side as the object. $m=-\frac{s^{\prime}}{s}=-\frac{-1.40 \mathrm{~cm}}{21.0 \mathrm{~cm}}=+0.0667 .\left|y^{\prime}\right|=|m| y=(0.0667)(3.80 \mathrm{~mm})=0.253 \mathrm{~mm}$.
Evaluate: The image is virtual, upright and smaller than the object.
34.9. Identify: The shell behaves as a spherical mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$, and its magnification is given by $m=-\frac{s^{\prime}}{s}$.

ExECUTE: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{s}=\frac{2}{-18.0 \mathrm{~cm}}-\frac{1}{-6.00 \mathrm{~cm}} \Rightarrow s=18.0 \mathrm{~cm}$ from the vertex.
$m=-\frac{s^{\prime}}{s}=-\frac{-6.00 \mathrm{~cm}}{18.0 \mathrm{~cm}}=\frac{1}{3} \Rightarrow y^{\prime}=\frac{1}{3}(1.5 \mathrm{~cm})=0.50 \mathrm{~cm}$. The image is 0.50 cm tall, erect, and virtual.
Evaluate: Since the magnification is less than one, the image is smaller than the object.
34.10. Identify: The bottom surface of the bowl behaves as a spherical convex mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$, and its magnification is given by $m=-\frac{s^{\prime}}{s}$.
EXECUTE: $\quad \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{s^{\prime}}=\frac{-2}{35 \mathrm{~cm}}-\frac{1}{90 \mathrm{~cm}} \Rightarrow s^{\prime}=-15 \mathrm{~cm}$ behind bowl.
$m=-\frac{s^{\prime}}{s}=\frac{15 \mathrm{~cm}}{90 \mathrm{~cm}}=0.167 \Rightarrow y^{\prime}=(0.167)(2.0 \mathrm{~cm})=0.33 \mathrm{~cm}$. The image is 0.33 cm tall, erect, and virtual.
Evaluate: Since the magnification is less than one, the image is smaller than the object.
34.11. Identify: We are dealing with a spherical mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$, and its magnification is given by $m=-\frac{s^{\prime}}{s}$.
EXECUTE: (a) $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{s^{\prime}}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{f s} \Rightarrow s^{\prime}=\frac{s f}{s-f}$. Also $m=-\frac{s^{\prime}}{s}=\frac{f}{f-s}$.
(b) The graph is given in Figure 34.11a.
(c) $s^{\prime}>0$ for $s>f, s<0$.
(d) $s^{\prime}<0$ for $0<s<f$.
(e) The image is at negative infinity, "behind" the mirror.
(f) At the focal point, $s=f$.
(g) The image is at the mirror, $s^{\prime}=0$.
(h) The graph is given in Figure 34.11b.
(i) Erect and larger if $0<s<f$.
(j) Inverted if $s>f$.
(k) The image is smaller if $s>2 f$ or $s<0$.
(l) As the object is moved closer and closer to the focal point, the magnification increases to infinite values. Evaluate: As the object crosses the focal point, both the image distance and the magnification undergo discontinuities.

(a)


Figure 34.11
34.12. Identify: $s^{\prime}=\frac{s f}{s-f}$ and $m=\frac{f}{f-s}$.

SET UP: With $f=-|f|, s^{\prime}=-\frac{s|f|}{s+|f|}$ and $m=\frac{|f|}{s+|f|}$.
Execute: The graphs are given in Figure 34.12.
(a) $s^{\prime}>0$ for $-|f|<s<0$.
(b) $s^{\prime}<0$ for $s<-|f|$ and $s<0$.
(c) If the object is at infinity, the image is at the outward going focal point.
(d) If the object is next to the mirror, then the image is also at the mirror
(e) The image is erect (magnification greater than zero) for $s>-|f|$.
(f) The image is inverted (magnification less than zero) for $s<-|f|$.
(g) The image is larger than the object (magnification greater than one) for $-2|f|<s<0$.
(h) The image is smaller than the object (magnification less than one) for $s>0$ and $s<-2|f|$.

Evaluate: For a real image $(s>0)$, the image formed by a convex mirror is always virtual and smaller than the object.


34.13. IDENTIFY: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and $m=\frac{y^{\prime}}{y}=-\frac{s^{\prime}}{s}$.

SET UP: $\quad m=+2.00$ and $s=1.25 \mathrm{~cm}$. An erect image must be virtual.
EXECUTE: (a) $s^{\prime}=\frac{s f}{s-f}$ and $m=-\frac{f}{s-f}$. For a concave mirror, $m$ can be larger than 1.00. For a convex mirror, $|f|=-f$ so $m=+\frac{|f|}{s+|f|}$ and $m$ is always less than 1.00. The mirror must be concave $(f>0)$.
(b) $\frac{1}{f}=\frac{s^{\prime}+s}{s s^{\prime}} . f=\frac{s s^{\prime}}{s+s^{\prime}} . m=-\frac{s^{\prime}}{s}=+2.00$ and $s^{\prime}=-2.00 s . f=\frac{s(-2.00 s)}{s-2.00 s}=+2.00 s=+2.50 \mathrm{~cm}$.
$R=2 f=+5.00 \mathrm{~cm}$.
(c) The principal ray diagram is drawn in Figure 34.13.

Evaluate: The principal-ray diagram agrees with the description from the equations.


Figure 34.13
34.14. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and $m=-\frac{s^{\prime}}{s}$.

SET UP: For a concave mirror, $R>0 . R=32.0 \mathrm{~cm}$ and $f=\frac{R}{2}=16.0 \mathrm{~cm}$.
EXECUTE: (a) $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \cdot s^{\prime}=\frac{s f}{s-f}=\frac{(12.0 \mathrm{~cm})(16.0 \mathrm{~cm})}{12.0 \mathrm{~cm}-16.0 \mathrm{~cm}}=-48.0 \mathrm{~cm} . \quad m=-\frac{s^{\prime}}{s}=-\frac{-48.0 \mathrm{~cm}}{12.0 \mathrm{~cm}}=+4.00$.
(b) $s^{\prime}=-48.0 \mathrm{~cm}$, so the image is 48.0 cm to the right of the mirror. $s^{\prime}<0$ so the image is virtual.
(c) The principal-ray diagram is sketched in Figure 34.14. The rules for principal rays apply only to paraxial rays. Principal ray 2 , that travels to the mirror along a line that passes through the focus, makes a large angle with the optic axis and is not described well by the paraxial approximation. Therefore, principal ray 2 is not included in the sketch.
Evaluate: A concave mirror forms a virtual image whenever $s<f$.


Figure 34.14
34.15. Identify: Apply Eq.(34.11), with $R \rightarrow \infty$. $\left|s^{\prime}\right|$ is the apparent depth.

SET UP The image and object are shown in Figure 34.15.


Figure 34.15

EXECUTE: $\quad s^{\prime}=-\frac{n_{b} s}{n_{a}}=-\frac{(1.00)(3.50 \mathrm{~cm})}{1.309}=-2.67 \mathrm{~cm}$
The apparent depth is 2.67 cm .
Evaluate: When the light goes from ice to air (larger to smaller $n$ ), it is bent away from the normal and the virtual image is closer to the surface than the object is.
34.16. Identify: The surface is flat so $R \rightarrow \infty$ and $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=0$.

SET UP: The light travels from the fish to the eye, so $n_{a}=1.333$ and $n_{b}=1.00$. When the fish is viewed, $s=7.0 \mathrm{~cm}$. The fish is $20.0 \mathrm{~cm}-7.0 \mathrm{~cm}=13.0 \mathrm{~cm}$ above the mirror, so the image of the fish is 13.0 cm below the mirror and $20.0 \mathrm{~cm}+13.0 \mathrm{~cm}=33.0 \mathrm{~cm}$ below the surface of the water. When the image is viewed, $s=33.0 \mathrm{~cm}$.
EXECUTE: (a) $s^{\prime}=-\left(\frac{n_{b}}{n_{a}}\right) s=-\left(\frac{1.00}{1.333}\right)(7.0 \mathrm{~cm})=-5.25 \mathrm{~cm}$. The apparent depth is 5.25 cm .
(b) $s^{\prime}=-\left(\frac{n_{b}}{n_{a}}\right) s=-\left(\frac{1.00}{1.333}\right)(33.0 \mathrm{~cm})=-24.8 \mathrm{~cm}$. The apparent depth of the image of the fish in the mirror is 24.8 cm .

Evaluate: In each case the apparent depth is less than the actual depth of what is being viewed.
34.17. IDENTIFY: $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} . m=-\frac{n_{a} s^{\prime}}{n_{b} s}$. Light comes from the fish to the person's eye.

SET UP: $\quad R=-14.0 \mathrm{~cm} . s=+14.0 \mathrm{~cm} . n_{a}=1.333$ (water). $n_{b}=1.00$ (air). Figure 34.17 shows the object and the refracting surface.
EXECUTE: (a) $\frac{1.333}{14.0 \mathrm{~cm}}+\frac{1.00}{s^{\prime}}=\frac{1.00-1.333}{-14.0 \mathrm{~cm}} . \quad s^{\prime}=-14.0 \mathrm{~cm} . \quad m=-\frac{(1.333)(-14.0 \mathrm{~cm})}{(1.00)(14.0 \mathrm{~cm})}=+1.33$.
The fish's image is 14.0 cm to the left of the bowl surface so is at the center of the bowl and the magnification is 1.33 .
(b) The focal point is at the image location when $s \rightarrow \infty . \frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} . n_{a}=1.00 . n_{b}=1.333 . R=+14.0 \mathrm{~cm}$. $\frac{1.333}{s^{\prime}}=\frac{1.333-1.00}{14.0 \mathrm{~cm}} . s^{\prime}=+56.0 \mathrm{~cm} . s^{\prime}$ is greater than the diameter of the bowl, so the surface facing the sunlight does not focus the sunlight to a point inside the bowl. The focal point is outside the bowl and there is no danger to the fish. Evaluate: In part (b) the rays refract when they exit the bowl back into the air so the image we calculated is not the final image.


Figure 34.17
34.18. IDENTIFY: Apply $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$.

SET UP: For a convex surface, $R>0 . R=+3.00 \mathrm{~cm} . n_{a}=1.00, n_{b}=1.60$.
EXECUTE: (a) $s \rightarrow \infty$. $\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} \cdot s^{\prime}=\left(\frac{n_{b}}{n_{b}-n_{a}}\right) R=\left(\frac{1.60}{1.60-1.00}\right)(+3.00 \mathrm{~cm})=+8.00 \mathrm{~cm}$. The image is
8.00 cm to the right of the vertex.
(b) $s=12.0 \mathrm{~cm} \cdot \frac{1.00}{12.0 \mathrm{~cm}}+\frac{1.60}{s^{\prime}}=\frac{1.60-1.00}{3.00 \mathrm{~cm}} . s^{\prime}=+13.7 \mathrm{~cm}$. The image is 13.7 cm to the right of the vertex.
(c) $s=2.00 \mathrm{~cm} \cdot \frac{1.00}{2.00 \mathrm{~cm}}+\frac{1.60}{s^{\prime}}=\frac{1.60-1.00}{3.00 \mathrm{~cm}} . s^{\prime}=-5.33 \mathrm{~cm}$. The image is 5.33 cm to the left of the vertex.

Evaluate: The image can be either real $\left(s^{\prime}>0\right)$ or virtual ( $s^{\prime}<0$ ), depending on the distance of the object from the refracting surface.
34.19. Identify: The hemispherical glass surface forms an image by refraction. The location of this image depends on the curvature of the surface and the indices of refraction of the glass and oil.
SET UP: The image and object distances are related to the indices of refraction and the radius of curvature by the equation $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$.

EXECUTE: $\quad \frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{1.45}{s}+\frac{1.60}{1.20 \mathrm{~m}}=\frac{0.15}{0.0300 \mathrm{~m}} \Rightarrow s=0.395 \mathrm{~cm}$
Evaluate: The presence of the oil changes the location of the image.
34.20. IDENTIFY: $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} . m=-\frac{n_{a} s^{\prime}}{n_{b} s}$.

SET UP: $\quad R=+4.00 \mathrm{~cm} . n_{a}=1.00 . n_{b}=1.60 . \quad s=24.0 \mathrm{~cm}$.
EXECUTE: $\frac{1}{24.0 \mathrm{~cm}}+\frac{1.60}{s^{\prime}}=\frac{1.60-1.00}{4.00 \mathrm{~cm}} . s^{\prime}=+14.8 \mathrm{~cm} . \quad m=-\frac{(1.00)(14.8 \mathrm{~cm})}{(1.60)(24.0 \mathrm{~cm})}=-0.385$.
$\left|y^{\prime}\right|=|m| y=(0.385)(1.50 \mathrm{~mm})=0.578 \mathrm{~mm}$. The image is 14.8 cm to the right of the vertex and is 0.578 mm tall. $m<0$, so the image is inverted.
Evaluate: The image is real.
34.21. Identify: Apply Eqs.(34.11) and (34.12). Calculate $s^{\prime}$ and $y^{\prime}$. The image is erect if $m>0$.

SET UP: The object and refracting surface are shown in Figure 34.21.


Figure 34.21
EXECUTE: $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$
$\frac{1.00}{24.0 \mathrm{~cm}}+\frac{1.60}{s^{\prime}}=\frac{1.60-1.00}{-4.00 \mathrm{~cm}}$
Multiplying by 24.0 cm gives $1.00+\frac{38.4}{s^{\prime}}=-3.60$
$\frac{38.4 \mathrm{~cm}}{s^{\prime}}=-4.60$ and $s^{\prime}=-\frac{38.4 \mathrm{~cm}}{4.60}=-8.35 \mathrm{~cm}$
Eq.(34.12): $m=-\frac{n_{a} s^{\prime}}{n_{b} s}=-\frac{(1.00)(-8.35 \mathrm{~cm})}{(1.60)(+24.0 \mathrm{~cm})}=+0.217$
$\left|y^{\prime}\right|=|m||y|=(0.217)(1.50 \mathrm{~mm})=0.326 \mathrm{~mm}$
Evaluate: The image is virtual $\left(s^{\prime}<0\right)$ and is 8.35 cm to the left of the vertex. The image is erect ( $m>0$ ) and is 0.326 mm tall. $R$ is negative since the center of curvature of the surface is on the incoming side.
34.22. IDENTIFY: The hemispherical glass surface forms an image by refraction. The location of this image depends on the curvature of the surface and the indices of refraction of the glass and liquid.
SET UP: The image and object distances are related to the indices of refraction and the radius of curvature by the equation $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$.
EXECUTE: $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{n_{a}}{14.0 \mathrm{~cm}}+\frac{1.60}{9.00 \mathrm{~cm}}=\frac{1.60-n_{a}}{4.00 \mathrm{~cm}} \Rightarrow n_{a}=1.24$.
Evaluate: The result is a reasonable refractive index for liquids.
34.23. IDENTIFY: Use $\frac{1}{f}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$ to calculate $f$. The apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and $m=\frac{y^{\prime}}{y}=-\frac{s^{\prime}}{s}$.

SET UP: $\quad R_{1} \rightarrow \infty . \quad R_{2}=-13.0 \mathrm{~cm}$. If the lens is reversed, $R_{1}=+13.0 \mathrm{~cm}$ and $R_{2} \rightarrow \infty$.
EXECUTE: (a) $\frac{1}{f}=(0.70)\left(\frac{1}{\infty}-\frac{1}{-13.0 \mathrm{~cm}}\right)=\frac{0.70}{13.0 \mathrm{~cm}}$ and $f=18.6 \mathrm{~cm} . \frac{1}{s^{\prime}}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{s f}$.
$s^{\prime}=\frac{s f}{s-f}=\frac{(22.5 \mathrm{~cm})(18.6 \mathrm{~cm})}{22.5 \mathrm{~cm}-18.6 \mathrm{~cm}}=107 \mathrm{~cm} . \quad m=-\frac{s^{\prime}}{s}=-\frac{107 \mathrm{~cm}}{22.5 \mathrm{~cm}}=-4.76$.
$y^{\prime}=m y=(-4.76)(3.75 \mathrm{~mm})=-17.8 \mathrm{~mm}$. The image is 107 cm to the right of the lens and is 17.8 mm tall. The image is real and inverted.
(b) $\frac{1}{f}=(n-1)\left(\frac{1}{13.0 \mathrm{~cm}}-\frac{1}{\infty}\right)$ and $f=18.6 \mathrm{~cm}$. The image is the same as in part (a).

Evaluate: Reversing a lens does not change the focal length of the lens.
34.24. Identify: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$. The sign of $f$ determines whether the lens is converging or diverging.

SET UP: $\quad s=16.0 \mathrm{~cm} . s^{\prime}=-12.0 \mathrm{~cm}$.
EXECUTE: (a) $f=\frac{s s^{\prime}}{s+s^{\prime}}=\frac{(16.0 \mathrm{~cm})(-12.0 \mathrm{~cm})}{16.0 \mathrm{~cm}+(-12.0 \mathrm{~cm})}=-48.0 \mathrm{~cm} . f<0$ and the lens is diverging.
(b) $m=-\frac{s^{\prime}}{s}=-\frac{-12.0 \mathrm{~cm}}{16.0 \mathrm{~cm}}=+0.750 .\left|y^{\prime}\right|=|m| y=(0.750)(8.50 \mathrm{~mm})=6.38 \mathrm{~mm} . \quad m>0$ and the image is erect.
(c) The principal-ray diagram is sketched in Figure 34.24.

Evaluate: A diverging lens always forms an image that is virtual, erect and reduced in size.


Figure 34.24
34.25. Identify: The liquid behaves like a lens, so the lensmaker's equation applies.

SET UP: The lensmaker's equation is $\frac{1}{s}+\frac{1}{s^{\prime}}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$, and the magnification of the lens is $m=-\frac{s^{\prime}}{s}$.
EXECUTE: (a) $\frac{1}{s}+\frac{1}{s^{\prime}}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right) \Rightarrow \frac{1}{24.0 \mathrm{~cm}}+\frac{1}{s^{\prime}}=(1.52-1)\left(\frac{1}{-7.00 \mathrm{~cm}}-\frac{1}{-4.00 \mathrm{~cm}}\right)$
$\Rightarrow s^{\prime}=71.2 \mathrm{~cm}$, to the right of the lens.
(b) $m=-\frac{s^{\prime}}{s}=-\frac{71.2 \mathrm{~cm}}{24.0 \mathrm{~cm}}=-2.97$

Evaluate: Since the magnification is negative, the image is inverted.
34.26. Identify: Apply $m=\frac{y^{\prime}}{y}=-\frac{s^{\prime}}{s}$ to relate $s^{\prime}$ and $s$ and then use $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$.

