POTENTIAL ENERGY AND ENERGY CONSERVATION

7

7.1. **IDENTIFY:** $U_{\text{grav}} = mgy$ so $\Delta U_{\text{grav}} = mg(y_2 - y_1)$

SET UP: $+y$ is upward.

EXECUTE: **(a)** $\Delta U = (75\text{ kg})(9.80\text{ m/s}^2)(2400\text{ m} - 1500\text{ m}) = +6.6\times10^5\text{ J}$

(b) $\Delta U = (75\text{ kg})(9.80\text{ m/s}^2)(1350\text{ m} - 2400\text{ m}) = -7.7\times10^5\text{ J}$

EVALUATE: U_{grav} increases when the altitude of the object increases.

7.2. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to the sack to find the force. $W = Fs\cos\phi$.

SET UP: The lifting force acts in the same direction as the sack's motion, so $\phi = 0°$

EXECUTE: **(a)** For constant speed, the net force is zero, so the required force is the sack's weight, $(5.00\text{ kg})(9.80\text{ m/s}^2) = 49.0\text{ N}$.

(b) $W = (49.0\text{ N})\,(15.0\text{ m}) = 735\text{ J}$. This work becomes potential energy.

EVALUATE: The results are independent of the speed.

7.3. **IDENTIFY:** Use the free-body diagram for the bag and Newton's first law to find the force the worker applies. Since the bag starts and ends at rest, $K_2 - K_1 = 0$ and $W_{\text{tot}} = 0$.

SET UP: A sketch showing the initial and final positions of the bag is given in Figure 7.3a. $\sin\phi = \dfrac{2.0\text{ m}}{3.5\text{ m}}$ and $\phi = 34.85°$. The free-body diagram is given in Figure 7.3b. $\vec{F}$ is the horizontal force applied by the worker. In the calculation of U_{grav} take $+y$ upward and $y = 0$ at the initial position of the bag.

EXECUTE: **(a)** $\sum F_y = 0$ gives $T\cos\phi = mg$ and $\sum F_x = 0$ gives $F = T\sin\phi$. Combining these equations to eliminate T gives $F = mg\tan\phi = (120\text{ kg})(9.80\text{ m/s}^2)\tan 34.85° = 820\text{ N}$.

(b) (i) The tension in the rope is radial and the displacement is tangential so there is no component of T in the direction of the displacement during the motion and the tension in the rope does no work. (ii) $W_{\text{tot}} = 0$ so

$W_{\text{worker}} = -W_{\text{grav}} = U_{\text{grav,2}} - U_{\text{grav,1}} = mg(y_2 - y_1) = (120\text{ kg})(9.80\text{ m/s}^2)(0.6277\text{ m}) = 740\text{ J}$.

EVALUATE: The force applied by the worker varies during the motion of the bag and it would be difficult to calculate W_{worker} directly.

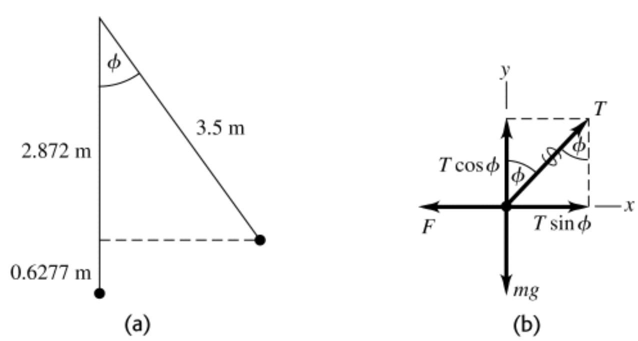


Figure 7.3

7.4. **IDENTIFY:** Only gravity does work on him from the point where he has just left the board until just before he enters the water, so Eq.(7.4) applies.

SET UP: Let point 1 be just after he leaves the board and point 2 be just before he enters the water. $+y$ is upward and $y = 0$ at the water.

EXECUTE: **(a)** $K_1 = 0$. $y_2 = 0$. $y_1 = 3.25\text{ m}$. $K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$ gives $U_{\text{grav},1} = K_2$ and $mgy_1 = \frac{1}{2}mv_2^2$.

$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\text{ m/s}^2)(3.25\text{ m})} = 7.98\text{ m/s}$.

(b) $v_1 = 2.50\text{ m/s}$, $y_2 = 0$, $y_1 = 3.25\text{ m}$. $K_1 + U_{\text{grav},1} = K_2$ and $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2$.

$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{(2.50\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(3.25\text{ m})} = 8.36\text{ m/s}$.

(c) $v_1 = 2.5\text{ m/s}$ and $v_2 = 8.36\text{ m/s}$, the same as in part (b).

EVALUATE: Kinetic energy depends only on the speed, not on the direction of the velocity.

7.5. **IDENTIFY** and **SET UP:** Use energy methods.

(a) $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Solve for K_2 and then use $K_2 = \frac{1}{2}mv_2^2$ to obtain v_2.

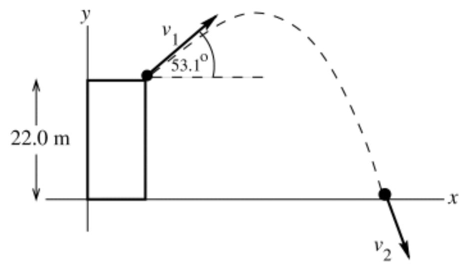


Figure 7.5

$W_{\text{other}} = 0$ (The only force on the ball while it is in the air is gravity.)

$K_1 = \frac{1}{2}mv_1^2$; $K_2 = \frac{1}{2}mv_2^2$

$U_1 = mgy_1$, $y_1 = 22.0\text{ m}$

$U_2 = mgy_2 = 0$, since $y_2 = 0$ for our choice of coordinates.

