PERIODIC MOTION

13

13.1. **IDENTIFY and SET UP:** The target variables are the period T and angular frequency ω. We are given the frequency f, so we can find these using Eqs.(13.1) and (13.2)

EXECUTE: **(a)** $f = 220$ Hz

$T = 1/f = 1/220 \text{ Hz} = 4.54 \times 10^{-3}$ s

$\omega = 2\pi f = 2\pi(220 \text{ Hz}) = 1380$ rad/s

(b) $f = 2(220 \text{ Hz}) = 440$ Hz

$T = 1/f = 1/440 \text{ Hz} = 2.27 \times 10^{-3}$ s (smaller by a factor of 2)

$\omega = 2\pi f = 2\pi(440 \text{ Hz}) = 2760$ rad/s (factor of 2 larger)

EVALUATE: The angular frequency is directly proportional to the frequency and the period is inversely proportional to the frequency.

13.2. **IDENTIFY and SET UP:** The amplitude is the maximum displacement from equilibrium. In one period the object goes from $x = +A$ to $x = -A$ and returns.

EXECUTE: **(a)** $A = 0.120$ m

(b) $0.800 \text{ s} = T/2$ so the period is 1.60 s

(c) $f = \dfrac{1}{T} = 0.625$ Hz

EVALUATE: Whenever the object is released from rest, its initial displacement equals the amplitude of its SHM.

13.3. **IDENTIFY:** The period is the time for one vibration and $\omega = \dfrac{2\pi}{T}$.

SET UP: The units of angular frequency are rad/s.

EXECUTE: The period is $\dfrac{0.50 \text{ s}}{440} = 1.14 \times 10^{-3}$ s and the angular frequency is $\omega = \dfrac{2\pi}{T} = 5.53 \times 10^{3}$ rad/s.

EVALUATE: There are 880 vibrations in 1.0 s, so $f = 880$ Hz. This is equal to $1/T$.

13.4. **IDENTIFY:** The period is the time for one cycle and the amplitude is the maximum displacement from equilibrium. Both these values can be read from the graph.

SET UP: The maximum x is 10.0 cm. The time for one cycle is 16.0 s.

EXECUTE: **(a)** $T = 16.0$ s so $f = \dfrac{1}{T} = 0.0625$ Hz.

(b) $A = 10.0$ cm.

(c) $T = 16.0$ s

(d) $\omega = 2\pi f = 0.393$ rad/s

EVALUATE: After one cycle the motion repeats.

13.5. **IDENTIFY:** This displacement is $\frac{1}{4}$ of a period.

SET UP: $T = 1/f = 0.200$ s.

EXECUTE: $t = 0.0500$ s

EVALUATE: The time is the same for $x = A$ to $x = 0$, for $x = 0$ to $x = -A$, for $x = -A$ to $x = 0$ and for $x = 0$ to $x = A$.

13.6. **IDENTIFY:** Apply Eq.(13.12).

SET UP: The period will be twice the interval between the times at which the glider is at the equilibrium position.

EXECUTE: $k = \omega^2 m = \left(\dfrac{2\pi}{T}\right)^2 m = \left(\dfrac{2\pi}{2(2.60 \text{ s})}\right)^2 (0.200 \text{ kg}) = 0.292$ N/m.

EVALUATE: $1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$, so $1 \text{ N/m} = 1 \text{ kg/s}^2$.

13.7. IDENTIFY and SET UP: Use Eq.(13.1) to calculate T, Eq.(13.2) to calculate ω, and Eq.(13.10) for m.

EXECUTE: **(a)** $T = 1/f = 1/6.00\text{ Hz} = 0.167\text{ s}$

(b) $\omega = 2\pi f = 2\pi(6.00\text{ Hz}) = 37.7\text{ rad/s}$

(c) $\omega = \sqrt{k/m}$ implies $m = k/\omega^2 = (120\text{ N/m})/(37.7\text{ rad/s})^2 = 0.0844\text{ kg}$

EVALUATE: We can verify that k/ω^2 has units of mass.

13.8. IDENTIFY: The mass and frequency are related by $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$.

SET UP: $f\sqrt{m} = \dfrac{\sqrt{k}}{2\pi} =$ constant, so $f_1\sqrt{m_1} = f_2\sqrt{m_2}$.

EXECUTE: **(a)** $m_1 = 0.750\text{ kg}$, $f_1 = 1.33\text{ Hz}$ and $m_2 = 0.750\text{ kg} + 0.220\text{ kg} = 0.970\text{ kg}$.

$$f_2 = f_1\sqrt{\frac{m_1}{m_2}} = (1.33\text{ Hz})\sqrt{\frac{0.750\text{ kg}}{0.970\text{ kg}}} = 1.17\text{ Hz}.$$

(b) $m_2 = 0.750\text{ kg} - 0.220\text{ kg} = 0.530\text{ kg}$. $f_2 = (1.33\text{ Hz})\sqrt{\dfrac{0.750\text{ kg}}{0.530\text{ kg}}} = 1.58\text{ Hz}$

EVALUATE: When the mass increases the frequency decreases and when the mass decreases the frequency increases.

13.9. IDENTIFY: Apply Eqs.(13.11) and (13.12).

SET UP: $f = 1/T$

EXECUTE: (a) $T = 2\pi\sqrt{\dfrac{0.500\text{ kg}}{140\text{ N/m}}} = 0.375\text{ s}$.

(b) $f = \dfrac{1}{T} = 2.66\text{ Hz}$. **(c)** $\omega = 2\pi f = 16.7\text{ rad/s}$.

EVALUATE: We can verify that $1\text{ kg/(N/m)} = 1\text{ s}^2$.

13.10. IDENTIFY and SET UP: Use Eqs. (13.13), (13.15), and (13.16).

EXECUTE: $f = 440\text{ Hz}$, $A = 3.0\text{ mm}$, $\phi = 0$

(a) $x = A\cos(\omega t + \phi)$

$\omega = 2\pi f = 2\pi(440\text{ hz}) = 2.76\times10^3\text{ rad/s}$

$x = (3.0\times10^{-3}\text{ m})\cos((2.76\times10^3\text{ rad/s})t)$

(b) $v_x = -\omega A\sin(\omega t + \phi)$

$v_{max} = \omega A = (2.76\times10^3\text{ rad/s})(3.0\times10^{-3}\text{ m}) = 8.3\text{ m/s}$ (maximum magnitude of velocity)

$a_x = -\omega^2 A\cos(\omega t + \phi)$

$a_{max} = \omega^2 A = (2.76\times10^3\text{ rad/s})^2(3.0\times10^{-3}\text{ m}) = 2.3\times10^4\text{ m/s}^2$ (maximum magnitude of acceleration)

(c) $a_x = -\omega^2 A\cos\omega t$

$da_x/dt = +\omega^3 A\sin\omega t = [2\pi(440\text{ Hz})]^3\,(3.0\times10^{-3}\text{ m})\sin([2.76\times10^3\text{ rad/s}]t) = (6.3\times10^7\text{ m/s}^3)\sin([2.76\times10^3\text{rad/s}]t)$

Maximum magnitude of the jerk is $\omega^3 A = 6.3\times10^7\text{ m/s}^3$

EVALUATE: The period of the motion is small, so the maximum acceleration and jerk are large.

13.11. IDENTIFY: Use Eq.(13.19) to calculate A. The initial position and velocity of the block determine ϕ. $x(t)$ is given by Eq.(13.13).

SET UP: $\cos\theta$ is zero when $\theta = \pm\pi/2$ and $\sin(\pi/2) = 1$.

EXECUTE: **(a)** From Eq. (13.19), $A = \left|\dfrac{v_0}{\omega}\right| = \left|\dfrac{v_0}{\sqrt{k/m}}\right| = 0.98\text{ m}$.

(b) Since $x(0) = 0$, Eq.(13.14) requires $\phi = \pm\dfrac{\pi}{2}$. Since the block is initially moving to the left, $v_{0x} < 0$ and Eq.(13.7) requires that sin $\phi > 0$, so $\phi = +\dfrac{\pi}{2}$.

(c) $\cos(\omega t + (\pi/2)) = -\sin\omega t$, so $x = (-0.98\text{ m})\sin((12.2\text{ rad/s})t)$.

EVALUATE: The $x(t)$ result in part (c) does give $x = 0$ at $t = 0$ and $x < 0$ for t slightly greater than zero.

13.12. IDENTIFY and SET UP: We are given k, m, x_0, and v_0. Use Eqs.(13.19), (13.18), and (13.13).

EXECUTE: **(a)** Eq.(13.19): $A = \sqrt{x_0^2 + v_{0x}^2/\omega^2} = \sqrt{x_0^2 + mv_{0x}^2/k}$

$$A = \sqrt{(0.200\text{ m})^2 + (2.00\text{ kg})(-4.00\text{ m/s})^2/(300\text{ N/m})} = 0.383\text{ m}$$

(b) Eq.(13.18): $\phi = \arctan(-v_{0x}/\omega x_0)$

$\omega = \sqrt{k/m} = \sqrt{(300\text{ N/m})/2.00\text{ kg}} = 12.25\text{ rad/s}$

$$\phi = \arctan\left(-\frac{(-4.00\text{ m/s})}{(12.25\text{ rad/s})(0.200\text{ m})}\right) = \arctan(+1.633) = 58.5° \ \text{(or 1.02 rad)}$$

(c) $x = A\cos(\omega t + \phi)$ gives $x = (0.383\text{ m})\cos([12.2\text{rad/s}]t + 1.02\text{ rad})$

EVALUATE: At $t = 0$ the block is displaced 0.200 m from equilibrium but is moving, so $A > 0.200$ m. According to Eq.(13.15), a phase angle ϕ in the range $0 < \phi < 90°$ gives $v_{0x} < 0$.

13.13. **IDENTIFY:** For SHM, $a_x = -\omega^2 x = -(2\pi f)^2 x$. Apply Eqs.(13.13), (13.15) and (13.16), with A and ϕ from Eqs.(13.18) and (13.19).

SET UP: $x = 1.1$ cm, $v_{0x} = -15$ cm/s. $\omega = 2\pi f$, with $f = 2.5$ Hz.

EXECUTE: **(a)** $a_x = -(2\pi(2.5\text{ Hz}))^2(1.1\times10^{-2}\text{ m}) = -2.71\text{ m/s}^2$.

(b) From Eq. (13.19) the amplitude is 1.46 cm, and from Eq. (13.18) the phase angle is 0.715 rad. The angular frequency is $2\pi f = 15.7$ rad/s, so $x = (1.46\text{ cm})\cos((15.7\text{ rad/s})t + 0.715\text{ rad})$,

$v_x = (-22.9\text{ cm/s})\sin((15.7\text{ rad/s})t + 0.715\text{ rad})$ and $a_x = (-359\text{ cm/s}^2)\cos((15.7\text{ rad/s})t + 0.715\text{ rad})$.

EVALUATE: We can verify that our equations for x, v_x and a_x give the specified values at $t = 0$.

13.14. **IDENTIFY** and **SET UP:** Calculate x using Eq.(13.13). Use T to calculate ω and x_0 to calculate ϕ.

EXECUTE: $x = 0$ at $t = 0$ implies that $\phi = \pm\pi/2$ rad

Thus $x = A\cos(\omega t \pm \pi/2)$.

$T = 2\pi/\omega$ so $\omega = 2\pi/T = 2\pi/1.20\text{ s} = 5.236\text{ rad/s}$

$x = (0.600\text{ m})\cos([5.236\text{ rad/s}][0.480\text{ s}] \pm \pi/2) = \mp 0.353\text{ m}$.

The distance of the object from the equilibrium position is 0.353 m.

EVALUATE: The problem doesn't specify whether the object is moving in the $+x$ or $-x$ direction at $t = 0$.

13.15. **IDENTIFY:** Apply $T = 2\pi\sqrt{\frac{m}{k}}$. Use the information about the empty chair to calculate k.

SET UP: When $m = 42.5$ kg, $T = 1.30$ s.

EXECUTE: Empty chair: $T = 2\pi\sqrt{\frac{m}{k}}$ gives $k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(42.5\text{ kg})}{(1.30\text{ s})^2} = 993\text{ N/m}$

With person in chair: $T = 2\pi\sqrt{\frac{m}{k}}$ gives $m = \frac{T^2 k}{4\pi^2} = \frac{(2.54\text{ s})^2(993\text{ N/m})}{4\pi^2} = 162$ kg and

$m_{\text{person}} = 162\text{ kg} - 42.5\text{ kg} = 120\text{ kg}$.

EVALUATE: For the same spring, when the mass increases, the period increases.

13.16. **IDENTIFY** and **SET UP:** Use Eq.(13.12) for T and Eq.(13.4) to relate a_x and k.

EXECUTE: $T = 2\pi\sqrt{m/k}$, $m = 0.400$ kg

Use $a_x = -2.70\text{ m/s}^2$ to calculate k: $-kx = ma_x$ gives $k = -\frac{ma_x}{x} = -\frac{(0.400\text{ kg})(-2.70\text{ m/s}^2)}{0.300\text{ m}} = +3.60\text{ N/m}$

$T = 2\pi\sqrt{m/k} = 2.09$ s

EVALUATE: a_x is negative when x is positive. ma_x/x has units of N/m and $\sqrt{m/k}$ has units of s.

13.17. **IDENTIFY:** $T = 2\pi\sqrt{\frac{m}{k}}$. $a_x = -\frac{k}{m}x$ so $a_{\max} = \frac{k}{m}A$. $F = -kx$.

SET UP: a_x is proportional to x so a_x goes through one cycle when the displacement goes through one cycle. From the graph, one cycle of a_x extends from $t = 0.10$ s to $t = 0.30$ s, so the period is $T = 0.20$ s. $k = 2.50\text{ N/cm} = 250\text{ N/m}$. From the graph the maximum acceleration is 12.0 m/s^2.

EXECUTE: **(a)** $T = 2\pi\sqrt{\frac{m}{k}}$ gives $m = k\left(\frac{T}{2\pi}\right)^2 = (250\text{ N/m})\left(\frac{0.20\text{ s}}{2\pi}\right)^2 = 0.253\text{ kg}$

(b) $A = \frac{ma_{\max}}{k} = \frac{(0.253\text{ kg})(12.0\text{ m/s}^2)}{250\text{ N/m}} = 0.0121\text{ m} = 1.21\text{ cm}$

(c) $F_{\max} = kA = (250\text{ N/m})(0.0121\text{ m}) = 3.03\text{ N}$

EVALUATE: We can also calculate the maximum force from the maximum acceleration:
$F_{max} = ma_{max} = (0.253\text{ kg})(12.0\text{ m/s}^2) = 3.04\text{ N},$ which agrees with our previous results.

13.18. IDENTIFY: The general expression for $v_x(t)$ is $v_x(t) = -\omega A\sin(\omega t + \phi)$. We can determine ω and A by comparing the equation in the problem to the general form.

SET UP: $\omega = 4.71\text{ rad/s}$. $\omega A = 3.60\text{ cm/s} = 0.0360\text{ m/s}$.

EXECUTE: **(a)** $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi\text{ rad}}{4.71\text{ rad/s}} = 1.33\text{ s}$

(b) $A = \dfrac{0.0360\text{ m/s}}{\omega} = \dfrac{0.0360\text{ m/s}}{4.71\text{ rad/s}} = 7.64\times10^{-3}\text{ m} = 7.64\text{ mm}$

(c) $a_{max} = \omega^2 A = (4.71\text{ rad/s})^2(7.64\times10^{-3}\text{ m}) = 0.169\text{ m/s}^2$

(d) $\omega = \sqrt{\dfrac{k}{m}}$ so $k = m\omega^2 = (0.500\text{ kg})(4.71\text{ rad/s})^2 = 11.1\text{ N/m}$.

EVALUATE: The overall positive sign in the expression for $v_x(t)$ and the factor of $-\pi/2$ both are related to the phase factor ϕ in the general expression.

13.19. IDENTIFY: Compare the specific $x(t)$ given in the problem to the general form of Eq.(13.13).

SET UP: $A = 7.40\text{ cm}$, $\omega = 4.16\text{ rad/s}$, and $\phi = -2.42\text{ rad}$.

EXECUTE: **(a)** $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4.16\text{ rad/s}} = 1.51\text{ s}$.

(b) $\omega = \sqrt{\dfrac{k}{m}}$ so $k = m\omega^2 = (1.50\text{ kg})(4.16\text{ rad/s})^2 = 26.0\text{ N/m}$

(c) $v_{max} = \omega A = (4.16\text{ rad/s})(7.40\text{ cm}) = 30.8\text{ cm/s}$

(d) $F_x = -kx$ so $F = kA = (26.0\text{ N/m})(0.0740\text{ m}) = 1.92\text{ N}$.