Set Up: Since the image is inverted, $y^{\prime}<0$ and $m<0$.
EXECUTE: $\quad m=\frac{y^{\prime}}{y}=\frac{-4.50 \mathrm{~cm}}{3.20 \mathrm{~cm}}=-1.406 . m=-\frac{s^{\prime}}{s}$ gives $s^{\prime}=+1.406 s . \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ gives
$\frac{1}{s}+\frac{1}{1.406 s}=\frac{1}{90.0 \mathrm{~cm}}$ and $s=154 \mathrm{~cm} . s^{\prime}=(1.406)(154 \mathrm{~cm})=217 \mathrm{~cm}$. The object is 154 cm to the left of the lens. The image is 217 cm to the right of the lens and is real.
Evaluate: For a single lens an inverted image is always real.
34.27. Identify: The thin-lens equation applies in this case.

SET UP: The thin-lens equation is $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$, and the magnification is $m=-\frac{s^{\prime}}{s}=\frac{y^{\prime}}{y}$.
EXECUTE: $\quad m=\frac{y^{\prime}}{y}=\frac{34.0 \mathrm{~mm}}{8.00 \mathrm{~mm}}=4.25=-\frac{s^{\prime}}{s}=-\frac{-12.0 \mathrm{~cm}}{s} \Rightarrow s=2.82 \mathrm{~cm}$. The thin-lens equation gives
$\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow f=3.69 \mathrm{~cm}$.
Evaluate: Since the focal length is positive, this is a converging lens. The image distance is negative because the object is inside the focal point of the lens.
34.28. Identify: Apply $m=-\frac{s^{\prime}}{s}$ to relate $s$ and $s^{\prime}$. Then use $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$.

Set Up: Since the image is to the right of the lens, $s^{\prime}>0 . s^{\prime}+s=6.00 \mathrm{~m}$.
EXECUTE: (a) $s^{\prime}=80.0 s$ and $s+s^{\prime}=6.00 \mathrm{~m}$ gives $81.00 s=6.00 \mathrm{~m}$ and $s=0.0741 \mathrm{~m} . s^{\prime}=5.93 \mathrm{~m}$.
(b) The image is inverted since both the image and object are real $\left(s^{\prime}>0, s>0\right)$.
(c) $\frac{1}{f}=\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{0.0741 \mathrm{~m}}+\frac{1}{5.93 \mathrm{~m}} \Rightarrow f=0.0732 \mathrm{~m}$, and the lens is converging.

Evaluate: The object is close to the lens and the image is much farther from the lens. This is typical for slide projectors.
34.29. Identify: Apply $\frac{1}{f}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$.

SET UP: For a distant object the image is at the focal point of the lens. Therefore, $f=1.87 \mathrm{~cm}$. For the doubleconvex lens, $R_{1}=+R$ and $R_{2}=-R$, where $R=2.50 \mathrm{~cm}$.
EXECUTE: $\quad \frac{1}{f}=(n-1)\left(\frac{1}{R}-\frac{1}{-R}\right)=\frac{2(n-1)}{R} . \quad n=\frac{R}{2 f}+1=\frac{2.50 \mathrm{~cm}}{2(1.87 \mathrm{~cm})}+1=1.67$.
Evaluate: $\quad f>0$ and the lens is converging. A double-convex lens is always converging.
34.30. Identify and Set Up: Apply $\frac{1}{f}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$

EXECUTE: We have a converging lens if the focal length is positive, which requires $\frac{1}{f}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)>0 \Rightarrow\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)>0$. This can occur in one of three ways:
(i) $R_{1}$ and $R_{2}$ both positive and $R_{1}<R_{2}$. (ii) $R_{1} \geq 0, R_{2} \leq 0$ (double convex and planoconvex).
(iii) $R_{1}$ and $R_{2}$ both negative and $\left|R_{1}\right|>\left|R_{2}\right|$ (meniscus). The three lenses in Figure 35.32a in the textbook fall into these categories.
We have a diverging lens if the focal length is negative, which requires
$\frac{1}{f}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)<0 \Rightarrow\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)<0$. This can occur in one of three ways:
(i) $R_{1}$ and $R_{2}$ both positive and $R_{1}>R_{2}$ (meniscus). (ii) $R_{1}$ and $R_{2}$ both negative and $\left|R_{2}\right|>\left|R_{1}\right|$. (iii) $R_{1} \leq 0, R_{2} \geq 0$ (planoconcave and double concave). The three lenses in Figure 34.32b in the textbook fall into these categories. Evaluate: The converging lenses in Figure 34.32a are all thicker at the center than at the edges. The diverging lenses in Figure 34.32b are all thinner at the center than at the edges.
34.31. Identify and Set Up: The equations $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and $m=-\frac{s^{\prime}}{s}$ apply to both thin lenses and spherical mirrors.

Execute: (a) The derivation of the equations in Exercise 34.11 is identical and one gets: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{s^{\prime}}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{f s} \Rightarrow s^{\prime}=\frac{s f}{s-f}$, and also $m=-\frac{s^{\prime}}{s}=\frac{f}{f-s}$.
(b) Again, one gets exactly the same equations for a converging lens rather than a concave mirror because the equations are identical. The difference lies in the interpretation of the results. For a lens, the outgoing side is not that on which the object lies, unlike for a mirror. So for an object on the left side of the lens, a positive image distance means that the image is on the right of the lens, and a negative image distance means that the image is on the left side of the lens.
(c) Again, for Exercise 34.12, the change from a convex mirror to a diverging lens changes nothing in the exercises, except for the interpretation of the location of the images, as explained in part (b) above.
Evaluate: Concave mirrors and converging lenses both have $f>0$. Convex mirrors and diverging lenses both have $f<0$.
34.32. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and $m=\frac{y^{\prime}}{y}=-\frac{s^{\prime}}{s}$.

SET UP: $\quad f=+12.0 \mathrm{~cm}$ and $s^{\prime}=-17.0 \mathrm{~cm}$.
EXECUTE: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{s}=\frac{1}{12.0 \mathrm{~cm}}-\frac{1}{-17.0 \mathrm{~cm}} \Rightarrow s=7.0 \mathrm{~cm}$.
$m=-\frac{s^{\prime}}{s}=-\frac{(-17.0)}{7.2}=+2.4 \Rightarrow y=\frac{y^{\prime}}{m}=\frac{0.800 \mathrm{~cm}}{+2.4}=+0.34 \mathrm{~cm}$, so the object is 0.34 cm tall, erect, same side as the image. The principal-ray diagram is sketched in Figure 34.32.

Evaluate: When the object is inside the focal point, a converging lens forms a virtual, enlarged image.


Figure 34.32
34.33. Identify: Use Eq.(34.16) to calculate the object distance $s . m$ calculated from Eq.(34.17) determines the size and orientation of the image.
SET UP: $f=-48.0 \mathrm{~cm}$. Virtual image 17.0 cm from lens so $s^{\prime}=-17.0 \mathrm{~cm}$.
Execute: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$, so $\frac{1}{s^{\prime}}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{s f}$
$s=\frac{s^{\prime} f}{s^{\prime}-f}=\frac{(-17.0 \mathrm{~cm})(-48.0 \mathrm{~cm})}{-17.0 \mathrm{~cm}-(-48.0 \mathrm{~cm})}=+26.3 \mathrm{~cm}$
$m=-\frac{s^{\prime}}{s}=-\frac{-17.0 \mathrm{~cm}}{+26.3 \mathrm{~cm}}=+0.646$
$m=\frac{y^{\prime}}{y}$ so $|y|=\frac{\left|y^{\prime}\right|}{|m|}=\frac{8.00 \mathrm{~mm}}{0.646}=12.4 \mathrm{~mm}$
The principal-ray diagram is sketched in Figure 34.33.
Evaluate: Virtual image, real object ( $s>0$ ) so image and object are on same side of lens.
$m>0$ so image is erect with respect to the object. The height of the object is 12.4 mm .


Figure 34.33
34.34. Identify: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$.

SET UP: The sign of $f$ determines whether the lens is converging or diverging. $s=16.0 \mathrm{~cm} . \quad s^{\prime}=+36.0 \mathrm{~cm}$. Use $m=-\frac{s^{\prime}}{s}$ to find the size and orientation of the image.
EXECUTE: (a) $f=\frac{s s^{\prime}}{s+s^{\prime}}=\frac{(16.0 \mathrm{~cm})(36.0 \mathrm{~cm})}{16.0 \mathrm{~cm}+36.0 \mathrm{~cm}}=11.1 \mathrm{~cm} . f>0$ and the lens is converging.
(b) $m=-\frac{s^{\prime}}{s}=-\frac{36.0 \mathrm{~cm}}{16.0 \mathrm{~cm}}=-2.25 .\left|y^{\prime}\right|=|m| y=(2.25)(8.00 \mathrm{~mm})=18.0 \mathrm{~mm} . m<0$ so the image is inverted.
(c) The principal-ray diagram is sketched in Figure 34.34.

Evaluate: The image is real so the lens must be converging.


Figure 34.34
34.35. Identify: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$.

SET UP: The image is to be formed on the film, so $s^{\prime}=+20.4 \mathrm{~cm}$.
EXECUTE: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{s}+\frac{1}{20.4 \mathrm{~cm}}=\frac{1}{20.0 \mathrm{~cm}} \Rightarrow s=1020 \mathrm{~cm}=10.2 \mathrm{~m}$.
Evaluate: The object distance is much greater than $f$, so the image is just outside the focal point of the lens.
34.36. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and $m=\frac{y^{\prime}}{y}=-\frac{s^{\prime}}{s}$.

Set Up: $\quad s=3.90 \mathrm{~m} . \quad f=0.085 \mathrm{~m}$.
EXECUTE: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{3.90 \mathrm{~m}}+\frac{1}{s^{\prime}}=\frac{1}{0.085 \mathrm{~m}} \Rightarrow s^{\prime}=0.0869 \mathrm{~m} . \quad y^{\prime}=-\frac{s^{\prime}}{s} y=-\frac{0.0869}{3.90} 1750 \mathrm{~mm}=-39.0 \mathrm{~mm}$, so it will not fit on the $24-\mathrm{mm} \times 36-\mathrm{mm}$ film.
Evaluate: The image is just outside the focal point and $s^{\prime} \approx f$. To have $y^{\prime}=36 \mathrm{~mm}$, so that the image will fit on the film, $s=-\frac{s^{\prime} y}{y^{\prime}} \approx-\frac{(0.085 \mathrm{~m})(1.75 \mathrm{~m})}{-0.036 \mathrm{~m}}=4.1 \mathrm{~m}$. The person would need to stand about 4.1 m from the lens.
34.37. Identify: $|m|=\left|\frac{s^{\prime}}{s}\right|$.

SET UP: $\quad s \gg f$, so $s^{\prime} \approx f$.
EXECUTE: (a) $|m|=\frac{s^{\prime}}{s} \approx \frac{f}{s} \Rightarrow|m|=\frac{28 \mathrm{~mm}}{200,000 \mathrm{~mm}}=1.4 \times 10^{-4}$.
(b) $|m|=\frac{s^{\prime}}{s} \approx \frac{f}{s} \Rightarrow|m|=\frac{105 \mathrm{~mm}}{200,000 \mathrm{~mm}}=5.3 \times 10^{-4}$.
(c) $|m|=\frac{s^{\prime}}{s} \approx \frac{f}{s} \Rightarrow|m|=\frac{300 \mathrm{~mm}}{200,000 \mathrm{~mm}}=1.5 \times 10^{-3}$.

Evaluate: The magnitude of the magnification increases when $f$ increases.
34.38. IDENTIFY: $\quad|m|=\left|\frac{s^{\prime}}{s}\right|=\frac{\left|y^{\prime}\right|}{y}$

SET UP: $\quad s \gg f$, so $s^{\prime} \approx f$.
EXECUTE: $\quad\left|y^{\prime}\right|=\frac{s^{\prime}}{s} y \approx \frac{f}{s} y=\frac{5.00 \mathrm{~m}}{9.50 \times 10^{3} \mathrm{~m}}(70.7 \mathrm{~m})=0.0372 \mathrm{~m}=37.2 \mathrm{~mm}$.
Evaluate: A very long focal length lens is needed to photograph a distant object.
34.39. Identify and Set Up: Find the lateral magnification that results in this desired image size. Use Eq.(34.17) to relate $m$ and $s^{\prime}$ and Eq.(34.16) to relate $s$ and $s^{\prime}$ to $f$.
EXECUTE: (a) We need $m=-\frac{24 \times 10^{-3} \mathrm{~m}}{160 \mathrm{~m}}=-1.5 \times 10^{-4}$. Alternatively, $m=-\frac{36 \times 10^{-3} \mathrm{~m}}{240 \mathrm{~m}}=-1.5 \times 10^{-4}$. $s \gg f$ so $s^{\prime} \approx f$
Then $m=-\frac{s^{\prime}}{s}=-\frac{f}{s}=-1.5 \times 10^{-4}$ and $f=\left(1.5 \times 10^{-4}\right)(600 \mathrm{~m})=0.090 \mathrm{~m}=90 \mathrm{~mm}$.
A smaller $f$ means a smaller $s^{\prime}$ and a smaller $m$, so with $f=85 \mathrm{~mm}$ the object's image nearly fills the picture area.
(b) We need $m=-\frac{36 \times 10^{-3} \mathrm{~m}}{9.6 \mathrm{~m}}=-3.75 \times 10^{-3}$. Then, as in part (a), $\frac{f}{s}=3.75 \times 10^{-3}$ and
$f=(40.0 \mathrm{~m})\left(3.75 \times 10^{-3}\right)=0.15 \mathrm{~m}=150 \mathrm{~mm}$. Therefore use the 135 mm lens.
Evaluate: When $s \gg f$ and $s^{\prime} \approx f, y^{\prime}=-f(y / s)$. For the mobile home $y / s$ is smaller so a larger $f$ is needed. Note that $m$ is very small; the image is much smaller than the object.
34.40. Identify: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ to each lens. The image of the first lens serves as the object for the second lens.

Set Up: For a distant object, $s \rightarrow \infty$
EXECUTE: (a) $s_{1}=\infty \Rightarrow s_{1}^{\prime}=f_{1}=12 \mathrm{~cm}$.
(b) $s_{2}=4.0 \mathrm{~cm}-12 \mathrm{~cm}=-8 \mathrm{~cm}$.
(c) $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{-8 \mathrm{~cm}}+\frac{1}{s_{2}^{\prime}}=\frac{1}{-12 \mathrm{~cm}} \Rightarrow s_{2}^{\prime}=24 \mathrm{~cm}$, to the right.
(d) $s_{1}=\infty \Rightarrow s_{1}^{\prime}=f_{1}=12 \mathrm{~cm} . s_{2}=8.0 \mathrm{~cm}-12 \mathrm{~cm}=-4 \mathrm{~cm} . \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{-4 \mathrm{~cm}}+\frac{1}{s_{2}^{\prime}}=\frac{1}{-12 \mathrm{~cm}} \Rightarrow s_{2}^{\prime}=6 \mathrm{~cm}$.

Evaluate: In each case the image of the first lens serves as a virtual object for the second lens, and $s_{2}<0$.
34.41. Identify: The $f$-number of a lens is the ratio of its focal length to its diameter. To maintain the same exposure, the amount of light passing through the lens during the exposure must remain the same.
Set Up: The $f$-number is $f / D$.
EXECUTE: (a) $f$-number $=\frac{f}{D} \Rightarrow f$-number $=\frac{180.0 \mathrm{~mm}}{16.36 \mathrm{~mm}} \Rightarrow f$-number $=f / 11$. (The $f$-number is an integer.)
(b) $f / 11$ to $f / 2.8$ is four steps of 2 in intensity, so one needs $1 / 16^{\text {th }}$ the exposure. The exposure should be $1 / 480 \mathrm{~s}=$ $2.1 \times 10^{-3} \mathrm{~s}=2.1 \mathrm{~ms}$.
Evaluate: When opening the lens from $f / 11$ to $f / 2.8$, the area increases by a factor of 16 , so 16 times as much light is allowed in. Therefore the exposure time must be decreased by a factor of $1 / 16$ to maintain the same exposure on the film or light receptors of a digital camera.
34.42. Identify and Set Up: The square of the aperture diameter is proportional to the length of the exposure time required.
ExECUTE: $\left(\frac{1}{30} \mathrm{~s}\right)\left(\frac{8 \mathrm{~mm}}{23.1 \mathrm{~mm}}\right)^{2} \approx\left(\frac{1}{250} \mathrm{~s}\right)$
Evaluate: An increase in the aperture diameter decreases the exposure time.
34.43. Identify and Set UP: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ to calculate $s^{\prime}$.

Execute: (a) A real image is formed at the film, so the lens must be convex.
(b) $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ so $\frac{1}{s^{\prime}}=\frac{s-f}{s f}$ and $s^{\prime}=\frac{s f}{s-f}$, with $f=+50.0 .0 \mathrm{~mm}$. For $s=45 \mathrm{~cm}=450 \mathrm{~mm}, \mathrm{~s}^{\prime}=56 \mathrm{~mm}$. For $s=\infty, s^{\prime}=f=50 \mathrm{~mm}$. The range of distances between the lens and film is 50 mm to 56 mm .
Evaluate: The lens is closer to the film when photographing more distant objects.
34.44. Identify: The projector lens can be modeled as a thin lens.

SET UP: The thin-lens equation is $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$, and the magnification of the lens is $m=-\frac{s^{\prime}}{s}$.
ExECUTE: (a) $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{f}=\frac{1}{0.150 \mathrm{~m}}+\frac{1}{9.00 \mathrm{~m}} \Rightarrow f=147.5 \mathrm{~mm}$, so use a $f=148 \mathrm{~mm}$ lens.
(b) $m=-\frac{s^{\prime}}{s} \Rightarrow|m|=60 \Rightarrow$ Area $=1.44 \mathrm{~m} \times 2.16 \mathrm{~m}$.