EXECUTE: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2$

$$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{(12.0\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(22.0\text{ m})} = 24.0\text{ m/s}$$

EVALUATE: The projection angle of $53.1°$ doesn't enter into the calculation. The kinetic energy depends only on the magnitude of the velocity; it is independent of the direction of the velocity.

(b) Nothing changes in the calculation. The expression derived in part (a) for v_2 is independent of the angle, so $v_2 = 24.0\text{ m/s}$, the same as in part (a).

(c) The ball travels a shorter distance in part (b), so in that case air resistance will have less effect.

7.6. **IDENTIFY:** The normal force does no work, so only gravity does work and Eq.(7.4) applies.

SET UP: $K_1 = 0$. The crate's initial point is at a vertical height of $d\sin\alpha$ above the bottom of the ramp.

EXECUTE: **(a)** $y_2 = 0$, $y_1 = d\sin\alpha$. $K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$ gives $U_{\text{grav},1} = K_2$. $mgd\sin\alpha = \frac{1}{2}mv_2^2$ and $v_2 = \sqrt{2gd\sin\alpha}$.

(b) $y_1 = 0$, $y_2 = -d\sin\alpha$. $K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$ gives $0 = K_2 + U_{\text{grav},2}$. $0 = \frac{1}{2}mv_2^2 + (-mgd\sin\alpha)$ and $v_2 = \sqrt{2gd\sin\alpha}$, the same as in part (a).

(c) The normal force is perpendicular to the displacement and does no work.

EVALUATE: When we use $U_{\text{grav}} = mgy$ we can take any point as $y = 0$ but we must take $+y$ to be upward.

7.7. **IDENTIFY:** Apply Eq.(7.7) to points 2 and 3. Take results from Example 7.6. $W_{\text{other}} = -fs$, the work done by friction.

SET UP: As in Example 7.6, $K_2 = 0$, $U_2 = 94\text{ J}$, and $U_3 = 0$.

EXECUTE: The work done by friction is $-(35\text{ N})(1.6\text{ m}) = -56\text{ J}$. $K_3 = 38\text{ J}$, and $v_3 = \sqrt{\dfrac{2(38\text{ J})}{12\text{ kg}}} = 2.5\text{ m/s}$.

EVALUATE: The value of v_3 we obtained is the same as calculated in Example 7.6. For the motion from point 2 to point 3, gravity does positive work, friction does negative work and the net work is positive.

7.8. **IDENTIFY** and **SET UP:** Apply Eq.(7.7) and consider how each term depends on the mass.

EXECUTE: The speed is v and the kinetic energy is $4K$. The work done by friction is proportional to the normal force, and hence to the mass, and so each term in Eq. (7.7) is proportional to the total mass of the crate, and the speed at the bottom is the same for any mass. The kinetic energy is proportional to the mass, and for the same speed but four times the mass, the kinetic energy is quadrupled.

EVALUATE: The same result is obtained if we apply $\sum \vec{F} = m\vec{a}$ to the motion. Each force is proportional to m and m divides out, so a is independent of m.

7.9. **IDENTIFY:** $W_{tot} = K_B - K_A$. The forces on the rock are gravity, the normal force and friction.

SET UP: Let $y = 0$ at point *B* and let $+y$ be upward. $y_A = R = 0.50$ m. The work done by friction is negative; $W_f = -0.22$ J. $K_A = 0$. The free-body diagram for the rock at point *B* is given in Figure 7.9. The acceleration of the rock at this point is $a_{rad} = v^2/R$, upward.

EXECUTE: **(a)** (i) The normal force is perpendicular to the displacement and does zero work.

(ii) $W_{grav} = U_{grav,A} - U_{grav,B} = mgy_A = (0.20\text{ kg})(9.80\text{ m/s}^2)(0.50\text{ m}) = 0.98\text{ J}$.

(b) $W_{tot} = W_n + W_f + W_{grav} = 0 + (-0.22\text{ J}) + 0.98\text{ J} = 0.76\text{ J}$. $W_{tot} = K_B - K_A$ gives $\frac{1}{2}mv_B^2 = W_{tot}$.

$$v_B = \sqrt{\frac{2W_{tot}}{m}} = \sqrt{\frac{2(0.76\text{ J})}{0.20\text{ kg}}} = 2.8\text{ m/s}.$$

(c) Gravity is constant and equal to *mg*. *n* is not constant; it is zero at *A* and not zero at *B*. Therefore, $f_k = \mu_k n$ is also not constant.

(d) $\sum F_y = ma_y$ applied to Figure 7.9 gives $n - mg = ma_{rad}$.

$$n = m\left(g + \frac{v^2}{R}\right) = (0.20\text{ kg})\left(9.80\text{ m/s}^2 + \frac{[2.8\text{ m/s}]^2}{0.50\text{ m}}\right) = 5.1\text{ N}.$$

EVALUATE: In the absence of friction, the speed of the rock at point *B* would be $\sqrt{2gR} = 3.1$ m/s. As the rock slides through point *B*, the normal force is greater than the weight $mg = 2.0$ N of the rock.

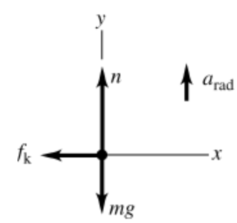


Figure 7.9

7.10. **IDENTIFY:** Only gravity does work, so Eq.(7.4) applies.

SET UP: Let point 1 be just after the rock leaves the thrower and point 2 be at the maximum height. Let $y_1 = 0$ and $+y$ be upward. $v_1 = v_0$. At the highest point, $v_2 = v_0\cos\theta$. $\sin^2\theta + \cos^2\theta = 1$.

