SOURCES OF MAGNETIC FIELD

28

28.1. **IDENTIFY** and **SET UP:** Use Eq.(28.2) to calculate $\vec{B}$ at each point.

$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$, since $\hat{r} = \frac{\vec{r}}{r}$.

$\vec{v} = (8.00\times10^6\ \text{m/s})\hat{j}$ and $\vec{r}$ is the vector from the charge to the point where the field is calculated.

EXECUTE: **(a)** $\vec{r} = (0.500\ \text{m})\hat{i},\ r = 0.500\ \text{m}$

$\vec{v}\times\vec{r} = vr\hat{j}\times\hat{i} = -vr\hat{k}$

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{qv}{r^2}\hat{k} = -(1\times10^{-7}\ \text{T}\cdot\text{m/A})\frac{(6.00\times10^{-6}\ \text{C})(8.00\times10^6\ \text{m/s})}{(0.500\ \text{m})^2}\hat{k}$$

$\vec{B} = -(1.92\times10^{-5}\ \text{T})\hat{k}$

(b) $\vec{r} = -(0.500\ \text{m})\hat{j},\ r = 0.500\ \text{m}$

$\vec{v}\times\vec{r} = -vr\hat{j}\times\hat{j} = 0$ and $\vec{B} = 0$.

(c) $\vec{r} = (0.500\ \text{m})\hat{k},\ r = 0.500\ \text{m}$

$\vec{v}\times\vec{r} = vr\hat{j}\times\hat{k} = vr\hat{i}$

$$\vec{B} = (1\times10^{-7}\ \text{T}\cdot\text{m/A})\frac{(6.00\times10^{-6}\ \text{C})(8.00\times10^6\ \text{m/s})}{(0.500\ \text{m})^2}\hat{i} = +(1.92\times10^{-5}\ \text{T})\hat{i}$$

(d) $\vec{r} = -(0.500\ \text{m})\hat{j} + (0.500\ \text{m})\hat{k},\ r = \sqrt{(0.500\ \text{m})^2 + (0.500\ \text{m})^2} = 0.7071\ \text{m}$

$\vec{v}\times\vec{r} = v(0.500\ \text{m})(-\hat{j}\times\hat{j} + \hat{j}\times\hat{k}) = (4.00\times10^6\ \text{m}^2/\text{s})\hat{i}$

$$\vec{B} = (1\times10^{-7}\ \text{T}\cdot\text{m/A})\frac{(6.00\times10^{-6}\ \text{C})(4.00\times10^6\ \text{m/s})}{(0.7071\ \text{m})^3}\hat{i} = +(6.79\times10^{-6}\ \text{T})\hat{i}$$

EVALUATE: At each point $\vec{B}$ is perpendicular to both $\vec{v}$ and $\vec{r}$. $B = 0$ along the direction of $\vec{v}$.

28.2. **IDENTIFY:** A moving charge creates a magnetic field as well as an electric field.

SET UP: The magnetic field caused by a moving charge is $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$, and its electric field is $E = \frac{1}{4\pi\epsilon_0}\frac{e}{r^2}$ since $q = e$.

EXECUTE: Substitute the appropriate numbers into the above equations.

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2} = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(1.60\times10^{-19}\text{C})(2.2\times10^6\text{m/s})\sin90^\circ}{(5.3\times10^{-11}\text{m})^2} = 13\ \text{T, out of the page.}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{e}{r^2} = \frac{(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})}{(5.3\times10^{-11}\ \text{m})^2} = 5.1\times10^{11}\ \text{N/C, toward the electron.}$$

EVALUATE: There are enormous fields within the atom!

28.3. **IDENTIFY:** A moving charge creates a magnetic field.

SET UP: The magnetic field due to a moving charge is $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$.

EXECUTE: Substituting numbers into the above equation gives

(a) $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2} = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(1.6\times10^{-19}\ \text{C})(3.0\times10^{7}\ \text{m/s})\sin 30°}{(2.00\times10^{-6}\ \text{m})^2}.$

$B = 6.00 \times 10^{-8}$ T, out of the paper, and it is the same at point B.

(b) $B = (1.00 \times 10^{-7}\ \text{T}\cdot\text{m/A})(1.60 \times 10^{-19}\ \text{C})(3.00 \times 10^{7}\ \text{m/s})/(2.00 \times 10^{-6}\ \text{m})^2$

$B = 1.20 \times 10^{-7}$ T, out of the page.

(c) $B = 0$ T since $\sin(180°) = 0$.

EVALUATE: Even at high speeds, these charges produce magnetic fields much less than the Earth's magnetic field.

28.4. IDENTIFY: Both moving charges produce magnetic fields, and the net field is the vector sum of the two fields.

SET UP: Both fields point out of the paper, so their magnitudes add, giving

$$B = B_{\text{alpha}} + B_{\text{el}} = \frac{\mu_0 v}{4\pi r^2}\left(e\sin 40° + 2e\sin 140°\right)$$

EXECUTE: Factoring out an e and putting in the numbers gives

$$B = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(1.60\times10^{-19}\text{C})(2.50\times10^{5}\text{m/s})}{(1.75\times10^{-9}\text{m})^2}\left(\sin 40° + 2\sin 140°\right)$$

$$B = 2.52\times10^{-3}\ \text{T} = 2.52\ \text{mT, out of the page.}$$

EVALUATE: At distances very close to the charges, the magnetic field is strong enough to be important.

28.5. IDENTIFY: Apply $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$.

SET UP: Since the charge is at the origin, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

EXECUTE: **(a)** $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{i}$; $\vec{v}\times\vec{r} = 0$, $B = 0$.

(b) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{j}$; $\vec{v}\times\vec{r} = vr\hat{k}$, $r = 0.500$ m.

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{|q|v}{r^2} = \frac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(6.80\times10^{5}\ \text{m/s})}{(0.500\ \text{m})^2} = 1.31\times10^{-6}\ \text{T}.$$

q is negative, so $\vec{B} = -(1.31\times10^{-6}\ \text{T})\hat{k}$.

(c) $\vec{v} = v\hat{i}$, $\vec{r} = (0.500\ \text{m})(\hat{i} + \hat{j})$; $\vec{v}\times\vec{r} = (0.500\ \text{m})v\hat{k}$, $r = 0.7071$ m.

$$B = \left(\frac{\mu_0}{4\pi}\right)\left(|q||\vec{v}\times\vec{r}|/r^3\right) = \frac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(0.500\ \text{m})(6.80\times10^{5}\ \text{m/s})}{(0.7071\ \text{m})^3}.$$

$B = 4.62\times10^{-7}$ T. $\vec{B} = -(4.62\times10^{-7}\ \text{T})\hat{k}$.

(d) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{k}$; $\vec{v}\times\vec{r} = -vr\hat{j}$, $r = 0.500$ m

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{|q|v}{r^2} = \frac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(6.80\times10^{5}\ \text{m/s})}{(0.500\ \text{m})^2} = 1.31\times10^{-6}\ \text{T}.$$

$\vec{B} = \left(1.31\times10^{-6}\ \text{T}\right)\hat{j}$.

EVALUATE: In each case, $\vec{B}$ is perpendicular to both $\vec{r}$ and $\vec{v}$.

28.6. IDENTIFY: Apply $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$. For the magnetic force, apply the results of Example 28.1, except here the two charges and velocities are different.

SET UP: In part (a), $r = d$ and $\vec{r}$ is perpendicular to $\vec{v}$ in each case, so $\frac{|\vec{v}\times\vec{r}|}{r^3} = \frac{v}{d^2}$. For calculating the force between the charges, $r = 2d$.

EXECUTE: **(a)** $B_{\text{total}} = B + B' = \frac{\mu_0}{4\pi}\left(\frac{qv}{d^2} + \frac{q'v'}{d^2}\right)$.

$$B = \frac{\mu_0}{4\pi}\left(\frac{(8.0\times10^{-6}\ \text{C})(4.5\times10^{6}\ \text{m/s})}{(0.120\ \text{m})^2} + \frac{(3.0\times10^{-6}\ \text{C})(9.0\times10^{6}\ \text{m/s})}{(0.120\ \text{m})^2}\right) = 4.38\times10^{-4}\ \text{T}.$$

The direction of $\vec{B}$ is into the page.

(b) Following Example 28.1 we can find the magnetic force between the charges:

$$F_B = \frac{\mu_0}{4\pi}\frac{qq'vv'}{r^2} = (10^{-7}\ \text{T}\cdot\text{m/A})\frac{(8.00\times10^{-6}\ \text{C})(3.00\times10^{-6}\ \text{C})(4.50\times10^{6}\ \text{m/s})(9.00\times10^{6}\ \text{m/s})}{(0.240\ \text{m})^2}$$

$F_B = 1.69\times10^{-3}$ N. The force on the upper charge points up and the force on the lower charge points down. The Coulomb force between the charges is $F_C = k\frac{q_1q_2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(8.0\times10^{-6}\ \text{C})(3.0\times10^{-6}\ \text{C})}{(0.240\ \text{m})^2} = 3.75\ \text{N}$.

The force on the upper charge points up and the force on the lower charge points down. The ratio of the Coulomb force to the magnetic force is $\frac{F_C}{F_B} = \frac{c^2}{v_1v_2} = \frac{3.75\ \text{N}}{1.69\times10^{-3}\ \text{N}} = 2.22\times10^3$; the Coulomb force is much larger.

(b) The magnetic forces are reversed in direction when the direction of only one velocity is reversed but the magnitude of the force is unchanged.

EVALUATE: When two charges have the same sign and move in opposite directions, the force between them is repulsive. When two charges of the same sign move in the same direction, the force between them is attractive.

28.7. **IDENTIFY:** Apply $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$. For the magnetic force on q', use $\vec{F}_B = q'\vec{v}\times\vec{B}_q$ and for the magnetic force on q use $\vec{F}_B = q\vec{v}\times\vec{B}_{q'}$.

SET UP: In part (a), $r = d$ and $\frac{|\vec{v}\times\vec{r}|}{r^3} = \frac{v}{d^2}$.

EXECUTE: **(a)** $q' = -q$; $B_q = \frac{\mu_0 qv}{4\pi d^2}$, into the page; $B_{q'} = \frac{\mu_0 qv'}{4\pi d^2}$, out of the page.

(i) $v' = \frac{v}{2}$ gives $B = \frac{\mu_0 qv}{4\pi d^2}\left(1-\tfrac{1}{2}\right) = \frac{\mu_0 qv}{4\pi(2d^2)}$, into the page. (ii) $v' = v$ gives $B = 0$.

(iii) $v' = 2v$ gives $B = \frac{\mu_0 qv}{4\pi d^2}$, out of the page.

(b) The force that q exerts on q' is given by $\vec{F} = q'\vec{v}'\times\vec{B}_q$, so $F = \frac{\mu_0 q^2 v'v}{4\pi(2d)^2}$. $\vec{B}_q$ is into the page, so the force on q' is toward q. The force that q' exerts on q is toward q'. The force between the two charges is attractive.

(c) $F_B = \frac{\mu_0 q^2 vv'}{4\pi(2d)^2}$, $F_C = \frac{q^2}{4\pi\epsilon_0(2d)^2}$ so $\frac{F_B}{F_C} = \mu_0\epsilon_0 vv' = \mu_0\epsilon_0(3.00\times10^5\ \text{m/s})^2 = 1.00\times10^{-6}$.

EVALUATE: When charges of opposite sign move in opposite directions, the force between them is attractive. For the values specified in part (c), the magnetic force between the two charges is much smaller in magnitude than the Coulomb force between them.

28.8. **IDENTIFY:** Both moving charges create magnetic fields, and the net field is the vector sum of the two. The magnetic force on a moving charge is $F_{mag} = qvB\sin\phi$ and the electrical force obeys Coulomb's law.

SET UP: The magnetic field due to a moving charge is $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$.

EXECUTE: **(a)** Both fields are into the page, so their magnitudes add, giving

$$B = B_e + B_p = \frac{\mu_0}{4\pi}\left(\frac{ev}{r_e^2}+\frac{ev}{r_p^2}\right)\sin 90°$$

$$B = \frac{\mu_0}{4\pi}\left(1.60\times10^{-19}\ \text{C}\right)\left(845{,}000\ \text{m/s}\right)\left[\frac{1}{(5.00\times10^{-9}\ \text{m})^2}+\frac{1}{(4.00\times10^{-9}\ \text{m})^2}\right]$$

$B = 1.39\times10^{-3}$ T = 1.39 mT, into the page.

(b) Using $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$, where $r = \sqrt{41}$ nm and $\phi = 180° - \arctan(5/4) = 128.7°$, we get

$$B = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(1.6\times10^{-19}\ \text{C})(845{,}000\ \text{m/s})\sin 128.7°}{(\sqrt{41}\times10^{-9}\ \text{m})^2} = 2.58\times10^{-4}\ \text{T},\ \text{into the page.}$$

(c) $F_{mag} = qvB\sin 90° = (1.60\times10^{-19}\ \text{C})(845{,}000\ \text{m/s})(2.58\times10^{-4}\ \text{T}) = 3.48\times10^{-17}\ \text{N}$, in the $+x$ direction.

$F_{elec} = (1/4\pi\epsilon_0)e^2/r^2 = \frac{(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(\sqrt{41}\times10^{-9}\ \text{m})^2} = 5.62\times10^{-12}\ \text{N}$, at 51.3° below the $+x$-axis measured clockwise.

EVALUATE: The electric force is much stronger than the magnetic force.

28.9. **IDENTIFY:** A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$.

EXECUTE: Applying the law of Biot and Savart gives

(a) $dB = \frac{4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(10.0\ \text{A})(0.00110\ \text{m})\sin 90°}{(0.0500\ \text{m})^2}$ = 4.40×10^{-7} T, out of the paper.

(b) The same as above, except $r = \sqrt{(5.00\ \text{cm})^2 + (14.0\ \text{cm})^2}$ and $\phi = \arctan(5/14) = 19.65°$, giving $dB = 1.67 \times 10^{-8}$ T, out of the page.

(c) $dB = 0$ since $\phi = 0°$.

EVALUATE: This is a very small field, but it comes from a very small segment of current.

28.10. **IDENTIFY:** Apply $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3}$.

SET UP: The magnitude of the field due to the current element is $dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$, where ϕ is the angle between $\vec{r}$ and the current direction.

EXECUTE: The magnetic field at the given points is:

$$dB_a = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{(200\ \text{A})(0.000100\ \text{m})}{(0.100\ \text{m})^2} = 2.00\times 10^{-6}\ \text{T}.$$

$$dB_b = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{(200\ \text{A})(0.000100\ \text{m})\sin 45°}{2(0.100\ \text{m})^2} = 0.705\times 10^{-6}\ \text{T}.$$

$$dB_c = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{(200\ \text{A})(0.000100\ \text{m})}{(0.100\ \text{m})^2} = 2.00\times 10^{-6}\ \text{T}.$$

$$dB_d = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{Idl\sin(0°)}{r^2} = 0.$$

$$dB_e = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{(200\ \text{A})(0.00100\ \text{m})}{3(0.100\ \text{m})^2}\frac{\sqrt{2}}{\sqrt{3}} = 0.545\times 10^{-6}\ \text{T}$$

The field vectors at each point are shown in Figure 28.10.

EVALUATE: In each case $d\vec{B}$ is perpendicular to the current direction.

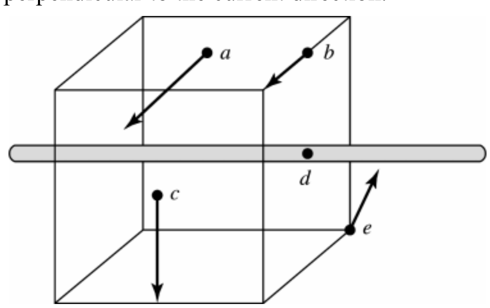


Figure 28.10

28.11. **IDENTIFY** and **SET UP:** The magnetic field produced by an infinitesimal current element is given by Eq.(28.6).

