

THE SECOND LAW OF THERMODYNAMICS

20.1. IDENTIFY: For a heat engine, $W = |Q_H| - |Q_C|$. $e = \frac{W}{Q_H}$. $Q_H > 0$, $Q_C < 0$.

SET UP: $W = 2200 \text{ J}$. $|Q_C| = 4300 \text{ J}$.

EXECUTE: (a) $Q_H = W + |Q_C| = 6500 \text{ J}$.

(b) $e = \frac{2200 \text{ J}}{6500 \text{ J}} = 0.34 = 34\%$.

EVALUATE: Since the engine operates on a cycle, the net Q equal the net W . But to calculate the efficiency we use the heat energy input, Q_H .

20.2. IDENTIFY: For a heat engine, $W = |Q_H| - |Q_C|$. $e = \frac{W}{Q_H}$. $Q_H > 0$, $Q_C < 0$.

SET UP: $|Q_H| = 9000 \text{ J}$. $|Q_C| = 6400 \text{ J}$.

EXECUTE: (a) $W = 9000 \text{ J} - 6400 \text{ J} = 2600 \text{ J}$.

(b) $e = \frac{W}{Q_H} = \frac{2600 \text{ J}}{9000 \text{ J}} = 0.29 = 29\%$.

EVALUATE: Since the engine operates on a cycle, the net Q equal the net W . But to calculate the efficiency we use the heat energy input, Q_H .

20.3. IDENTIFY and SET UP: The problem deals with a heat engine. $W = +3700 \text{ W}$ and $Q_H = +16,100 \text{ J}$. Use Eq.(20.4) to calculate the efficiency e and Eq.(20.2) to calculate $|Q_C|$. Power = W/t .

EXECUTE: (a) $e = \frac{\text{work output}}{\text{heat energy input}} = \frac{W}{Q_H} = \frac{3700 \text{ J}}{16,100 \text{ J}} = 0.23 = 23\%$.

(b) $W = Q = |Q_H| - |Q_C|$

Heat discarded is $|Q_C| = |Q_H| - W = 16,100 \text{ J} - 3700 \text{ J} = 12,400 \text{ J}$.

(c) Q_H is supplied by burning fuel; $Q_H = mL_c$ where L_c is the heat of combustion.

$m = \frac{Q_H}{L_c} = \frac{16,100 \text{ J}}{4.60 \times 10^4 \text{ J/g}} = 0.350 \text{ g}$.

(d) $W = 3700 \text{ J}$ per cycle

In $t = 1.00 \text{ s}$ the engine goes through 60.0 cycles.

$P = W/t = 60.0(3700 \text{ J})/1.00 \text{ s} = 222 \text{ kW}$

$P = (2.22 \times 10^5 \text{ W})(1 \text{ hp}/746 \text{ W}) = 298 \text{ hp}$

EVALUATE: $Q_C = -12,400 \text{ J}$. In one cycle $Q_{\text{tot}} = Q_C + Q_H = 3700 \text{ J}$. This equals W_{tot} for one cycle.

20.4. IDENTIFY: $W = |Q_H| - |Q_C|$. $e = \frac{W}{Q_H}$. $Q_H > 0$, $Q_C < 0$.

SET UP: For 1.00 s, $W = 180 \times 10^3 \text{ J}$.

EXECUTE: (a) $Q_H = \frac{W}{e} = \frac{180 \times 10^3 \text{ J}}{0.280} = 6.43 \times 10^5 \text{ J}$.

(b) $|Q_C| = |Q_H| - W = 6.43 \times 10^5 \text{ J} - 1.80 \times 10^5 \text{ J} = 4.63 \times 10^5 \text{ J}$.

EVALUATE: Of the $6.43 \times 10^5 \text{ J}$ of heat energy supplied to the engine each second, $1.80 \times 10^5 \text{ J}$ is converted to mechanical work and the remaining $4.63 \times 10^5 \text{ J}$ is discarded into the low temperature reservoir.

20.5. IDENTIFY: $W = |Q_H| - |Q_C|$. $e = \frac{W}{Q_H}$. $Q_H > 0$, $Q_C < 0$. Dividing by t gives equivalent equations for the rate of heat flows and power output.

SET UP: $W/t = 330 \text{ MW}$. $|Q_H|/t = 1300 \text{ MW}$.

EXECUTE: (a) $e = \frac{W}{Q_H} = \frac{W/t}{Q_H/t} = \frac{330 \text{ MW}}{1300 \text{ MW}} = 0.25 = 25\%$.

(b) $|Q_C| = |Q_H| - W$ so $|Q_C|/t = |Q_H|/t - W/t = 1300 \text{ MW} - 330 \text{ MW} = 970 \text{ MW}$.

EVALUATE: The equations for e and W have the same form when written in terms of power output and rate of heat flow.

20.6. IDENTIFY: Apply $e = 1 - \frac{1}{r^{\gamma-1}}$. $e = 1 - \frac{|Q_C|}{|Q_H|}$.

SET UP: In part (b), $Q_H = 10,000 \text{ J}$. The heat discarded is $|Q_C|$.

EXECUTE: (a) $e = 1 - \frac{1}{9.50^{0.40}} = 0.594 = 59.4\%$.

(b) $|Q_C| = |Q_H|(1 - e) = (10,000 \text{ J})(1 - 0.594) = 4060 \text{ J}$.

EVALUATE: The work output of the engine is $W = |Q_H| - |Q_C| = 10,000 \text{ J} - 4060 \text{ J} = 5940 \text{ J}$

20.7. IDENTIFY: $e = 1 - \frac{1}{r^{\gamma-1}}$.

SET UP: $\gamma = 1.40$ and $e = 0.650$.

EXECUTE: $\frac{1}{r^{\gamma-1}} = 1 - e = 0.350$. $r^{0.40} = \frac{1}{0.350}$ and $r = 13.8$.

EVALUATE: e increases when r increases.

20.8. IDENTIFY: $e = 1 - r^{1-\gamma}$

SET UP: r is the compression ratio.

EXECUTE: (a) $e = 1 - (8.8)^{-0.40} = 0.581$, which rounds to 58%.

(b) $e = 1 - (9.6)^{-0.40} = 0.595$ an increase of 1.4%.

EVALUATE: An increase in r gives an increase in e .

20.9. IDENTIFY and SET UP: For the refrigerator $K = 2.10$ and $Q_C = +3.4 \times 10^4 \text{ J}$. Use Eq.(20.9) to calculate $|W|$ and then Eq.(20.2) to calculate Q_H .

(a) **EXECUTE:** Performance coefficient $K = Q_C/|W|$ (Eq.20.9)

$|W| = Q_C/K = 3.40 \times 10^4 \text{ J}/2.10 = 1.62 \times 10^4 \text{ J}$

(b) **SET UP:** The operation of the device is illustrated in Figure 20.9

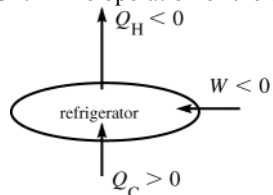


Figure 20.9

EXECUTE:

$$W = Q_C + Q_H$$

$$Q_H = W - Q_C$$

$$Q_H = -1.62 \times 10^4 \text{ J} - 3.40 \times 10^4 \text{ J} = -5.02 \times 10^4 \text{ J}$$

(negative because heat goes out of the system)

EVALUATE $|Q_H| = |W| + |Q_C|$. The heat $|Q_H|$ delivered to the high temperature reservoir is greater than the heat taken in from the low temperature reservoir.

20.10. IDENTIFY: $K = \frac{|Q_C|}{|W|}$ and $|Q_H| = |Q_C| + |W|$.

SET UP: The heat removed from the room is $|Q_C|$ and the heat delivered to the hot outside is $|Q_H|$.

$|W| = (850 \text{ J/s})(60.0 \text{ s}) = 5.10 \times 10^4 \text{ J}$.

EXECUTE: (a) $|Q_C| = K|W| = (2.9)(5.10 \times 10^4 \text{ J}) = 1.48 \times 10^5 \text{ J}$

(b) $|Q_H| = |Q_C| + |W| = 1.48 \times 10^5 \text{ J} + 5.10 \times 10^4 \text{ J} = 1.99 \times 10^5 \text{ J}$.

EVALUATE: (c) $|Q_H| = |Q_C| + |W|$, so $|Q_H| > |Q_C|$.

20.11. IDENTIFY and SET UP: Apply Eq.(20.2) to the cycle and calculate $|W|$ and then $P = |W|/t$. Section 20.4 shows that $EER = (3.413)K$.

(a) The operation of the device is illustrated in Figure 20.11.

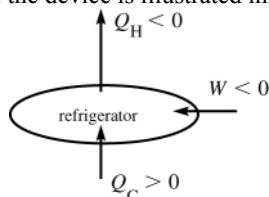


Figure 20.11

EXECUTE:

$$Q_C = +9.80 \times 10^4 \text{ J}$$

$$Q_H = -1.44 \times 10^5 \text{ J}$$

$$W = Q_C + Q_H = +9.80 \times 10^4 \text{ J} - 1.44 \times 10^5 \text{ J} = -4.60 \times 10^4 \text{ J}$$

$$P = W/t = -4.60 \times 10^4 \text{ J}/60.0 \text{ s} = -767 \text{ W}$$

(b) $EER = (3.413)K$

$$K = |Q_C|/|W| = 9.80 \times 10^4 \text{ J}/4.60 \times 10^4 \text{ J} = 2.13$$

$$EER = (3.413)(2.13) = 7.27$$

EVALUATE: W negative means power is consumed, not produced, by the device.

$$|Q_H| = |W| + |Q_C|.$$

20.12. IDENTIFY: $|Q_H| = |Q_C| + |W|$. $K = \frac{|Q_C|}{W}$.

SET UP: For water, $c_w = 4190 \text{ J/kg} \cdot \text{K}$ and $L_f = 3.34 \times 10^5 \text{ J/kg}$. For ice, $c_{ice} = 2010 \text{ J/kg} \cdot \text{K}$.

EXECUTE: (a) $Q = mc_{ice}\Delta T_{ice} - mL_f + mc_w\Delta T_w$.

$$Q = (1.80 \text{ kg})([2010 \text{ J/kg} \cdot \text{K}][-5.0 \text{ C}^\circ] - 3.34 \times 10^5 \text{ J/kg} + [4190 \text{ J/kg} \cdot \text{K}][-25.0 \text{ C}^\circ]) = -8.08 \times 10^5 \text{ J}$$

$Q = -8.08 \times 10^5 \text{ J}$. Q is negative for the water since heat is removed from it.

$$(b) |Q_C| = 8.08 \times 10^5 \text{ J}. W = \frac{|Q_C|}{K} = \frac{8.08 \times 10^5 \text{ J}}{2.40} = 3.37 \times 10^5 \text{ J}.$$

$$(c) |Q_H| = 8.08 \times 10^5 \text{ J} + 3.37 \times 10^5 \text{ J} = 1.14 \times 10^6 \text{ J}.$$

EVALUATE: For this device, $Q_C > 0$ and $Q_H < 0$. More heat is rejected to the room than is removed from the water.

20.13. IDENTIFY: Use Eq.(20.2) to calculate $|W|$. Since it is a Carnot device we can use Eq.(20.13) to relate the heat flows out of the reservoirs. The reservoir temperatures can be used in Eq.(20.14) to calculate e .

(a) **SET UP:** The operation of the device is sketched in Figure 20.13.

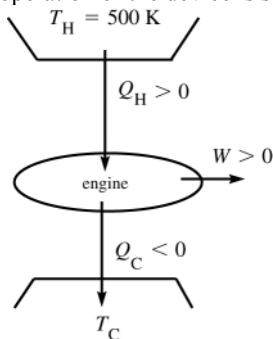


Figure 20.13

EXECUTE:

$$W = Q_C + Q_H$$

$$W = -335 \text{ J} + 550 \text{ J} = 215 \text{ J}$$

(b) For a Carnot cycle, $\frac{|Q_C|}{|Q_H|} = \frac{T_C}{T_H}$ (Eq.20.13)

$$T_C = T_H \frac{|Q_C|}{|Q_H|} = 620 \text{ K} \left(\frac{335 \text{ J}}{550 \text{ J}} \right) = 378 \text{ K}$$

(c) $e(\text{Carnot}) = 1 - T_C/T_H = 1 - 378 \text{ K}/620 \text{ K} = 0.390 = 39.0\%$

EVALUATE: We could use the underlying definition of e (Eq.20.4):

$$e = W/Q_H = (215 \text{ J})/(550 \text{ J}) = 39\%, \text{ which checks.}$$

20.14. IDENTIFY: $|W| = |Q_H| - |Q_C|$. $Q_C < 0$, $Q_H > 0$. $e = \frac{W}{Q_H}$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$.

SET UP: $T_C = 300 \text{ K}$, $T_H = 520 \text{ K}$. $|Q_H| = 6.45 \times 10^3 \text{ J}$.

EXECUTE: (a) $Q_C = -Q_H \left(\frac{T_C}{T_H} \right) = -(6.45 \times 10^3 \text{ J}) \left(\frac{300 \text{ K}}{520 \text{ K}} \right) = -3.72 \times 10^3 \text{ J}$.

(b) $|W| = |Q_H| - |Q_C| = 6.45 \times 10^3 \text{ J} - 3.72 \times 10^3 \text{ J} = 2.73 \times 10^3 \text{ J}$

(c) $e = \frac{W}{Q_H} = \frac{2.73 \times 10^3 \text{ J}}{6.45 \times 10^3 \text{ J}} = 0.423 = 42.3\%$.

EVALUATE: We can verify that $e = 1 - T_C/T_H$ also gives $e = 42.3\%$.