EXECUTE: $K_1 + U_{grav,1} = K_2 + U_{grav,2}$ gives $\frac{1}{2}mv_0^2 = \frac{1}{2}m(v_0\cos\theta)^2 + mgy_2$. $y_2 = \frac{v_0^2}{2g}(1-\cos^2\theta) = \frac{v_0^2\sin^2\theta}{2g}$, was to be shown.

EVALUATE: The initial kinetic energy is independent of the angle θ but the kinetic energy at the maximum height depends on θ, so the maximum height depends on θ.

7.11. **IDENTIFY:** Apply Eq.(7.7) to the motion of the car.

SET UP: Take $y = 0$ at point *A*. Let point 1 be *A* and point 2 be *B*.

$$K_1 + U_1 + W_{other} = K_2 + U_2$$

EXECUTE: $U_1 = 0,\ U_2 = mg(2R) = 28{,}224\text{ J},\ W_{other} = W_f$

$K_1 = \frac{1}{2}mv_1^2 = 37{,}500\text{ J},\ K_2 = \frac{1}{2}mv_2^2 = 3840\text{ J}$

The work-energy relation then gives $W_f = K_2 + U_2 - K_1 = -5400$ J.

EVALUATE: Friction does negative work. The final mechanical energy $(K_2 + U_2 = 32{,}064\text{ J})$ is less than the initial mechanical energy $(K_1 + U_1 = 37{,}500\text{ J})$ because of the energy removed by friction work.

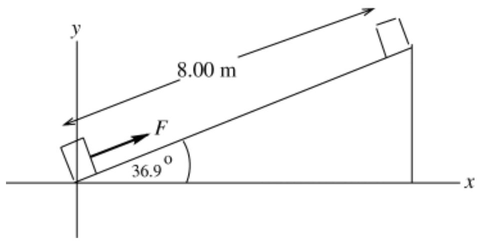
7.12. **IDENTIFY:** Only gravity does work, so apply Eq.(7.5).

SET UP: $v_1 = 0$, so $\frac{1}{2}mv_2^2 = mg(y_1 - y_2)$.

EXECUTE: Tarzan is lower than his original height by a distance $y_1 - y_2 = l(\cos 30° - \cos 45°)$ so his speed is $v = \sqrt{2gl(\cos 30° - \cos 45°)} = 7.9$ m/s, a bit quick for conversation.

EVALUATE: The result is independent of Tarzan's mass.

7.13.



$y_1 = 0$
$y_2 = (8.00\text{ m})\sin 36.9°$
$y_2 = 4.80\text{ m}$

Figure 7.13a

(a) IDENTIFY and **SET UP:** $\vec{F}$ is constant so Eq.(6.2) can be used. The situation is sketched in Figure 7.13a.

EXECUTE: $W_F = (F\cos\phi)s = (110\text{ N})(\cos 0°)(8.00\text{ m}) = 880\text{ J}$

EVALUATE: $\vec{F}$ is in the direction of the displacement and does positive work.

(b) IDENTIFY and **SET UP:** Calculate W using Eq.(6.2) but first must calculate the friction force. Use the free-body diagram for the oven sketched in Figure 7.13b to calculate the normal force n; then the friction force can be calculated from $f_k = \mu_k n$. For this calculation use coordinates parallel and perpendicular to the incline.

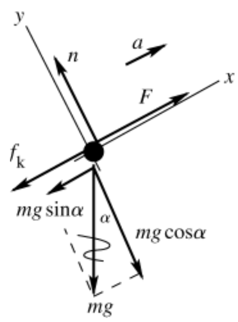


Figure 7.13b

EXECUTE: $\sum F_y = ma_y$

$n - mg\cos 36.9° = 0$

$n = mg\cos 36.9°$

$f_k = \mu_k n = \mu_k mg\cos 36.9°$

$f_k = (0.25)(10.0\text{ kg})(9.80\text{ m/s}^2)\cos 36.9° = 19.6\text{ N}$

$W_f = (f_k\cos\phi)s = (19.6\text{ N})(\cos 180°)(8.00\text{ m}) = -157\text{ J}$

EVALUATE: Friction does negative work.

(c) IDENTIFY and **SET UP:** $U = mgy$; take $y = 0$ at the bottom of the ramp.

EXECUTE: $\Delta U = U_2 - U_1 = mg(y_2 - y_1) = (10.0\text{ kg})(9.80\text{ m/s}^2)(4.80\text{ m} - 0) = 470\text{ J}$

EVALUATE: The object moves upward and U increases.

(d) IDENTIFY and **SET UP:** Use Eq.(7.7). Solve for ΔK.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$\Delta K = K_2 - K_1 = U_1 - U_2 + W_{\text{other}}$

$\Delta K = W_{\text{other}} - \Delta U$

$W_{\text{other}} = W_F + W_f = 880\text{ J} - 157\text{ J} = 723\text{ J}$

$\Delta U = 470\text{ J}$

Thus $\Delta K = 723\text{ J} - 470\text{ J} = 253\text{ J}$.

EVALUATE: W_{other} is positive. Some of W_{other} goes to increasing U and the rest goes to increasing K.

(e) IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to the oven. Solve for $\vec{a}$ and then use a constant acceleration equation to calculate v_2.

