24

CAPACITANCE AND DIELECTRICS

24.1. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$

SET UP: $1\ \mu\text{F} = 10^{-6}\ \text{F}$

EXECUTE: $Q = CV_{ab} = (7.28\times10^{-6}\ \text{F})(25.0\ \text{V}) = 1.82\times10^{-4}\ \text{C} = 182\ \mu\text{C}$

EVALUATE: One plate has charge $+Q$ and the other has charge $-Q$.

24.2. **IDENTIFY** and **SET UP:** $C = \dfrac{\epsilon_0 A}{d}$, $C = \dfrac{Q}{V}$ and $V = Ed$.

(a) $C = \epsilon_0 \dfrac{A}{d} = \epsilon_0 \dfrac{0.00122\ \text{m}^2}{0.00328\ \text{m}} = 3.29\ \text{pF}$

(b) $V = \dfrac{Q}{C} = \dfrac{4.35\times10^{-8}\ \text{C}}{3.29\times10^{-12}\ \text{F}} = 13.2\ \text{kV}$

(c) $E = \dfrac{V}{d} = \dfrac{13.2\times10^{3}\ \text{V}}{0.00328\ \text{m}} = 4.02\times10^{6}\ \text{V/m}$

EVALUATE: The electric field is uniform between the plates, at points that aren't close to the edges.

24.3. **IDENTIFY** and **SET UP:** It is a parallel-plate air capacitor, so we can apply the equations of Sections 24.1.

EXECUTE: **(a)** $C = \dfrac{Q}{V_{ab}}$ so $V_{ab} = \dfrac{Q}{C} = \dfrac{0.148\times10^{-6}\ \text{C}}{245\times10^{-12}\ \text{F}} = 604\ \text{V}$

(b) $C = \dfrac{\epsilon_0 A}{d}$ so $A = \dfrac{Cd}{\epsilon_0} = \dfrac{(245\times10^{-12}\ \text{F})(0.328\times10^{-3}\ \text{m})}{8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 9.08\times10^{-3}\ \text{m}^2 = 90.8\ \text{cm}^2$

(c) $V_{ab} = Ed$ so $E = \dfrac{V_{ab}}{d} = \dfrac{604\ \text{V}}{0.328\times10^{-3}\ \text{m}} = 1.84\times10^{6}\ \text{V/m}$

(d) $E = \dfrac{\sigma}{\epsilon_0}$ so $\sigma = E\epsilon_0 = (1.84\times10^{6}\ \text{V/m})(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 1.63\times10^{-5}\ \text{C/m}^2$

EVALUATE: We could also calculate σ directly as *Q/A*. $\sigma = \dfrac{Q}{A} = \dfrac{0.148\times10^{-6}\ \text{C}}{9.08\times10^{-3}\ \text{m}^2} = 1.63\times10^{-5}\ \text{C/m}^2$, which checks.

24.4. **IDENTIFY:** $C = \epsilon_0 \dfrac{A}{d}$ when there is air between the plates.

SET UP: $A = (3.0\times10^{-2}\ \text{m})^2$ is the area of each plate.

EXECUTE: $C = \dfrac{(8.854\times10^{-12}\ \text{F/m})(3.0\times10^{-2}\ \text{m})^2}{5.0\times10^{-3}\ \text{m}} = 1.59\times10^{-12}\ \text{F} = 1.59\ \text{pF}$

EVALUATE: *C* increases when *A* increases and *C* increases when *d* decreases.

24.5. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\epsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, $V_{ab} = 12.0\ \text{V}$.

EXECUTE: **(a)** $Q = CV_{ab} = (10.0\times10^{-6}\ \text{F})(12.0\ \text{V}) = 1.20\times10^{-4}\ \text{C} = 120\ \mu\text{C}$

(b) When *d* is doubled *C* is halved, so *Q* is halved. $Q = 60\ \mu\text{C}$.

(c) If *r* is doubled, *A* increases by a factor of 4. *C* increases by a factor of 4 and *Q* increases by a factor of 4. $Q = 480\ \mu\text{C}$.

EVALUATE: When the plates are moved apart, less charge on the plates is required to produce the same potential difference. With the separation of the plates constant, the electric field must remain constant to produce the same potential difference. The electric field depends on the surface charge density, σ. To produce the same σ, more charge is required when the area increases.

24.6. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\epsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, enough charge flows onto the plates to make $V_{ab} = 12.0\text{ V}$.

EXECUTE: **(a)** 12.0 V

(b) (i) When d is doubled, C is halved. $V_{ab} = \dfrac{Q}{C}$ and Q is constant, so V doubles. $V = 24.0\text{ V}$.

(ii) When r is doubled, A increases by a factor of 4. V decreases by a factor of 4 and $V = 3.0\text{ V}$.

EVALUATE: The electric field between the plates is $E = Q/\epsilon_0 A$. $V_{ab} = Ed$. When d is doubled E is unchanged and V doubles. When A is increased by a factor of 4, E decreases by a factor of 4 so V decreases by a factor of 4.

24.7. **IDENTIFY:** $C = \dfrac{\epsilon_0 A}{d}$. Solve for d.

SET UP: Estimate $r = 1.0\text{ cm}$. $A = \pi r^2$.

EXECUTE: $C = \dfrac{\epsilon_0 A}{d}$ so $d = \dfrac{\epsilon_0 \pi r^2}{C} = \dfrac{\epsilon_0 \pi (0.010\text{ m})^2}{1.00\times 10^{-12}\text{ F}} = 2.8\text{ mm}$.

EVALUATE: The separation between the pennies is nearly a factor of 10 smaller than the diameter of a penny, so it is a reasonable approximation to treat them as infinite sheets.

24.8. **INCREASE:** $C = \dfrac{Q}{V_{ab}}$. $V_{ab} = Ed$. $C = \dfrac{\epsilon_0 A}{d}$.

SET UP: We want $E = 1.00\times 10^4\text{ N/C}$ when $V = 100\text{ V}$.

EXECUTE: **(a)** $d = \dfrac{V_{ab}}{E} = \dfrac{1.00\times 10^2\text{ V}}{1.00\times 10^4\text{ N/C}} = 1.00\times 10^{-2}\text{ m} = 1.00\text{ cm}$.

$A = \dfrac{Cd}{\epsilon_0} = \dfrac{(5.00\times 10^{-12}\text{ F})(1.00\times 10^{-2}\text{ m})}{8.854\times 10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2)} = 5.65\times 10^{-3}\text{ m}^2$. $A = \pi r^2$ so $r = \sqrt{\dfrac{A}{\pi}} = 4.24\times 10^{-2}\text{ m} = 4.24\text{ cm}$.

(b) $Q = CV_{ab} = (5.00\times 10^{-12}\text{ F})(1.00\times 10^2\text{ V}) = 5.00\times 10^{-10}\text{ C} = 500\text{ pC}$

EVALUATE: $C = \dfrac{\epsilon_0 A}{d}$. We could have a larger d, along with a larger A, and still achieve the required C without exceeding the maximum allowed E.

24.9. **IDENTIFY:** Apply the results of Example 24.4. $C = Q/V$.

SET UP: $r_a = 0.50\text{ mm}$, $r_b = 5.00\text{ mm}$

EXECUTE: **(a)** $C = \dfrac{L2\pi\epsilon_0}{\ln(r_b/r_a)} = \dfrac{(0.180\text{ m})2\pi\epsilon_0}{\ln(5.00/0.50)} = 4.35\times 10^{-12}\text{ F}$.

(b) $V = Q/C = (10.0\times 10^{-12}\text{ C})/(4.35\times 10^{-12}\text{ F}) = 2.30\text{ V}$

EVALUATE: $\dfrac{C}{L} = 24.2\text{ pF}$. This value is similar to those in Example 24.4. The capacitance is determined entirely by the dimensions of the cylinders.

24.10. **IDENTIFY:** Capacitance depends on the geometry of the object.

(a) SET UP: The capacitance of a cylindrical capacitor is $C = \dfrac{2\pi\epsilon_0 L}{\ln\left(r_b/r_a\right)}$. Solving for r_b gives $r_b = r_a e^{2\pi\epsilon_0 L/C}$.

EXECUTE: Substituting in the numbers for the exponent gives

$$\frac{2\pi\left(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)(0.120\text{ m})}{3.67\times 10^{-11}\text{ F}} = 0.182$$

Now use this value to calculate r_b: $r_b = r_a e^{0.182} = (0.250\text{ cm})e^{0.182} = 0.300\text{ cm}$

(b) SET UP: For any capacitor, $C = Q/V$ and $\lambda = Q/L$. Combining these equations and substituting the numbers gives $\lambda = Q/L = CV/L$.

EXECUTE: Numerically we get

$$\lambda = \frac{CV}{L} = \frac{\left(3.67\times 10^{-11}\text{ F}\right)\left(125\text{ V}\right)}{0.120\text{ m}} = 3.82\times 10^{-8}\text{ C/m} = 38.2\text{ nC/m}$$

EVALUATE: The distance between the surfaces of the two cylinders would be only 0.050 cm, which is just 0.50 mm. These cylinders would have to be carefully constructed.

24.11. **IDENTIFY** and **SET UP:** Use the expression for C/L derived in Example 24.4. Then use Eq.(24.1) to calculate Q.

EXECUTE: **(a)** From Example 24.4, $\dfrac{C}{L}=\dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$

$$\frac{C}{L}=\frac{2\pi\left(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}{\ln(3.5\ \text{mm}/1.5\ \text{mm})}=6.57\times10^{-11}\ \text{F/m}=66\ \text{pF/m}$$

(b) $C=\left(6.57\times10^{-11}\ \text{F/m}\right)(2.8\ \text{m})=1.84\times10^{-10}\ \text{F}$.

$$Q=CV=\left(1.84\times10^{-10}\ \text{F}\right)\left(350\times10^{-3}\ \text{V}\right)=6.4\times10^{-11}\ \text{C}=64\ \text{pC}$$

The conductor at higher potential has the positive charge, so there is +64 pC on the inner conductor and -64 pC on the outer conductor.

EVALUATE: C depends only on the dimensions of the capacitor. Q and V are proportional.

24.12. **IDENTIFY:** Apply the results of Example 24.3. $C=Q/V$.

SET UP: $r_a=15.0$ cm. Solve for r_b.

EXECUTE: **(a)** For two concentric spherical shells, the capacitance is $C=\dfrac{1}{k}\left(\dfrac{r_ar_b}{r_b-r_a}\right)$. $kCr_b-kCr_a=r_ar_b$ and

$$r_b=\frac{kCr_a}{kC-r_a}=\frac{k(116\times10^{-12}\ \text{F})(0.150\ \text{m})}{k(116\times10^{-12}\ \text{F})-0.150\ \text{m}}=0.175\ \text{m}.$$

(b) $V=220$ V and $Q=CV=(116\times10^{-12}\ \text{F})(220\ \text{V})=2.55\times10^{-8}\ \text{C}$.

EVALUATE: A parallel-plate capacitor with $A=4\pi r_ar_b=0.33\ \text{m}^2$ and $d=r_b-r_a=2.5\times10^{-2}$ m has $C=\dfrac{\epsilon_0A}{d}=117$ pF, in excellent agreement with the value of C for the spherical capacitor.

24.13. **IDENTIFY:** We can use the definition of capacitance to find the capacitance of the capacitor, and then relate the capacitance to geometry to find the inner radius.

(a) SET UP: By the definition of capacitance, $C = Q/V$.

EXECUTE: $C=\dfrac{Q}{V}=\dfrac{3.30\times10^{-9}\ \text{C}}{2.20\times10^{2}\ \text{V}}=1.50\times10^{-11}\ \text{F}=15.0\ \text{pF}$

(b) SET UP: The capacitance of a spherical capacitor is $C=4\pi\epsilon_0\dfrac{r_ar_b}{r_b-r_a}$.

EXECUTE: Solve for r_a and evaluate using C = 15.0 pF and r_b = 4.00 cm, giving r_a = 3.09 cm.

(c) SET UP: We can treat the inner sphere as a point-charge located at its center and use Coulomb's law, $E=\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$.

EXECUTE: $E=\dfrac{\left(9.00\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.30\times10^{-9}\ \text{C}\right)}{(0.0309\ \text{m})^2}=3.12\times10^{4}\ \text{N/C}$

EVALUATE: Outside the capacitor, the electric field is zero because the charges on the spheres are equal in magnitude but opposite in sign.

24.14. **IDENTIFY:** The capacitors between b and c are in parallel. This combination is in series with the 15 pF capacitor.

SET UP: Let $C_1=15$ pF, $C_2=9.0$ pF and $C_3=11$ pF.

EXECUTE: **(a)** For capacitors in parallel, $C_{\text{eq}}=C_1+C_2+\cdots$ so $C_{23}=C_2+C_3=20$ pF

(b) $C_1=15$ pF is in series with $C_{23}=20$ pF. For capacitors in series, $\dfrac{1}{C_{\text{eq}}}=\dfrac{1}{C_1}+\dfrac{1}{C_2}+\cdots$ so $\dfrac{1}{C_{123}}=\dfrac{1}{C_1}+\dfrac{1}{C_{23}}$ and

$$C_{123}=\frac{C_1C_{23}}{C_1+C_{23}}=\frac{(15\ \text{pF})(20\ \text{pF})}{15\ \text{pF}+20\ \text{pF}}=8.6\ \text{pF}.$$

EVALUATE: For capacitors in parallel the equivalent capacitance is larger than any of the individual capacitors. For capacitors in series the equivalent capacitance is smaller than any of the individual capacitors.

24.15. **IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

SET UP: Do parts **(a)** and **(b)** together. The capacitor network is drawn in Figure 24.15a.

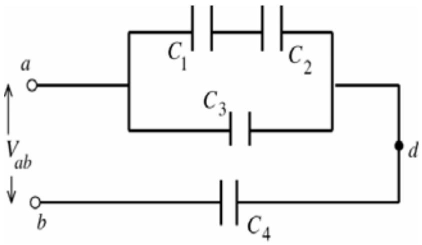
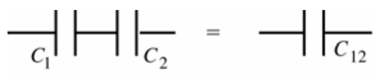


Figure 24.15a

$C_1 = C_2 = C_3 = C_4 = 400\ \mu\text{F}$
$V_{ab} = 28.0\ \text{V}$

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_1 and C_2 are in series and are equivalent to C_{12} (Figure 24.15b).

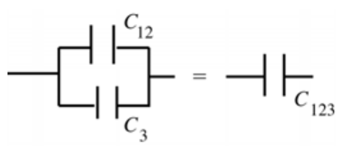


$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Figure 24.15b

$$C_{12} = \frac{C_1C_2}{C_1 + C_2} = \frac{(4.00\times10^{-6}\ \text{F})(4.00\times10^{-6}\ \text{F})}{4.00\times10^{-6}\ \text{F} + 4.00\times10^{-6}\ \text{F}} = 2.00\times10^{-6}\ \text{F}$$

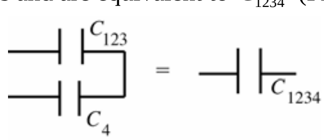
C_{12} and C_3 are in parallel and are equivalent to C_{123} (Figure 24.15c).



$C_{123} = C_{12} + C_3$
$C_{123} = 2.00\times10^{-6}\ \text{F} + 4.00\times10^{-6}\ \text{F}$
$C_{123} = 6.00\times10^{-6}\ \text{F}$

Figure 24.15c

C_{123} and C_4 are in series and are equivalent to C_{1234} (Figure 24.15d).

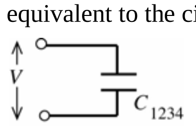


$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4}$$

Figure 24.15d

$$C_{1234} = \frac{C_{123}C_4}{C_{123} + C_4} = \frac{(6.00\times10^{-6}\ \text{F})(4.00\times10^{-6}\ \text{F})}{6.00\times10^{-6}\ \text{F} + 4.00\times10^{-6}\ \text{F}} = 2.40\times10^{-6}\ \text{F}$$

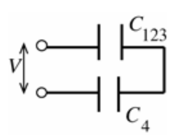
The circuit is equivalent to the circuit shown in Figure 24.15e.