Evaluate: The lens must produce a real image to be viewed on the screen. Since the magnification comes out negative, the slides to be viewed must be placed upside down in the tray.
34.45. (a) IDENTIFY: The purpose of the corrective lens is to take an object 25 cm from the eye and form a virtual image at the eye's near point. Use Eq.(34.16) to solve for the image distance when the object distance is 25 cm .
SET UP: $\frac{1}{f}=+2.75$ diopters means $f=+\frac{1}{2.75} \mathrm{~m}=+0.3636 \mathrm{~m}$ (converging lens)
$f=36.36 \mathrm{~cm} ; s=25 \mathrm{~cm} ; s^{\prime}=$ ?
Execute: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ so
$s^{\prime}=\frac{s f}{s-f}=\frac{(25 \mathrm{~cm})(36.36 \mathrm{~cm})}{25 \mathrm{~cm}-36.36 \mathrm{~cm}}=-80.0 \mathrm{~cm}$
The eye's near point is 80.0 cm from the eye.
(b) Identify: The purpose of the corrective lens is to take an object at infinity and form a virtual image of it at the eye's far point. Use Eq.(34.16) to solve for the image distance when the object is at infinity.
SET UP: $\frac{1}{f}=-1.30$ diopters means $f=-\frac{1}{1.30} \mathrm{~m}=-0.7692 \mathrm{~m}$ (diverging lens)
$f=-76.92 \mathrm{~cm} ; s=\infty ; s^{\prime}=$ ?
EXECUTE: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and $s=\infty$ says $\frac{1}{s^{\prime}}=\frac{1}{f}$ and $s^{\prime}=f=-76.9 \mathrm{~cm}$. The eye's far point is 76.9 cm from the eye.

Evaluate: In each case a virtual image is formed by the lens. The eye views this virtual image instead of the object. The object is at a distance where the eye can't focus on it, but the virtual image is at a distance where the eye can focus.
34.46. IDENTIFY: $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$

SET UP: $\quad n_{a}=1.00, n_{b}=1.40 . s=40.0 \mathrm{~cm}, s^{\prime}=2.60 \mathrm{~cm}$.
EXECUTE: $\frac{1}{40.0 \mathrm{~cm}}+\frac{1.40}{2.60 \mathrm{~cm}}=\frac{0.40}{R}$ and $R=0.710 \mathrm{~cm}$.
Evaluate: The cornea presents a convex surface to the object, so $R>0$.
34.47. Identify: In each case the lens forms a virtual image at a distance where the eye can focus. Power in diopters equals $1 / f$, where $f$ is in meters.
SET UP: In part (a), $s=25 \mathrm{~cm}$ and in part (b), $s \rightarrow \infty$.
EXECUTE: (a) $\frac{1}{f}=\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{0.25 \mathrm{~m}}+\frac{1}{-0.600 \mathrm{~m}} \Rightarrow$ power $=\frac{1}{f}=+2.33$ diopters.
(b) $\frac{1}{f}=\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{\infty}+\frac{1}{-0.600 \mathrm{~m}} \Rightarrow$ power $=\frac{1}{f}=-1.67$ diopters.

Evaluate: A converging lens corrects the near vision and a diverging lens corrects the far vision.
34.48. Identify: When the object is at the focal point, $M=\frac{25.0 \mathrm{~cm}}{f}$. In part (b), apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ to calculate $s$ for $s^{\prime}=-25.0 \mathrm{~cm}$.
SET UP: Our calculation assumes the near point is 25.0 cm from the eye.
EXECUTE: (a) Angular magnification $M=\frac{25.0 \mathrm{~cm}}{f}=\frac{25.0 \mathrm{~cm}}{6.00 \mathrm{~cm}}=4.17$.
(b) $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{s}+\frac{1}{-25.0 \mathrm{~cm}}=\frac{1}{6.00 \mathrm{~cm}} \Rightarrow s=4.84 \mathrm{~cm}$.

Evaluate: In part (b), $\theta^{\prime}=\frac{y}{s}, \theta=\frac{y}{25.0 \mathrm{~cm}}$ and $M=\frac{25.0 \mathrm{~cm}}{s}=\frac{25.0 \mathrm{~cm}}{4.84 \mathrm{~cm}}=5.17 . M$ is greater when the image is at the near point than when the image is at infinity.
34.49. Identify: Use Eqs.(34.16) and (34.17) to calculate $s$ and $y^{\prime}$.
(a) SET UP: $\quad f=8.00 \mathrm{~cm} ; s^{\prime}=-25.0 \mathrm{~cm} ; s=$ ?
$\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$, so $\frac{1}{s}=\frac{1}{f}-\frac{1}{s^{\prime}}=\frac{s^{\prime}-f}{s^{\prime} f}$
EXECUTE: $\quad s=\frac{s^{\prime} f}{s^{\prime}-f}=\frac{(-25.0 \mathrm{~cm})(+8.00 \mathrm{~cm})}{-25.0 \mathrm{~cm}-8.00 \mathrm{~cm}}=+6.06 \mathrm{~cm}$
(b) $m=-\frac{s^{\prime}}{s}=-\frac{-25.0 \mathrm{~cm}}{6.06 \mathrm{~cm}}=+4.125$
$|m|=\frac{\left|y^{\prime}\right|}{|y|}$ so $\left|y^{\prime}\right|=|m||y|=(4.125)(1.00 \mathrm{~mm})=4.12 \mathrm{~mm}$
Evaluate: The lens allows the object to be much closer to the eye than the near point. The lense allows the eye to view an image at the near point rather than the object.
34.50. Identify: For a thin lens, $-\frac{s^{\prime}}{s}=\frac{y^{\prime}}{y}$, so $\left|\frac{y^{\prime}}{s^{\prime}}\right|=\left|\frac{y}{s}\right|$, and the angular size of the image equals the angular size of the object.
SET UP: The object has angular size $\theta=\frac{y}{f}$, with $\theta$ in radians.
ExECUTE: $\quad \theta=\frac{y}{f} \Rightarrow f=\frac{y}{\theta}=\frac{2.00 \mathrm{~mm}}{0.025 \mathrm{rad}}=80.0 \mathrm{~mm}=8.00 \mathrm{~cm}$.
Evaluate: If the insect is at the near point of a normal eye, its angular size is $\frac{2.00 \mathrm{~mm}}{250 \mathrm{~mm}}=0.0080 \mathrm{rad}$.
34.51. IDENTIFY: The thin-lens equation applies to the magnifying lens.

SET UP: The thin-lens equation is $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$.

ExECUTE: The image is behind the lens, so $s^{\prime}<0$. The thin-lens equation gives

$$
\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{s}=\frac{1}{5.00 \mathrm{~cm}}-\frac{1}{-25.0 \mathrm{~cm}} \Rightarrow s=4.17 \mathrm{~cm}, \text { on the same side of the lens as the ant. }
$$

Evaluate: Since $s^{\prime}<0$, the image will be erect.
34.52. Identify: Apply Eq.(34.24).

SET UP: $\quad s_{1}^{\prime}=160 \mathrm{~mm}+5.0 \mathrm{~mm}=165 \mathrm{~mm}$
EXECUTE: (a) $M=\frac{(250 \mathrm{~mm}) s_{1}^{\prime}}{f_{1} f_{2}}=\frac{(250 \mathrm{~mm})(165 \mathrm{~mm})}{(5.00 \mathrm{~mm})(26.0 \mathrm{~mm})}=317$.
(b) The minimum separation is $\frac{0.10 \mathrm{~mm}}{M}=\frac{0.10 \mathrm{~mm}}{317}=3.15 \times 10^{-4} \mathrm{~mm}$.

Evaluate: The angular size of the image viewed by the eye when looking through the microscope is 317 times larger than if the object is viewed at the near-point of the unaided eye.
34.53. (a) Identify and SET Up:


Figure 34.53
Final image is at $\infty$ so the object for the eyepiece is at its focal point. But the object for the eyepiece is the image of the objective so the image formed by the objective is $19.7 \mathrm{~cm}-1.80 \mathrm{~cm}=17.9 \mathrm{~cm}$ to the right of the lens.
Apply Eq.(34.16) to the image formation by the objective, solve for the object distance $s$.
$f=0.800 \mathrm{~cm} ; s^{\prime}=17.9 \mathrm{~cm} ; s=$ ?
$\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$, so $\frac{1}{s}=\frac{1}{f}-\frac{1}{s^{\prime}}=\frac{s^{\prime}-f}{s^{\prime} f}$
EXECUTE: $\quad s=\frac{s^{\prime} f}{s^{\prime}-f}=\frac{(17.9 \mathrm{~cm})(+0.800 \mathrm{~cm})}{17.9 \mathrm{~cm}-0.800 \mathrm{~cm}}=+8.37 \mathrm{~mm}$
(b) Set Up: Use Eq.(34.17).

EXECUTE: $\quad m_{1}=-\frac{s^{\prime}}{s}=-\frac{17.9 \mathrm{~cm}}{0.837 \mathrm{~cm}}=-21.4$
The linear magnification of the objective is 21.4.
(c) SET Up: Use Eq.(34.23): $\quad M=m_{1} M_{2}$

EXECUTE: $\quad M_{2}=\frac{25 \mathrm{~cm}}{f_{2}}=\frac{25 \mathrm{~cm}}{1.80 \mathrm{~cm}}=13.9$
$M=m_{1} M_{2}=(-21.4)(13.9)=-297$
Evaluate: $M$ is not accurately given by $(25 \mathrm{~cm}) s_{1}^{\prime} / f_{1} f_{2}=311$, because the object is not quite at the focal point of the objective ( $s_{1}=0.837 \mathrm{~cm}$ and $f_{1}=0.800 \mathrm{~cm}$ ).
34.54. Identify: Eq.(34.24) can be written $M=\left|m_{1}\right| M_{2}=\left|\frac{s_{1}^{\prime}}{f_{1}}\right| M_{2}$.

SET UP: $\quad s_{1}^{\prime}=f_{1}+120 \mathrm{~mm}$
EXECUTE: $\quad f=16 \mathrm{~mm}: s^{\prime}=120 \mathrm{~mm}+16 \mathrm{~mm}=136 \mathrm{~mm} ; s=16 \mathrm{~mm} .\left|m_{1}\right|=\frac{s^{\prime}}{s}=\frac{136 \mathrm{~mm}}{16 \mathrm{~mm}}=8.5$.
$f=4 \mathrm{~mm}: s^{\prime}=120 \mathrm{~mm}+4 \mathrm{~mm}=124 \mathrm{~mm} ; s=4 \mathrm{~mm} \Rightarrow\left|m_{1}\right|=\frac{s^{\prime}}{s}=\frac{124 \mathrm{~mm}}{4 \mathrm{~mm}}=31$.
$f=1.9 \mathrm{~mm}: s^{\prime}=120 \mathrm{~mm}+1.9 \mathrm{~mm}=122 \mathrm{~mm} ; s=1.9 \mathrm{~mm} \Rightarrow\left|m_{1}\right|=\frac{s^{\prime}}{s}=\frac{122 \mathrm{~mm}}{1.9 \mathrm{~mm}}=64$.
The eyepiece magnifies by either 5 or 10 , so:
(a) The maximum magnification occurs for the $1.9-\mathrm{mm}$ objective and 10x eyepiece:
$M=\left|m_{1}\right| M_{\mathrm{e}}=(64)(10)=640$.
(b) The minimum magnification occurs for the 16-mm objective and 5 x eyepiece:
$M=\left|m_{1}\right| M_{\mathrm{e}}=(8.5)(5)=43$.
Evaluate: The smaller the focal length of the objective, the greater the overall magnification.
34.55. $\quad$ IDENTIFY: $f$-number $=f / D$

SET UP: $\quad D=1.02 \mathrm{~m}$
EXECUTE: $\frac{f}{D}=19.0 \Rightarrow f=(19.0) D=(19.0)(1.02 \mathrm{~m})=19.4 \mathrm{~m}$.
Evaluate: Camera lenses can also have an $f$-number of 19.0. For a camera lens, both the focal length and lens diameter are much smaller, but the $f$-number is a measure of their ratio.
34.56. Identify: For a telescope, $M=-\frac{f_{1}}{f_{2}}$.

SET UP: $\quad f_{2}=9.0 \mathrm{~cm}$. The distance between the two lenses equals $f_{1}+f_{2}$.
EXECUTE: $\quad f_{1}+f_{2}=1.80 \mathrm{~m} \Rightarrow f_{1}=1.80 \mathrm{~m}-0.0900 \mathrm{~m}=1.71 \mathrm{~m} . \quad M=-\frac{f_{1}}{f_{2}}=-\frac{171}{9.00}=-19.0$.
Evaluate: For a telescope, $f_{1} \gg f_{2}$.
34.57. (a) Identify and Set Up: Use Eq.(34.24), with $f_{1}=95.0 \mathrm{~cm}$ (objective) and $f_{2}=15.0 \mathrm{~cm}$ (eyepiece).

EXECUTE: $\quad M=-\frac{f_{1}}{f_{2}}=-\frac{95.0 \mathrm{~cm}}{15.0 \mathrm{~cm}}=-6.33$
(b) Identify and Set Up: Use Eq.(34.17) to calculate $y^{\prime}$.

SET UP: $\quad s=3.00 \times 10^{3} \mathrm{~m}$
$s^{\prime}=f_{1}=95.0 \mathrm{~cm}$ (since $s$ is very large, $\left.s^{\prime} \approx f\right)$
EXECUTE: $\quad m=-\frac{s^{\prime}}{s}=-\frac{0.950 \mathrm{~m}}{3.00 \times 10^{3} \mathrm{~m}}=-3.167 \times 10^{-4}$
$\left|y^{\prime}\right|=|m||y|=\left(3.167 \times 10^{-4}\right)(60.0 \mathrm{~m})=0.0190 \mathrm{~m}=1.90 \mathrm{~cm}$
(c) Identify: Use Eq. (34.21) and the angular magnification $M$ obtained in part (a) to calculate $\theta^{\prime}$. The angular size $\theta$ of the image formed by the objective (object for the eyepiece) is its height divided by its distance from the objective.
EXECUTE: The angular size of the object for the eyepiece is $\theta=\frac{0.0190 \mathrm{~m}}{0.950 \mathrm{~m}}=0.0200 \mathrm{rad}$.
(Note that this is also the angular size of the object for the objective: $\theta=\frac{60.0 \mathrm{~m}}{3.00 \times 10^{3} \mathrm{~m}}=0.0200 \mathrm{rad}$. For a thin lens the object and image have the same angular size and the image of the objective is the object for the eyepiece.)
$M=\frac{\theta^{\prime}}{\theta}$ (Eq.34.21) so the angular size of the image is $\theta^{\prime}=M \theta=-(6.33)(0.0200 \mathrm{rad})=-0.127 \mathrm{rad}$ (The minus sign shows that the final image is inverted.)
Evaluate: The lateral magnification of the objective is small; the image it forms is much smaller than the object. But the total angular magnification is larger than 1.00 ; the angular size of the final image viewed by the eye is 6.33 times larger than the angular size of the original object, as viewed by the unaided eye.
34.58. Identify: The angle subtended by Saturn with the naked eye is the same as the angle subtended by the image of Saturn formed by the objective lens (see Fig. 34.53 in the textbook).
SET UP: The angle subtended by Saturn is $\theta=\frac{\text { diameter of Saturn }}{\text { distance to Saturn }}=\frac{y^{\prime}}{f_{1}}$.
EXECUTE: Putting in the numbers gives $\theta=\frac{y^{\prime}}{f_{1}}=\frac{1.7 \mathrm{~mm}}{18 \mathrm{~m}}=\frac{0.0017 \mathrm{~m}}{18 \mathrm{~m}}=9.4 \times 10^{-5} \mathrm{rad}=0.0054^{\circ}$
Evaluate: The angle subtended by the final image, formed by the eyepiece, would be much larger than $0.0054^{\circ}$.
34.59. IDENTIFY: $f=R / 2$ and $M=-\frac{f_{1}}{f_{2}}$.

SET UP: For object and image both at infinity, $f_{1}+f_{2}$ equals the distance $d$ between the two mirrors. $f_{2}=1.10 \mathrm{~cm} . R_{1}=1.30 \mathrm{~m}$.
EXECUTE: (a) $f_{1}=\frac{R_{1}}{2}=0.650 \mathrm{~m} \Rightarrow d=f_{1}+f_{2}=0.661 \mathrm{~m}$.
(b) $|M|=\frac{f_{1}}{f_{2}}=\frac{0.650 \mathrm{~m}}{0.011 \mathrm{~m}}=59.1$.

Evaluate: For a telescope, $f_{1} \gg f_{2}$.
34.60. Identify: The primary mirror forms an image which then acts as the object for the secondary mirror. SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$.
ExECUTE: For the first image (formed by the primary mirror):

$$
\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{s^{\prime}}=\frac{1}{2.5 \mathrm{~m}}-\frac{1}{\infty} \Rightarrow s^{\prime}=2.5 \mathrm{~m} .
$$

For the second image (formed by the secondary mirror), the distance between the two vertices is $x$. Assuming that the image formed by the primary mirror is to the right of the secondary mirror, the object distance is $s=x-2.5 \mathrm{~m}$ and the image distance is $s^{\prime}=x+0.15 \mathrm{~m}$. Therefore we have

$$
\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{x-2.5 \mathrm{~m}}+\frac{1}{x+0.15 \mathrm{~m}}=\frac{1}{-1.5 \mathrm{~m}}
$$

The positive root of the quadratic equation gives $x=1.7 \mathrm{~m}$, which is the distance between the vertices.
Evaluate: Some light is blocked by the secondary mirror, but usually not enough to make much difference.
34.61. Identify and Set UP: For a plane mirror $s^{\prime}=-s . v=\frac{d s}{d t}$ and $v^{\prime}=\frac{d s^{\prime}}{d t}$, so $v^{\prime}=-v$.

EXECUTE: The velocities of the object and image relative to the mirror are equal in magnitude and opposite in direction. Thus both you and your image are receding from the mirror surface at $2.40 \mathrm{~m} / \mathrm{s}$, in opposite directions. Your image is therefore moving at $4.80 \mathrm{~m} / \mathrm{s}$ relative to you.
Evaluate: The result derives from the fact that for a plane mirror the image is the same distance behind the mirror as the object is in front of the mirror.
34.62. Identify: Apply the law of reflection.

SET UP: The image of one mirror can serve as the object for the other mirror.
Execute: (a) There are three images formed, as shown in Figure 34.62a.
(b) The paths of rays for each image are sketched in Figure 34.62b.

Evaluate: Our results agree with Figure 34.9 in the textbook.