SET UP: We can use the free-body diagram that is in part (b):

$\sum F_x = ma_x$

$F - f_k - mg\sin 36.9° = ma$

EXECUTE: $a = \dfrac{F - f_k - mg\sin 36.9°}{m} = \dfrac{110\text{ N} - 19.6\text{ N} - (10\text{ kg})(9.80\text{ m/s}^2)\sin 36.9°}{10.0\text{ kg}} = 3.16\text{ m/s}^2$

SET UP: $v_{1x} = 0$, $a_x = 3.16\text{ m/s}^2$, $x - x_0 = 8.00\text{ m}$, $v_{2x} = ?$

$v_{2x}^2 = v_{1x}^2 + 2a_x(x - x_0)$

EXECUTE: $v_{2x} = \sqrt{2a_x(x - x_0)} = \sqrt{2(3.16\text{ m/s}^2)(8.00\text{ m})} = 7.11\text{ m/s}^2$

Then $\Delta K = K_2 - K_1 = \frac{1}{2}mv_2^2 = \frac{1}{2}(10.0\text{ kg})(7.11\text{ m/s})^2 = 253\text{ J}$.

EVALUATE: This agrees with the result calculated in part (d) using energy methods.

7.14. **IDENTIFY:** Only gravity does work, so apply Eq.(7.4). Use $\sum \vec{F} = m\vec{a}$ to calculate the tension.

SET UP: Let $y = 0$ at the bottom of the arc. Let point 1 be when the string makes a $45°$ angle with the vertical and point 2 be where the string is vertical. The rock moves in an arc of a circle, so it has radial acceleration $a_{\text{rad}} = v^2/r$

EXECUTE: **(a)** At the top of the swing, when the kinetic energy is zero, the potential energy (with respect to the bottom of the circular arc) is $mgl(1-\cos\,\theta)$, where l is the length of the string and θ is the angle the string makes with the vertical. At the bottom of the swing, this potential energy has become kinetic energy, so $mgl(1-\cos\theta) = \frac{1}{2}mv^2$, or $v = \sqrt{2gl(1-\cos\theta)} = \sqrt{2(9.80\text{ m/s}^2)\,(0.80\text{ m})\,(1-\cos 45°)} = 2.1\text{ m/s}$.

(b) At $45°$ from the vertical, the speed is zero, and there is no radial acceleration; the tension is equal to the radial component of the weight, or $mg\cos\theta = (0.12\text{ kg})\,(9.80\text{ m/s}^2)\cos 45° = 0.83\text{ N}$.

(c) At the bottom of the circle, the tension is the sum of the weight and the mass times the radial acceleration,

$$mg + mv_2^2/l = mg(1 + 2(1-\cos 45°)) = 1.9\text{ N}$$

EVALUATE: When the string passes through the vertical, the tension is greater than the weight because the acceleration is upward.

7.15. **IDENTIFY:** Apply $U_{\text{el}} = \frac{1}{2}kx^2$.

SET UP: $kx = F$, so $U = \frac{1}{2}Fx$, where F is the magnitude of force required to stretch or compress the spring a distance x.

EXECUTE: **(a)** $(1/2)(800\text{ N})(0.200\text{ m}) = 80.0\text{ J}$.

(b) The potential energy is proportional to the square of the compression or extension; $(80.0\text{ J})\,(0.050\text{ m}/0.200\text{ m})^2 = 5.0\text{ J}$.

EVALUATE: We could have calculated $k = \dfrac{F}{x} = \dfrac{800\text{ N}}{0.200\text{ m}} = 4000\text{ N/m}$ and then used $U_{\text{el}} = \frac{1}{2}kx^2$ directly.

7.16. **IDENTIFY:** Use the information given in the problem with $F = kx$ to find k. Then $U_{\text{el}} = \frac{1}{2}kx^2$.

SET UP: x is the amount the spring is stretched. When the weight is hung from the spring, $F = mg$.

EXECUTE: $k = \dfrac{F}{x} = \dfrac{mg}{x} = \dfrac{(3.15\text{ kg})(9.80\text{ m/s}^2)}{0.1340\text{ m} - 0.1200\text{ m}} = 2205\text{ N/m}$.

$x = \pm\sqrt{\dfrac{2U_{\text{el}}}{k}} = \pm\sqrt{\dfrac{2(10.0\text{ J})}{2205\text{ N/m}}} = \pm 0.0952\text{ m} = \pm 9.52\text{ cm}$. The spring could be either stretched 9.52 cm or compressed 9.52 cm. If it were stretched, the total length of the spring would be $12.00\text{ cm} + 9.52\text{ cm} = 21.52\text{ cm}$. If it were compressed, the total length of the spring would be $12.00\text{ cm} - 9.52\text{ cm} = 2.48\text{ cm}$.

EVALUATE: To stretch or compress the spring 9.52 cm requires a force $F = kx = 210\text{ N}$.

7.17. **IDENTIFY:** Apply $U_{\text{el}} = \frac{1}{2}kx^2$.

SET UP: $U_0 = \frac{1}{2}kx_0^2$. x is the distance the spring is stretched or compressed.

EXECUTE: **(a)** (i) $x = 2x_0$ gives $U_{\text{el}} = \frac{1}{2}k(2x_0)^2 = 4(\frac{1}{2}kx_0^2) = 4U_0$. (ii) $x = x_0/2$ gives $U_{\text{el}} = \frac{1}{2}k(x_0/2)^2 = \frac{1}{4}(\frac{1}{2}kx_0^2) = U_0/4$.

(b) (i) $U = 2U_0$ gives $\frac{1}{2}kx^2 = 2(\frac{1}{2}kx_0^2)$ and $x = x_0\sqrt{2}$. (ii) $U = U_0/2$ gives $\frac{1}{2}kx^2 = \frac{1}{2}(\frac{1}{2}kx_0^2)$ and $x = x_0/\sqrt{2}$.

EVALUATE: U is proportional to x^2 and x is proportional to $\sqrt{U}$.

7.18. **IDENTIFY:** Apply Eq.(7.13).

SET UP: Initially and at the highest point, $v = 0$, so $K_1 = K_2 = 0$. $W_{\text{other}} = 0$.