$V_{1234} = V = 28.0\ \text{V}$
$Q_{1234} = C_{1234}V = (2.40\times10^{-6}\ \text{F})(28.0\ \text{V}) = 67.2\ \mu\text{C}$

Figure 24.15e

Now build back up the original circuit, step by step. C_{1234} represents C_{123} and C_4 in series (Figure 24.15f).



$Q_{123} = Q_4 - Q_{1234} = 67.2\ \mu\text{C}$
(charge same for capacitors in series)

Figure 24.15f

Then $V_{123} = \dfrac{Q_{123}}{C_{123}} = \dfrac{67.2\ \mu\text{C}}{6.00\ \mu\text{F}} = 11.2\ \text{V}$

$$V_4 = \frac{Q_4}{C_4} = \frac{67.2\ \mu\text{C}}{4.00\ \mu\text{F}} = 16.8\ \text{V}$$

Note that $V_4 + V_{123} = 16.8\ \text{V} + 11.2\ \text{V} = 28.0\ \text{V}$, as it should.

Next consider the circuit as written in Figure 24.15g.

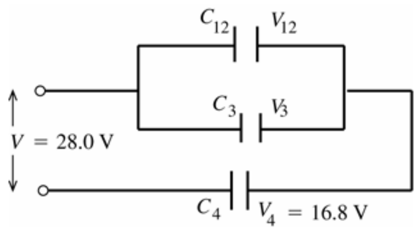


Figure 24.15g

$V_3 = V_{12} = 28.0 \text{ V} - V_4$

$V_3 = 11.2 \text{ V}$

$Q_3 = C_3V_3 = (4.00\ \mu\text{F})(11.2 \text{ V})$

$Q_3 = 44.8\ \mu\text{C}$

$Q_{12} - C_{12}V_{12} = (2.00\ \mu\text{F})(11.2 \text{ V})$

$Q_{12} = 22.4\ \mu\text{C}$

Finally, consider the original circuit, as shown in Figure 24.15h.

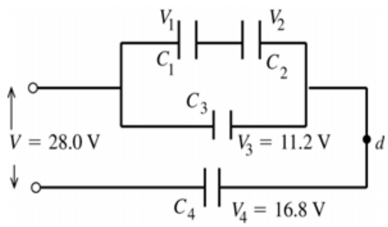


Figure 24.15h

$Q_1 = Q_2 = Q_{12} = 22.4\ \mu\text{C}$

(charge same for capacitors in series)

$V_1 = \dfrac{Q_1}{C_1} = \dfrac{22.4\ \mu\text{C}}{4.00\ \mu\text{F}} = 5.6 \text{ V}$

$V_2 = \dfrac{Q_2}{C_2} = \dfrac{22.4\ \mu\text{C}}{4.00\ \mu\text{F}} = 5.6 \text{ V}$

Note that $V_1 + V_2 = 11.2 \text{ V}$, which equals V_3 as it should.

Summary: $Q_1 = 22.4\ \mu\text{C},\ V_1 = 5.6 \text{ V}$

$Q_2 = 22.4\ \mu\text{C},\ V_2 = 5.6 \text{ V}$

$Q_3 = 44.8\ \mu\text{C},\ V_3 = 11.2 \text{ V}$

$Q_4 = 67.2\ \mu\text{C},\ V_4 = 16.8 \text{ V}$

(c) $V_{ad} = V_3 = 11.2 \text{ V}$

EVALUATE: $V_1 + V_2 + V_4 = V$, or $V_3 + V_4 = V$. $Q_1 = Q_2$, $Q_1 + Q_3 = Q_4$ and $Q_4 = Q_{1234}$.

24.16. **IDENTIFY:** The two capacitors are in series. The equivalent capacitance is given by $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

SET UP: For capacitors in series the charges are the same and the potentials add to give the potential across the network.

EXECUTE: **(a)** $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{(3.0\times10^{-6}\text{ F})} + \dfrac{1}{(5.0\times10^{-6}\text{ F})} = 5.33\times10^5 \text{ F}^{-1}$. $C_{\text{eq}} = 1.88\times10^{-6}$ F. Then

$Q = VC_{\text{eq}} = (52.0 \text{ V})(1.88\times10^{-6}\text{ F}) = 9.75\times10^{-5}$ C. Each capacitor has charge 9.75×10^{-5} C.

(b) $V_1 = Q/C_1 = 9.75\times10^{-5} \text{ C}/3.0\times10^{-6} \text{ F} = 32.5 \text{ V}$.

$V_2 = Q/C_2 = 9.75\times10^{-5} \text{ C}/5.0\times10^{-6} \text{ F} = 19.5 \text{ V}$.

EVALUATE: $V_1 + V_2 = 52.0$ V, which is equal to the applied potential V_{ab}. The capacitor with the smaller *C* has the larger *V*.

24.17. **IDENTIFY:** The two capacitors are in parallel so the voltage is the same on each, and equal to the applied voltage V_{ab}.

SET UP: Do parts **(a)** and **(b)** together. The network is sketched in Figure 24.17.

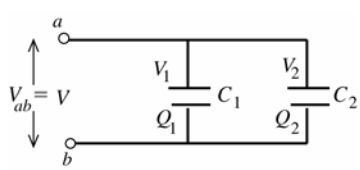


Figure 24.17

EXECUTE: $V_1 = V_2 = V$

$V_1 = 52.0 \text{ V}$

$V_2 = 52.0 \text{ V}$

$C = Q/V$ so $Q = CV$

$Q_1 = C_1V_1 = (3.00\ \mu\text{F})(52.0 \text{ V}) = 156\ \mu\text{C}$. $Q_2 = C_2V_2 = (5.00\ \mu\text{F})(52.0 \text{ V}) = 260\ \mu\text{C}$.

EVALUATE: To produce the same potential difference, the capacitor with the larger *C* has the larger *Q*.

24.18. **IDENTIFY:** For capacitors in parallel the voltages are the same and the charges add. For capacitors in series, the charges are the same and the voltages add. $C = Q/V$.

SET UP: C_1 and C_2 are in parallel and C_3 is in series with the parallel combination of C_1 and C_2.

EXECUTE: **(a)** C_1 and C_2 are in parallel and so have the same potential across them:

$V_1 = V_2 = \dfrac{Q_2}{C_2} = \dfrac{40.0\times10^{-6}\text{ C}}{3.00\times10^{-6}\text{ F}} = 13.33\text{ V}$. Therefore, $Q_1 = V_1C_1 = (13.33\text{ V})(3.00\times10^{-6}\text{ F}) = 80.0\times10^{-6}\text{ C}$. Since C_3 is in series with the parallel combination of C_1 and C_2, its charge must be equal to their combined charge: $C_3 = 40.0\times10^{-6}\text{ C} + 80.0\times10^{-6}\text{ C} = 120.0\times10^{-6}\text{ C}$.

(b) The total capacitance is found from $\dfrac{1}{C_{\text{tot}}} = \dfrac{1}{C_{12}} + \dfrac{1}{C_3} = \dfrac{1}{9.00\times10^{-6}\text{ F}} + \dfrac{1}{5.00\times10^{-6}\text{ F}}$ and $C_{\text{tot}} = 3.21\ \mu\text{F}$.

$V_{ab} = \dfrac{Q_{\text{tot}}}{C_{\text{tot}}} = \dfrac{120.0\times10^{-6}\text{ C}}{3.21\times10^{-6}\text{ F}} = 37.4\text{ V}$.

EVALUATE: $V_3 = \dfrac{Q_3}{C_3} = \dfrac{120.0\times10^{-6}\text{ C}}{5.00\times10^{-6}\text{ F}} = 24.0\text{ V}$. $V_{ab} = V_1 + V_3$.

24.19. **IDENTIFY** and **SET UP:** Use the rules for *V* for capacitors in series and parallel: for capacitors in parallel the voltages are the same and for capacitors in series the voltages add.

EXECUTE: $V_1 = Q_1/C_1 = (150\ \mu\text{C})/(3.00\ \mu\text{F}) = 50\text{ V}$

C_1 and C_2 are in parallel, so $V_2 = 50\text{ V}$

$V_3 = 120\text{ V} - V_1 = 70\text{ V}$

EVALUATE: Now that we know the voltages, we could also calculate *Q* for the other two capacitors.

24.20. **IDENTIFY** and **SET UP:** $C = \dfrac{\epsilon_0 A}{d}$. For two capacitors in series, $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

EXECUTE: $C_{\text{eq}} = \left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)^{-1} = \left(\dfrac{d_1}{\epsilon_0 A} + \dfrac{d_2}{\epsilon_0 A}\right)^{-1} = \dfrac{\epsilon_0 A}{d_1 + d_2}$. This shows that the combined capacitance for two capacitors in series is the same as that for a capacitor of area *A* and separation $(d_1 + d_2)$.

EVALUATE: C_{eq} is smaller than either C_1 or C_2.

24.21. **IDENTIFY** and **SET UP:** $C = \dfrac{\epsilon_0 A}{d}$. For two capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$.

EXECUTE: $C_{\text{eq}} = C_1 + C_2 = \dfrac{\epsilon_0 A_1}{d} + \dfrac{\epsilon_0 A_2}{d} = \dfrac{\epsilon_0 (A_1 + A_2)}{d}$. So the combined capacitance for two capacitors in parallel is that of a single capacitor of their combined area $(A_1 + A_2)$ and common plate separation *d*.

EVALUATE: C_{eq} is larger than either C_1 or C_2.

24.22. **IDENTIFY:** Simplify the network by replacing series and parallel combinations of capacitors by their equivalents.

SET UP: For capacitors in series the voltages add and the charges are the same; $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots$ For capacitors in parallel the voltages are the same and the charges add; $C_{\text{eq}} = C_1 + C_2 + \cdots$ $C = \dfrac{Q}{V}$.

EXECUTE: **(a)** The equivalent capacitance of the $5.0\ \mu\text{F}$ and $8.0\ \mu\text{F}$ capacitors in parallel is $13.0\ \mu\text{F}$. When these two capacitors are replaced by their equivalent we get the network sketched in Figure 24.22. The equivalent capacitance of these three capacitors in series is $3.47\ \mu\text{F}$.

(b) $Q_{\text{tot}} = C_{\text{tot}}V = (3.47\ \mu\text{F})(50.0\text{ V}) = 174\ \mu\text{C}$

(c) Q_{tot} is the same as *Q* for each of the capacitors in the series combination shown in Figure 24.22, so *Q* for each of the capacitors is $174\ \mu\text{C}$.

EVALUATE: The voltages across each capacitor in Figure 24.22 are $V_{10} = \dfrac{Q_{\text{tot}}}{C_{10}} = 17.4\text{ V}$, $V_{13} = \dfrac{Q_{\text{tot}}}{C_{13}} = 13.4\text{ V}$ and $V_9 = \dfrac{Q_{\text{tot}}}{C_9} = 19.3\text{ V}$. $V_{10} + V_{13} + V_9 = 17.4\text{ V} + 13.4\text{ V} + 19.3\text{ V} = 50.1\text{ V}$. The sum of the voltages equals the applied voltage, apart from a small difference due to rounding.

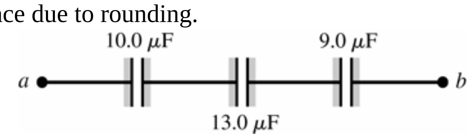


Figure 24.22

24.23. **IDENTIFY:** Refer to Figure 24.10b in the textbook. For capacitors in parallel, $C_{eq} = C_1 + C_2 + \cdots$. For capacitors in series, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$.

SET UP: The $11\ \mu F$, $4\ \mu F$ and replacement capacitor are in parallel and this combination is in series with the $9.0\ \mu F$ capacitor.

EXECUTE: $\frac{1}{C_{eq}} = \frac{1}{8.0\ \mu F} = \left(\frac{1}{(11 + 4.0 + x)\ \mu F} + \frac{1}{9.0\ \mu F}\right)$. $(15 + x)\ \mu F = 72\ \mu F$ and $x = 57\ \mu F$.

EVALUATE: Increasing the capacitance of the one capacitor by a large amount makes a small increase in the equivalent capacitance of the network.

24.24. **IDENTIFY:** Apply $C = Q/V$. $C = \frac{\epsilon_0 A}{d}$. The work done to double the separation equals the change in the stored energy.

SET UP: $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$.

EXECUTE: **(a)** $V = Q/C = (2.55\ \mu C)/(920 \times 10^{-12}\ F) = 2770\ V$

(b) $C = \frac{\epsilon_0 A}{d}$ says that since the charge is kept constant while the separation doubles, that means that the capacitance halves and the voltage doubles to 5540 V.

(c) $U = \frac{Q^2}{2C} = \frac{(2.55 \times 10^{-6}\ C)^2}{2(920 \times 10^{-12}\ F)} = 3.53 \times 10^{-3}\ J$. When if the separation is doubled while Q stays the same, the capacitance halves, and the energy stored doubles. So the amount of work done to move the plates equals the difference in energy stored in the capacitor, which is 3.53×10^{-3} J.

EVALUATE: The oppositely charged plates attract each other and positive work must be done by an external force to pull them farther apart.

24.25. **IDENTIFY** and **SET UP:** The energy density is given by Eq.(24.11): $u = \frac{1}{2}\epsilon_0 E^2$. Use $V = Ed$ to solve for E.

EXECUTE: Calculate $E: E = \frac{V}{d} = \frac{400\ V}{5.00 \times 10^{-3}\ m} = 8.00 \times 10^4\ V/m$.

Then $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\left(8.854 \times 10^{-12}\ C^2/N \cdot m^2\right)\left(8.00 \times 10^4\ V/m\right)^2 = 0.0283\ J/m^3$

EVALUATE: E is smaller than the value in Example 24.8 by about a factor of 6 so u is smaller by about a factor of $6^2 = 36$.

24.26. **IDENTIFY:** $C = \frac{Q}{V_{ab}}$. $C = \frac{\epsilon_0 A}{d}$. $V_{ab} = Ed$. The stored energy is $\frac{1}{2}QV$.

SET UP: $d = 1.50 \times 10^{-3}\ m$. $1\ \mu C = 10^{-6}\ C$

EXECUTE: **(a)** $C = \frac{0.0180 \times 10^{-6}\ C}{200\ V} = 9.00 \times 10^{-11}\ F = 90.0\ pF$

(b) $C = \frac{\epsilon_0 A}{d}$ so $A = \frac{Cd}{\epsilon_0} = \frac{(9.00 \times 10^{-11}\ F)(1.50 \times 10^{-3}\ m)}{8.854 \times 10^{-12}\ C^2/(N \cdot m^2)} = 0.0152\ m^2$.

(c) $V = Ed = (3.0 \times 10^6\ V/m)(1.50 \times 10^{-3}\ m) = 4.5 \times 10^3\ V$

(d) Energy $= \frac{1}{2}QV = \frac{1}{2}(0.0180 \times 10^{-6}\ C)(200\ V) = 1.80 \times 10^{-6}\ J = 1.80\ \mu J$

EVALUATE: We could also calculate the stored energy as $\frac{Q^2}{2C} = \frac{(0.0180 \times 10^{-6}\ C)^2}{2(9.00 \times 10^{-11}\ F)} = 1.80\ \mu J$.

24.27. **IDENTIFY:** The energy stored in a charged capacitor is $\frac{1}{2}CV^2$.