(e) $x(t)$ evaluated at $t = 1.00\text{ s}$ gives $x = -0.0125\text{ m}$. $v_x = -\omega A\sin(\omega t + \phi) = 30.4\text{ cm/s}$.
$a_x = -kx/m = -\omega^2 x = +0.216\text{ m/s}^2$.

EVALUATE: The maximum speed occurs when $x = 0$ and the maximum force is when $x = \pm A$.

13.20. IDENTIFY: Apply $x(t) = A\cos(\omega t + \phi)$

SET UP: $x = A$ at $t = 0$, so $\phi = 0$. $A = 6.00\text{ cm}$. $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.300\text{ s}} = 20.9\text{ rad/s}$, so
$x(t) = (6.00\text{ cm})\cos([20.9\text{ rad/s}]t)$.

EXECUTE: $t = 0$ at $x = 6.00\text{ cm}$. $x = -1.50\text{ cm}$ when $-1.50\text{ cm} = (6.00\text{ cm})\cos([20.9\text{ rad/s}]t)$.
$t = \left(\dfrac{1}{20.9\text{ rad/s}}\right)\arccos\left(-\dfrac{1.50\text{ cm}}{6.00\text{ cm}}\right) = 0.0872\text{ s}$. It takes 0.0872 s.

EVALUATE: It takes $t = T/4 = 0.075\text{ s}$ to go from $x = 6.00\text{ cm}$ to $x = 0$ and 0.150 s to go from $x = +6.00\text{ cm}$ to $x = -6.00\text{ cm}$. Our result is between these values, as it should be.

13.21. IDENTIFY: $v_{max} = \omega A = 2\pi fA$. $K_{max} = \frac{1}{2}mv_{max}^2$

SET UP: The fly has the same speed as the tip of the tuning fork.

EXECUTE: **(a)** $v_{max} = 2\pi fA = 2\pi(392\text{ Hz})(0.600\times10^{-3}\text{ m}) = 1.48\text{ m/s}$

(b) $K_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(0.0270\times10^{-3}\text{ kg})(1.48\text{ m/s})^2 = 2.96\times10^{-5}\text{ J}$

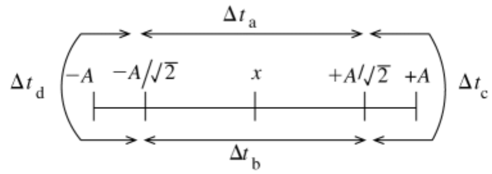
EVALUATE: v_{max} is directly proportional to the frequency and to the amplitude of the motion.

13.22. IDENTIFY and SET UP: Use Eq.(13.21) to relate K and U. U depends on x and K depends on v_x.

EXECUTE: **(a)** $U + K = E,$ so $U = K$ says that $2U = E$
$2\left(\frac{1}{2}kx^2\right) = \frac{1}{2}kA^2$ and $x = \pm A/\sqrt{2};$ magnitude is $A/\sqrt{2}$
But $U = K$ also implies that $2K = E$
$2(\frac{1}{2}mv_x^2) = \frac{1}{2}kA^2$ and $v_x = \pm\sqrt{k/m}A/\sqrt{2} = \pm\omega A/\sqrt{2};$ magnitude is $\omega A/\sqrt{2}.$

(b) In one cycle x goes from A to 0 to $-A$ to 0 to $+A$. Thus $x = +A\sqrt{2}$ twice and $x = -A/\sqrt{2}$ twice in each cycle. Therefore, $U = K$ four times each cycle. The time between $U = K$ occurrences is the time Δt_a for $x_1 = +A/\sqrt{2}$ to

$x_2 = -A\sqrt{2}$, time Δt_b for $x_1 = -A/\sqrt{2}$ to $x_2 = +A/\sqrt{2}$, time Δt_c for $x_1 = +A/\sqrt{2}$ to $x_2 = +A\sqrt{2}$, or the time Δt_d for $x_1 = -A/\sqrt{2}$ to $x_2 = -A/\sqrt{2}$, as shown in Figure 13.22.



$\Delta t_a = \Delta t_b$

$\Delta t_c = \Delta t_d$

Figure 13.22

Calculation of Δt_a :

Specify *x* in $x = A\cos\omega t$ (choose $\phi = 0$ so $x = A$ at $t = 0$) and solve for *t*.

$x_1 = +A/\sqrt{2}$ implies $A/\sqrt{2} = A\cos(\omega t_1)$

$\cos\omega t_1 = 1/\sqrt{2}$ so $\omega t_1 = \arccos(1/\sqrt{2}) = \pi/4$ rad

$t_1 = \pi/4\omega$

$x_2 = -A/\sqrt{2}$ implies $-A/\sqrt{2} = A\cos(\omega t_2)$

$\cos\omega t_2 = -1/\sqrt{2}$ so $\omega t_1 = 3\pi/4$ rad

$t_2 = 3\pi/4\omega$

$\Delta t_a = t_2 - t_1 = 3\pi/4\omega - \pi/4\omega = \pi/2\omega$ (Note that this is $T/4$, one fourth period.)

Calculation of Δt_d :

$x_1 = -A/\sqrt{2}$ implies $t_1 = 3\pi/4\omega$

$x_2 = -A/\sqrt{2}$, t_2 is the *next* time after t_1 that gives $\cos\omega t_2 = -1/\sqrt{2}$

Thus $\omega t_2 = \omega t_1 + \pi/2 = 5\pi/4$ and $t_2 = 5\pi/4\omega$

$\Delta t_d = t_2 - t_1 = 5\pi/4\omega - 3\pi/4\omega = \pi/2\omega$, so is the same as Δt_a.

Therfore the occurrences of $K = U$ are equally spaced in time, with a time interval between them of $\pi/2\omega$.

EVALUATE: This is one-fourth *T*, as it must be if there are 4 equally spaced occurrences each period.

(c) EXECUTE: $x = A/2$ and $U + K = E$

$K = E - U = \frac{1}{2}kA^2 - \frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}k(A/2)^2 = \frac{1}{2}kA^2 - \frac{1}{8}kA^2 = 3kA^2/8$

Then $\dfrac{K}{E} = \dfrac{3kA^2/8}{\frac{1}{2}kA^2} = \dfrac{3}{4}$ and $\dfrac{U}{E} = \dfrac{\frac{1}{8}kA^2}{\frac{1}{2}kA^2} = \dfrac{1}{4}$

EVALUATE: At $x = 0$ all the energy is kinetic and at $x = \pm A$ all the energy is potiential. But $K = U$ does not occur at $x = \pm A/2$, since *U* is not linear in *x*.

13.23. **IDENTIFY:** Velocity and position are related by $E = \frac{1}{2}kA^2 = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$. Acceleration and position are related by $-kx = ma_x$.

SET UP: The maximum speed is at $x = 0$ and the maximum magnitude of acceleration is at $x = \pm A$,

EXECUTE: **(a)** For $x = 0$, $\frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$ and $v_{max} = A\sqrt{\dfrac{k}{m}} = (0.040\text{ m})\sqrt{\dfrac{450\text{ N/m}}{0.500\text{ kg}}} = 1.20\text{ m/s}$

(b) $v_x = \pm\sqrt{\dfrac{k}{m}}\sqrt{A^2 - x^2} = \pm\sqrt{\dfrac{450\text{ N/m}}{0.500\text{ kg}}}\sqrt{(0.040\text{ m})^2 - (0.015\text{ m})^2} = \pm 1.11\text{ m/s}$.

The speed is $v = 1.11\text{ m/s}$.

(c) For $x = \pm A$, $a_{max} = \dfrac{k}{m}A = \left(\dfrac{450\text{ N/m}}{0.500\text{ kg}}\right)(0.040\text{ m}) = 36\text{ m/s}^2$

(d) $a_x = -\dfrac{kx}{m} = -\dfrac{(450\text{ N/m})(-0.015\text{ m})}{0.500\text{ kg}} = +13.5\text{ m/s}^2$

(e) $E = \frac{1}{2}kA^2 = \frac{1}{2}(450\text{ N/m})(0.040\text{ m})^2 = 0.360\text{ J}$

EVALUATE: The speed and acceleration at $x = -0.015$ m are less than their maximum values.

13.24. **IDENTIFY** and **SET UP:** a_x is related to *x* by Eq.(13.4) and v_x is related to *x* by Eq.(13.21). a_x is a maximum when $x = \pm A$ and v_x is a maximum when $x = 0$. *t* is related to *x* by Eq.(13.13).

EXECUTE: **(a)** $-kx = ma_x$ so $a_x = -(k/m)x$ (Eq.13.4). But the maximum $|x|$ is A, so $a_{max} = (k/m)A = \omega^2 A$.

$f = 0.850$ Hz implies $\omega = \sqrt{k/m} = 2\pi f = 2\pi(0.850 \text{ Hz}) = 5.34$ rad/s.

$a_{max} = \omega^2 A = (5.34 \text{ rad/s})^2(0.180 \text{ m}) = 5.13 \text{ m/s}^2$.

$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$v_x = v_{max}$ when $x = 0$ so $\frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$

$v_{max} = \sqrt{k/m}A = \omega A = (5.34 \text{ rad/s})(0.180 \text{ m}) = 0.961$ m/s

(b) $a_x = -(k/m)x = -\omega^2 x = -(5.34 \text{ rad/s})^2(0.090 \text{ m}) = -2.57 \text{ m/s}^2$

$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ says that $v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2} = \pm\omega\sqrt{A^2 - x^2}$

$v_x = \pm(5.34 \text{ rad/s})\sqrt{(0.180 \text{ m})^2 - (0.090 \text{ m})^2} = \pm 0.832$ m/s

The speed is 0.832 m/s.

(c) $x = A\cos(\omega t + \phi)$

Let $\phi = -\pi/2$ so that $x = 0$ at $t = 0$.

Then $x = A\cos(\omega t - \pi/2) = A\sin(\omega t)$ [Using the trig identity $\cos(a - \pi/2) = \sin a$]

Find the time t that gives $x = 0.120$ m.

$0.120 \text{ m} = (0.180 \text{ m})\sin(\omega t)$

$\sin\omega t = 0.6667$

$t = \arcsin(0.6667)/\omega = 0.7297 \text{ rad}/(5.34 \text{ rad/s}) = 0.137$ s

EVALUATE: It takes one-fourth of a period for the object to go from $x = 0$ to $x = A = 0.180$ m. So the time we have calculated should be less than $T/4$. $T = 1/f = 1/0.850 \text{ Hz} = 1.18$ s, $T/4 = 0.295$ s, and the time we calculated is less than this. Note that the a_x and v_x we calculated in part (b) are smaller in magnitude than the maximum values we calculated in part (b).

(d) The conservation of energy equation relates v and x and $F = ma$ relates a and x. So the speed and acceleration can be found by energy methods but the time cannot.

Specifying x uniquely determines a_x but determines only the magnitude of v_x; at a given x the object could be moving either in the $+x$ or $-x$ direction.

13.25. **IDENTIFY:** Use the results of Example 13.15 and also that $E = \frac{1}{2}kA^2$.

SET UP: In the example, $A_2 = A_1\sqrt{\frac{M}{M+m}}$ and now we want $A_2 = \frac{1}{2}A_1$. Therefore, $\frac{1}{2} = \sqrt{\frac{M}{M+m}}$, or $m = 3M$. For the energy, $E_2 = \frac{1}{2}kA_2^2$, but since $A_2 = \frac{1}{2}A_1$, $E_2 = \frac{1}{4}E_1$, and $\frac{3}{4}E_1$ is lost to heat.

EVALUATE: The putty and the moving block undergo a totally inelastic collision and the mechanical energy of the system decreases.

13.26. **IDENTIFY** and **SET UP:** Use Eq.(13.21). $x = \pm A\omega$ when $v_x = 0$ and $v_x = \pm v_{max}$ when $x = 0$.

EXECUTE: **(a)** $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$E = \frac{1}{2}(0.150 \text{ kg})(0.300 \text{ m/s})^2 + \frac{1}{2}(300 \text{ N/m})(0.012 \text{ m})^2 = 0.0284$ J

(b) $E = \frac{1}{2}kA^2$ so $A = \sqrt{2E/k} = \sqrt{2(0.0284 \text{ J})/300 \text{ N/m}} = 0.014$ m

(c) $E = \frac{1}{2}mv_{max}^2$ so $v_{max} = \sqrt{2E/m} = \sqrt{2(0.0284 \text{ J})/0.150 \text{ kg}} = 0.615$ m/s

EVALUATE: The total energy E is constant but is transferred between kinetic and potential energy during the motion.

13.27. **IDENTIFY:** Conservation of energy says $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ and Newton's second law says $-kx = ma_x$.

SET UP: Let $+x$ be to the right. Let the mass of the object be m.

EXECUTE: $k = -\frac{ma_x}{x} = -m\left(\frac{-8.40 \text{ m/s}^2}{0.600 \text{ m}}\right) = (14.0 \text{ s}^{-2})m$.

$A = \sqrt{x^2 + (m/k)v^2} = \sqrt{(0.600 \text{ m})^2 + \left(\frac{m}{[14.0 \text{ s}^{-2}]m}\right)(2.20 \text{ m/s})^2} = 0.840$ m. The object will therefore travel $0.840 \text{ m} - 0.600 \text{ m} = 0.240$ m to the right before stopping at its maximum amplitude.

EVALUATE: The acceleration is not constant and we cannot use the constant acceleration kinematic equations.

13.28. **IDENTIFY:** When the box has its maximum speed all of the energy of the system is in the form of kinetic energy. When the stone is removed the oscillating mass is decreased and the speed of the remaining mass is unchanged. The period is given by $T = 2\pi\sqrt{\frac{m}{k}}$.

SET UP: The maximum speed is $v_{max} = \omega A = \sqrt{\frac{k}{m}}A$. With the stone in the box $m = 8.64$ kg and $A = 0.0750$ m.

EXECUTE: **(a)** $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{5.20 \text{ kg}}{375 \text{ N/m}}} = 0.740 \text{ s}$

(b) Just before the stone is removed, the speed is $v_{max} = \sqrt{\frac{375 \text{ N/m}}{8.64 \text{ kg}}}(0.0750 \text{ m}) = 0.494 \text{ m/s}$. The speed of the box isn't altered by removing the stone but the mass on the spring decreases to 5.20 kg. The new amplitude is $A = \sqrt{\frac{m}{k}}v_{max} = \sqrt{\frac{5.20 \text{ kg}}{375 \text{ N/m}}}(0.494 \text{ m/s}) = 0.0582 \text{ m}$. The new amplitude can also be calculated as $\sqrt{\frac{5.20 \text{ kg}}{8.64 \text{ kg}}}(0.0750 \text{ m}) = 0.0582 \text{ m}$.

(c) $T = 2\pi\sqrt{\frac{m}{k}}$. The force constant remains the same. *m* decreases, so *T* decreases.

EVALUATE: After the stone is removed, the energy left in the system is $\frac{1}{2}m_{box}v_{max}^2 = \frac{1}{2}(5.20 \text{ kg})(0.494 \text{ m/s})^2 = 0.6345 \text{ J}$. This then is the energy stored in the spring at its maximum extension or compression and $\frac{1}{2}kA^2 = 0.6345 \text{ J}$. This gives the new amplitude to be 0.0582 m, in agreement with our previous calculation.

13.29. **IDENTIFY:** Work in an inertial frame moving with the vehicle after the engines have shut off. The acceleration before engine shut-off determines the amount the spring is initially stretched. The initial speed of the ball relative to the vehicle is zero.

SET UP: Before the engine shut-off the ball has acceleration $a = 5.00 \text{ m/s}^2$.

EXECUTE: **(a)** $F_x = -kx = ma_x$ gives $A = \frac{ma}{k} = \frac{(3.50 \text{ kg})(5.00 \text{ m/s}^2)}{225 \text{ N/m}} = 0.0778 \text{ m}$. This is the amplitude of the subsequent motion.

(b) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{225 \text{ N/m}}{3.50 \text{ kg}}} = 1.28 \text{ Hz}$

(c) Energy conservation gives $\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$ and $v_{max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{225 \text{ N/m}}{3.50 \text{ kg}}}(0.0778 \text{ m}) = 0.624 \text{ m/s}$.

EVALUATE: During the simple harmonic motion of the ball its maximum acceleration, when $x = \pm A$, continues to have magnitude 5.00 m/s^2.

13.30. **IDENTIFY:** Use the amount the spring is stretched by the weight of the fish to calculate the force constant *k* of the spring. $T = 2\pi\sqrt{m/k}$. $v_{max} = \omega A = 2\pi fA.$

SET UP: When the fish hangs at rest the upward spring force $|F_x| = kx$ equals the weight *mg* of the fish. $f = 1/T$. The amplitude of the SHM is 0.0500 m.