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\vec{l}\times\hat{r}}{r^2}$ As in Example 28.2 use this equation for the finite 0.500-mm segment of wire since the $\Delta l = 0.500$ mm length is much smaller than the distances to the field points.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\vec{r}}{r^3}$$

I is in the $+z$-direction, so $\Delta\vec{l} = \left(0.500\times 10^{-3}\ \text{m}\right)\hat{k}$

EXECUTE: **(a)** Field point is at $x = 2.00$ m, $y = 0$, $z = 0$ so the vector $\vec{r}$ from the source point (at the origin) to the field point is $\vec{r} = (2.00\ \text{m})\hat{i}$.

$$\Delta\vec{l}\times\vec{r} = \left(0.500\times 10^{-3}\ \text{m}\right)(2.00\ \text{m})\hat{k}\times\hat{i} = +\left(1.00\times 10^{-3}\ \text{m}^2\right)\hat{j}$$

$$\vec{B} = \frac{\left(1\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(4.00\ \text{A})\left(1.00\times 10^{-3}\ \text{m}^2\right)}{(2.00\ \text{m})^3}\hat{j} = \left(5.00\times 10^{-11}\ \text{T}\right)\hat{j}$$

(b) $\vec{r} = (2.00\text{ m})\hat{\boldsymbol{j}},\ r = 2.00\text{ m}.$

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\text{ m})(2.00\text{ m})\hat{\boldsymbol{k}} \times \hat{\boldsymbol{j}} = -(1.00\times10^{-3}\text{ m}^2)\hat{\boldsymbol{i}}$

$$\vec{B} = -\frac{(1\times10^{-7}\text{ T}\cdot\text{m/A})(4.00\text{ A})(1.00\times10^{-3}\text{ m}^2)}{(2.00\text{ m})^3}\hat{\boldsymbol{i}} = -(5.00\times10^{-11}\text{ T})\hat{\boldsymbol{i}}$$

(c) $\vec{r} = (2.00\text{ m})(\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}}),\ r = \sqrt{2}(2.00\text{ m}).$

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\text{ m})(2.00\text{ m})\hat{\boldsymbol{k}} \times (\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}}) = (1.00\times10^{-3}\text{ m}^2)(\hat{\boldsymbol{j}} - \hat{\boldsymbol{i}})$

$$\vec{B} = \frac{(1\times10^{-7}\text{ T}\cdot\text{m/A})(4.00\text{ A})(1.00\times10^{-3}\text{ m}^2)}{\left[\sqrt{2}(2.00\text{ m})\right]^3}(\hat{\boldsymbol{j}} - \hat{\boldsymbol{i}}) = (-1.77\times10^{-11}\text{ T})(\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}})$$

(d) $\vec{r} = (2.00\text{ m})\hat{\boldsymbol{k}},\ r = 2.00\text{ m}.$

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\text{ m})(2.00\text{ m})\hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}} = 0;\ \vec{B} = 0.$

EVALUATE: At each point $\vec{B}$ is perpendicular to both $\vec{r}$ and $\Delta\vec{l}$. $B = 0$ along the length of the wire.

28.12. **IDENTIFY:** A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$.

Both fields are into the page, so their magnitudes add.

EXECUTE: Applying the law of Biot and Savart for the 12.0-A current gives

$$dB = \frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{4\pi}\frac{(12.0\text{ A})(0.00150\text{ m})\left(\frac{2.50\text{ cm}}{8.00\text{ cm}}\right)}{(0.0800\text{ m})^2} = 8.79\times10^{-8}\text{ T}$$

The field from the 24.0-A segment is twice this value, so the total field is 2.64×10^{-7} T, into the page.

EVALUATE: The rest of each wire also produces field at *P*. We have calculated just the field from the two segments that are indicated in the problem.

28.13. **IDENTIFY:** A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$. Both fields are into the page, so their magnitudes add.

EXECUTE: Applying the Biot and Savart law, where $r = \frac{1}{2}\sqrt{(3.00\text{ cm})^2 + (3.00\text{ cm})^2} = 2.121$ cm, we have

$$dB = 2\frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{4\pi}\frac{(28.0\text{ A})(0.00200\text{ m})\sin 45.0^\circ}{(0.02121\text{ m})^2} = 1.76\times10^{-5}\text{ T, into the paper.}$$

EVALUATE: Even though the two wire segments are at right angles, the magnetic fields they create are in the same direction.

28.14. **IDENTIFY:** A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$. All four fields are of equal magnitude and into the page, so their magnitudes add.

EXECUTE: $dB = 4\frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{4\pi}\frac{(15.0\text{ A})(0.00120\text{ m})\sin 90^\circ}{(0.0500\text{ m})^2} = 2.88\times10^{-6}$ T, into the page.

EVALUATE: A small current element causes a small magnetic field.

28.15. **IDENTIFY:** We can model the lightning bolt and the household current as very long current-carrying wires.

SET UP: The magnetic field produced by a long wire is $B = \frac{\mu_0 I}{2\pi r}$.

EXECUTE: Substituting the numerical values gives

(a) $B = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(20{,}000\text{ A})}{2\pi(5.0\text{ m})} = 8\times10^{-4}\text{ T}$

(b) $B = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(10\text{ A})}{2\pi(0.050\text{ m})} = 4.0\times10^{-5}\text{ T}.$

EVALUATE: The field from the lightning bolt is about 20 times as strong as the field from the household current.

28.16. **IDENTIFY:** The long current-carrying wire produces a magnetic field.

SET UP: The magnetic field due to a long wire is $B = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: First find the current: $I = (3.50 \times 10^{18}\ \text{el/s})(1.60 \times 10^{-19}\ \text{C/el}) = 0.560\ \text{A}$

Now find the magnetic field: $\dfrac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(0.560\ \text{A})}{2\pi(0.0400\ \text{m})} = 2.80 \times 10^{-6}\ \text{T}$

Since electrons are negative, the conventional current runs from east to west, so the magnetic field above the wire points toward the north.

EVALUATE: This magnetic field is much less than that of the Earth, so any experiments involving such a current would have to be shielded from the Earth's magnetic field, or at least would have to take it into consideration.

28.17. **IDENTIFY:** The long current-carrying wire produces a magnetic field.

SET UP: The magnetic field due to a long wire is $B = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: First solve for the current, then substitute the numbers using the above equation.

(a) Solving for the current gives

$$I = 2\pi rB/\mu_0 = 2\pi(0.0200\ \text{m})(1.00\times 10^{-4}\ \text{T})/(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}) = 10.0\ \text{A}$$

(b) The earth's horizontal field points northward, so at all points directly above the wire the field of the wire would point northward.

(c) At all points directly east of the wire, its field would point northward.

EVALUATE: Even though the Earth's magnetic field is rather weak, it requires a fairly large current to cancel this field.

28.18. **IDENTIFY:** For each wire $B = \dfrac{\mu_0 I}{2\pi r}$ (Eq.28.9), and the direction of $\vec{B}$ is given by the right-hand rule (Fig. 28.6 in the textbook). Add the field vectors for each wire to calculate the total field.

(a) SET UP: The two fields at this point have the directions shown in Figure 28.18a.

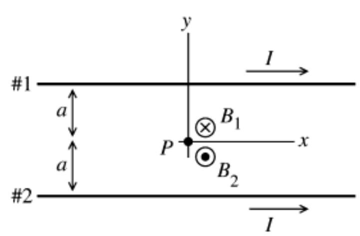


Figure 28.18a

EXECUTE: At point P midway between the two wires the fields $\vec{B}_1$ and $\vec{B}_2$ due to the two currents are in opposite directions, so $B = B_2 - B_1$.

But $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi a}$, so $B = 0$.

(b) SET UP: The two fields at this point have the directions shown in Figure 28.18b.

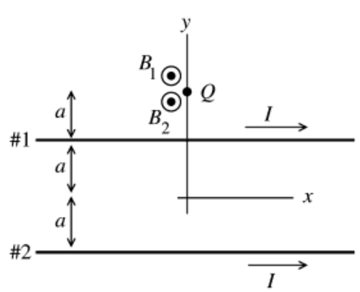


Figure 28.18b

EXECUTE: At point Q above the upper wire $\vec{B}_1$ and $\vec{B}_2$ are both directed out of the page ($+z$-direction), so $B = B_1 + B_2$.

$B_1 = \dfrac{\mu_0 I}{2\pi a}$, $B_2 = \dfrac{\mu_0 I}{2\pi(3a)}$

$B = \dfrac{\mu_0 I}{2\pi a}\left(1 + \tfrac{1}{3}\right) = \dfrac{2\mu_0 I}{3\pi a}$; $\vec{B} = \dfrac{2\mu_0 I}{3\pi a}\hat{k}$

(c) SET UP: The two fields at this point have the directions shown in Figure 28.18c.

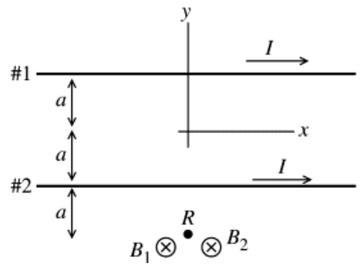


Figure 28.18c

EXECUTE: At point *R* below the lower wire $\vec{B}_1$ and $\vec{B}_2$ are both directed into the page ($-z$-direction), so $B = B_1 + B_2$.

$$B_1 = \frac{\mu_0 I}{2\pi(3a)},\ B_2 = \frac{\mu_0 I}{2\pi a}$$

$$B_1 = \frac{\mu_0 I}{2\pi a}\left(1+\tfrac{1}{3}\right) = \frac{2\mu_0 I}{3\pi a};\ \vec{B} = -\frac{2\mu_0 I}{3\pi a}\hat{k}$$

EVALUATE: In the figures we have drawn, $\vec{B}$ due to each wire is out of the page at points above the wire and into the page at points below the wire. If the two field vectors are in opposite directions the magnitudes subtract.

28.19. **IDENTIFY:** The total magnetic field is the vector sum of the constant magnetic field and the wire's magnetic field.

SET UP: For the wire, $B_{\text{wire}} = \frac{\mu_0 I}{2\pi r}$ and the direction of B_{wire} is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook. $\vec{B}_0 = (1.50\times10^{-6}\text{ T})\hat{i}$.

EXECUTE: **(a)** At (0, 0, 1 m), $\vec{B} = \vec{B}_0 - \frac{\mu_0 I}{2\pi r}\hat{i} = (1.50\times10^{-6}\text{ T})\hat{i} - \frac{\mu_0(8.00\text{ A})}{2\pi(1.00\text{ m})}\hat{i} = -(1.0\times10^{-7}\text{ T})\hat{i}$.

(b) At (1 m, 0, 0), $\vec{B} = \vec{B}_0 + \frac{\mu_0 I}{2\pi r}\hat{k} = (1.50\times10^{-6}\text{ T})\hat{i} + \frac{\mu_0(8.00\text{ A})}{2\pi(1.00\text{ m})}\hat{k}$.

$\vec{B} = (1.50\times10^{-6}\text{ T})\hat{i} + (1.6\times10^{-6}\text{ T})\hat{k} = 2.19\times10^{-6}\text{ T}$, at $\theta = 46.8°$ from *x* to *z*.

(c) At (0, 0, –0.25 m), $\vec{B} = \vec{B}_0 + \frac{\mu_0 I}{2\pi r}\hat{i} = (1.50\times10^{-6}\text{ T})\hat{i} + \frac{\mu_0(8.00\text{ A})}{2\pi(0.25\text{ m})}\hat{i} = (7.9\times10^{-6}\text{ T})\hat{i}$.

EVALUATE: At point *c* the two fields are in the same direction and their magnitudes add. At point *a* they are in opposite directions and their magnitudes subtract. At point *b* the two fields are perpendicular.

28.20. **IDENTIFY** and **SET UP:** The magnitude of $\vec{B}$ is given by Eq.(28.9) and the direction is given by the right-hand rule.

(a) EXECUTE: Viewed from above, the current is in the direction shown in Figure 28.20.

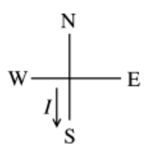


Figure 28.20

Directly below the wire the direction of the magnetic field due to the current in the wire is east.

$$B = \frac{\mu_0 I}{2\pi r} = (2\times10^{-7}\text{ T}\cdot\text{m/A})\left(\frac{800\text{ A}}{5.50\text{ m}}\right) = 2.91\times10^{-5}\text{ T}$$

(b) EVALUATE: *B* from the current is nearly equal in magnitude to the earth's field, so, yes, the current really is a problem.

28.21. **IDENTIFY:** $B = \frac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule in Section 20.7.

SET UP: Call the wires *a* and *b*, as indicated in Figure 28.21. The magnetic fields of each wire at points P_1 and P_2 are shown in Figure 28.21a. The fields at point 3 are shown in Figure 28.21b.

EXECUTE: **(a)** At P_1, $B_a = B_b$ and the two fields are in opposite directions, so the net field is zero.

(b) $B_a = \frac{\mu_0 I}{2\pi r_a}$. $B_b = \frac{\mu_0 I}{2\pi r_b}$. $\vec{B}_a$ and $\vec{B}_b$ are in the same direction so

$$B = B_a + B_b = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r_a} + \frac{1}{r_b}\right) = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(4.00\text{ A})}{2\pi}\left[\frac{1}{0.300\text{ m}} + \frac{1}{0.200\text{ m}}\right] = 6.67\times10^{-6}\text{ T}$$

$\vec{B}$ has magnitude $6.67\ \mu\text{T}$ and is directed toward the top of the page.

(c) In Figure 28.21b, $\vec{B}_a$ is perpendicular to $\vec{r}_a$ and $\vec{B}_b$ is perpendicular to $\vec{r}_b$. $\tan\theta = \dfrac{5\text{ cm}}{20\text{ cm}}$ and $\theta = 14.04°$.

$r_a = r_b = \sqrt{(0.200\text{ m})^2 + (0.050\text{ m})^2} = 0.206\text{ m}$ and $B_a = B_b$.

$$B = B_a\cos\theta + B_b\cos\theta = 2B_a\cos\theta = 2\left(\frac{\mu_0 I}{2\pi r_a}\right)\cos\theta = \frac{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(4.0\text{ A})\cos 14.04°}{2\pi(0.206\text{ m})} = 7.54\ \mu\text{T}$$

B has magnitude $7.53\ \mu\text{T}$ and is directed to the left.

EVALUATE: At points directly to the left of both wires the net field is directed toward the bottom of the page.

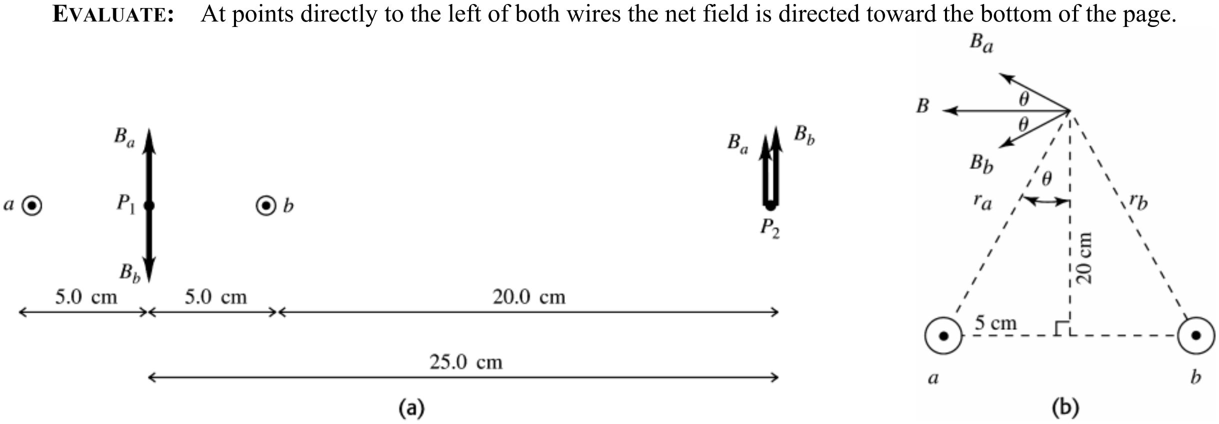


Figure 28.21

28.22. **IDENTIFY:** Use Eq.(28.9) and the right-hand rule to determine points where the fields of the two wires cancel.