SET UP: $1\ \mu F = 10^{-6}\ F$

EXECUTE: $\frac{1}{2}CV^2 = \frac{1}{2}(450 \times 10^{-6}\ F)(295\ V)^2 = 19.6\ J$

EVALUATE: Thermal energy is generated in the wire at the rate I^2R, where I is the current in the wire. When the capacitor discharges there is a flow of charge that corresponds to current in the wire.

24.28. **IDENTIFY:** After the two capacitors are connected they must have equal potential difference, and their combined charge must add up to the original charge.

SET UP: $C = Q/V$. The stored energy is $U = \dfrac{Q^2}{2C} = \dfrac{1}{2}CV^2$

EXECUTE: **(a)** $Q = CV_0$.

(b) $V = \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2}$ and also $Q_1 + Q_2 = Q = CV_0$. $C_1 = C$ and $C_2 = \dfrac{C}{2}$ so $\dfrac{Q_1}{C} = \dfrac{Q_2}{(C/2)}$ and $Q_2 = \dfrac{Q_1}{2}$. $Q = \dfrac{3}{2}Q_1$.

$Q_1 = \dfrac{2}{3}Q$ and $V = \dfrac{Q_1}{C} = \dfrac{2}{3}\dfrac{Q}{C} = \dfrac{2}{3}V_0$.

(c) $U = \dfrac{1}{2}\left(\dfrac{Q_1^2}{C_1} + \dfrac{Q_2^2}{C_2}\right) = \dfrac{1}{2}\left[\dfrac{(\frac{2}{3}Q)^2}{C} + \dfrac{2(\frac{1}{3}Q)^2}{C}\right] = \dfrac{1}{3}\dfrac{Q^2}{C} = \dfrac{1}{3}CV_0^2$

(d) The original U was $U = \frac{1}{2}CV_0^2$, so $\Delta U = -\dfrac{1}{6}CV_0^2$.

(e) Thermal energy of capacitor, wires, etc., and electromagnetic radiation.

EVALUATE: The original charge of the charged capacitor must distribute between the two capacitors to make the potential the same across each capacitor. The voltage V for each after they are connected is less than the original voltage V_0 of the charged capacitor.

24.29. **IDENTIFY** and **SET UP:** Combine Eqs. (24.9) and (24.2) to write the stored energy in terms of the separation between the plates.

EXECUTE: **(a)** $U = \dfrac{Q^2}{2C}$; $C = \dfrac{\epsilon_0 A}{x}$ so $U = \dfrac{xQ^2}{2\epsilon_0 A}$

(b) $x \to x + dx$ gives $U = \dfrac{(x+dx)Q^2}{2\epsilon_0 A}$

$$dU = \frac{(x+dx)Q^2}{2\epsilon_0 A} - \frac{xQ^2}{2\epsilon_0 A} = \left(\frac{Q^2}{2\epsilon_0 A}\right)dx$$

(c) $dW = F\ dx = dU$, so $F = \dfrac{Q^2}{2\epsilon_0 A}$

(d) EVALUATE: The capacitor plates and the field between the plates are shown in Figure 24.29a.

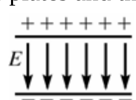


Figure 24.29a

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$
$$F = \tfrac{1}{2}QE, \text{ not } QE$$

The reason for the difference is that E is the field due to <u>both</u> plates. If we consider the positive plate only and calculate its electric field using Gauss's law (Figure 24.29b):

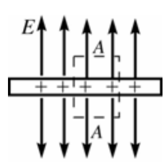


Figure 24.29b

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$
$$2EA = \frac{\sigma A}{\epsilon_0}$$
$$E = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2\epsilon_0 A}$$

The force this field exerts on the other plate, that has charge $-Q$, is $F = \dfrac{Q^2}{2\epsilon_0 A}$.

24.30. **IDENTIFY:** $C = \dfrac{\varepsilon_0 A}{d}$. The stored energy can be expressed either as $\dfrac{Q^2}{2C}$ or as $\dfrac{CV^2}{2}$, whichever is more convenient for the calculation.

SET UP: Since d is halved, C doubles.

EXECUTE: **(a)** If the separation distance is halved while the charge is kept fixed, then the capacitance increases and the stored energy, which was 8.38 J, decreases since $U = Q^2/2C$. Therefore the new energy is 4.19 J.

(b) If the voltage is kept fixed while the separation is decreased by one half, then the doubling of the capacitance leads to a doubling of the stored energy to 16.8 J, using $U = CV^2/2$, when V is held constant throughout.

EVALUATE: When the capacitor is disconnected, the stored energy decreases because of the positive work done by the attractive force between the plates. When the capacitor remains connected to the battery, $Q = CV$ tells us that the charge on the plates increases. The increased stored energy comes from the battery when it puts more charge onto the plates.

24.31. **IDENTIFY** and **SET UP:** $C = \dfrac{Q}{V}$. $U = \frac{1}{2}CV^2$.

EXECUTE: **(a)** $Q = CV = (5.0\ \mu\text{F})(1.5\ \text{V}) = 7.5\ \mu\text{C}$. $U = \frac{1}{2}CV^2 = \frac{1}{2}(5.0\ \mu\text{F})(1.5\ \text{V})^2 = 5.62\ \mu\text{J}$

(b) $U = \frac{1}{2}CV^2 = \frac{1}{2}C(Q/C)^2 = Q^2/2C$. $Q = \sqrt{2CU} = \sqrt{2(5.0\times10^{-6}\ \text{F})(1.0\ \text{J})} = 3.2\times10^{-3}\ \text{C}$.

$V = \dfrac{Q}{C} = \dfrac{3.2\times10^{-3}\ \text{C}}{5.0\times10^{-6}\ \text{F}} = 640\ \text{V}$.

EVALUATE: The stored energy is proportional to Q^2 and to V^2.

24.32. **IDENTIFY:** The two capacitors are in series. $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots$ $C = \dfrac{Q}{V}$. $U = \frac{1}{2}CV^2$.

SET UP: For capacitors in series the voltages add and the charges are the same.

EXECUTE: **(a)** $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$ so $C_{\text{eq}} = \dfrac{C_1C_2}{C_1 + C_2} = \dfrac{(150\ \text{nF})(120\ \text{nF})}{150\ \text{nF} + 120\ \text{nF}} = 66.7\ \text{nF}$.

$Q = CV = (66.7\ \text{nF})(36\ \text{V}) = 2.4\times10^{-6}\ \text{C} = 2.4\ \mu\text{C}$

(b) $Q = 2.4\ \mu\text{C}$ for each capacitor.

(c) $U = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}(66.7\times10^{-9}\ \text{F})(36\ \text{V})^2 = 43.2\ \mu\text{J}$

(d) We know C and Q for each capacitor so rewrite U in terms of these quantities. $U = \frac{1}{2}CV^2 = \frac{1}{2}C(Q/C)^2 = Q^2/2C$

150 nF: $U = \dfrac{(2.4\times10^{-6}\ \text{C})^2}{2(150\times10^{-9}\ \text{F})} = 19.2\ \mu\text{J}$; 120 nF: $U = \dfrac{(2.4\times10^{-6}\ \text{C})^2}{2(120\times10^{-9}\ \text{F})} = 24.0\ \mu\text{J}$

Note that $19.2\ \mu\text{J} + 24.0\ \mu\text{J} = 43.2\ \mu\text{J}$, the total stored energy calculated in part (c).

(e) 150 nF: $V = \dfrac{Q}{C} = \dfrac{2.4\times10^{-6}\ \text{C}}{150\times10^{-9}\ \text{F}} = 16\ \text{V}$; 120 nF: $V = \dfrac{Q}{C} = \dfrac{2.4\times10^{-6}\ \text{C}}{120\times10^{-9}\ \text{F}} = 20\ \text{V}$

Note that these two voltages sum to 36 V, the voltage applied across the network.

EVALUATE: Since Q is the same the capacitor with smaller C stores more energy ($U = Q^2/2C$) and has a larger voltage ($V = Q/C$).

24.33. **IDENTIFY:** The two capacitors are in parallel. $C_{\text{eq}} = C_1 + C_2$. $C = \dfrac{Q}{V}$. $U = \frac{1}{2}CV^2$.

SET UP: For capacitors in parallel, the voltages are the same and the charges add.

EXECUTE: **(a)** $C_{\text{eq}} = C_1 + C_2 = 35\ \text{nF} + 75\ \text{nF} = 110\ \text{nF}$. $Q_{\text{tot}} = C_{\text{eq}}V = (110\times10^{-9}\ \text{F})(220\ \text{V}) = 24.2\ \mu\text{C}$

(b) $V = 220\ \text{V}$ for each capacitor.

35 nF: $Q_{35} = C_{35}V = (35\times10^{-9}\ \text{F})(220\ \text{V}) = 7.7\ \mu\text{C}$; 75 nF: $Q_{75} = C_{75}V = (75\times10^{-9}\ \text{F})(220\ \text{V}) = 16.5\ \mu\text{C}$. Note that $Q_{35} + Q_{75} = Q_{\text{tot}}$.

(c) $U_{\text{tot}} = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}(110\times10^{-9}\ \text{F})(220\ \text{V})^2 = 2.66\ \text{mJ}$

(d) 35 nF: $U_{35} = \frac{1}{2}C_{35}V^2 = \frac{1}{2}(35\times10^{-9}\ \text{F})(220\ \text{V})^2 = 0.85\ \text{mJ}$;

75 nF: $U_{75} = \frac{1}{2}C_{75}V^2 = \frac{1}{2}(75\times10^{-9}\ \text{F})(220\ \text{V})^2 = 1.81\ \text{mJ}$. Since V is the same the capacitor with larger C stores more energy.

(e) 220 V for each capacitor.

EVALUATE: The capacitor with the larger C has the larger Q.

24.34. **IDENTIFY:** Capacitance depends on the geometry of the object.

(a) SET UP: The potential difference between the core and tube is $V = \dfrac{\lambda}{2\pi\epsilon_0}\ln\left(r_b/r_a\right)$. Solving for the linear charge density gives $\lambda = \dfrac{2\pi\epsilon_0 V}{\ln\left(r_b/r_a\right)} = \dfrac{4\pi\epsilon_0 V}{2\ln\left(r_b/r_a\right)}$.

EXECUTE: Using the given values gives $\lambda = \dfrac{6.00\,\text{V}}{2\left(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\ln\left(\dfrac{2.00}{1.20}\right)} = 6.53\times10^{-10}\ \text{C/m}$

(b) SET UP: $Q = \lambda L$

EXECUTE: $Q = (6.53\times10^{-10}\ \text{C/m})(0.350\ \text{m}) = 2.29\times10^{-10}\ \text{C}$

(c) SET UP: The definition of capacitance is $C = Q/V$.

EXECUTE: $C = \dfrac{2.29\times10^{10}\ \text{C}}{6.00\ \text{V}} = 3.81\times10^{-11}\ \text{F}$

(d) SET UP: The energy stored in a capacitor is $U = \frac{1}{2}CV^2$.

EXECUTE: $U = \frac{1}{2}(3.81\times10^{-11}\ \text{F})(6.00\ \text{V})^2 = 6.85\times10^{-10}\ \text{J}$

EVALUATE: The stored energy could be converted to heat or other forms of energy.

24.35. IDENTIFY: $U = \frac{1}{2}QV$. Solve for *Q*. $C = Q/V$.

SET UP: Example 24.4 shows that for a cylindrical capacitor, $\dfrac{C}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$.

EXECUTE: **(a)** $U = \frac{1}{2}QV$ gives $Q = \dfrac{2U}{V} = \dfrac{2(3.20\times10^{-9}\ \text{J})}{4.00\ \text{V}} = 1.60\times10^{-9}$ C.

(b) $\dfrac{C}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$. $\dfrac{r_b}{r_a} = \exp(2\pi\epsilon_0 L/C) = \exp(2\pi\epsilon_0 LV/Q) = \exp(2\pi\epsilon_0(15.0\ \text{m})(4.00\ \text{V})/(1.60\times10^{-9}\ \text{C})) = 8.05.$

The radius of the outer conductor is 8.05 times the radius of the inner conductor.

EVALUATE: When the ratio r_b/r_a increases, C/L decreases and less charge is stored for a given potential difference.

24.36. IDENTIFY: Apply Eq.(24.11).

SET UP: Example 24.3 shows that $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$ between the conducting shells and that $\dfrac{Q}{4\pi\epsilon_0} = \left(\dfrac{r_a r_b}{r_b - r_a}\right)V_{ab}$.

EXECUTE: $E = \left(\dfrac{r_a r_b}{r_b - r_a}\right)\dfrac{V_{ab}}{r^2} = \left(\dfrac{[0.125\ \text{m}][0.148\ \text{m}]}{0.148\ \text{m} - 0.125\ \text{m}}\right)\dfrac{120\ \text{V}}{r^2} = \dfrac{96.5\ \text{V}\cdot\text{m}}{r^2}$

(a) For $r = 0.126$ m, $E = 6.08\times10^3$ V/m. $u = \frac{1}{2}\epsilon_0 E^2 = 1.64\times10^{-4}\ \text{J/m}^3$.

(b) For $r = 0.147$ m, $E = 4.47\times10^3$ V/m. $u = \frac{1}{2}\epsilon_0 E^2 = 8.85\times10^{-5}\ \text{J/m}^3$.

EVALUATE: **(c)** No, the results of parts (a) and (b) show that the energy density is not uniform in the region between the plates. *E* decreases as *r* increases, so *u* decreases also.

24.37. IDENTIFY: Use the rules for series and for parallel capacitors to express the voltage for each capacitor in terms of the applied voltage. Express *U*, *Q*, and *E* in terms of the capacitor voltage.

SET UP: Le the applied voltage be *V*. Let each capacitor have capacitance *C*. $U = \frac{1}{2}CV^2$ for a single capacitor with voltage *V*.

EXECUTE: **(a) series**

Voltage across each capacitor is $V/2$. The total energy stored is $U_s = 2\left(\frac{1}{2}C[V/2]^2\right) = \frac{1}{4}CV^2$

parallel

Voltage across each capacitor is *V*. The total energy stored is $U_p = 2\left(\frac{1}{2}CV^2\right) = CV^2$

$U_p = 4U_s$

(b) $Q = CV$ for a single capacitor with voltage *V*. $Q_s = 2(C[V/2]) = CV$; $Q_p = 2(CV) = 2CV$; $Q_p = 2Q_s$

(c) $E = V/d$ for a capacitor with voltage *V*. $E_s = V/2d$; $E_p = V/d$; $E_p = 2E_s$

EVALUATE: The parallel combination stores more energy and more charge since the voltage for each capacitor is larger for parallel. More energy stored and larger voltage for parallel means larger electric field in the parallel case.

24.38. IDENTIFY: $V = Ed$ and $C = Q/V$. With the dielectric present, $C = KC_0$.

SET UP: $V = Ed$ holds both with and without the dielectric.

EXECUTE: **(a)** $V = Ed = (3.00\times10^4\ \text{V/m})(1.50\times10^{-3}\ \text{m}) = 45.0\ \text{V}$.

$Q = C_0 V = (5.00\times10^{-12}\ \text{F})(45.0\ \text{V}) = 2.25\times10^{-10}\ \text{C}$.

(b) With the dielectric, $C = KC_0 = (2.70)(5.00\ \text{pF}) = 13.5\ \text{pF}$. *V* is still 45.0 V, so

$Q = CV = (13.5\times10^{-12}\ \text{F})(45.0\ \text{V}) = 6.08\times10^{-10}\ \text{C}$.

EVALUATE: The presence of the dielectric increases the amount of charge that can be stored for a given potential difference and electric field between the plates. *Q* increases by a factor of *K*.

24.39. **IDENTIFY** and **SET UP:** Q is constant so we can apply Eq.(24.14). The charge density on each surface of the dielectric is given by Eq.(24.16).