20.15. IDENTIFY: $e = \frac{W}{Q_H}$ for any engine. For the Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$.

SET UP: $T_C = 20.0^\circ\text{C} + 273.15 \text{ K} = 293.15 \text{ K}$

EXECUTE: (a) $Q_H = \frac{W}{e} = \frac{2.5 \times 10^4 \text{ J}}{0.59} = 4.24 \times 10^4 \text{ J}$

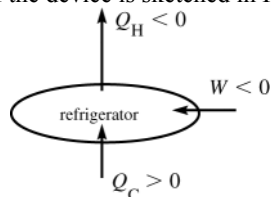
(b) $W = Q_H + Q_C$ so $Q_C = W - Q_H = 2.5 \times 10^4 \text{ J} - 4.24 \times 10^4 \text{ J} = -1.74 \times 10^4 \text{ J}$.

$T_H = -T_C \frac{Q_H}{Q_C} = -(293.15 \text{ K}) \left(\frac{4.24 \times 10^4 \text{ J}}{-1.74 \times 10^4 \text{ J}} \right) = 714 \text{ K} = 441^\circ\text{C}$.

EVALUATE: For a heat engine, $W > 0$, $Q_H > 0$ and $Q_C < 0$.

20.16. IDENTIFY and SET UP: The device is a Carnot refrigerator. We can use Eqs.(20.2) and (20.13).

(a) The operation of the device is sketched in Figure 20.16.



$$T_H = 24.0^\circ\text{C} = 297 \text{ K}$$

$$T_C = 0.0^\circ\text{C} = 273 \text{ K}$$

Figure 20.16

The amount of heat taken out of the water to make the liquid \rightarrow solid phase change is

$Q = -mL_f = -(85.0 \text{ kg})(334 \times 10^3 \text{ J/kg}) = -2.84 \times 10^7 \text{ J}$. This amount of heat must go into the working substance of the refrigerator, so $Q_C = +2.84 \times 10^7 \text{ J}$. For Carnot cycle $|Q_C|/|Q_H| = T_C/T_H$

EXECUTE: $|Q_H| = |Q_C|(T_H/T_C) = 2.84 \times 10^7 \text{ J}(297 \text{ K}/273 \text{ K}) = 3.09 \times 10^7 \text{ J}$

(b) $W = Q_C + Q_H = +2.84 \times 10^7 \text{ J} - 3.09 \times 10^7 \text{ J} = -2.5 \times 10^6 \text{ J}$

EVALUATE: W is negative because this much energy must be supplied to the refrigerator rather than obtained from it. Note that in Eq.(20.13) we must use Kelvin temperatures.

20.17. IDENTIFY: $|Q_H| = |W| + |Q_C|$. $Q_H < 0$, $Q_C > 0$. $K = \frac{|Q_C|}{|W|}$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$.

SET UP: $T_C = 270 \text{ K}$, $T_H = 320 \text{ K}$. $|Q_C| = 415 \text{ J}$.

EXECUTE: (a) $Q_H = -\left(\frac{T_H}{T_C} \right) Q_C = -\left(\frac{320 \text{ K}}{270 \text{ K}} \right) (415 \text{ J}) = -492 \text{ J}$.

(b) For one cycle, $|W| = |Q_H| - |Q_C| = 492 \text{ J} - 415 \text{ J} = 77 \text{ J}$. $P = \frac{(165)(77 \text{ J})}{60 \text{ s}} = 212 \text{ W}$.

(c) $K = \frac{|Q_C|}{|W|} = \frac{415 \text{ J}}{77 \text{ J}} = 5.4$.

EVALUATE: The amount of heat energy $|Q_H|$ delivered to the high-temperature reservoir is greater than the amount of heat energy $|Q_C|$ removed from the low-temperature reservoir.

20.18. IDENTIFY: $|W| = |Q_H| - |Q_C|$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$, where the temperatures must be in kelvins.

SET UP: $-10.0^\circ\text{C} = 263.15\text{ K}$, $25.0^\circ\text{C} = 298.15\text{ K}$, $0.0^\circ\text{C} = 273.15\text{ K}$ and $-25.0^\circ\text{C} = 248.15\text{ K}$.

EXECUTE: (a) The heat is discarded at a higher temperature, and a refrigerator is required. $|Q_H| = |Q_C|(T_H/T_C)$ and $|W| = |Q_C|((T_H/T_C) - 1) = (5.00 \times 10^3\text{ J})((298.15\text{ K}/263.15\text{ K}) - 1) = 665\text{ J}$.

(b) Again, the device is a refrigerator, and $|W| = (5.00 \times 10^3\text{ J})((273.15\text{ K}/263.15\text{ K}) - 1) = 190\text{ J}$.

(c) The device is an engine; the heat is taken from the hot reservoir, and the work done by the engine is $|W| = (5.00 \times 10^3\text{ J})(1 - (248.15\text{ K}/263.15\text{ K})) = 285\text{ J}$.

EVALUATE: For a refrigerator work must be supplied to the device. For a heat engine, there is mechanical work output from the device.

20.19. IDENTIFY: The theoretical maximum performance coefficient is $K_{\text{Carnot}} = \frac{T_C}{T_H - T_C}$. $K = \frac{|Q_C|}{|W|}$. $|Q_C|$ is the heat removed from the water to convert it to ice. For the water, $|Q| = mc_w\Delta T + mL_f$.

SET UP: $T_C = -5.0^\circ\text{C} = 268\text{ K}$. $T_H = 20.0^\circ\text{C} = 293\text{ K}$. $c_w = 4190\text{ J/kg}\cdot\text{K}$ and $L_f = 334 \times 10^3\text{ J/kg}$.

EXECUTE: (a) In one year the freezer operates $(5\text{ h/day})(365\text{ days}) = 1825\text{ h}$.

$$P = \frac{730\text{ kWh}}{1825\text{ h}} = 0.400\text{ kW} = 400\text{ W}.$$

$$(b) K_{\text{Carnot}} = \frac{268\text{ K}}{293\text{ K} - 268\text{ K}} = 10.7$$

(c) $|W| = Pt = (400\text{ W})(3600\text{ s}) = 1.44 \times 10^6\text{ J}$. $|Q_C| = K|W| = 1.54 \times 10^7\text{ J}$. $|Q| = mc_w\Delta T + mL_f$ gives

$$m = \frac{|Q_C|}{c_w\Delta T + L_f} = \frac{1.54 \times 10^7\text{ J}}{(4190\text{ J/kg}\cdot\text{K})(20.0\text{ K}) + 334 \times 10^3\text{ J/kg}} = 36.9\text{ kg}.$$

EVALUATE: For any actual device, $K < K_{\text{Carnot}}$, $|Q_C|$ is less than we calculated and the freezer makes less ice in one hour than the mass we calculated in part (c).

20.20. IDENTIFY: The total work that must be done is $W_{\text{tot}} = mg\Delta y$. $|W| = |Q_H| - |Q_C|$. $Q_H > 0$, $W > 0$ and $Q_C < 0$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$,

SET UP: $T_C = 373\text{ K}$, $T_H = 773\text{ K}$. $|Q_H| = 250\text{ J}$.

EXECUTE: $Q_C = -Q_H\left(\frac{T_C}{T_H}\right) = -(250\text{ J})\left(\frac{373\text{ K}}{773\text{ K}}\right) = -121\text{ J}$. $|W| = 250\text{ J} - 121\text{ J} = 129\text{ J}$. This is the work done in

one cycle. $W_{\text{tot}} = (500\text{ kg})(9.80\text{ m/s}^2)(100\text{ m}) = 4.90 \times 10^5\text{ J}$. The number of cycles required is

$$\frac{W_{\text{tot}}}{|W|} = \frac{4.90 \times 10^5\text{ J}}{129\text{ J/cycle}} = 3.80 \times 10^3\text{ cycles}.$$

EVALUATE: In $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$, the temperatures must be in kelvins.

20.21. IDENTIFY: $e = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ and $e = 1 - \frac{T_C}{T_H}$.

SET UP: $T_H = 800\text{ K}$. $Q_C = -3000\text{ J}$.

EXECUTE: For a heat engine, $Q_H = -Q_C/(1 - e) = -(-3000\text{ J})/(1 - 0.600) = 7500\text{ J}$, and then

$$W = eQ_H = (0.600)(7500\text{ J}) = 4500\text{ J}.$$

EVALUATE: This does not make use of the given value of T_H . If T_H is used, then $T_C = T_H(1 - e) = (800\text{ K})(1 - 0.600) = 320\text{ K}$ and $Q_H = -Q_C T_H/T_C$, which gives the same result.

20.22. IDENTIFY: $W = Q_C + Q_H$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$. For the ice to liquid water phase transition, $Q = mL_f$.

SET UP: For water, $L_f = 334 \times 10^3\text{ J/kg}$

EXECUTE: $Q_C = -mL_f = -(0.0400 \text{ kg})(334 \times 10^3 \text{ J/kg}) = -1.336 \times 10^4 \text{ J}$. $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ gives

$$Q_H = -(T_H/T_C)Q_C = -(-1.336 \times 10^4 \text{ J})[(373.15 \text{ K})/(273.15 \text{ K})] = +1.825 \times 10^4 \text{ J}. \quad W = Q_C + Q_H = 4.89 \times 10^3 \text{ J}.$$

EVALUATE: For a heat engine, Q_C is negative and Q_H is positive. The heat that comes out of the engine ($Q < 0$) goes into the ice ($Q > 0$).

20.23. IDENTIFY: The power output is $P = \frac{W}{t}$. The theoretical maximum efficiency is $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$. $e = \frac{W}{Q_H}$.

SET UP: $Q_H = 1.50 \times 10^4 \text{ J}$. $T_C = 350 \text{ K}$. $T_H = 650 \text{ K}$. $1 \text{ hp} = 746 \text{ W}$.

EXECUTE: $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{350 \text{ K}}{650 \text{ K}} = 0.4615$. $W = eQ_H = (0.4615)(1.50 \times 10^4 \text{ J}) = 6.923 \times 10^3 \text{ J}$; this is the

work output in one cycle. $P = \frac{W}{t} = \frac{(240)(6.923 \times 10^3 \text{ J})}{60.0 \text{ s}} = 2.77 \times 10^4 \text{ W} = 37.1 \text{ hp}$.

EVALUATE: We could also use $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ to calculate $Q_C = -\left(\frac{T_C}{T_H}\right)Q_H = -\left(\frac{350 \text{ K}}{650 \text{ K}}\right)(1.50 \times 10^4 \text{ J}) = -8.08 \times 10^3 \text{ J}$.

Then $W = Q_C + Q_H = 6.92 \times 10^3 \text{ J}$, the same as previously calculated.

20.24. IDENTIFY and SET UP: $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$. $K_{\text{Carnot}} = \frac{T_C}{T_H - T_C}$.

EXECUTE: (a) $T_C = T_H(1 - e)$. $K = \frac{T_H(1 - e)}{T_H - T_H(1 - e)} = \frac{1 - e}{e}$.

EVALUATE: (b) When $e \rightarrow 1$, $K \rightarrow 0$. When $e \rightarrow 0$, $K \rightarrow \infty$.

$e \rightarrow 1$ when $|Q_C| \ll |Q_H|$. $|Q_C|$ is small in this limit. That is good for an engine since $|Q_C|$ is wasted. But it is bad for a refrigerator since $|Q_C|$ is what is useful. $e \rightarrow 0$ when $|Q_C| \rightarrow |Q_H|$ and $|W|$ is very small. That is bad for an engine but good for a refrigerator.

20.25. IDENTIFY: $\Delta S = \frac{Q}{T}$ for each object, where T must be in kelvins. The temperature of each object remains constant.

SET UP: For water, $L_f = 3.34 \times 10^5 \text{ J/kg}$.

EXECUTE: (a) The heat flow into the ice is $Q = mL_f = (0.350 \text{ kg})(3.34 \times 10^5 \text{ J/kg}) = 1.17 \times 10^5 \text{ J}$. The heat flow occurs at $T = 273 \text{ K}$, so $\Delta S = \frac{Q}{T} = \frac{1.17 \times 10^5 \text{ J}}{273 \text{ K}} = 429 \text{ J/K}$. Q is positive and ΔS is positive.

(b) $Q = -1.17 \times 10^5 \text{ J}$ flows out of the heat source, at $T = 298 \text{ K}$. $\Delta S = \frac{Q}{T} = \frac{-1.17 \times 10^5 \text{ J}}{298 \text{ K}} = -393 \text{ J/K}$. Q is negative and ΔS is negative.

(c) $\Delta S_{\text{tot}} = 429 \text{ J/K} + (-393 \text{ J/K}) = +36 \text{ J/K}$.

EVALUATE: For the total isolated system, $\Delta S > 0$ and the process is irreversible.

20.26. IDENTIFY: Apply $Q_{\text{system}} = 0$ to calculate the final temperature. $Q = mc\Delta T$. Example 20.6 shows that $\Delta S = mc \ln(T_2/T_1)$ when an object undergoes a temperature change.

SET UP: For water $c = 4190 \text{ J/kg} \cdot \text{K}$. Boiling water has $T = 100.0^\circ\text{C} = 373 \text{ K}$.

EXECUTE: (a) The heat transfer between 100°C water and 30°C water occurs over a finite temperature difference and the process is irreversible.

(b) $(270 \text{ kg})c(T_2 - 30.0^\circ\text{C}) + (5.00 \text{ kg})c(T_2 - 100^\circ\text{C}) = 0$. $T_2 = 31.27^\circ\text{C} = 304.42 \text{ K}$.

(c) $\Delta S = (270 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{304.42 \text{ K}}{303.15 \text{ K}}\right) + (5.00 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{304.42 \text{ K}}{373.15 \text{ K}}\right)$.

$\Delta S = 4730 \text{ J/K} + (-4265 \text{ J/K}) = +470 \text{ J/K}$.

EVALUATE: $\Delta S_{\text{system}} > 0$, as it should for an irreversible process.

20.27. IDENTIFY: Both the ice and the room are at a constant temperature, so $\Delta S = \frac{Q}{T}$. For the melting phase transition, $Q = mL_f$. Conservation of energy requires that the quantity of heat that goes into the ice is the amount of heat that comes out of the room.

SET UP: For ice, $L_f = 334 \times 10^3 \text{ J/kg}$. When heat flows into an object, $Q > 0$, and when heat flows out of an object, $Q < 0$.