(a) SET UP: The only place where the magnetic fields of the two wires are in opposite directions is between the wires, in the plane of the wires. Consider a point a distance *x* from the wire carrying $I_2 = 75.0$ A. B_{tot} will be zero where $B_1 = B_2$.

EXECUTE: $\dfrac{\mu_0 I_1}{2\pi(0.400\text{ m} - x)} = \dfrac{\mu_0 I_2}{2\pi x}$

$I_2(0.400\text{ m} - x) = I_1 x;\ I_1 = 25.0\text{ A},\ I_2 = 75.0\text{ A}$

$x = 0.300$ m; $B_{tot} = 0$ along a line 0.300 m from the wire carrying 75.0 A and 0.100 m from the wire carrying current 25.0 A.

(b) SET UP: Let the wire with $I_1 = 25.0$ A be 0.400 m above the wire with $I_2 = 75.0$ A. The magnetic fields of the two wires are in opposite directions in the plane of the wires and at points above both wires or below both wires. But to have $B_1 = B_2$ must be closer to wire #1 since $I_1 < I_2$, so can have $B_{tot} = 0$ only at points above both wires. Consider a point a distance *x* from the wire carrying $I_1 = 25.0$ A. B_{tot} will be zero where $B_1 = B_2$.

EXECUTE: $\dfrac{\mu_0 I_1}{2\pi x} = \dfrac{\mu_0 I_2}{2\pi(0.400\text{ m} + x)}$

$I_2 x = I_1(0.400\text{ m} + x);\ x = 0.200\text{ m}$

$B_{tot} = 0$ along a line 0.200 m from the wire carrying current 25.0 A and 0.600 m from the wire carrying current $I_2 = 75.0$ A.

EVALUATE: For parts (a) and (b) the locations of zero field are in different regions. In each case the points of zero field are closer to the wire that has the smaller current.

28.23. **IDENTIFY:** The net magnetic field at the center of the square is the vector sum of the fields due to each wire.

SET UP: For each wire, $B = \dfrac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook.

EXECUTE: **(a)** and **(b)** $B = 0$ since the magnetic fields due to currents at opposite corners of the square cancel.

(c) The fields due to each wire are sketched in Figure 28.23.

$$B = B_a\cos45° + B_b\cos45° + B_c\cos45° + B_d\cos45° = 4B_a\cos45° = 4\left(\frac{\mu_0 I}{2\pi r}\right)\cos45°.$$

$r = \sqrt{(10\text{ cm})^2 + (10\text{ cm})^2} = 10\sqrt{2}\text{ cm} = 0.10\sqrt{2}\text{ m}$, so

$$B = 4\frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(100\text{ A})}{2\pi(0.10\sqrt{2}\text{ m})}\cos45° = 4.0\times10^{-4}\text{ T, to the left.}$$

EVALUATE: In part (c), if all four currents are reversed in direction, the net field at the center of the square would be to the right.

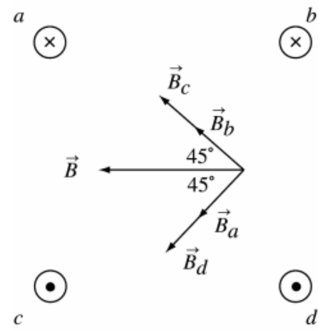
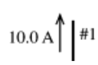


Figure 28.23

28.24. **IDENTIFY:** Use Eq.(28.9) and the right-hand rule to determine the field due to each wire. Set the sum of the four fields equal to zero and use that equation to solve for the field and the current of the fourth wire.

SET UP: The three known currents are shown in Figure 28.24.



$\vec{B}_1 \otimes, \vec{B}_2 \otimes, \vec{B}_3 \odot$

$$B = \frac{\mu_0 I}{2\pi r};\ r = 0.200\text{ m for each wire}$$

Figure 28.24

EXECUTE: Let $\odot$ be the positive *z*-direction. $I_1 = 10.0\text{ A}$, $I_2 = 8.0\text{ A}$, $I_3 = 20.0\text{ A}$. Then $B_1 = 1.00\times10^{-5}\text{ T}$, $B_2 = 0.80\times10^{-5}\text{ T}$, and $B_3 = 2.00\times10^{-5}\text{ T}$.

$$B_{1z} = -1.00\times10^{-5}\text{ T}, B_{2z} = -0.80\times10^{-5}\text{ T}, B_{3z} = +2.00\times10^{-5}\text{ T}$$

$$B_{1z} + B_{2z} + B_{3z} + B_{4z} = 0$$

$$B_{4z} = -(B_{1z} + B_{2z} + B_{3z}) = -2.0\times10^{-6}\text{ T}$$

To give $\vec{B}_4$ in the $\otimes$ direction the current in wire 4 must be toward the bottom of the page.

$$B_4 = \frac{\mu_0 I}{2\pi r}\text{ so } I_4 = \frac{rB_4}{(\mu_0/2\pi)} = \frac{(0.200\text{ m})(2.0\times10^{-6}\text{ T})}{(2\times10^{-7}\text{ T}\cdot\text{m/A})} = 2.0\text{ A}$$

EVALUATE: The fields of wires #2 and #3 are in opposite directions and their net field is the same as due to a current 20.0 A – 8.0 A = 12.0 A in one wire. The field of wire #4 must be in the same direction as that of wire #1, and $10.0\text{ A} + I_4 = 12.0\text{ A}$.

28.25. **IDENTIFY:** Apply Eq.(28.11).

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: **(a)** $F = \dfrac{\mu_0 I_1 I_2 L}{2\pi r} = \dfrac{\mu_0(5.00\text{ A})(2.00\text{ A})(1.20\text{ m})}{2\pi(0.400\text{ m})} = 6.00\times10^{-6}\text{ N}$, and the force is repulsive since the currents are in opposite directions.

(b) Doubling the currents makes the force increase by a factor of four to $F = 2.40\times10^{-5}\text{ N}$.

EVALUATE: Doubling the current in a wire doubles the magnetic field of that wire. For fixed magnetic field, doubling the current in a wire doubles the force that the magnetic field exerts on the wire.

28.26. **IDENTIFY:** Apply Eq.(28.11).

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: **(a)** $\dfrac{F}{L} = \dfrac{\mu_0 I_1 I_2}{2\pi r}$ gives $I_2 = \dfrac{F}{L}\dfrac{2\pi r}{\mu_0 I_1} = (4.0\times10^{-5}\text{ N/m})\dfrac{2\pi(0.0250\text{ m})}{\mu_0(0.60\text{ A})} = 8.33\text{ A}$.

(b) The two wires repel so the currents are in opposite directions.

EVALUATE: The force between the two wires is proportional to the product of the currents in the wires.

28.27. **IDENTIFY:** The lamp cord wires are two parallel current-carrying wires, so they must exert a magnetic force on each other.

SET UP: First find the current in the cord. Since it is connected to a light bulb, the power consumed by the bulb is $P = IV$. Then find the force per unit length using $F/L = \dfrac{\mu_0}{2\pi}\dfrac{I'I}{r}$.

EXECUTE: For the light bulb, 100 W = I(120 V) gives $I = 0.833$ A. The force per unit length is

$$F/L = \frac{4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}}{2\pi}\frac{(0.833\ \text{A})^2}{0.003\ \text{m}} = 4.6\times 10^{-5}\ \text{N/m}$$

Since the currents are in opposite directions, the force is repulsive.

EVALUATE: This force is too small to have an appreciable effect for an ordinary cord.

28.28. **IDENTIFY:** Apply Eq.(28.11) for the force from each wire.

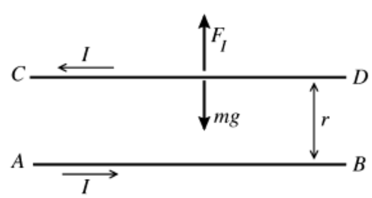
SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: On the top wire $\dfrac{F}{L} = \dfrac{\mu_0 I^2}{2\pi}\left(\dfrac{1}{d} - \dfrac{1}{2d}\right) = \dfrac{\mu_0 I^2}{4\pi d}$, upward. On the middle wire, the magnetic forces cancel so the net force is zero. On the bottom wire $\dfrac{F}{L} = \dfrac{\mu_0 I^2}{2\pi}\left(-\dfrac{1}{d} + \dfrac{1}{2d}\right) = \dfrac{\mu_0 I^2}{4\pi d}$, downward.

EVALUATE: The net force on the middle wire is zero because at the location of the middle wire the net magnetic field due to the other two wires is zero.

28.29. **IDENTIFY:** The wire CD rises until the upward force F_I due to the currents balances the downward force of gravity.

SET UP: The forces on wire CD are shown in Figure 28.29.



Currents in opposite directions so the force is repulsive and F_I is upward, as shown.

Figure 28.29

Eq.(28.11) says $F_I = \dfrac{\mu_0 I^2 L}{2\pi h}$ where L is the length of wire CD and h is the distance between the wires.

EXECUTE: $mg = \lambda L g$

Thus $F_I - mg = 0$ says $\dfrac{\mu_0 I^2 L}{2\pi h} = \lambda L g$ and $h = \dfrac{\mu_0 I^2}{2\pi g\lambda}$.

EVALUATE: The larger I is or the smaller λ is, the larger h will be.

28.30. **IDENTIFY:** The magnetic field at the center of a circular loop is $B = \dfrac{\mu_0 I}{2R}$. By symmetry each segment of the loop that has length Δl contributes equally to the field, so the field at the center of a semicircle is $\frac{1}{2}$ that of a full loop.

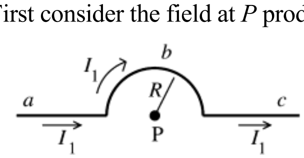
SET UP: Since the straight sections produce no field at P, the field at P is $B = \dfrac{\mu_0 I}{4R}$.

EXECUTE: $B = \dfrac{\mu_0 I}{4R}$. The direction of $\vec{B}$ is given by the right-hand rule: $\vec{B}$ is directed into the page.

EVALUATE: For a quarter-circle section of wire the magnetic field at its center of curvature is $B = \dfrac{\mu_0 I}{8R}$.

28.31. **IDENTIFY:** Calculate the magnetic field vector produced by each wire and add these fields to get the total field.

SET UP: First consider the field at P produced by the current I_1 in the upper semicircle of wire. See Figure 28.31a.

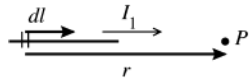


Consider the three parts of this wire
a: long straight section,
b: semicircle
c: long, straight section

Figure 28.31a

Apply the Biot-Savart law $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l}\times\hat{r}}{r^2} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l}\times\vec{r}}{r^3}$ to each piece.

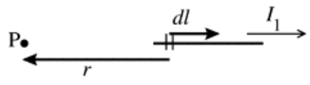
EXECUTE: <u>part *a*</u> See Figure 28.31b.



$d\vec{l} \times \vec{r} = 0,$
so $dB = 0$

Figure 28.31b

The same is true for all the infinitesimal segments that make up this piece of the wire, so $B = 0$ for this piece.
<u>part *c*</u> See Figure 28.31c.



(c)

$d\vec{l} \times \vec{r} = 0,$
so $dB = 0$ and $B = 0$ for this piece.

Figure 28.31c

<u>part *b*</u> See Figure 28.31d.

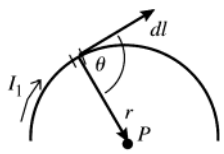


Figure 28.31d

$d\vec{l} \times \vec{r}$ is directed into the paper for all infinitesimal segments that make up this semicircular piece, so $\vec{B}$ is directed into the paper and $B = \int dB$ (the vector sum of the $d\vec{B}$ is obtained by adding their magnitudes since they are in the same direction).

$\left|d\vec{l} \times \vec{r}\right| = r\,dl\sin\theta$. The angle θ between $d\vec{l}$ and $\vec{r}$ is 90° and $r = R$, the radius of the semicircle. Thus $\left|d\vec{l} \times \vec{r}\right| = R\,dl$

$$dB = \frac{\mu_0}{4\pi}\frac{I\left|d\vec{l} \times \vec{r}\right|}{r^3} = \frac{\mu_0 I_1}{4\pi}\frac{R}{R^3}dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)dl$$

$$B = \int dB = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)\int dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)(\pi R) = \frac{\mu_0 I_1}{4R}$$

(We used that $\int dl$ is equal to πR, the length of wire in the semicircle.) We have shown that the two straight sections make zero contribution to $\vec{B}$, so $B_1 = \mu_0 I_1/4R$ and is directed into the page.

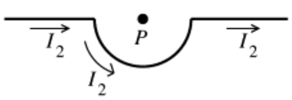


Figure 28.31e

For current in the direction shown in Figure 28.31e, a similar analysis gives $B_2 = \mu_0 I_2/4R$, out of the paper

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions, so the magnitude of the net field at P is $B = \left|B_1 - B_2\right| = \dfrac{\mu_0\left|I_1 - I_2\right|}{4R}$.

EVALUATE: When $I_1 = I_2$, $B = 0$.

28.32. **IDENTIFY:** Apply Eq.(28.16).

SET UP: At the center of the coil, $x = 0$. a is the radius of the coil, 0.0240 m.

EXECUTE: **(a)** $B_x = \mu_0 NI/2a$, so $I = \dfrac{2aB_x}{\mu_0 N} = \dfrac{2(0.024\text{ m})\,(0.0580\text{ T})}{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})\,(800)} = 2.77\text{ A}$

(b) At the center, $B_c = \mu_0 NI/2a$. At a distance x from the center,

$$B_x = \frac{\mu_0 NIa^2}{2(x^2+a^2)^{3/2}} = \left(\frac{\mu_0 NI}{2a}\right)\left(\frac{a^3}{(x^2+a^2)^{3/2}}\right) = B_c\left(\frac{a^3}{(x^2+a^2)^{3/2}}\right).\ B_x = \tfrac{1}{2}B_c \text{ says } \frac{a^3}{(x^2+a^2)^{3/2}} = \tfrac{1}{2}, \text{ and } (x^2+a^2)^3 = 4a^6.$$

Since $a = 0.024$ m, $x = 0.0184$ m.

EVALUATE: As shown in Figure 28.41 in the textbook, the field has its largest magnitude at the center of the coil and decreases with distance along the axis from the center.

28.33. **IDENTIFY:** Apply Eq.(28.16).

SET UP: At the center of the coil, $x = 0$. a is the radius of the coil, 0.020 m.

EXECUTE: **(a)** $B_{\text{center}} = \dfrac{\mu_0 NI}{2a} = \dfrac{\mu_0(600)\,(0.500\text{ A})}{2(0.020\text{ m})} = 9.42 \times 10^{-3}\text{ T}.$

(b) $B(x) = \dfrac{\mu_0 NIa^2}{2(x^2+a^2)^{3/2}}$. $B(0.08\text{ m}) = \dfrac{\mu_0(600)(0.500\text{ A})(0.020\text{ m})^2}{2((0.080\text{ m})^2+(0.020\text{ m})^2)^{3/2}} = 1.34 \times 10^{-4}\text{ T}.$

EVALUATE: As shown in Figure 28.41 in the textbook, the field has its largest magnitude at the center of the coil and decreases with distance along the axis from the center.

28.34. IDENTIFY and SET UP: The magnetic field at a point on the axis of N circular loops is given by $B_x = \dfrac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}}$. Solve for N and set $x = 0.0600$ m.

EXECUTE: $N = \dfrac{2B_x(x^2+a^2)^{3/2}}{\mu_0 I a^2} = \dfrac{2(6.39\times10^{-4}\text{ T})[(0.0600\text{ m})^2+(0.0600\text{ m})^2]^{3/2}}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(2.50\text{ A})(0.0600\text{ m})^2} = 69.$

EVALUATE: At the center of the coil the field is $B_x = \dfrac{\mu_0 N I}{2a} = 1.8\times10^{-3}$ T. The field 6.00 cm from the center is a factor of $1/2^{3/2}$ times smaller.