EXECUTE: **(a)** In going from rest in the slingshot's pocket to rest at the maximum height, the potential energy stored in the rubber band is converted to gravitational potential energy; $U = mgy = (10\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)\,(22.0\text{ m}) = 2.16\text{ J}$.

(b) Because gravitational potential energy is proportional to mass, the larger pebble rises only 8.8 m.

(c) The lack of air resistance and no deformation of the rubber band are two possible assumptions.

EVALUATE: The potential energy stored in the rubber band depends on k for the rubber band and the maximum distance it is stretched.

7.19. IDENTIFY and SET UP: Use energy methods. There are changes in both elastic and gravitational potential energy; elastic; $U = \frac{1}{2}kx^2$, gravitational: $U = mgy$.

EXECUTE: **(a)** $U = \frac{1}{2}kx^2$ so $x = \sqrt{\frac{2U}{k}} = \sqrt{\frac{2(3.20\text{ J})}{1600\text{ N/m}}} = 0.0632\text{ m} = 6.32\text{ cm}$

(b) Points 1 and 2 in the motion are sketched in Figure 7.19.

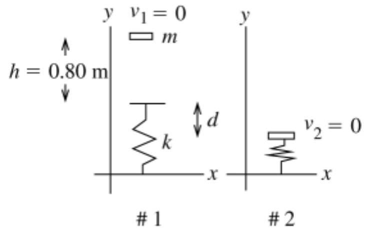


Figure 7.19

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$W_{\text{other}} = 0$ (Only work is that done by gravity and spring force)

$K_1 = 0,\ \ K_2 = 0$

$y = 0$ at final position of book

$U_1 = mg(h + d),\ \ U_2 = \frac{1}{2}kd^2$

$0 + mg(h + d) + 0 = \frac{1}{2}kd^2$

The original gravitational potential energy of the system is converted into potential energy of the compressed spring.

$\frac{1}{2}kd^2 - mgd - mgh = 0$

$$d = \frac{1}{k}\left(mg \pm \sqrt{(mg)^2 + 4\left(\frac{1}{2}k\right)(mgh)}\right)$$

d must be positive, so $d = \frac{1}{k}\left(mg + \sqrt{(mg)^2 + 2kmgh}\right)$

$$d = \frac{1}{1600\text{ N/m}}((1.20\text{ kg})(9.80\text{ m/s}^2) + \sqrt{((1.20\text{ kg})(9.80\text{ m/s}^2))^2 + 2(1600\text{ N/m})(1.20\text{ kg})(9.80\text{ m/s}^2)(0.80\text{ m})}$$

$d = 0.0074\text{ m} + 0.1087\text{ m} = 0.12\text{ m} = 12\text{ cm}$

EVALUATE: It was important to recognize that the total displacement was $h + d$; gravity continues to do work as the book moves against the spring. Also note that with the spring compressed 0.12 m it exerts an upward force (192 N) greater than the weight of the book (11.8 N). The book will be accelerated upward from this position.

7.20. IDENTIFY: Use energy methods. There are changes in both elastic and gravitational potential energy.

SET UP: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Points 1 and 2 in the motion are sketched in Figure 7.20.

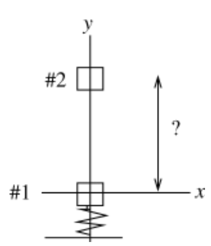


Figure 7.20

The spring force and gravity are the only forces doing work on the cheese, so $W_{\text{other}} = 0$ and $U = U_{\text{grav}} + U_{\text{el}}$.

EXECUTE: Cheese released from rest implies $K_1 = 0$.

At the maximum height $v_2 = 0$ so $K_2 = 0$.

$U_1 = U_{1,\text{el}} + U_{1,\text{grav}}$

$y_1 = 0$ implies $U_{1,\text{grav}} = 0$

$U_{1,\text{el}} = \frac{1}{2}kx_1^2 = \frac{1}{2}(1800\text{ N/m})(0.15\text{ m})^2 = 20.25\text{ J}$

(Here x_1 refers to the amount the spring is stretched or compressed when the cheese is at position 1; it is *not* the x-coordinate of the cheese in the coordinate system shown in the sketch.)

$U_2 = U_{2,\text{el}} + U_{2,\text{grav}}$

$U_{2,\text{grav}} = mgy_2$, where y_2 is the height we are solving for. $U_{2,\text{el}} = 0$ since now the spring is no longer compressed. Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $U_{1,\text{el}} = U_{2,\text{grav}}$

$$y_2 = \frac{20.25\text{ J}}{mg} = \frac{20.25\text{ J}}{(1.20\text{ kg})(9.80\text{ m/s}^2)} = 1.72\text{ m}$$

EVALUATE: The description in terms of energy is very simple; the elastic potential energy originally stored in the spring is converted into gravitational potential energy of the system.

7.21. **IDENTIFY:** Apply Eq.(7.13).

SET UP: $W_{\text{other}} = 0$. As in Example 7.7, $K_1 = 0$ and $U_1 = 0.0250$ J.

EXECUTE: For $v_2 = 0.20$ m/s, $K_2 = 0.0040$ J. $U_2 = 0.0210\text{ J} = \frac{1}{2}kx^2$, and $x = \pm\sqrt{\frac{2(0.0210\text{ J})}{5.00\text{ N/m}}} = \pm 0.092$ m. The glider has this speed when the spring is stretched 0.092 m or compressed 0.092 m.

EVALUATE: Example 7.7 showed that $v_x = 0.30$ m/s when $x = 0.0800$ m. As x increases, v_x decreases, so our result of $v_x = 0.20$ m/s at $x = 0.092$ m is consistent with the result in the example.