EXECUTE: (a) Irreversible because heat will not spontaneously flow out of 15 kg of water into a warm room to freeze the water.

$$\text{(b) } \Delta S = \Delta S_{\text{ice}} + \Delta S_{\text{room}} = \frac{mL_f}{T_{\text{ice}}} + \frac{-mL_f}{T_{\text{room}}} = \frac{(15.0 \text{ kg})(334 \times 10^3 \text{ J/kg})}{273 \text{ K}} + \frac{-(15.0 \text{ kg})(334 \times 10^3 \text{ J/kg})}{293 \text{ K}}. \quad \Delta S = +1250 \text{ J/K}.$$

EVALUATE: This result is consistent with the answer in (a) because $\Delta S > 0$ for irreversible processes.

- 20.28. IDENTIFY:** $Q = mc\Delta T$ for the water. Example 20.6 shows that $\Delta S = mc \ln(T_2/T_1)$ when an object undergoes a temperature change. $\Delta S = Q/T$ for an isothermal process.

SET UP: For water, $c = 4190 \text{ J/kg} \cdot \text{K}$. $85.0^\circ\text{C} = 358.2 \text{ K}$. $20.0^\circ\text{C} = 293.2 \text{ K}$.

EXECUTE: (a) $\Delta S = mc \ln\left(\frac{T_2}{T_1}\right) = (0.250 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{293.2 \text{ K}}{358.2 \text{ K}}\right) = -210 \text{ J/K}$. Heat comes out of the water and its entropy decreases.

(b) $Q = mc\Delta T = (0.250 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(-65.0 \text{ K}) = -6.81 \times 10^4 \text{ J}$. The amount of heat that goes into the air is $+6.81 \times 10^4 \text{ J}$. For the air, $\Delta S = \frac{Q}{T} = \frac{+6.81 \times 10^4 \text{ J}}{293.1 \text{ K}} = +232 \text{ J/K}$. $\Delta S_{\text{system}} = -210 \text{ J/K} + 232 \text{ J/K} = +22 \text{ J/K}$.

EVALUATE: $\Delta S_{\text{system}} > 0$ and the process is irreversible.

- 20.29. IDENTIFY:** The process is at constant temperature, so $\Delta S = \frac{Q}{T}$. $\Delta U = Q - W$.

SET UP: For an isothermal process of an ideal gas, $\Delta U = 0$ and $Q = W$. For a compression, $\Delta V < 0$ and $W < 0$.

EXECUTE: $Q = W = -1850 \text{ J}$. $\Delta S = \frac{-1850 \text{ J}}{293 \text{ K}} = -6.31 \text{ J/K}$.

EVALUATE: The entropy change of the gas is negative. Heat must be removed from the gas during the compression to keep its temperature constant and therefore the gas is not an isolated system.

- 20.30. IDENTIFY and SET UP:** The initial and final states are at the same temperature, at the normal boiling point of 4.216 K. Calculate the entropy change for the irreversible process by considering a reversible isothermal process that connects the same two states, since ΔS is path independent and depends only on the initial and final states. For the reversible isothermal process we can use Eq.(20.18).

The heat flow for the helium is $Q = -mL_v$, negative since in condensation heat flows out of the helium. The heat of vaporization L_v is given in Table 17.4 and is $L_v = 20.9 \times 10^3 \text{ J/kg}$.

EXECUTE: $Q = -mL_v = -(0.130 \text{ kg})(20.9 \times 10^3 \text{ J/kg}) = -2717 \text{ J}$
 $\Delta S = Q/T = -2717 \text{ J}/4.216 \text{ K} = -644 \text{ J/K}$.

EVALUATE: The system we considered is the 0.130 kg of helium; ΔS is the entropy change of the helium. This is not an isolated system since heat must flow out of it into some other material. Our result that $\Delta S < 0$ doesn't violate the 2nd law since it is not an isolated system. The material that receives the heat that flows out of the helium would have a positive entropy change and the total entropy change would be positive.

- 20.31. IDENTIFY:** Each phase transition occurs at constant temperature and $\Delta S = \frac{Q}{T}$. $Q = mL_v$.

SET UP: For vaporization of water, $L_v = 2256 \times 10^3 \text{ J/kg}$.

EXECUTE: (a) $\Delta S = \frac{Q}{T} = \frac{mL_v}{T} = \frac{(1.00 \text{ kg})(2256 \times 10^3 \text{ J/kg})}{(373.15 \text{ K})} = 6.05 \times 10^3 \text{ J/K}$. Note that this is the change of

entropy of the water as it changes to steam.

(b) The magnitude of the entropy change is roughly five times the value found in Example 20.5.

EVALUATE: Water is less ordered (more random) than ice, but water is far less random than steam; a consideration of the density changes indicates why this should be so.

- 20.32. IDENTIFY:** The phase transition occurs at constant temperature and $\Delta S = \frac{Q}{T}$. $Q = mL_v$. The mass of one mole is the molecular mass M .

SET UP: For water, $L_v = 2256 \times 10^3 \text{ J/kg}$. For N_2 , $M = 28.0 \times 10^{-3} \text{ kg/mol}$, the boiling point is 77.34 K and

$L_v = 201 \times 10^3 \text{ J/kg}$. For silver (Ag), $M = 107.9 \times 10^{-3} \text{ kg/mol}$, the boiling point is 2466 K and $L_v = 2336 \times 10^3 \text{ J/kg}$.

For mercury (Hg), $M = 200.6 \times 10^{-3} \text{ kg/mol}$, the boiling point is 630 K and $L_v = 272 \times 10^3 \text{ J/kg}$.

EXECUTE: (a) $\Delta S = \frac{Q}{T} = \frac{mL_v}{T} = \frac{(18.0 \times 10^{-3} \text{ kg})(2256 \times 10^3 \text{ J/kg})}{(373.15 \text{ K})} = 109 \text{ J/K}$.

(b) $N_2: \frac{(28.0 \times 10^{-3} \text{ kg})(201 \times 10^3 \text{ J/kg})}{(77.34 \text{ K})} = 72.8 \text{ J/K}$. Ag: $\frac{(107.9 \times 10^{-3} \text{ kg})(2336 \times 10^3 \text{ J/kg})}{(2466 \text{ K})} = 102.2 \text{ J/K}$.

Hg: $\frac{(200.6 \times 10^{-3} \text{ kg})(272 \times 10^3 \text{ J/kg})}{(630 \text{ K})} = 86.6 \text{ J/K}$

(c) The results are the same order of magnitude, all around 100 J/K.

EVALUATE: The entropy change is a measure of the increase in randomness when a certain number (one mole) goes from the liquid to the vapor state. The entropy per particle for any substance in a vapor state is expected to be roughly the same, and since the randomness is much higher in the vapor state (see Exercise 20.31), the entropy change per molecule is roughly the same for these substances.

- 20.33. IDENTIFY:** During the phase transition the gallium is at a constant temperature equal to the melting point of gallium. Your hand is at a constant temperature of $98.6^\circ\text{F} = 37.0^\circ\text{C} = 310.1 \text{ K}$. Heat $|Q| = mL_f$ flows out of your hand and into the gallium. For heat flow at constant temperature, $\Delta S = \frac{Q}{T}$.

SET UP: For gallium, $L_f = 8.04 \times 10^4 \text{ J/kg}$ and the melting point is $29.8^\circ\text{C} = 303.0 \text{ K}$.

EXECUTE: $|Q| = mL_f = (25.0 \times 10^{-3} \text{ kg})(8.04 \times 10^4 \text{ J/kg}) = 2.01 \times 10^3 \text{ J}$. For your hand,

$\Delta S = \frac{Q}{T} = \frac{-2.01 \times 10^3 \text{ J}}{310.1 \text{ K}} = -6.48 \text{ J/K}$. Heat flows out of your hand, Q is negative, and ΔS is negative. For the

gallium, $\Delta S = \frac{Q}{303.0 \text{ K}}$. The temperature of the gallium is less than that of your hand and $|Q|$ is the same, so the

magnitude of the entropy change of the gallium is greater than the magnitude of the entropy change of your hand.

EVALUATE: For the gallium, $\Delta S > 0$, so $\Delta S_{\text{system}} > 0$ and the process is irreversible.

- 20.34. IDENTIFY:** Apply Eq.(20.23) and follow the procedure used in Example 20.11.

SET UP: After the partition is punctured each molecule has equal probability of being on each side of the box. The probability of two independent events occurring simultaneously is the product of the probabilities of each separate event.

EXECUTE: (a) On the average, each half of the box will contain half of each type of molecule, 250 of nitrogen and 50 of oxygen.

(b) See Example 20.11. The total change in entropy is

$\Delta S = kN_1 \ln(2) + kN_2 \ln(2) = (N_1 + N_2)k \ln(2) = (600)(1.381 \times 10^{-23} \text{ J/K}) \ln(2) = 5.74 \times 10^{-21} \text{ J/K}$.

(c) The probability is $(1/2)^{500} \times (1/2)^{100} = (1/2)^{600} = 2.4 \times 10^{-181}$, and is not likely to happen. The numerical result for part (c) above may not be obtained directly on some standard calculators. For such calculators, the result may be found by taking the log base ten of 0.5 and multiplying by 600, then adding 181 and then finding 10 to the power of the sum. The result is then $10^{-181} \times 10^{0.87} = 2.4 \times 10^{-181}$.

EVALUATE: The contents of the box constitutes an isolated system. $\Delta S > 0$ and the process is irreversible.

- 20.35. (a) IDENTIFY and SET UP:** The velocity distribution of Eq.(18.32) depends only on T , so in an isothermal process it does not change.

(b) **EXECUTE:** Calculate the change in the number of available microscopic states and apply Eq.(20.23).

Following the reasoning of Example 20.11, the number of possible positions available to each molecule is altered by a factor of 3 (becomes larger). Hence the number of microscopic states the gas occupies at volume $3V$ is

$w_2 = (3)^N w_1$, where N is the number of molecules and w_1 is the number of possible microscopic states at the start of the process, where the volume is V . Then, by Eq.(20.23),

$\Delta S = k \ln(w_2 / w_1) = k \ln(3)^N = Nk \ln(3) = nN_A k \ln(3) = nR \ln(3)$

$\Delta S = (2.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K}) \ln(3) = +18.3 \text{ J/K}$

(c) **IDENTIFY and SET UP:** For an isothermal reversible process $\Delta S = Q/T$.

EXECUTE: Calculate W and then use the first law to calculate Q .

$\Delta T = 0$ implies $\Delta U = 0$, since system is an ideal gas.

Then by $\Delta U = Q - W$, $Q = W$.

For an isothermal process, $W = \int_{V_1}^{V_2} p \, dV = \int_{V_1}^{V_2} (nRT/V) \, dV = nRT \ln(V_2/V_1)$

Thus $Q = nRT \ln(V_2/V_1)$ and $\Delta S = Q/T = nR \ln(V_2/V_1)$

$\Delta S = (2.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K}) \ln(3V_1/V_1) = +18.3 \text{ J/K}$

EVALUATE: This is the same result as obtained in part (b).

20.36. IDENTIFY: Example 20.8 shows that for a free expansion, $\Delta S = nR \ln(V_2/V_1)$.

SET UP: $V_1 = 2.40 \text{ L} = 2.40 \times 10^{-3} \text{ m}^3$

EXECUTE: $\Delta S = (0.100 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K}) \ln\left(\frac{425 \text{ m}^3}{2.40 \times 10^{-3} \text{ m}^3}\right) = 10.0 \text{ J/K}$

EVALUATE: $\Delta S_{\text{system}} > 0$ and the free expansion is irreversible.

20.37. IDENTIFY: $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$. $W = Q_H + Q_C$. $e = \frac{W}{Q_H}$.

SET UP: $pV = nRT$; the 300 K isotherm lies below the 400 K isotherm in the pV -diagram.

EXECUTE: (a) $e_{\text{Carnot}} = 1 - \frac{400 \text{ K}}{500 \text{ K}} = 0.200 = 20.0\%$.

(b) $Q_H = \frac{W}{e} = \frac{2000 \text{ J}}{0.200} = 10,000 \text{ J}$. $|Q_C| = |Q_H| - |W| = 10,000 \text{ J} - 2000 \text{ J} = 8000 \text{ J}$.

(c) The 500 K and 400 K isotherms and the Carnot cycle operating between those isotherms are sketched in Figure 20.37.

(d) The 300 K isotherm and the Carnot cycle operating between the 500 K and 300 K isotherms are also sketched in Figure 20.37.

(e) The cycle with $T_C = 300 \text{ K}$ encloses more area than the cycle with $T_C = 400 \text{ K}$.

(f) Less work is done on the gas during the compression at lower temperature, so less heat is ejected to keep the internal energy and temperature constant.

EVALUATE: For $T_C = 300 \text{ K}$, $e_{\text{Carnot}} = 0.400$. $W = eQ_H = (0.400)(10,000 \text{ J}) = 4000 \text{ J}$. $|Q_C| = 6000 \text{ J}$.

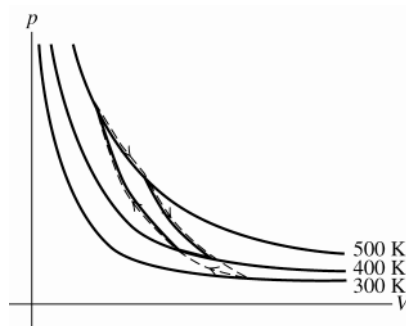


Figure 20.37

20.38. IDENTIFY: $W = Q_C + Q_H$. Since it is a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$. The heat required to melt the ice is $Q = mL_f$.

SET UP: For water, $L_f = 334 \times 10^3 \text{ J/kg}$. $Q_H > 0$, $Q_C < 0$. $Q_C = -mL_f$. $T_H = 527^\circ\text{C} = 800.15 \text{ K}$.

EXECUTE: (a) $Q_H = +400 \text{ J}$, $W = +300 \text{ J}$. $Q_C = W - Q_H = -100 \text{ J}$.