28.35. IDENTIFY: Apply Ampere's law.

SET UP: $\mu_0 = 4\pi\times10^{-7}\text{ T}\cdot\text{m/A}$

EXECUTE: **(a)** $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} = 3.83\times10^{-4}\text{ T}\cdot\text{m}$ and $I_{\text{encl}} = 305$ A.

(b) $-3.83\times10^{-4}\text{ T}\cdot\text{m}$ since at each point on the curve the direction of $d\vec{l}$ is reversed.

EVALUATE: The line integral $\oint \vec{B}\cdot d\vec{l}$ around a closed path is proportional to the net current that is enclosed by the path.

28.36. IDENTIFY: Apply Ampere's law.

SET UP: From the right-hand rule, when going around the path in a counterclockwise direction currents out of the page are positive and currents into the page are negative.

EXECUTE: Path a: $I_{\text{encl}} = 0 \Rightarrow \oint \vec{B}\cdot d\vec{l} = 0.$

Path b: $I_{\text{encl}} = -I_1 = -4.0\text{ A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = -\mu_0(4.0\text{ A}) = -5.03\times10^{-6}\text{ T}\cdot\text{m}.$

Path c: $I_{\text{encl}} = -I_1 + I_2 = -4.0\text{ A} + 6.0\text{ A} = 2.0\text{ A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0(2.0\text{ A}) = 2.51\times10^{-6}\text{ T}\cdot\text{m}$

Path d: $I_{\text{encl}} = -I_1 + I_2 + I_3 = 4.0\text{ A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = +\mu_0(4.0\text{ A}) = 5.03\times10^{-6}\text{ T}\cdot\text{m}.$

EVALUATE: If we instead went around each path in the clockwise direction, the sign of the line integral would be reversed.

28.37. IDENTIFY: Apply Ampere's law.

SET UP: To calculate the magnetic field at a distance r from the center of the cable, apply Ampere's law to a circular path of radius r. By symmetry, $\oint \vec{B}\cdot d\vec{l} = B(2\pi r)$ for such a path.

EXECUTE: **(a)** For $a < r < b$, $I_{\text{encl}} = I \Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 I \Rightarrow B2\pi r = \mu_0 I \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}.$

(b) For $r > c$, the enclosed current is zero, so the magnetic field is also zero.

EVALUATE: A useful property of coaxial cables for many applications is that the current carried by the cable doesn't produce a magnetic field outside the cable.

28.38. IDENTIFY: Apply Ampere's law to calculate $\vec{B}$.

(a) SET UP: For $a < r < b$ the end view is shown in Figure 28.38a.

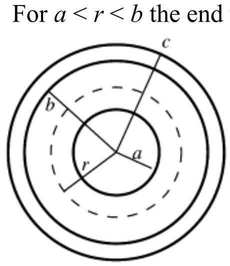


Figure 28.38a

Apply Ampere's law to a circle of radius r, where $a < r < b$. Take currents I_1 and I_2 to be directed into the page. Take this direction to be positive, so go around the integration path in the clockwise direction.

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$

$\oint \vec{B}\cdot d\vec{l} = B(2\pi r),\ I_{\text{encl}} = I_1$

Thus $B(2\pi r) = \mu_0 I_1$ and $B = \dfrac{\mu_0 I_1}{2\pi r}$

(b) SET UP: r > c: See Figure 28.38b.

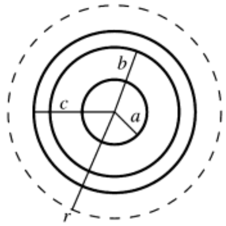


Figure 28.38b

Apply Ampere's law to a circle of radius r, where $r > c$. Both currents are in the positive direction.

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$

$\oint \vec{B} \cdot d\vec{l} = B(2\pi r),\ I_{\text{encl}} = I_1 + I_2$

Thus $B(2\pi r) = \mu_0(I_1 + I_2)$ and $B = \dfrac{\mu_0(I_1 + I_2)}{2\pi r}$

EVALUATE: For $a < r < b$ the field is due only to the current in the central conductor. For $r > c$ both currents contribute to the total field.

28.39. **IDENTIFY:** The largest value of the field occurs at the surface of the cylinder. Inside the cylinder, the field increases linearly from zero at the center, and outside the field decreases inversely with distance from the central axis of the cylinder.

SET UP: At the surface of the cylinder, $B = \dfrac{\mu_0 I}{2\pi R}$, inside the cylinder, Eq. 28.21 gives $B = \dfrac{\mu_0 I}{2\pi}\dfrac{r}{R^2}$, and outside the field is $B = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: For points inside the cylinder, the field is half its maximum value when $\dfrac{\mu_0 I}{2\pi}\dfrac{r}{R^2} = \dfrac{1}{2}\left(\dfrac{\mu_0 I}{2\pi R}\right)$, which gives $r = R/2$. Outside the cylinder, we have $\dfrac{\mu_0 I}{2\pi r} = \dfrac{1}{2}\left(\dfrac{\mu_0 I}{2\pi R}\right)$, which gives $r = 2R$.

EVALUATE: The field has half its maximum value at all points on cylinders coaxial with the wire but of radius $R/2$ and of radius $2R$.

28.40. **IDENTIFY:** $B = \mu_0 n I = \dfrac{\mu_0 N I}{L}$

SET UP: $L = 0.150\ \text{m}$

EXECUTE: $B = \dfrac{\mu_0 (600)\ (8.00\ \text{A})}{(0.150\ \text{m})} = 0.0402\ \text{T}$

EVALUATE: The field near the center of the solenoid is independent of the radius of the solenoid, as long as the radius is much less than the length.

28.41. **(a) IDENTIFY** and **SET UP:** The magnetic field near the center of a long solenoid is given by Eq.(28.23), $B = \mu_0 n I$.

EXECUTE: Turns per unit length $n = \dfrac{B}{\mu_0 I} = \dfrac{0.0270\ \text{T}}{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(12.0\ \text{A})} = 1790\ \text{turns/m}$

(b) $N = nL = (1790\ \text{turns/m})(0.400\ \text{m}) = 716\ \text{turns}$

Each turn of radius R has a length $2\pi R$ of wire. The total length of wire required is $N(2\pi R) = (716)(2\pi)(1.40 \times 10^{-2}\ \text{m}) = 63.0\ \text{m}$.

EVALUATE: A large length of wire is required. Due to the length of wire the solenoid will have appreciable resistance.

28.42. **IDENTIFY** and **SET UP:** At the center of a long solenoid $B = \mu_0 n I = \mu_0 \dfrac{N}{L} I$.

EXECUTE: $I = \dfrac{BL}{\mu_0 N} = \dfrac{(0.150\ \text{T})(1.40\ \text{m})}{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(4000)} = 41.8\ \text{A}$

EVALUATE: The magnetic field inside the solenoid is independent of the radius of the solenoid, if the radius is much less than the length, as is the case here.

28.43. **IDENTIFY** and **SET UP:** Use the appropriate expression for the magnetic field produced by each current configuration.

EXECUTE: **(a)** $B=\dfrac{\mu_0 I}{2\pi r}$ so $I=\dfrac{2\pi B}{\mu_0}=\dfrac{2\pi(2.00\times10^{-2}\text{ m})(37.2\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}=3.72\times10^{6}\text{ A}$.

(b) $B=\dfrac{N\mu_0 I}{2R}$ so $I=\dfrac{2RB}{N\mu_0}=\dfrac{2(0.210\text{ m})(37.2\text{ T})}{(100)(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})}=1.24\times10^{5}\text{ A}$.

(c) $B=\mu_0\dfrac{N}{L}I$ so $I=\dfrac{BL}{\mu_0 N}=\dfrac{(37.2\text{ T})(0.320\text{ m})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(40{,}000)}=237\text{ A}$.

EVALUATE: Much less current is needed for the solenoid, because of its large number of turns per unit length.

28.44. **IDENTIFY:** Example 28.10 shows that outside a toroidal solenoid there is no magnetic field and inside it the magnetic field is given by $B=\dfrac{\mu_0 NI}{2\pi r}$.

SET UP: The torus extends from $r_1=15.0$ cm to $r_2=18.0$ cm.

EXECUTE: **(a)** $r=0.12$ m, which is outside the torus, so $B=0$.

(b) $r=0.16$ m, so $B=\dfrac{\mu_0 NI}{2\pi r}=\dfrac{\mu_0(250)(8.50\text{ A})}{2\pi(0.160\text{ m})}=2.66\times10^{-3}\text{ T}$.

(c) $r=0.20$ m, which is outside the torus, so $B=0$.

EVALUATE: The magnetic field inside the torus is proportional to $1/r$, so it varies somewhat over the cross-section of the torus.

28.45. **IDENTIFY:** Example 28.10 shows that inside a toroidal solenoid, $B=\dfrac{\mu_0 NI}{2\pi r}$.

SET UP: $r=0.070$ m

EXECUTE: $B=\dfrac{\mu_0 NI}{2\pi r}=\dfrac{\mu_0(600)(0.650\text{ A})}{2\pi(0.070\text{ m})}=1.11\times10^{-3}\text{ T}$.

EVALUATE: If the radial thickness of the torus is small compared to its mean diameter, B is approximately uniform inside its windings.

28.46. **IDENTIFY:** Use Eq.(28.24), with μ_0 replaced by $\mu=K_m\mu_0$, with $K_m=80$.

SET UP: The contribution from atomic currents is the difference between B calculated with μ and B calculated with μ_0.

EXECUTE: **(a)** $B=\dfrac{\mu NI}{2\pi r}=\dfrac{K_m\mu_0 NI}{2\pi r}=\dfrac{\mu_0(80)(400)(0.25\text{ A})}{2\pi(0.060\text{ m})}=0.0267\text{ T}$.

(b) The amount due to atomic currents is $B'=\dfrac{79}{80}B=\dfrac{79}{80}(0.0267\text{ T})=0.0263\text{ T}$.

EVALUATE: The presence of the core greatly enhances the magnetic field produced by the solenoid.

28.47. **IDENTIFY** and **SET UP:** $B=\dfrac{K_m\mu_0 NI}{2\pi r}$ (Eq.28.24, with μ_0 replaced by $K_m\mu_0$)

EXECUTE: **(a)** $K_m=1400$

$$I=\frac{2\pi rB}{\mu_0 K_m N}=\frac{(2.90\times10^{-2}\text{ m})(0.350\text{ T})}{(2\times10^{-7}\text{ T}\cdot\text{m/A})(1400)(500)}=0.0725\text{ A}$$

(b) $K_m=5200$

$$I=\frac{2\pi rB}{\mu_0 K_m N}=\frac{(2.90\times10^{-2}\text{ m})(0.350\text{ T})}{(2\times10^{-7}\text{ T}\cdot\text{m/A})(5200)(500)}=0.0195\text{ A}$$

EVALUATE: If the solenoid were air-filled instead, a much larger current would be required to produce the same magnetic field.

28.48. **IDENTIFY:** Apply $B=\dfrac{K_m\mu_0 NI}{2\pi r}$.

SET UP: K_m is the relative permeability and $\chi_m=K_m-1$ is the magnetic susceptibility.

EXECUTE: **(a)** $K_m=\dfrac{2\pi rB}{\mu_0 NI}=\dfrac{2\pi(0.2500\text{ m})(1.940\text{ T})}{\mu_0(500)(2.400\text{ A})}=2021$.

(b) $\chi_m=K_m-1=2020$.

EVALUATE: Without the magnetic material the magnetic field inside the windings would be $B/2021=9.6\times10^{-4}\text{ T}$. The presence of the magnetic material greatly enhances the magnetic field inside the windings.

28.49. **IDENTIFY:** The magnetic field from the solenoid alone is $B_0 = \mu_0 nI$. The total magnetic field is $B = K_m B_0$. M is given by Eq.(28.29).

SET UP: $n = 6000$ turns/m

EXECUTE: **(a)** (i) $B_0 = \mu_0 nI = \mu_0(6000\ \text{m}^{-1})(0.15\ \text{A}) = 1.13\times10^{-3}\ \text{T}$.

(ii) $M = \dfrac{K_m - 1}{\mu_0}B_0 = \dfrac{5199}{\mu_0}(1.13\times10^{-3}\ \text{T}) = 4.68\times10^{6}\ \text{A/m}$.

(iii) $B = K_m B_0 = (5200)(1.13\times10^{-3}\ \text{T}) = 5.88\ \text{T}$.

(b) The directions of $\vec{B}$, $\vec{B}_0$ and $\vec{M}$ are shown in Figure 28.49. Silicon steel is paramagnetic and $\vec{B}_0$ and $\vec{M}$ are in the same direction.

EVALUATE: The total magnetic field is much larger than the field due to the solenoid current alone.

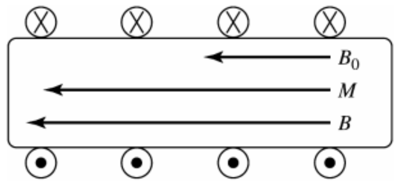


Figure 28.49

28.50. **IDENTIFY:** Curie's law (Eq.28.32) says that $1/M$ is proportional to T, so $1/\chi_m$ is proportional to T.

SET UP: The graph of $1/\chi_m$ versus the Kelvin temperature is given in Figure 28.50.

EXECUTE: The material does obey Curie's law because the graph in Figure 28.50 is a straight line. $M = C\dfrac{B}{T}$ and $M = \dfrac{B - B_0}{\mu_0}$ says that $\chi_m = \dfrac{C\mu_0}{T}$. $1/\chi_m = \dfrac{T}{C\mu_0}$ and the slope of $1/\chi_m$ versus T is $1/(C\mu_0)$. Therefore, from the graph the Curie constant is $C = \dfrac{1}{\mu_0(\text{slope})} = \dfrac{1}{\mu_0(5.13\ \text{K}^{-1})} = 1.55\times10^{5}\ \text{K}\cdot\text{A/T}\cdot\text{m}$.

EVALUATE: For this material Curie's law is valid over a wide temperature range.

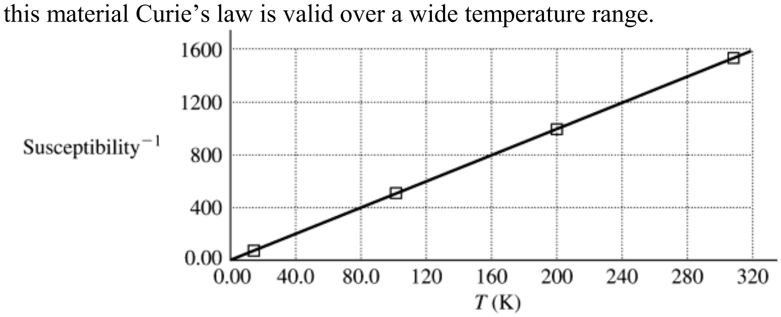


Figure 28.50

28.51. **IDENTIFY:** Moving charges create magnetic fields. The net field is the vector sum of the two fields. A charge moving in an external magnetic field feels a force.

(a) SET UP: The magnetic field due to a moving charge is $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$. Both fields are into the paper, so their magnitudes add, giving $B_{\text{net}} = B + B' = \dfrac{\mu_0}{4\pi}\left(\dfrac{qv\sin\phi}{r^2} + \dfrac{q'v'\sin\phi'}{r'^2}\right)$.

EXECUTE: Substituting numbers gives

$$B_{\text{net}} = \frac{\mu_0}{4\pi}\left[\frac{(8.00\ \mu\text{C})(9.00\times10^{4}\ \text{m/s})\sin 90^\circ}{(0.300\ \text{m})^2} + \frac{(5.00\ \mu\text{C})(6.50\times10^{4}\ \text{m/s})\sin 90^\circ}{(0.400\ \text{m})^2}\right]$$

$B_{\text{net}} = 1.00\times10^{-6}\ \text{T} = 1.00\,\mu\text{T}$, into the paper.

(b) SET UP: The magnetic force on a moving charge is $\vec{F} = q\vec{v}\times\vec{B}$, and the magnetic field of charge q' at the location of charge q is into the page. The force on q is

$$\vec{F} = q\vec{v}\times\vec{B}' = (qv)\hat{i}\times\frac{\mu_0}{4\pi}\frac{q\vec{v}'\times\hat{r}}{r^2} = (qv)\hat{i}\times\left(\frac{\mu_0}{4\pi}\frac{qv'\sin\phi}{r^2}\right)(-\hat{k}) = \left(\frac{\mu_0}{4\pi}\frac{qq'vv'\sin\phi}{r^2}\right)\hat{j}$$

where ϕ is the angle between $\vec{v}'$ and $\hat{r}'$.