EXECUTE: $E = \dfrac{E_0}{K}$ so $K = \dfrac{E_0}{E} = \dfrac{3.20\times10^5\ \text{V/m}}{2.50\times10^5\ \text{V/m}} = 1.28$

(a) $\sigma_i = \sigma(1-1/K)$

$\sigma = \epsilon_0 E_0 = (8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.20\times10^5\ \text{N/C}) = 2.833\times10^{-6}\ \text{C/m}^2$

$\sigma_i = (2.833\times10^{-6}\ \text{C/m}^2)(1-1/1.28) = 6.20\times10^{-7}\ \text{C/m}^2$

(b) As calculated above, $K = 1.28$.

EVALUATE: The surface charges on the dielectric produce an electric field that partially cancels the electric field produced by the charges on the capacitor plates.

24.40. **IDENTIFY:** Capacitance depends on geometry, and the introduction of a dielectric increases the capacitance.

SET UP: For a parallel-plate capacitor, $C = K\epsilon_0 A/d$.

EXECUTE: **(a)** Solving for d gives

$$d = \frac{K\epsilon_0 A}{C} = \frac{(3.0)(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.22\ \text{m})(0.28\ \text{m})}{1.0\times10^{-9}\ \text{F}} = 1.64\times10^{-3}\ \text{m} = 1.64\ \text{mm}.$$

Dividing this result by the thickness of a sheet of paper gives $\dfrac{1.64\ \text{mm}}{0.20\ \text{mm/sheet}} \approx 8\ \text{sheets}$.

(b) Solving for the area of the plates gives $A = \dfrac{Cd}{K\epsilon_0} = \dfrac{(1.0\times10^{-9}\ \text{F})(0.012\ \text{m})}{(3.0)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 0.45\ \text{m}^2$.

(c) Teflon has a smaller dielectric constant (2.1) than the posterboard, so she will need more area to achieve the same capacitance.

EVALUATE: The use of dielectric makes it possible to construct reasonable-sized capacitors since the dielectric increases the capacitance by a factor of K.

24.41. **IDENTIFY** and **SET UP:** For a parallel-plate capacitor with a dielectric we can use the equation $C = K\epsilon_0 A/d$. Minimum A means smallest possible d. d is limited by the requirement that E be less than $1.60\times10^7\ \text{V/m}$ when V is as large as 5500 V.

EXECUTE: $V = Ed$ so $d = \dfrac{V}{E} = \dfrac{5500\ \text{V}}{1.60\times10^7\ \text{V/m}} = 3.44\times10^{-4}\ \text{m}$

Then $A = \dfrac{Cd}{K\epsilon_0} = \dfrac{(1.25\times10^{-9}\ \text{F})(3.44\times10^{-4}\ \text{m})}{(3.60)(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 0.0135\ \text{m}^2.$

EVALUATE: The relation $V = Ed$ applies with or without a dielectric present. A would have to be larger if there were no dielectric.

24.42. **IDENTIFY** and **SET UP:** Adapt the derivation of Eq.(24.1) to the situation where a dielectric is present.

EXECUTE: Placing a dielectric between the plates just results in the replacement of ϵ for ϵ_0 in the derivation of Equation (24.20). One can follow exactly the procedure as shown for Equation (24.11).

EVALUATE: The presence of the dielectric increases the energy density for a given electric field.

24.43. **IDENTIFY:** The permittivity ϵ of a material is related to its dielectric constant by $\epsilon = K\epsilon_0$. The maximum voltage is related to the maximum possible electric field before dielectric breakdown by $V_{\text{max}} = E_{\text{max}} d$. $E = \dfrac{E_0}{K} = \dfrac{\sigma}{K\epsilon_0}$, where σ is the surface charge density on each plate. The induced surface charge density on the surface of the dielectric is given by $\sigma_{\text{i}} = \sigma(1-1/K)$.

SET UP: From Table 24.2, for polystyrene $K = 2.6$ and the dielectric strength (maximum allowed electric field) is $2\times10^7\ \text{V/m}$.

EXECUTE: **(a)** $\epsilon = K\epsilon_0 = (2.6)\epsilon_0 = 2.3\times10^{-11}\ \text{C}^2/\text{N}\cdot\text{m}^2$

(b) $V_{\text{max}} = E_{\text{max}} d = (2.0\times10^7\ \text{V/m})(2.0\times10^{-3}\ \text{m}) = 4.0\times10^4\ \text{V}$

(c) $E = \dfrac{\sigma}{K\epsilon_0}$ and $\sigma = \epsilon E = (2.3\times10^{-11}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.0\times10^7\ \text{V/m}) = 0.46\times10^{-3}\ \text{C/m}^2.$

$$\sigma_{\text{i}} = \sigma\left(1-\frac{1}{K}\right) = (0.46\times10^{-3}\ \text{C/m}^2)(1-1/2.6) = 2.8\times10^{-4}\ \text{C/m}^2.$$

EVALUATE: The net surface charge density is $\sigma_{\text{net}} = \sigma - \sigma_{\text{i}} = 1.8\times10^{-4}\ \text{C/m}^2$ and the electric field between the plates is $E = \sigma_{\text{net}}/\epsilon_0$.

24.44. **IDENTIFY:** $C = Q/V$. $C = KC_0$. $V = Ed$.

SET UP: Table 24.1 gives $K = 3.1$ for mylar.

EXECUTE: **(a)** $\Delta Q = Q - Q_0 = (K-1)Q_0 = (K-1)C_0V_0 = (2.1)(2.5\times10^{-7}\ \text{F})(12\ \text{V}) = 6.3\times10^{-6}\ \text{C}$.

(b) $\sigma_\text{i} = \sigma(1-1/K)$ so $Q_i = Q(1-1/K) = (9.3\times10^{-6}\ \text{C})(1-1/3.1) = 6.3\times10^{-6}\ \text{C}$.

(c) The addition of the mylar doesn't affect the electric field since the induced charge cancels the additional charge drawn to the plates.

EVALUATE: $E = V/d$ and V is constant so E doesn't change when the dielectric is inserted.

24.45. **(a) IDENTIFY** and **SET UP:** Since the capacitor remains connected to the power supply the potential difference doesn't change when the dielectric is inserted. Use Eq.(24.9) to calculate V and combine it with Eq.(24.12) to obtain a relation between the stored energies and the dielectric constant and use this to calculate K.

EXECUTE: Before the dielectric is inserted $U_0 = \frac{1}{2}C_0V^2$ so $V = \sqrt{\dfrac{2U_0}{C_0}} = \sqrt{\dfrac{2\left(1.85\times10^{-5}\ \text{J}\right)}{360\times10^{-9}\ \text{F}}} = 10.1\ \text{V}$

(b) $K = C/C_0$

$U_0 = \frac{1}{2}C_0V^2,\ U = \frac{1}{2}CV^2$ so $C/C_0 = U/U_0$

$$K = \frac{U}{U_0} = \frac{1.85\times10^{-5}\ \text{J} + 2.32\times10^{-5}\ \text{J}}{1.85\times10^{-5}\ \text{J}} = 2.25$$

EVALUATE: K increases the capacitance and then from $U = \frac{1}{2}CV^2$, with V constant an increase in C gives an increase in U.

24.46. **IDENTIFY:** $C = KC_0$. $C = Q/V$. $V = Ed$.

SET UP: Since the capacitor remains connected to the battery the potential between the plates of the capacitor doesn't change.

EXECUTE: **(a)** The capacitance changes by a factor of K when the dielectric is inserted. Since V is unchanged (the battery is still connected), $\dfrac{C_\text{after}}{C_\text{before}} = \dfrac{Q_\text{after}}{Q_\text{before}} = \dfrac{45.0\ \text{pC}}{25.0\ \text{pC}} = K = 1.80$.

(b) The area of the plates is $\pi r^2 = \pi(0.0300\ \text{m})^2 = 2.827\times10^{-3}\text{m}^2$ and the separation between them is thus $d = \dfrac{\epsilon_0 A}{C} = \dfrac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\ \text{m}^2)}{12.5\times10^{-12}\ \text{F}} = 2.00\times10^{-3}\ \text{m}$. Before the dielectric is inserted, $C = \dfrac{\epsilon_0 A}{d} = \dfrac{Q}{V}$ and $V = \dfrac{Qd}{\epsilon_0 A} = \dfrac{(25.0\times10^{-12}\ \text{C})(2.00\times10^{-3}\ \text{m})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\ \text{m}^2)} = 2.00\ \text{V}$. The battery remains connected, so the potential difference is unchanged after the dielectric is inserted.

(c) Before the dielectric is inserted, $E = \dfrac{Q}{\epsilon_0 A} = \dfrac{25.0\times10^{-12}\ \text{C}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\ \text{m}^2)} = 1000\ \text{N/C}$

Again, since the voltage is unchanged after the dielectric is inserted, the electric field is also unchanged.

EVALUATE: $E = \dfrac{V}{d} = \dfrac{2.00\ \text{V}}{2.00\times10^{-3}\ \text{m}} = 1000\ \text{N/C}$, whether or not the dielectric is present. This agrees with the result in part (c). The electric field has this value at any point between the plates. We need d to calculate E because V is the potential difference between points separated by distance d.

24.47. **IDENTIFY:** $C = KC_0$. $U = \frac{1}{2}CV^2$.

SET UP: $C_0 = 12.5\ \mu\text{F}$ is the value of the capacitance without the dielectric present.

EXECUTE: **(a)** With the dielectric, $C = (3.75)(12.5\ \mu\text{F}) = 46.9\ \mu\text{F}$.

before: $U = \frac{1}{2}C_0V^2 = \frac{1}{2}(12.5\times10^{-6}\ \text{F})(24.0\ \text{V})^2 = 3.60\ \text{mJ}$

after: $U = \frac{1}{2}CV^2 = \frac{1}{2}(46.9\times10^{-6}\ \text{F})(24.0\ \text{V})^2 = 13.5\ \text{mJ}$

(b) $\Delta U = 13.5\ \text{mJ} - 3.6\ \text{mJ} = 9.9\ \text{mJ}$. The energy increased.

EVALUATE: The power supply must put additional charge on the plates to maintain the same potential difference when the dielectric is inserted. $U = \frac{1}{2}QV$, so the stored energy increases.

24.48. **IDENTIFY:** Gauss's law in dielectrics has the same form as in vacuum except that the electric field is multiplied by a factor of K and the charge enclosed by the Gaussian surface is the free charge. The capacitance of an object depends on its geometry.

(a) SET UP: The capacitance of a parallel-plate capacitor is $C = K\epsilon_0 A/d$ and the charge on its plates is $Q = CV$.

EXECUTE: First find the capacitance:

$$C = \frac{K\epsilon_0 A}{d} = \frac{(2.1)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)}{1.00\times10^{-3}\ \text{m}} = 4.18\times10^{-10}\ \text{F}.$$

Now find the charge on the plates: $Q = CV = (4.18\times10^{-10}\ \text{F})(12.0\ \text{V}) = 5.02\times10^{-9}\ \text{C}$.

(b) SET UP: Gauss's law within the dielectric gives $KEA = Q_{\text{free}}/\epsilon_0$.

EXECUTE: Solving for E gives

$$E = \frac{Q_{\text{free}}}{KA\epsilon_0} = \frac{5.02\times10^{-9}\ \text{C}}{(2.1)(0.0225\ \text{m}^2)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 1.20\times10^4\ \text{N/C}$$

(c) SET UP: Without the Teflon and the voltage source, the charge is unchanged but the potential increases, so $C = \epsilon_0 A/d$ and Gauss's law now gives $EA = Q/\epsilon_0$.

EXECUTE: First find the capacitance:

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)}{1.00\times10^{-3}\ \text{m}} = 1.99\times10^{-10}\ \text{F}.$$

The potential difference is $V = \frac{Q}{C} = \frac{5.02\times10^{-9}\ \text{C}}{1.99\times10^{-10}\ \text{F}} = 25.2\ \text{V}$. From Gauss's law, the electric field is

$$E = \frac{Q}{\epsilon_0 A} = \frac{5.02\times10^{-9}\ \text{C}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)} = 2.52\times10^4\ \text{N/C}.$$

EVALUATE: The dielectric reduces the electric field inside the capacitor because the electric field due to the dipoles of the dielectric is opposite to the external field due to the free charge on the plates.

24.49. IDENTIFY: Apply Eq.(24.23) to calculate *E*. *V* = *Ed* and *C* = *Q/V* apply whether there is a dielectric between the plates or not.

(a) SET UP: Apply Eq.(24.23) to the dashed surface in Figure 24.49:

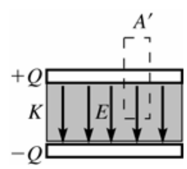


Figure 24.49

EXECUTE: $\oint K\vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\epsilon_0}$

$\oint K\vec{E}\cdot d\vec{A} = KEA'$

since $E = 0$ outside the plates

$Q_{\text{encl-free}} = \sigma A' = (Q/A)A'$

Thus $KEA' = \frac{(Q/A)A'}{\epsilon_0}$ and $E = \frac{Q}{\epsilon_0 AK}$

(b) $V = Ed = \frac{Qd}{\epsilon_0 AK}$

(c) $C = \frac{Q}{V} = \frac{Q}{(Qd/\epsilon_0 AK)} = K\frac{\epsilon_0 A}{d} = KC_0.$

EVALUATE: Our result shows that $K = C/C_0$, which is Eq.(24.12).

24.50. IDENTIFY: $C = \frac{\epsilon_0 A}{d}$. $C = Q/V$. $V = Ed$. $U = \frac{1}{2}CV^2$.

SET UP: With the battery disconnected, *Q* is constant. When the separation *d* is doubled, *C* is halved.

EXECUTE: **(a)** $C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 (0.16\ \text{m})^2}{4.7\times10^{-3}\ \text{m}} = 4.8\times10^{-11}\ \text{F}$

(b) $Q = CV = (4.8\times10^{-11}\ \text{F})(12\ \text{V}) = 0.58\times10^{-9}\ \text{C}$

(c) $E = V/d = (12\ \text{V})/(4.7\times10^{-3}\ \text{m}) = 2550\ \text{V/m}$

(d) $U = \frac{1}{2}CV^2 = \frac{1}{2}(4.8\times10^{-11}\ \text{F})(12\ \text{V})^2 = 3.46\times10^{-9}\ \text{J}$

(e) If the battery is disconnected, so the charge remains constant, and the plates are pulled further apart to 0.0094 m, then the calculations above can be carried out just as before, and we find: (a) $C = 2.41\times10^{-11}\ \text{F}$ (b) $Q = 0.58\times10^{-9}\ \text{C}$

(c) $E = 2550\ \text{V/m}$ (d) $U = \frac{Q^2}{2C} = \frac{(0.58\times10^{-9}\ \text{C})^2}{2(2.41\times10^{-11}\ \text{F})} = 6.91\times10^{-9}\ \text{J}$

EVALUATE: *Q* is unchanged. $E = \frac{Q}{\epsilon_0 A}$ so *E* is unchanged. *U* doubles because *C* is halved. The additional stored energy comes from the work done by the force that pulled the plates apart.

24.51. **IDENTIFY** and **SET UP:** If the capacitor remains connected to the battery, the battery keeps the potential difference between the plates constant by changing the charge on the plates.

EXECUTE: **(a)** $C = \dfrac{\epsilon_0 A}{d}$

$$C = \frac{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.16\ \text{m})^2}{9.4\times10^{-3}\ \text{m}} = 2.4\times10^{-11}\ \text{F} = 24\ \text{pF}$$

(b) Remains connected to the battery says that V stays 12 V. $Q = CV = (2.4\times10^{-11}\ \text{F})(12\ \text{V}) = 2.9\times10^{-10}\ \text{C}$

(c) $E = \dfrac{V}{d} = \dfrac{12\ \text{V}}{9.4\times10^{-3}\ \text{m}} = 1.3\times10^{3}\ \text{V/m}$

(d) $U = \tfrac{1}{2}QV = \tfrac{1}{2}(2.9\times10^{-10}\ \text{C})(12.0\ \text{V}) = 1.7\times10^{-9}\ \text{J}$

EVALUATE: Increasing the separation decreases C. With V constant, this means that Q decreases and U decreases. Q decreases and $E = Q/\epsilon_0 A$ so E decreases. We come to the same conclusion from $E = V/d$.