$T_C = -T_H(Q_C/Q_H) = -(800.15 \text{ K})[(-100 \text{ J})/(400 \text{ J})] = +200 \text{ K} = -73^\circ\text{C}$

(b) The total Q_C required is $-mL_f = -(10.0 \text{ kg})(334 \times 10^3 \text{ J/kg}) = -3.34 \times 10^6 \text{ J}$. Q_C for one cycle is -100 J , so

the number of cycles required is $\frac{-3.34 \times 10^6 \text{ J}}{-100 \text{ J/cycle}} = 3.34 \times 10^4$ cycles.

EVALUATE: The results depend only on the maximum temperature of the gas, not on the number of moles or the maximum pressure.

20.39. IDENTIFY: $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$, where T_C and T_H must be in kelvins.

SET UP: $T_C = -90.0^\circ\text{C} = 183 \text{ K}$.

EXECUTE: (a) $T_H = \frac{T_C}{1-e}$. For $e = 0.400$, $T_H = \frac{183 \text{ K}}{1-0.400} = 305 \text{ K}$. For $e = 0.450$, $T_H = \frac{183 \text{ K}}{1-0.450} = 333 \text{ K}$. T_H

must be increased $28 \text{ K} = 28 \text{ C}^\circ$.

(b) $T_C = (1-e)T_H = (1-0.450)(305 \text{ K}) = 168 \text{ K}$. T_C must be decreased $15 \text{ K} = 15 \text{ C}^\circ$.

EVALUATE: A Kelvin degree is the same size as a Celsius degree, so a temperature change ΔT has the same numerical value whether it is expressed in K or in C° .

20.40. IDENTIFY: Use the ideal gas law to calculate p and V for each state. Use the first law and specific expressions for Q , W , and ΔU for each process. Use Eq.(20.4) to calculate e . Q_{H} is the net heat flow into the gas.

SET UP: $\gamma = 1.40$

$C_V = R/(\gamma - 1) = 20.79 \text{ J/mol} \cdot \text{K}$; $C_p = C_V + R = 29.10 \text{ J/mol} \cdot \text{K}$. The cycle is sketched in Figure 20.40.

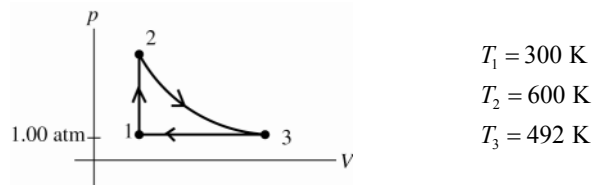


Figure 20.40

$$T_1 = 300 \text{ K}$$

$$T_2 = 600 \text{ K}$$

$$T_3 = 492 \text{ K}$$

EXECUTE: (a) point 1

$$p_1 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa} \quad (\text{given}); \quad pV = nRT;$$

$$V_1 = \frac{nRT_1}{p_1} = \frac{(0.350 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})}{1.013 \times 10^5 \text{ Pa}} = 8.62 \times 10^{-3} \text{ m}^3$$

point 2

$$\text{process } 1 \rightarrow 2 \text{ at constant volume so } V_2 = V_1 = 8.62 \times 10^{-3} \text{ m}^3$$

$$pV = nRT \text{ and } n, R, V \text{ constant implies } p_1/T_1 = p_2/T_2$$

$$p_2 = p_1(T_2/T_1) = (1.00 \text{ atm})(600 \text{ K}/300 \text{ K}) = 2.00 \text{ atm} = 2.03 \times 10^5 \text{ Pa}$$

point 3

Consider the process $3 \rightarrow 1$, since it is simpler than $2 \rightarrow 3$.

$$\text{Process } 3 \rightarrow 1 \text{ is at constant pressure so } p_3 = p_1 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

$$pV = nRT \text{ and } n, R, p \text{ constant implies } V_1/T_1 = V_3/T_3$$

$$V_3 = V_1(T_3/T_1) = (8.62 \times 10^{-3} \text{ m}^3)(492 \text{ K}/300 \text{ K}) = 14.1 \times 10^{-3} \text{ m}^3$$

(b) process $1 \rightarrow 2$

constant volume ($\Delta V = 0$)

$$Q = nC_V\Delta T = (0.350 \text{ mol})(20.79 \text{ J/mol} \cdot \text{K})(600 \text{ K} - 300 \text{ K}) = 2180 \text{ J}$$

$$\Delta V = 0 \text{ and } W = 0. \text{ Then } \Delta U = Q - W = 2180 \text{ J}$$

process $2 \rightarrow 3$

Adiabatic means $Q = 0$.

$$\Delta U = nC_V\Delta T \text{ (any process), so}$$

$$\Delta U = (0.350 \text{ mol})(20.79 \text{ J/mol} \cdot \text{K})(492 \text{ K} - 600 \text{ K}) = -780 \text{ J}$$

Then $\Delta U = Q - W$ gives $W = Q - \Delta U = +780 \text{ J}$. (It is correct for W to be positive since ΔV is positive.)

process $3 \rightarrow 1$

For constant pressure

$$W = p\Delta V = (1.013 \times 10^5 \text{ Pa})(8.62 \times 10^{-3} \text{ m}^3 - 14.1 \times 10^{-3} \text{ m}^3) = -560 \text{ J}$$

or $W = nR\Delta T = (0.350 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 492 \text{ K}) = -560 \text{ J}$, which checks. (It is correct for W to be negative, since ΔV is negative for this process.)

$$Q = nC_p\Delta T = (0.350 \text{ mol})(29.10 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 492 \text{ K}) = -1960 \text{ J}$$

$$\Delta U = Q - W = -1960 \text{ J} - (-560 \text{ J}) = -1400 \text{ J}$$

or $\Delta U = nC_V\Delta T = (0.350 \text{ mol})(20.79 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 492 \text{ K}) = -1400 \text{ J}$, which checks

$$\text{(c) } W_{\text{net}} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + W_{3 \rightarrow 1} = 0 + 780 \text{ J} - 560 \text{ J} = +220 \text{ J}$$

$$\text{(d) } Q_{\text{net}} = Q_{1 \rightarrow 2} + Q_{2 \rightarrow 3} + Q_{3 \rightarrow 1} = 2180 \text{ J} + 0 - 1960 \text{ J} = +220 \text{ J}$$

$$\text{(e) } e = \frac{\text{work output}}{\text{heat energy input}} = \frac{W}{Q_{\text{H}}} = \frac{220 \text{ J}}{2180 \text{ J}} = 0.101 = 10.1\%$$

$$e(\text{Carnot}) = 1 - T_C/T_H = 1 - 300 \text{ K}/600 \text{ K} = 0.500.$$

EVALUATE: For a cycle $\Delta U = 0$, so by $\Delta U = Q - W$ it must be that $Q_{\text{net}} = W_{\text{net}}$ for a cycle. We can also check that $\Delta U_{\text{net}} = 0$: $\Delta U_{\text{net}} = \Delta U_{1 \rightarrow 2} + \Delta U_{2 \rightarrow 3} + \Delta U_{3 \rightarrow 1} = 2180 \text{ J} - 1050 \text{ J} - 1130 \text{ J} = 0$
 $e < e(\text{Carnot})$, as it must.

20.41. IDENTIFY: $pV = nRT$, so pV is constant when T is constant. Use the appropriate expression to calculate Q and W for each process in the cycle. $e = \frac{W}{Q_{\text{H}}}$.

SET UP: For an ideal diatomic gas, $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$.

EXECUTE: (a) $p_a V_a = 2.0 \times 10^3 \text{ J}$. $p_b V_b = 2.0 \times 10^3 \text{ J}$. $pV = nRT$ so $p_a V_a = p_b V_b$ says $T_a = T_b$.

(b) For an isothermal process, $Q = W = nRT \ln(V_2/V_1)$. ab is a compression, with $V_b < V_a$, so $Q < 0$ and heat is rejected. bc is at constant pressure, so $Q = nC_p \Delta T = \frac{C_p}{R} p \Delta V$. ΔV is positive, so $Q > 0$ and heat is absorbed. cd is

at constant volume, so $Q = nC_V \Delta T = \frac{C_V}{R} V \Delta p$. Δp is negative, so $Q < 0$ and heat is rejected.

$$\text{(c) } T_a = \frac{p_a V_a}{nR} = \frac{2.0 \times 10^3 \text{ J}}{(1.00)(8.314 \text{ J/mol} \cdot \text{K})} = 241 \text{ K}. \quad T_b = \frac{p_b V_b}{nR} = T_a = 241 \text{ K}.$$

$$T_c = \frac{p_c V_c}{nR} = \frac{4.0 \times 10^3 \text{ J}}{(1.00)(8.314 \text{ J/mol} \cdot \text{K})} = 481 \text{ K}.$$

$$\text{(d) } Q_{ab} = nRT \ln\left(\frac{V_b}{V_a}\right) = (1.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(241 \text{ K}) \ln\left(\frac{0.0050 \text{ m}^3}{0.010 \text{ m}^3}\right) = -1.39 \times 10^3 \text{ J}.$$

$$Q_{bc} = nC_p \Delta T = (1.00)\left(\frac{7}{2}\right)(8.314 \text{ J/mol} \cdot \text{K})(241 \text{ K}) = 7.01 \times 10^3 \text{ J}.$$

$$Q_{ca} = nC_V \Delta T = (1.00)\left(\frac{5}{2}\right)(8.314 \text{ J/mol} \cdot \text{K})(-241 \text{ K}) = -5.01 \times 10^3 \text{ J}. \quad Q_{\text{net}} = Q_{ab} + Q_{bc} + Q_{ca} = 610 \text{ J}.$$

$$W_{\text{net}} = Q_{\text{net}} = 610 \text{ J}.$$

$$\text{(e) } e = \frac{W}{Q_{\text{H}}} = \frac{610 \text{ J}}{7.01 \times 10^3 \text{ J}} = 0.087 = 8.7\%$$

EVALUATE: We can calculate W for each process in the cycle. $W_{ab} = Q_{ab} = -1.39 \times 10^3 \text{ J}$.

$W_{bc} = p \Delta V = (4.0 \times 10^5 \text{ Pa})(0.0050 \text{ m}^3) = 2.00 \times 10^3 \text{ J}$. $W_{ca} = 0$. $W_{\text{net}} = W_{ab} + W_{bc} + W_{ca} = 610 \text{ J}$, which does equal Q_{net} .

20.42. (a) IDENTIFY and SET UP: Combine Eqs.(20.13) and (20.2) to eliminate Q_C and obtain an expression for Q_{H} in terms of W , T_C , and T_{H} .

$$W = 1.00 \text{ J}, \quad T_C = 268.15 \text{ K}, \quad T_{\text{H}} = 290.15 \text{ K}$$

For the heat pump $Q_C > 0$ and $Q_{\text{H}} < 0$

$$\text{EXECUTE: } W = Q_C + Q_{\text{H}}; \text{ combining this with } \frac{Q_C}{Q_{\text{H}}} = -\frac{T_C}{T_{\text{H}}} \text{ gives } Q_{\text{H}} = \frac{W}{1 - T_C/T_{\text{H}}} = \frac{1.00 \text{ J}}{1 - (268.15/290.15)} = 13.2 \text{ J}$$

(b) Electrical energy is converted directly into heat, so an electrical energy input of 13.2 J would be required.

(c) EVALUATE: From part (a), $Q_{\text{H}} = \frac{W}{1 - T_C/T_{\text{H}}}$. Q_{H} decreases as T_C decreases. The heat pump is less efficient as the temperature difference through which the heat has to be “pumped” increases. In an engine, heat flows from T_{H} to T_C and work is extracted. The engine is more efficient the larger the temperature difference through which the heat flows.

20.43. IDENTIFY: $T_b = T_c$ and is equal to the minimum temperature. Use the ideal gas law to calculate T_a . Apply the appropriate expression to calculate Q for each process. $e = \frac{W}{Q_{\text{H}}}$. $\Delta U = 0$ for a complete cycle and for an

isothermal process of an ideal gas.

SET UP: For helium, $C_V = 3R/2$ and $C_p = 5R/2$. The maximum efficiency is for a Carnot cycle, and

$$e_{\text{Carnot}} = 1 - T_C/T_{\text{H}}.$$

EXECUTE: (a) $Q_{\text{in}} = Q_{ab} + Q_{bc}$. $Q_{\text{out}} = Q_{ca}$. $T_{\text{max}} = T_b = T_c = 327^\circ\text{C} = 600\text{ K}$.

$$\frac{p_a V_a}{T_a} = \frac{p_b V_b}{T_b} \rightarrow T_a = \frac{p_a T_b}{p_b} = \frac{1}{3}(600\text{ K}) = 200\text{ K}.$$

$$p_b V_b = nRT_b \rightarrow V_b = \frac{nRT_b}{p_b} = \frac{(2\text{ moles})(8.31\text{ J/mol}\cdot\text{K})(600\text{ K})}{3.0 \times 10^5\text{ Pa}} = 0.0332\text{ m}^3.$$

$$\frac{p_b V_b}{T_b} = \frac{p_c V_c}{T_c} \rightarrow V_c = V_b \frac{p_b}{p_c} = (0.0332\text{ m}^3) \left(\frac{3}{1}\right) = 0.0997\text{ m}^3 = V_a.$$

$$Q_{ab} = nC_V \Delta T_{ab} = (2\text{ mol}) \left(\frac{3}{2}\right) (8.31\text{ J/mol}\cdot\text{K})(400\text{ K}) = 9.97 \times 10^3\text{ J}$$

$$Q_{bc} = W_{bc} = \int_b^c p dV = \int_b^c \frac{nRT_b}{V} dV = nRT_b \ln \frac{V_c}{V_b} = nRT_b \ln 3.$$

$$Q_{bc} = (2.00\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(600\text{ K}) \ln 3 = 1.10 \times 10^4\text{ J}. \quad Q_{\text{in}} = Q_{ab} + Q_{bc} = 2.10 \times 10^4\text{ J}.$$

$$Q_{\text{out}} = Q_{ca} = nC_p \Delta T_{ca} = (2.00\text{ mol}) \left(\frac{5}{2}\right) (8.31\text{ J/mol}\cdot\text{K})(400\text{ K}) = 1.66 \times 10^4\text{ J}.$$

(b) $Q = \Delta U + W = 0 + W \rightarrow W = Q_{\text{in}} - Q_{\text{out}} = 2.10 \times 10^4\text{ J} - 1.66 \times 10^4\text{ J} = 4.4 \times 10^3\text{ J}.$

$$e = W/Q_{\text{in}} = \frac{4.4 \times 10^3\text{ J}}{2.10 \times 10^4\text{ J}} = 0.21 = 21\%.$$

(c) $e_{\text{max}} = e_{\text{Carnot}} = 1 - \frac{T_c}{T_h} = 1 - \frac{200\text{ K}}{600\text{ K}} = 0.67 = 67\%$

EVALUATE: The thermal efficiency of this cycle is about one-third of the efficiency of a Carnot cycle that operates between the same two temperatures.