EXECUTE: **(a)** $mg = kx$ so $k = \frac{mg}{x} = \frac{(65.0 \text{ kg})(9.80 \text{ m/s}^2)}{0.120 \text{ m}} = 5.31 \times 10^3 \text{ N/m}$.

(b) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{65.0 \text{ kg}}{5.31 \times 10^3 \text{ N/m}}} = 0.695 \text{ s}$.

(c) $v_{max} = 2\pi fA = \frac{2\pi A}{T} = \frac{2\pi(0.0500 \text{ m})}{0.695 \text{ s}} = 0.452 \text{ m/s}$

EVALUATE: Note that *T* depends only on *m* and *k* and is independent of the distance the fish is pulled down. But v_{max} does depend on this distance.

13.31. **IDENTIFY:** Initially part of the energy is kinetic energy and part is potential energy in the stretched spring. When $x = \pm A$ all the energy is potential energy and when the glider has its maximum speed all the energy is kinetic energy. The total energy of the system remains constant during the motion.

SET UP: Initially $v_x = \pm 0.815 \text{ m/s}$ and $x = \pm 0.0300 \text{ m}$.

EXECUTE: **(a)** Initially the energy of the system is
$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.175\text{ kg})(0.815\text{ m/s})^2 + \frac{1}{2}(155\text{ N/m})(0.0300\text{ m})^2 = 0.128\text{ J}$. $\frac{1}{2}kA^2 = E$ and
$A = \sqrt{\dfrac{2E}{k}} = \sqrt{\dfrac{2(0.128\text{ J})}{155\text{ N/m}}} = 0.0406\text{ m} = 4.06\text{ cm}$.

(b) $\frac{1}{2}mv_{\max}^2 = E$ and $v_{\max} = \sqrt{\dfrac{2E}{m}} = \sqrt{\dfrac{2(0.128\text{ J})}{0.175\text{ kg}}} = 1.21\text{ m/s}$.

(c) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{155\text{ N/m}}{0.175\text{ kg}}} = 29.8\text{ rad/s}$

EVALUATE: The amplitude and the maximum speed depend on the total energy of the system but the angular frequency is independent of the amount of energy in the system and just depends on the force constant of the spring and the mass of the object.

13.32. **IDENTIFY:** $K = \frac{1}{2}mv^2$, $U_{\text{grav}} = mgy$ and $U_{\text{el}} = \frac{1}{2}kx^2$.

SET UP: At the lowest point of the motion, the spring is stretched an amount $2A$.

EXECUTE: **(a)** At the top of the motion, the spring is unstretched and so has no potential energy, the cat is not moving and so has no kinetic energy, and the gravitational potential energy relative to the bottom is $2mgA = 2(4.00\text{ kg})(9.80\text{ m/s}^2)(0.050\text{ m}) = 3.92\text{ J}$. This is the total energy, and is the same total for each part.

(b) $U_{\text{grav}} = 0, K = 0,$ so $U_{\text{spring}} = 3.92\text{ J}$.

(c) At equilibrium the spring is stretched half as much as it was for part (a), and so $U_{\text{spring}} = \frac{1}{4}(3.92\text{ J}) = 0.98\text{ J}$,
$U_{\text{grav}} = \frac{1}{2}(3.92\text{ J}) = 1.96\text{ J},$ and so $K = 0.98\text{ J}.$

EVALUATE: During the motion, work done by the forces transfers energy among the forms kinetic energy, gravitational potential energy and elastic potential energy.

13.33. **IDENTIFY:** The location of the equilibrium position, the position where the downward gravity force is balanced by the upward spring force, changes when the mass of the suspended object changes.

SET UP: At the equilibrium position, the spring is stretched a distance d. The amplitude is the maximum distance of the object from the equilibrium position.

EXECUTE: **(a)** The force of the glue on the lower ball is the upward force that accelerates that ball upward. The upward acceleration of the two balls is greatest when they have the greatest downward displacement, so this is when the force of the glue must be greatest.

(b) With both balls, the distance d_1 that the spring is stretched at equilibrium is given by $kd_1 = (1.50\text{ kg} + 2.00\text{ kg})g$ and $d_1 = 20.8\text{ cm}$. At the lowest point the spring is stretched $20.8\text{ cm} + 15.0\text{ cm} = 35.8\text{ cm}$. After the 1.50 kg ball falls off the distance d_2 that the spring is stretched at equilibrium is given by $kd_2 = (2.00\text{ kg})g$ and $d_2 = 11.9\text{ cm}$.
The new amplitude is $35.8\text{ cm} - 11.9\text{ cm} = 23.9\text{ cm}$. The new frequency is $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} = \dfrac{1}{2\pi}\sqrt{\dfrac{165\text{ N/m}}{2.00\text{ kg}}} = 1.45\text{ Hz}$.

EVALUATE: The potential energy stored in the spring doesn't change when the lower ball comes loose.

13.34. **IDENTIFY:** The torsion constant κ is defined by $\tau_z = -\kappa\theta$. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{\kappa}{I}}$ and $T = 1/f$. $\theta(t) = \Theta\cos(\omega t + \phi)$.

SET UP: For the disk, $I = \frac{1}{2}MR^2$. $\tau_z = -FR$. At $t = 0$, $\theta = \Theta = 3.34° = 0.0583\text{ rad}$, so $\phi = 0$.

EXECUTE: **(a)** $\kappa = -\dfrac{\tau_z}{\theta} = -\dfrac{-FR}{0.0583\text{ rad}} = +\dfrac{(4.23\text{ N})(0.120\text{ m})}{0.0583\text{ rad}} = 8.71\text{ N}\cdot\text{m/rad}$

(b) $f = \dfrac{1}{2\pi}\sqrt{\dfrac{\kappa}{I}} = \dfrac{1}{2\pi}\sqrt{\dfrac{2\kappa}{MR^2}} = \dfrac{1}{2\pi}\sqrt{\dfrac{2(8.71\text{ N}\cdot\text{m/rad})}{(6.50\text{ kg})(0.120\text{ m})^2}} = 2.71\text{ Hz}$. $T = 1/f = 0.461\text{ s}$.

(c) $\omega = 2\pi f = 13.6\text{ rad/s}$. $\theta(t) = (3.34°)\cos([13.6\text{ rad/s}]t)$.

EVALUATE: The frequency and period are independent of the initial angular displacement, so long as this displacement is small.

13.35. **IDENTIFY** and **SET UP:** The number of ticks per second tells us the period and therefore the frequency. We can use a formula form Table 9.2 to calculate I. Then Eq.(13.24) allows us to calculate the torsion constant κ.

EXECUTE: Ticks four times each second implies 0.25 s per tick. Each tick is half a period, so $T = 0.50\text{ s}$ and $f = 1/T = 1/0.50\text{ s} = 2.00\text{ Hz}$

(a) Thin rim implies $I = MR^2$ (from Table 9.2). $I = (0.900\times10^{-3}\text{ kg})(0.55\times10^{-2}\text{ m})^2 = 2.7\times10^{-8}\text{ kg}\cdot\text{m}^2$

(b) $T = 2\pi\sqrt{I/\kappa}$ so $\kappa = I(2\pi/T)^2 = (2.7\times10^{-8}\text{ kg}\cdot\text{m}^2)(2\pi/0.50\text{ s})^2 = 4.3\times10^{-6}\text{ N}\cdot\text{m/rad}$

EVALUATE: Both I and κ are small numbers.

13.36. **IDENTIFY:** Eq.(13.24) and $T = 1/f$ says $T = 2\pi\sqrt{\dfrac{I}{\kappa}}$.

SET UP: $I = \frac{1}{2}mR^2$.

EXECUTE: Solving Eq. (13.24) for κ in terms of the period,

$$\kappa = \left(\frac{2\pi}{T}\right)^2 I = \left(\frac{2\pi}{1.00\text{ s}}\right)^2 ((1/2)(2.00\times10^{-3}\text{ kg})(2.20\times10^{-2}\text{ m})^2) = 1.91\times10^{-5}\text{ N}\cdot\text{m/rad.}$$

EVALUATE: The longer the period, the smaller the torsion constant.

13.37. **IDENTIFY:** $f = \dfrac{1}{2\pi}\sqrt{\dfrac{\kappa}{I}}$.

SET UP: $f = 125/(265\text{ s})$, the number of oscillations per second.

EXECUTE: $I = \dfrac{\kappa}{(2\pi f)^2} = \dfrac{0.450\text{ N}\cdot\text{m/rad}}{\left(2\pi(125)/(265\text{ s})\right)^2} = 0.0152\text{ kg}\cdot\text{m}^2.$

EVALUATE: For a larger I, f is smaller.

13.38. **IDENTIFY:** $\theta(t)$ is given by $\theta(t) = \Theta\cos(\omega t + \phi)$. Evaluate the derivatives specified in the problem.

SET UP: $d(\cos\omega t)/dt = -\omega\sin\omega t$. $d(\sin\omega t)/dt = \omega\cos\omega t$. $\sin^2\theta + \cos^2\theta = 1$

In this problem, $\varphi = 0$.

EXECUTE: **(a)** $\omega = \dfrac{d\theta}{dt} = -\omega\,\Theta\sin(\omega\, t)$ and $\dfrac{d^2\theta}{dt^2} = -\omega^2\,\Theta\cos(\omega\, t)$.

(b) When the angular displacement is Θ, $\Theta = \Theta\cos(\omega t)$. This occurs at $t = 0$, so $\omega = 0$. $\alpha = -\omega^2\Theta$. When the angular displacement is $\Theta/2$, $\dfrac{\Theta}{2} = \Theta\cos(\omega t)$, or $\frac{1}{2} = \cos(\omega t)$. $\omega = \dfrac{-\omega\,\Theta\sqrt{3}}{2}$ since $\sin(\omega t) = \dfrac{\sqrt{3}}{2}$. $\alpha = \dfrac{-\omega^2\Theta}{2}$, since $\cos(\omega t) = 1/2$.

EVALUATE: $\cos(\omega t) = \frac{1}{2}$ when $\omega t = \pi/3\text{ rad} = 60^\circ$. At this t, $\cos(\omega t)$ is decreasing and θ is decreasing, as required. There are other, larger values of ωt for which $\theta = \Theta/2$, but θ is increasing.

13.39. **IDENTIFY** and **SET UP:** Follow the procedure outlined in the problem.

EXECUTE: Eq.(13.25): $U = U_0[(R_0/r)^{12} - 2(R_0/r)^6]$. Let $r = R_0 + x$.

$$U = U_0\left[\left(\frac{R_0}{R_0 + x}\right)^{12} - 2\left(\frac{R_0}{R_0 + x}\right)^6\right] = U_0\left[\left(\frac{1}{1 + x/R_0}\right)^{12} - 2\left(\frac{1}{1 + x/R_0}\right)^6\right]$$

$$\left(\frac{1}{1 + x/R_0}\right)^{12} = (1 + x/R_0)^{-12};\quad |x/R_0| << 1$$

Apply Eq.(13.28) with $n = -12$ and $u = +x/R_0$:

$$\left(\frac{1}{1 + x/R_0}\right)^{12} = 1 - 12x/R_0 + 66x^2/R_0^2 - \ldots$$

For $\left(\dfrac{1}{1 + x/R_0}\right)^6$ apply Eq.(13.28) with $n = -6$ and $u = +x/R_0$:

$$\left(\frac{1}{1 + x/R_0}\right)^6 = 1 - 6x/R_0 + 15x^2/R_0^2 - \ldots$$

Thus $U = U_0(1 - 12x/R_0 + 66x^2/R_0^2 - 2 + 12x/R_0 - 30x^2/R_0^2) = -U_0 + 36U_0x^2/R_0^2$. This is in the form $U = \frac{1}{2}kx^2 - U_0$ with $k = 72U_0/R_0^2$, which is the same as the force constant in Eq.(13.29).

EVALUATE: $F_x = -dU/dx$ so $U(x)$ contains an additive constant that can be set to any value we wish. If $U_0 = 0$ then $U = 0$ when $x = 0$.

13.40. **IDENTIFY:** Example 13.7 tells us that $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{(m/2)}}$.

SET UP: $1\text{ u} = 1.66\times10^{-27}\text{ kg}$

EXECUTE: $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{(m/2)}} = \dfrac{1}{2\pi}\sqrt{\dfrac{2(580\text{ N/m})}{(1.008)(1.66\times10^{-27}\text{ kg})}} = 1.33\times10^{14}\text{ Hz.}$

EVALUATE: This frequency is much larger than *f* calculated in Example 13.7. Here *m* is smaller by a factor of 1/40 but *k* is smaller by a factor of 1/700.

13.41. IDENTIFY: $T = 2\pi\sqrt{L/g}$ is the time for one complete swing.

SET UP: The motion from the maximum displacement on either side of the vertical to the vertical position is one-fourth of a complete swing.

EXECUTE: **(a)** To the given precision, the small-angle approximation is valid. The highest speed is at the bottom of the arc, which occurs after a quarter period, $\frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{L}{g}} = 0.25\text{ s}$.

(b) The same as calculated in (a), 0.25 s. The period is independent of amplitude.

EVALUATE: For small amplitudes of swing, the period depends on *L* and *g*.

13.42. IDENTIFY: Since the rope is long compared to the height of a person, the system can be modeled as a simple pendulum. Since the amplitude is small, the period of the motion is $T = 2\pi\sqrt{\frac{L}{g}}$.

SET UP: From his initial position to his lowest point is one-fourth of a cycle. He returns to this lowest point in time $T/2$ from when he was previously there.

EXECUTE: **(a)** $T = 2\pi\sqrt{\frac{6.50\text{ m}}{9.80\text{ m/s}^2}} = 5.12\text{ s}$. $t = T/4 = 1.28\text{ s}$.

(b) $t = 3T/4 = 3.84\text{ s}$.

EVALUATE: The period is independent of his mass.

13.43. IDENTIFY: Since the cord is much longer than the height of the object, the system can be modeled as a simple pendulum. We will assume the amplitude of swing is small, so that $T = 2\pi\sqrt{\frac{L}{g}}$.

SET UP: The number of swings per second is the frequency $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$.

EXECUTE: $f = \frac{1}{2\pi}\sqrt{\frac{9.80\text{ m/s}^2}{1.50\text{ m}}} = 0.407\text{ swings per second}$.

EVALUATE: The period and frequency are both independent of the mass of the object.

13.44. IDENTIFY: Use Eq.(13.34) to relate the period to *g*.

SET UP: Let the period on earth be $T_E = 2\pi\sqrt{L/g_E}$, where $g_E = 9.80\text{ m/s}^2$, the value on earth.

Let the period on Mars be $T_M = 2\pi\sqrt{L/g_M}$, where $g_M = 3.71\text{ m/s}^2$, the value on Mars.

We can eliminate *L*, which we don't know, by taking a ratio:

EXECUTE: $\frac{T_M}{T_E} = 2\pi\sqrt{\frac{L}{g_M}}\frac{1}{2\pi}\sqrt{\frac{g_E}{L}} = \sqrt{\frac{g_E}{g_M}}$.

$$T_M = T_E\sqrt{\frac{g_E}{g_M}} = (1.60\text{ s})\sqrt{\frac{9.80\text{ m/s}^2}{3.71\text{ m/s}^2}} = 2.60\text{ s}.$$

EVALUATE: Gravity is weaker on Mars so the period of the pendulum is longer there.

13.45. IDENTIFY and **SET UP:** The bounce frequency is given by Eq.(13.11) and the pendulum frequency by Eq.(13.33). Use the relation between these two frequencies that is specified in the problem to calculate the equilibrium length *L* of the spring, when the apple hangs at rest on the end of the spring.

EXECUTE: vertical SHM: $f_b = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

pendulum motion (small amplitude): $f_p = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

The problem specifies that $f_p = \frac{1}{2}f_b$.

$$\frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2}\frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$g/L = k/4m$ so $L = 4gm/k = 4w/k = 4(1.00\text{ N})/1.50\text{ N/m} = 2.67\text{ m}$

EVALUATE: This is the *stretched* length of the spring, its length when the apple is hanging from it. (Note: Small angle of swing means *v* is small as the apple passes through the lowest point, so a_{rad} is small and the component of *mg* perpendicular to the spring is small. Thus the amount the spring is stretched changes very little as the apple swings back and forth.)

IDENTIFY: Use Newton's second law to calculate the distance the spring is stretched form its unstretched length when the apple hangs from it.

SET UP: The free-body diagram for the apple hanging at rest on the end of the spring is given in Figure 13.45.