24.52. **IDENTIFY:** $C = KC_0 = K\epsilon_0 \dfrac{A}{d}$. $V = Ed$ for a parallel plate capacitor; this equation applies whether or not a dielectric is present.

SET UP: $A = 1.0\ \text{cm}^2 = 1.0\times10^{-4}\ \text{m}^2$.

EXECUTE: **(a)** $C = (10)\dfrac{(8.85\times10^{-12}\ \text{F/m})(1.0\times10^{-4}\ \text{m}^2)}{7.5\times10^{-9}\ \text{m}} = 1.18\ \mu\text{F}$ per cm^2.

(b) $E = \dfrac{V}{K} = \dfrac{85\ \text{mV}}{7.5\times10^{-9}\ \text{m}} = 1.13\times10^{7}\ \text{V/m}$.

EVALUATE: The dielectric material increases the capacitance. If the dielectric were not present, the same charge density on the faces of the membrane would produce a larger potential difference across the membrane.

24.53. **IDENTIFY:** $P = E/t$, where E is the total light energy output. The energy stored in the capacitor is $U = \tfrac{1}{2}CV^2$.

SET UP: $E = 0.95U$

EXECUTE: **(a)** The power output is 600 W, and 95% of the original energy is converted, so

$E = Pt = (2.70\times10^{5}\ \text{W})(1.48\times10^{-3}\ \text{s}) = 400\ \text{J}$. $E_0 = \dfrac{400\ \text{J}}{0.95} = 421\ \text{J}$.

(b) $U = \tfrac{1}{2}CV^2$ so $C = \dfrac{2U}{V^2} = \dfrac{2(421\ \text{J})}{(125\ \text{V})^2} = 0.054\ \text{F}$.

EVALUATE: For a given V, the stored energy increases linearly with C.

24.54. **IDENTIFY:** $C = \dfrac{\epsilon_0 A}{d}$

SET UP: $A = 4.2\times10^{-5}\ \text{m}^2$. The original separation between the plates is $d = 0.700\times10^{-3}\ \text{m}$. d' is the separation between the plates at the new value of C.

EXECUTE: $C_0 = \dfrac{A\epsilon_0}{d} = \dfrac{(4.20\times10^{-5}\ \text{m}^2)\epsilon_0}{7.00\times10^{-4}\ \text{m}} = 5.31\times10^{-13}\ \text{F}$. The new value of C is $C = C_0 + 0.25\ \text{pF} = 7.81\times10^{-13}\ \text{F}$.

But $C = \dfrac{A\epsilon_0}{d'}$, so $d' = \dfrac{A\epsilon_0}{C} = \dfrac{(4.20\times10^{-5}\ \text{m}^2)\epsilon_0}{7.81\times10^{-13}\ \text{F}} = 4.76\times10^{-4}\ \text{m}$. Therefore the key must be depressed by a distance of $7.00\times10^{-4}\ \text{m} - 4.76\times10^{-4}\ \text{m} = 0.224\ \text{mm}$.

EVALUATE: When the key is depressed, d decreases and C increases.

24.55. **IDENTIFY:** Example 24.4 shows that $C = \dfrac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$ for a cylindrical capacitor.

SET UP: $\ln(1+x) \approx x$ when x is small. The area of each conductor is approximately $A = 2\pi r_a L$.

EXECUTE: **(a)** $d << r_a$: $C = \dfrac{2\pi\epsilon_0 L}{\ln(r_b/r_a)} = \dfrac{2\pi\epsilon_0 L}{\ln((d+r_a)/r_a)} = \dfrac{2\pi\epsilon_0 L}{\ln(1+d/r_a)} \approx \dfrac{2\pi r_a L\epsilon_0}{d} = \dfrac{\epsilon_0 A}{d}$

EVALUATE: **(b)** At the scale of part (a) the cylinders appear to be flat, and so the capacitance should appear like that of flat plates.

24.56. **IDENTIFY:** Initially the capacitors are connected in parallel to the source and we can calculate the charges Q_1 and Q_2 on each. After they are reconnected to each other the total charge is $Q = Q_2 - Q_1$. $U = \tfrac{1}{2}CV^2 = \dfrac{Q^2}{2C}$.

SET UP: After they are reconnected, the charges add and the voltages are the same, so $C_{\text{eq}} = C_1 + C_2$, as for capacitors in parallel.

EXECUTE: Originally $Q_1 = C_1V_1 = (9.0\ \mu\text{F})\ (28\ \text{V}) = 2.52\times10^{-4}\ \text{C}$ and $Q_2 = C_2V_2 = (4.0\ \mu\text{F})(28\ \text{V}) = 1.12\times10^{-4}\ \text{C}$. $C_{\text{eq}} = C_1 + C_2 = 13.0\ \mu\text{F}$. The original energy stored is $U = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}(13.0\times10^{-6}\ \text{F})(28\ \text{V})^2 = 5.10\times10^{-3}\ \text{J}$. Disconnect and flip the capacitors, so now the total charge is $Q = Q_2 - Q_1 = 1.4\times10^{-4}$ C and the equivalent capacitance is still the same, $C_{\text{eq}} = 13.0\ \mu\text{F}$. The new energy stored is $U = \dfrac{Q^2}{2C_{\text{eq}}} = \dfrac{(1.4\times10^{-4}\ \text{C})^2}{2(13.0\times10^{-6}\ \text{F})} = 7.54\times10^{-4}\ \text{J}$. The change in stored energy is $\Delta U = 7.45\times10^{-4}\ \text{J} - 5.10\times10^{-3}\ \text{J} = -4.35\times10^{-3}\ \text{J}$.

EVALUATE: When they are reconnected, charge flows and thermal energy is generated and energy is radiated as electromagnetic waves.

24.57. IDENTIFY: Simplify the network by replacing series and parallel combinations by their equivalent. The stored energy in a capacitor is $U = \frac{1}{2}CV^2$.

SET UP: For capacitors in series the voltages add and the charges are the same; $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots$. For capacitors in parallel the voltages are the same and the charges add; $C_{\text{eq}} = C_1 + C_2 + \cdots$ $C = \dfrac{Q}{V}$. $U = \frac{1}{2}CV^2$.

EXECUTE: (a) Find C_{eq} for the network by replacing each series or parallel combination by its equivalent. The successive simplified circuits are shown in Figure 24.57a–c.

$U_{\text{tot}} = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}(2.19\times10^{-6}\ \text{F})(12.0\ \text{V})^2 = 1.58\times10^{-4}\ \text{J} = 158\ \mu\text{J}$

(b) From Figure 24.57c, $Q_{\text{tot}} = C_{\text{eq}}V = (2.19\times10^{-6}\ \text{F})(12.0\ \text{V}) = 2.63\times10^{-5}\ \text{C}$. From Figure 24.57b, $Q_{\text{tot}} = 2.63\times10^{-5}\ \text{C}$.

$V_{4.8} = \dfrac{Q_{4.8}}{C_{4.8}} = \dfrac{2.63\times10^{-5}\ \text{C}}{4.80\times10^{-6}\ \text{F}} = 5.48\ \text{V}$. $U_{4.8} = \frac{1}{2}CV^2 = \frac{1}{2}(4.80\times10^{-6}\ \text{F})(5.48\ \text{V})^2 = 7.21\times10^{-5}\ \text{J} = 72.1\ \mu\text{J}$

This one capacitor stores nearly half the total stored energy.

EVALUATE: $U = \dfrac{Q^2}{2C}$. For capacitors in series the capacitor with the smallest C stores the greatest amount of energy.

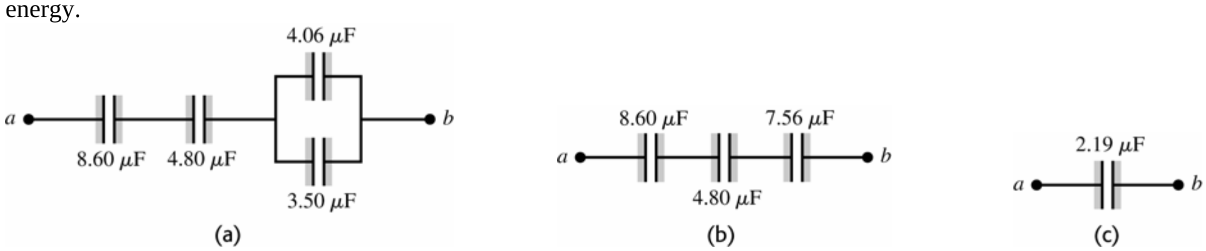


Figure 24.57

24.58. IDENTIFY: Apply the rules for combining capacitors in series and parallel. For capacitors in series the voltages add and in parallel the voltages are the same.

SET UP: When a capacitor is a moderately good conductor it can be replaced by a wire and the potential across it is zero.

EXECUTE: (a) A network that has the desired properties is sketched in Figure 24.58a. $C_{\text{eq}} = \dfrac{C}{2} + \dfrac{C}{2} = C$. The total capacitance is the same as each individual capacitor, and the voltage is spilt over each so that $V = 480$ V.

(b) If one capacitor is a moderately good conductor, then it can be treated as a "short" and thus removed from the circuit, and one capacitor will have greater than 600 V across it.

EVALUATE: An alternative solution is two in parallel in series with two in parallel, as sketched in Figure 24.58b.

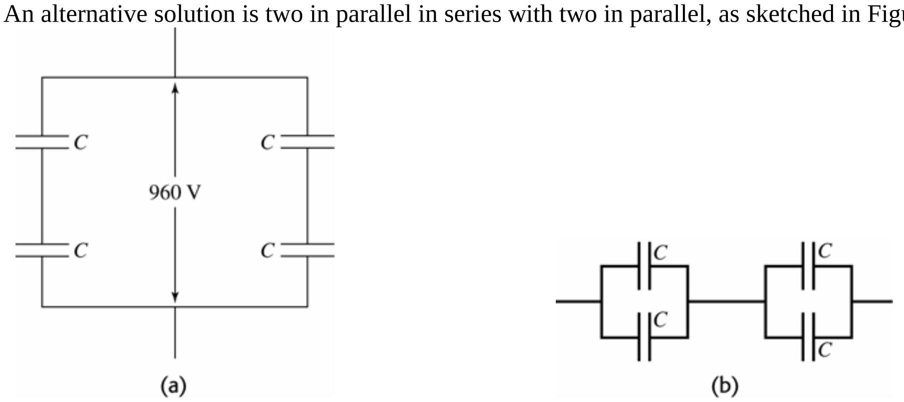
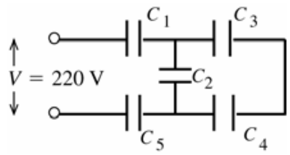


Figure 24.58

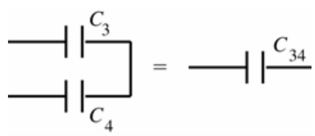
24.59. **(a) IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents.
SET UP: The network is sketched in Figure 24.59a.



$$C_1 = C_5 = 8.4\ \mu\text{F}$$
$$C_2 = C_3 = C_4 = 4.2\ \mu\text{F}$$

Figure 24.59a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_3 and C_4 are in series and can be replaced by C_{34} (Figure 24.59b):

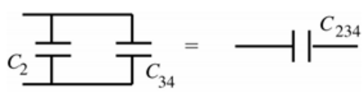


$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4}$$
$$\frac{1}{C_{34}} = \frac{C_3 + C_4}{C_3 C_4}$$

Figure 24.59b

$$C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{(4.2\ \mu\text{F})(4.2\ \mu\text{F})}{4.2\ \mu\text{F} + 4.2\ \mu\text{F}} = 2.1\ \mu\text{F}$$

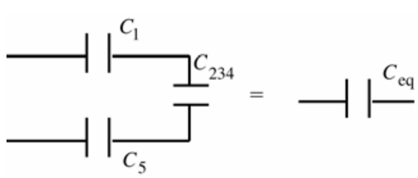
C_2 and C_{34} are in parallel and can be replaced by their equivalent (Figure 24.59c):



$$C_{234} = C_2 + C_{34}$$
$$C_{234} = 4.2\ \mu\text{F} + 2.1\ \mu\text{F}$$
$$C_{234} = 6.3\ \mu\text{F}$$

Figure 24.59c

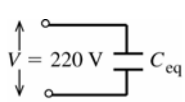
C_1, C_5 and C_{234} are in series and can be replaced by C_{eq} (Figure 24.59d):



$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_5} + \frac{1}{C_{234}}$$
$$\frac{1}{C_{\text{eq}}} = \frac{2}{8.4\ \mu\text{F}} + \frac{1}{6.3\ \mu\text{F}}$$
$$C_{\text{eq}} = 2.5\ \mu\text{F}$$

Figure 24.59d

EVALUATE: For capacitors in series the equivalent capacitor is smaller than any of those in series. For capacitors in parallel the equivalent capacitance is larger than any of those in parallel.
(b) IDENTIFY and **SET UP:** In each equivalent network apply the rules for *Q* and *V* for capacitors in series and parallel; start with the simplest network and work back to the original circuit.
EXECUTE: The equivalent circuit is drawn in Figure 24.59e.



$$Q_{\text{eq}} = C_{\text{eq}} V$$
$$Q_{\text{eq}} = (2.5\ \mu\text{F})(220\ \text{V}) = 550\ \mu\text{C}$$

Figure 24.59e

$Q_1 = Q_5 = Q_{234} = 550\ \mu\text{C}$ (capacitors in series have same charge)

$$V_1 = \frac{Q_1}{C_1} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}$$

$$V_5 = \frac{Q_5}{C_5} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}$$

$$V_{234} = \frac{Q_{234}}{C_{234}} = \frac{550\ \mu\text{C}}{6.3\ \mu\text{F}} = 87\ \text{V}$$

Now draw the network as in Figure 24.59f.

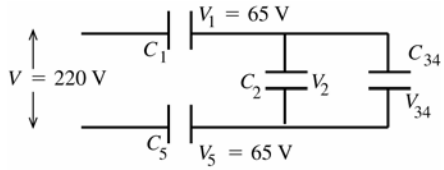


Figure 24.59f

$V_2 = V_{34} = V_{234} = 87\text{ V}$

capacitors in parallel have the same potential

$Q_2 = C_2V_2 = (4.2\ \mu\text{F})(87\text{ V}) = 370\ \mu\text{C}$

$Q_{34} = C_{34}V_{34} = (2.1\ \mu\text{F})(87\text{ V}) = 180\ \mu\text{C}$

Finally, consider the original circuit (Figure 24.59g).

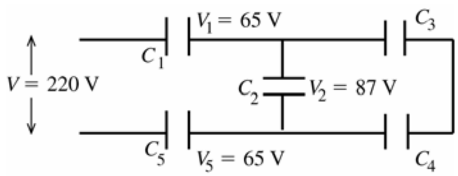


Figure 24.59g

$Q_3 = Q_4 = Q_{34} = 180\ \mu\text{C}$

capacitors in series have the same charge

$V_3 = \dfrac{Q_3}{C_3} = \dfrac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\text{ V}$

$V_4 = \dfrac{Q_4}{C_4} = \dfrac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\text{ V}$

Summary: $Q_1 = 550\ \mu\text{C},\ V_1 = 65\text{ V}$

$Q_2 = 370\ \mu\text{C},\ V_2 = 87\text{ V}$

$Q_3 = 180\ \mu\text{C},\ V_3 = 43\text{ V}$

$Q_4 = 180\ \mu\text{C},\ V_4 = 43\text{ V}$

$Q_5 = 550\ \mu\text{C},\ V_5 = 65\text{ V}$

EVALUATE: $V_3 + V_4 = V_2$ and $V_1 + V_2 + V_5 = 220\text{ V}$ (apart from some small rounding error)

$Q_1 = Q_2 + Q_3$ and $Q_5 = Q_2 + Q_4$

24.60. **IDENTIFY:** Apply the rules for combining capacitors in series and in parallel.