20.44. IDENTIFY: For a Carnot engine, $\frac{Q_c}{Q_h} = -\frac{T_c}{T_h}$. $e_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$. $|W| = |Q_h| - |Q_c|$. $Q_h > 0$, $Q_c < 0$. $pV = nRT$.

SET UP: The work done by the engine each cycle is $mg\Delta y$, with $m = 15.0\text{ kg}$ and $\Delta y = 2.00\text{ m}$. $T_h = 773\text{ K}$. $Q_h = 500\text{ J}$.

EXECUTE: (a) The pV diagram is sketched in Figure 20.44.

(b) $W = mg\Delta y = (15.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 294\text{ J}$. $|Q_c| = |Q_h| - |W| = 500\text{ J} - 294\text{ J} = 206\text{ J}$, and $Q_c = -206\text{ J}$.

$$T_c = -T_h \left(\frac{Q_c}{Q_h}\right) = -(773\text{ K}) \left(\frac{-206\text{ J}}{500\text{ J}}\right) = 318\text{ K} = 45^\circ\text{C}.$$

(c) $e = 1 - \frac{T_c}{T_h} = 1 - \frac{318\text{ K}}{773\text{ K}} = 0.589 = 58.9\%$.

(d) $|Q_c| = 206\text{ J}$.

(e) The maximum pressure is for state a . This is also where the volume is a minimum, so

$$V_a = 5.00\text{ L} = 5.00 \times 10^{-3}\text{ m}^3. \quad T_a = T_h = 773\text{ K}. \quad p_a = \frac{nRT_a}{V_a} = \frac{(2.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(773\text{ K})}{5.00 \times 10^{-3}\text{ m}^3} = 2.57 \times 10^6\text{ Pa}.$$

EVALUATE: We can verify that $e = \frac{W}{Q_h}$ gives the same value for e as calculated in part (c).

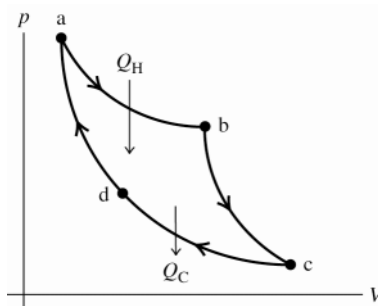


Figure 20.44

20.45. IDENTIFY: $e_{\max} = e_{\text{Carnot}} = 1 - T_C/T_H$. $e = \frac{W}{Q_H} = \frac{W/t}{Q_H/t}$. $W = Q_H + Q_C$ so $\frac{W}{t} = \frac{Q_C}{t} + \frac{Q_H}{t}$. For a temperature change

$$Q = mc\Delta T.$$

SET UP: $T_H = 300.15 \text{ K}$, $T_C = 279.15 \text{ K}$. For water, $\rho = 1000 \text{ kg/m}^3$, so a mass of 1 kg has a volume of 1 L. For water, $c = 4190 \text{ J/kg}\cdot\text{K}$.

EXECUTE: (a) $e = 1 - \frac{279.15 \text{ K}}{300.15 \text{ K}} = 7.0\%$.

(b) $\frac{Q_H}{t} = \frac{P_{\text{out}}}{e} = \frac{210 \text{ kW}}{0.070} = 3.0 \text{ MW}$. $\frac{Q_C}{t} = \frac{Q_H}{t} - \frac{W}{t} = 3.0 \text{ MW} - 210 \text{ kW} = 2.8 \text{ MW}$.

(c) $\frac{m}{t} = \frac{|Q_C|/t}{c\Delta T} = \frac{(2.8 \times 10^6 \text{ W})(3600 \text{ s/h})}{(4190 \text{ J/kg}\cdot\text{K})(4 \text{ K})} = 6 \times 10^5 \text{ kg/h} = 6 \times 10^5 \text{ L/h}$.

EVALUATE: The efficiency is small since T_C and T_H don't differ greatly.

20.46. IDENTIFY: Use Eq.(20.4) to calculate e .

SET UP: The cycle is sketched in Figure 20.46.

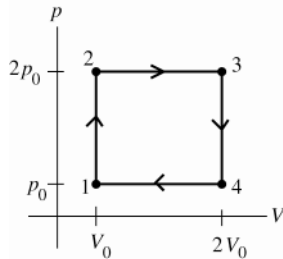


Figure 20.46

$$C_V = 5R/2$$

$$\text{for an ideal gas } C_p = C_V + R = 7R/2$$

SET UP: Calculate Q and W for each process.

process 1 \rightarrow 2

$$\Delta V = 0 \text{ implies } W = 0$$

$$\Delta V = 0 \text{ implies } Q = nC_V\Delta T = nC_V(T_2 - T_1)$$

But $pV = nRT$ and V constant says $p_1V = nRT_1$ and $p_2V = nRT_2$.

Thus $(p_2 - p_1)V = nR(T_2 - T_1)$; $V\Delta p = nR\Delta T$ (true when V is constant).

Then $Q = nC_V\Delta T = nC_V(V\Delta p/nR) = (C_V/R)V\Delta p = (C_V/R)V_0(2p_0 - p_0) = (C_V/R)p_0V_0$. ($Q > 0$; heat is absorbed by the gas.)

process 2 \rightarrow 3

$$\Delta p = 0 \text{ so } W = p\Delta V = p(V_3 - V_2) = 2p_0(2V_0 - V_0) = 2p_0V_0 \text{ (} W \text{ is positive since } V \text{ increases.)}$$

$$\Delta p = 0 \text{ implies } Q = nC_p\Delta T = nC_p(T_2 - T_1)$$

But $pV = nRT$ and p constant says $pV_1 = nRT_1$ and $pV_2 = nRT_2$.

Thus $p(V_2 - V_1) = nR(T_2 - T_1)$; $p\Delta V = nR\Delta T$ (true when p is constant).

Then $Q = nC_p\Delta T = nC_p(p\Delta V/nR) = (C_p/R)p\Delta V = (C_p/R)2p_0(2V_0 - V_0) = (C_p/R)2p_0V_0$. ($Q > 0$; heat is absorbed by the gas.)

process 3 \rightarrow 4

$$\Delta V = 0 \text{ implies } W = 0$$

$$\Delta V = 0 \text{ so}$$

$$Q = nC_V\Delta T = nC_V(V\Delta p/nR) = (C_V/R)(2V_0)(p_0 - 2p_0) = -2(C_V/R)p_0V_0$$

($Q < 0$ so heat is rejected by the gas.)

process 4 \rightarrow 1

$$\Delta p = 0 \text{ so } W = p\Delta V = p(V_1 - V_4) = p_0(V_0 - 2V_0) = -p_0V_0 \text{ (} W \text{ is negative since } V \text{ decreases)}$$

$$\Delta p = 0 \text{ so } Q = nC_p\Delta T = nC_p(p\Delta V/nR) = (C_p/R)p\Delta V = (C_p/R)p_0(V_0 - 2V_0) = -(C_p/R)p_0V_0 \text{ (} Q < 0 \text{ so heat is}$$

rejected by the gas.)

total work performed by the gas during the cycle:

$$W_{\text{tot}} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + W_{3 \rightarrow 4} + W_{4 \rightarrow 1} = 0 + 2p_0V_0 + 0 - p_0V_0 = p_0V_0$$

(Note that W_{tot} equals the area enclosed by the cycle in the pV -diagram.)

total heat absorbed by the gas during the cycle (Q_{H}):

Heat is absorbed in processes $1 \rightarrow 2$ and $2 \rightarrow 3$.

$$Q_{\text{H}} = Q_{1 \rightarrow 2} + Q_{2 \rightarrow 3} = \frac{C_V}{R} p_0V_0 + 2 \frac{C_p}{R} p_0V_0 = \left(\frac{C_V + 2C_p}{R} \right) p_0V_0$$

$$\text{But } C_p = C_V + R \text{ so } Q_{\text{H}} = \frac{C_V + 2(C_V + R)}{R} p_0V_0 = \left(\frac{3C_V + 2R}{R} \right) p_0V_0.$$

total heat rejected by the gas during the cycle (Q_{C}):

Heat is rejected in processes $3 \rightarrow 4$ and $4 \rightarrow 1$.

$$Q_{\text{C}} = Q_{3 \rightarrow 4} + Q_{4 \rightarrow 1} = -2 \frac{C_V}{R} p_0V_0 - \frac{C_p}{R} p_0V_0 = - \left(\frac{2C_V + C_p}{R} \right) p_0V_0$$

$$\text{But } C_p = C_V + R \text{ so } Q_{\text{C}} = - \frac{2C_V + (C_V + R)}{R} p_0V_0 = - \left(\frac{3C_V + R}{R} \right) p_0V_0.$$

efficiency

$$e = \frac{W}{Q_{\text{H}}} = \frac{p_0V_0}{\left(\frac{3C_V + 2R}{R} \right) p_0V_0} = \frac{R}{3C_V + 2R} = \frac{R}{3(5R/2) + 2R} = \frac{2}{19}.$$

$$e = 0.105 = 10.5\%$$

$$\text{EVALUATE: As a check on the calculations note that } Q_{\text{C}} + Q_{\text{H}} = - \left(\frac{3C_V + R}{R} \right) p_0V_0 + \left(\frac{3C_V + 2R}{R} \right) p_0V_0 = p_0V_0 = W,$$

as it should.

20.47. IDENTIFY: Use $pV = nRT$. Apply the expressions for Q and W that apply to each type of process. $e = \frac{W}{Q_{\text{H}}}$.

SET UP: For O_2 , $C_V = 20.85 \text{ J/mol} \cdot \text{K}$ and $C_p = 29.17 \text{ J/mol} \cdot \text{K}$.

EXECUTE: (a) $p_1 = 2.00 \text{ atm}$, $V_1 = 4.00 \text{ L}$, $T_1 = 300 \text{ K}$.

$$p_2 = 2.00 \text{ atm}. \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}. \quad V_2 = \left(\frac{T_2}{T_1} \right) V_1 = \left(\frac{450 \text{ K}}{300 \text{ K}} \right) (4.00 \text{ L}) = 6.00 \text{ L}.$$

$$V_3 = 6.00 \text{ L}. \quad \frac{p_2}{T_2} = \frac{p_3}{T_3}. \quad p_3 = \left(\frac{T_3}{T_2} \right) p_2 = \left(\frac{250 \text{ K}}{450 \text{ K}} \right) (2.00 \text{ atm}) = 1.11 \text{ atm}$$

$$V_4 = 4.00 \text{ L}. \quad p_3V_3 = p_4V_4. \quad p_4 = p_3 \left(\frac{V_3}{V_4} \right) = (1.11 \text{ atm}) \left(\frac{6.00 \text{ L}}{4.00 \text{ L}} \right) = 1.67 \text{ atm}.$$

These processes are shown in Figure 20.47.

$$\text{(b) } n = \frac{p_1V_1}{RT_1} = \frac{(2.00 \text{ atm})(4.00 \text{ L})}{(0.08206 \text{ L} \cdot \text{atm/mol} \cdot \text{K})(300 \text{ K})} = 0.325 \text{ mol}$$

$$\text{process } 1 \rightarrow 2: W = p\Delta V = nR\Delta T = (0.325 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(150 \text{ K}) = 405 \text{ J}.$$

$$Q = nC_p\Delta T = (0.325 \text{ mol})(29.17 \text{ J/mol} \cdot \text{K})(150 \text{ K}) = 1422 \text{ J}.$$

$$\text{process } 2 \rightarrow 3: W = 0. \quad Q = nC_V\Delta T = (0.325 \text{ mol})(20.85 \text{ J/mol} \cdot \text{K})(-200 \text{ K}) = -1355 \text{ J}.$$

$$\text{process } 3 \rightarrow 4: \Delta U = 0 \text{ and } Q = W = nRT_3 \ln \left(\frac{V_4}{V_3} \right) = (0.325 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(250 \text{ K}) \ln \left(\frac{4.00 \text{ L}}{6.00 \text{ L}} \right) = -274 \text{ J}.$$

$$\text{process } 4 \rightarrow 1: W = 0. \quad Q = nC_V\Delta T = (0.325 \text{ mol})(20.85 \text{ J/mol} \cdot \text{K})(50 \text{ K}) = 339 \text{ J}.$$

$$\text{(c) } W = 405 \text{ J} - 274 \text{ J} = 131 \text{ J}$$

$$\text{(d) } e = \frac{W}{Q_{\text{H}}} = \frac{131 \text{ J}}{1422 \text{ J} + 339 \text{ J}} = 0.0744 = 7.44\%.$$

$$e_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} = 1 - \frac{250 \text{ K}}{450 \text{ K}} = 0.444 = 44.4\%; \quad e_{\text{Carnot}} \text{ is much larger.}$$

EVALUATE: $Q_{\text{tot}} = +1422 \text{ J} + (-1355 \text{ J}) + (-274 \text{ J}) + 339 \text{ J} = 132 \text{ J}$. This is equal to W_{tot} , apart from a slight difference due to rounding. For a cycle, $W_{\text{tot}} = Q_{\text{tot}}$, since $\Delta U = 0$.