EXECUTE: Substituting numbers gives

$$\vec{F} = \frac{\mu_0}{4\pi}\left[\frac{(8.00\times10^{-6}\text{ C})(5.00\times10^{-6}\text{ C})(9.00\times10^{-6}\text{ m/s})(6.50\times10^{-6}\text{ m/s})}{(0.500\text{ m})^2}\left(\frac{0.400}{0.500}\right)\right]\hat{j}$$

$$\vec{F} = (7.49\times10^{-8}\text{ N})\hat{j}.$$

EVALUATE: These are small fields and small forces, but if the charge has small mass, the force can affect its motion.

28.52. IDENTIFY: The wire creates a magnetic field near it, and the moving electron feels a force due to this field.

SET UP: The magnetic field due to the wire is $B = \frac{\mu_0 I}{2\pi r}$, and the force on a moving charge is $F = qvB\sin\phi$.

EXECUTE: $F = qvB\sin\phi = (ev\mu_0 I\sin\phi)/2\pi r$. Substituting numbers gives

$F = (1.60\times10^{-19}\text{ C})(6.00\times10^{4}\text{ m/s})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(2.50\text{ A})(\sin 90°)/[2\pi(0.0450\text{ m})]$

$F = 1.07\times10^{-19}$ N

From the right hand rule for the cross product, the direction of $\vec{v}\times\vec{B}$ is opposite to the current, but since the electron is negative, the force is in the same direction as the current.

EVALUATE: This force is small at an everyday level, but it would give the electron an acceleration of about 10^{11} m/s^2.

28.53. IDENTIFY: Find the force that the magnetic field of the wire exerts on the electron.

SET UP: The force on a moving charge has magnitude $F = |q|vB\sin\phi$ and direction given by the right-hand rule.

For a long straight wire, $B = \frac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: **(a)** $a = \frac{F}{m} = \frac{|q|vB\sin\phi}{m} = \frac{ev}{m}\left(\frac{\mu_0 I}{2\pi r}\right)$

$$a = \frac{(1.6\times10^{-17}\text{C})(2.50\times10^{5}\text{ m/s})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(25.0\text{ A})}{(9.11\times10^{-31}\text{ kg})(2\pi)(0.0200\text{ m})} = 1.1\times10^{13}\text{ m/s}^2,$$

away from the wire.

(b) The electric force must balance the magnetic force. $eE = evB$, and

$E = vB = v\frac{\mu_0 I}{2\pi r} = \frac{(250,000\text{ m/s})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(25.0\text{ A})}{2\pi(0.0200\text{ m})} = 62.5\text{ N/C}$. The magnetic force is directed away from the wire so the force from the electric field must be toward the wire. Since the charge of the electron is negative, the electric field must be directed away from the wire to produce a force in the desired direction.

EVALUATE: **(c)** $mg = (9.11\times10^{-31}\text{ kg})(9.8\text{ m/s}^2) \approx 10^{-29}\text{ N}$. $F_{\text{el}} = eE = (1.6\times10^{-19}\text{ C})(62.5\text{ N/C}) \approx 10^{-17}\text{ N}$.

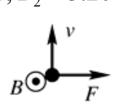
$F_{\text{el}} \approx 10^{12}\, F_{\text{grav}}$, so we can neglect gravity.

28.54. IDENTIFY: Use Eq.(28.9) and the right-hand rule to calculate the magnetic field due to each wire. Add these field vectors to calculate the net field and then use Eq.(27.2) to calculate the force.

SET UP: Let the wire connected to the $25.0\ \Omega$ resistor be #2 and the wire connected to the $10.0\ \Omega$ resistor be #1. Both I_1 and I_2 are directed toward the right in the figure, so at the location of the proton B_2 is $\otimes$ and $B_1 = \odot$.

$B_1 = \frac{\mu_0 I_1}{2\pi r}$ and $B_2 = \frac{\mu_0 I_2}{2\pi r}$, with $r = 0.0250$ m. $I_1 = (100.0\text{ V})/(10.0\ \Omega) = 10.0\text{ A}$ and $I_2 = (100.0\text{ V})/(25.0\ \Omega) = 4.00\text{ A}$

EXECUTE: $B_1 = 8.00\times10^{-5}\text{ T}$, $B_2 = 3.20\times10^{-5}\text{ T}$ and $B = B_1 - B_2 = 4.80\times10^{-5}\text{ T}$ and in the direction $\odot$.



Force is to the right.

Figure 28.54

$F = qvB = (1.602\times10^{-19}\text{ C})(650\times10^{3}\text{ m/s})(4.80\times10^{-5}\text{ T}) = 5.00\times10^{-18}\text{ N}$

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$. The magnetic force is much larger than the gravity force on the proton.

28.55. IDENTIFY: Find the net magnetic field due to the two loops at the location of the proton and then find the force these fields exert on the proton.

SET UP: For a circular loop, the field on the axis, a distance x from the center of the loop is $B = \frac{\mu_0 IR^2}{2(R^2+x^2)^{3/2}}$.

$R = 0.200$ m and $x = 0.125$ m.

EXECUTE: The fields add, so $B = B_1 + B_2 = 2B_1 = 2\left[\dfrac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}}\right]$.

$$B = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1.50\ \text{A})(0.200\ \text{m})^2}{[(0.200\ \text{m})^2 + (0.125\ \text{m})^2]^{3/2}} = 5.75 \times 10^{-6}\ \text{T}.$$

$F = |q|vB\sin\phi = (1.6\times10^{-19}\ \text{C})(2400\ \text{m/s})(5.75\times10^{-6}\ \text{T})\sin 90° = 2.21\times10^{-21}\ \text{N}$, perpendicular to the line ab and to the velocity.

EVALUATE: The weight of a proton is $w = mg = 1.6\times10^{-24}$ N, so the force from the loops is much greater than the gravity force on the proton.

28.56. IDENTIFY: The net magnetic field is the vector sum of the fields due to each wire.

SET UP: $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: **(a)** The currents are the same so points where the two fields are equal in magnitude are equidistant from the two wires. The net field is zero along the dashed line shown in Figure 28.56a.
(b) For the magnitudes of the two fields to be the same at a point, the point must be 3 times closer to the wire with the smaller current. The net field is zero along the dashed line shown in Figure 28.56b.
(c) As in (a), the points are equidistant from both wires. The net field is zero along the dashed line shown in Figure 28.56c.

EVALUATE: The lines of zero net field consist of points at which the fields of the two wires have opposite directions and equal magnitudes.

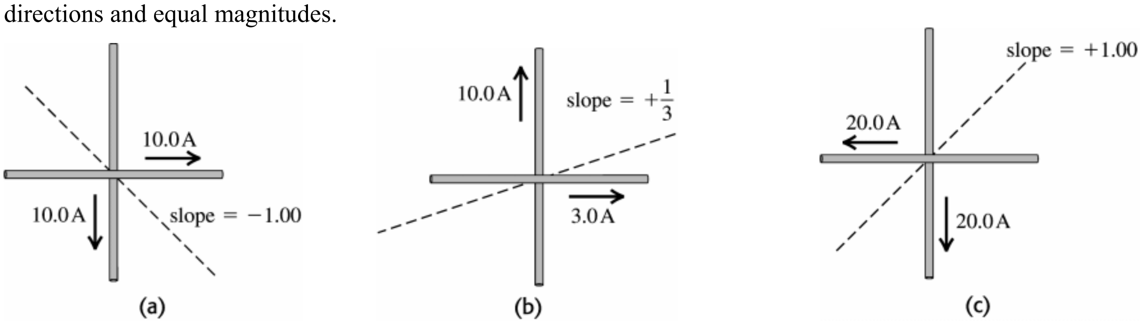


Figure 28.56

28.57. IDENTIFY: $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}_0 \times \hat{r}}{r^2}$

SET UP: $\hat{r} = \hat{i}$ and $r = 0.250$ m, so $\vec{v}_0 \times \hat{r} = v_{0z}\hat{j} - v_{0y}\hat{k}$.

EXECUTE: $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q}{r^2}\left(v_{0z}\hat{j} - v_{0y}\hat{k}\right) = \left(6.00\times10^{-6}\ \text{T}\right)\hat{j}$. $v_{0y} = 0$. $\dfrac{\mu_0}{4\pi}\dfrac{q}{r^2}v_{0z} = 6.00\times10^{-6}$ T and

$v_{0z} = \dfrac{4\pi(6.00\times10^{-6}\ \text{T})(0.25\ \text{m})^2}{\mu_0(-7.20\times10^{-3}\ \text{C})} = -521$ m/s. $v_{0x} = \pm\sqrt{v_0^2 - v_{0y}{}^2 - v_{0z}{}^2} = \pm\sqrt{(800\ \text{m/s})^2 - (-521\ \text{m/s})^2} = \pm 607$ m/s.

The sign of v_{0x} isn't determined.

(b) Now $\hat{r} = \hat{j}$ and $r = 0.250$ m. $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}_0\times\hat{r}}{r^2} = \dfrac{\mu_0}{4\pi}\dfrac{q}{r^2}\left(v_{0x}\hat{k} - v_{0z}\hat{i}\right)$.

$$B = \frac{\mu_0}{4\pi}\frac{|q|}{r^2}\sqrt{v_{0x}^2 + v_{0z}^2} = \frac{\mu_0}{4\pi}\frac{|q|}{r^2}v_0 = \frac{\mu_0}{4\pi}\frac{(7.20\times10^{-3}\ \text{C})}{(0.250\ \text{m})^2}800\ \text{m/s} = 9.20\times10^{-6}\ \text{T}.$$

EVALUATE: The magnetic field in part (b) doesn't depend on the sign of v_{0x}.

28.58. IDENTIFY and SET UP: $\vec{B} = B_0(x/a)\hat{i}$

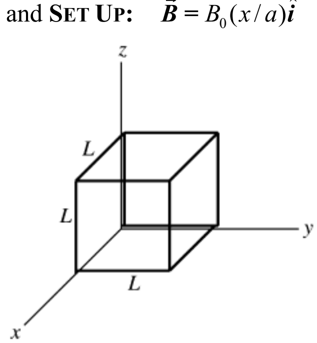


Figure 28.58

Apply Gauss's law for magnetic fields to a cube with side length L, one corner at the origin, and sides parallel to the x, y and z axes, as shown in Figure 28.58.

EXECUTE: Since $\vec{B}$ is parallel to the *x*-axis the only sides that have nonzero flux are the front side (parallel to the *yz*-plane at $x = L$) and the back side (parallel to the *yz*-plane at $x = 0$.)

front $\Phi_B = \int \vec{B}\cdot d\vec{A} = B_0(x/a)\int dA(\hat{i}\cdot\hat{i}) = B_0(x/a)\int dA$

$x = L$ on this face so $\vec{B}\cdot d\vec{A} = B_0(L/a)dA$

$\Phi_B = B_0(L/a)\int dA = B_0(L/a)L^2 = B_0(L^3/a)$

back On the back face $x = 0$ so $B = 0$ and $\Phi_B = 0$. The total flux through the cubical Gaussian surface is $\Phi_B = B_0(L^3/a)$.

EVALUATE: This violates Eq.(27.8), which says that $\Phi_B = 0$ for any closed surface. The claimed $\vec{B}$ is impossible because it has been shown to violate Gauss's law for magnetism.

28.59. **IDENTIFY:** Use Eq.(28.9) and the right-hand rule to calculate the magnitude and direction of the magnetic field at *P* produced by each wire. Add these two field vectors to find the net field.

(a) SET UP: The directions of the fields at point *P* due to the two wires are sketched in Figure 28.59a.

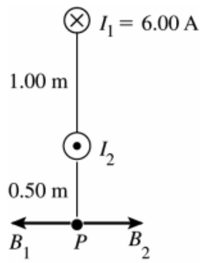


Figure 28.59a

EXECUTE: $\vec{B}_1$ and $\vec{B}_2$ must be equal and opposite for the resultant field at *P* to be zero. $\vec{B}_2$ is to the right so I_2 is out of the page.

$$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{\mu_0}{2\pi}\left(\frac{6.00\text{ A}}{1.50\text{ m}}\right) \qquad B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{\mu_0}{2\pi}\left(\frac{I_2}{0.50\text{ m}}\right)$$

$B_1 = B_2$ says $\frac{\mu_0}{2\pi}\left(\frac{6.00\text{ A}}{1.50\text{ m}}\right) = \frac{\mu_0}{2\pi}\left(\frac{I_2}{0.50\text{ m}}\right)$

$$I_2 = \left(\frac{0.50\text{ m}}{1.50\text{ m}}\right)(6.00\text{ A}) = 2.00\text{ A}$$

(b) SET UP: The directions of the fields at point *Q* are sketched in Figure 28.59b.

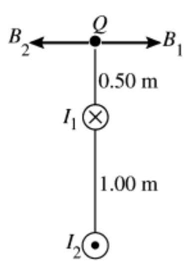


Figure 28.59b

EXECUTE: $B_1 = \frac{\mu_0 I_1}{2\pi r_1}$

$B_1 = (2\times10^{-7}\text{ T}\cdot\text{m/A})\left(\frac{6.00\text{ A}}{0.50\text{ m}}\right) = 2.40\times10^{-6}\text{ T}$

$B_2 = \frac{\mu_0 I_2}{2\pi r_2}$

$B_2 = (2\times10^{-7}\text{ T}\cdot\text{m/A})\left(\frac{2.00\text{ A}}{1.50\text{ m}}\right) = 2.67\times10^{-7}\text{ T}$

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions and $B_1 > B_2$ so

$B = B_1 - B_2 = 2.40\times10^{-6}\text{ T} - 2.67\times10^{-7}\text{ T} = 2.13\times10^{-6}\text{ T}$, and $\vec{B}$ is to the right.

(c) SET UP: The directions of the fields at point *S* are sketched in Figure 28.59c.

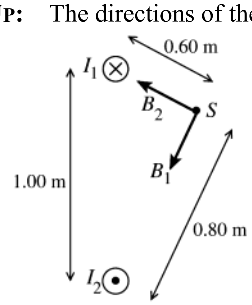


Figure 28.59c

EXECUTE: $B_1 = \frac{\mu_0 I_1}{2\pi r_1}$

$B_1 = (2\times10^{-7}\text{ T}\cdot\text{m/A})\left(\frac{6.00\text{ A}}{0.60\text{ m}}\right) = 2.00\times10^{-6}\text{ T}$

$B_2 = \frac{\mu_0 I_2}{2\pi r_2}$

$B_2 = (2\times10^{-7}\text{ T}\cdot\text{m/A})\left(\frac{2.00\text{ A}}{0.80\text{ m}}\right) = 5.00\times10^{-7}\text{ T}$

$\vec{B}_1$ and $\vec{B}_2$ are right angles to each other, so the magnitude of their resultant is given by

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(2.00\times10^{-6}\text{ T})^2 + (5.00\times10^{-7}\text{ T})^2} = 2.06\times10^{-6}\text{ T}$$

EVALUATE: The magnetic field lines for a long, straight wire are concentric circles with the wire at the center. The magnetic field at each point is tangent to the field line, so $\vec{B}$ is perpendicular to the line from the wire to the point where the field is calculated.

28.60. **IDENTIFY:** Find the vector sum of the magnetic fields due to each wire.

SET UP: For a long straight wire $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule and is perpendicular to the line from the wire to the point where then field is calculated.

EXECUTE: **(a)** The magnetic field vectors are shown in Figure 28.60a.

(b) At a position on the x-axis $B_{\text{net}} = 2\dfrac{\mu_0 I}{2\pi r}\sin\theta = \dfrac{\mu_0 I}{\pi\sqrt{x^2+a^2}}\dfrac{a}{\sqrt{x^2+a^2}} = \dfrac{\mu_0 Ia}{\pi(x^2+a^2)}$, in the positive x-direction.