SET UP: With the switch open each pair of $3.00\ \mu\text{F}$ and $6.00\ \mu\text{F}$ capacitors are in series with each other and each pair is in parallel with the other pair. When the switch is closed each pair of $3.00\ \mu\text{F}$ and $6.00\ \mu\text{F}$ capacitors are in parallel with each other and the two pairs are in series.

EXECUTE: **(a)** With the switch open $C_{\text{eq}} = \left(\left(\dfrac{1}{3\ \mu\text{F}} + \dfrac{1}{6\ \mu\text{F}}\right)^{-1} + \left(\dfrac{1}{3\ \mu\text{F}} + \dfrac{1}{6\ \mu\text{F}}\right)^{-1}\right) = 4.00\ \mu\text{F}$.

$Q_{\text{total}} = C_{\text{eq}}V = (4.00\ \mu\text{F})\ (210\text{ V}) = 8.40\times10^{-4}\text{ C}$. By symmetry, each capacitor carries 4.20×10^{-4} C. The voltages are then calculated via $V = Q/C$. This gives $V_{ad} = Q/C_3 = 140\text{ V}$ and $V_{ac} = Q/C_6 = 70\text{ V}$.

$V_{cd} = V_{ad} - V_{ac} = 70\text{ V}$.

(b) When the switch is closed, the points *c* and *d* must be at the same potential, so the equivalent capacitance is

$C_{\text{eq}} = \left(\dfrac{1}{(3.00+6.00)\ \mu\text{F}} + \dfrac{1}{(3.00+6.00)\ \mu\text{F}}\right)^{-1} = 4.5\ \mu\text{F}$. $Q_{\text{total}} = C_{\text{eq}}V = (4.50\ \mu\text{F})(210\text{ V}) = 9.5\times10^{-4}\text{ C}$, and each capacitor has the same potential difference of 105 V (again, by symmetry).

(c) The only way for the sum of the positive charge on one plate of C_2 and the negative charge on one plate of C_1 to change is for charge to flow through the switch. That is, the quantity of charge that flows through the switch is equal to the change in $Q_2 - Q_1$. With the switch open, $Q_1 = Q_2$ and $Q_2 - Q_1 = 0$. After the switch is closed, $Q_2 - Q_1 = 315\ \mu\text{C}$, so $315\ \mu\text{C}$ of charge flowed through the switch.

EVALUATE: When the switch is closed the charge must redistribute to make points *c* and *d* be at the same potential.

24.61. **(a) IDENTIFY:** Replace the three capacitors in series by their equivalent. The charge on the equivalent capacitor equals the charge on each of the original capacitors.

SET UP: The three capacitors can be replaced by their equivalent as shown in Figure 24.61a.

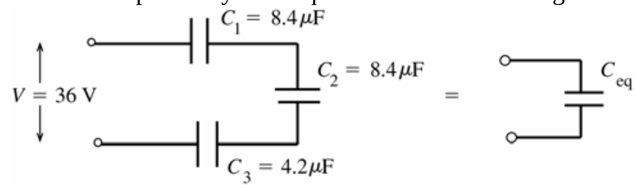


Figure 24.61a

EXECUTE: $C_3 = C_1/2$ so $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} = \dfrac{4}{8.4\ \mu\text{F}}$ and $C_{eq} = 8.4\ \mu\text{F}/4 = 2.1\ \mu\text{F}$

$Q = C_{eq}V = (2.1\ \mu\text{F})(36\ \text{V}) = 76\ \mu\text{C}$

The three capacitors are in series so they each have the same charge: $Q_1 = Q_2 = Q_3 = 76\ \mu\text{C}$

EVALUATE: The equivalent capacitance for capacitors in series is smaller than each of the original capacitors.

(b) IDENTIFY and **SET UP:** Use $U = \frac{1}{2}QV$. We know each Q and we know that $V_1 + V_2 + V_3 = 36$ V.

EXECUTE: $U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3$

But $Q_1 = Q_2 = Q_3 = Q$ so $U = \frac{1}{2}Q(V_1 + V_2 + V_3)$

But also $V_1 + V_2 + V_3 = V = 36$ V, so $U = \frac{1}{2}QV = \frac{1}{2}(76\ \mu\text{C})(36\ \text{V}) = 1.4\times10^{-3}$ J.

EVALUATE: We could also use $U = Q^2/2C$ and calculate U for each capacitor.

(c) IDENTIFY: The charges on the plates redistribute to make the potentials across each capacitor the same.

SET UP: The capacitors before and after they are connected are sketched in Figure 24.61b.

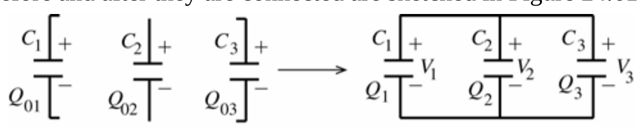


Figure 24.61b

EXECUTE: The total positive charge that is available to be distributed on the upper plates of the three capacitors is $Q_0 = Q_{01} + Q_{02} + Q_{03} = 3(76\ \mu\text{C}) = 228\ \mu\text{C}$. Thus $Q_1 + Q_2 + Q_3 = 228\ \mu\text{C}$. After the circuit is completed the charge distributes to make $V_1 = V_2 = V_3$. $V = Q/C$ and $V_1 = V_2$ so $Q_1/C_1 = Q_2/C_2$ and then $C_1 = C_2$ says $Q_1 = Q_2$. $V_1 = V_3$ says $Q_1/C_1 = Q_3/C_3$ and $Q_1 = Q_3(C_1/C_3) = Q_3(8.4\ \mu\text{F}/4.2\ \mu\text{F}) = 2Q_3$

Using $Q_2 = Q_1$ and $Q_1 = 2Q_3$ in the above equation gives $2Q_3 + 2Q_3 + Q_3 = 228\ \mu\text{C}$.

$5Q_3 = 228\ \mu\text{C}$ and $Q_3 = 45.6\ \mu\text{C}$, $Q_1 = Q_2 = 91.2\ \mu\text{C}$

Then $V_1 = \dfrac{Q_1}{C_1} = \dfrac{91.2\ \mu\text{C}}{8.4\ \mu\text{F}} = 11$ V, $V_2 = \dfrac{Q_2}{C_2} = \dfrac{91.2\ \mu\text{C}}{8.4\ \mu\text{F}} = 11$ V, and $V_3 = \dfrac{Q_3}{C_3} = \dfrac{45.6\ \mu\text{C}}{4.2\ \mu\text{F}} = 11$ V.

The voltage across each capacitor in the parallel combination is 11 V.

(d) $U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3$.

But $V_1 = V_2 = V_3$ so $U = \frac{1}{2}V_1(Q_1 + Q_2 + Q_3) = \frac{1}{2}(11\ \text{V})(228\ \mu\text{C}) = 1.3\times10^{-3}$ J.

EVALUATE: This is less than the original energy of 1.4×10^{-3} J. The stored energy has decreased, as in Example 24.7.

24.62. **IDENTIFY:** $C = \dfrac{\epsilon_0 A}{d}$. $C = \dfrac{Q}{V}$. $V = Ed$. $U = \frac{1}{2}QV$.

SET UP: $d = 3.0\times10^3$ m. $A = \pi r^2$, with $r = 1.0\times10^3$ m.

EXECUTE: **(a)** $C = \dfrac{\epsilon_0 A}{d} = \dfrac{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\pi(1.0\times10^3\ \text{m})^2}{3.0\times10^3\ \text{m}} = 9.3\times10^{-9}$ F.

(b) $V = \dfrac{Q}{C} = \dfrac{20\ \text{C}}{9.3\times10^{-9}\ \text{F}} = 2.2\times10^9$ V

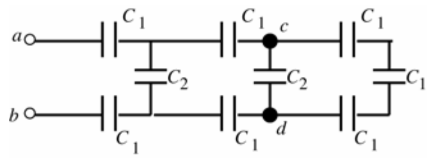
(c) $E = \dfrac{V}{d} = \dfrac{2.2\times10^9\ \text{V}}{3.0\times10^3\ \text{m}} = 7.3\times10^5$ V/m

(d) $U = \frac{1}{2}QV = \frac{1}{2}(20\ \text{C})(2.2\times10^9\ \text{V}) = 2.2\times10^{10}$ J

EVALUATE: Thunderclouds involve very large potential differences and large amounts of stored energy.

24.63. **IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

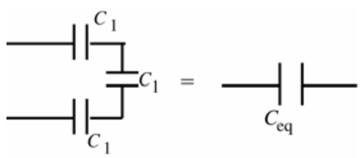
(a) SET UP: The network is sketched in Figure 24.63a.



$C_1 = 6.9\ \mu\text{F}$
$C_2 = 4.6\ \mu\text{F}$

Figure 24.63a

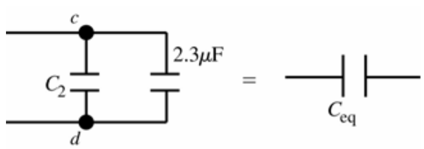
EXECUTE: Simplify the network by replacing the capacitor combinations by their equivalents. Make the replacement shown in Figure 24.63b.



$$\frac{1}{C_{eq}} = \frac{3}{C_1}$$
$$C_{eq} = \frac{C_1}{3} = \frac{6.9\ \mu\text{F}}{3} = 2.3\ \mu\text{F}$$

Figure 24.63b

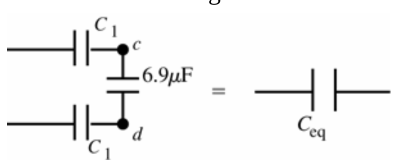
Next make the replacement shown in Figure 24.63c.



$$C_{eq} = 2.3\ \mu\text{F} + C_2$$
$$C_{eq} = 2.3\ \mu\text{F} + 4.6\ \mu\text{F} = 6.9\ \mu\text{F}$$

Figure 24.63c

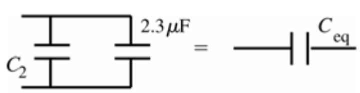
Make the replacement shown in Figure 24.63d.



$$\frac{1}{C_{eq}} = \frac{2}{C_1} + \frac{1}{6.9\ \mu\text{F}} = \frac{3}{6.9\ \mu\text{F}}$$
$$C_{eq} = 2.3\ \mu\text{F}$$

Figure 24.63d

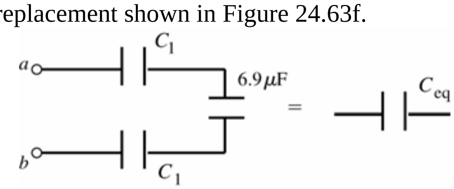
Make the replacement shown in Figure 24.63e.



$$C_{eq} = C_2 + 2.3\ \mu\text{F} = 4.6\ \mu\text{F} + 2.3\ \mu\text{F}$$
$$C_{eq} = 6.9\ \mu\text{F}$$

Figure 24.63e

Make the replacement shown in Figure 24.63f.



$$\frac{1}{C_{eq}} = \frac{2}{C_1} + \frac{1}{6.9\ \mu\text{F}} = \frac{3}{6.9\ \mu\text{F}}$$
$$C_{eq} = 2.3\ \mu\text{F}$$

Figure 24.63f

(b) Consider the network as drawn in Figure 24.63g.

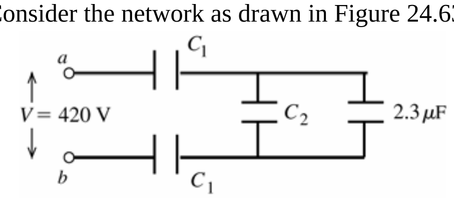


Figure 24.63g

From part (a) $2.3\ \mu\text{F}$ is the equivalent capacitance of the rest of the network.

The equivalent network is shown in Figure 24.63h.

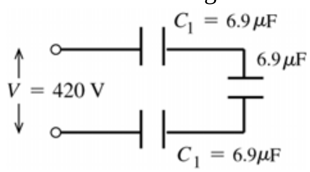


Figure 24.63h

The capacitors are in series, so all three capacitors have the same Q.

But here all three have the same C, so by $V = Q/C$ all three must have the same V. The three voltages must add to 420 V, so each capacitor has $V = 140$ V. The $6.9\ \mu\text{F}$ to the right is the equivalent of C_2 and the $2.3\ \mu\text{F}$ capacitor in parallel, so $V_2 = 140\ \text{V}$. (Capacitors in parallel have the same potential difference.) Hence $Q_1 = C_1V_1 = (6.9\ \mu\text{F})(140\ \text{V}) = 9.7\times10^{-4}\ \text{C}$ and $Q_2 = C_2V_2 = (4.6\ \mu\text{F})(140\ \text{V}) = 6.4\times10^{-4}\ \text{C}$.

(c) From the potentials deduced in part (b) we have the situation shown in Figure 24.63i.

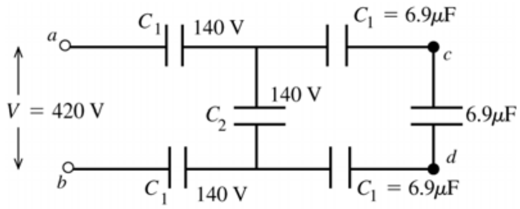


Figure 24.63i

From part (a) $6.9\ \mu\text{F}$ is the equivalent capacitance of the rest of the network.

The three right-most capacitors are in series and therefore have the same charge. But their capacitances are also equal, so by $V = Q/C$ they each have the same potential difference. Their potentials must sum to 140 V, so the potential across each is 47 V and $V_{cd} = 47$ V.

EVALUATE: In each capacitor network the rules for combining V for capacitors in series and parallel are obeyed. Note that $V_{cd} < V$, in fact $V - 2(140\ \text{V}) - 2(47\ \text{V}) = V_{cd}$.

24.64. **IDENTIFY:** Find the total charge on the capacitor network when it is connected to the battery. This is the amount of charge that flows through the signal device when the switch is closed.

SET UP: For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots$

EXECUTE: $C_{\text{equiv}} = C_1 + C_2 + C_3 = 60.0\ \mu\text{F}$. $Q = CV = (60.0\ \mu\text{F})(120\ \text{V}) = 7200\ \mu\text{C}$.

EVALUATE: More charge is stored by the three capacitors in parallel than would be stored in each capacitor used alone.

24.65. **(a) IDENTIFY** and **SET UP:** Q is constant. $C = KC_0$; use Eq.(24.1) to relate the dielectric constant K to the ratio of the voltages without and with the dielectric.

EXECUTE: With the dielectric: $V = Q/C = Q/(KC_0)$

without the dielectric: $V_0 = Q/C_0$

$V_0/V = K$, so $K = (45.0\ \text{V})/(11.5\ \text{V}) = 3.91$

EVALUATE: Our analysis agrees with Eq.(24.13).

(b) IDENTIFY: The capacitor can be treated as equivalent to two capacitors C_1 and C_2 in parallel, one with area 2A/3 and air between the plates and one with area A/3 and dielectric between the plates.

SET UP: The equivalent network is shown in Figure 24.65.