34.63. Identify: Apply the law of reflection for rays from the feet to the eyes and from the top of the head to the eyes. SET UP: In Figure 34.63, ray 1 travels from the feet of the woman to her eyes and ray 2 travels from the top of her head to her eyes. The total height of the woman is $h$.
ExECUTE: The two angles labeled $\theta_{1}$ are equal because of the law of reflection, as are the two angles labeled
$\theta_{2}$. Since these angles are equal, the two distances labeled $y_{1}$ are equal and the two distances labeled $y_{2}$ are equal.
The height of the woman is $h_{\mathrm{w}}=2 y_{1}+2 y_{2}$. As the drawing shows, the height of the mirror is $h_{\mathrm{m}}=y_{1}+y_{2}$.
Comparing, we find that $h_{\mathrm{m}}=h_{\mathrm{w}} / 2$. The minimum height required is half the height of the woman.

Evaluate: The height of the image is the same as the height of the woman, so the height of the image is twice the height of the mirror.


Figure 34.63
34.64. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{2}{R}$ and $m=-\frac{s^{\prime}}{s}$.

SET UP: Since the image is projected onto the wall it is real and $s^{\prime}>0 . m=-\frac{s^{\prime}}{s}$ so $m$ is negative and $m=-2.25$. The object, mirror and wall are sketched in Figure 34.64. The sketch shows that $s^{\prime}-s=400 \mathrm{~cm}$.
EXECUTE: $\quad m=-2.25=-\frac{s^{\prime}}{s}$ and $s^{\prime}=2.25 s . s^{\prime}-s=2.25 s-s=400 \mathrm{~cm}$ and $s=320 \mathrm{~cm}$.
$s^{\prime}=400 \mathrm{~cm}+320 \mathrm{~cm}=720 \mathrm{~cm}$. The mirror should be 7.20 m from the wall. $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{2}{R} \cdot \frac{1}{320 \mathrm{~cm}}+\frac{1}{720 \mathrm{~cm}}=\frac{2}{R}$. $R=4.43 \mathrm{~m}$.
Evaluate: The focal length of the mirror is $f=R / 2=222 \mathrm{~cm} . s>f$, as it must if the image is to be real.


## Figure 34.64

34.65. Identify: We are given the image distance, the image height, and the object height. Use Eq.(34.7) to calculate the object distance $s$. Then use Eq.(34.4) to calculate $R$.
(a) SET UP: Image is to be formed on screen so is real image; $s^{\prime}>0$. Mirror to screen distance is 8.00 m , so $s^{\prime}=+800 \mathrm{~cm} . m=-\frac{s^{\prime}}{s}<0$ since both $s$ and $s^{\prime}$ are positive.

EXECUTE: $\quad|m|=\frac{\left|y^{\prime}\right|}{|y|}=\frac{36.0 \mathrm{~m}}{0.600 \mathrm{~cm}}=60.0$ and $m=-60.0$. Then $m=-\frac{s^{\prime}}{s}$ gives $s=-\frac{s^{\prime}}{m}=-\frac{800 \mathrm{~cm}}{-60.0}=+13.3 \mathrm{~cm}$.
(b) $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{2}{R}$, so $\frac{2}{R}=\frac{s+s^{\prime}}{s s^{\prime}}$
$R=2\left(\frac{s s^{\prime}}{s+s^{\prime}}\right)=2\left(\frac{(13.3 \mathrm{~cm})(800 \mathrm{~cm})}{800 \mathrm{~cm}+13.3 \mathrm{~cm}}\right)=26.2 \mathrm{~cm}$
Evaluate: $R$ is calculated to be positive, which is correct for a concave mirror. Also, in part (a) $s$ is calculated to be positive, as it should be for a real object.
34.66. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ to calculate $s^{\prime}$ and then use $m=-\frac{s^{\prime}}{s}=\frac{y^{\prime}}{y}$ to find the height of the image.

SET UP: For a convex mirror, $R<0$, so $R=-18.0 \mathrm{~cm}$ and $f=\frac{R}{2}=-9.00 \mathrm{~cm}$.
EXECUTE: (a) $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} . s^{\prime}=\frac{s f}{s-f}=\frac{(1300 \mathrm{~cm})(-9.00 \mathrm{~cm})}{1300 \mathrm{~cm}-(-9.00 \mathrm{~cm})}=-8.94 \mathrm{~cm} . \quad m=-\frac{s^{\prime}}{s}=-\frac{-8.94 \mathrm{~cm}}{1300 \mathrm{~cm}}=6.88 \times 10^{-3}$.
$\left|y^{\prime}\right|=|m| y=\left(6.88 \times 10^{-3}\right)(1.5 \mathrm{~m})=0.0103 \mathrm{~m}=1.03 \mathrm{~cm}$.
(b) The height of the image is much less than the height of the car, so the car appears to be farther away than its actual distance.
Evaluate: The image formed by a convex mirror is always virtual and smaller than the object.
34.67. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{2}{R}$ and $m=-\frac{s^{\prime}}{s}$.

SET UP: $\quad R=+19.4 \mathrm{~cm}$.
EXECUTE: (a) $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{2}{R} \Rightarrow \frac{1}{8.0 \mathrm{~cm}}+\frac{1}{s^{\prime}}=\frac{2}{19.4 \mathrm{~cm}} \Rightarrow s^{\prime}=-46 \mathrm{~cm}$, so the image is virtual.
(b) $m=-\frac{s^{\prime}}{s}=-\frac{-46}{8.0}=5.8$, so the image is erect, and its height is $y^{\prime}=(5.8) y=(5.8)(5.0 \mathrm{~mm})=29 \mathrm{~mm}$.

Evaluate: (c) When the filament is 8 cm from the mirror, the image is virtual and cannot be projected onto a wall.
34.68. Identify: Combine $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{2}{R}$ and $m=-\frac{s^{\prime}}{s}$.

SET UP: $\quad m=+2.50 . \quad R>0$.
EXECUTE: $\quad m=-\frac{s^{\prime}}{s}=+2.50 . \quad s^{\prime}=-2.50 s . \quad \frac{1}{s}+\frac{1}{-2.50 s}=\frac{2}{R} . \quad \frac{0.600}{s}=\frac{2}{R}$ and $s=0.300 R$.
$s^{\prime}=-2.50 s=(-2.50)(0.300 R)=-0.750 R$. The object is a distance of $0.300 R$ in front of the mirror and the image is a distance of $0.750 R$ behind the mirror.
Evaluate: For a single mirror an erect image is always virtual.
34.69. Identify and Set Up: Apply Eqs.(34.6) and (34.7). For a virtual object $s<0$. The image is real if $s^{\prime}>0$.

Execute: (a) Convex implies $R<0 ; R=-24.0 \mathrm{~cm} ; f=R / 2=-12.0 \mathrm{~cm}$
$\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$, so $\frac{1}{s^{\prime}}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{s f}$
$s^{\prime}=\frac{s f}{s-f}=\frac{(-12.0 \mathrm{~cm}) s}{s+12.0 \mathrm{~cm}}$
$s$ is negative, so write as $s=-|s| ; s^{\prime}=+\frac{(12.0 \mathrm{~cm})|s|}{12.0 \mathrm{~cm}-|s|}$. Thus $s^{\prime}>0$ (real image) for $|s|<12.0 \mathrm{~cm}$. Since $s$ is negative this means $-12.0 \mathrm{~cm}<s<0$. A real image is formed if the virtual object is closer to the mirror than the focus.
(b) $m=-\frac{s^{\prime}}{s}$; real image implies $s^{\prime}>0$; virtual object implies $s<0$. Thus $m>0$ and the image is erect.
(c) The principal-ray diagram is given in Figure 34.69.


Figure 34.69
Evaluate: For a real object, only virtual images are formed by a convex mirror. The virtual object considered in this problem must have been produced by some other optical element, by another lens or mirror in addition to the convex one we considered.
34.70. Identify: Apply $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$, with $R \rightarrow \infty$ since the surfaces are flat.

SET UP: The image formed by the first interface serves as the object for the second interface.
Execute: For the water-benzene interface to get the apparent water depth:

$$
\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=0 \Rightarrow \frac{1.33}{6.50 \mathrm{~cm}}+\frac{1.50}{s^{\prime}}=0 \Rightarrow s^{\prime}=-7.33 \mathrm{~cm} .
$$

For the benzene-air interface, to get the total apparent distance to the bottom:

$$
\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=0 \Rightarrow \frac{1.50}{(7.33 \mathrm{~cm}+2.60 \mathrm{~cm})}+\frac{1}{s^{\prime}}=0 \Rightarrow s^{\prime}=-6.62 \mathrm{~cm} .
$$

Evaluate: At the water-benzene interface the light refracts into material of greater refractive index and the overall effect is that the apparent depth is greater than the actual depth.
34.71. Identify: The focal length is given by $\frac{1}{f}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$.

SET UP: $\quad R_{1}= \pm 4.0 \mathrm{~cm}$ or $\pm 8.0 \mathrm{~cm} . R_{2}= \pm 8.0 \mathrm{~cm}$ or $\pm 4.0 \mathrm{~cm}$. The signs are determined by the location of the center of curvature for each surface.
ExECUTE: $\frac{1}{f}=(0.60)\left(\frac{1}{ \pm 4.00 \mathrm{~cm}}-\frac{1}{ \pm 8.00 \mathrm{~cm}}\right)$, so $f= \pm 4.44 \mathrm{~cm}, \pm 13.3 \mathrm{~cm}$. The possible lens shapes are sketched in Figure 34.71.
$f_{1}=+13.3 \mathrm{~cm} ; f_{2}=+4.44 \mathrm{~cm} ; f_{3}=4.44 \mathrm{~cm} ; f_{4}=-13.3 \mathrm{~cm} ; f_{5}=-13.3 \mathrm{~cm} ; f_{6}=+13.3 \mathrm{~cm} ;$
$f_{7}=-4.44 \mathrm{~cm} ; f_{8}=-4.44 \mathrm{~cm}$.
Evaluate: $f$ is the same whether the light travels through the lens from right to left or left to right, so for the pairs $(1,6),(4,5)$ and $(7,8)$ the focal lengths are the same.

1

2

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Figure 34.71
34.72. Identify: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and the concept of principal rays.

SET UP: $s=10.0 \mathrm{~cm}$. If extended backwards the ray comes from a point on the optic axis 18.0 cm from the lens and the ray is parallel to the optic axis after it passes through the lens.
Execute: (a) The ray is bent toward the optic axis by the lens so the lens is converging.
(b) The ray is parallel to the optic axis after it passes through the lens so it comes from the focal point; $f=18.0 \mathrm{~cm}$.
(c) The principal ray diagram is drawn in Figure 34.72. The diagram shows that the image is 22.5 cm to the left of the lens.
(d) $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ gives $s^{\prime}=\frac{s f}{s-f}=\frac{(10.0 \mathrm{~cm})(18.0 \mathrm{~cm})}{10.0 \mathrm{~cm}-18.0 \mathrm{~cm}}=-22.5 \mathrm{~cm}$. The calculated image position agrees with the principal ray diagram.
Evaluate: The image is virtual. A converging lens produces a virtual image when the object is inside the focal point.


Figure 34.72
34.73. IDENTIFY: Since the truck is moving toward the mirror, its image will also be moving toward the mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$, where $f=R / 2$.
EXECUTE: Since the mirror is convex, $f=R / 2=(-1.50 \mathrm{~m}) / 2=-0.75 \mathrm{~m}$. Applying the equation for a spherical mirror gives $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow s^{\prime}=\frac{f s}{s-f}$.
Using the chain rule from calculus and the fact that $v=d s / d t$, we have $v^{\prime}=\frac{d s^{\prime}}{d t}=\frac{d s^{\prime}}{d s} \frac{d s}{d t}=v \frac{f^{2}}{(s-f)^{2}}$
Solving for $v$ gives $v=v^{\prime}\left(\frac{s-f}{f}\right)^{2}=(1.5 \mathrm{~m} / \mathrm{s})\left[\frac{2.0 \mathrm{~m}-(-0.75 \mathrm{~m})}{-0.75 \mathrm{~m}}\right]^{2}=20.2 \mathrm{~m} / \mathrm{s}$.
This is the velocity of the truck relative to the mirror, so the truck is approaching the mirror at $20.2 \mathrm{~m} / \mathrm{s}$. You are traveling at $25 \mathrm{~m} / \mathrm{s}$, so the truck must be traveling at $25 \mathrm{~m} / \mathrm{s}+20.2 \mathrm{~m} / \mathrm{s}=45 \mathrm{~m} / \mathrm{s}$ relative to the highway.
Evaluate: Even though the truck and car are moving at constant speed, the image of the truck is not moving at constant speed because its location depends on the distance from the mirror to the truck.
34.74. Identify: In this context, the microscope just looks at an image or object. Apply $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=0$ to the image formed by refraction at the top surface of the second plate. In this calculation the object is the bottom surface of the second plate.
SET UP: The thickness of the second plate is $2.50 \mathrm{~mm}+0.78 \mathrm{~mm}$, and this is $s$. The image is 2.50 mm below the top surface, so $s^{\prime}=-2.50 \mathrm{~mm}$.
EXECUTE: $\quad \frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=0 \Rightarrow \frac{n}{s}+\frac{1}{s^{\prime}}=0 \Rightarrow n=-\frac{s}{s^{\prime}}=-\frac{2.50 \mathrm{~mm}+0.780 \mathrm{~mm}}{-2.50 \mathrm{~mm}}=1.31$.
Evaluate: The object and image distances are measured from the front surface of the second plate, and the image is virtual.
34.75. Identify and Set Up: In part (a) use $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ to evaluate $d s^{\prime} / d s$. Compare to $m=-\frac{s^{\prime}}{s}$. In part (b) use $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{2}{R}$ to find the location of the image of each face of the cube.
EXECUTE: (a) $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and taking its derivative with respect to $s$ we have $0=\frac{d}{d s}\left(\frac{1}{s}+\frac{1}{s^{\prime}}-\frac{1}{f}\right)=-\frac{1}{s^{2}}-\frac{1}{s^{\prime 2}} \frac{d s^{\prime}}{d s}$ and $\frac{d s^{\prime}}{d s}=-\frac{s^{\prime 2}}{s^{2}}=-m^{2}$. But $\frac{d s^{\prime}}{d s}=m^{\prime}$, so $m^{\prime}=-m^{2}$. Images are always inverted longitudinally.
(b) (i) Front face: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{2}{R} \Rightarrow \frac{1}{200.000 \mathrm{~cm}}+\frac{1}{s^{\prime}}=\frac{2}{150.000 \mathrm{~cm}} \Rightarrow s^{\prime}=120.00 \mathrm{~cm}$.

Rear face: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{2}{R} \Rightarrow \frac{1}{200.100 \mathrm{~cm}}+\frac{1}{s^{\prime}}=\frac{2}{150.000 \mathrm{~cm}} \Rightarrow s^{\prime}=119.96 \mathrm{~cm}$.
(ii) $m=-\frac{s^{\prime}}{s}=-\frac{120.000}{200.000}=-0.600 . \quad m^{\prime}=-m^{2}=-(-0.600)^{2}=-0.360$.
(iii) The two faces perpendicular to the axis (the front and rear faces): squares with side length 0.600 mm . The four faces parallel to the axis (the side faces): rectangles with sides of length 0.360 mm parallel to the axis and 0.600 mm perpendicular to the axis.

Evaluate: Since the lateral and longitudinal magnifications have different values the image of the cube is not a cube.
34.76. IDENTIFY: $m^{\prime}=d s^{\prime} / d s$ and $m=-\frac{n_{a} s^{\prime}}{n_{b} s}$.

SET UP: Use $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$ to evaluate $d s^{\prime} / d s$.
EXECUTE: $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$ and taking its derivative with respect to $s$ we have
$0=\frac{d}{d s}\left(\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}-\frac{n_{b}-n_{a}}{R}\right)=-\frac{n_{a}}{s^{2}}-\frac{n_{b}}{s^{\prime 2}} \frac{d s^{\prime}}{d s}$ and $\frac{d s^{\prime}}{d s}=-\frac{s^{\prime 2}}{s^{2}} \frac{n_{a}}{n_{b}}=-\left(\frac{s^{\prime 2}}{s^{2}} \frac{n_{a}^{2}}{n_{b}^{2}}\right) \frac{n_{b}}{n_{a}}=-m^{2} \frac{n_{b}}{n_{a}}$.
But $\frac{d s^{\prime}}{d s}=m^{\prime}$, so $m^{\prime}=-m^{2} \frac{n_{b}}{n_{a}}$.
Evaluate: $\quad m^{\prime}$ is always negative. This means that images are always inverted longitudinally.
34.77. Identify and Set Up: Rays that pass through the hole are undeflected. All other rays are blocked. $m=-\frac{s^{\prime}}{s}$.

Execute: (a) The ray diagram is drawn in Figure 34.77. The ray shown is the only ray from the top of the object that reaches the film, so this ray passes through the top of the image. An inverted image is formed on the far side of the box, no matter how far this side is from the pinhole and no matter how far the object is from the pinhole.
(b) $s=1.5 \mathrm{~m} . \quad s^{\prime}=20.0 \mathrm{~cm} . \quad m=-\frac{s^{\prime}}{s}=-\frac{20.0 \mathrm{~cm}}{150 \mathrm{~cm}}=-0.133 . y^{\prime}=m y=(-0.133)(18 \mathrm{~cm})=-2.4 \mathrm{~cm}$. The image is 2.4 cm tall.

Evaluate: A defect of this camera is that not much light energy passes through the small hole each second, so long exposure times are required.


Figure 34.77
34.78. IDENTIFY: Apply $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$ and $m=-\frac{n_{a} s^{\prime}}{n_{b} s}$ to each refraction. The overall magnification is $m=m_{1} m_{2}$.

SET UP: For the first refraction, $R=+6.0 \mathrm{~cm}, n_{a}=1.00$ and $n_{b}=1.60$. For the second refraction, $R=-12.0 \mathrm{~cm}$, $n_{a}=1.60$ and $n_{b}=1.00$.
ExECUTE: (a) The image from the left end acts as the object for the right end of the rod.
(b) $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{1}{23.0 \mathrm{~cm}}+\frac{1.60}{s^{\prime}}=\frac{0.60}{6.0 \mathrm{~cm}} \Rightarrow s^{\prime}=28.3 \mathrm{~cm}$.