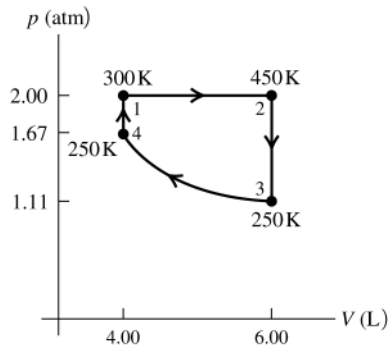


Figure 20.47

20.48. IDENTIFY and SET UP: For the constant pressure processes ab and cd calculate W and use the first law to calculate Q . Calculate Q_{tot} and use that $W_{\text{tot}} = Q_{\text{tot}}$ for a cycle. The coefficient of performance is given by Eq.(20.9); Q_c is the net heat that goes into the system. The cycle is sketched in Figure 20.48.

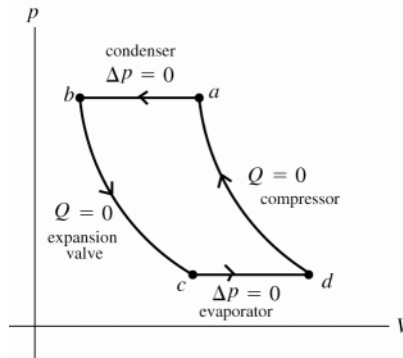


Figure 20.48

EXECUTE: (a) process $c \rightarrow d$

$$\Delta U = U_d - U_c = 1657 \times 10^3 \text{ J} - 1005 \times 10^3 \text{ J} = 6.52 \times 10^5 \text{ J}$$

$$W = \int_{V_c}^{V_d} p dV = p\Delta V \text{ (since is a constant pressure process)}$$

$$W = (363 \times 10^3 \text{ Pa})(0.4513 \text{ m}^3 - 0.2202 \text{ m}^3) = +8.39 \times 10^4 \text{ J} \text{ (positive since process is an expansion)}$$

$$\Delta U = Q - W \text{ so } Q = \Delta U + W = 6.52 \times 10^5 \text{ J} + 8.39 \times 10^4 \text{ J} = 7.36 \times 10^5 \text{ J}.$$

(Q positive so heat goes into the coolant)

(b) process $a \rightarrow b$

$$\Delta U = U_b - U_a = 1171 \times 10^3 \text{ J} - 1969 \times 10^3 \text{ J} = -7.98 \times 10^5 \text{ J}$$

$$W = p\Delta V = (2305 \times 10^3 \text{ Pa})(0.00946 \text{ m}^3 - 0.0682 \text{ m}^3) = -1.35 \times 10^5 \text{ J}$$

(negative since $\Delta V < 0$ for the process)

$$Q = \Delta U + W = -7.98 \times 10^5 \text{ J} - 1.35 \times 10^5 \text{ J} = -9.33 \times 10^5 \text{ J}$$

(negative so heat comes out of coolant).

(c) The coolant cannot be treated as an ideal gas, so we can't calculate W for the adiabatic processes. But $\Delta U = 0$ (for cycle) so $W_{\text{net}} = Q_{\text{net}}$.

$$Q = 0 \text{ for the two adiabatic processes, so } Q_{\text{net}} = Q_{cd} + Q_{ab} = 7.36 \times 10^5 \text{ J} - 9.33 \times 10^5 \text{ J} = -1.97 \times 10^5 \text{ J}$$

Thus $W_{\text{net}} = -1.97 \times 10^5 \text{ J}$ (negative since work is done on the coolant, the working substance).

(d) $K = Q_c / |W| = (+7.36 \times 10^5 \text{ J}) / (+1.97 \times 10^5 \text{ J}) = 3.74$.

EVALUATE: $W_{\text{net}} < 0$ when the cycle is taken in the counterclockwise direction, as is the case here.

20.49. IDENTIFY: Use $\Delta U = Q - W$ and the appropriate expressions for Q , W and ΔU for each type of process.

$pV = nRT$ relates ΔT to p and V values. $e = \frac{W}{Q_H}$, where Q_H is the heat that enters the gas during the cycle.

SET UP: For a monatomic ideal gas, $C_p = \frac{5}{2}R$ and $C_v = \frac{3}{2}R$.

(a) ab : The temperature changes by the same factor as the volume, and so

$$Q = nC_p\Delta T = \frac{C_p}{R}p_a(V_a - V_b) = (2.5)(3.00 \times 10^5 \text{ Pa})(0.300 \text{ m}^3) = 2.25 \times 10^5 \text{ J}.$$

The work $p\Delta V$ is the same except for the factor of $\frac{5}{2}$, so $W = 0.90 \times 10^5 \text{ J}$.

$$\Delta U = Q - W = 1.35 \times 10^5 \text{ J}.$$

bc : The temperature now changes in proportion to the pressure change, and

$$Q = \frac{3}{2}(p_c - p_b)V_b = (1.5)(-2.00 \times 10^5 \text{ Pa})(0.800 \text{ m}^3) = -2.40 \times 10^5 \text{ J}, \text{ and the work is zero}$$

$$(\Delta V = 0). \Delta U = Q - W = -2.40 \times 10^5 \text{ J}.$$

ca : The easiest way to do this is to find the work done first; W will be the negative of area in the p - V plane bounded by the line representing the process ca and the verticals from points a and c . The area of this trapezoid is $\frac{1}{2}(3.00 \times 10^5 \text{ Pa} + 1.00 \times 10^5 \text{ Pa})(0.800 \text{ m}^3 - 0.500 \text{ m}^3) = 6.00 \times 10^4 \text{ J}$ and so the work is $-0.60 \times 10^5 \text{ J}$. ΔU must be $1.05 \times 10^5 \text{ J}$ (since $\Delta U = 0$ for the cycle, anticipating part (b)), and so Q must be $\Delta U + W = 0.45 \times 10^5 \text{ J}$.

(b) See above; $Q = W = 0.30 \times 10^5 \text{ J}$, $\Delta U = 0$.

(c) The heat added, during process ab and ca , is $2.25 \times 10^5 \text{ J} + 0.45 \times 10^5 \text{ J} = 2.70 \times 10^5 \text{ J}$ and the efficiency is

$$e = \frac{W}{Q_H} = \frac{0.30 \times 10^5}{2.70 \times 10^5} = 0.111 = 11.1\%.$$

EVALUATE: For any cycle, $\Delta U = 0$ and $Q = W$.

20.50. IDENTIFY: Use the appropriate expressions for Q , W and ΔU for each process. $e = W/Q_H$ and $e_{\text{Carnot}} = 1 - T_C/T_H$.

SET UP: For this cycle, $T_H = T_2$ and $T_C = T_1$

EXECUTE: (a) ab : For the isothermal process, $\Delta T = 0$ and $\Delta U = 0$.

$$W = nRT_1 \ln(V_b/V_a) = nRT_1 \ln(1/r) = -nRT_1 \ln(r) \text{ and } Q = W = -nRT_1 \ln(r).$$

bc : For the isochoric process, $\Delta V = 0$ and $W = 0$. $Q = \Delta U = nC_v\Delta T = nC_v(T_2 - T_1)$.

cd : As in the process ab , $\Delta U = 0$ and $W = Q = nRT_2 \ln(r)$.

da : As in process bc , $\Delta V = 0$ and $W = 0$; $\Delta U = Q = nC_v(T_1 - T_2)$.

(b) The values of Q for the processes are the negatives of each other.

(c) The net work for one cycle is $W_{\text{net}} = nR(T_2 - T_1)\ln(r)$, and the heat added (neglecting the heat exchanged during the isochoric expansion and compression, as mentioned in part (b)) is $Q_{\text{cd}} = nRT_2 \ln(r)$, and the efficiency is

$e = \frac{W_{\text{net}}}{Q_{\text{cd}}} = 1 - (T_1/T_2)$. This is the same as the efficiency of a Carnot-cycle engine operating between the two temperatures.

EVALUATE: For a Carnot cycle two steps in the cycle are isothermal and two are adiabatic and all the heat flow occurs in the isothermal processes. For the Stirling cycle all the heat flow is also in the isothermal steps, since the net heat flow in the two constant volume steps is zero.

20.51. IDENTIFY: The efficiency of the composite engine is $e_{12} = \frac{W_1 + W_2}{Q_{H1}}$, where Q_{H1} is the heat input to the first engine

and W_1 and W_2 are the work outputs of the two engines. For any heat engine, $W = Q_C + Q_H$, and for a Carnot engine,

$$\frac{Q_{\text{low}}}{Q_{\text{high}}} = -\frac{T_{\text{low}}}{T_{\text{high}}}, \text{ where } Q_{\text{low}} \text{ and } Q_{\text{high}} \text{ are the heat flows at the two reservoirs that have temperatures } T_{\text{low}} \text{ and } T_{\text{high}}.$$

SET UP: $Q_{\text{high},2} = -Q_{\text{low},1}$, $T_{\text{low},1} = T'$, $T_{\text{high},1} = T_H$, $T_{\text{low},2} = T_C$ and $T_{\text{high},2} = T'$.

EXECUTE: $e_{12} = \frac{W_1 + W_2}{Q_{H1}} = \frac{Q_{\text{high},1} + Q_{\text{low},1} + Q_{\text{high},2} + Q_{\text{low},2}}{Q_{\text{high},1}}$. Since $Q_{\text{high},2} = -Q_{\text{low},1}$, this reduces to $e_{12} = 1 + \frac{Q_{\text{low},2}}{Q_{\text{high},1}}$.

$$Q_{\text{low},2} = -Q_{\text{high},2} \frac{T_{\text{low},2}}{T_{\text{high},2}} = Q_{\text{low},1} \frac{T_C}{T'} = -Q_{\text{high},1} \left(\frac{T_{\text{low},1}}{T_{\text{high},1}} \right) \frac{T_C}{T'} = -Q_{\text{high},1} \left(\frac{T'}{T_H} \right) \frac{T_C}{T'}. \text{ This gives } e_{12} = 1 - \frac{T_C}{T_H}.$$

The efficiency of the composite system is the same as that of the original engine.

EVALUATE: The overall efficiency is independent of the value of the intermediate temperature T' .

20.52. IDENTIFY: $e = \frac{W}{Q_H}$. 1 day = 8.64×10^4 s. For the river water, $Q = mc\Delta T$, where the heat that goes into the water

is the heat Q_C rejected by the engine. The density of water is 1000 kg/m^3 . When an object undergoes a temperature change, $\Delta S = mc \ln(T_2/T_1)$.

SET UP: $18.0^\circ\text{C} = 291.1 \text{ K}$. $18.5^\circ\text{C} = 291.6 \text{ K}$.

EXECUTE: (a) $Q_H = \frac{W}{e}$ so $P_H = \frac{P_W}{e} = \frac{1000 \text{ MW}}{0.40} = 2.50 \times 10^3 \text{ MW}$.

(b) The heat input in one day is $(2.50 \times 10^9 \text{ W})(8.64 \times 10^4 \text{ s}) = 2.16 \times 10^{14} \text{ J}$. The mass of coal used per day is $\frac{2.16 \times 10^{14} \text{ J}}{2.65 \times 10^7 \text{ J/kg}} = 8.15 \times 10^6 \text{ kg}$.

(c) $|Q_H| = |W| + |Q_C|$. $|Q_C| = |Q_H| - |W|$. $P_C = P_H - P_W = 2.50 \times 10^3 \text{ MW} - 1000 \text{ MW} = 1.50 \times 10^3 \text{ MW}$.

(d) The heat input to the river is $1.50 \times 10^9 \text{ J/s}$. $Q = mc\Delta T$ and $\Delta T = 0.5 \text{ C}^\circ$ gives

$m = \frac{Q}{c\Delta T} = \frac{1.50 \times 10^9 \text{ J}}{(4190 \text{ J/kg} \cdot \text{K})(0.5 \text{ K})} = 7.16 \times 10^5 \text{ kg}$. $V = \frac{m}{\rho} = 716 \text{ m}^3$. The river flow rate must be $716 \text{ m}^3/\text{s}$.

(e) In one second, $7.16 \times 10^5 \text{ kg}$ of water goes from 291.1 K to 291.6 K .

$\Delta S = mc \ln\left(\frac{T_2}{T_1}\right) = (7.16 \times 10^5 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{291.6 \text{ K}}{291.1 \text{ K}}\right) = 5.1 \times 10^6 \text{ J/K}$.

EVALUATE: The entropy of the river increases because heat flows into it. The mass of coal used per second is huge.

20.53. (a) IDENTIFY and SET UP: Calculate e from Eq.(20.6), Q_C from Eq.(20.4) and then W from Eq.(20.2).

EXECUTE: $e = 1 - 1/(r^{\gamma-1}) = 1 - 1/(10.6^{0.4}) = 0.6111$

$e = (Q_H + Q_C)/Q_H$ and we are given $Q_H = 200 \text{ J}$; calculate Q_C .

$Q_C = (e - 1)Q_H = (0.6111 - 1)(200 \text{ J}) = -78 \text{ J}$ (negative since corresponds to heat leaving)

Then $W = Q_C + Q_H = -78 \text{ J} + 200 \text{ J} = 122 \text{ J}$. (Positive, in agreement with Fig. 20.6.)

EVALUATE: $Q_H, W > 0$, and $Q_C < 0$ for an engine cycle.

(b) **IDENTIFY and SET UP:** The stroke times the bore equals the change in volume. The initial volume is the final volume V times the compression ratio r . Combining these two expressions gives an equation for V . For each cylinder of area $A = \pi(d/2)^2$ the piston moves 0.864 m and the volume changes from rV to V , as shown in Figure 20.53a.