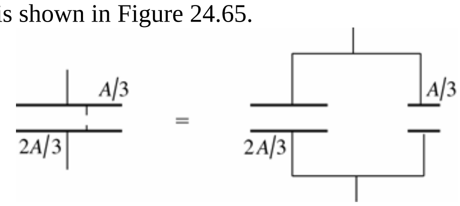


Figure 24.65

EXECUTE: Let $C_0 = \epsilon_0 A/d$ be the capacitance with only air between the plates. $C_1 = KC_0/3$, $C_2 = 2C_0/3$;

$C_{\text{eq}} = C_1 + C_2 = (C_0/3)(K+2)$

$$V = \frac{Q}{C_{\text{eq}}} = \frac{Q}{C_0}\left(\frac{3}{K+2}\right) = V_0\left(\frac{3}{K+2}\right) = (45.0\ \text{V})\left(\frac{3}{5.91}\right) = 22.8\ \text{V}$$

EVALUATE: The voltage is reduced by the dielectric. The voltage reduction is less when the dielectric doesn't completely fill the volume between the plates.

24.66. **IDENTIFY:** This situation is analogous to having two capacitors C_1 in series, each with separation $\frac{1}{2}(d-a)$.

SET UP: For capacitors in series, $\frac{1}{C_{\text{eq}}}=\frac{1}{C_1}+\frac{1}{C_2}$.

EXECUTE: **(a)** $C=\left(\frac{1}{C_1}+\frac{1}{C_1}\right)^{-1}=\frac{1}{2}C_1=\frac{1}{2}\frac{\epsilon_0 A}{(d-a)/2}=\frac{\epsilon_0 A}{d-a}$

(b) $C=\frac{\epsilon_0 A}{d-a}=\frac{\epsilon_0 A}{d}\frac{d}{d-a}=C_0\frac{d}{d-a}$

(c) As $a\to 0$, $C\to C_0$. The metal slab has no effect if it is very thin. And as $a\to d$, $C\to\infty$. $V=Q/C$. $V=Ey$ is the potential difference between two points separated by a distance y parallel to a uniform electric field. When the distance is very small, it takes a very large field and hence a large Q on the plates for a given potential difference. Since $Q=CV$ this corresponds to a very large C.

24.67. **(a) IDENTIFY:** The conductor can be at some potential V, where $V = 0$ far from the conductor. This potential depends on the charge Q on the conductor so we can define $C = Q/V$ where C will not depend on V or Q.
(b) SET UP: Use the expression for the potential at the surface of the sphere in the analysis in part (a).
EXECUTE: For any point on a solid conducting sphere $V=Q/4\pi\epsilon_0 R$ if $V=0$ at $r\to\infty$.

$$C=\frac{Q}{V}=Q\left(\frac{4\pi\epsilon_0 R}{Q}\right)=4\pi\epsilon_0 R$$

(c) $C=4\pi\epsilon_0 R=4\pi\left(8.854\times10^{-12}\ \text{F/m}\right)\left(6.38\times10^{6}\ \text{m}\right)=7.10\times10^{-4}\ \text{F}=710\ \mu\text{F}$.

EVALUATE: The capacitance of the earth is about seven times larger than the largest capacitances in this range. The capacitance of the earth is quite small, in view of its large size.

24.68. **IDENTIFY:** The electric field energy density is $\frac{1}{2}\epsilon_0 E^2$. For a capacitor $U=\frac{Q^2}{2C}$.

SET UP: For a solid conducting sphere of radius R, $E=0$ for $r<R$ and $E=\frac{Q}{4\pi\epsilon_0 r^2}$ for $r>R$.

EXECUTE: **(a)** $r<R$: $u=\frac{1}{2}\epsilon_0 E^2=0$.

(b) $r>R$: $u=\frac{1}{2}\epsilon_0 E^2=\frac{1}{2}\epsilon_0\left(\frac{Q}{4\pi\epsilon_0 r^2}\right)^2=\frac{Q^2}{32\pi^2\epsilon_0 r^4}$.

(c) $U=\int u\,dV=4\pi\int_R^\infty r^2u\,dr=\frac{Q^2}{8\pi\epsilon_0}\int_R^\infty\frac{dr}{r^2}=\frac{Q^2}{8\pi\epsilon_0 R}$.

(d) This energy is equal to $\frac{1}{2}\frac{Q^2}{4\pi\epsilon_0 R}$ which is just the energy required to assemble all the charge into a spherical distribution. (Note that being aware of double counting gives the factor of 1/2 in front of the familiar potential energy formula for a charge Q a distance R from another charge Q.)

EVALUATE: **(e)** From Equation (24.9), $U=\frac{Q^2}{2C}$. $U=\frac{Q^2}{8\pi\epsilon_0 R}$ from part (c), $C=4\pi\epsilon_0 R$, as in Problem (24.67).

24.69. **IDENTIFY:** We model the earth as a spherical capacitor.

SET UP: The capacitance of the earth is $C=4\pi\epsilon_0\frac{r_ar_b}{r_b-r_a}$ and, the charge on it is $Q = CV$, and its stored energy is $U=\frac{1}{2}CV^2$.

EXECUTE: **(a)** $C=\frac{1}{9.00\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}\frac{\left(6.38\times10^{6}\ \text{m}\right)\left(6.45\times10^{6}\ \text{m}\right)}{6.45\times10^{6}\ \text{m}-6.38\times10^{6}\ \text{m}}=6.5\times10^{-2}\ \text{F}$

(b) $Q=CV=\left(6.54\times10^{-2}\ \text{F}\right)(350{,}000\ \text{V})=2.3\times10^{4}\ \text{C}$

(c) $U=\frac{1}{2}CV^2=\frac{1}{2}\left(6.54\times10^{-2}\ \text{F}\right)(350{,}000\ \text{V})^2=4.0\times10^{9}\ \text{J}$

EVALUATE: While the capacitance of the earth is larger than ordinary laboratory capacitors, capacitors much larger than this, such as 1 F, are readily available.

24.70. **IDENTIFY:** The electric field energy density is $u = \frac{1}{2}\epsilon_0 E^2$. $U = \dfrac{Q^2}{2C}$.

SET UP: For this charge distribution, $E = 0$ for $r < r_a$, $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for $r_a < r < r_b$ and $E = 0$ for $r > r_b$.

Example 24.4 shows that $\dfrac{U}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b / r_a)}$ for a cylindrical capacitor.

EXECUTE: **(a)** $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 \left(\dfrac{\lambda}{2\pi\epsilon_0 r}\right)^2 = \dfrac{\lambda^2}{8\pi^2\epsilon_0 r^2}$

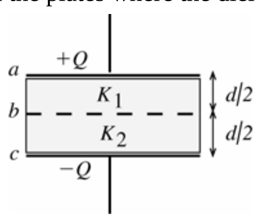
(b) $U = \int u dV = 2\pi L \int u r dr = \dfrac{L\lambda^2}{4\pi\epsilon_0}\displaystyle\int_{r_a}^{r_b} \dfrac{dr}{r}$ and $\dfrac{U}{L} = \dfrac{\lambda^2}{4\pi\epsilon_0}\ln(r_b / r_a)$.

(c) Using Equation (24.9), $U = \dfrac{Q^2}{2C} = \dfrac{Q^2}{4\pi\epsilon_0 L}\ln(r_b / r_a) = \dfrac{\lambda^2 L}{4\pi\epsilon_0}\ln(r_b / r_a)$. This agrees with the result of part (b).

EVALUATE: We could have used the results of part (b) and $U = \dfrac{Q^2}{2C}$ to calculate U/L and would obtain the same result as in Example 24.4.

24.71. **IDENTIFY:** $C = Q/V$, so we need to calculate the effect of the dielectrics on the potential difference between the plates.

SET UP: Let the potential of the positive plate be V_a, the potential of the negative plate be V_c, and the potential midway between the plates where the dielectrics meet be V_b, as shown in Figure 24.71.



$$C = \frac{Q}{V_a - V_c} = \frac{Q}{V_{ac}}.$$
$$V_{ac} = V_{ab} + V_{bc}.$$

Figure 24.71

EXECUTE: The electric field in the absence of any dielectric is $E_0 = \dfrac{Q}{\epsilon_0 A}$. In the first dielectric the electric field is reduced to $E_1 = \dfrac{E_0}{K_1} = \dfrac{Q}{K_1\epsilon_0 A}$ and $V_{ab} = E_1\left(\dfrac{d}{2}\right) = \dfrac{Qd}{K_1 2\epsilon_0 A}$. In the second dielectric the electric field is reduced to $E_2 = \dfrac{E_0}{K_2} = \dfrac{Q}{K_2\epsilon_0 A}$ and $V_{bc} = E_2\left(\dfrac{d}{2}\right) = \dfrac{Qd}{K_2 2\epsilon_0 A}$. Thus $V_{ac} = V_{ab} + V_{bc} = \dfrac{Qd}{K_1 2\epsilon_0 A} + \dfrac{Qd}{K_2 2\epsilon_0 A} = \dfrac{Qd}{2\epsilon_0 A}\left(\dfrac{1}{K_1} + \dfrac{1}{K_2}\right)$.

$V_{ac} = \dfrac{Qd}{2\epsilon_0 A}\left(\dfrac{K_1 + K_2}{K_1 K_2}\right)$. This gives $C = \dfrac{Q}{V_{ac}} = Q\left(\dfrac{2\epsilon_0 A}{Qd}\right)\left(\dfrac{K_1 K_2}{K_1 + K_2}\right) = \dfrac{2\epsilon_0 A}{d}\left(\dfrac{K_1 K_2}{K_1 + K_2}\right)$.

EVALUATE: An equivalent way to calculate C is to consider the capacitor to be two in series, one with dielectric constant K_1 and the other with dielectric constant K_2 and both with plate separation $d/2$. (Can imagine inserting a thin conducting plate between the dielectric slabs.)

$$C_1 = K_1 \frac{\epsilon_0 A}{d/2} = 2K_1 \frac{\epsilon_0 A}{d}$$

$$C_2 = K_2 \frac{\epsilon_0 A}{d/2} = 2K_2 \frac{\epsilon_0 A}{d}$$

Since they are in series the total capacitance C is given by $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$ so $C = \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{2\epsilon_0 A}{d}\left(\dfrac{K_1 K_2}{K_1 + K_2}\right)$

24.72. **IDENTIFY:** This situation is analogous to having two capacitors in parallel, each with an area $A/2$.

SET UP: For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$. For a parallel-plate capacitor with plates of area $A/2$, $C = \dfrac{\epsilon_0 (A/2)}{d}$.

EXECUTE: $C_{\text{eq}} = C_1 + C_2 = \dfrac{\epsilon_0 A/2}{d} + \dfrac{\epsilon_0 A/2}{d} = \dfrac{\epsilon_0 A}{2d}(K_1 + K_2)$

EVALUATE: If $K_1 = K_2$, $C_{\text{eq}} = K\dfrac{\epsilon_0 A}{d}$, which is Eq.(24.19).

24.73. **IDENTIFY** and **SET UP:** Show the transformation from one circuit to the other:

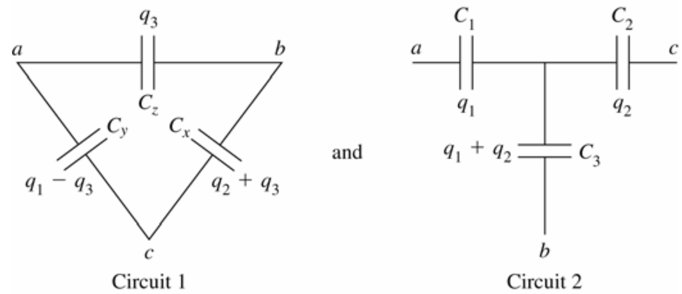


Figure 24.73a

EXECUTE: **(a)** Consider the two networks shown in Figure 24.73a. From Circuit 1: $V_{ac} = \dfrac{q_1 - q_3}{C_y}$ and $V_{bc} = \dfrac{q_2 + q_3}{C_x}$.

q_3 is derived from V_{ab}: $V_{ab} = \dfrac{q_3}{C_z} = \dfrac{q_1 - q_3}{C_y} = \dfrac{q_2 - q_3}{C_x}$. This gives $q_3 = \dfrac{C_x C_y C_z}{C_x + C_y + C_z}\left(\dfrac{q_1}{C_y} - \dfrac{q_2}{C_x}\right) \equiv K\left(\dfrac{q_1}{C_y} - \dfrac{q_2}{C_x}\right)$.

From Circuit 2: $V_{ac} = \dfrac{q_1}{C_1} + \dfrac{q_1 + q_2}{C_3} = q_1\left(\dfrac{1}{C_1} + \dfrac{1}{C_3}\right) + q_2\dfrac{1}{C_3}$ and $V_{bc} = \dfrac{q_2}{C_2} + \dfrac{q_1 + q_2}{C_3} = q_1\dfrac{1}{C_3} + q_2\left(\dfrac{1}{C_2} + \dfrac{1}{C_3}\right)$. Setting the coefficients of the charges equal to each other in matching potential equations from the two circuits results in three independent equations relating the two sets of capacitances. The set of equations are $\dfrac{1}{C_1} = \dfrac{1}{C_y}\left(1 - \dfrac{1}{KC_y} - \dfrac{1}{KC_x}\right)$,

$\dfrac{1}{C_2} = \dfrac{1}{C_x}\left(1 - \dfrac{1}{KC_y} - \dfrac{1}{KC_x}\right)$ and $\dfrac{1}{C_3} = \dfrac{1}{KC_y C_x}$. From these, subbing in the expression for K, we get

$C_1 = (C_xC_y + C_yC_z + C_zC_x)/C_x$, $C_2 = (C_xC_y + C_yC_z + C_zC_x)/C_y$ and $C_3 = (C_xC_y + C_yC_z + C_zC_x)/C_z$.

(b) Using the transformation of part (a) we have the equivalent networks shown in Figure 24.73b:

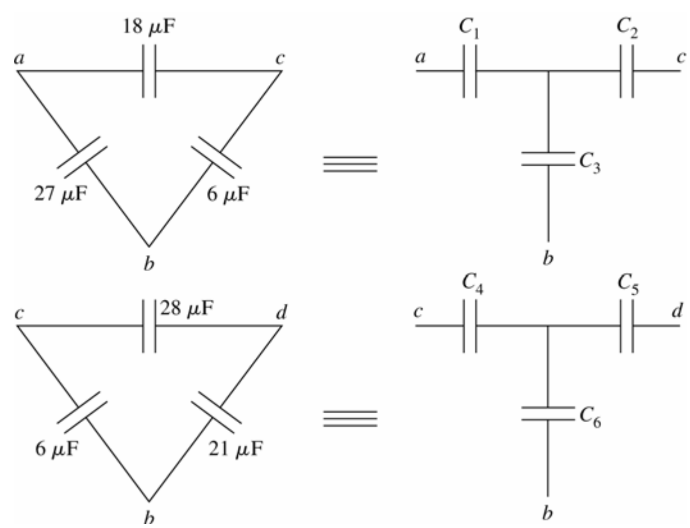


Figure 24.73b

$C_1 = 126\ \mu\text{F}$, $C_2 = 28\ \mu\text{F}$, $C_3 = 42\ \mu\text{F}$, $C_4 = 42\ \mu\text{F}$, $C_5 = 147\ \mu\text{F}$ and $C_6 = 32\ \mu\text{F}$. The total equivalent capacitance is $C_{\text{eq}} = \left(\dfrac{1}{72\ \mu\text{F}} + \dfrac{1}{126\ \mu\text{F}} + \dfrac{1}{34.8\ \mu\text{F}} + \dfrac{1}{147\ \mu\text{F}} + \dfrac{1}{72\ \mu\text{F}}\right)^{-1} = 14.0\ \mu\text{F}$, where the $34.8\ \mu\text{F}$ comes from

$$34.8\ \mu\text{F} = \left(\left(\frac{1}{42\ \mu\text{F}} + \frac{1}{32\ \mu\text{F}}\right)^{-1} + \left(\frac{1}{28\ \mu\text{F}} + \frac{1}{42\ \mu\text{F}}\right)^{-1}\right).$$

(c) The circuit diagram can be redrawn as shown in Figure 25.73c. The overall charge is given by $Q = C_{\text{eq}}V = (14.0\ \mu\text{F})(36\ \text{V}) = 5.04\times10^{-4}\ \text{C}$. And this is also the charge on the $72\ \mu\text{F}$ capacitors, so

$V_{72} = \dfrac{5.04\times10^{-4}\ \text{C}}{72\times10^{-6}\ \text{F}} = 7.0\ \text{V}$.