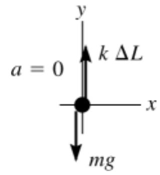


Figure 13.45

EXECUTE: $\sum F_y = ma_y$

$k\Delta L - mg = 0$

$\Delta L = mg/k = w/k = 1.00\text{ N}/1.50\text{ N/m} = 0.667\text{ m}$

Thus the unstretched length of the spring is $2.67\text{ m} - 0.67\text{ m} = 2.00\text{ m}$.

EVALUATE: The spring shortens to its unstretched length when the apple is removed.

13.46. **IDENTIFY:** $a_{\tan} = L\alpha$, $a_{rad} = L\omega^2$ and $a = \sqrt{a_{\tan}^2 + a_{rad}^2}$. Apply conservation of energy to calculate the speed in part (c).

SET UP: Just after the sphere is released, $\omega = 0$ and $a_{rad} = 0$. When the rod is vertical, $a_{\tan} = 0$.

EXECUTE: **(a)** The forces and acceleration are shown in Figure 13.46a. $a_{rad} = 0$ and $a = a_{\tan} = g\sin\theta$.

(b) The forces and acceleration are shown in Figure 13.46b.

(c) The forces and acceleration are shown in Figure 13.46c. $U_i = K_f$ gives $mgL(1-\cos\Theta) = \frac{1}{2}mv^2$ and $v = \sqrt{2gL(1-\cos\Theta)}$.

EVALUATE: As the rod moves toward the vertical, v increases, a_{rad} increases and $a_{\tan}$ decreases.

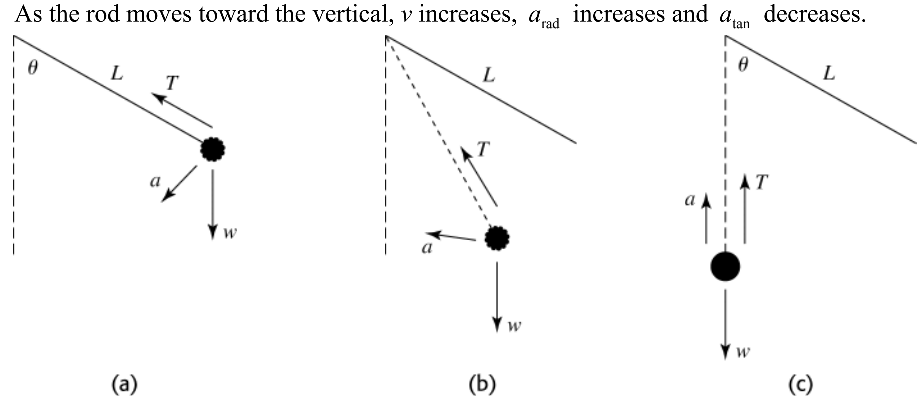


Figure 13.46

13.47. **IDENTIFY:** Apply $T = 2\pi\sqrt{L/g}$

SET UP: The period of the pendulum is $T = (136\text{ s})/100 = 1.36\text{ s}$.

EXECUTE: $g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(0.500\text{ m})}{(1.36\text{ s})^2} = 10.7\text{ m/s}^2$.

EVALUATE: The same pendulum on earth, where g is smaller, would have a larger period.

13.48. **IDENTIFY:** If a small amplitude is assumed, $T = 2\pi\sqrt{\frac{L}{g}}$.

SET UP: The fourth term in Eq.(13.35) would be $\frac{1^2\cdot 3^2\cdot 5^2}{2^2\cdot 4^2\cdot 6^2}\sin^6\frac{\Theta}{2}$.

EXECUTE: **(a)** $T = 2\pi\sqrt{\frac{2.00\text{ m}}{9.80\text{ m/s}^2}} = 2.84\text{ s}$

(b) $T = (2.84\text{ s})\left(1 + \frac{1}{4}\sin^2 15.0° + \frac{9}{64}\sin^4 15.0° + \frac{225}{2305}\sin^6 15.0°\right) = 2.89\text{ s}$

(c) Eq.(13.35) is more accurate. Eq.(13.34) is in error by $\frac{2.84\text{ s} - 2.89\text{ s}}{2.89\text{ s}} = -2\%$.

EVALUATE: As Figure 13.22 in Section 13.5 shows, the approximation $F_\theta = -mg\theta$ is larger in magnitude than the true value as θ increases. Eq.(13.34) therefore over-estimates the restoring force and this results in a value of T that is smaller than the actual value.

13.49. **IDENTIFY:** $T = 2\pi\sqrt{I/mgd}$.

SET UP: $d = 0.200\text{ m}$. $T = (120\text{ s})/100$.

EXECUTE: $I = mgd\left(\frac{T}{2\pi}\right)^2 = (1.80\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})\left(\frac{120\text{ s}/100}{2\pi}\right)^2 = 0.129\text{ kg.m}^2.$

EVALUATE: If the rod were uniform, its center of gravity would be at its geometrical center and it would have length $l = 0.400\text{ m}$. For a uniform rod with an axis at one end, $I = \frac{1}{3}ml^2 = 0.096\text{ kg}\cdot\text{m}^2$. The value of *I* for the actual rod is about 34% larger than this value.

13.50. **IDENTIFY:** $T = 2\pi\sqrt{I/mgd}$

SET UP: From the parallel axis theorem, the moment of inertia of the hoop about the nail is $I = MR^2 + MR^2 = 2MR^2$. $d = R$.

EXECUTE: Solving for *R*, $R = gT^2/8\pi^2 = 0.496$ m.

EVALUATE: A simple pendulum of length $L = R$ has period $T = 2\pi\sqrt{R/g}$. The hoop has a period that is larger by a factor of $\sqrt{2}$.

13.51. **IDENTIFY:** For a physical pendulum, $T = 2\pi\sqrt{I/mgd}$ and for a simple pendulum $T = 2\pi\sqrt{L/g}$.

SET UP: For the situation described, $I = mL^2$ and $d = L$.

EXECUTE: $T = 2\pi\sqrt{\frac{mL^2}{mgL}} = 2\pi\sqrt{L/g}$, so the two expressions are the same.

EVALUATE: Eq.(13.39) applies to any pendulum and reduces to Eq.(13.34) when the conditions for the object to be a simple pendulum are satisfied.

13.52. **IDENTIFY:** Apply Eq.(13.39) to calculate *I* and conservation of energy to calculate the maximum angular speed, Ω_{max} .

SET UP: $d = 0.250\text{ m}$. In part (b), $y_i = d(1-\cos\Theta)$, with $\Theta = 0.400\text{ rad}$ and $y_f = 0$.

EXECUTE: **(a)** Solving Eq.(13.39) for *I*,

$$I = \left(\frac{T}{2\pi}\right)^2 mgd = \left(\frac{0.940\text{ s}}{2\pi}\right)^2 (1.80\text{ kg})(9.80\text{ m/s}^2)(0.250\text{ m}) = 0.0987\text{ kg}\cdot\text{m}^2.$$

(b) The small-angle approximation will not give three-figure accuracy for $\Theta = 0.400$ rad. From energy considerations, $mgd(1-\cos\Theta) = \frac{1}{2}I\Omega_{max}^2$. Expressing Ω_{max} in terms of the period of small-angle oscillations, this becomes

$$\Omega_{max} = \sqrt{2\left(\frac{2\pi}{T}\right)^2(1-\cos\Theta)} = \sqrt{2\left(\frac{2\pi}{0.940\text{ s}}\right)^2(1-\cos(0.400\text{ rad}))} = 2.66\text{ rad/s}.$$

EVALUATE: The time for the motion in part (b) is $t = T/4$, so $\Omega_{av} = \Delta\theta/\Delta t = (0.400\text{ rad})/(0.235\text{ s}) = 1.70\text{ rad/s}$. Ω increases during the motion and the final Ω is larger than the average Ω .

13.53. **IDENTIFY:** Pendulum *A* can be treated as a simple pendulum. Pendulum *B* is a physical pendulum.

SET UP: For pendulum *B* the distance *d* from the axis to the center of gravity is $3L/4$. $I = \frac{1}{3}(m/2)L^2$ for a bar of mass *m*/2 and the axis at one end. For a small ball of mass *m*/2 at a distance *L* from the axis, $I_{ball} = (m/2)L^2$.

EXECUTE: Pendulum *A*: $T_A = 2\pi\sqrt{\frac{L}{g}}$.

Pendulum *B*: $I = I_{bar} + I_{ball} = \frac{1}{3}(m/2)L^2 + (m/2)L^2 = \frac{2}{3}mL^2$.

$T_B = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{2}{3}mL^2}{mg(3L/4)}} = 2\pi\sqrt{\frac{L}{g}}\sqrt{\frac{2}{3}\cdot\frac{4}{3}} = \sqrt{\frac{8}{9}}\left(2\pi\sqrt{\frac{L}{g}}\right) = 0.943T_A$. The period is longer for pendulum *A*.

EVALUATE: Example 13.9 shows that for the bar alone, $T = \sqrt{\frac{2}{3}}T_A = 0.816T_A$. Adding the ball of equal mass to the end of the rod increases the period compared to that for the rod alone.

13.54. **IDENTIFY:** The ornament is a physical pendulum: $T = 2\pi\sqrt{I/mgd}$ (Eq.13.39). T is the target variable.

SET UP: $I = 5MR^2/3$, the moment of inertia about an axis at the edge of the sphere. d is the distance from the axis to the center of gravity, which is at the center of the sphere, so $d = R$.

EXECUTE: $T = 2\pi\sqrt{5/3}\sqrt{R/g} = 2\pi\sqrt{5/3}\sqrt{0.050\text{ m}/(9.80\text{m/s}^2)} = 0.58\text{ s}.$

EVALUATE: A simple pendulum of length $R = 0.050$ m has period 0.45 s; the period of the physical pendulum is longer.

13.55. **IDENTIFY:** Pendulum A can be treated as a simple pendulum. Pendulum B is a physical pendulum. Use the parallel-axis theorem to find the moment of inertia of the ball in B for an axis at the top of the string.

SET UP: For pendulum B the center of gravity is at the center of the ball, so $d = L$. For a solid sphere with an axis through its center, $I_{\text{cm}} = \frac{2}{5}MR^2$. $R = L/2$ and $I_{\text{cm}} = \frac{1}{10}ML^2$.

EXECUTE: Pendulum A: $T_A = 2\pi\sqrt{\dfrac{L}{g}}$.

Pendulum B: The parallel-axis theorem says $I = I_{\text{cm}} + ML^2 = \frac{11}{10}ML^2$.

$T = 2\pi\sqrt{\dfrac{I}{mgd}} = 2\pi\sqrt{\dfrac{11ML^2}{10MgL}} = \sqrt{\dfrac{11}{10}}\left(2\pi\sqrt{\dfrac{L}{g}}\right) = \sqrt{\dfrac{11}{10}}T_A = 1.05T_A$. It takes pendulum B longer to complete a swing.

EVALUATE: The center of the ball is the same distance from the top of the string for both pendulums, but the mass is distributed differently and I is larger for pendulum B, even though the masses are the same.

13.56. **IDENTIFY:** If the system is critically damped or overdamped it doesn't oscillate. With no damping, $\omega = \sqrt{m/k}$.

With underdamping, the angular frequency has the smaller value $\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}}$.

SET UP: $m = 2.20$ kg, $k = 250.0$ N/m. $T' = \dfrac{2\pi}{\omega'}$ and $\omega' = \dfrac{2\pi}{T'} = \dfrac{2\pi}{0.615\text{ s}} = 10.22$ rad/s.

EXECUTE: **(a)** $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{250.0\text{ N/m}}{2.20\text{ kg}}} = 10.66$ rad/s. $\omega' < \omega$ so the system is damped. $\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}}$ gives

$b = 2m\sqrt{\dfrac{k}{m} - \omega'^2} = 2(2.20\text{ kg})\sqrt{\dfrac{250.0\text{ N/m}}{2.20\text{ kg}} - (10.22\text{ rad/s})^2} = 13.3$ kg/s.

(b) Since the motion has a period the system oscillates and is underdamped.

EVALUATE: The critical value of the damping constant is $b = 2\sqrt{km} = 2\sqrt{(250.0\text{ N/m})(2.20\text{ kg})} = 46.9$ kg/s. In this problem b is much less than its critical value.

13.57. **IDENTIFY** and **SET UP:** Use Eq.(13.43) to calculate ω', and then $f' = \omega'/2\pi$.

(a) EXECUTE: $\omega' = \sqrt{(k/m) - (b^2/4m^2)} = \sqrt{\dfrac{2.50\text{ N/m}}{0.300\text{ kg}} - \dfrac{(0.900\text{ kg/s})^2}{4(0.300\text{ kg})^2}} = 2.47$ rad/s

$f' = \omega'/2\pi = (2.47\text{ rad/s})/2\pi = 0.393$ Hz

(b) IDENTIFY and **SET UP:** The condition for critical damping is $b = 2\sqrt{km}$ (Eq.13.44)

EXECUTE: $b = 2\sqrt{(2.50\text{ N/m})(0.300\text{ kg})} = 1.73$ kg/s

EVALUATE: The value of b in part (a) is less than the critical damping value found in part (b). With no damping, the frequency is $f = 0.459$ Hz; the damping reduces the oscillation frequency.

13.58. **IDENTIFY:** From Eq.(13.42) $A_2 = A_1 \exp\left(-\dfrac{b}{2m}t\right)$.

SET UP: $\ln(e^{-x}) = -x$

EXECUTE: $b = \dfrac{2m}{t}\ln\left(\dfrac{A_1}{A_2}\right) = \dfrac{2(0.050\text{ kg})}{(5.00\text{ s})}\ln\left(\dfrac{0.300\text{ m}}{0.100\text{ m}}\right) = 0.0220$ kg/s.

EVALUATE: As a check, note that the oscillation frequency is the same as the undamped frequency to 4.8×10^{-3}%, so Eq. (13.42) is valid.

13.59. **IDENTIFY:** $x(t)$ is given by Eq.(13.42). $v_x = dx/dt$ and $a_x = dv_x/dt$.

SET UP: $d(\cos\omega' t)/dt = -\omega'\sin\omega' t$. $d(\sin\omega' t)/dt = \omega'\cos\omega' t$. $d(e^{-\alpha t})/dt = -\alpha e^{-\alpha t}$.

EXECUTE: **(a)** With $\phi = 0$, $x(0) = A$.

(b) $v_x = \dfrac{dx}{dt} = Ae^{-(b/2m)t}\left[-\dfrac{b}{2m}\cos\omega' t - \omega'\sin\omega' t\right]$, and at $t=0$, $v_x = -Ab/2m$; the graph of x versus t near $t=0$ slopes down.

(c) $a_x = \dfrac{dv_x}{dt} = Ae^{-(b/2m)t}\left[\left(\dfrac{b^2}{4m^2} - \omega'^2\right)\cos\omega' t + \dfrac{\omega' b}{2m}\sin\omega' t\right]$, and at $t=0$, $a_x = A\left(\dfrac{b^2}{4m^2} - \omega'^2\right) = A\left(\dfrac{b^2}{2m^2} - \dfrac{k}{m}\right)$.

(Note that this is $(-bv_0 - kx_0)/m$.) This will be negative if $b < \sqrt{2km}$, zero if $b = \sqrt{2km}$ and positive if $b > \sqrt{2km}$. The graph in the three cases will be curved down, not curved, or curved up, respectively.

EVALUATE: $a_x(0) = 0$ corresponds to the situation of critical damping.

13.60. **IDENTIFY:** Apply Eq.(13.46).

SET UP: $\omega_d = \sqrt{k/m}$ corresponds to resonance, and in this case Eq.(13.46) reduces to $A = F_{max}/b\omega_d$.

EXECUTE: **(a)** $A_1/3$

(b) $2A_1$

EVALUATE: Note that the resonance frequency is independent of the value of *b*. (See Figure 13.28 in the textbook).

13.61 **IDENTIFY** and **SET UP:** Apply Eq.(13.46): $A = \dfrac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$

EXECUTE: **(a)** Consider the special case where $k - m\omega_d^2 = 0$, so $A = F_{max}/b\omega_d$ and $b = F_{max}/A\omega_d$. Units of $\dfrac{F_{max}}{A\omega_d}$ are $\dfrac{\text{kg}\cdot\text{m/s}^2}{(\text{m})(\text{s}^{-1})} = \text{kg/s}$. For units consistency the units of *b* must be kg/s.

(b) Units of $\sqrt{km}$: $[(\text{N/m})\text{kg}]^{1/2} = (\text{N kg/m})^{1/2} = [(\text{kg}\cdot\text{m/s}^2)(\text{kg})/\text{m}]^{1/2} = (\text{kg}^2/\text{s}^2)^{1/2} = \text{kg/s}$, the same as the units for *b*.

(c) For $\omega_d = \sqrt{k/m}$ (at resonance) $A = (F_{max}/b)\sqrt{m/k}$.