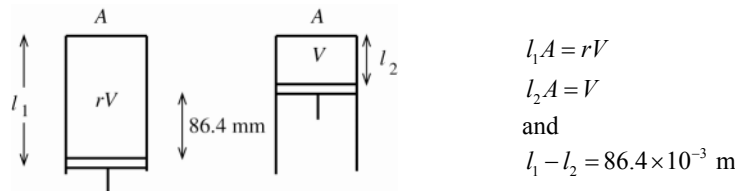


Figure 20.53a

EXECUTE: $l_1A - l_2A = rV - V$ and $(l_1 - l_2)A = (r - 1)V$

$V = \frac{(l_1 - l_2)A}{r - 1} = \frac{(86.4 \times 10^{-3} \text{ m})\pi(41.25 \times 10^{-3} \text{ m})^2}{10.6 - 1} = 4.811 \times 10^{-5} \text{ m}^3$

At point a the volume is $rV = 10.6(4.811 \times 10^{-5} \text{ m}^3) = 5.10 \times 10^{-4} \text{ m}^3$.

(c) **IDENTIFY and SET UP:** The processes in the Otto cycle are either constant volume or adiabatic. Use the Q_H that is given to calculate ΔT for process bc . Use Eq.(19.22) and $pV = nRT$ to relate p, V and T for the adiabatic processes ab and cd .

EXECUTE: point a: $T_a = 300 \text{ K}$, $p_a = 8.50 \times 10^4 \text{ Pa}$, and $V_a = 5.10 \times 10^{-4} \text{ m}^3$

point b: $V_b = V_a/r = 4.81 \times 10^{-5} \text{ m}^3$. Process $a \rightarrow b$ is adiabatic, so $T_a V_a^{\gamma-1} = T_b V_b^{\gamma-1}$.

$T_a (rV)^{\gamma-1} = T_b V^{\gamma-1}$

$T_b = T_a r^{\gamma-1} = 300 \text{ K}(10.6)^{0.4} = 771 \text{ K}$

$pV = nRT$ so $pV/T = nR = \text{constant}$, so $p_a V_a/T_a = p_b V_b/T_b$

$p_b = p_a (V_a/V_b)(T_b/T_a) = (8.50 \times 10^4 \text{ Pa})(rV/V)(771 \text{ K}/300 \text{ K}) = 2.32 \times 10^6 \text{ Pa}$

point c: Process $b \rightarrow c$ is at constant volume, so $V_c = V_b = 4.81 \times 10^{-5} \text{ m}^3$

$Q_H = nC_V \Delta T = nC_V (T_c - T_b)$. The problem specifies $Q_H = 200 \text{ J}$; use to calculate T_c . First use the p, V, T values at point a to calculate the number of moles n .

$$n = \frac{pV}{RT} = \frac{(8.50 \times 10^4 \text{ Pa})(5.10 \times 10^{-4} \text{ m}^3)}{(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})} = 0.01738 \text{ mol}$$

$$\text{Then } T_c - T_b = \frac{Q_H}{nC_V} = \frac{200 \text{ J}}{(0.01738 \text{ mol})(20.5 \text{ J/mol} \cdot \text{K})} = 561.3 \text{ K, and } T_c = T_b + 561.3 \text{ K} = 771 \text{ K} + 561 \text{ K} = 1332 \text{ K}$$

$$p/T = nR/V = \text{constant so } p_b/T_b = p_c/T_c$$

$$p_c = p_b(T_c/T_b) = (2.32 \times 10^6 \text{ Pa})(1332 \text{ K}/771 \text{ K}) = 4.01 \times 10^6 \text{ Pa}$$

point d: $V_d = V_a = 5.10 \times 10^{-4} \text{ m}^3$

process $c \rightarrow d$ is adiabatic, so $T_d V_d^{\gamma-1} = T_c V_c^{\gamma-1}$

$$T_d (rV)^{\gamma-1} = T_c V^{\gamma-1}$$

$$T_d = T_c / r^{\gamma-1} = 1332 \text{ K} / 10.6^{0.4} = 518 \text{ K}$$

$$p_c V_c / T_c = p_d V_d / T_d$$

$$p_d = p_c (V_c / V_d) (T_d / T_c) = (4.01 \times 10^6 \text{ Pa})(V / rV)(518 \text{ K} / 1332 \text{ K}) = 1.47 \times 10^5 \text{ Pa}$$

EVALUATE: Can look at process $d \rightarrow a$ as a check.

$Q_C = nC_V (T_a - T_d) = (0.01738 \text{ mol})(20.5 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 518 \text{ K}) = -78 \text{ J}$, which agrees with part (a). The cycle is sketched in Figure 20.53b.

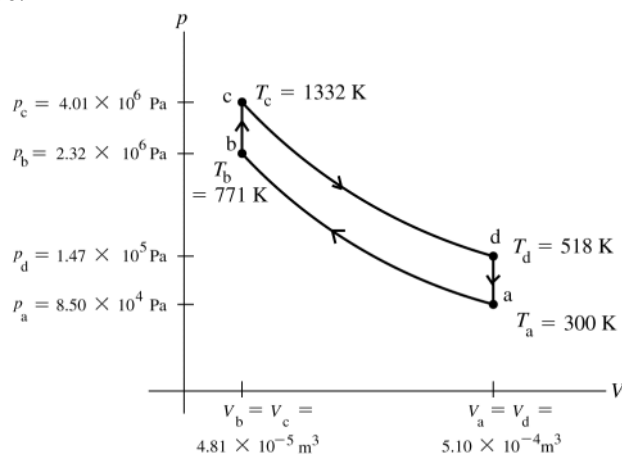


Figure 20.53b

(d) IDENTIFY and SET UP: The Carnot efficiency is given by Eq.(20.14). T_H is the highest temperature reached in the cycle and T_C is the lowest.

EXECUTE: From part (a) the efficiency of this Otto cycle is $e = 0.611 = 61.1\%$.

The efficiency of a Carnot cycle operating between 1332 K and 300 K is

$$e(\text{Carnot}) = 1 - T_C / T_H = 1 - 300 \text{ K} / 1332 \text{ K} = 0.775 = 77.5\%, \text{ which is larger.}$$

EVALUATE: The 2nd law requires that $e \leq e(\text{Carnot})$, and our result obeys this law.

20.54. IDENTIFY: $K = \frac{|Q_C|}{|W|}$. $|Q_H| = |Q_C| + |W|$. The heat flows for the inside and outside air occur at constant T , so

$$\Delta S = Q/T.$$

SET UP: $21.0^\circ\text{C} = 294.1 \text{ K}$. $35.0^\circ\text{C} = 308.1 \text{ K}$.

EXECUTE: (a) $|Q_C| = K|W|$. $P_C = KP_W = (2.80)(800 \text{ W}) = 2.24 \times 10^3 \text{ W}$.

(b) $P_H = P_C + P_W = 2.24 \times 10^3 \text{ W} + 800 \text{ W} = 3.04 \times 10^3 \text{ W}$.

(c) In $1 \text{ h} = 3600 \text{ s}$, $Q_H = P_H t = 1.094 \times 10^7 \text{ J}$. $\Delta S_{\text{out}} = \frac{Q_H}{T_H} = \frac{1.094 \times 10^7 \text{ J}}{308.1 \text{ K}} = 3.55 \times 10^4 \text{ J/K}$.

(d) $Q_C = P_C t = 8.064 \times 10^6 \text{ J}$. Heat Q_C is removed from the inside air.

$$\Delta S_{\text{in}} = \frac{-Q_C}{T_C} = \frac{-8.064 \times 10^6 \text{ J}}{294.1 \text{ K}} = -2.74 \times 10^4 \text{ J/K}. \quad \Delta S_{\text{net}} = \Delta S_{\text{out}} + \Delta S_{\text{in}} = 8.1 \times 10^3 \text{ J/K}.$$

EVALUATE: The increase in the entropy of the outside air is greater than the entropy decrease of the air in the room.

20.55. IDENTIFY and SET UP: Use Eq.(20.13) for an infinitesimal heat flow dQ_H from the hot reservoir and use that expression with Eq.(20.19) to relate ΔS_H , the entropy change of the hot reservoir, to $|Q_C|$

(a) **EXECUTE:** Consider an infinitesimal heat flow dQ_H that occurs when the temperature of the hot reservoir is T' :

$$dQ_C = -(T_C/T')dQ_H$$

$$\int dQ_C = -T_C \int \frac{dQ_H}{T'}$$

$$|Q_C| = T_C \left| \int \frac{dQ_H}{T'} \right| = T_C |\Delta S_H|$$

(b) The 1.00 kg of water (the high-temperature reservoir) goes from 373 K to 273 K.

$$Q_H = mc\Delta T = (1.00 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(100 \text{ K}) = 4.19 \times 10^5 \text{ J}$$

$$\Delta S_H = mc \ln(T_2/T_1) = (1.00 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln(273/373) = -1308 \text{ J/K}$$

The result of part (a) gives $|Q_C| = (273 \text{ K})(1308 \text{ J/K}) = 3.57 \times 10^5 \text{ J}$

Q_C comes out of the engine, so $Q_C = -3.57 \times 10^5 \text{ J}$

Then $W = Q_C + Q_H = -3.57 \times 10^5 \text{ J} + 4.19 \times 10^5 \text{ J} = 6.2 \times 10^4 \text{ J}$.

(c) 2.00 kg of water goes from 323 K to 273 K

$$Q_H = -mc\Delta T = (2.00 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(50 \text{ K}) = 4.19 \times 10^5 \text{ J}$$

$$\Delta S_H = mc \ln(T_2/T_1) = (2.00 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln(272/323) = -1.41 \times 10^3 \text{ J/K}$$

$$Q_C = -T_C |\Delta S_H| = -3.85 \times 10^5 \text{ J}$$

$$W = Q_C + Q_H = 3.4 \times 10^4 \text{ J}$$

(d) **EVALUATE:** More work can be extracted from 1.00 kg of water at 373 K than from 2.00 kg of water at 323 K even though the energy that comes out of the water as it cools to 273 K is the same in both cases. The energy in the 323 K water is less available for conversion into mechanical work.

20.56. IDENTIFY: The maximum power that can be extracted is the total kinetic energy K of the mass of air that passes over the turbine blades in time t .

SET UP: The volume of a cylinder of diameter d and length L is $(\pi d^2/4)L$. Kinetic energy is $\frac{1}{2}mv^2$.

EXECUTE: (a) The cylinder described contains a mass of air $m = \rho(\pi d^2/4)L$, and so the total kinetic energy is

$K = \rho(\pi/8)d^2Lv^2$. This mass of air will pass by the turbine in a time $t = L/v$, and so the maximum power is

$P = \frac{K}{t} = \rho(\pi/8)d^2v^3$. Numerically, the product $\rho_{\text{air}}(\pi/8) \approx 0.5 \text{ kg/m}^3 = 0.5 \text{ W} \cdot \text{s}^3/\text{m}^5$. This completes the proof.

$$(b) v = \left(\frac{P/e}{kd^2} \right)^{1/3} = \left(\frac{(3.2 \times 10^6 \text{ W})/(0.25)}{(0.5 \text{ W} \cdot \text{s}^3/\text{m}^5)(97 \text{ m}^2)} \right)^{1/3} = 14 \text{ m/s} = 50 \text{ km/h}.$$

(c) Wind speeds tend to be higher in mountain passes.

EVALUATE: The maximum power is proportional to v^3 , so increases rapidly with increase in wind speed.

20.57. IDENTIFY: For a Carnot device, $\frac{T_C}{T_H} = -\frac{Q_C}{Q_H}$. $W = Q_H + Q_C$.

SET UP: $Q_C = 1000 \text{ J}$. $10.0^\circ\text{C} = 283.1 \text{ K}$. $35.0^\circ\text{C} = 308.1 \text{ K}$. $15.0^\circ\text{C} = 288.1 \text{ K}$.

$$\text{EXECUTE: (a) } Q_H = -\left(\frac{T_H}{T_C} \right) Q_C = -\left(\frac{308.1 \text{ K}}{283.1 \text{ K}} \right) (1000 \text{ J}) = -1.088 \times 10^3 \text{ J}. \quad W = 1000 \text{ J} + (-1.088 \times 10^3 \text{ J}) = -88 \text{ J}.$$

$$(b) \text{ Now } Q_H = -\left(\frac{288.1 \text{ K}}{283.1 \text{ K}} \right) (1000 \text{ J}) = -1.018 \times 10^3 \text{ J}. \quad W = 1000 \text{ J} + (-1.018 \times 10^3 \text{ J}) = -18 \text{ J}.$$

(c) The pV -diagrams for the two Carnot cycles are sketched in Figure 20.57.

EVALUATE: More work must be done to move the heat energy through a greater temperature difference.

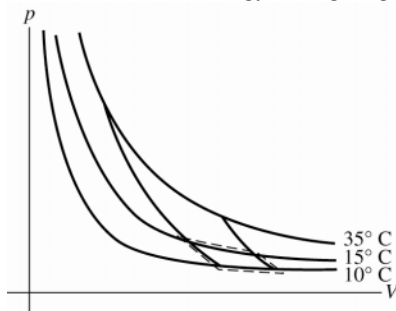


Figure 20.57

20.58. IDENTIFY and SET UP: First use the methods of Chapter 17 to calculate the final temperature T of the system.

EXECUTE: 0.600 kg of water (cools from 45.0°C to T)

$$Q = mc\Delta T = (0.600 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(T - 45.0^\circ\text{C}) = (2514 \text{ J/K})T - 1.1313 \times 10^5 \text{ J}$$

0.0500 kg of ice (warms to 0°C , melts, and water warms from 0°C to T)

$$Q = mc_{\text{ice}}(0^\circ\text{C} - (-15.0^\circ\text{C})) + mL_f + mc_{\text{water}}(T - 0^\circ\text{C})$$

$$Q = 0.0500 \text{ kg} \left[(2100 \text{ J/kg} \cdot \text{K})(15.0^\circ\text{C}) + 334 \times 10^3 \text{ J/kg} + (4190 \text{ J/kg} \cdot \text{K})(T - 0^\circ\text{C}) \right]$$

$$Q = 1575 \text{ J} + 1.67 \times 10^4 \text{ J} + (209.5 \text{ J/K})T = 1.828 \times 10^4 \text{ J} + (209.5 \text{ J/K})T$$

$$Q_{\text{system}} = 0 \text{ gives } (2514 \text{ J/K})T - 1.1313 \times 10^5 \text{ J} + 1.828 \times 10^4 \text{ J} + (209.5 \text{ J/K})T = 0$$

$$(2.724 \times 10^3 \text{ J/K})T = 9.485 \times 10^4 \text{ J}$$

$$T = (9.485 \times 10^4 \text{ J}) / (2.724 \times 10^3 \text{ J/K}) = 34.83^\circ\text{C} = 308 \text{ K}$$

EVALUATE: The final temperature must lie between -15.0°C and 45.0°C . A final temperature of 34.8°C is consistent with only liquid water being present at equilibrium.