So the second object distance is $s_{2}=40.0 \mathrm{~cm}-28.3 \mathrm{~cm}=11.7 \mathrm{~cm} . m_{1}=-\frac{n_{a} s^{\prime}}{n_{b} s}=-\frac{28.3}{(1.60)(23.0)}=-0.769$.
(c) The object is real and inverted.
(d) $\frac{n_{a}}{s_{2}}+\frac{n_{b}}{s_{2}^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{1.60}{11.7 \mathrm{~cm}}+\frac{1}{s_{2}^{\prime}}=\frac{-0.60}{-12.0 \mathrm{~cm}} \Rightarrow s^{\prime}=-11.5 \mathrm{~cm}$.
$m_{2}=-\frac{n_{a} s^{\prime}}{n_{b} s}=-\frac{(1.60)(-11.5)}{11.7}=1.57 \Rightarrow m=m_{1} m_{2}=(-0.769)(1.57)=-1.21$.
(e) The final image is virtual, and inverted.
(f) $y^{\prime}=(1.50 \mathrm{~mm})(-1.21)=-1.82 \mathrm{~mm}$.

Evaluate: The first image is to the left of the second surface, so it serves as a real object for the second surface, with positive object distance.
34.79. Identify: Apply Eqs.(34.11) and (34.12) to the refraction as the light enters the rod and as it leaves the rod. The image formed by the first surface serves as the object for the second surface. The total magnification is $m_{\mathrm{tot}}=m_{1} m_{2}$, where $m_{1}$ and $m_{2}$ are the magnifications for each surface.
SET UP: The object and rod are shown in Figure 34.79.


Figure 34.79
(a) image formed by refraction at first surface (left end of rod):
$s=+23.0 \mathrm{~cm} ; n_{a}=1.00 ; n_{b}=1.60 ; R=+6.00 \mathrm{~cm}$
$\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$
EXECUTE: $\frac{1}{23.0 \mathrm{~cm}}+\frac{1.60}{s^{\prime}}=\frac{1.60-1.00}{6.00 \mathrm{~cm}}$
$\frac{1.60}{s^{\prime}}=\frac{1}{10.0 \mathrm{~cm}}-\frac{1}{23.0 \mathrm{~cm}}=\frac{23-10}{230 \mathrm{~cm}}=\frac{13}{230 \mathrm{~cm}}$
$s^{\prime}=1.60\left(\frac{230 \mathrm{~cm}}{13}\right)=+28.3 \mathrm{~cm}$; image is 28.3 cm to right of first vertex.
This image serves as the object for the refraction at the second surface (right-hand end of rod). It is
$28.3 \mathrm{~cm}-25.0 \mathrm{~cm}=3.3 \mathrm{~cm}$ to the right of the second vertex. For the second surface $s=-3.3 \mathrm{~cm}$ (virtual object).
(b) Evaluate: Object is on side of outgoing light, so is a virtual object.
(c) SET UP: Image formed by refraction at second surface (right end of rod):
$s=-3.3 \mathrm{~cm} ; n_{a}=1.60 ; n_{b}=1.00 ; R=-12.0 \mathrm{~cm}$
$\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$
EXECUTE: $\frac{1.60}{-3.3 \mathrm{~cm}}+\frac{1.00}{s^{\prime}}=\frac{1.00-1.60}{-12.0 \mathrm{~cm}}$
$s^{\prime}=+1.9 \mathrm{~cm} ; s^{\prime}>0$ so image is 1.9 cm to right of vertex at right-hand end of rod.
(d) $s^{\prime}>0$ so final image is real.

Magnification for first surface:
$m=-\frac{n_{a} s^{\prime}}{n_{b} s}=-\frac{(1.60)(+28.3 \mathrm{~cm})}{(1.00)(+23.0 \mathrm{~cm})}=-0.769$
Magnification for second surface:
$m=-\frac{n_{a} s^{\prime}}{n_{b} s}=-\frac{(1.60)(+1.9 \mathrm{~cm})}{(1.00)(-3.3 \mathrm{~cm})}=+0.92$
The overall magnification is $m_{\text {tot }}=m_{1} m_{2}=(-0.769)(+0.92)=-0.71 \quad m_{\text {tot }}<0$ so final image is inverted with respect to the original object.
(e) $y^{\prime}=m_{\text {tot }} y=(-0.71)(1.50 \mathrm{~mm})=-1.06 \mathrm{~mm}$

The final image has a height of 1.06 mm .
Evaluate: The two refracting surfaces are not close together and Eq.(34.18) does not apply.
34.80. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and $m=\frac{y^{\prime}}{y}=-\frac{s^{\prime}}{s}$. The type of lens determines the sign of $f$. The sign of $s^{\prime}$ determines whether the image is real or virtual.
SET UP: $\quad s=+8.00 \mathrm{~cm} . s^{\prime}=-3.00 \mathrm{~cm} . s^{\prime}$ is negative because the image is on the same side of the lens as the object.
EXECUTE: (a) $\frac{1}{f}=\frac{s+s^{\prime}}{s s^{\prime}}$ and $f=\frac{s s^{\prime}}{s+s^{\prime}}=\frac{(8.00 \mathrm{~cm})(-3.00 \mathrm{~cm})}{8.00 \mathrm{~cm}-3.00 \mathrm{~cm}}=-4.80 \mathrm{~cm} . f$ is negative so the lens is diverging.
(b) $m=-\frac{s^{\prime}}{s}=-\frac{-3.00 \mathrm{~cm}}{8.00 \mathrm{~cm}}=+0.375 . y^{\prime}=m y=(0.375)(6.50 \mathrm{~mm})=2.44 \mathrm{~mm} . s^{\prime}<0$ and the image is virtual.

Evaluate: A converging lens can also form a virtual image, if the object distance is less than the focal length. But in that case $\left|s^{\prime}\right|>s$ and the image would be farther from the lens than the object is.
34.81. Identify: $\quad \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$. The type of lens determines the sign of $f . m=\frac{y^{\prime}}{y}=-\frac{s^{\prime}}{s}$. The sign of $s^{\prime}$ depends on whether the image is real or virtual. $s=16.0 \mathrm{~cm}$.
Set Up: $\quad s^{\prime}=-22.0 \mathrm{~cm} ; s^{\prime}$ is negative because the image is on the same side of the lens as the object.
EXECUTE: (a) $\frac{1}{f}=\frac{s+s^{\prime}}{s s^{\prime}}$ and $f=\frac{s s^{\prime}}{s+s^{\prime}}=\frac{(16.0 \mathrm{~cm})(-22.0 \mathrm{~cm})}{16.0 \mathrm{~cm}-22.0 \mathrm{~cm}}=+58.7 \mathrm{~cm} . f$ is positive so the lens is converging.
(b) $m=-\frac{s^{\prime}}{s}=-\frac{-22.0 \mathrm{~cm}}{16.0 \mathrm{~cm}}=1.38 . \quad y^{\prime}=m y=(1.38)(3.25 \mathrm{~mm})=4.48 \mathrm{~mm} . s^{\prime}<0$ and the image is virtual.

Evaluate: A converging lens forms a virtual image when the object is closer to the lens than the focal point.
34.82. Identify: Apply $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$. Use the image distance when viewed from the flat end to determine the refractive index $n$ of the rod.
SET UP: When viewing from the flat end, $n_{a}=n, n_{b}=1.00$ and $R \rightarrow \infty$. When viewing from the curved end, $n_{a}=n, n_{b}=1.00$ and $R=-10.0 \mathrm{~cm}$.
EXECUTE: $\quad \frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=0 \Rightarrow \frac{n}{15.0 \mathrm{~cm}}+\frac{1}{-9.50 \mathrm{~cm}}=0 \Rightarrow n=\frac{15.0}{9.50}=1.58$. When viewed from the curved end of the $\operatorname{rod} \frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{n}{s}+\frac{1}{s^{\prime}}=\frac{1-n}{R} \Rightarrow \frac{1.58}{15.0 \mathrm{~cm}}+\frac{1}{s^{\prime}}=\frac{-0.58}{-10.0 \mathrm{~cm}}$, and $s^{\prime}=-21.1 \mathrm{~cm}$. The image is 21.1 cm within the rod from the curved end.
Evaluate: In each case the image is virtual and on the same side of the surface as the object.
34.83. (a) Identify: Apply Snell's law to the refraction of a ray at each side of the beam to find where these rays strike the table.
SET UP: The path of a ray is sketched in Figure 34.83.


Figure 34.83
The width of the incident beam is exaggerated in the sketch, to make it easier to draw. Since the diameter of the beam is much less than the radius of the hemisphere, angles $\theta_{a}$ and $\theta_{b}$ are small. The diameter of the circle of light formed on the table is $2 x$. Note the two right triangles containing the angles $\theta_{a}$ and $\theta_{b}$.
$r=0.190 \mathrm{~cm}$ is the radius of the incident beam.
$R=12.0 \mathrm{~cm}$ is the radius of the glass hemisphere.
ExECUTE: $\quad \theta_{a}$ and $\theta_{b}$ small imply $x \approx x^{\prime} ; \sin \theta_{a}=\frac{r}{R}, \sin \theta_{b}=\frac{x^{\prime}}{R} \approx \frac{x}{R}$
Snell's law: $n_{a} \sin \theta_{a}=n_{b} \sin \theta_{b}$
Using the above expressions for $\sin \theta_{a}$ and $\sin \theta_{b}$ gives $n_{a} \frac{r}{R}=n_{b} \frac{x}{R}$
$n_{a} r=n_{b} x$ so $x=\frac{n_{a} r}{n_{b}}=\frac{1.00(0.190 \mathrm{~cm})}{1.50}=0.1267 \mathrm{~cm}$
The diameter of the circle on the table is $2 x=2(0.1267 \mathrm{~cm})=0.253 \mathrm{~cm}$.
(b) Evaluate: $\quad R$ divides out of the expression; the result for the diameter of the spot is independent of the radius $R$ of the hemisphere. It depends only on the diameter of the incident beam and the index of refraction of the glass.
34.84. Identify and Set Up: Treating each of the goblet surfaces as spherical surfaces, we have to pass, from left to right, through four interfaces. Apply $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$ to each surface. The image formed by one surface serves as the object for the next surface.
EXECUTE: (a) For the empty goblet:
$\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{1}{\infty}+\frac{1.50}{s_{1}^{\prime}}=\frac{0.50}{4.00 \mathrm{~cm}} \Rightarrow s_{1}^{\prime}=12 \mathrm{~cm}$.
$s_{2}=0.60 \mathrm{~cm}-12 \mathrm{~cm}=-11.4 \mathrm{~cm} \Rightarrow \frac{1.50}{-11.4 \mathrm{~cm}}+\frac{1}{s_{2}^{\prime}}=\frac{-0.50}{3.40 \mathrm{~cm}} \Rightarrow s_{2}^{\prime}=-64.6 \mathrm{~cm}$.
$s_{3}=64.6 \mathrm{~cm}+6.80 \mathrm{~cm}=71.4 \mathrm{~cm} \Rightarrow \frac{1}{71.4 \mathrm{~cm}}+\frac{1.50}{s_{3}^{\prime}}=\frac{0.50}{-3.40 \mathrm{~cm}} \Rightarrow s_{3}^{\prime}=-9.31 \mathrm{~cm}$.
$s_{4}=9.31 \mathrm{~cm}+0.60 \mathrm{~cm}=9.91 \mathrm{~cm} \Rightarrow \frac{1.50}{9.91 \mathrm{~cm}}+\frac{1}{s_{4}^{\prime}}=\frac{-0.50}{-4.00 \mathrm{~cm}} \Rightarrow s_{4}^{\prime}=-37.9 \mathrm{~cm}$. The final image is
$37.9 \mathrm{~cm}-2(4.0 \mathrm{~cm})=29.9 \mathrm{~cm}$ to the left of the goblet.
(b) For the wine-filled goblet:
$\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{1}{\infty}+\frac{1.50}{s_{1}^{\prime}}=\frac{0.50}{4.00 \mathrm{~cm}} \Rightarrow s_{1}^{\prime}=12 \mathrm{~cm}$.
$s_{2}=0.60 \mathrm{~cm}-12 \mathrm{~cm}=-11.4 \mathrm{~cm} \Rightarrow \frac{1.50}{-11.4 \mathrm{~cm}}+\frac{1.37}{s_{2}^{\prime}}=\frac{-0.13}{3.40 \mathrm{~cm}} \Rightarrow s_{2}^{\prime}=14.7 \mathrm{~cm}$.
$s_{3}=6.80 \mathrm{~cm}-14.7 \mathrm{~cm}=-7.9 \mathrm{~cm} \Rightarrow \frac{1.37}{-7.9 \mathrm{~cm}}+\frac{1.50}{s_{3}^{\prime}}=\frac{0.13}{-3.40 \mathrm{~cm}} \Rightarrow s_{3}^{\prime}=11.1 \mathrm{~cm}$.
$s_{4}=0.60 \mathrm{~cm}-11.1 \mathrm{~cm}=-10.5 \mathrm{~cm} \Rightarrow \frac{1.50}{-10.5 \mathrm{~cm}}+\frac{1}{s_{4}^{\prime}}=\frac{-0.50}{-4.00 \mathrm{~cm}} \Rightarrow s_{4}^{\prime}=3.73 \mathrm{~cm}$. The final image is 3.73 cm to the right of the goblet.
Evaluate: If the object for a surface is on the outgoing side of the light, then the object is virtual and the object distance is negative.
34.85. IDENTIFY: The image formed by refraction at the surface of the eye is located by $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$.

SET UP: $\quad n_{a}=1.00, n_{b}=1.35 . R>0$. For a distant object, $s \approx \infty$ and $\frac{1}{s} \approx 0$.
EXECUTE: (a) $s \approx \infty$ and $s^{\prime}=2.5 \mathrm{~cm}: \frac{1.35}{2.5 \mathrm{~cm}}=\frac{1.35-1.00}{R}$ and $R=0.648 \mathrm{~cm}=6.48 \mathrm{~mm}$.
(b) $R=0.648 \mathrm{~cm}$ and $s=25 \mathrm{~cm}: \frac{1.00}{25 \mathrm{~cm}}+\frac{1.35}{s^{\prime}}=\frac{1.35-1.00}{0.648} \cdot \frac{1.35}{s^{\prime}}=0.500$ and $s^{\prime}=2.70 \mathrm{~cm}=27.0 \mathrm{~mm}$. The image is formed behind the retina.
(c) Calculate $s^{\prime}$ for $s \approx \infty$ and $R=0.50 \mathrm{~cm}: \frac{1.35}{s^{\prime}}=\frac{1.35-1.00}{0.50 \mathrm{~cm}} . s^{\prime}=1.93 \mathrm{~cm}=19.3 \mathrm{~mm}$. The image is formed in front of the retina.
Evaluate: The cornea alone cannot achieve focus of both close and distant objects.
34.86. IDENTIFY: Apply $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$ and $m=-\frac{n_{a} s^{\prime}}{n_{b} s}$ to each surface. The overall magnification is $m=m_{1} m_{2}$. The image formed by the first surface is the object for the second surface.
SET UP: For the first surface, $n_{a}=1.00, n_{b}=1.60$ and $R=+15.0 \mathrm{~cm}$. For the second surface, $n_{a}=1.60$, $n_{b}=1.00$ and $R=\rightarrow \infty$.
EXECUTE: (a) $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{1}{12.0 \mathrm{~cm}}+\frac{1.60}{s^{\prime}}=\frac{0.60}{15.0 \mathrm{~cm}} \Rightarrow s^{\prime}=-36.9 \mathrm{~cm}$. The object distance for the far end of the rod is $50.0 \mathrm{~cm}-(-36.9 \mathrm{~cm})=86.9 \mathrm{~cm}$. The final image is 4.3 cm to the left of the vertex of the hemispherical surface. $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{1.60}{86.9 \mathrm{~cm}}+\frac{1}{s^{\prime}}=0 \Rightarrow s^{\prime}=-54.3 \mathrm{~cm}$.
(b) The magnification is the product of the two magnifications:
$m_{1}=-\frac{n_{a} s^{\prime}}{n_{b} s}=-\frac{-36.9}{(1.60)(12.0)}=1.92, m_{2}=1.00 \Rightarrow m=m_{1} m_{2}=1.92$.
Evaluate: The final image is virtual, erect and larger than the object.
34.87. Identify: Apply Eq.(34.11) to the image formed by refraction at the front surface of the sphere.

SET UP: Let $n_{g}$ be the index of refraction of the glass. The image formation is shown in Figure 34.87.


Figure 34.87
$s=\infty$
$s^{\prime}=+2 r$, where $r$ is the
radius of the sphere

$$
n_{a}=1.00, n_{b}=n_{g}, R=+r
$$

$\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$
EXECUTE: $\frac{1}{\infty}+\frac{n_{g}}{2 r}=\frac{n_{g}-1.00}{r}$
$\frac{n_{g}}{2 r}=\frac{n_{g}}{r}-\frac{1}{r} ; \frac{n_{g}}{2 r}=\frac{1}{r}$ and $n_{g}=2.00$
Evaluate: The required refractive index of the glass does not depend on the radius of the sphere.
34.88. IDENTIFY: Apply $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$ to each surface. The image of the first surface is the object for the second surface. The relation between $s_{1}^{\prime}$ and $s_{2}$ involves the length $d$ of the rod.
Set UP: For the first surface, $n_{a}=1.00, n_{b}=1.55$ and $R=+6.00 \mathrm{~cm}$. For the second surface, $n_{a}=1.55$, $n_{b}=1.00$ and $R=-6.00 \mathrm{~cm}$.
ExECUTE: We have images formed from both ends. From the first surface:
$\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{1}{25.0 \mathrm{~cm}}+\frac{1.55}{s^{\prime}}=\frac{0.55}{6.00 \mathrm{~cm}} \Rightarrow s^{\prime}=30.0 \mathrm{~cm}$.
This image becomes the object for the second end: $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{1.55}{d-30.0 \mathrm{~cm}}+\frac{1}{65.0 \mathrm{~cm}}=\frac{-0.55}{-6.00 \mathrm{~cm}}$.
$d-30.0 \mathrm{~cm}=20.3 \mathrm{~cm} \Rightarrow d=50.3 \mathrm{~cm}$.
Evaluate: The final image is real. The first image is 20.3 cm to the right of the second surface and serves as a real object.
34.89. Identify: The first lens forms an image which then acts as the object for the second lens.

SET UP: The thin-lens equation is $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and the magnification is $m=-\frac{s^{\prime}}{s}$.
EXECUTE: (a) For the first lens: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{5.00 \mathrm{~cm}}+\frac{1}{s^{\prime}}=\frac{1}{-15.0 \mathrm{~cm}} \Rightarrow s^{\prime}=-3.75 \mathrm{~cm}$, to the left of the lens (virtual image).
(b) For the second lens, $s=12.0 \mathrm{~cm}+3.75 \mathrm{~cm}=15.75 \mathrm{~cm}$.
$\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{15.75 \mathrm{~cm}}+\frac{1}{s^{\prime}}=\frac{1}{15.0 \mathrm{~cm}} \Rightarrow s^{\prime}=315 \mathrm{~cm}$, or 332 cm from the object.
(c) The final image is real.
(d) $m=-\frac{s^{\prime}}{s}, m_{1}=0.750, m_{2}=-20.0, m_{\text {total }}=-15.0 \Rightarrow y^{\prime}=-6.00 \mathrm{~cm}$, inverted.