(i) $b = 0.2\sqrt{km}$

$$A = F_{max}\sqrt{\frac{m}{k}}\frac{1}{0.2\sqrt{km}} = \frac{F_{max}}{0.2k} = 5.0\frac{F_{max}}{k}.$$

(ii) $b = 0.4\sqrt{km}$

$$A = F_{max}\sqrt{\frac{m}{k}}\frac{1}{0.4\sqrt{km}} = \frac{F_{max}}{0.4k} = 2.5\frac{F_{max}}{k}.$$

EVALUATE: Both these results agree with what is shown in Figure 13.28 in the textbook. As *b* increases the maximum amplitude decreases.

13.62. **IDENTIFY:** Calculate the resonant frequency and compare to 35 Hz.

SET UP: ω in rad/s is related to *f* in Hz by $\omega = 2\pi f$.

EXECUTE: The resonant frequency is $\sqrt{k/m} = \sqrt{(2.1\times10^6 \text{ N/m})/108 \text{ kg}} = 139 \text{ rad/s} = 22.2 \text{ Hz}$, and this package does not meet the criterion.

EVALUATE: To make the package meet the requirement, increase the resonant frequency by increasing the force constant *k*.

13.63. **IDENTIFY:** $ma_x = -kx$ so $a_{max} = \dfrac{k}{m}A = \omega^2 A$ is the magnitude of the acceleration when $x = \pm A$. $v_{max} = \sqrt{\dfrac{k}{m}}A = \omega A$.

$P = \dfrac{W}{t} = \dfrac{\Delta K}{t}$.

SET UP: $A = 0.0500$ m. $\omega = 3500 \text{ rpm} = 366.5 \text{ rad/s}$.

EXECUTE: **(a)** $a_{max} = \omega^2 A = (366.5 \text{ rad/s})^2(0.0500 \text{ m}) = 6.72\times10^3 \text{ m/s}^2$

(b) $F_{max} = ma_{max} = (0.450 \text{ kg})(6.72\times10^3 \text{ m/s}^2) = 3.02\times10^3 \text{ N}$

(c) $v_{max} = \omega A = (366.5 \text{ rad/s})(0.0500 \text{ m}) = 18.3 \text{ m/s}$. $K_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(0.450 \text{ kg})(18.3 \text{ m/s})^2 = 75.4 \text{ J}$

(d) $P = \dfrac{\frac{1}{2}mv^2}{t}$. $t = \dfrac{T}{4} = \dfrac{2\pi}{4\omega} = 4.286\times10^{-3} \text{ s}$. $P = \dfrac{(0.450 \text{ kg})(18.3 \text{ m/s})^2}{2(4.286\times10^{-3} \text{ s})} = 1.76\times10^4 \text{ W}$.

(e) a_{max} is proportional to ω^2, so F_{max} increases by a factor of 4, to 1.21×10^4 N. v_{max} is proportional to ω, so v_{max} doubles, to 36.6 m/s, and K_{max} increases by a factor of 4, to 302 J. In part (d). *t* is halved and *K* is quadrupled, so P_{max} increases by a factor of 8 and becomes 1.41×10^5 W.

EVALUATE: For a given amplitude, the maximum acceleration and maximum velocity increase when the frequency of the motion increases and the period decreases.

13.64. IDENTIFY: $T = 2\pi\sqrt{\dfrac{m}{k}}$. The period changes when the mass changes.

SET UP: M is the mass of the empty car and the mass of the loaded car is $M + 250\text{ kg}$.

EXECUTE: The period of the empty car is $T_E = 2\pi\sqrt{\dfrac{M}{k}}$. The period of the loaded car is $T_L = 2\pi\sqrt{\dfrac{M+250\text{ kg}}{k}}$.

$$k = \frac{(250\text{ kg})(9.80\text{ m/s}^2)}{4.00\times10^{-2}\text{ m}} = 6.125\times10^4\text{ N/m}$$

$$M = \left(\frac{T_L}{2\pi}\right)^2 k - 250\text{ kg} = \left(\frac{1.08\text{ s}}{2\pi}\right)^2 (6.125\times10^4\text{ N/m}) - 250\text{ kg} = 1.56\times10^3\text{ kg}.\; T_E = 2\pi\sqrt{\frac{1.56\times10^3\text{ kg}}{6.125\times10^4\text{ N/m}}} = 1.00\text{ s}.$$

EVALUATE: When the mass decreases, the period decreases.

13.65. IDENTIFY and SET UP: Use Eqs. (13.12), (13.21) and (13.22) to relate the various quantities to the amplitude.

EXECUTE: (a) $T = 2\pi\sqrt{m/k}$; independent of A so period doesn't change

$f = 1/T$; doesn't change

$\omega = 2\pi f$; doesn't change

(b) $E = \frac{1}{2}kA^2$ when $x = \pm A$. When A is halved E decreases by a factor of 4; $E_2 = E_1/4$.

(c) $v_{max} = \omega A = 2\pi f A$

$v_{max,1} = 2\pi f A_1, \quad v_{max,2} = 2\pi f A_2 \quad$ (f doesn't change)

Since $A_2 = \frac{1}{2}A_1, v_{max,2} = 2\pi f(\frac{1}{2}A_1) = \frac{1}{2}2\pi f A_1 = \frac{1}{2}v_{max,1}; v_{max}$ is one-half as great

(d) $v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2}$

$x = \pm A_1/4$ gives $v_x = \pm\sqrt{k/m}\sqrt{A^2 - A_1^2/16}$

With the original amplitude $v_{1x} = \pm\sqrt{k/m}\sqrt{A_1^2 - A_1^2/16} = \pm\sqrt{15/16}(\sqrt{k/m})A_1$

With the reduced amplitude $v_{2x} = \pm\sqrt{k/m}\sqrt{A_2^2 - A_1^2/16} = \pm\sqrt{k/m}\sqrt{(A_1/2)^2 - A_1^2/16} = \pm\sqrt{3/16}(\sqrt{k/m})A_1$

$v_{1x}/v_{2x} = \sqrt{15/3} = \sqrt{5}$, so $v_2 = v_1/\sqrt{5}$; the speed at this x is $1/\sqrt{5}$ times as great.

(e) $U = \frac{1}{2}kx^2$; same x so same U.

$K = \frac{1}{2}mv_x^2;\; K_1 = \frac{1}{2}mv_{1x}^2$

$K_2 = \frac{1}{2}mv_{2x}^2 = \frac{1}{2}m(v_{1x}/\sqrt{5})^2 = \frac{1}{5}(\frac{1}{2}mv_{1x}^2) = K_1/5$; 1/5 times as great.

EVALUATE: Reducing A reduces the total energy but doesn't affect the period and the frequency.

13.66. (a) IDENTIFY and SET UP: Combine Eqs. (13.12) and (13.21) to relate v_x and x to T.

EXECUTE: $T = 2\pi\sqrt{m/k}$

We are given information about v_x at a particular x. The expression relating these two quantities comes from conservation of energy: $\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

We can solve this equation for $\sqrt{m/k}$, and then use that result to calculate T. $mv_x^2 = k(A^2 - x^2)$

$$\sqrt{\frac{m}{k}} = \frac{\sqrt{A^2 - x^2}}{v_x} = \frac{\sqrt{(0.100\text{ m})^2 - (0.060\text{ m})^2}}{0.300\text{ m/s}} = 0.267\text{ s}$$

Then $T = 2\pi\sqrt{m/k} = 2\pi(0.267\text{ s}) = 1.68\text{ s}.$

(b) IDENTIFY and SET UP: We are asked to relate x and v_x, so use conservation of energy equation:

$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$kx^2 = kA^2 - mv_x^2$

$x = \sqrt{A^2 - (m/k)v_x^2} = \sqrt{(0.100\text{ m})^2 - (0.267\text{ s})^2(0.160\text{ m/s})^2} = 0.090\text{ m}$

EVALUATE: Smaller $|v_x|$ means larger x.

(c) IDENTIFY: If the slice doesn't slip the maximum acceleration of the plate (Eq.13.4) equals the maximum acceleration of the slice, which is determined by applying Newton's 2nd law to the slice.

SET UP: For the plate, $-kx = ma_x$ and $a_x = -(k/m)x$. The maximum $|x|$ is A, so $a_{max} = (k/m)A$. If the carrot slice doesn't slip then the static friction force must be able to give it this much acceleration. The free-body diagram for the carrot slice (mass m') is given in Figure 13.66.

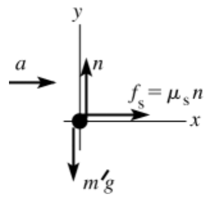


Figure 13.66

EXECUTE: $\sum F_y = ma_y$

$n - m'g = 0$

$n = m'g$

$\sum F_x = ma_x$

$\mu_s n = m'a$

$\mu_s m'g = m'a$ and $a = \mu_s g$

But we require that $a = a_{max} = (k/m)A = \mu_s g$ and $\mu_s = \frac{k}{m}\frac{A}{g} = \left(\frac{1}{0.267\ \text{s}}\right)^2\left(\frac{0.100\ \text{m}}{9.80\ \text{m/s}^2}\right) = 0.143$

EVALUATE: We can write this as $\mu_s = \omega^2 A/g$. More friction is required if the frequency or the amplitude is increased.

13.67. IDENTIFY: The largest downward acceleration the ball can have is g whereas the downward acceleration of the tray depends on the spring force. When the downward acceleration of the tray is greater than g, then the ball leaves the tray. $y(t) = A\cos(\omega t + \phi)$.

SET UP: The downward force exerted by the spring is $F = kd$, where d is the distance of the object above the equilibrium point. The downward acceleration of the tray has magnitude $\frac{F}{m} = \frac{kd}{m}$, where m is the total mass of the ball and tray. $x = A$ at $t = 0$, so the phase angle ϕ is zero and $+x$ is downward.

EXECUTE: **(a)** $\frac{kd}{m} = g$ gives $d = \frac{mg}{k} = \frac{(1.775\ \text{kg})(9.80\ \text{m/s}^2)}{185\ \text{N/m}} = 9.40\ \text{cm}$. This point is 9.40 cm above the equilibrium point so is $9.40\ \text{cm} + 15.0\ \text{cm} = 24.4\ \text{cm}$ above point A.

(b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{185\ \text{N/m}}{1.775\ \text{kg}}} = 10.2\ \text{rad/s}$. The point in (a) is above the equilibrium point so $x = -9.40\ \text{cm}$.

$x = A\cos(\omega t)$ gives $\omega t = \arccos\left(\frac{x}{A}\right) = \arccos\left(\frac{-9.40\ \text{cm}}{15.0\ \text{cm}}\right) = 2.25\ \text{rad}$. $t = \frac{2.25\ \text{rad}}{10.2\ \text{rad/s}} = 0.221\ \text{s}$.

(c) $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$ gives $v = \sqrt{\frac{k}{m}(A^2 - x^2)} = \sqrt{\frac{185\ \text{N/m}}{1.775\ \text{kg}}([0.150\ \text{m}]^2 - [-0.0940\ \text{m}]^2)} = 1.19\ \text{m/s}$.

EVALUATE: The period is $T = 2\pi\sqrt{\frac{m}{k}} = 0.615\ \text{s}$. To go from the lowest point to the highest point takes time $T/2 = 0.308\ \text{s}$. The time in (b) is less than this, as it should be.

13.68. IDENTIFY: In SHM, $a_{max} = \frac{k}{m_{tot}}A$. Apply $\sum \vec{F} = m\vec{a}$ to the top block.

SET UP: The maximum acceleration of the lower block can't exceed the maximum acceleration that can be given to the other block by the friction force.

EXECUTE: For block m, the maximum friction force is $f_s = \mu_s n = \mu_s mg$. $\sum F_x = ma_x$ gives $\mu_s mg = ma$ and $a = \mu_s g$. Then treat both blocks together and consider their simple harmonic motion. $a_{max} = \left(\frac{k}{M+m}\right)A$. Set $a_{max} = a$ and solve for A: $\mu_s g = \left(\frac{k}{M+m}\right)A$ and $A = \frac{\mu_s g(M+m)}{k}$.

EVALUATE: If A is larger than this the spring gives the block with mass M a larger acceleration than friction can give the other block, and the first block accelerates out from underneath the other block.

13.69. IDENTIFY: Apply conservation of linear momentum to the collision and conservation of energy to the motion after the collision. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ and $T = \frac{1}{f}$.

SET UP: The object returns to the equilibrium position in time $T/2$.

EXECUTE: **(a)** Momentum conservation during the collision: $mv_0 = (2m)V$. $V = \frac{1}{2}v_0 = \frac{1}{2}(2.00\ \text{m/s}) = 1.00\ \text{m/s}$.

Energy conservation after the collision: $\frac{1}{2}MV^2 = \frac{1}{2}kx^2$.

$$x = \sqrt{\frac{MV^2}{k}} = \sqrt{\frac{(20.0\ \text{kg})(1.00\ \text{m/s})^2}{80.0\ \text{N/m}}} = 0.500\ \text{m (amplitude)}$$

$$\omega = 2\pi f = \sqrt{k/M}.\ f = \frac{1}{2\pi}\sqrt{k/M} = \frac{1}{2\pi}\sqrt{\frac{80.0\ \text{N/m}}{20.0\ \text{kg}}} = 0.318\ \text{Hz}.\ T = \frac{1}{f} = \frac{1}{0.318\ \text{Hz}} = 3.14\ \text{s}.$$

(b) It takes $1/2$ period to first return: $\frac{1}{2}(3.14\ \text{s}) = 1.57\ \text{s}$

EVALUATE: The total mechanical energy of the system determines the amplitude. The frequency and period depend only on the force constant of the spring and the mass that is attached to the spring.

13.70. **IDENTIFY:** The upward acceleration of the rocket produces an effective downward acceleration for objects in its frame of reference that is equal to $g' = a + g$.

SET UP: The amplitude is the maximum displacement from equilibrium and is unaffected by the motion of the rocket. The period is affected and is given by $T = 2\pi\sqrt{\frac{L}{g'}}$.

EXECUTE: The amplitude is $8.50°$. $T = 2\pi\sqrt{\frac{1.10\ \text{m}}{4.00\ \text{m/s}^2 + 9.80\ \text{m/s}^2}} = 1.77\ \text{s}$.

EVALUATE: For a pendulum of the same length and with its point of support at rest relative to the earth, $T = 2\pi\sqrt{\frac{L}{g}} = 2.11\ \text{s}$. The upward acceleration decreases the period of the pendulum. If the rocket were instead accelerating downward, the period would be greater than 2.11 s.

13.71. **IDENTIFY:** The object oscillates as a physical pendulum, so $f = \frac{1}{2\pi}\sqrt{\frac{m_{\text{object}}gd}{I}}$. Use the parallel-axis theorem, $I = I_{\text{cm}} + Md^2$, to find the moment of inertia of each stick about an axis at the hook.

SET UP: The center of mass of the square object is at its geometrical center, so its distance from the hook is $L\cos 45° = L/\sqrt{2}$. The center of mass of each stick is at its geometrical center. For each stick, $I_{\text{cm}} = \frac{1}{12}mL^2$.

EXECUTE: The parallel-axis theorem gives I for each stick for an axis at the center of the square to be $\frac{1}{12}mL^2 + m(L/2)^2 = \frac{1}{3}mL^2$ and the total I for this axis is $\frac{4}{3}mL^2$. For the entire object and an axis at the hook, applying the parallel-axis theorem again to the object of mass $4m$ gives $I = \frac{4}{3}mL^2 + 4m(L/\sqrt{2})^2 = \frac{10}{3}mL^2$.

$$f = \frac{1}{2\pi}\sqrt{\frac{m_{\text{object}}gd}{I}} = \frac{1}{2\pi}\sqrt{\frac{4mgL/\sqrt{2}}{\frac{10}{3}mL^2}} = \sqrt{\frac{6}{5\sqrt{2}}}\left(\frac{1}{2\pi}\sqrt{\frac{g}{L}}\right) = 0.921\left(\frac{1}{2\pi}\sqrt{\frac{g}{L}}\right).$$

EVALUATE: Just as for a simple pendulum, the frequency is independent of the mass. A simple pendulum of length L has frequency $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$ and this object has a frequency that is slightly less than this.

13.72. **IDENTIFY:** Conservation of energy says $K + U = E$.

SET UP: $U = \frac{1}{2}kx^2$ and $E = U_{\max} = \frac{1}{2}kA^2$.

EXECUTE: **(a)** The graph is given in Figure 13.72. The following answers are found algebraically, to be used as a check on the graphical method.

(b) $A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.200\ \text{J})}{(10.0\ \text{N/m})}} = 0.200\ \text{m}$.