7.22. **IDENTIFY** and **SET UP:** Use energy methods. The elastic potential energy changes. In part (a) solve for K_2 and from this obtain v_2. In part (b) solve for U_1 and from this obtain x_1.

(a) $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

point 1: the glider is at its initial position, where $x_1 = 0.100$ m and $v_1 = 0$

point 2: the glider is at $x = 0$

EXECUTE: $K_1 = 0$ (released from rest), $K_2 = \frac{1}{2}mv_2^2$

$U_1 = \frac{1}{2}kx_1^2$, $U_2 = 0$, $W_{\text{other}} = 0$ (only the spring force does work)

Thus $\frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2$. (The initial potential energy of the stretched spring is converted entirely into kinetic energy of the glider.)

$$v_2 = x_1\sqrt{\frac{k}{m}} = (0.100\text{ m})\sqrt{\frac{5.00\text{ N/m}}{0.200\text{ kg}}} = 0.500\text{ m/s}$$

(b) The maximum speed occurs at $x = 0$, so the same equation applies.

$$\tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2$$

$$x_1 = v_2\sqrt{\frac{m}{k}} = 2.50\text{ m/s}\sqrt{\frac{0.200\text{ kg}}{5.00\text{ N/m}}} = 0.500\text{ m}$$

EVALUATE: Elastic potential energy is converted into kinetic energy. A larger x_1 gives a larger v_2.

7.23. **IDENTIFY:** Only the spring does work and Eq.(7.11) applies. $a = \frac{F}{m} = \frac{-kx}{m}$, where F is the force the spring exerts on the mass.

SET UP: Let point 1 be the initial position of the mass against the compressed spring, so $K_1 = 0$ and $U_1 = 11.5$ J. Let point 2 be where the mass leaves the spring, so $U_{\text{el},2} = 0$.

EXECUTE: **(a)** $K_1 + U_{\text{el},1} = K_2 + U_{\text{el},2}$ gives $U_{\text{el},1} = K_2$. $\frac{1}{2}mv_2^2 = U_{\text{el},1}$ and $v_2 = \sqrt{\frac{2U_{\text{el},1}}{m}} = \sqrt{\frac{2(11.5\text{ J})}{2.50\text{ kg}}} = 3.03$ m/s.

K is largest when U_{el} is least and this is when the mass leaves the spring. The mass achieves its maximum speed of 3.03 m/s as it leaves the spring and then slides along the surface with constant speed.

(b) The acceleration is greatest when the force on the mass is the greatest, and this is when the spring has its maximum compression. $U_{\text{el}} = \frac{1}{2}kx^2$ so $x = -\sqrt{\frac{2U_{\text{el}}}{k}} = -\sqrt{\frac{2(11.5\text{ J})}{2500\text{ N/m}}} = -0.0959$ m. The minus sign indicates compression. $F = -kx = ma_x$ and $a_x = -\frac{kx}{m} = -\frac{(2500\text{ N/m})(-0.0959\text{ m})}{2.50\text{ kg}} = 95.9\text{ m/s}^2$.

EVALUATE: If the end of the spring is displaced to the left when the spring is compressed, then a_x in part (b) is to the right, and vice versa.

7.24. **(a) IDENTIFY** and **SET UP:** Use energy methods. Both elastic and gravitational potential energy changes. Work is done by friction.

Choose point 1 as in Example 7.9 and let that be the origin, so $y_1 = 0$. Let point 2 be 1.00 m below point 1, so $y_2 = -1.00$ m.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(2000\text{ kg})(25\text{ m/s})^2 = 625{,}000\text{ J},\; U_1 = 0$

$W_{\text{other}} = -f\left|y_2\right| = -(17{,}000\text{ N})(1.00\text{ m}) = -17{,}000\text{ J}$

$K_2 = \frac{1}{2}mg_2^2$

$U_2 = U_{2,\text{grav}} + U_{2,\text{el}} = mgy_2 + \frac{1}{2}ky_2^2$

$U_2 = (2000\text{ kg})(9.80\text{ m/s}^2)(-1.00\text{ m}) + \frac{1}{2}(1.41\times 10^5\text{ N/m})(1.00\text{ m})^2$

$U_2 = -19{,}600\text{ J} + 70{,}500\text{ J} = +50{,}900\text{ J}$

Thus $625{,}000\text{ J} - 17{,}000\text{ J} = \frac{1}{2}mv_2^2 + 50{,}900\text{ J}$

$\frac{1}{2}mv_2^2 = 557{,}100\text{ J}$

$$v_2 = \sqrt{\frac{2(557{,}100\text{ J})}{2000\text{ kg}}} = 23.6\text{ m/s}$$

EVALUATE: The elevator stops after descending 3.00 m. After descending 1.00 m it is still moving but has slowed down.

(b) IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the elevator. We know the forces and can solve for $\vec{a}$.

SET UP: The free-body diagram for the elevator is given in Figure 7.24.

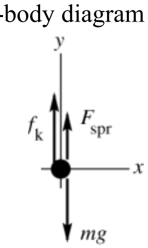


Figure 7.24

EXECUTE: $F_{\text{spr}} = kd$, where d is the distance the spring is compressed

$\sum F_y = ma_y$

$f_k + F_{\text{spr}} - mg = ma$

$f_k + kd - mg = ma$

$$a = \frac{f_k + kd - mg}{m} = \frac{17{,}000\text{ N} + (1.41\times 10^5\text{ N/m})(1.00\text{ m}) - (2000\text{ kg})(9.80\text{ m/s}^2)}{2000\text{ kg}} = 69.2\text{ m/s}^2$$

We calculate that a is positive, so the acceleration is upward.