(c) The graph of B versus x/a is given in Figure 28.60b.

EVALUATE: **(d)** The magnetic field is a maximum at the origin, $x = 0$.

(e) When $x >> a$, $B \approx \dfrac{\mu_0 Ia}{\pi x^2}$.

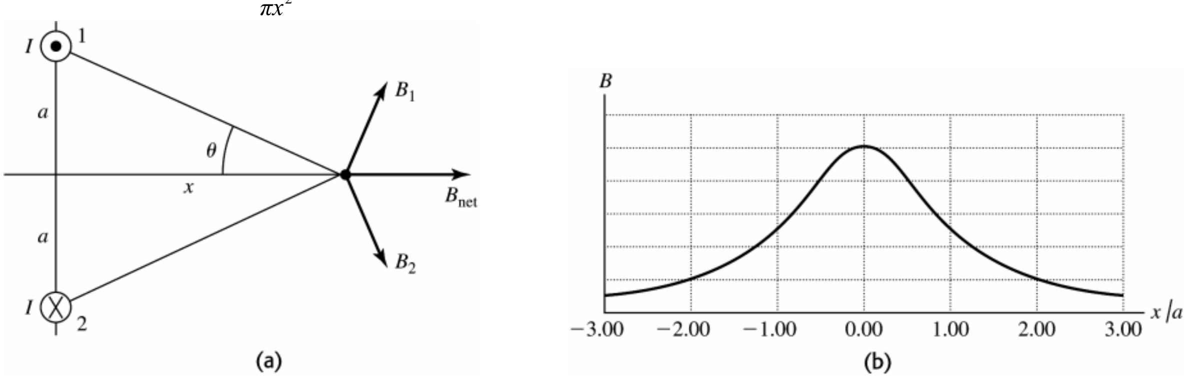


Figure 28.60

28.61. **IDENTIFY:** Apply $F = IlB\sin\phi$, with the magnetic field at point P that is calculated in problem 28.60.

SET UP: The net field of the first two wires at the location of the third wire is $B = \dfrac{\mu_0 Ia}{\pi(x^2+a^2)}$, in the $+x$-direction.

EXECUTE: **(a)** Wire is carrying current into the page, so it feels a force in the $-y$-direction .

$$\frac{F}{L} = IB = I\left(\frac{\mu_0 Ia}{\pi(x^2+a^2)}\right) = \frac{\mu_0(6.00\text{ A})^2(0.400\text{ m})}{\pi\left((0.600\text{ m})^2+(0.400\text{ m})^2\right)} = 1.11\times10^{-5}\text{ N/m}.$$

(b) If the wire carries current out of the page then the force felt will be in the opposite direction as in part (a). Thus the force will be 1.11×10^{-5} N/m, in the $+y$-direction.

EVALUATE: We could also calculate the force exerted by each of the first two wires and find the vector sum of the two forces.

28.62. **IDENTIFY:** The wires repel each other since they carry currents in opposite directions, so the wires will move away from each other until the magnetic force is just balanced by the force due to the spring.

SET UP: The force of the spring is kx and the magnetic force on each wire is $F_{\text{mag}} = \dfrac{\mu_0}{2\pi}\dfrac{I^2 L}{x}$.

EXECUTE: Call x the distance the springs each stretch. On each wire, $F_{\text{spr}} = F_{\text{mag}}$, and there are two spring forces on each wire. Therefore $2kx = \dfrac{\mu_0}{2\pi}\dfrac{I^2 L}{x}$, which gives $x = \sqrt{\dfrac{\mu_0}{2\pi}\dfrac{I^2 L}{k}}$.

EVALUATE: Since $\mu_0/2\pi$ is small, x will likely be much less than the length of the wires.

28.63. **IDENTIFY:** Apply $\sum\vec{F} = \mathbf{0}$ to one of the wires. The force one wire exerts on the other depends on I so $\sum\vec{F} = \mathbf{0}$ gives two equations for the two unknowns T and I.

SET UP: The force diagram for one of the wires is given in Figure 28.63.

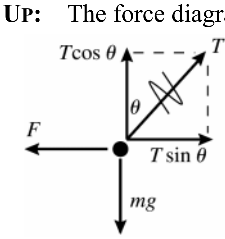


Figure 28.63

The force one wire exerts on the other is $F = \left(\dfrac{\mu_0 I^2}{2\pi r}\right)L$, where $r = 2(0.040\text{ m})\sin\theta = 8.362\times10^{-3}$ m is the distance between the two wires.

EXECUTE: $\sum F_y = 0$ gives $T\cos\theta = mg$ and $T = mg/\cos\theta$

$\sum F_x = 0$ gives $F = T\sin\theta = (mg/\cos\theta)\sin\theta = mg\tan\theta$

And $m = \lambda L$, so $F = \lambda Lg\tan\theta$

$$\left(\frac{\mu_0 I^2}{2\pi r}\right)L = \lambda Lg\tan\theta$$

$$I = \sqrt{\frac{\lambda gr\tan\theta}{(\mu_0/2\pi)}}$$

$$I = \sqrt{\frac{(0.0125\text{ kg/m})(9.80\text{ m/s})^2(\tan\ 6.00°)(8.362\times10^{-3}\text{ m})}{2\times10^{-7}\,\text{T}\cdot\text{m/A}}} = 23.2\text{ A}$$

EVALUATE: Since the currents are in opposite directions the wires repel. When I is increased, the angle θ from the vertical increases; a large current is required even for the small displacement specified in this problem.

28.64. **IDENTIFY:** Consider the forces on each side of the loop.

SET UP: The forces on the left and right sides cancel. The forces on the top and bottom segments of the loop are in opposite directions, so the magnitudes subtract.

EXECUTE: $F = F_{\text{t}} - F_{\text{b}} = \left(\frac{\mu_0 I_{\text{wire}}}{2\pi}\right)\left(\frac{Il}{r_{\text{t}}} - \frac{Il}{r_{\text{b}}}\right) = \frac{\mu_0 IlI_{\text{wire}}}{2\pi}\left(\frac{1}{r_{\text{r}}} - \frac{1}{r_{\text{l}}}\right)$.

$F = \frac{\mu_0(5.00\text{ A})(0.200\text{ m})(14.0\text{ A})}{2\pi}\left(\frac{1}{0.100\text{ m}} - \frac{1}{0.026\text{ m}}\right) = 7.97\times10^{-5}$ N. The force on the top segment is away from the wire, so the net force is away from the wire.

EVALUATE: The net force on a current loop in a uniform magnetic field is zero, but the magnetic field of the wire is not uniform, it is stronger closer to the wire.

28.65. **IDENTIFY:** Find the magnetic field of the first loop at the location of the second loop and apply $\tau = |\vec{\mu}\times\vec{B}|$ and $U = -\vec{\mu}\cdot\vec{B}$ to find μ and U.

SET UP: Since x is much larger than a', assume B is uniform over the second loop and equal to its value on the axis of the first loop.

EXECUTE: **(a)** $x >> a \Rightarrow B = \frac{N\mu_0 Ia^2}{2(x^2+a^2)^{3/2}} \approx \frac{N\mu_0 Ia^2}{2x^3}$.

$$\tau = |\vec{\mu}\times\vec{B}| = \mu B\sin\theta = (N'I'A')\left(\frac{N\mu_0 Ia^2}{2x^3}\right)\sin\theta = \frac{NN'\mu_0\pi I\ I'a^2a'^2\sin\theta}{2x^3}$$

(b) $U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\theta = -(N'I'\pi a'^2)\left(\frac{N\mu_0 Ia^2}{2x^3}\right)\cos\theta = -\frac{NN'\mu_0\pi II'a^2a'^2\cos\theta}{2x^3}$.

EVALUATE: **(c)** Having $x >> a$ allows us to simplify the form of the magnetic field, whereas assuming $x >> a'$ means we can assume that the magnetic field from the first loop is constant over the second loop.

28.66. **IDENTIFY:** Apply $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$.

SET UP: The two straight segments produce zero field at P. The field at the center of a circular loop of radius R is $B = \frac{\mu_0 I}{2R}$, so the field at the center of curvature of a semicircular loop is $B = \frac{\mu_0 I}{4R}$.

EXECUTE: The semicircular loop of radius a produces field out of the page at P and the semicircular loop of radius b produces field into the page. Therefore, $B = B_a - B_b = \frac{1}{2}\left(\frac{\mu_0 I}{2}\right)\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\mu_0 I}{4a}\left(1 - \frac{a}{b}\right)$, out of page.

EVALUATE: If $a = b$, $B = 0$.

28.67. **IDENTIFY:** Find the vector sum of the fields due to each loop.

SET UP: For a single loop $B = \frac{\mu_0 Ia^2}{2(x^2+a^2)^{3/2}}$. Here we have two loops, each of N turns, and measuring the field along the x-axis from between them means that the "x" in the formula is different for each case:

EXECUTE:

$$\text{Left coil: } x \to x + \frac{a}{2} \Rightarrow B_{\text{l}} = \frac{\mu_0 NIa^2}{2((x+a/2)^2+a^2)^{3/2}}.$$

$$\text{Right coil: } x \to x - \frac{a}{2} \Rightarrow B_{\text{r}} = \frac{\mu_0 NIa^2}{2((x-a/2)^2+a^2)^{3/2}}.$$

So, the total field at a point a distance x from the point between them is

$$B = \frac{\mu_0 N I a^2}{2}\left(\frac{1}{((x+a/2)^2+a^2)^{3/2}} + \frac{1}{((x-a/2)^2+a^2)^{3/2}}\right).$$

(b) B versus x is graphed in Figure 28.67. Figure 28.67a is the total field and Figure 27.67b is the field from the right-hand coil.

(c) At point P, $x = 0$ and $B = \dfrac{\mu_0 N I a^2}{2}\left(\dfrac{1}{((a/2)^2+a^2)^{3/2}} + \dfrac{1}{((-a/2)^2+a^2)^{3/2}}\right) = \dfrac{\mu_0 N I a^2}{(5a^2/4)^{3/2}} = \left(\dfrac{4}{5}\right)^{3/2}\dfrac{\mu_0 N I}{a}$

(d) $B = \left(\dfrac{4}{5}\right)^{3/2}\dfrac{\mu_0 N I}{a} = \left(\dfrac{4}{5}\right)^{3/2}\dfrac{\mu_0(300)(6.00\text{ A})}{(0.080\text{ m})} = 0.0202\text{ T}.$

(e) $\dfrac{dB}{dx} = \dfrac{\mu_0 N I a^2}{2}\left(\dfrac{-3(x+a/2)}{((x+a/2)^2+a^2)^{5/2}} + \dfrac{-3(x-a/2)}{((x-a/2)^2+a^2)^{5/2}}\right)$. At $x = 0$,

$$\left.\frac{dB}{dx}\right|_{x=0} = \frac{\mu_0 N I a^2}{2}\left(\frac{-3(a/2)}{((a/2)^2+a^2)^{5/2}} + \frac{-3(-a/2)}{((-a/2)^2+a^2)^{5/2}}\right) = 0.$$

$$\frac{d^2B}{dx^2} = \frac{\mu_0 N I a^2}{2}\left(\frac{-3}{((x+a/2)^2+a^2)^{5/2}} + \frac{6(x+a/2)^2(5/2)}{((x+a/2)^2+a^2)^{7/2}} + \frac{-3}{((x-a/2)^2+a^2)^{5/2}} + \frac{6(x-a/2)^2(5/2)}{((x-a/2)^2+a^2)^{7/2}}\right)$$

At $x = 0$, $\left.\dfrac{d^2B}{dx^2}\right|_{x=0} = \dfrac{\mu_0 N I a^2}{2}\left(\dfrac{-3}{((a/2)^2+a^2)^{5/2}} + \dfrac{6(a/2)^2(5/2)}{((a/2)^2+a^2)^{7/2}} + \dfrac{-3}{((a/2)^2+a^2)^{5/2}} + \dfrac{6(-a/2)^2(5/2)}{((a/2)^2+a^2)^{7/2}}\right) = 0.$

EVALUATE: Since both first and second derivatives are zero, the field can only be changing very slowly.

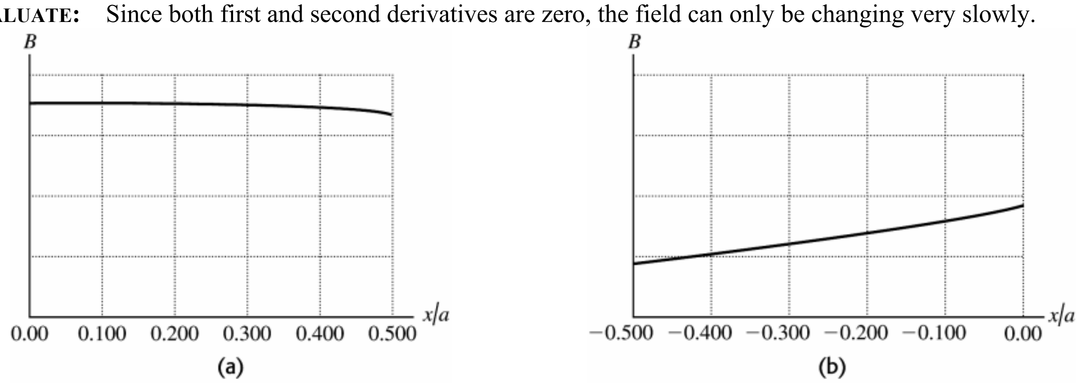


Figure 28.67

28.68. **IDENTIFY:** A current-carrying wire produces a magnetic field, but the strength of the field depends on the shape of the wire.

SET UP: The magnetic field at the center of a circular wire of radius a is $B = \mu_0 I/2a$, and the field a distance x from the center of a straight wire of length $2a$ is $B = \dfrac{\mu_0 I}{4\pi}\dfrac{2a}{x\sqrt{x^2+a^2}}$.

EXECUTE: **(a)** Since the diameter $D = 2a$, we have $B = \mu_0 I/2a = \mu_0 I/D$.

(b) In this case, the length of the wire is equal to the diameter of the circle, so $2a = \pi D$, giving $a = \pi D/2$, and $x = D/2$. Therefore $B = \dfrac{\mu_0 I}{4\pi}\dfrac{2(\pi D/2)}{(D/2)\sqrt{D^2/4+\pi^2 D^2/4}} = \dfrac{\mu_0 I}{D\sqrt{1+\pi^2}}$.

EVALUATE: The field in part (a) is greater by a factor of $\sqrt{1+\pi^2}$. It is reasonable that the field due to the circular wire is greater than the field due to the straight wire because more of the current is close to point A for the circular wire than it is for the straight wire.

28.69. **IDENTIFY:** Apply $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I d\vec{l} \times \hat{r}}{r^2}$.

SET UP: The contribution from the straight segments is zero since $d\vec{l} \times \vec{r} = 0$. The magnetic field from the curved wire is just one quarter of a full loop.

EXECUTE: $B = \frac{1}{4}\left(\dfrac{\mu_0 I}{2R}\right) = \dfrac{\mu_0 I}{8R}$ and is directed out of the page.

EVALUATE: It is very simple to calculate B at point P but it would be much more difficult to calculate B at other points.

28.70. IDENTIFY: Apply $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$.

SET UP: The horizontal wire yields zero magnetic field since $d\vec{l} \times \vec{r} = 0$. The vertical current provides the magnetic field of half of an infinite wire. (The contributions from all infinitesimal pieces of the wire point in the same direction, so there is no vector addition or components to worry about.)

EXECUTE: $B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi R}\right) = \frac{\mu_0 I}{4\pi R}$ and is directed out of the page.

EVALUATE: In the equation preceding Eq.(28.8) the limits on the integration are 0 to a rather than $-a$ to a and this introduces a factor of $\frac{1}{2}$ into the expression for B.

28.71. (a) IDENTIFY: Consider current density J for a small concentric ring and integrate to find the total current in terms of α and R.

SET UP: We can't say $I = JA = J\pi R^2$, since J varies across the cross section.