(c) $\frac{E}{4} = 0.050\ \text{J}$.

(d) $U = \frac{1}{2}E$. $x = \frac{A}{\sqrt{2}} = 0.141\ \text{m}$.

(e) From Eq. (13.18), using $v_0 = -\sqrt{\frac{2K_0}{m}}$ and $x_0 = \sqrt{\frac{2U_0}{k}}$, $-\frac{v_0}{\omega x_0} = \frac{\sqrt{(2K_0/m)}}{\sqrt{(k/m)}\sqrt{(2U_0/k)}} = \sqrt{\frac{K_0}{U_0}} = \sqrt{0.429}$ and

$\phi = \arctan\left(\sqrt{0.429}\right) = 0.580\ \text{rad}$.

EVALUATE: The dependence of U on x is not linear and $U = \frac{1}{2}U_{max}$ does not occur at $x = \frac{1}{2}x_{max}$.

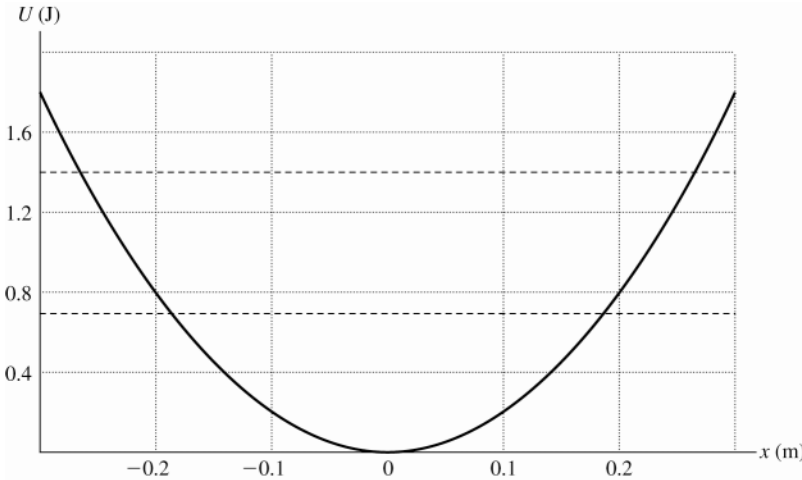


Figure 13.72

13.73. **IDENTIFY:** $T = 2\pi\sqrt{\frac{m}{k}}$ so the period changes because the mass changes.

SET UP: $\frac{dm}{dt} = -2.00\times10^{-3}$ kg/s. The rate of change of the period is $\frac{dT}{dt}$.

EXECUTE: **(a)** When the bucket is half full, $m = 7.00$ kg. $T = 2\pi\sqrt{\frac{7.00\text{ kg}}{125\text{ N/m}}} = 1.49$ s.

(b) $\frac{dT}{dt} = \frac{2\pi}{\sqrt{k}}\frac{d}{dt}(m^{1/2}) = \frac{2\pi}{\sqrt{k}}\frac{1}{2}m^{-1/2}\frac{dm}{dt} = \frac{\pi}{\sqrt{mk}}\frac{dm}{dt}$.

$\frac{dT}{dt} = \frac{\pi}{\sqrt{(7.00\text{ kg})(125\text{ N/m})}}(-2.00\times10^{-3}\text{ kg/s}) = -2.12\times10^{-4}$ s per s. $\frac{dT}{dt}$ is negative; the period is getting shorter.

(c) The shortest period is when all the water has leaked out and $m = 2.00$ kg. Then $T = 0.795$ s.

EVALUATE: The rate at which the period changes is not constant but instead increases in time, even though the rate at which the water flows out is constant.

13.74. **IDENTIFY:** Use $F_x = -kx$ to determine k for the wire. Then $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

SET UP: $F = mg$ moves the end of the wire a distance Δl.

EXECUTE: The force constant for this wire is $k = \frac{mg}{\Delta l}$, so $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta l}} = \frac{1}{2\pi}\sqrt{\frac{9.80\text{ m/s}^2}{2.00\times10^{-3}\text{ m}}} = 11.1$ Hz.

EVALUATE: The frequency is independent of the additional distance the ball is pulled downward, so long as that distance is small.

13.75. **IDENTIFY** and **SET UP:** Measure x from the equilibrium position of the object, where the gravity and spring forces balance. Let $+x$ be downward.

(a) Use conservation of energy (Eq.13.21) to relate v_x and x. Use Eq. (13.12) to relate T to k/m.

EXECUTE: $\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

For $x = 0, \frac{1}{2}mv_x^2 = \frac{1}{2}kA^2$ and $v = A\sqrt{k/m}$, just as for horizontal SHM. We can use the period to calculate $\sqrt{k/m}: T = 2\pi\sqrt{m/k}$ implies $\sqrt{k/m} = 2\pi/T$. Thus $v = 2\pi A/T = 2\pi(0.100\text{ m})/4.20\text{ s} = 0.150$ m/s.

(b) IDENTIFY and **SET UP:** Use Eq.(13.4) to relate a_x and x.

EXECUTE: $ma_x = -kx$ so $a_x = -(k/m)x$

$+x$-direction is downward, so here $x = -0.050$ m

$a_x = -(2\pi/T)^2(-0.050\text{ m}) = +(2\pi/4.20\text{ s})^2(0.050\text{ m}) = 0.112\text{ m/s}^2$ (positive, so direction is downward)

(c) IDENTIFY and **SET UP:** Use Eq.(13.13) to relate x and t. The time asked for is twice the time it takes to go from $x = 0$ to $x = +0.050$ m.

EXECUTE: $x(t) = A\cos(\omega t + \phi)$

Let $\phi = -\pi/2$, so $x = 0$ at $t = 0$. Then $x = A\cos(\omega t - \pi/2) = A\sin\omega t = A\sin(2\pi t/T)$. Find the time t that gives $x = +0.050$ m: $0.050 \text{ m} = (0.100 \text{ m})\sin(2\pi t/T)$

$2\pi t/T = \arcsin(0.50) = \pi/6$ and $t = T/12 = 4.20 \text{ s}/12 = 0.350$ s

The time asked for in the problem is twice this, 0.700 s.

(d) IDENTIFY: The problem is asking for the distance d that the spring stretches when the object hangs at rest from it. Apply Newton's 2nd law to the object.

SET UP: The free-body diagram for the object is given in Figure 13.75.

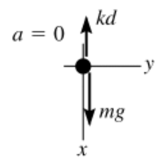


Figure 13.75

EXECUTE: $\sum F_x = ma_x$

$mg - kd = 0$

$d = (m/k)g$

But $\sqrt{k/m} = 2\pi/T$ (part (a)) and $m/k = (T/2\pi)^2$

$$d = \left(\frac{T}{2\pi}\right)^2 g = \left(\frac{4.20 \text{ s}}{2}\right)^2 (9.80 \text{ m/s}^2) = 4.38 \text{ m}.$$

EVALUATE: When the displacement is upward (part (b)), the acceleration is downward. The mass of the partridge is never entered into the calculation. We used just the ratio k/m, that is determined from T.

13.76. IDENTIFY: $x(t) = A\cos(\omega t + \phi)$, $v_x = -A\omega\sin(\omega t + \phi)$ and $a_x = -\omega^2 x$. $\omega = 2\pi/T$.

SET UP: $x = A$ when $t = 0$ gives $\phi = 0$.

EXECUTE: $x = (0.240 \text{ m})\cos\left(\frac{2\pi t}{1.50 \text{ s}}\right)$. $v_x = -\left(\frac{2\pi(0.240 \text{ m})}{(1.50 \text{ s})}\right)\sin\left(\frac{2\pi t}{1.50 \text{ s}}\right) = -(1.00530 \text{ m/s})\sin\left(\frac{2\pi t}{1.50 \text{ s}}\right)$.

$$a_x = -\left(\frac{2\pi}{1.50 \text{ s}}\right)^2 (0.240 \text{ m})\cos\left(\frac{2\pi t}{1.50 \text{ s}}\right) = -(4.2110 \text{ m/s}^2)\cos\left(\frac{2\pi t}{1.50 \text{ s}}\right).$$

(a) Substitution gives $x = -0.120$ m, or using $t = \frac{T}{3}$ gives $x = A\cos 120° = \frac{-A}{2}$.

(b) Substitution gives $ma_x = +(0.0200 \text{ kg})(2.106 \text{ m/s}^2) = 4.21\times10^{-2}$ N, in the $+x$-direction.

(c) $t = \frac{T}{2\pi}\arccos\left(\frac{-3A/4}{A}\right) = 0.577$ s.

(d) Using the time found in part (c), $v = 0.665$ m/s.

EVALUATE: We could also calculate the speed in part (d) from the conservation of energy expression, Eq.(13.22).

13.77. IDENTIFY: Apply conservation of linear momentum to the collision between the steak and the pan. Then apply conservation of energy to the motion after the collision to find the amplitude of the subsequent SHM. Use Eq.(13.12) to calculate the period.

(a) SET UP: First find the speed of the steak just before it strikes the pan. Use a coordinate system with $+y$ downward.

$v_{0y} = 0$ (released from the rest); $y - y_0 = 0.40$ m; $a_y = +9.80 \text{ m/s}^2$; $v_y = ?$

$v_y^2 = v_{0_y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_y = +\sqrt{2a_y(y - y_0)} = +\sqrt{2(9.80 \text{ m/s}^2)(0.40 \text{ m})} = +2.80$ m/s

SET UP: Apply conservation of momentum to the collision between the steak and the pan. After the collision the steak and the pan are moving together with common velocity v_2. Let A be the steak and B be the pan. The system before and after the collision is shown in Figure 13.77.

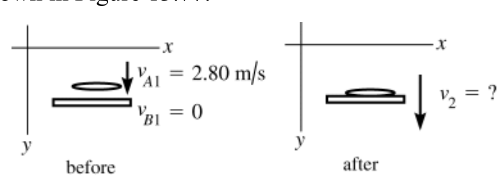


Figure 13.77

EXECUTE: P_y conserved: $m_A v_{A1y} + m_B v_{B1y} = (m_A + m_B)v_{2y}$

$m_A v_{A1} = (m_A + m_B)v_2$

$$v_2 = \left(\frac{m_A}{m_A + m_B}\right)v_{A1} = \left(\frac{2.2\text{ kg}}{2.2\text{ kg} + 0.20\text{ kg}}\right)(2.80\text{ m/s}) = 2.57\text{ m/s}$$

(b) SET UP: Conservation of energy applied to the SHM gives: $\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}kA^2$ where v_0 and x_0 are the initial speed and displacement of the object and where the displacement is measured from the equilibrium position of the object.

EXECUTE: The weight of the steak will stretch the spring an additional distance d given by $kd = mg$ so

$d = \dfrac{mg}{k} = \dfrac{(2.2\text{ kg})(9.80\text{ m/s}^2)}{400\text{ N/m}} = 0.0539\text{ m}.$ So just after the steak hits the pan, before the pan has had time to move, the steak plus pan is 0.0539 m above the equilibrium position of the combined object. Thus $x_0 = 0.0539$ m. From part (a) $v_0 = 2.57$ m/s, the speed of the combined object just after the collision. Then $\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}kA^2$ gives

$$A = \sqrt{\frac{mv_0^2 + kx_0^2}{k}} = \sqrt{\frac{2.4\text{ kg}(2.57\text{ m/s})^2 + (400\text{ N/m})(0.0539\text{ m})^2}{400\text{ N/m}}} = 0.21\text{ m}$$

(c) $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{\dfrac{2.4\text{ kg}}{400\text{ N/m}}} = 0.49\text{ s}$

EVALUATE: The amplitude is less than the initial height of the steak above the pan because mechanical energy is lost in the inelastic collision.

13.78. **IDENTIFY:** $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$. Use energy considerations to find the new amplitude.

SET UP: $f = 0.600$ Hz, $m = 400$ kg; $f = \frac{1}{2}\sqrt{\dfrac{k}{m}}$ gives $k = 5685$ N/m. This is the effective force constant of the two springs.

(a) After the gravel sack falls off, the remaining mass attached to the springs is 225 kg. The force constant of the springs is unaffected, so $f = 0.800$ Hz. To find the new amplitude use energy considerations to find the distance downward that the beam travels after the gravel falls off. Before the sack falls off, the amount x_0 that the spring is stretched at equilibrium is given by $mg - kx_0$, so $x_0 = mg/k = (400\text{ kg})(9.80\text{ m/s}^2)/(5685\text{ N/m}) = 0.6895\text{ m}$. The maximum upward displacement of the beam is $A = 0.400$ m above this point, so at this point the spring is stretched 0.2895 m. With the new mass, the mass 225 kg of the beam alone, at equilibrium the spring is stretched $mg/k = (225\text{ kg})(9.80\text{ m/s}^2)/(5685\text{ N/m}) = 0.6895\text{ m}$. The new amplitude is therefore $0.3879\text{ m} - 0.2895\text{ m} = 0.098\text{ m}$. The beam moves 0.098 m above and below the new equilibrium position. Energy calculations show that $v = 0$ when the beam is 0.098 m above and below the equilibrium point.

(b) The remaining mass and the spring constant is the same in part (a), so the new frequency is again 0.800 Hz. The sack falls off when the spring is stretched 0.6895 m. And the speed of the beam at this point is $v = A\sqrt{k/m} =$ $(0.400\text{ m})\sqrt{(5685\text{ N/m})/(400\text{ kg})} = 1.508\text{ m/s}$. Take $y = 0$ at this point. The total energy of the beam at this point, just after the sack falls off, is $E = K + U_{\text{el}} + U_{\text{grav}} = \frac{1}{2}(225\text{ kg})(1.508\text{ m/s}^2) + \frac{1}{2}(5695\text{ N/m})(0.6895\text{ m})^2 + 0 = 1608\text{ J}$. Let this be point 1. Let point 2 be where the beam has moved upward a distance d and where $v = 0$. $E_2 = \frac{1}{2}k\left(0.6985\text{ m} - d\right)^2 + mgd$. $E_1 = E_2$ gives $d = 0.7275$ m. At this end point of motion the spring is compressed 0.7275 m – 0.6895 m = 0.0380 m. At the new equilibrium position the spring is stretched 0.3879 m, so the new amplitude is 0.3789 m + 0.0380 m = 0.426 m. Energy calculations show that v is also zero when the beam is 0.426 m below the equilibrium position.

EVALUATE: The new frequency is independent of the point in the motion at which the bag falls off. The new amplitude is smaller than the original amplitude when the sack falls off at the maximum upward displacement of the beam. The new amplitude is larger than the original amplitude when the sack falls off when the beam has maximum speed.

13.79. **IDENTIFY** and **SET UP:** Use Eq.(13.12) to calculate g and use Eq.(12.4) applied to Newtonia to relate g to the mass of the planet.

EXECUTE: The pendulum swings through $\frac{1}{2}$ cycle in 1.42 s, so T = 2.84 s. L = 1.85 m. Use T to find g:

$T = 2\pi\sqrt{L/\text{g}}$ so $\text{g} = L(2\pi/T)^2 = 9.055\text{ m/s}^2$

Use g to find the mass M_p of Newtonia: $g = GM_\text{p}/R_\text{p}^2$

$2\pi R_\text{p} = 5.14\times10^7$ m, so $R_\text{p} = 8.18\times10^6$ m

$$m_\text{p} = \frac{gR_\text{p}^2}{G} = 9.08\times10^{24}\text{ kg}$$

EVALUATE: g is similar to that at the surface of the earth. The radius of Newtonia is a little less than earth's radius and its mass is a little more.

13.80. **IDENTIFY:** $F_x = -kx$ allows us to calculate k. $T = 2\pi\sqrt{m/k}$. $x(t) = A\cos(\omega t + \phi)$. $F_{net} = -kx$.

SET UP: Let $\phi = \pi/2$ so $x(t) = A\sin(\omega t)$. At $t = 0$, $x = 0$ and the object is moving downward. When the object is below the equilibrium position, F_{spring} is upward.

EXECUTE: **(a)** Solving Eq. (13.12) for m, and using $k = \dfrac{F}{\Delta l}$

$$m = \left(\frac{T}{2\pi}\right)^2 \frac{F}{\Delta l} = \left(\frac{1}{2\pi}\right)^2 \frac{40.0\text{ N}}{0.250\text{ m}} = 4.05\text{ kg}.$$

(b) $t = (0.35)T$, and so $x = -A\sin[2\pi(0.35)] = -0.0405$ m. Since $t > T/4$, the mass has already passed the lowest point of its motion, and is on the way up.

(c) Taking upward forces to be positive, $F_{spring} - mg = -kx$, where x is the displacement from equilibrium, so $F_{spring} = -(160\text{ N/m})(-0.030\text{ m}) + (4.05\text{ kg})(9.80\text{ m/s}^2) = 44.5\text{ N}$.