Evaluate: Note that the total magnification is the product of the individual magnifications.
34.90. Identify and SET UP: Use $\frac{1}{f}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$ to calculate the focal length of the lenses. The image formed by the first lens serves at the object for the second lens. $m_{\text {tot }}=m_{1} m_{2} . \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ gives $s^{\prime}=\frac{s f}{s-f}$.
ExECUTE: (a) $\frac{1}{f}=(0.60)\left(\frac{1}{12.0 \mathrm{~cm}}-\frac{1}{28.0 \mathrm{~cm}}\right)$ and $f=+35.0 \mathrm{~cm}$.
Lens 1: $f_{1}=+35.0 \mathrm{~cm} . \quad s_{1}=+45.0 \mathrm{~cm} . \quad s_{1}^{\prime}=\frac{s_{1} f_{1}}{s_{1}-f_{1}}=\frac{(45.0 \mathrm{~cm})(35.0 \mathrm{~cm})}{45.0 \mathrm{~cm}-35.0 \mathrm{~cm}}=+158 \mathrm{~cm}$.
$m_{1}=-\frac{s_{1}^{\prime}}{s_{1}}=-\frac{158 \mathrm{~cm}}{45.0 \mathrm{~cm}}=-3.51 .\left|y_{1}^{\prime}\right|=\left|m_{1}\right| y_{1}=(3.51)(5.00 \mathrm{~mm})=17.6 \mathrm{~mm}$. The image of the first lens is 158 cm to the right of lens 1 and is 17.6 mm tall.
(b) The image of lens 1 is $315 \mathrm{~cm}-158 \mathrm{~cm}=157 \mathrm{~cm}$ to the left of lens $2 . f_{2}=+35.0 \mathrm{~cm} . s_{2}=+157 \mathrm{~cm}$.
$s_{2}{ }^{\prime}=\frac{s_{2} f_{2}}{s_{2}-f_{2}}=\frac{(157 \mathrm{~cm})(35.0 \mathrm{~cm})}{157 \mathrm{~cm}-35.0 \mathrm{~cm}}=+45.0 \mathrm{~cm} . \quad m_{2}=-\frac{s_{2}{ }^{\prime}}{s_{2}}=-\frac{45.0 \mathrm{~cm}}{157 \mathrm{~cm}}=-0.287$.
$m_{\mathrm{tot}}=m_{1} m_{2}=(-3.51)(-0.287)=+1.00$. The final image is 45.0 cm to the right of lens 2 . The final image is 5.00 mm tall. $m_{\mathrm{tot}}>0$. So the final image is erect.
Evaluate: The final image is real. It is erect because each lens produces an inversion of the image, and two inversions return the image to the orientation of the object.
34.91. Identify and Set Up: Apply Eq.(34.16) for each lens position. The lens to screen distance in each case is the image distance. There are two unknowns, the original object distance $x$ and the focal length $f$ of the lens. But each lens position gives an equation, so there are two equations for these two unknowns. The object, lens and screen before and after the lens is moved are shown in Figure 34.91.


Figure 34.91
$s=x+4.00 \mathrm{~cm} ; s^{\prime}=22.0 \mathrm{~cm}$
$\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ gives $\frac{1}{x+4.00 \mathrm{~cm}}+\frac{1}{22.0 \mathrm{~cm}}=\frac{1}{f}$
ExECUTE: Equate these two expressions for $1 / f$ :
$\frac{1}{x}+\frac{1}{30.0 \mathrm{~cm}}=\frac{1}{x+4.00 \mathrm{~cm}}+\frac{1}{22.0 \mathrm{~cm}}$
$\frac{1}{x}-\frac{1}{x+4.00 \mathrm{~cm}}=\frac{1}{22.0 \mathrm{~cm}}-\frac{1}{30.0 \mathrm{~cm}}$
$\frac{x+4.00 \mathrm{~cm}-x}{x(x+4.00 \mathrm{~cm})}=\frac{30.0-22.0}{660 \mathrm{~cm}}$ and $\frac{4.00 \mathrm{~cm}}{x(x+4.00 \mathrm{~cm})}=\frac{8}{660 \mathrm{~cm}}$
$x^{2}+(4.00 \mathrm{~cm}) x-330 \mathrm{~cm}^{2}=0$ and $x=\frac{1}{2}(-4.00 \pm \sqrt{16.0+4(330)}) \mathrm{cm}$
$x$ must be positive so $x=\frac{1}{2}(-4.00+36.55) \mathrm{cm}=16.28 \mathrm{~cm}$
Then $\frac{1}{x}+\frac{1}{30.0 \mathrm{~cm}}=\frac{1}{f}$ and $\frac{1}{f}=\frac{1}{16.28 \mathrm{~cm}}+\frac{1}{30.0 \mathrm{~cm}}$
$f=+10.55 \mathrm{~cm}$, which rounds to $10.6 \mathrm{~cm} . f>0$; the lens is converging.
Evaluate: We can check that $s=16.28 \mathrm{~cm}$ and $f=10.55 \mathrm{~cm}$ gives $s^{\prime}=30.0 \mathrm{~cm}$ and that $s=(16.28+4.0) \mathrm{cm}=20.28 \mathrm{~cm}$ and $f=10.55 \mathrm{~cm}$ gives $s^{\prime}=22.0 \mathrm{~cm}$.
34.92. IdENTIFY and SET UP: Apply $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$.

EXECUTE: (a) $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{n_{a}}{f}+\frac{n_{b}}{\infty}=\frac{n_{b}-n_{a}}{R}$ and $\frac{n_{a}}{\infty}+\frac{n_{b}}{f^{\prime}}=\frac{n_{b}-n_{a}}{R}$. $\frac{n_{a}}{f}=\frac{n_{b}-n_{a}}{R}$ and $\frac{n_{b}}{f^{\prime}}=\frac{n_{b}-n_{a}}{R}$. Therefore, $\frac{n_{a}}{f}=\frac{n_{b}}{f^{\prime}}$ and $n_{a} / n_{b}=\frac{f}{f^{\prime}}$.
(b) $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{n_{b} f}{s f^{\prime}}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}\left(1-f / f^{\prime}\right)}{R}$. Therefore, $\frac{f}{s}+\frac{f^{\prime}}{s^{\prime}}=\frac{f^{\prime}\left(1-f / f^{\prime}\right)}{R}=\frac{f^{\prime}-f}{R}=1$.

Evaluate: For a thin lens the first and second focal lengths are equal.
34.93. (a) Identify: Use Eq.(34.6) to locate the image formed by each mirror. The image formed by the first mirror serves as the object for the 2nd mirror.

SET UP: The positions of the object and the two mirrors are shown in Figure 34.93a.


$$
\begin{aligned}
& |R|=0.360 \mathrm{~m} \\
& |f|=|R| / 2=0.180 \mathrm{~m}
\end{aligned}
$$

Figure 34.93a
EXECUTE: Image formed by convex mirror (mirror \#1):
convex means $f_{1}=-0.180 \mathrm{~m} ; s_{1}=L-x$
$s_{1}^{\prime}=\frac{s_{1} f_{1}}{s_{1}-f_{1}}=\frac{(L-x)(-0.180 \mathrm{~m})}{L-x+0.180 \mathrm{~m}}=-(0.180 \mathrm{~m})\left(\frac{0.600 \mathrm{~m}-x}{0.780 \mathrm{~m}-x}\right)<0$
The image is $(0.180 \mathrm{~m})\left(\frac{0.600 \mathrm{~m}-x}{0.780 \mathrm{~m}-x}\right)$ to the left of mirror \#1 so is
$0.600 \mathrm{~m}+(0.180 \mathrm{~m})\left(\frac{0.600 \mathrm{~m}-x}{0.780 \mathrm{~m}-x}\right)=\frac{0.576 \mathrm{~m}^{2}-(0.780 \mathrm{~m}) x}{0.780 \mathrm{~m}-x}$ to the left of mirror \#2.
Image formed by concave mirror (mirror \#2);
concave implies $f_{2}=+0.180 \mathrm{~m}$
$s_{2}=\frac{0.576 \mathrm{~m}^{2}-(0.780 \mathrm{~m}) x}{0.780 \mathrm{~m}-x}$
Rays return to the source implies $s_{2}^{\prime}=x$. Using these expressions in $s_{2}=\frac{s_{2}^{\prime} f_{2}}{s_{2}^{\prime}-f_{2}}$ gives
$\frac{0.576 \mathrm{~m}^{2}-(0.780 \mathrm{~m}) x}{0.780 \mathrm{~m}-x}=\frac{(0.180 \mathrm{~m}) x}{x-0.180 \mathrm{~m}}$
$0.600 x^{2}-(0.576 \mathrm{~m}) x+0.10368 \mathrm{~m}^{2}=0$
$x=\frac{1}{1.20}\left(0.576 \pm \sqrt{(0.576)^{2}-4(0.600)(0.10368)}\right) \mathrm{m}=\frac{1}{1.20}(0.576 \pm 0.288) \mathrm{m}$
$x=0.72 \mathrm{~m}$ (impossible; can't have $x>L=0.600 \mathrm{~m}$ ) or $x=0.24 \mathrm{~m}$.
(b) SET Up: Which mirror is \#1 and which is \#2 is now reversed form part (a). This is shown in Figure 34.93b.


Figure 34.93b
ExECUTE: Image formed by concave mirror (mirror \#1): concave means $f_{1}=+0.180 \mathrm{~m} ; s_{1}=x$
$s_{1}^{\prime}=\frac{s_{1} f_{1}}{s_{1}-f_{1}}=\frac{(0.180 \mathrm{~m}) x}{x-0.180 \mathrm{~m}}$
The image is $\frac{(0.180 \mathrm{~m}) x}{x-0.180 \mathrm{~m}}$ to the left of mirror \#1, so $s_{2}=0.600 \mathrm{~m}-\frac{(0.180 \mathrm{~m}) x}{x-0.180 \mathrm{~m}}=\frac{(0.420 \mathrm{~m}) x-0.180 \mathrm{~m}^{2}}{x-0.180 \mathrm{~m}}$
Image formed by convex mirror (mirror \#2):
convex means $f_{2}=-0.180 \mathrm{~m}$
rays return to the source means $s_{2}^{\prime}=L-x=0.600 \mathrm{~m}-x$
$\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ gives
$\frac{x-0.180 \mathrm{~m}}{(0.420 \mathrm{~m}) x-0.180 \mathrm{~m}^{2}}+\frac{1}{0.600 \mathrm{~m}-x}=-\frac{1}{0.180 \mathrm{~m}}$
$\frac{x-0.180 \mathrm{~m}}{(0.420 \mathrm{~m}) x-0.180 \mathrm{~m}^{2}}=-\left(\frac{0.780 \mathrm{~m}-x}{0.180 \mathrm{~m}^{2}-(0.180 \mathrm{~m}) x}\right)$
$0.600 x^{2}-(0.576 \mathrm{~m}) x+0.1036 \mathrm{~m}^{2}=0$
This is the same quadratic equation as obtained in part (a), so again $x=0.24 \mathrm{~m}$.

Evaluate: For $x=0.24 \mathrm{~m}$ the image is at the location of the source, both for rays that initially travel from the source toward the left and for rays that travel from the source toward the right.
34.94. Identify: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ gives $s^{\prime}=\frac{s f}{s-f}$, for both the mirror and the lens.

SET UP: For the second image, the image formed by the mirror serves as the object for the lens. For the mirror, $f_{\mathrm{m}}=+10.0 \mathrm{~cm}$. For the lens, $f=32.0 \mathrm{~cm}$. The center of curvature of the mirror is $R=2 f_{\mathrm{m}}=20.0 \mathrm{~cm}$ to the right of the mirror vertex.
EXECUTE: (a) The principal-ray diagrams from the two images are sketched in Figures 34.94a-b. In Figure 34.94 b , only the image formed by the mirror is shown. This image is at the location of the candle so the principal ray diagram that shows the image formation when the image of the mirror serves as the object for the lens is analogous to that in Figure 34.94a and is not drawn.
(b) Image formed by the light that passes directly through the lens: The candle is 85.0 cm to the left of the lens. $s^{\prime}=\frac{s f}{s-f}=\frac{(85.0 \mathrm{~cm})(32.0 \mathrm{~cm})}{85.0 \mathrm{~cm}-32.0 \mathrm{~cm}}=+51.3 \mathrm{~cm} . m=-\frac{s^{\prime}}{s}=-\frac{51.3 \mathrm{~cm}}{85.0 \mathrm{~cm}}=-0.604$. This image is 51.3 cm to the right of the lens. $s^{\prime}>0$ so the image is real. $m<0$ so the image is inverted. Image formed by the light that first reflects off the mirror: First consider the image formed by the mirror. The candle is 20.0 cm to the right of the mirror, so $s=+20.0 \mathrm{~cm} . s^{\prime}=\frac{s f}{s-f}=\frac{(20.0 \mathrm{~cm})(10.0 \mathrm{~cm})}{20.0 \mathrm{~cm}-10.0 \mathrm{~cm}}=20.0 \mathrm{~cm} . m_{1}=-\frac{s_{1}{ }^{\prime}}{s_{1}}=-\frac{20.0 \mathrm{~cm}}{20.0 \mathrm{~cm}}=-1.00$. The image formed by the mirror is at the location of the candle, so $s_{2}=+85.0 \mathrm{cmand} s_{2}{ }^{\prime}=51.3 \mathrm{~cm} . m_{2}=-0.604 . m_{\mathrm{tot}}=m_{1} m_{2}=$ $(-1.00)(-0.604)=0.604$. The second image is 51.3 cm to the right of the lens. $s_{2}{ }^{\prime}>0$, so the final image is real. $m_{\text {tot }}>0$, so the final image is erect.
Evaluate: The two images are at the same place. They are the same size. One is erect and one is inverted.


Figure 34.94
34.95. Identify: Apply $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$ to each case.

SET UP: $s=20.0 \mathrm{~cm} . R>0$. Use $s^{\prime}=+9.12 \mathrm{~cm}$ to find $R$. For this calculation, $n_{a}=1.00$ and $n_{b}=1.55$. Then repeat the calculation with $n_{a}=1.33$.
EXECUTE: $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$ gives $\frac{1.00}{20.0 \mathrm{~cm}}+\frac{1.55}{9.12 \mathrm{~cm}}=\frac{1.55-1.00}{R} . R=2.50 \mathrm{~cm}$.

Then $\frac{1.33}{20.0 \mathrm{~cm}}+\frac{1.55}{s^{\prime}}=\frac{1.55-1.33}{2.50 \mathrm{~cm}}$ gives $s^{\prime}=-72.1 \mathrm{~cm}$. The image is 72.1 cm to the left of the surface vertex.
Evaluate: With the rod in air the image is real and with the rod in water the image is virtual.
34.96. Identify: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ to each lens. The image formed by the first lens serves as the object for the second lens. The focal length of the lens combination is defined by $\frac{1}{s_{1}}+\frac{1}{s_{2}^{\prime}}=\frac{1}{f}$. In part (b) use $\frac{1}{f}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$ to calculate $f$ for the meniscus lens and for the $\mathrm{CCl}_{4}$, treated as a thin lens.
SET Up: With two lenses of different focal length in contact, the image distance from the first lens becomes exactly minus the object distance for the second lens.
EXECUTE: (a) $\frac{1}{s_{1}}+\frac{1}{s_{1}^{\prime}}=\frac{1}{f_{1}} \Rightarrow \frac{1}{s_{1}^{\prime}}=\frac{1}{f_{1}}-\frac{1}{s_{1}}$ and $\frac{1}{s_{2}}+\frac{1}{s_{2}^{\prime}}=\frac{1}{-s_{1}^{\prime}}+\frac{1}{s_{2}^{\prime}}=\left(\frac{1}{s_{1}}-\frac{1}{f_{1}}\right)+\frac{1}{s_{2}^{\prime}}=\frac{1}{f_{2}}$. But overall for the lens system, $\frac{1}{s_{1}}+\frac{1}{s_{2}^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{f}=\frac{1}{f_{2}}+\frac{1}{f_{1}}$.
(b) With carbon tetrachloride sitting in a meniscus lens, we have two lenses in contact. All we need in order to calculate the system's focal length is calculate the individual focal lengths, and then use the formula from part (a).
For the meniscus lens $\frac{1}{f_{\mathrm{m}}}=\left(n_{b}-n_{a}\right)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)=(0.55)\left(\frac{1}{4.50 \mathrm{~cm}}-\frac{1}{9.00 \mathrm{~cm}}\right)=0.061 \mathrm{~cm}^{-1}$ and $f_{\mathrm{m}}=16.4 \mathrm{~cm}$.
For the $\mathrm{CCl}_{4}: \frac{1}{f_{\mathrm{w}}}=\left(n_{b}-n_{a}\right)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)=(0.46)\left(\frac{1}{9.00 \mathrm{~cm}}-\frac{1}{\infty}\right)=0.051 \mathrm{~cm}^{-1}$ and $f_{\mathrm{w}}=19.6 \mathrm{~cm}$.
$\frac{1}{f}=\frac{1}{f_{\mathrm{w}}}+\frac{1}{f_{\mathrm{m}}}=0.112 \mathrm{~cm}^{-1}$ and $f=8.93 \mathrm{~cm}$.
Evaluate: $\quad f=\frac{f_{1} f_{2}}{f_{1}+f_{2}}$, so $f$ for the combination is less than either $f_{1}$ or $f_{2}$.
34.97. Identify: Apply Eq.(34.11) with $R \rightarrow \infty$ to the refraction at each surface. For refraction at the first surface the point $P$ serves as a virtual object. The image formed by the first refraction serves as the object for the second refraction.
SET UP: The glass plate and the two points are shown in Figure 37.97.

plane faces means $R \rightarrow \infty$ and

Figure 34.97
EXECUTE: refraction at the first (left-hand) surface of the piece of glass:
The rays converging toward point $P$ constitute a virtual object for this surface, so
$s=-14.4 \mathrm{~cm}$.
$n_{a}=1.00, n_{b}=1.60$.
$s^{\prime}=-\frac{1.60}{1.00}(-14.4 \mathrm{~cm})=+23.0 \mathrm{~cm}$
This image is 23.0 cm to the right of the first surface so is a distance $23.0 \mathrm{~cm}-t$ to the right of the second surface. This image serves as a virtual object for the second surface.
refraction at the second (right-hand) surface of the piece of glass:
The image is at $P^{\prime}$ so $s^{\prime}=14.4 \mathrm{~cm}+0.30 \mathrm{~cm}-t=14.7 \mathrm{~cm}-t . \quad s=-(23.0 \mathrm{~cm}-t) ; n_{a}=1.60 ; n_{b}=1.00 \quad s^{\prime}=-\frac{n_{b}}{n_{a}} s$ gives $14.7 \mathrm{~cm}-t=-\left(\frac{1.00}{1.60}\right)(-[23.0 \mathrm{~cm}-t]) .14 .7 \mathrm{~cm}-t=+14.4 \mathrm{~cm}-0.625 t$.
$0.375 t=0.30 \mathrm{~cm}$ and $t=0.80 \mathrm{~cm}$
Evaluate: The overall effect of the piece of glass is to diverge the rays and move their convergence point to the right. For a real object, refraction at a plane surface always produces a virtual image, but with a virtual object the image can be real.
34.98. Identify: Apply the two equations $\frac{n_{a}}{s_{1}}+\frac{n_{b}}{s_{1}^{\prime}}=\frac{n_{b}-n_{a}}{R_{1}}$ and $\frac{n_{b}}{s_{2}}+\frac{n_{c}}{s_{2}^{\prime}}=\frac{n_{c}-n_{b}}{R_{2}}$.