EVALUATE: The velocity is downward and the acceleration is upward, so the elevator is slowing down at this point. Note that $a = 7.1g$; this is unacceptably high for an elevator.

7.25. **IDENTIFY:** Apply Eq.(7.13) and $F = ma$.

SET UP: $W_{\text{other}} = 0$. There is no change in U_{grav}. $K_1 = 0$, $U_2 = 0$.

EXECUTE: $\frac{1}{2}kx^2 = \frac{1}{2}mv_x^2$. The relations for m, v_x, k and x are $kx^2 = mv_x^2$ and $kx = 5mg$.

Dividing the first equation by the second gives $x = \dfrac{v_x^2}{5g}$, and substituting this into the second gives $k = 25\dfrac{mg^2}{v_x^2}$.

(a) $k = 25\dfrac{(1160\text{ kg})(9.80\text{ m/s}^2)^2}{(2.50\text{ m/s})^2} = 4.46\times 10^5\text{ N/m}$

(b) $x = \dfrac{(2.50\text{ m/s})^2}{5(9.80\text{ m/s}^2)} = 0.128\text{ m}$

EVALUATE: Our results for k and x do give the required values for a_x and v_x:

$$a_x = \frac{kx}{m} = \frac{(4.46\times 10^5\text{ N/m})(0.128\text{ m})}{1160\text{ kg}} = 49.2\text{ m/s}^2 = 5.0g \text{ and } v_x = x\sqrt{\frac{k}{m}} = 2.5\text{ m/s}.$$

7.26. **IDENTIFY:** $W_{\text{grav}} = mg\cos\phi$.

SET UP: When he moves upward, $\phi = 180°$ and when he moves downward, $\phi = 0°$. When he moves parallel to the ground, $\phi = 90°$.

EXECUTE: **(a)** $W_{\text{grav}} = (75\text{ kg})(9.80\text{ m/s}^2)(7.0\text{ m})\cos 180° = -5100\text{ J}$.

(b) $W_{\text{grav}} = (75\text{ kg})(9.80\text{ m/s}^2)(7.0\text{ m})\cos 0° = +5100\text{ J}$.

(c) $\phi = 90°$ in each case and $W_{\text{grav}} = 0$ in each case.

(d) The total work done on him by gravity during the round trip is $-5100\text{ J} + 5100\text{ J} = 0$.

(e) Gravity is a conservative force since the total work done for a round trip is zero.

EVALUATE: The gravity force is independent of the position and motion of the object. When the object moves upward gravity does negative work and when the object moves downward gravity does positive work.

7.27. **IDENTIFY:** Apply $W_{f_k} = f_k s\cos\phi$. $f_k = \mu_k n$.

SET UP: For a circular trip the distance traveled is $d = 2\pi r$. At each point in the motion the friction force and the displacement are in opposite directions and $\phi = 180°$. Therefore, $W_{f_k} = -f_k d = -f_k(2\pi r)$. $n = mg$ so $f_k = \mu_k mg$.

EXECUTE: **(a)** $W_{f_k} = -\mu_k mg 2\pi r = -(0.250)(10.0\text{ kg})(9.80\text{ m/s}^2)(2\pi)(2.00\text{ m}) = -308\text{ J}$.

(b) The distance along the path doubles so the work done doubles and becomes -616 J.

(c) The work done for a round trip displacement is not zero and friction is a nonconservative force.

EVALUATE: The direction of the friction force depends on the direction of motion of the object and that is why friction is a nonconservative force.

7.28. **IDENTIFY** and **SET UP:** The force is not constant so we must use Eq.(6.14) to calculate *W*. The properties of work done by a conservative force are described in Section 7.3.

$$W = \int_1^2 \vec{F}\cdot d\vec{l}, \quad \vec{F} = -\alpha x^2\hat{i}$$

EXECUTE: **(a)** $d\vec{l} = dy\hat{j}$ (x is constant; the displacement is in the $+y$-direction)

$\vec{F}\cdot d\vec{l} = 0$ (since $\hat{i}\cdot\hat{j} = 0$) and thus $W = 0$.

(b) $d\vec{l} = dx\hat{i}$

$\vec{F}\cdot d\vec{l} = (-\alpha x^2\hat{i})\cdot(dx\hat{i}) = -\alpha x^2\, dx$

$$W = \int_{x_1}^{x_2}(-\alpha x^2)dx = -\tfrac{1}{3}\alpha x^3\Big|_{x_1}^{x_2} = -\tfrac{1}{3}\alpha(x_2^3 - x_1^3) = -\frac{12\text{ N/m}^2}{3}((0.300\text{ m})^3 - (0.10\text{ m})^3) = -0.10\text{ J}$$

(c) $d\vec{l} = dx\hat{i}$ as in part (b), but now $x_1 = 0.30\text{ m}$ and $x_2 = 0.10\text{ m}$

$$W = -\tfrac{1}{3}\alpha(x_2^3 - x_1^3) = +0.10\text{ J}$$

(d) **EVALUATE:** The total work for the displacement along the *x*-axis from 0.10 m to 0.30 m and then back to 0.10 m is the sum of the results of parts (b) and (c), which is zero. The total work is zero when the starting and ending points are the same, so the force is conservative.

EXECUTE: $W_{x_1\to x_2} = -\tfrac{1}{3}\alpha(x_2^3 - x_1^3) = \tfrac{1}{3}\alpha x_1^3 - \tfrac{1}{3}\alpha x_2^3$

The definition of the potential energy function is $W_{x_1\to x_2} = U_1 - U_2$. Comparison of the two expressions for *W* gives $U = \tfrac{1}{3}\alpha x^3$. This does correspond to $U = 0$ when $x = 0$.