EVALUATE: When the object is below the equilibrium position the net force is upward and the upward spring force is larger in magnitude than the downward weight of the object.

13.81. **IDENTIFY:** Use Eq.(13.13) to relate x and t. $T = 3.5$ s.

SET UP: The motion of the raft is sketched in Figure 13.81.

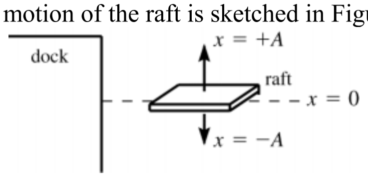


Figure 13.81

Let the raft be at $x = +A$ when $t = 0$. Then $\phi = 0$ and $x(t) = A\cos\omega t$.

EXECUTE: Calculate the time it takes the raft to move from $x = +A = +0.200$ m to $x = A - 0.100\text{ m} = 0.100$ m. Write the equation for $x(t)$ in terms of T rather than ω: $\omega = 2\pi/T$ gives that $x(t) = A\cos(2\pi t/T)$

$x = A$ at $t = 0$

$x = 0.100$ m implies $0.100\text{ m} = (0.200\text{ m})\cos(2\pi t/T)$

$\cos(2\pi t/T) = 0.500$ so $2\pi t/T = \arccos(0.500) = 1.047$ rad

$t = (T/2\pi)(1.047\text{ rad}) = (3.5\text{ s}/2\pi)(1.047\text{ rad}) = 0.583$ s

This is the time for the raft to move down from $x = 0.200$ m to $x = 0.100$ m. But people can also get off while the raft is moving up from $x = 0.100$ m to $x = 0.200$ m, so during each period of the motion the time the people have to get off is $2t = 2(0.583\text{ s}) = 1.17$ s.

EVALUATE: The time to go from $x = 0$ to $x = A$ and return is $T/2 = 1.75$ s. The time to go from $x = A/2$ to A and return is less than this.

13.82. **IDENTIFY:** $T = 2\pi/\omega$. $F_r(r) = -kr$ to determine k.

SET UP: Example 12.10 derives $F_r(r) = -\dfrac{GM_E m}{R_E^3} r$.

EXECUTE: $a_r = F_r/m$ is in the form of Eq.(13.8), with x replaced by r, so the motion is simple harmonic.

$k = \dfrac{GM_E m}{R_E^3}$. $\omega^2 = \dfrac{k}{m} = \dfrac{GM_E}{R_E^3} = \dfrac{g}{R_E}$. The period is then $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{R_E}{g}} = 2\pi\sqrt{\dfrac{6.38\times10^6\text{ m}}{9.80\text{ m/s}^2}} = 5070$ s, or 84.5 min.

EVALUATE: The period is independent of the mass of the object but does depend on R_E, which is also the amplitude of the motion.

13.83. **IDENTIFY:** If $F_{net} = kx$, then $\omega = \sqrt{\dfrac{k}{m}}$. Calculate F_{net}. If it is of this form, calculate k.

SET UP: The gravitational force between two point masses is $F_g = G\dfrac{m_1 m_2}{r^2}$ and is attractive. The forces on M are sketched in Figure 13.83.

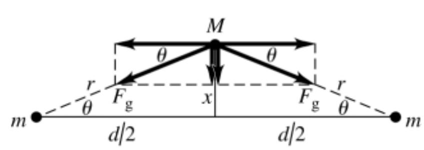


Figure 13.83

EXECUTE: **(a)** $r=\sqrt{(d/2)^2+x^2}\approx d/2$, if $x<<d$. $\tan\theta=\dfrac{x}{d/2}=\dfrac{2x}{d}$. The net force is toward the original position of M and has magnitude $F_{\text{net}}=2G\dfrac{mM}{(d/2)^2}\sin\theta$. Since θ is small, $\sin\theta\approx\tan\theta=\dfrac{2x}{d}$ and $F_{\text{net}}=\left(\dfrac{16GmM}{d^3}\right)x$. This is a restoring force.

(b) Comparing the result in part (a) to $F_{\text{net}}=kx$ gives $k=\dfrac{16GmM}{d^3}$. $\omega=\sqrt{\dfrac{k}{m}}=\dfrac{4}{d}\sqrt{\dfrac{GM}{d}}$. $T=\dfrac{2\pi}{\omega}=\dfrac{\pi d}{2}\sqrt{\dfrac{d}{GM}}$.

(c) $T=\dfrac{\pi(0.250\text{ m})}{2}\sqrt{\dfrac{0.250\text{ m}}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(100\text{ kg})}}=2.40\times10^3\text{ s}=40\text{ min}$. This period is short enough that a patient person could measure it. The experiment would have to be done such that the gravitational forces are much larger than any other forces on M. The gravitational forces are very weak, so other forces, such as friction, forces from air currents, etc., would have to be kept extremely small.

(d) If M is displaced toward one of the fixed masses there is a net force on M toward that mass and therefore away from the equilibrium position of M. The net force is not a restoring force and M would not oscillate, it would continue to move in the direction in which it was displaced.

EVALUATE: The period is very long because the restoring force is very small.

13.84. **IDENTIFY:** $U(x)-U(x_0)=\int_{x_0}^{x}F_x dx$. In part (b) follow the steps outlined in the hint.

SET UP: In part (a), let $x_0=0$ and $U(x_0)=U(0)=0$. The time for the object to go from $x=0$ to $x=A$ is $T/4$.

EXECUTE: **(a)** $U=-\int_0^x F_x dx=c\int_0^x x^3dx=\dfrac{c}{4}x^4$.

(b) From conservation of energy, $\frac{1}{2}mv_x^2=\dfrac{c}{4}(A^4-x^4)$. $v_x=\dfrac{dx}{dt}$, so $\dfrac{dx}{\sqrt{A^4-x^4}}=\sqrt{\dfrac{c}{2m}}dt$. Integrating from 0 to A with respect to x and from 0 to $T/4$ with respect to t, $\int_0^A\dfrac{dx}{\sqrt{A^4-x^4}}=\sqrt{\dfrac{c}{2m}}\dfrac{T}{4}$ To use the hint, let $u=\dfrac{x}{A}$, so that $dx=a\ du$ and the upper limit of the u-integral is $u=1$. Factoring A^2 out of the square root,

$\dfrac{1}{A}\int_0^1\dfrac{du}{\sqrt{1-u^4}}=\dfrac{1.31}{A}=\sqrt{\dfrac{c}{32m}}T$, which may be expressed as $T=\dfrac{7.41}{A}\sqrt{\dfrac{m}{c}}$.

(c) The period does depend on amplitude, and the motion is not simple harmonic.

EVALUATE: Simple harmonic motion requires $F_x=-kx$, where k is a constant, and that is not the case here.

13.85. **IDENTIFY:** Find the x-component of the vector $\vec{v}_Q$ in Figure 13.6a in the textbook.

SET UP: $v_x=-v_{\tan}\sin\theta$ and $\theta=\omega t+\phi$.

EXECUTE: $v_x=-v_{\tan}\sin\theta$. Substituting for $v_{\tan}$ and θ gives Eq.(13.15).

EVALUATE: At $t=0$, Q is on the x-axis and has zero component of velocity. This corresponds to $v_x=0$ in Eq.(13.15).

13.86. **IDENTIFY:** $mV_{\text{cm}}=P_{\text{tot}}$. $K=p^2/2m$ for a single object and the total kinetic energy of the two masses is just the sum of their individual kinetic energies.

SET UP: Momentum is a vector and kinetic energy is a scalar.

EXECUTE: **(a)** For the center of mass to be at rest, the total momentum must be zero, so the momentum vectors must be of equal magnitude but opposite directions, and the momenta can be represented as $\vec{p}$ and $-\vec{p}$.

(b) $K_{\text{tot}}=2\dfrac{p^2}{2m}=\dfrac{p^2}{2(m/2)}$.

(c) The argument of part (a) is valid for any masses. The kinetic energy is

$K_{\text{tot}}=\dfrac{p^2}{2m_1}+\dfrac{p^2}{2m_2}=\dfrac{p^2}{2}\left(\dfrac{m_1+m_2}{m_1m_2}\right)=\dfrac{p^2}{2(m_1m_2/(m_1+m_2))}$.

EVALUATE: If $m_1=m_2=m$, the reduced mass is $m/2$. If $m_1>>m_2$, then the reduced mass is m_2.

13.87. **IDENTIFY:** $F_r=-dU/dr$. The equilibrium separation r_{eq} is given by $F(r_{\text{eq}})=0$. The force constant k is defined by $F_r=-kx$. $f=\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$, where m is the reduced mass.

SET UP: $d(r^{-n})/dr = -nr^{-(n+1)}$, for $n \geq 1$.

EXECUTE: **(a)** $F_r = -\dfrac{dU}{dr} = A\left[\left(\dfrac{R_0^7}{r^9}\right) - \dfrac{1}{r^2}\right].$

(b) Setting the above expression for F_r equal to zero, the term in square brackets vanishes, so that $\dfrac{R_0^7}{r_{eq}^9} = \dfrac{1}{r_{eq}^2}$, or

$R_0^7 = r_{eq}^7$, and $r_{eq} = R_0$.

(c) $U(R_0) = -\dfrac{7A}{8R_0} = -7.57 \times 10^{-19}$ J.

(d) The above expression for F_r can be expressed as

$$F_r = \frac{A}{R_0^2}\left[\left(\frac{r}{R_0}\right)^{-9} - \left(\frac{r}{R_0}\right)^{-2}\right] = \frac{A}{R_0^2}\left[(1+(x/R_0))^{-9} - (1+(x/R_0))^{-2}\right]$$

$$F_r \approx \frac{A}{R_0^2}[(1-9(x/R_0)) - (1-2(x/R_0))] = \frac{A}{R_0^2}(-7\ x/R_0) = -\left(\frac{7A}{R_0^3}\right)x.$$

(e) $f = \dfrac{1}{2\pi}\sqrt{k/m} = \dfrac{1}{2\pi}\sqrt{\dfrac{7A}{R_0^3 m}} = 8.39 \times 10^{12}$ Hz.

EVALUATE: The force constant depends on the parameters A and R_0 in the expression for $U(r)$. The minus sign in the expression in part (d) shows that for small displacements from equilibrium, F_r is a restoring force.

13.88. **IDENTIFY:** Apply $\sum \tau_z = I_{cm}\alpha_z$ and $\sum F_x = Ma_{cm-x}$ to the cylinders. Solve for a_{cm-x}. Compatre to Eq.(13.8) to find the angular frequency and period, $T = 2\pi\omega$.

SET UP:

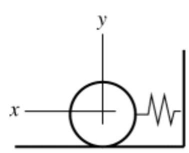


Figure 13.88a

Let the origin of coordinate be at the center of the cylinders when they are at their equilibrium position.

The free-body diagram for the cylinders when they are displaced a distance x to the left is given in Figure 13.88b.

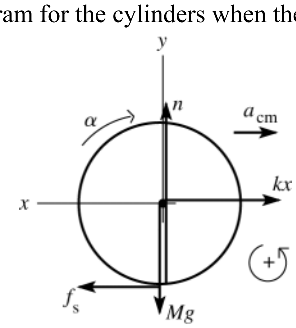


Figure 13.88b

EXECUTE:

$\sum \tau_z = I_{cm}\alpha_z$

$f_s R = (\tfrac{1}{2}MR^2)\alpha$

$f_s = \tfrac{1}{2}MR\alpha$

But $R\alpha = a_{cm}$ so

$f_s = \tfrac{1}{2}Ma_{cm}$

$\sum F_x = ma_x$

$f_s - kx = -Ma_{cm}$

$\tfrac{1}{2}Ma_{cm} - kx = -Ma_{cm}$

$kx = \tfrac{3}{2}Ma_{cm}$

$(2k/3M)x = a_{cm}$

Eq. (13.8): $a_x = -\omega^2 x$ (The minus sign says that x and a_x have opposite directions, as our diagram shows.) Our result for a_{cm} is of this form, with $\omega^2 = 2k/3M$ and $\omega = \sqrt{2k/3M}$. Thus $T = 2\pi/\omega = 2\pi\sqrt{3M/2k}$.

EVALUATE: If there were no friction and the cylinder didn't roll, the period would be $2\pi\sqrt{M/k}$. The period when there is rolling without slipping is larger than this.

13.89. **IDENTIFY:** Apply conservation of energy to the motion before and after the collision. Apply conservation of linear momentum to the collision. After the collision the system moves as a simple pendulum. If the maximum angular displacement is small, $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$.

SET UP: In the motion before and after the collision there is energy conversion between gravitational potential energy mgh, where h is the height above the lowest point in the motion, and kinetic energy.

EXECUTE: Energy conservation during downward swing: $m_2gh_0 = \frac{1}{2}m_2v^2$ and

$v = \sqrt{2gh_0} = \sqrt{2(9.8\text{ m/s}^2)(0.100\text{ m})} = 1.40\text{ m/s}$.

Momentum conservation during collision: $m_2v = (m_2 + m_3)V$ and $V = \frac{m_2v}{m_2 + m_3} = \frac{(2.00\text{ kg})(1.40\text{ m/s})}{5.00\text{ kg}} = 0.560\text{ m/s}$.

Energy conservation during upward swing: $Mgh_\text{f} = \frac{1}{2}MV^2$ and $h_\text{f} = V^2/2g = \frac{(0.560\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 0.0160\text{ m} = 1.60\text{ cm}$.

Figure 13.89 shows how the maximum angular displacement is calculated from h_f. $\cos\theta = \frac{48.4\text{ cm}}{50.0\text{ cm}}$ and $\theta = 14.5°$.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = \frac{1}{2\pi}\sqrt{\frac{9.80\text{ m/s}^2}{0.500\text{ m}}} = 0.705\text{ Hz}.$$

EVALUATE: $14.5° = 0.253\text{ rad}$. $\sin(0.253\text{ rad}) = 0.250$. $\sin\theta \approx \theta$ and Eq.(13.34) is accurate.

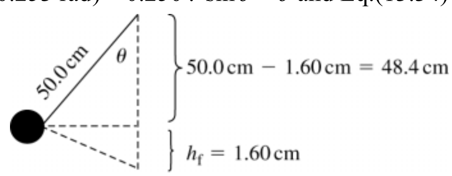


Figure 13.89

13.90. **IDENTIFY:** $T = 2\pi\sqrt{I/mgd}$

SET UP: The model for the leg is sketched in Figure 13.90. $T = 2\pi\sqrt{I/mgd}$, $m = 3M$. $d = y_\text{cg} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$. For a rod with the axis at one end, $I = \frac{1}{3}ML^2$. For a rod with the axis at its center, $I = \frac{1}{12}ML^2$.

EXECUTE: $d = \frac{2M([1.55\text{ m}]/2) + M(1.55\text{ m} + [1.55\text{ m}]/2)}{3M} = 1.292\text{ m}$. $I + I_1 + I_2$.

$I_1 = \frac{1}{3}(2M)(1.55\text{ m})^2 = (1.602\text{ m}^2)M$. $I_{2,\text{cm}} = \frac{1}{12}M(1.55\text{ m})^2$. The parallel-axis theorem (Eq. 9.19) gives

$I_2 = I_{2,\text{cm}} + M(1.55\text{ m} + [1.55\text{ m}]/2)^2 = (5.06\text{ m}^2)M$. $I = I_1 + I_2 = (7.208\text{ m}^2)M$. Then

$$T = 2\pi\sqrt{I/mgd} = 2\pi\sqrt{\frac{(7.208\text{ m}^2)M}{(3M)(9.80\text{ m/s}^2)(1.292\text{ m})}} = 2.74\text{ s}.$$

EVALUATE: This is a little smaller than $T = 2.9$ s found in Example 13.10.

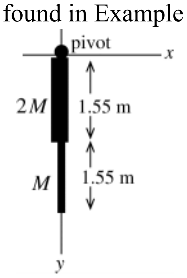


Figure 13.90

13.91: **IDENTIFY:** The motion is simple harmonic if the equation of motion for the angular oscillations is of the form $\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta$, and in this case the period is $T = 2\pi\sqrt{I/\kappa}$

SET UP: For a slender rod pivoted about its center, $I = \frac{1}{12}ML^2$

EXECUTE: The torque on the rod about the pivot is $\tau = -\left(k\frac{L}{2}\theta\right)\frac{L}{2}$. $\tau = I\alpha = I\frac{d^2\theta}{dt^2}$ gives

$\frac{d^2\theta}{dt^2} = -k\frac{L^2/4}{I}\theta = -\frac{3k}{M}\theta$. $\frac{d^2\theta}{dt^2}$ is proportional to θ and the motion is angular SHM. $\frac{\kappa}{I} = \frac{3k}{M}$, $T = 2\pi\sqrt{\frac{M}{3k}}$.