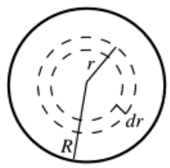


Figure 28.71

To integrate J over the cross section of the wire divide the wire cross section up into thin concentric rings of radius r and width dr, as shown in Figure 28.71.

EXECUTE: The area of such a ring is dA, and the current through it is $dI = J\,dA$; $dA = 2\pi r\,dr$ and

$dI = J\,dA = \alpha r(2\pi r\,dr) = 2\pi\alpha r^2 dr$

$I = \int dI = 2\pi\alpha\int_0^R r^2 dr = 2\pi\alpha(R^3/3)$ so $\alpha = \frac{3I}{2\pi R^3}$

(b) IDENTIFY and SET UP: (i) $r \le R$

Apply Ampere's law to a circle of radius $r < R$. Use the method of part (a) to find the current enclosed by the Ampere's law path.

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \oint B\,dl = B\oint dl = B(2\pi r)$, by the symmetry and direction of $\vec{B}$. The current passing through the path is $I_{\text{encl}} = \int dl$, where the integration is from 0 to r. $I_{\text{encl}} = 2\pi\alpha\int_0^r r^2 dr = \frac{2\pi\alpha r^3}{3} - \frac{2\pi}{3}\left(\frac{3I}{2\pi R^3}\right)r^3 = \frac{Ir^3}{R^3}$. Thus $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives $B(2\pi r) = \mu_0\left(\frac{Ir^3}{R^3}\right)$ and $B = \frac{\mu_0 I r^2}{2\pi R^3}$

(ii) **IDENTIFY and SET UP:** $r \ge R$

Apply Ampere's law to a circle of radius $r > R$.

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \oint B\,dl = B\oint dl = B(2\pi r)$

$I_{\text{encl}} = I$; all the current in the wire passes through this path. Thus $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives $B(2\pi r) = \mu_0 I$ and $B = \frac{\mu_0 I}{2\pi r}$

EVALUATE: Note that at $r = R$ the expression in (i) (for $r \le R$) gives $B = \frac{\mu_0 I}{2\pi R}$. At $r = R$ the expression in (ii) (for $r \ge R$) gives $B = \frac{\mu_0 I}{2\pi R}$, which is the same.

28.72. IDENTIFY: Apply Ampere's law to a circle of radius r in each case.

SET UP: Assume that the currents are uniform over the cross sections of the conductors.

EXECUTE: **(a)** $r < a \Rightarrow I_{\text{encl}} = I\left(\frac{A_r}{A_a}\right) = I\left(\frac{r^2}{a^2}\right)$. $\oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I\left(\frac{r^2}{a^2}\right)$ and $B = \frac{\mu_0 I r}{2\pi a^2}$. When $r = a$, $B = \frac{\mu_0 I}{2\pi a}$, which is just what was found in part (a) of Exercise 28.37.

(b) $b < r < c \Rightarrow I_{\text{encl}} = I - I\left(\frac{A_{b\to r}}{A_{b\to c}}\right) = I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right)$. $\oint \vec{B}\cdot d\vec{l} = B2\pi r = \mu_0 I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) = \mu_0 I\left(\frac{c^2 - r^2}{c^2 - b^2}\right)$ and

$B = \frac{\mu_0 I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right)$. When $r = b$, $B = \frac{\mu_0 I}{2\pi b}$, just as in part (a) of Exercise 28.37 and when $r = c$, $B = 0$, just as in part (b) of Exercise 28.37.

EVALUATE: Unlike E, B is not zero within the conductors. B varies across the cross section of each conductor.

28.73. **IDENTIFY:** Apply $\oint \vec{B}\cdot d\vec{A} = 0$.

SET UP: Take the closed gaussian surface to be a cylinder whose axis coincides with the wire.

EXECUTE: If there is a magnetic field component in the z-direction, it must be constant because of the symmetry of the wire. Therefore the contribution to a surface integral over a closed cylinder, encompassing a long straight wire will be zero: no flux through the barrel of the cylinder, and equal but opposite flux through the ends. The radial field will have no contribution through the ends, but through the barrel:

$0 = \oint \vec{B}\cdot d\vec{A} = \oint \vec{B}_r\cdot d\vec{A} = \int_{\text{barrel}} \vec{B}_r\cdot d\vec{A} = \int_{\text{barrel}} B_r dA = B_r A_{\text{barrel}} = 0$. Therefore, $B_r = 0$.

EVALUATE: The magnetic field of a long straight wire is everywhere tangent to a circular area whose plane is perpendicular to the wire, with the wire passing through the center of the circular area. This field produces zero flux through the cylindrical gaussian surface.

28.74. **IDENTIFY:** Apply Ampere's law to a circular path of radius r.

SET UP: Assume the current is uniform over the cross section of the conductor.

EXECUTE: **(a)** $r < a \Rightarrow I_{\text{encl}} = 0 \Rightarrow B = 0$.

(b) $a < r < b \Rightarrow I_{\text{encl}} = I\left(\frac{A_{a\to r}}{A_{a\to b}}\right) = I\left(\frac{\pi(r^2 - a^2)}{\pi(b^2 - a^2)}\right) = I\frac{(r^2 - a^2)}{(b^2 - a^2)}$. $\oint \vec{B}\cdot d\vec{l} = B2\pi r = \mu_0 I\frac{(r^2 - a^2)}{(b^2 - a^2)}$ and $B = \frac{\mu_0 I}{2\pi r}\frac{(r^2 - a^2)}{(b^2 - a^2)}$.

(c) $r > b \Rightarrow I_{\text{encl}} = I$. $\oint \vec{B}\cdot d\vec{l} = B2\pi r = \mu_0 I$ and $B = \frac{\mu_0 I}{2\pi r}$.

EVALUATE: The expression in part (b) gives $B = 0$ at $r = a$ and this agrees with the result of part (a). The expression in part (b) gives $B = \frac{\mu_0 I}{2\pi b}$ at $r = b$ and this agrees with the result of part (c).

28.75. **IDENTIFY:** Use Ampere's law to find the magnetic field at $r = 2a$ from the axis. The analysis of Example 28.9 shows that the field outside the cylinder is the same as for a long, straight wire along the axis of the cylinder.

SET UP:

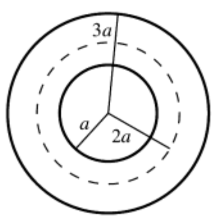


Figure 28.75

EXECUTE: Apply Ampere's law to a circular path of radius $2a$, as shown in Figure 28.75.

$B(2\pi) = \mu_0 I_{\text{encl}}$

$I_{\text{encl}} = I\left(\frac{(2a)^2 - a^2}{(3a)^2 - a^2}\right) = 3I/8$

$B = \frac{3}{16}\frac{\mu_0 I}{2\pi a}$; this is the magnetic field inside the metal at a distance of $2a$ from the cylinder axis. Outside the cylinder, $B = \frac{\mu_0 I}{2\pi r}$. The value of r where these two fields are equal is given by $1/r = 3/(16a)$ and $r = 16a/3$.

EVALUATE: For $r < 3a$, as r increases the magnetic field increases from zero at $r = 0$ to $\mu_0 I/(2\pi(3a))$ at $r = 3a$. For $r > 3a$ the field decreases as r increases so it is reasonable for there to be a $r > 3a$ where the field is the same as at $r = 2a$.

28.76. **IDENTIFY:** The net field is the vector sum of the fields due to the circular loop and to the long straight wire.

SET UP: For the long wire, $B = \frac{\mu_0 I_1}{2\pi D}$, and for the loop, $B = \frac{\mu_0 I_2}{2R}$.

EXECUTE: At the center of the circular loop the current I_2 generates a magnetic field that is into the page, so the current I_1 must point to the right. For complete cancellation the two fields must have the same magnitude:

$\frac{\mu_0 I_1}{2\pi D} = \frac{\mu_0 I_2}{2R}$. Thus, $I_1 = \frac{\pi D}{R} I_2$.

EVALUATE: If I_1 is to the left the two fields add.

28.77. **IDENTIFY:** Use the current density J to find dI through a concentric ring and integrate over the appropriate cross section to find the current through that cross section. Then use Ampere's law to find $\vec{B}$ at the specified distance from the center of the wire.

(a) SET UP:

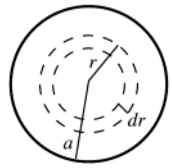


Figure 28.77a

Divide the cross section of the cylinder into thin concentric rings of radius r and width dr, as shown in Figure 28.77a. The current through each ring is $dI = J\,dA = J2\pi r\,dr$.

EXECUTE: $dI = \dfrac{2I_0}{\pi a^2}\left[1-(r/a)^2\right]2\pi r\ dr = \dfrac{4I_0}{a^2}\left[1-(r/a)^2\right]r\ dr$. The total current I is obtained by integrating dI over the cross section $I = \int_0^a dI = \left(\dfrac{4I_0}{a^2}\right)\int_0^a \left(1-r^2/a^2\right)r\ dr = \left(\dfrac{4I_0}{a^2}\right)\left[\dfrac{1}{2}r^2 - \dfrac{1}{4}r^4/a^2\right]_0^a = I_0$, as was to be shown.

(b) SET UP: Apply Ampere's law to a path that is a circle of radius $r > a$, as shown in Figure 28.77b.

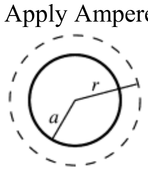


Figure 28.77b

$$\oint \vec{B}\cdot d\vec{l} = B(2\pi r)$$

$I_{\text{encl}} = I_0$ (the path encloses the entire cylinder)

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ says $B(2\pi r) = \mu_0 I_0$ and $B = \dfrac{\mu_0 I_0}{2\pi r}$.

(c) SET UP:

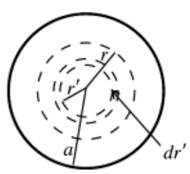


Figure 28.77c

Divide the cross section of the cylinder into concentric rings of radius r' and width dr', as was done in part (a). See Figure 28.77c. The current dI through each ring is $dI = \dfrac{4I_0}{a^2}\left[1-\left(\dfrac{r'}{a}\right)^2\right]r'\ dr'$

EXECUTE: The current I is obtained by integrating dI from $r' = 0$ to $r' = r$:

$$I = \int dI = \frac{4I_0}{a^2}\int_0^r \left[1-\left(\frac{r'}{a}\right)^2\right]r'\ dr' = \frac{4I_0}{a^2}\left[\tfrac{1}{2}(r')^2 - \tfrac{1}{4}(r')^4/a^2\right]_0^r$$

$$I = \frac{4I_0}{a^2}(r^2/2 - r^4/4a^2) = \frac{I_0 r^2}{a^2}\left(2-\frac{r^2}{a^2}\right)$$

(d) SET UP: Apply Ampere's law to a path that is a circle of radius $r < a$, as shown in Figure 28.77d.

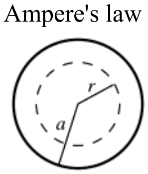


Figure 28.77d

$$\oint \vec{B}\cdot d\vec{l} = B(2\pi r)$$

$I_{\text{encl}} = \dfrac{I_0 r^2}{a^2}\left(2-\dfrac{r^2}{a^2}\right)$ (from part (c))

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ says $B(2\pi r) = \mu_0 \dfrac{I_0 r^2}{a^2}(2 - r^2/a^2)$ and $B = \dfrac{\mu_0 I_0}{2\pi}\dfrac{r}{a^2}(2 - r^2/a^2)$

EVALUATE: Result in part (b) evaluated at $r = a$: $B = \dfrac{\mu_0 I_0}{2\pi a}$. Result in part (d) evaluated at $r = a$: $B = \dfrac{\mu_0 I_0}{2\pi}\dfrac{a}{a^2}(2 - a^2/a^2) = \dfrac{\mu_0 I_0}{2\pi a}$. The two results, one for $r > a$ and the other for $r < a$, agree at $r = a$.

28.78. **IDENTIFY:** Apply Ampere's law to a circle of radius r.

SET UP: The current within a radius r is $I = \int \vec{J}\cdot d\vec{A}$, where the integration is over a disk of radius r.

EXECUTE: **(a)** $I_0 = \int \vec{J} \cdot d\vec{A} = \int \left(\frac{b}{r}e^{(r-a)/\delta}\right) r dr d\theta = 2\pi b \int_0^a e^{(r-a)/\delta} dr = 2\pi b\delta\, e^{(r-a)/\delta}\Big|_0^a = 2\pi b\delta\left(1 - e^{-a/\delta}\right).$

$I_0 = 2\pi(600\text{ A/m})(0.025\text{ m})(1 - e^{(0.050/0.025)}) = 81.5\text{ A}.$

(b) For $r \ge a$, $\oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0$ and $B = \frac{\mu_0 I_0}{2\pi r}$.

(c) For $r \le a$, $I(r) = \int \vec{J} \cdot d\vec{A} = \int \left(\frac{b}{r'}e^{(r'-a)/\delta}\right) r' dr' d\theta = 2\pi b \int_0^r e^{(r-a)/\delta} dr = 2\pi b\delta e^{(r'-a)/\delta}\Big|_0^r$.

$I(r) = 2\pi b\delta(e^{(r'-a)/\delta} - e^{-a/\delta}) = 2\pi b\delta e^{-a/\delta}(e^{r/\delta} - 1)$ and $I(r) = I_0 \frac{(e^{r/\delta} - 1)}{(e^{a/\delta} - 1)}$.

(d) For $r \le a$, $\oint \vec{B} \cdot d\vec{l} = B(r)2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0 \frac{(e^{r/\delta} - 1)}{(e^{a/\delta} - 1)}$ and $B = \frac{\mu_0 I_0 (e^{r/\delta} - 1)}{2\pi r(e^{a/\delta} - 1)}$.

(e) At $r = \delta = 0.025$ m, $B = \frac{\mu_0 I_0 (e-1)}{2\pi\delta(e^{a/\delta} - 1)} = \frac{\mu_0(81.5\text{ A})}{2\pi(0.025\text{ m})} \frac{(e-1)}{(e^{0.050/0.025} - 1)} = 1.75 \times 10^{-4}$ T.

At $r = a = 0.050$ m, $B = \frac{\mu_0 I_0}{2\pi a} \frac{(e^{a/\delta} - 1)}{(e^{a/\delta} - 1)} = \frac{\mu_0(81.5\text{ A})}{2\pi(0.050\text{ m})} = 3.26 \times 10^{-4}$ T.

At $r = 2a = 0.100$ m, $B = \frac{\mu_0 I_0}{2\pi r} = \frac{\mu_0(81.5\text{ A})}{2\pi(0.100\text{ m})} = 1.63 \times 10^{-4}$ T.

EVALUATE: At points outside the cylinder, the magnetic field is the same as that due to a long wire running along the axis of the cylinder.

28.79. **IDENTIFY:** Evaluate the integral as specified in the problem.

SET UP: Eq.(28.15) says $B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$.

EXECUTE: $\int_{-\infty}^{\infty} B_x dx = \int_{-\infty}^{\infty} \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} dx = \frac{\mu_0 I}{2} \int_{-\infty}^{\infty} \frac{1}{((x/a)^2 + 1)^{3/2}} d(x/a).$

$$B = \frac{\mu_0 I}{2} \int_{-\infty}^{\infty} \frac{dz}{(z^2 + 1)^{3/2}} \Rightarrow \int_{-\infty}^{\infty} B_x dx = \frac{\mu_0 I}{2} \int_{-\pi/2}^{\pi/2} \cos\theta d\theta = \frac{\mu_0 I}{2} (\sin\theta)\Big|_{-\pi/2}^{\pi/2} = \mu_0 I,$$

where we used the substitution $z = \tan\theta$ to go from the first to second line.

EVALUATE: This is just what Ampere's Law tells us to expect if we imagine the loop runs along the x-axis closing on itself at infinity: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$.

28.80. **IDENTIFY:** Follow the procedure specified in the problem.

SET UP: The field and integration path are sketched in Figure 28.80.

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = 0$ (no currents in the region). Using the figure, let $\vec{B} = B_0 \hat{i}$ for $y < 0$ and $B = 0$ for $y > 0$.