SET UP: $\quad n_{a}=n_{\text {liq }}=n_{c}, n_{b}=n$, and $s_{1}^{\prime}=-s_{2}$.
EXECUTE: (a) $\frac{n_{\text {liq }}}{s_{1}}+\frac{n}{s_{1}^{\prime}}=\frac{n-n_{\mathrm{liq}}}{R_{1}}$ and $\frac{n}{-s_{1}^{\prime}}+\frac{n_{\mathrm{liq}}}{s_{2}^{\prime}}=\frac{n_{\mathrm{liq}}-n}{R_{2}} \cdot \frac{1}{s_{1}}+\frac{1}{s_{2}^{\prime}}=\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f^{\prime}}=\left(n / n_{\mathrm{liq}}-1\right)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$.
(b) Comparing the equations for focal length in and out of air we have:

$$
f(n-1)=f^{\prime}\left(n / n_{\mathrm{liq}}-1\right)=f^{\prime}\left(\frac{n-n_{\mathrm{liq}}}{n_{\mathrm{liq}}}\right) \Rightarrow f^{\prime}=\left[\frac{n_{\mathrm{liq}}(n-1)}{n-n_{\mathrm{liq}}}\right] f .
$$

Evaluate: When $n_{\text {liq }}=1, f^{\prime}=f$, as it should.
34.99. Identify: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$.

SET UP: The image formed by the converging lens is 30.0 cm from the converging lens, and becomes a virtual object for the diverging lens at a position 15.0 cm to the right of the diverging lens. The final image is projected $15 \mathrm{~cm}+19.2 \mathrm{~cm}=34.2 \mathrm{~cm}$ from the diverging lens.
EXECUTE: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{-15.0 \mathrm{~cm}}+\frac{1}{34.2 \mathrm{~cm}}=\frac{1}{f} \Rightarrow f=-26.7 \mathrm{~cm}$.
Evaluate: Our calculation yields a negative value of $f$, which should be the case for a diverging lens.
34.100. Identify: The spherical mirror forms an image of the object. It forms another image when the image of the plane mirror serves as an object.
SET UP: For the convex mirror $f=-24.0 \mathrm{~cm}$. The image formed by the plane mirror is 10.0 cm to the right of the plane mirror, so is $20.0 \mathrm{~cm}+10.0 \mathrm{~cm}=30.0 \mathrm{~cm}$ from the vertex of the spherical mirror.
Execute: The first image formed by the spherical mirror is the one where the light immediately strikes its surface, without bouncing from the plane mirror.
$\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{10.0 \mathrm{~cm}}+\frac{1}{s^{\prime}}=\frac{1}{-24.0 \mathrm{~cm}} \Rightarrow s^{\prime}=-7.06 \mathrm{~cm}$, and the image height
is $y^{\prime}=-\frac{s^{\prime}}{s} y=-\frac{-7.06}{10.0}(0.250 \mathrm{~cm})=0.177 \mathrm{~cm}$.
The second image is of the plane mirror image is located 30.0 cm from the vertex of the spherical mirror. $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{30.0 \mathrm{~cm}}+\frac{1}{s^{\prime}}=\frac{1}{-24.0 \mathrm{~cm}} \Rightarrow s^{\prime}=-13.3 \mathrm{~cm}$ and the image height is
$y^{\prime}=-\frac{s^{\prime}}{s} y=-\frac{-13.3}{30.0}(0.250 \mathrm{~cm})=0.111 \mathrm{~cm}$.
Evaluate: Other images are formed by additional reflections from the two mirrors.
34.101. Identify: In the sketch in Figure 34.101 the light travels upward from the object. Apply Eq.(34.11) with $R \rightarrow \infty$ to the refraction at each surface. The image formed by the first surface serves as the object for the second surface.
SET UP: The locations of the object and the glass plate are shown in Figure 34.101.


For a plane (flat) surface

$$
\begin{aligned}
& R \rightarrow \infty \text { so } \frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=0 \\
& s^{\prime}=-\frac{n_{b}}{n_{a}} s
\end{aligned}
$$

ExECUTE: First refraction ( air $\rightarrow$ glass):
$n_{a}=1.00 ; n_{b}=1.55 ; s=6.00 \mathrm{~cm}$
$s^{\prime}=-\frac{n_{b}}{n_{a}} s=-\frac{1.55}{1.00}(6.00 \mathrm{~cm})=-9.30 \mathrm{~cm}$
The image is 9.30 cm below the lower surface of the glass, so is $9.30 \mathrm{~cm}+3.50 \mathrm{~cm}=12.8 \mathrm{~cm}$ below the upper surface.

Second refraction (glass $\rightarrow$ air):
$n_{a}=1.55 ; n_{b}=1.00 ; s=+12.8 \mathrm{~cm}$
$s^{\prime}=-\frac{n_{b}}{n_{a}} s=-\frac{1.00}{1.55}(12.8 \mathrm{~cm})=-8.26 \mathrm{~cm}$
The image of the page is 8.26 cm below the top surface of the glass plate and therefore $9.50 \mathrm{~cm}-8.26 \mathrm{~cm}=1.24 \mathrm{~cm}$ above the page.
Evaluate: The image is virtual. If you view the object by looking down from above the plate, the image of the page that you see is closer to your eye than the page is.
34.102. IDENTIFY: Light refracts at the front surface of the lens, refracts at the glass-water interface, reflects from the plane mirror and passes through the two interfaces again, now traveling in the opposite direction.
SET Up: Use the focal length in air to find the radius of curvature $R$ of the lens surfaces.
EXECUTE: (a) $\frac{1}{f}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right) \Rightarrow \frac{1}{40 \mathrm{~cm}}=0.52\left(\frac{2}{R}\right) \Rightarrow R=41.6 \mathrm{~cm}$.
At the air-lens interface: $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{1}{70.0 \mathrm{~cm}}+\frac{1.52}{s_{1}^{\prime}}=\frac{0.52}{41.6 \mathrm{~cm}}$ and $s_{1}^{\prime}=-851 \mathrm{~cm}$ and $s_{2}=851 \mathrm{~cm}$.
At the lens-water interface: $\Rightarrow \frac{1.52}{851 \mathrm{~cm}}+\frac{1.33}{s_{2}^{\prime}}=\frac{-0.187}{-41.6 \mathrm{~cm}}$ and $s_{2}^{\prime}=491 \mathrm{~cm}$.
The mirror reflects the image back (since there is just 90 cm between the lens and mirror.) So, the position of the image is 401 cm to the left of the mirror, or 311 cm to the left of the lens.
At the water-lens interface: $\Rightarrow \frac{1.33}{-311 \mathrm{~cm}}+\frac{1.52}{s_{3}^{\prime}}=\frac{0.187}{41.6 \mathrm{~cm}}$ and $s_{3}^{\prime}=+173 \mathrm{~cm}$.
At the lens-air interface: $\Rightarrow \frac{1.52}{-173 \mathrm{~cm}}+\frac{1}{s_{4}^{\prime}}=\frac{-0.52}{-41.6 \mathrm{~cm}}$ and $s_{4}^{\prime}=+47.0 \mathrm{~cm}$, to the left of lens.
$m=m_{1} m_{2} m_{3} m_{4}=\left(\frac{n_{a 1} s_{1}^{\prime}}{n_{b 1} s_{1}}\right)\left(\frac{n_{a 2} s_{2}^{\prime}}{n_{b 2} s_{2}}\right)\left(\frac{n_{a 3} s_{3}^{\prime}}{n_{b 3} s_{3}}\right)\left(\frac{n_{a 4} s_{4}^{\prime}}{n_{b 4} s_{4}}\right)=\left(\frac{-851}{70}\right)\left(\frac{491}{-851}\right)\left(\frac{+173}{-311}\right)\left(\frac{+47.0}{-173}\right)=-1.06$.
(Note all the indices of refraction cancel out.)
(b) The image is real.
(c) The image is inverted.
(d) The final height is $y^{\prime}=m y=(1.06)(4.00 \mathrm{~mm})=4.24 \mathrm{~mm}$.

Evaluate: The final image is real even though it is on the same side of the lens as the object!
34.103. Identify: The camera lens can be modeled as a thin lens that forms an image on the film.

SET UP: The thin-lens equation is $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$, and the magnification of the lens is $m=-\frac{s^{\prime}}{s}$.
EXECUTE: (a) $m=-\frac{s^{\prime}}{s}=\frac{y^{\prime}}{y}=\frac{1}{4} \frac{(0.0360 \mathrm{~m})}{(12.0 \mathrm{~m})} \Rightarrow s^{\prime}=\left(7.50 \times 10^{-4}\right) s$,

$$
\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{s}+\frac{1}{\left(7.50 \times 10^{-4}\right) s}=\frac{1}{s}\left(1+\frac{1}{7.50 \times 10^{-4}}\right)=\frac{1}{f}=\frac{1}{0.0350 \mathrm{~m}} \Rightarrow s=46.7 \mathrm{~m} .
$$

(b) To just fill the frame, the magnification must be $3.00 \times 10^{-3}$ so:

$$
\frac{1}{s}\left(1+\frac{1}{3.00 \times 10^{-3}}\right)=\frac{1}{f}=\frac{1}{0.0350 \mathrm{~m}} \Rightarrow s=11.7 \mathrm{~m}
$$

Since the boat is originally 46.7 m away, the distance you must move closer to the boat is $46.7 \mathrm{~m}-11.7 \mathrm{~m}=35.0 \mathrm{~m}$.
Evaluate: This result seems to imply that if you are 4 times as far, the image is $1 / 4$ as large on the film. However this result is only an approximation, and would not be true for very close distances. It is a better approximation for large distances.
34.104. Identify: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and $m=-\frac{s^{\prime}}{s}$.

SET UP: $\quad s+s^{\prime}=18.0 \mathrm{~cm}$
EXECUTE: (a) $\frac{1}{18.0 \mathrm{~cm}-s^{\prime}}+\frac{1}{s^{\prime}}=\frac{1}{3.00 \mathrm{~cm}} \cdot\left(s^{\prime}\right)^{2}-(18.0 \mathrm{~cm}) s^{\prime}+54.0 \mathrm{~cm}^{2}=0$ so $s^{\prime}=14.2 \mathrm{~cm}$ or 3.80 cm . $s=3.80 \mathrm{~cm}$ or 14.2 cm , so the screen must either be 3.80 cm or 14.2 cm from the object.
(b) $s=3.80 \mathrm{~cm}: m=-\frac{s^{\prime}}{s}=-\frac{3.80}{14.2}=-0.268 . \quad s=14.2 \mathrm{~cm}: m=-\frac{s^{\prime}}{s}=-\frac{14.2}{3.80}=-3.74$.

Evaluate: Since the image is projected onto the screen, the image is real and $s^{\prime}$ is positive. We assumed this when we wrote the condition $s+s^{\prime}=18.0 \mathrm{~cm}$.
34.105. Identify: Apply Eq.(34.16) to calculate the image distance for each lens. The image formed by the 1 st lens serves as the object for the 2 nd lens, and the image formed by the 2 nd lens serves as the object for the 3 rd lens. SET UP: The positions of the object and lenses are shown in Figure 34.105.


$$
\begin{aligned}
& \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \\
& \frac{1}{s^{\prime}}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{s f} \\
& s^{\prime}=\frac{s f}{s-f}
\end{aligned}
$$

Figure 34.105
Execute: lens \#1
$s=+80.0 \mathrm{~cm} ; f=+40.0 \mathrm{~cm}$
$s^{\prime}=\frac{s f}{s-f}=\frac{(+80.0 \mathrm{~cm})(+40.0 \mathrm{~cm})}{+80.0 \mathrm{~cm}-40.0 \mathrm{~cm}}=+80.0 \mathrm{~cm}$
The image formed by the first lens is 80.0 cm to the right of the first lens, so it is $80.0 \mathrm{~cm}-52.0 \mathrm{~cm}=28.0 \mathrm{~cm}$ to the right of the second lens.
lens \#2
$s=-28.0 \mathrm{~cm} ; f=+40.0 \mathrm{~cm}$
$s^{\prime}=\frac{s f}{s-f}=\frac{(-28.0 \mathrm{~cm})(+40.0 \mathrm{~cm})}{-28.0 \mathrm{~cm}-40.0 \mathrm{~cm}}=+16.47 \mathrm{~cm}$
The image formed by the second lens is 16.47 cm to the right of the second lens, so it is $52.0 \mathrm{~cm}-16.47 \mathrm{~cm}=35.53 \mathrm{~cm}$ to the left of the third lens.
lens \#3
$s=+35.53 \mathrm{~cm} ; f=+40.0 \mathrm{~cm}$
$s^{\prime}=\frac{s f}{s-f}=\frac{(+35.53 \mathrm{~cm})(+40.0 \mathrm{~cm})}{+35.53 \mathrm{~cm}-40.0 \mathrm{~cm}}=-318 \mathrm{~cm}$
The final image is 318 cm to the left of the third lens, so it is $318 \mathrm{~cm}-52 \mathrm{~cm}-52 \mathrm{~cm}-80 \mathrm{~cm}=134 \mathrm{~cm}$ to the left of the object.
Evaluate: We used the separation between the lenses and the sign conventions for $s$ and $s^{\prime}$ to determine the object distances for the 2 nd and 3 rd lenses. The final image is virtual since the final $s^{\prime}$ is negative.
34.106. Identify: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ and calculate $s^{\prime}$ for each $s$.

Set Up: $f=90 \mathrm{~mm}$
EXECUTE: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{1300 \mathrm{~mm}}+\frac{1}{s^{\prime}}=\frac{1}{90 \mathrm{~mm}} \Rightarrow s^{\prime}=96.7 \mathrm{~mm}$.
$\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{6500 \mathrm{~mm}}+\frac{1}{s^{\prime}}=\frac{1}{90 \mathrm{~mm}} \Rightarrow s^{\prime}=91.3 \mathrm{~mm}$.
$\Rightarrow \Delta s^{\prime}=96.7 \mathrm{~mm}-91.3 \mathrm{~mm}=5.4 \mathrm{~mm}$ toward the film
Evaluate: $\quad s^{\prime}=\frac{s f}{s-f}$. For $f>0$ and $s>f, s^{\prime}$ decreases as $s$ increases.
34.107. Identify and Set Up: The generalization of Eq.(34.22) is $M=\frac{\text { near point }}{f}$, so $f=\frac{\text { near point }}{M}$.

Execute: (a) age 10, near point $=7 \mathrm{~cm}$
$f=\frac{7 \mathrm{~cm}}{2.0}=3.5 \mathrm{~cm}$
(b) age 30 , near point $=14 \mathrm{~cm}$
$f=\frac{14 \mathrm{~cm}}{2.0}=7.0 \mathrm{~cm}$
(c) age 60 , near point $=200 \mathrm{~cm}$
$f=\frac{200 \mathrm{~cm}}{2.0}=100 \mathrm{~cm}$
(d) $f=3.5 \mathrm{~cm}$ (from part (a)) and near point $=200 \mathrm{~cm}$ (for 60-year-old)
$M=\frac{200 \mathrm{~cm}}{3.5 \mathrm{~cm}}=57$
(e) Evaluate: No. The reason $f=3.5 \mathrm{~cm}$ gives a larger $M$ for a 60 -year-old than for a 10 -year-old is that the eye of the older person can't focus on as close an object as the younger person can. The unaided eye of the 60-year-old must view a much smaller angular size, and that is why the same $f$ gives a much larger $M$. The angular size of the image depends only on $f$ and is the same for the two ages.
34.108. Identify: Use $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ to calculate $s$ that gives $s^{\prime}=-25 \mathrm{~cm} . \quad M=\frac{\theta^{\prime}}{\theta}$.

SET UP: Let the height of the object be $y$, so $\theta^{\prime}=\frac{y}{s}$ and $\theta=\frac{y}{25 \mathrm{~cm}}$.
EXECUTE: (a) $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{s}+\frac{1}{-25 \mathrm{~cm}}=\frac{1}{f} \Rightarrow s=\frac{f(25 \mathrm{~cm})}{f+25 \mathrm{~cm}}$.
(b) $\theta^{\prime}=\arctan \left(\frac{y}{s}\right)=\arctan \left(\frac{y(f+25 \mathrm{~cm})}{f(25 \mathrm{~cm})}\right) \approx \frac{y(f+25 \mathrm{~cm})}{f(25 \mathrm{~cm})}$.
(c) $M=\frac{\theta^{\prime}}{\theta}=\frac{y(f+25 \mathrm{~cm})}{f(25 \mathrm{~cm})} \frac{1}{y / 25 \mathrm{~cm}}=\frac{f+25 \mathrm{~cm}}{f}$.
(d) If $f=10 \mathrm{~cm} \Rightarrow M=\frac{10 \mathrm{~cm}+25 \mathrm{~cm}}{10 \mathrm{~cm}}=3.5$. This is 1.4 times greater than the magnification obtained if the image
if formed at infinity $\left(M_{\infty}=\frac{25 \mathrm{~cm}}{f}=2.5\right)$.
Evaluate: (e) Having the first image form just within the focal length puts one in the situation described above, where it acts as a source that yields an enlarged virtual image. If the first image fell just outside the second focal point, then the image would be real and diminished.
34.109. Identify: Apply $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$. The near point is at infinity, so that is where the image must be formed for any objects that are close.
SET UP: The power in diopters equals $\frac{1}{f}$, with $f$ in meters.
Execute: $\frac{1}{f}=\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{24 \mathrm{~cm}}+\frac{1}{-\infty}=\frac{1}{0.24 \mathrm{~m}}=4.17$ diopters.
Evaluate: To focus on closer objects, the power must be increased.
34.110. Identify: Apply $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$.