EVALUATE: The expression we used for the torque, $\tau = -\left(k\frac{L}{2}\theta\right)\frac{L}{2}$, is valid only when θ is small enough for $\sin\theta \approx \theta$ and $\cos\theta \approx 1$.

13.92. **IDENTIFY** and **SET UP:** Eq. (13.39) gives the period for the bell and Eq. (13.34) gives the period for the clapper.
EXECUTE: The bell swings as a physical pendulum so its period of oscillation is given by

$T = 2\pi\sqrt{I/mgd} = 2\pi\sqrt{18.0\text{ kg}\cdot\text{m}^2/(34.0\text{ kg})(9.80\text{ m/s}^2)(0.60\text{ m})} = 1.885\text{ s}$

The clapper is a simple pendulum so its period is given by $T = 2\pi\sqrt{L/g}$.

Thus $L = g(T/2\pi)^2 = (9.80\text{ m/s}^2)(1.885\text{ s}/2\pi)^2 = 0.88\text{ m}$.

EVALUATE: If the cm of the bell were at the geometrical center of the bell, the bell would extend 1.20 m from the pivot, so the clapper is well inside tbe bell.

13.93. **IDENTIFY:** The object oscillates as a physical pendulum, with $f = \frac{1}{2\pi}\sqrt{\frac{Mgd}{I}}$, where M is the total mass of the object.

SET UP: The moment of inertia about the pivot is $2(1/3)ML^2 = (2/3)\,ML^2$, and the center of gravity when balanced is a distance $d = L/(2\sqrt{2})$ below the pivot .

EXECUTE: The frequency is $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{6g}{4\sqrt{2}L}} = \frac{1}{4\pi}\sqrt{\frac{6g}{\sqrt{2}L}}$.

EVALUATE: If $f_{sp} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$ is the frequency for a simple pendulum of length L, $f = \frac{1}{2}\sqrt{\frac{6}{\sqrt{2}}}f_{sp} = 1.03 f_{sp}$.

13.94. **IDENTIFY** and **SET UP:** Use Eq. (13.34) for the simple pendulum. Use a physical pendulum (Eq.13.39) for the pendulum in the case.
EXECUTE: (a) $T = 2\pi\sqrt{L/g}$ and $L = g(T/2\pi)^2 = (9.80\text{ m/s}^2)(4.00\text{ s}/2\pi)^2 = 3.97\text{ m}$

(b) Use a uniform slender rod of mass M and length $L = 0.50$ m. Pivot the rod about an axis that is a distance d above the center of the rod. The rod will oscillate as a physical pendulum with period $T = 2\pi\sqrt{I/Mgd}$.

Choose d so that $T = 4.00$ s.

$I = I_{cm} + Md^2 = \frac{1}{12}ML^2 + Md^2 = M(\frac{1}{12}L^2 + d^2)$

$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{M(\frac{1}{12}L^2 + d^2)}{Mgd}} = 2\pi\sqrt{\frac{\frac{1}{12}L^2 + d^2}{gd}}$.

Solve for d and set $L = 0.50$ m and $T = 4.00$ s:

$gd(T/2\pi)^2 = \frac{1}{12}L^2 + d^2$

$d^2 - (T/2\pi)^2 gd + L^2/12 = 0$

$d^2 - (4.00\text{ s}/2\pi)^2(9.80\text{ m/s}^2)d + (0.50\text{ m})^2/12 = 0$

$d^2 - 3.9718d + 0.020833 = 0$

The quadratic formula gives

$d = \frac{1}{2}[3.9718 \pm \sqrt{(3.9718)^2 - 4(0.020833)}]\text{ m}$

$d = (1.9859 \pm 1.9806)\text{ m}$ so $d = 3.97$ m or $d = 0.0053$ m.

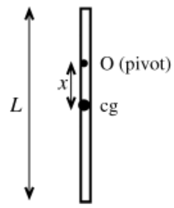
The maximum value d can have is $L/2 = 0.25$ m, so the answer we want is $d = 0.0053\text{ m} = 0.53\text{ cm}$.

Therefore, take a slender rod of length 0.50 m and pivot it about an axis that is 0.53 cm above its center.

EVALUATE: Note that $T \to \infty$ as $d \to 0$ (pivot at center of rod) and that if the pivot is at the top of rod then $d = L/2$ and $T = 2\pi\sqrt{\frac{\frac{1}{12}L^2 + \frac{1}{4}L^2}{Lg/2}} = 2\pi\sqrt{\frac{L}{g}\frac{4}{6}} = 2\pi\sqrt{\frac{2L}{3g}} = 2\pi\sqrt{\frac{2(0.50\text{ m})}{3(9.80\text{ m/s}^2)}} = 1.16$ s, which is less than the desired 4.00 s. Thus it is reasonable to expect that there is a value of d between 0 and $L/2$ for which $T = 4.00$ s.

13.95. **IDENTIFY:** The angular frequency is given by Eq.(13.38). Use the parallel-axis theorem to calculate I in terms of x.

(a) SET UP:



$\omega = \sqrt{mgd/I}$

Figure 13.95

$d = x,$ the distance from the cg of the object (which is at its geometrical center) from the pivot

EXECUTE: I is the moment of inertia about the axis of rotation through O. By the parallel axis theorem

$I_0 = md^2 + I_{\text{cm}}$. $I_{\text{cm}} = \frac{1}{12}mL^2$ (Table 9.2), so $I_0 = mx^2 + \frac{1}{12}mL^2$. $\omega = \sqrt{\dfrac{mgx}{mx^2 + \frac{1}{12}mL^2}} = \sqrt{\dfrac{gx}{x^2 + L^2/12}}$

(b) The maximum ω as x varies occurs when $d\omega/dx = 0$. $\dfrac{d\omega}{dx} = 0$ gives $\sqrt{g}\dfrac{d}{dx}\left(\dfrac{x^{1/2}}{(x^2 + L^2/12)^{1/2}}\right) = 0.$

$$\frac{\frac{1}{2}x^{-1/2}}{(x^2 + L^2/12)^{1/2}} - \frac{1}{2}\frac{2x}{(x^2 + L^2/12)^{3/2}}(x^{1/2}) = 0$$

$$x^{-1/2} - \frac{2x^{3/2}}{x^2 + L^2/12} = 0$$

$x^2 + L^2/12 = 2x^2$ so $x = L/\sqrt{12}$. Get maximum ω when the pivot is a distance $L/\sqrt{12}$ above the center of the rod.

(c) To answer this question we need an expression for ω_{max}:

In $\omega = \sqrt{\dfrac{gx}{x^2 + L^2/12}}$ substitute $x = L/\sqrt{12}$.

$$\omega_{\text{max}} = \sqrt{\frac{g(L/\sqrt{12})}{L^2/12 + L^2/12}} = \frac{g^{1/2}(12)^{-1/4}}{(L/6)^{1/2}} = \sqrt{g/L}(12)^{-1/4}(6)^{1/2} = \sqrt{g/L}(3)^{1/4}$$

$\omega_{\text{max}}^2 = (g/L)\sqrt{3}$ and $L = g\sqrt{3}/\omega_{\text{max}}^2$

$\omega_{\text{max}} = 2\pi$ rad/s gives $L = \dfrac{(9.80\text{ m/s}^2)\sqrt{3}}{(2\pi\text{ rad/s})^2} = 0.430$ m.

EVALUATE: $\omega \to 0$ as $x \to 0$ and $\omega \to \sqrt{3g/(2L)} = 1.225\sqrt{g/L}$ when $x \to L/2$. ω_{max} is greater than the $x = L/2$ value. A simple pendulum has $\omega = \sqrt{g/L}$; ω_{max} is greater than this.

13.96. **IDENTIFY:** Calculate F_{net} and define k_{eff} by $F_{\text{net}} = -k_{\text{eff}}x$. $T = 2\pi\sqrt{m/k_{\text{eff}}}$.

SET UP: If the elongations of the springs are x_1 and x_2, they must satisfy $x_1 + x_2 = 0.200$ m

EXECUTE: **(a)** The net force on the block at equilibrium is zero, and so $k_1x_1 = k_2x_2$ and one spring (the one with $k_1 = 2.00$ N/m) must be stretched three times as much as the one with $k_2 = 6.00$ N/m . The sum of the elongations is 0.200 m, and so one spring stretches 0.150 m and the other stretches 0.050 m, and so the equilibrium lengths are 0.350 m and 0.250 m.

(b) When the block is displaced a distance x to the right, the net force on the block is $-k_1(x_1 + x) + k_2(x_2 - x) = [k_1x_1 - k_2x_2] - (k_1 + k_2)x$. From the result of part (a), the term in square brackets is zero, and so the net force is $-(k_1 + k_2)x$, the effective spring constant is $k_{\text{eff}} = k_1 + k_2$ and the period of vibration is

$$T = 2\pi\sqrt{\frac{0.100\text{ kg}}{8.00\text{ N/m}}} = 0.702\text{ s}.$$

EVALUATE: The motion is the same as if the block were attached to a single spring that has force constant k_{eff}.

13.97. **IDENTIFY:** In each situation, imagine the mass moves a distance Δx, the springs move distances Δx_1 and Δx_2, with forces $F_1 = -k_1\Delta x_1$, $F_2 = -k_2\Delta x_2$.

SET UP: Let Δx_1 and Δx_2 be positive if the springs are stretched, negative if compressed.

EXECUTE: **(a)** $\Delta x = \Delta x_1 = \Delta x_2$, $F = F_1 + F_2 = -(k_1 + k_2)\Delta x$, so $k_{\text{eff}} = k_1 + k_2$.

(b) Despite the orientation of the springs, and the fact that one will be compressed when the other is extended, $\Delta x = \Delta x_1 - \Delta x_2$ and both spring forces are in the same direction. The above result is still valid; $k_{\text{eff}} = k_1 + k_2$.

(c) For massless springs, the force on the block must be equal to the tension in any point of the spring combination, and $F = F_1 = F_2$. $\Delta x_1 = -\dfrac{F}{k_1}$, $\Delta x_2 = -\dfrac{F}{k_2}$, $\Delta x = -\left(\dfrac{1}{k_1} + \dfrac{1}{k_2}\right) F = -\dfrac{k_1 + k_2}{k_1 k_2} F$ and $k_{\text{eff}} = \dfrac{k_1 k_2}{k_1 + k_2}$.

(d) The result of part (c) shows that when a spring is cut in half, the effective spring constant doubles, and so the frequency increases by a factor of $\sqrt{2}$.

EVALUATE: In cases (a) and (b) the effective force constant is greater than either k_1 or k_2 and in case (c) it is less.

13.98. **IDENTIFY:** Follow the procedure specified in the hint.

SET UP: $T = 2\pi\sqrt{L/g}$

EXECUTE: **(a)** $T + \Delta T \approx 2\pi\sqrt{L}\left(g^{-1/2} - \dfrac{1}{2} g^{-3/2} \Delta g\right) = T - T\dfrac{\Delta g}{2g}$, so $\Delta T = -(1/2)(T/g)\Delta g$.

(b) The clock runs slow; $\Delta T > 0$, $\Delta g < 0$ and $g + \Delta g = g\left(1 - \dfrac{2\Delta T}{T}\right) = (9.80 \text{ m/s}^2)\left(1 - \dfrac{2(4.00 \text{ s})}{(86{,}400 \text{ s})}\right) = 9.7991 \text{ m/s}^2$.

EVALUATE: The result in part (a) says that T increases when g decreases, and the magnitude of the fractional change in T is one-half of the magnitude of the fractional change in g.

13.99. **IDENTIFY:** Follow the procedure specified in the hint.

SET UP: Denote the position of a piece of the spring by l; $l = 0$ is the fixed point and $l = L$ is the moving end of the spring. Then the velocity of the point corresponding to l, denoted u, is $u(l) = v\dfrac{l}{L}$ (when the spring is moving, l will be a function of time, and so u is an implicit function of time).

(a) $dm = \dfrac{M}{L} dl$, and so $dK = \dfrac{1}{2} dm\, u^2 = \dfrac{1}{2}\dfrac{Mv^2}{L^3} l^2\, dl$ and $K = \int dK = \dfrac{Mv^2}{2L^3}\displaystyle\int_0^L l^2\, dl = \dfrac{Mv^2}{6}$.

(b) $mv\dfrac{dv}{dt} + kx\dfrac{dx}{dt} = 0$, or $ma + kx = 0$, which is Eq. (13.4).

(c) m is replaced by $\dfrac{M}{3}$, so $\omega = \sqrt{\dfrac{3k}{M}}$ and $M' = \dfrac{M}{3}$.

EVALUATE: The effective mass of the spring is only one-third of its actual mass.

13.100. **IDENTIFY:** $T = 2\pi\sqrt{I/mgd}$

SET UP: With $I = (1/3)ML^2$ and $d = L/2$ in Eq. (13.39), $T_0 = 2\pi\sqrt{2L/3g}$. With the added mass, $I = M\left((L^2/3) + y^2\right)$, $m = 2M$ and $d = (L/4) + y/2$. $T = 2\pi\sqrt{(L^2/3 + y^2)/(g(L/2 + y))}$ and $r = \dfrac{T}{T_0} = \sqrt{\dfrac{L^2 + 3y^2}{L^2 + 2yL}}$. The graph of the ratio r versus y is given in Figure 13.100.

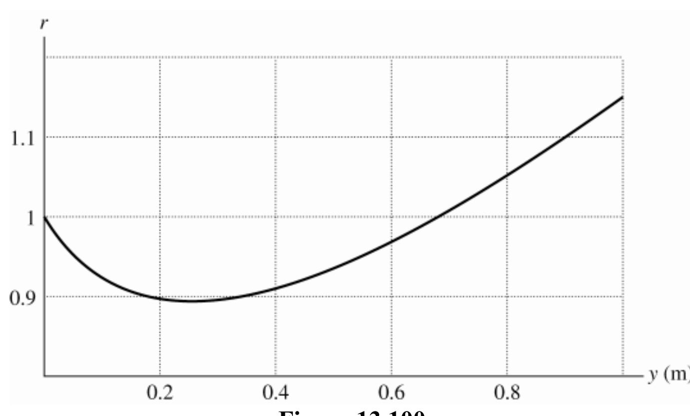


Figure 13.100

(b) From the expression found in part (a), $T = T_0$ when $y = \frac{2}{3}L$. At this point, a simple pendulum with length y would have the same period as the meter stick without the added mass; the two bodies oscillate with the same period and do not affect the other's motion.

EVALUATE: Adding the mass can either increase or decrease the period, depending on where the added mass is placed.

13.101. **IDENTIFY:** Eq.(13.39) says $T = 2\pi\sqrt{I/mgd}$.

SET UP: Let the two distances from the center of mass be d_1 and d_2. There are then two relations of the form of Eq. (13.39); with $I_1 = I_{cm} + md_1^2$ and $I_2 = I_{cm} + md_2^2$.

EXECUTE: These relations may be rewritten as $mgd_1T^2 = 4\pi^2\left(I_{cm} + md_1^2\right)$ and $mgd_2T^2 = 4\pi^2\left(I_{cm} + md_2^2\right)$.

Subtracting the expressions gives $mg\left(d_1 - d_2\right)T^2 = 4\pi^2 m\left(d_1^2 - d_2^2\right) = 4\pi^2 m\left(d_1 - d_2\right)\left(d_1 + d_2\right)$. Dividing by the common factor of $m\left(d_1 - d_2\right)$ and letting $d_1 + d_2 = L$ gives the desired result.

EVALUATE: The procedure works in practice only if both pivot locations give rise to SHM for small oscillations.

13.102. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the mass, with $a = a_{rad} = r\omega^2$.

SET UP: The spring, when stretched, provides an inward force.

EXECUTE: Using $\omega'^2 l$ for the magnitude of the inward radial acceleration, $m\omega'^2 l = k\left(l - l_0\right)$, or $l = \dfrac{kl_0}{k - m\omega'^2}$.

(b) The spring will tend to become unboundedly long.

EXECUTE: As resonance is approached and *l* becomes very large, both the spring force and the radial acceleration become large.

13.103. **IDENTIFY:** For a small displacement *x*, the force constant *k* is defined by $F_x = -kx$.

SET UP: Let $r = R_0 + x$, so that $r - R_0 = x$ and $F = A[e^{-2bx} - e^{-bx}]$.

EXECUTE: When *x* is small compared to b^{-1}, expanding the exponential function gives

$F \approx A\,[(1 - 2bx) - (1 - bx)] = -Abx$, corresponding to a force constant of $Ab = 579$ N/m .

EVALUATE: Our result is very close to the value given in Exercise 13.40.