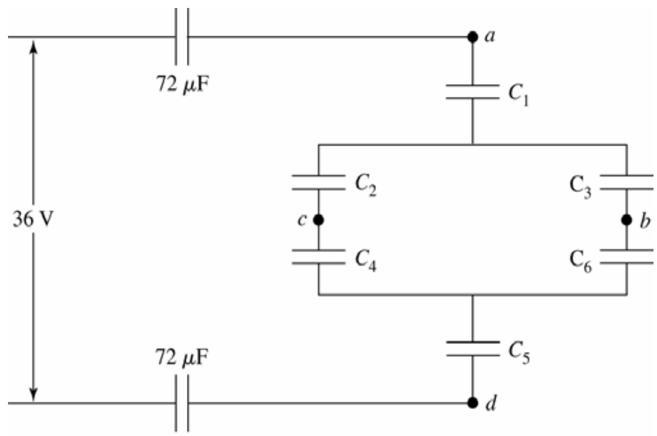


Figure 24.73c

Next we will find the voltage over the numbered capacitors, and their associated voltages. Then those voltages will be changed back into voltage of the original capacitors, and then their charges. $Q_{C_1} = Q_{C_5} = Q_{72} = 5.04\times10^{-4}\ \text{C}$.

$V_{C_5} = \dfrac{5.04\times10^{-4}\ \text{C}}{147\times10^{-6}\ \text{F}} = 3.43\ \text{V}$ and $V_{C_1} = \dfrac{5.04\times10^{-4}\text{C}}{126\times10^{-6}\ \text{F}} = 4.00\ \text{V}$. Therefore,

$V_{C_2C_4} = V_{C_3C_8} = (36.0 - 7.00 - 7.00 - 4.00 - 3.43)\ \text{V} = 14.6\ \text{V}$. But $C_{\text{eq}}(C_2C_4) = \left(\dfrac{1}{C_2} + \dfrac{1}{C_4}\right)^{-1} = 16.8\ \mu\text{F}$ and

$C_{\text{eq}}(C_3C_6) = \left(\dfrac{1}{C_3} + \dfrac{1}{C_6}\right)^{-1} = 18.2\ \mu\text{F}$, so $Q_{C_2} = Q_{C_4} = V_{C_2C_4}C_{\text{eq}(C_2C_4)} = 2.45\times10^{-4}\ \text{C}$ and

$Q_{C_3} = Q_{C_6} = V_{C_3C_6}C_{\text{eq}(C_3C_6)} = 2.64\times10^{-4}\ \text{C}$. Then $V_{C_2} = \dfrac{Q_{C_2}}{C_2} = 8.8\ \text{V}$, $V_{C_3} = \dfrac{Q_{C_3}}{C_3} = 6.3\ \text{V}$, $V_{C_4} = \dfrac{Q_{C_4}}{C_4} = 5.8\ \text{V}$ and

$V_{C_6} = \dfrac{Q_{C_6}}{C_6} = 8.3\ \text{V}$. $V_{ac} = V_{C_1} + V_{C_2} = V_{18} = 13\ \text{V}$ and $Q_{18} = C_{18}V_{18} = 2.3\times10^{-4}\ \text{C}$. $V_{ab} = V_{C_1} + V_{C_3} = V_{27} = 10\ \text{V}$ and $Q_{27} = C_{27}V_{27} = 2.8\times10^{-4}\ \text{C}$. $V_{cd} = V_{C_4} + V_{C_5} = V_{28} = 9\ \text{V}$ and $Q_{28} = C_{28}V_{28} = 2.6\times10^{-4}\ \text{C}$. $V_{bd} = V_{C_5} + V_{C_6} = V_{21} = 12\ \text{V}$ and $Q_{21} = C_{21}V_{21} = 2.5\times10^{-4}\ \text{C}$. $V_{bc} = V_{C_3} - V_{C_2} = V_6 = 2.5\ \text{V}$ and $Q_6 = C_6V_6 = 1.5\times10^{-5}\ \text{C}$.

EVALUATE: Note that $2V_{72} + V_{18} + V_{28} = 2(7.0\ \text{V}) + 13\ \text{V} + 9\ \text{V} = 36\ \text{V},$ as it should.

24.74. IDENTIFY: The force on one plate is due to the electric field of the other plate. The electrostatic force must be balanced by the forces from the springs.

SET UP: The electric field due to one plate is $E = \dfrac{\sigma}{2\epsilon_0}$. The force exerted by a spring compressed a distance $z_0 - z$ from equilibrium is $k(z_0 - z)$.

EXECUTE: **(a)** The force between the two parallel plates is $F = qE = \dfrac{q\sigma}{2\epsilon_0} = \dfrac{q^2}{2\epsilon_0 A} = \dfrac{(CV)^2}{2\epsilon_0 A} = \dfrac{\epsilon_0^2 A^2}{z^2}\dfrac{V^2}{2\epsilon_0 A} = \dfrac{\epsilon_0 A V^2}{2z^2}$.

(b) When $V = 0,$ the separation is just $z_0.$ When $V \neq 0$, the total force from the four springs must equal the electrostatic force calculated in part (a). $F_{4\,\text{springs}} = 4k(z_0 - z) = \dfrac{\epsilon_0 A V^2}{2z^2}$ and $2z^3 - 2z^3 z_0 + \dfrac{\epsilon_0 A V^2}{4k} = 0$.

(c) For $A = 0.300\ \text{m}^2$, $z_0 = 1.2\times10^{-3}\ \text{m}$, $k = 25\ \text{N/m}$ and $V = 120\ \text{V}$, so $2z^3 - (2.4\times10^{-3}\ \text{m})z^2 + 3.82\times10^{-10}\ \text{m}^3 = 0$. The physical solutions to this equation are $z = 0.537$ mm and 1.014 mm.

EVALUATE: **(d)** Stable equilibrium occurs if a slight displacement from equilibrium yields a force back toward the equilibrium point. If one evaluates the forces at small displacements from the equilibrium positions above, the 1.014 mm separation is seen to be stable, but not the 0.537 mm separation.

24.75. **IDENTIFY:** The system can be considered to be two capacitors in parallel, one with plate area $L(L-x)$ and air between the plates and one with area Lx and dielectric filling the space between the plates.

SET UP: $C=\dfrac{K\epsilon_0 A}{d}$ for a parallel-plate capacitor with plate area *A*.

EXECUTE: **(a)** $C=\dfrac{\epsilon_0}{D}((L-x)L+xKL)=\dfrac{\epsilon_0 L}{D}(L+(K-1)x)$

(b) $dU=\frac{1}{2}(dC)V^2$, where $C=C_0+\dfrac{\epsilon_0 L}{D}(-dx+dxK)$, with $C_0=\dfrac{\epsilon_0 L}{D}(L+(K-1)x)$. This gives $dU=\frac{1}{2}\left(\dfrac{\epsilon_0 L dx}{D}(K-1)\right)V^2=\dfrac{(K-1)\epsilon_0 V^2 L}{2D}dx$.

(c) If the charge is kept constant on the plates, then $Q=\dfrac{\epsilon_0 LV}{D}(L+(K-1)x)$ and $U=\frac{1}{2}CV^2=\frac{1}{2}C_0V^2\left(\dfrac{C}{C_0}\right)$.

$U\approx\dfrac{C_0V^2}{2}\left(1-\dfrac{\epsilon_0 L}{DC_0}(K-1)dx\right)$ and $\Delta U=U-U_0=-\dfrac{(K-1)\epsilon_0 V^2 L}{2D}dx$.

(d) Since $dU=-Fdx=-\dfrac{(K-1)\epsilon_0 V^2 L}{2D}dx$, the force is in the opposite direction to the motion dx, meaning that the slab feels a force pushing it out.

EVALUATE: **(e)** When the plates are connected to the battery, the plates plus slab are not an isolated system. In addition to the work done on the slab by the charges on the plates, energy is also transferred between the battery and the plates. Comparing the results for dU in part (c) to $dU=-Fdx$ gives $F=\dfrac{(K-1)\epsilon_0 V^2 L}{2D}$.

24.76. **IDENTIFY:** $C=Q/V$. Apply Gauss's law and the relation between potential difference and electric field.

SET UP: Each conductor is an equipotential surface. $V_a-V_b=\int_{r_0}^{r_b}\vec{E}_U\cdot d\vec{r}=\int_{r_a}^{r_b}\vec{E}_L\cdot d\vec{r}$, so $E_U=E_L$, where these are the fields between the upper and lower hemispheres. The electric field is the same in the air space as in the dielectric.

EXECUTE: **(a)** For a normal spherical capacitor with air between the plates, $C_0=4\pi\epsilon_0\left(\dfrac{r_ar_b}{r_b-r_a}\right)$. The capacitor in this problem is equivalent to two parallel capacitors, C_L and C_U, each with half the plate area of the normal capacitor. $C_L=\dfrac{KC_0}{2}=2\pi K\epsilon_0\left(\dfrac{r_ar_b}{r_b-r_a}\right)$ and $C_U=\dfrac{C_0}{2}=2\pi\epsilon_0\left(\dfrac{r_ar_b}{r_b-r_a}\right)$. $C=C_U+C_L=2\pi\epsilon_0(1+K)\left(\dfrac{r_ar_b}{r_b-r_a}\right)$.

(b) Using a hemispherical Gaussian surface for each respective half, $E_L\dfrac{4\pi r^2}{2}=\dfrac{Q_L}{K\epsilon_0}$, so $E_L=\dfrac{Q_L}{2\pi K\epsilon_0 r^2}$, and $E_U\dfrac{4\pi r^2}{2}=\dfrac{Q_U}{\epsilon_0}$, so $E_U=\dfrac{Q_U}{2\pi\epsilon_0 r^2}$. But $Q_L=VC_L$ and $Q_U=VC_U$. Also, $Q_L+Q_U=Q$. Therefore, $Q_L=\dfrac{VC_0K}{2}=KQ_U$ and $Q_U=\dfrac{Q}{1+K}$, $Q_L=\dfrac{KQ}{1+K}$. This gives $E_L=\dfrac{KQ}{1+K}\dfrac{1}{2\pi K\epsilon_0 r^2}=\dfrac{2}{1+K}\dfrac{Q}{4\pi\epsilon_0 r^2}$ and

$E_U=\dfrac{Q}{1+K}\dfrac{1}{2\pi K\epsilon_0 r^2}=\dfrac{2}{1+K}\dfrac{Q}{4\pi K\epsilon_0 r^2}$. We do find that $E_U=E_L$.

(c) The free charge density on upper and lower hemispheres are: $(\sigma_{f,r_a})_U=\dfrac{Q_U}{2\pi r_a^2}=\dfrac{Q}{2\pi r_a^2(1+K)}$ and

$(\sigma_{f,r_b})_U=\dfrac{Q_U}{2\pi r_b^2}=\dfrac{Q}{2\pi r_b^2(1+K)}$; $(\sigma_{f,r_a})_L=\dfrac{Q_L}{2\pi r_a^2}=\dfrac{KQ}{2\pi r_a^2(1+K)}$ and $(\sigma_{f,r_b})_L=\dfrac{Q_L}{2\pi r_b^2}=\dfrac{KQ}{2\pi r_b^2(1+K)}$.

(d) $\sigma_{i,r_a}=\sigma_{f,r_a}(1-1/K)=\left(\dfrac{(K-1)}{K}\right)\dfrac{Q}{2\pi r_a^2}\left(\dfrac{K}{K+1}\right)=\left(\dfrac{K-1}{K+1}\right)\dfrac{Q}{2\pi r_a^2}$

$\sigma_{i,r_b}=\sigma_{f,r_b}(1-1/K)=\left(\dfrac{(K-1)}{K}\right)\dfrac{Q}{2\pi r_b^2}\left(\dfrac{K}{K+1}\right)=\left(\dfrac{K-1}{K+1}\right)\dfrac{Q}{2\pi r_b^2}$

(e) There is zero bound charge on the flat surface of the dielectric-air interface, or else that would imply a circumferential electric field, or that the electric field changed as we went around the sphere.

EVALUATE: The charge is not equally distributed over the surface of each conductor. There must be more charge on the lower half, by a factor of *K*, because the polarization of the dielectric means more free charge is needed on the lower half to produce the same electric field.

24.77. **IDENTIFY:** The object is equivalent to two identical capacitors in parallel, where each has the same area *A*, plate separation *d* and dielectric with dielectric constant *K*.

SET UP: For each capacitor in the parallel combination, $C = \dfrac{\epsilon_0 A}{d}$.

EXECUTE: **(a)** The charge distribution on the plates is shown in Figure 24.77.

(b) $C = 2\left(\dfrac{\epsilon_0 A}{d}\right) = \dfrac{2(4.2)\epsilon_0(0.120\text{ m})^2}{4.5\times10^{-4}\text{ m}} = 2.38\times10^{-9}\text{ F}$.

EVALUATE: If two of the plates are separated by both sheets of paper to form a capacitor, $C = \dfrac{\epsilon_0 A}{2d} = \dfrac{2.38\times10^{-9}\text{ F}}{4}$, smaller by a factor of 4 compared to the capacitor in the problem.

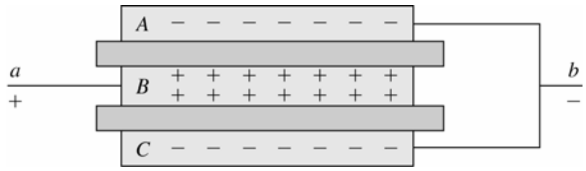


Figure 24.77

24.78. **IDENTIFY:** As in Problem 24.72, the system is equivalent to two capacitors in parallel. One of the capacitors has plate separation *d*, plate area $w(L-h)$ and air between the plates. The other has the same plate separation *d*, plate area *wh* and dielectric constant *K*.

SET UP: Define K_{eff} by $C_{\text{eq}} = \dfrac{K_{\text{eff}}\epsilon_0 A}{d}$, where $A = wL$. For two capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$.

EXECUTE: **(a)** The capacitors are in parallel, so $C = \dfrac{\epsilon_0 w(L-h)}{d} + \dfrac{K\epsilon_0 wh}{d} = \dfrac{\epsilon_0 wL}{d}\left(1 + \dfrac{Kh}{L} - \dfrac{h}{L}\right)$. This gives $K_{\text{eff}} = \left(1 + \dfrac{Kh}{L} - \dfrac{h}{L}\right)$.

(b) For gasoline, with $K = 1.95$: $\dfrac{1}{4}$ full: $K_{\text{eff}}\left(h = \dfrac{L}{4}\right) = 1.24$; $\dfrac{1}{2}$ full: $K_{\text{eff}}\left(h = \dfrac{L}{2}\right) = 1.48$; $\dfrac{3}{4}$ full: $K_{\text{eff}}\left(h = \dfrac{3L}{4}\right) = 1.71$.

(c) For methanol, with $K = 33$: $\dfrac{1}{4}$ full: $K_{\text{eff}}\left(h = \dfrac{L}{4}\right) = 9$; $\dfrac{1}{2}$ full: $K_{\text{eff}}\left(h = \dfrac{L}{2}\right) = 17$; $\dfrac{3}{4}$ full: $K_{\text{eff}}\left(h = \dfrac{3L}{4}\right) = 25$.

(d) This kind of fuel tank sensor will work best for methanol since it has the greater range of K_{eff} values.

EVALUATE: When $h = 0$, $K_{\text{eff}} = 1$. When $h = L$, $K_{\text{eff}} = K$.