IDENTIFY and SET UP: Now we can calculate the entropy changes. Use $\Delta S = Q/T$ for phase changes and the method of Example 20.6 to calculate ΔS for temperature changes.

EXECUTE: ice: The process takes ice at -15°C and produces water at 34.8°C . Calculate ΔS for a reversible process between these two states, in which heat is added very slowly. ΔS is path independent, so ΔS for a reversible process is the same as ΔS for the actual (irreversible) process as long as the initial and final states are the same.

$$\Delta S = \int_1^2 dQ/T, \text{ where } T \text{ must be in kelvins}$$

$$\text{For a temperature change } dQ = mc dT \text{ so } \Delta S = \int_{T_1}^{T_2} (mc/T) dT = mc \ln(T_2/T_1).$$

For a phase change, since it occurs at constant T ,

$$\Delta S = \int_1^2 dQ/T = Q/T = \pm mL/T.$$

$$\text{Therefore } \Delta S_{\text{ice}} = mc_{\text{ice}} \ln(273 \text{ K}/258 \text{ K}) + mL_f/273 \text{ K} + mc_{\text{water}} \ln(308 \text{ K}/273 \text{ K})$$

$$\Delta S_{\text{ice}} = (0.0500 \text{ kg})[(2100 \text{ J/kg} \cdot \text{K}) \ln(273 \text{ K}/258 \text{ K}) + (334 \times 10^3 \text{ J/kg})/273 \text{ K} + (4190 \text{ J/kg} \cdot \text{K}) \ln(308 \text{ K}/273 \text{ K})]$$

$$\Delta S_{\text{ice}} = 5.93 \text{ J/K} + 61.17 \text{ J/K} + 25.27 \text{ J/K} = 92.4 \text{ J/K}$$

$$\text{water: } \Delta S_{\text{water}} = mc \ln(T_2/T_1) = (0.600 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln(308 \text{ K}/318 \text{ K}) = -80.3 \text{ J/K}$$

$$\text{For the system, } \Delta S = \Delta S_{\text{ice}} + \Delta S_{\text{water}} = 92.4 \text{ J/K} - 80.3 \text{ J/K} = +12 \text{ J/K}$$

EVALUATE: Our calculation gives $\Delta S > 0$, as it must for an irreversible process of an isolated system.

20.59. IDENTIFY: Apply Eq.(20.19). From the derivation of Eq. (20.6), $T_b = r^{\gamma-1}T_a$ and $T_c = r^{\gamma-1}T_d$.

SET UP: For a constant volume process, $dQ = nC_V dT$.

EXECUTE: (a) For a constant-volume process for an ideal gas, where the temperature changes from T_1 to T_2 ,

$$\Delta S = nC_V \int_{T_1}^{T_2} \frac{dT}{T} = nC_V \ln\left(\frac{T_2}{T_1}\right). \text{ The entropy changes are } nC_V \ln(T_c/T_b) \text{ and } nC_V \ln(T_a/T_d).$$

(b) The total entropy change for one cycle is the sum of the entropy changes found in part (a); the other processes in the cycle are adiabatic, with $Q=0$ and $\Delta S=0$. The total is then

$$\Delta S = nC_V \ln \frac{T_c}{T_b} + nC_V \ln \frac{T_a}{T_d} = nC_V \ln \left(\frac{T_c T_a}{T_b T_d} \right). \frac{T_c T_a}{T_b T_d} = \frac{r^{\gamma-1} T_d T_a}{r^{\gamma-1} T_d T_a} = 1. \ln(1) = 0, \text{ so } \Delta S = 0.$$

(c) The system is not isolated, and a zero change of entropy for an irreversible system is certainly possible.

EVALUATE: In an irreversible process for an isolated system, $\Delta S > 0$. But the entropy change for some of the components of the system can be negative or zero.

- 20.60. IDENTIFY:** For a reversible isothermal process, $\Delta S = \frac{Q}{T}$. For a reversible adiabatic process, $Q = 0$ and $\Delta S = 0$.

The Carnot cycle consists of two reversible isothermal processes and two reversible adiabatic processes.

SET UP: Use the results for the Stirling cycle from Problem 20.50.

EXECUTE: (a) The graph is given in Figure 20.60.

(b) For a reversible process, $dS = \frac{dQ}{T}$, and so $dQ = T dS$, and $Q = \int dQ = \int T dS$, which is the area under the curve in the TS plane.

(c) Q_H is the area under the rectangle bounded by the horizontal part of the rectangle at T_H and the verticals. $|Q_C|$ is the area bounded by the horizontal part of the rectangle at T_C and the verticals. The net work is then $Q_H - |Q_C|$, the area bounded by the rectangle that represents the process. The ratio of the areas is the ratio of the lengths of the vertical sides of the respective rectangles, and the efficiency is $e = \frac{W}{Q_H} = \frac{T_H - T_C}{T_H}$.

(d) As explained in Problem 20.50, the substance that mediates the heat exchange during the isochoric expansion and compression does not leave the system, and the diagram is the same as in part (a). As found in that problem, the ideal efficiency is the same as for a Carnot-cycle engine.

EVALUATE: The derivation of e_{Carnot} using the concept of entropy is much simpler than the derivation in Section 20.6, but yields the same result.

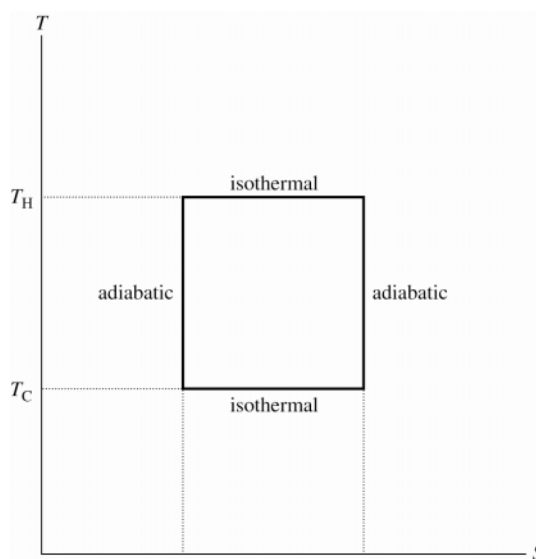


Figure 20.60

- 20.61. IDENTIFY:** The temperatures of the ice-water mixture and of the boiling water are constant, so $\Delta S = \frac{Q}{T}$. The heat

flow for the melting phase transition of the ice is $Q = +mL_f$.

SET UP: For water, $L_f = 3.34 \times 10^5$ J/kg.

EXECUTE: (a) The heat that goes into the ice-water mixture is

$Q = mL_f = (0.160 \text{ kg})(3.34 \times 10^5 \text{ J/kg}) = 5.34 \times 10^4$ J. This is same amount of heat leaves the boiling water, so

$$\Delta S = \frac{Q}{T} = \frac{-5.34 \times 10^4 \text{ J}}{373 \text{ K}} = -143 \text{ J/K.}$$

$$(b) \Delta S = \frac{Q}{T} = \frac{5.34 \times 10^4 \text{ J}}{273 \text{ K}} = +196 \text{ J/K}$$

(c) For any segment of the rod, the net heat flow is zero, so $\Delta S = 0$.

$$(d) \Delta S_{\text{tot}} = -143 \text{ J/K} + 196 \text{ J/K} = +53 \text{ J/K.}$$

EVALUATE: The heat flow is irreversible, since the system is isolated and the total entropy change is positive.

- 20.62. IDENTIFY:** Use the expression derived in Example 20.6 for the entropy change in a temperature change.

SET UP: For water, $c = 4190$ J/kg \cdot K. $20^\circ\text{C} = 293.15$ K, $65^\circ\text{C} = 338.15$ K and $120^\circ\text{C} = 393.15$ K.

EXECUTE: (a) $\Delta S = mc \ln(T_2/T_1) = (250 \times 10^{-3} \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln(338.15 \text{ K}/293.15 \text{ K}) = 150 \text{ J/K.}$

$$(b) \Delta S = \frac{-mc\Delta T}{T_{\text{element}}} = \frac{-(250 \times 10^{-3} \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(338.15 \text{ K} - 293.15 \text{ K})}{393.15 \text{ K}} = -120 \text{ J/K.}$$

(c) The sum of the result of parts (a) and (b) is $\Delta S_{\text{system}} = 30 \text{ J/K.}$

EVALUATE: (d) Heating a liquid is not reversible. Whatever the energy source for the heating element, heat is being delivered at a higher temperature than that of the water, and the entropy loss of the source will be less in magnitude than the entropy gain of the water. The net entropy change is positive.

- 20.63. IDENTIFY:** Use the expression derived in Example 20.6 for the entropy change in a temperature change. For the value of T for which ΔS is a maximum, $d(\Delta S)/dT = 0$.

SET UP: The heat flow for a temperature change is $Q = mc\Delta T$

EXECUTE: (a) As in Example 20.10, the entropy change of the first object is $m_1c_1\ln(T/T_1)$ and that of the second is $m_2c_2\ln(T'/T_2)$, and so the net entropy change is as given. Neglecting heat transfer to the surroundings,

$Q_1 + Q_2 = 0$, $m_1c_1(T - T_1) + m_2c_2(T' - T_2) = 0$, which is the given expression.

(b) Solving the energy-conservation relation for T' and substituting into the expression for ΔS gives

$$\Delta S = m_1c_1\ln\left(\frac{T}{T_1}\right) + m_2c_2\ln\left(1 - \frac{m_1c_1}{m_2c_2}\left(\frac{T}{T_2} - \frac{T_1}{T_2}\right)\right).$$

Differentiating with respect to T and setting the derivative equal to 0 gives $0 = \frac{m_1c_1}{T} + \frac{(m_2c_2)(m_1c_1/m_2c_2)(-1/T_2)}{\left(1 - (m_1c_1/m_2c_2)\left(\frac{T}{T_2} - \frac{T_1}{T_2}\right)\right)}$. This may be solved for $T = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$. Using this value for T

in the conservation of energy expression in part (a) and solving for T' gives $T' = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$. Therefore,

$T = T'$ when ΔS is a maximum.

EVALUATE: (c) The final state of the system will be that for which no further entropy change is possible. If $T < T'$, it is possible for the temperatures to approach each other while increasing the total entropy, but when $T = T'$, no further spontaneous heat exchange is possible.

- 20.64. IDENTIFY:** Calculate Q_C and Q_H in terms of p and V at each point. Use the ideal gas law and the pressure-volume relation for adiabatic processes for an ideal gas. $e = 1 - \frac{|Q_C|}{|Q_H|}$.

SET UP: For an ideal gas, $C_p = C_v + R$, and taking air to be diatomic, $C_p = \frac{7}{2}R$, $C_v = \frac{5}{2}R$ and $\gamma = \frac{7}{5}$.

EXECUTE: Referring to Figure 20.7 in the textbook, $Q_H = n\frac{7}{2}R(T_c - T_b) = \frac{7}{2}(p_cV_c - p_bV_b)$. Similarly, $Q_C = n\frac{5}{2}R(p_aV_a - p_dV_d)$. What needs to be done is to find the relations between the product of the pressure and the volume at the four points. For an ideal gas, $\frac{p_cV_c}{T_c} = \frac{p_bV_b}{T_b}$ so $p_cV_c = p_bV_b\left(\frac{T_c}{T_b}\right)$. For a compression ratio r , and given

that for the Diesel cycle the process ab is adiabatic, $p_bV_b = p_aV_a\left(\frac{V_b}{V_a}\right)^{\gamma-1} = p_aV_a r^{\gamma-1}$. Similarly, $p_dV_d = p_cV_c\left(\frac{V_d}{V_c}\right)^{\gamma-1}$.

Note that the last result uses the fact that process da is isochoric, and $V_d = V_a$; also, $p_c = p_b$ (process bc is isobaric),

and so $V_c = V_b\left(\frac{T_c}{T_a}\right)$. Then,

$$\frac{V_c}{V_a} = \frac{T_c}{T_b} \cdot \frac{V_b}{V_a} = \frac{T_b}{T_a} \cdot \frac{T_a}{T_b} \cdot \frac{V_a}{V_b} = \frac{T_c}{T_a} \cdot \left(\frac{T_a V_a^{\gamma-1}}{T_b V_b^{\gamma-1}}\right) \left(\frac{V_a}{V_b}\right)^{-\gamma} = \frac{T_c}{T_a} r^\gamma$$

Combining the above results, $p_dV_d = p_aV_a\left(\frac{T_c}{T_a}\right)^\gamma r^{\gamma-\gamma^2}$. Substitution of the above results into Eq. (20.4) gives

$$e = 1 - \frac{5}{7} \frac{\left[\left(\frac{T_c}{T_a}\right)^\gamma r^{\gamma-\gamma^2} - 1\right]}{\left[\left(\frac{T_c}{T_a}\right) - r^{\gamma-1}\right]}.$$

- (b) $e = 1 - \frac{1}{1.4} \frac{\left[(5.002)r^{-0.56} - 1\right]}{\left[(3.167) - r^{0.40}\right]}$, where $\frac{T_c}{T_a} = 3.167$ and $\gamma = 1.40$ have been used. Substitution of $r = 21.0$ yields $e = 0.708 = 70.8\%$.

EVALUATE: The efficiency for an Otto cycle with $r = 21.0$ and $\gamma = 1.40$ is $e = 1 - r^{1-\gamma} = 1 - (21.0)^{-0.40} = 70.4\%$. This is very close to the value for the Diesel cycle.