Then $\int_{abcde} \vec{B} \cdot d\vec{l} = B_{ab}L - B_{cd}L = 0$. $B_{cd} = 0$, so $B_{ab}L = 0$. But we have assumed that $B_{ab} \ne 0$. This is a contradiction and violates Ampere's Law.

EVALUATE: It is often convenient to approximate B as confined to a particular region of space, but this result tells us that the boundary of such a region isn't sharp.

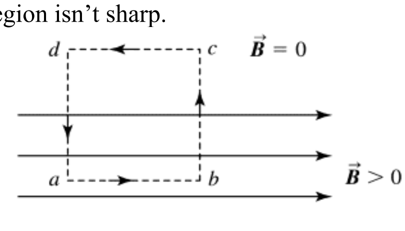


Figure 28.80

28.81. **IDENTIFY:** Use what we know about the magnetic field of a long, straight conductor to deduce the symmetry of the magnetic field. Then apply Ampere's law to calculate the magnetic field at a distance a above and below the current sheet.

SET UP: Do parts (a) and (b) together.

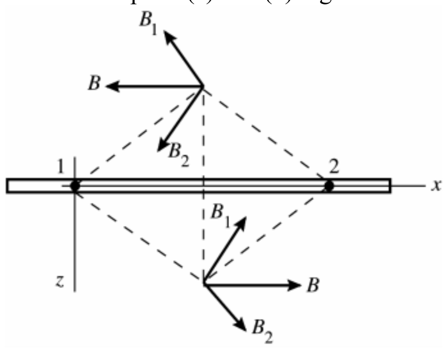


Figure 28.81a

Consider the individual currents in pairs, where the currents in each pair are equidistant on either side of the point where $\vec{B}$ is being calculated. Figure 28.81a shows that for each pair the z-components cancel, and that above the sheet the field is in the $-x$-direction and that below the sheet it is in the $+x$-direction.

Also, by symmetry the magnitude of $\vec{B}$ a distance a above the sheet must equal the magnitude of $\vec{B}$ a distance a below the sheet. Now that we have deduced the symmetry of $\vec{B}$, apply Ampere's law. Use a path that is a rectangle, as shown in Figure 28.81b.

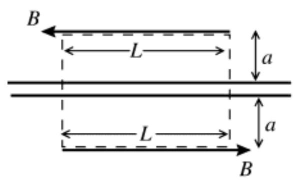


Figure 28.81b

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

I is directed out of the page, so for I to be positive the integral around the path is taken in the counterclockwise direction.

EXECUTE: Since $\vec{B}$ is parallel to the sheet, on the sides of the rectangle that have length $2a$, $\oint \vec{B} \cdot d\vec{l} = 0$. On the long sides of length L, $\vec{B}$ is parallel to the side, in the direction we are integrating around the path, and has the same magnitude, B, on each side. Thus $\oint \vec{B} \cdot d\vec{l} = 2BL$. n conductors per unit length and current I out of the page in each conductor gives $I_{\text{encl}} = InL$. Ampere's law then gives $2BL = \mu_0 InL$ and $B = \frac{1}{2}\mu_0 In$.

EVALUATE: Note that B is independent of the distance a from the sheet. Compare this result to the electric field due to an infinite sheet of charge (Example 22.7).

28.82. IDENTIFY: Find the vector sum of the fields due to each sheet.

SET UP: Problem 28.81 shows that for an infinite sheet $B = \frac{1}{2}\mu_0 In$. If I is out of the page, $\vec{B}$ is to the left above the sheet and to the right below the sheet. If I is into the page, $\vec{B}$ is to the right above the sheet and to the left below the sheet. B is independent of the distance from the sheet. The directions of the two fields at points P, R and S are shown in Figure 28.82.

EXECUTE: **(a)** Above the two sheets, the fields cancel (since there is no dependence upon the distance from the sheets).

(b) In between the sheets the two fields add up to yield $B = \mu_0 nI$, to the right.

(c) Below the two sheets, their fields again cancel (since there is no dependence upon the distance from the sheets).

EVALUATE: The two sheets with currents in opposite directions produce a uniform field between the sheets and zero field outside the two sheets. This is analogous to the electric field produced by large parallel sheets of charge of opposite sign.

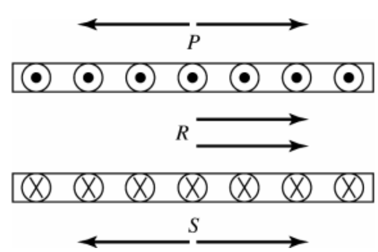


Figure 28.82

28.83. IDENTIFY and SET UP: Use Eq.(28.28) to calculate the total magnetic moment of a volume V of the iron. Use the density and atomic mass of iron to find the number of atoms in this volume and use that to find the magnetic dipole moment per atom.

EXECUTE: $M = \frac{\mu_{\text{total}}}{V}$, so $\mu_{\text{total}} = MV$ The average magnetic moment per atom is $\mu_{\text{atom}} = \mu_{\text{total}} / N = MV / N$, where N is the number of atoms in volume V. The mass of volume V is $m = \rho V$, where ρ is the density. ($\rho_{\text{iron}} = 7.8\times10^3\ \text{kg/m}^3$). The number of moles of iron in volume V is

$$n = \frac{m}{55.847\times10^{-3}\ \text{kg/mol}} = \frac{\rho V}{55.847\times10^{-3}\ \text{kg/mol}},$$ where $55.847\times10^{-3}\ \text{kg/mol}$ is the atomic mass of iron from appendix D. $N = nN_{\text{A}}$, where $N_{\text{A}} = 6.022\times10^{23}$ atoms/mol is Avogadro's number. Thus

$$N = nN_{\text{A}} = \frac{\rho V N_{\text{A}}}{55.847\times10^{-3}\ \text{kg/mol}}.$$

$$\mu_{\text{atom}} = \frac{MV}{N} = MV\left(\frac{55.847\times10^{-3}\ \text{kg/mol}}{\rho V N_{\text{A}}}\right) = \frac{M(55.847\times10^{-3}\ \text{kg/mol})}{\rho N_{\text{A}}}.$$

$$\mu_{\text{atom}} = \frac{(6.50\times10^4\ \text{A/m})(55.847\times10^{-3}\ \text{kg/mol})}{(7.8\times10^3\ \text{kg/m}^3)(6.022\times10^{23}\ \text{atoms/mol})}$$

$$\mu_{\text{atom}} = 7.73\times10^{-25}\ \text{A}\cdot\text{m}^2 = 7.73\times10^{-25}\ \text{J/T}$$

$\mu_{\text{B}} = 9.274\times10^{-24}\ \text{A}\cdot\text{m}^2$, so $\mu_{\text{atom}} = 0.0834\mu_{\text{B}}$.

EVALUATE: The magnetic moment per atom is much less than one Bohr magneton. The magnetic moments of each electron in the iron must be in different directions and mostly cancel each other.

28.84. **IDENTIFY:** The force on the cube of iron must equal the weight of the iron cube. The weight is proportional to the density and the magnetic force is proportional to μ, which is in turn proportional to K_{m}.

SET UP: The densities if iron, aluminum and silver are $\rho_{\text{Fe}} = 7.8\times10^3\ \text{kg/m}^3$, $\rho_{\text{Al}} = 2.7\times10^3\ \text{kg/m}^3$ and $\rho_{\text{Ag}} = 10.5\times10^3\ \text{kg/m}^3$. The relative permeabilities of iron, aluminum and silver are $K_{\text{Fe}} = 1400$, $K_{\text{Al}} = 1.00022$ and $K_{\text{Ag}} = 1.00 - 2.6\times10^{-5}$.

EXECUTE: **(a)** The microscopic magnetic moments of an initially unmagnetized ferromagnetic material experience torques from a magnet that aligns the magnetic domains with the external field, so they are attracted to the magnet. For a paramagnetic material, the same attraction occurs because the magnetic moments align themselves parallel to the external field. For a diamagnetic material, the magnetic moments align antiparallel to the external field so it is like two magnets repelling each other.

(b) The magnet can just pick up the iron cube so the force it exerts is $F_{\text{Fe}} = m_{\text{Fe}}g = \rho_{\text{Fe}}a^3g = (7.8\times10^3\ \text{kg/m}^3)(0.020\ \text{m})^3(9.8\ \text{m/s}^2) = 0.612\ \text{N}$. If the magnet tries to lift the aluminum cube of the same dimensions as the iron block, then the upward force felt by the cube is $F_{\text{Al}} = \frac{K_{\text{Al}}}{K_{\text{Fe}}}(0.612\ \text{N}) = \frac{1.000022}{1400}(0.612\ \text{N}) = 4.37\times10^{-4}\ \text{N}$. The weight of the aluminum cube is $W_{\text{Al}} = m_{\text{Al}}g = \rho_{\text{Al}}a^3g = (2.7\times10^3\ \text{kg/m}^3)(0.020\ \text{m})^3(9.8\ \text{m/s}^2) = 0.212\ \text{N}$. Therefore, the ratio of the magnetic force on the aluminum cube to the weight of the cube is $\frac{4.37\times10^{-4}\ \text{N}}{0.212\ \text{N}} = 2.1\times10^{-3}$ and the magnet cannot lift it.

(c) If the magnet tries to lift a silver cube of the same dimensions as the iron block, then the downward force felt by the cube is $F_{\text{Al}} = \frac{K_{\text{Ag}}}{K_{\text{Fe}}}(0.612\ \text{N}) = \frac{(1.00 - 2.6\times10^{-5})}{1400}(0.612\ \text{N}) = 4.37\times10^{-4}\ \text{N}$. But the weight of the silver cube is $W_{\text{Ag}} = m_{\text{Ag}}g = \rho_{\text{Ag}}a^3g = (10.5\times10^3\ \text{kg/m}^3)(0.020\ \text{m})^3(9.8\ \text{m/s}^2) = 0.823\ \text{N}$. So the ratio of the magnetic force on the silver cube to the weight of the cube is $\frac{4.37\times10^{-4}\ \text{N}}{0.823\ \text{N}} = 5.3\times10^{-4}$ and the magnet's effect would not be noticeable.

EVALUATE: Silver is diamagnetic and is repelled by the magnet. Aluminum is paramagnetic and is attracted by the magnet. But for both these materials the force is much less that the force on a similar cube of ferromagnetic iron.

28.85. **IDENTIFY:** The current-carrying wires repel each other magnetically, causing them to accelerate horizontally. Since gravity is vertical, it plays no initial role.

SET UP: The magnetic force per unit length is $\frac{F}{L} = \frac{\mu_0}{2\pi}\frac{I^2}{d}$, and the acceleration obeys the equation $F/L = m/L\ a$. The rms current over a short discharge time is $I_0/\sqrt{2}$.

EXECUTE: **(a)** First get the force per unit length:

$$\frac{F}{L}=\frac{\mu_0}{2\pi}\frac{I^2}{d}=\frac{\mu_0}{2\pi d}\left(\frac{I_0}{\sqrt{2}}\right)^2=\frac{\mu_0}{4\pi d}\left(\frac{V}{R}\right)^2=\frac{\mu_0}{4\pi d}\left(\frac{Q_0}{RC}\right)^2$$

Now apply Newton's second law using the result above: $\frac{F}{L}=\frac{m}{L}a=\lambda a=\frac{\mu_0}{4\pi d}\left(\frac{Q_0}{RC}\right)^2$. Solving for a gives

$a=\frac{\mu_0 Q_0^2}{4\pi\lambda R^2C^2d}$. From the kinematics equation $v_x=v_{0x}+a_xt$, we have $v_0=at=aRC=\frac{\mu_0Q_0^2}{4\pi\lambda RCd}$

(b) Conservation of energy gives $\frac{1}{2}mv_0^2=mgh$ and $h=\frac{v_0^2}{2g}=\frac{\left(\frac{\mu_0Q_0^2}{4\pi\lambda RCd}\right)^2}{2g}=\frac{1}{2g}\left(\frac{\mu_0Q_0^2}{4\pi\lambda RCd}\right)^2$.

EVALUATE: Once the wires have swung apart, we would have to consider gravity in applying Newton's second law.

28.86. **IDENTIFY:** Approximate the moving belt as an infinite current sheet.

SET UP: Problem 28.81 shows that $B=\frac{1}{2}\mu_0In$ for an infinite current sheet. Let L be the width of the sheet, so $n=I/L$.

EXECUTE: The amount of charge on a length Δx of the belt is $\Delta Q=L\Delta x\sigma$, so $I=\frac{\Delta Q}{\Delta t}=L\frac{\Delta x}{\Delta t}\sigma=Lv\sigma.$

Approximating the belt as an infinite sheet $B=\frac{\mu_0 I}{2L}=\frac{\mu_0 v\sigma}{2}$. $\vec{B}$ is directed out of the page, as shown in Figure 28.86.

EVALUATE: The field is uniform above the sheet, for points close enough to the sheet for it to be considered infinite.

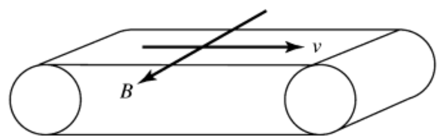


Figure 28.86

28.87. **IDENTIFY:** The rotating disk produces concentric rings of current. Calculate the field due to each ring and integrate over the surface of the disk to find the total field.

SET UP: At the center of a circular ring carrying current I, $B=\frac{\mu_0 I}{2r}$.

EXECUTE: The charge on a ring of radius r is $q=\sigma A=\sigma 2\pi r dr=\frac{2Qrdr}{a^2}$. If the disk rotates at n turns per second, then the current from that ring is $dI=\frac{dq}{dt}=ndq=\frac{2Qnrdr}{a^2}$. Therefore, $dB=\frac{\mu_0 I}{2r}=\frac{\mu_0}{2r}\frac{2Qnrdr}{a^2}=\frac{\mu_0 nQdr}{a^2}$.

We integrate out from the center to the edge of the disk and find $B=\int_0^a dB=\int_0^a\frac{\mu_0 nQdr}{a^2}=\frac{\mu_0 nQ}{a}$.

EVALUATE: The magnetic field is proportional to the total charge on the disk and to its rotation rate.

28.88. **IDENTIFY:** There are two parts to the magnetic field: that from the half loop and that from the straight wire segment running from $-a$ to a.

SET UP: Apply Eq.(28.14). Let the ϕ be the angle that locates dl around the ring.

EXECUTE: $B_x(ring)=\frac{1}{2}B_{\text{loop}}=-\frac{\mu_0 Ia^2}{4(x^2+a^2)^{3/2}}$.

$dB_y(ring)=dB\sin\theta\sin\phi=\frac{\mu_0 I}{4\pi}\frac{dl}{(x^2+a^2)}\frac{x}{(x^2+a^2)^{1/2}}\sin\phi=\frac{\mu_0 Iax\sin\phi\, d\phi}{4\pi(x^2+a^2)^{3/2}}$ and

$B_y(ring)=\int_0^\pi dB_y(ring)=\int_0^\pi\frac{\mu_0 Iax\sin\phi d\phi}{4\pi(x^2+a^2)^{3/2}}=\frac{\mu_0 Iax}{4\pi(x^2+a^2)^{3/2}}\cos\phi\Big|_0^\pi=-\frac{\mu_0 Iax}{2\pi(x^2+a^2)^{3/2}}$.

$B_y(rod)=\frac{\mu_0 Ia}{2\pi x(x^2+a^2)^{1/2}}$, using Eq. (28.8). The total field components are:

$B_x=-\frac{\mu_0 Ia^2}{4(x^2+a^2)^{3/2}}$ and $B_y=\frac{\mu_0 Ia}{2\pi x(x^2+a^2)^{1/2}}\left(1-\frac{x^2}{x^2+a^2}\right)=\frac{\mu_0 Ia^3}{2\pi x(x^2+a^2)^{3/2}}$.

EVALUATE: $B_y=-\frac{2}{\pi}\frac{a}{x}B_x$. B_y decreases faster than B_x as x increases. For very small x, $B_x=-\frac{\mu_0 I}{4a}$ and $B_y=\frac{\mu_0 I}{2\pi a}$.

In this limit B_x is the field at the center of curvature of a semicircle and B_y is the field of a long straight wire.