SET UP: $\quad n_{a}=1.00, n_{b}=1.40$.
EXECUTE: $\quad \frac{1}{36.0 \mathrm{~cm}}+\frac{1.40}{s^{\prime}}=\frac{0.40}{0.75 \mathrm{~cm}} \Rightarrow s^{\prime}=2.77 \mathrm{~cm}$.
Evaluate: This distance is greater than the normal eye, which has a cornea vertex to retina distance of about 2.6 cm .
34.111. Identify: Use similar triangles in Figure 34.63 in the textbook and Eq.(34.16) to derive the expressions called for in the problem.
(a) Set Up: The effect of the converging lens on the ray bundle is sketched in Figure 34.111.


Figure 34.111a

Execute: From similar triangles in Figure 34.111a,

$$
\frac{r_{0}}{f_{1}}=\frac{r_{0}^{\prime}}{f_{1}-d}
$$

Thus $r_{0}^{\prime}=\left(\frac{f_{1}-d}{f_{1}}\right) r_{0}$, as was to be shown.
(b) SET UP: The image at the focal point of the first lens, a distance $f_{1}$ to the right of the first lens, serves as the object for the second lens. The image is a distance $f_{1}-d$ to the right of the second lens, so $s_{2}=-\left(f_{1}-d\right)=d-f_{1}$.
EXECUTE: $\quad s_{2}^{\prime}=\frac{s_{2} f_{2}}{s_{2}-f_{2}}=\frac{\left(d-f_{1}\right) f_{2}}{d-f_{1}-f_{2}}$
$f_{2}<0$ so $\left|f_{2}\right|=-f_{2}$ and $s_{2}^{\prime}=\frac{\left(f_{1}-d\right)\left|f_{2}\right|}{\left|f_{2}\right|-f_{1}+d}$, as was to be shown.
(c) Set Up: The effect of the diverging lens on the ray bundle is sketched in Figure 34.111b.


Figure 34.111b

From the results of part (a), $\frac{r_{0}}{r_{0}^{\prime}}=\frac{f_{1}}{f_{1}-d}$. Combining the two results gives $\frac{f_{1}}{f_{1}-d}=\frac{f}{s_{2}^{\prime}}$ $f=s_{2}^{\prime}\left(\frac{f_{1}}{f_{1}-d}\right)=\frac{\left(f_{1}-d\right)\left|f_{2}\right| f_{1}}{\left(\left|f_{2}\right|-f_{1}+d\right)\left(f_{1}-d\right)}=\frac{f_{1}\left|f_{2}\right|}{\left|f_{2}\right|-f_{1}+d}$, as was to be shown.
(d) SET UP: Put the numerical values into the expression derived in part (c).

ExEcute: $\quad f=\frac{f_{1}\left|f_{2}\right|}{\left|f_{2}\right|-f_{1}+d}$
$f_{1}=12.0 \mathrm{~cm},\left|f_{2}\right|=18.0 \mathrm{~cm}$, so $f=\frac{216 \mathrm{~cm}^{2}}{6.0 \mathrm{~cm}+d}$
$d=0$ gives $f=36.0 \mathrm{~cm}$; maximum $f$
$d=4.0 \mathrm{~cm}$ gives $f=21.6 \mathrm{~cm}$; minimum $f$
$f=30.0 \mathrm{~cm}$ says $30.0 \mathrm{~cm}=\frac{216 \mathrm{~cm}^{2}}{6.0 \mathrm{~cm}+d}$
$6.0 \mathrm{~cm}+d=7.2 \mathrm{~cm}$ and $d=1.2 \mathrm{~cm}$
Evaluate: Changing $d$ produces a range of effective focal lengths. The effective focal length can be both smaller and larger than $f_{1}+\left|f_{2}\right|$.
34.112. IdENTIFY: $|M|=\frac{\theta^{\prime}}{\theta} . \quad \theta=\left|\frac{y_{1}^{\prime}}{f_{1}}\right|$, and $\theta^{\prime}=\left|\frac{y_{2}^{\prime}}{s_{2}^{\prime}}\right|$. This gives $|M|=\left|\frac{y_{2}^{\prime}}{s_{2}^{\prime}} \cdot \frac{f_{1}^{\prime}}{y_{1}^{\prime}}\right|$.

SET UP: Since the image formed by the objective is used as the object for the eyepiece, $y_{1}^{\prime}=y_{2}$.
ExECUTE: $\quad|M|=\left|\frac{y_{2}^{\prime}}{s_{2}^{\prime}} \cdot \frac{f_{1}}{y_{2}}\right|=\left|\frac{y_{2}^{\prime}}{y_{2}} \cdot \frac{f_{1}}{s_{2}^{\prime}}\right|=\left|\frac{s_{2}^{\prime}}{s_{2}} \cdot \frac{f_{1}}{s_{2}^{\prime}}\right|=\left|\frac{f_{1}}{s_{2}}\right|$. Therefore, $s_{2}=\frac{f_{1}}{|M|}=\frac{48.0 \mathrm{~cm}}{36}=1.33 \mathrm{~cm}$, and this is just outside the eyepiece focal point.
Now the distance from the mirror vertex to the lens is $f_{1}+s_{2}=49.3 \mathrm{~cm}$, and so
$\frac{1}{s_{2}}+\frac{1}{s_{2}^{\prime}}=\frac{1}{f_{2}} \Rightarrow s_{2}^{\prime}=\left(\frac{1}{1.20 \mathrm{~cm}}-\frac{1}{1.33 \mathrm{~cm}}\right)^{-1}=12.3 \mathrm{~cm}$. Thus we have a final image which is real and 12.3 cm from the eyepiece. (Take care to carry plenty of figures in the calculation because two close numbers are subtracted.)
Evaluate: Eq.(34.25) gives $|M|=40$, somewhat larger than $|M|$ for this telescope.
34.113. Identify and Set Up: The image formed by the objective is the object for the eyepiece. The total lateral magnification is $m_{\text {tot }}=m_{1} m_{2} \cdot f_{1}=8.00 \mathrm{~mm}$ (objective); $f_{2}=7.50 \mathrm{~cm}$ (eyepiece)
(a) The locations of the object, lenses and screen are shown in Figure 34.113.


Figure 34.113
Execute: Find the object distance $s_{1}$ for the objective:
$s_{1}^{\prime}=+18.0 \mathrm{~cm}, f_{1}=0.800 \mathrm{~cm}, s_{1}=$ ?
$\frac{1}{s_{1}}+\frac{1}{s_{1}^{\prime}}=\frac{1}{f_{1}}$, so $\frac{1}{s_{1}}=\frac{1}{f_{1}}-\frac{1}{s_{1}^{\prime}}=\frac{s_{1}^{\prime}-f_{1}}{s_{1}^{\prime} f_{1}}$
$s_{1}=\frac{s_{1}^{\prime} f_{1}}{s_{1}^{\prime}-f_{1}}=\frac{(18.0 \mathrm{~cm})(0.800 \mathrm{~cm})}{18.0 \mathrm{~cm}-0.800 \mathrm{~cm}}=0.8372 \mathrm{~cm}$
Find the object distance $s_{2}$ for the eyepiece:
$s_{2}^{\prime}=+200 \mathrm{~cm}, f_{2}=7.50 \mathrm{~cm}, s_{2}=$ ?
$\frac{1}{s_{2}}+\frac{1}{s_{2}^{\prime}}=\frac{1}{f_{2}}$
$s_{2}=\frac{s_{2}^{\prime} f_{2}}{s_{2}^{\prime}-f_{2}}=\frac{(200 \mathrm{~cm})(7.50 \mathrm{~cm})}{200 \mathrm{~cm}-7.50 \mathrm{~cm}}=7.792 \mathrm{~cm}$
Now we calculate the magnification for each lens:
$m_{1}=-\frac{s_{1}^{\prime}}{s_{1}}=-\frac{18.0 \mathrm{~cm}}{0.8372 \mathrm{~cm}}=-21.50$
$m_{2}=-\frac{s_{2}^{\prime}}{s_{2}}=-\frac{200 \mathrm{~cm}}{7.792 \mathrm{~cm}}=-25.67$
$m_{\text {tot }}=m_{1} m_{2}=(-21.50)(-25.67)=552$.
(b) From the sketch we can see that the distance between the two lenses is $s_{1}^{\prime}+s_{2}=18.0 \mathrm{~cm}+7.792 \mathrm{~cm}=25.8 \mathrm{~cm}$.

Evaluate: The microscope is not being used in the conventional way; it merely serves as a two-lens system. In particular, the final image formed by the eyepiece in the problem is real, not virtual as is the case normally for a microscope. Eq.(34.23) does not apply here, and in any event gives the angular not the lateral magnification.
34.114. Identify: For $u$ and $u^{\prime}$ as defined in Figure 34.64 in the textbook, $M=\frac{u^{\prime}}{u}$.

SET UP: $f_{2}$ is negative. From Figure 34.64, the length of the telescope is $f_{1}+f_{2}$.
EXECUTE: (a) From the figure, $u=\frac{y}{f_{1}}$ and $u^{\prime}=\frac{y}{\left|f_{2}\right|}=-\frac{y}{f_{2}}$. The angular magnification is $M=\frac{u^{\prime}}{u}=-\frac{f_{1}}{f_{2}}$.
(b) $M=-\frac{f_{1}}{f_{2}} \Rightarrow f_{2}=-\frac{f_{1}}{M}=-\frac{95.0 \mathrm{~cm}}{6.33}=-15.0 \mathrm{~cm}$.
(c) The length of the telescope is $95.0 \mathrm{~cm}-15.0 \mathrm{~cm}=80.0 \mathrm{~cm}$, compared to the length of 110 cm for the telescope in Exercise 34.57.
Evaluate: An advantage of this construction is that the telescope is somewhat shorter.
34.115. Identify: Use $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ to calculate $s^{\prime}$ (the distance of each point from the lens), for points $A, B$ and $C$.

SET UP: The object and lens are shown in Figure 34.115a.
EXECUTE: (a) For point $C: \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{45.0 \mathrm{~cm}}+\frac{1}{s^{\prime}}=\frac{1}{20.0 \mathrm{~cm}} \Rightarrow s^{\prime}=36.0 \mathrm{~cm}$.
$y^{\prime}=-\frac{s^{\prime}}{s} y=-\frac{36.0}{45.0}(15.0 \mathrm{~cm})=-12.0 \mathrm{~cm}$, so the image of point $C$ is 36.0 cm to the right of the lens, and
12.0 cm below the axis.

For point $A: s=45.0 \mathrm{~cm}+8.00 \mathrm{~cm}\left(\cos 45^{\circ}\right)=50.7 \mathrm{~cm}$.
$\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{50.7 \mathrm{~cm}}+\frac{1}{s^{\prime}}=\frac{1}{20.0 \mathrm{~cm}} \Rightarrow s^{\prime}=33.0 \mathrm{~cm} . y^{\prime}=-\frac{s^{\prime}}{s} y=-\frac{33.0}{45.0}\left(15.0 \mathrm{~cm}-8.00 \mathrm{~cm}\left(\sin 45^{\circ}\right)\right)=-6.10 \mathrm{~cm}$, so the image of point $A$ is 33.0 cm to the right of the lens, and 6.10 cm below the axis.

For point $B: s=45.0 \mathrm{~cm}-8.00 \mathrm{~cm}\left(\cos 45^{\circ}\right)=39.3 \mathrm{~cm} . \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \Rightarrow \frac{1}{39.3 \mathrm{~cm}}+\frac{1}{s^{\prime}}=\frac{1}{20.0 \mathrm{~cm}} \Rightarrow s^{\prime}=40.7 \mathrm{~cm}$. $y^{\prime}=-\frac{s^{\prime}}{s} y=-\frac{40.7}{39.3}\left(15.0 \mathrm{~cm}+8.00 \mathrm{~cm}\left(\sin 45^{\circ}\right)\right)=-21.4 \mathrm{~cm}$, so the image of point $B$ is 40.7 cm to the right of the lens, and 21.4 cm below the axis. The image is shown in Figure 34.115b.
(b) The length of the pencil is the distance from point $A$ to $B$ :
$L=\sqrt{\left(x_{A}-x_{B}\right)^{2}+\left(y_{A}-y_{B}\right)^{2}}=\sqrt{(33.0 \mathrm{~cm}-40.7 \mathrm{~cm})^{2}+(6.10 \mathrm{~cm}-21.4 \mathrm{~cm})^{2}}=17.1 \mathrm{~cm}$
Evaluate: The image is below the optic axis and is larger than the object.

(a)

(b)

Figure 34.115
34.116. Identify and Set Up: Consider the ray diagram drawn in Figure 34.116.

EXECUTE: (a) Using the diagram and law of sines, $\frac{\sin \theta}{(R-f)}=\frac{\sin \alpha}{g}$ but $\sin \theta=\frac{h}{R}=\sin \alpha$ (law of reflection), $g=(R-f)$. Bisecting the triangle: $\cos \theta=\frac{R / 2}{(R-f)} \Rightarrow R \cos \theta-f \cos \theta=\frac{R}{2}$. $f=\frac{R}{2}\left[2-\frac{1}{\cos \theta}\right]=f_{0}\left[2-\frac{1}{\cos \theta}\right] . f_{0}=\frac{R}{2}$ is the value of $f$ for $\theta$ near zero (incident ray near the axis). When $\theta$ increases, $(2-1 / \cos \theta)$ decreases and $f$ decreases.
(b) $\frac{f-f_{0}}{f_{0}}=-0.02 \Rightarrow \frac{f}{f_{0}}=0.98$ so $2-\frac{1}{\cos \theta}=0.98 . \cos \theta=\frac{1}{2-0.98}=0.98$ and $\theta=11.4^{\circ}$.

Evaluate: For $\theta=45^{\circ}, f=0.586 f_{0}$, and $f$ approaches zero as $\theta$ approaches $60^{\circ}$.


Figure 34.116
34.117. Identify: The distance between image and object can be calculated by taking the derivative of the separation distance and minimizing it.
SET UP: For a real image $s^{\prime}>0$ and the distance between the object and the image is $D=s+s^{\prime}$. For a real image must have $s>f$.
ExECUTE: $\quad D=s+s^{\prime}$ but $s^{\prime}=\frac{s f}{s-f} \Rightarrow D=s+\frac{s f}{s-f}=\frac{s^{2}}{s-f}$.
$\frac{d D}{d s}=\frac{d}{d s}\left(\frac{s^{2}}{s-f}\right)=\frac{2 s}{s-f}-\frac{s^{2}}{(s-f)^{2}}=\frac{s^{2}-2 s f}{(s-f)^{2}}=0 . s^{2}-2 s f=0 . s(s-2 f)=0 . s=2 f$ is the solution for which $s>f$. For $s=2 f, s^{\prime}=2 f$. Therefore, the minimum separation is $2 f+2 f=4 f$.
(b) A graph of $D / f$ versus $s / f$ is sketched in Figure 34.117. Note that the minimum does occur for $D=4 f$.

Evaluate: If, for example, $s=3 f / 2$, then $s^{\prime}=3 f$ and $D=s+s^{\prime}=4.5 f$, greater than the minimum value.

34.118. Identify and Set Up: For a plane mirror, $s^{\prime}=-s$.

EXECUTE: (a) By the symmetry of image production, any image must be the same distance $D$ as the object from the mirror intersection point. But if the images and the object are equal distances from the mirror intersection, they lie on a circle with radius equal to $D$.
(b) The center of the circle lies at the mirror intersection as discussed above.
(c) The diagram is sketched in Figure 34.118.

Evaluate: To see the image, light from the object must be able to reflect from each mirror and reach the person's eyes.


Figure 34.118
34.119. IDENTIFY: Apply $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$ to refraction at the cornea to find where the object for the cornea must be in order for the image to be at the retina. Then use $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ to calculate $f$ so that the lens produces an image of a distant object at this point.
SET UP: For refraction at the cornea, $n_{a}=1.33$ and $n_{b}=1.40$. The distance from the cornea to the retina in this model of the eye is 2.60 cm . From Problem $34.46, R=0.71 \mathrm{~cm}$.
Execute: (a) People with normal vision cannot focus on distant objects under water because the image is unable to be focused in a short enough distance to form on the retina. Equivalently, the radius of curvature of the normal eye is about five or six times too great for focusing at the retina to occur.
(b) When introducing glasses, let's first consider what happens at the eye:
$\frac{n_{a}}{s_{2}}+\frac{n_{b}}{s_{2}^{\prime}}=\frac{n_{b}-n_{a}}{R} \Rightarrow \frac{1.33}{s_{2}}+\frac{1.40}{2.6 \mathrm{~cm}}=\frac{0.07}{0.71 \mathrm{~cm}} \Rightarrow s_{2}=-3.02 \mathrm{~cm}$. That is, the object for the cornea must be 3.02 cm behind the cornea. Now, assume the glasses are 2.00 cm in front of the eye, so $s_{1}^{\prime}=2.00 \mathrm{~cm}+s_{2}=5.02 \mathrm{~cm}$.
$\frac{1}{s_{1}}+\frac{1}{s_{1}^{\prime}}=\frac{1}{f_{1}^{\prime}}$ gives $\frac{1}{\infty}+\frac{1}{5.02 \mathrm{~cm}}=\frac{1}{f_{1}^{\prime}}$ and $f_{1}^{\prime}=5.02 \mathrm{~cm}$. This is the focal length in water, but to get it in air, we use the formula from Problem 34.98: $f_{1}=f_{1}^{\prime}\left[\frac{n-n_{\text {liq }}}{n_{\text {liq }}(n-1)}\right]=(5.02 \mathrm{~cm})\left[\frac{1.52-1.333}{1.333(1.52-1)}\right]=1.35 \mathrm{~cm}$.
Evaluate: A converging lens is needed.