EVALUATE: In part (a) the work done is zero because the force and displacement are perpendicular. In part (b) the force is directed opposite to the displacement and the work done is negative. In part (c) the force and displacement are in the same direction and the work done is positive.

7.29. **IDENTIFY:** Since the force is constant, use $W = Fs\cos\phi$.

SET UP: For both displacements, the direction of the friction force is opposite to the displacement and $\phi = 180°$.

EXECUTE: **(a)** When the book moves to the left, the friction force is to the right, and the work is $-(1.2\text{ N})(3.0\text{ m}) = -3.6\text{ J}$.

(b) The friction force is now to the left, and the work is again -3.6 J.

(c) -7.2 J.

(d) The net work done by friction for the round trip is not zero, and friction is not a conservative force.

EVALUATE: The direction of the friction force depends on the motion of the object. For the gravity force, which is conservative, the force does not depend on the motion of the object.

7.30. **IDENTIFY** and **SET UP:** The friction force is constant during each displacement and Eq.(6.2) can be used to calculate work, but the direction of the friction force can be different for different displacements.

$f = \mu_k mg = (0.25)(1.5\text{ kg})(9.80\text{ m/s}^2) = 3.675\text{ N}$; direction of $\vec{f}$ is opposite to the motion.

EXECUTE: **(a)** The path of the book is sketched in Figure 7.30a.

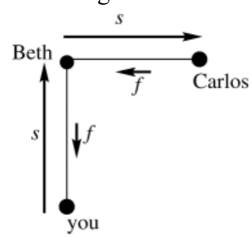


Figure 7.30a

For the motion from you to Beth the friction force is directed opposite to the displacement $\vec{s}$ and $W_1 = -fs = -(3.675\text{ N})(8.0\text{ m}) = -29.4\text{ J}$.

For the motion from Beth to Carlos the friction force is again directed opposite to the displacement and $W_2 = -29.4\text{ J}$.

$$W_{\text{tot}} = W_1 + W_2 = -29.4\text{ J} - 29.4\text{ J} = -59\text{ J}$$

(b) The path of the book is sketched in Figure 7.30b.

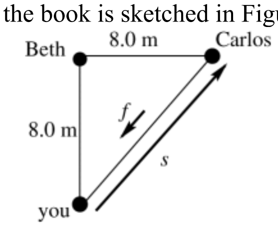


Figure 7.30b

$$s = \sqrt{2(8.0\text{ m})^2} = 11.3\text{ m}$$

$\vec{f}$ is opposite to $\vec{s}$, so $W = -fs = -(3.675\text{ N})(11.3\text{ m}) = -42\text{ J}$

(c)

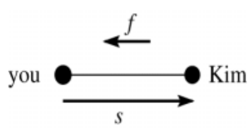


Figure 7.30c

For the motion from you to Kim

$W = -fs$

$W = -(3.675\text{ N})(8.0\text{ m}) = -29.4\text{ J}$

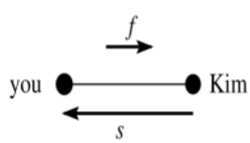


Figure 7.30d

For the motion from Kim to you

$W = -fs = -29.4\text{ J}$

The total work for the round trip is $-29.4\text{ J} - 29.4\text{ J} = -59\text{ J}$.

(d) EVALUATE: Parts (a) and (b) show that for two different paths between you and Carlos, the work done by friction is different. Part (c) shows that when the starting and ending points are the same, the total work is not zero. Both these results show that the friction force is nonconservative.

7.31. **IDENTIFY:** The work done by a spring on an object attached to its end when the object moves from x_i to x_f is $W = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$. This result holds for any x_i and x_f.

SET UP: Assume for simplicity that x_1, x_2 and x_3 are all positive, corresponding to the spring being stretched.

EXECUTE: **(a)** $\frac{1}{2}k(x_1^2 - x_2^2)$

(b) $-\frac{1}{2}k(x_1^2 - x_2^2)$. The total work is zero; the spring force is conservative.

(c) From x_1 to x_3, $W = -\frac{1}{2}k(x_3^2 - x_1^2)$. From x_3 to x_2, $W = -\frac{1}{2}k(x_2^2 - x_3^2)$. The net work is $-\frac{1}{2}k(x_2^2 - x_1^2)$. This is the same as the result of part (a).

EVALUATE: The results of part (c) illustrate that the work done by a conservative force is path independent.

7.32. **IDENTIFY and SET UP:** Use Eq.(7.17) to calculate the force from $U(x)$. Use coordinates where the origin is at one atom. The other atom then has coordinate x.

EXECUTE:

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left(-\frac{C_6}{x^6}\right) = +C_6\frac{d}{dx}\left(\frac{1}{x^6}\right) = -\frac{6C_6}{x^7}$$

The minus sign mean that F_x is directed in the $-x$-direction, toward the origin. The force has magnitude $6C_6/x^7$ and is attractive.

EVALUATE: U depends only on x so $\vec{F}$ is along the x-axis; it has no y or z components.

7.33. **IDENTIFY:** Apply Eq.(7.16).

SET UP: The sign of F_x indicates its direction.

EXECUTE: $F_x = -\dfrac{dU}{dx} = -4\alpha x^3 = -(4.8\ \text{J/m}^4)x^3$. $F_x(-0.800\ \text{m}) = -(4.8\ \text{J/m}^4)(-0.80\ \text{m})^3 = 2.46\ \text{N}$. The force is in the $+x$-direction.

EVALUATE: $F_x > 0$ when $x < 0$ and $F_x < 0$ when $x > 0$, so the force is always directed towards the origin.

7.34. **IDENTIFY:** Apply $F(x) = -\dfrac{dU(x)}{dx}$.

SET UP: $\dfrac{d(1/x)}{dx} = -\dfrac{1}{x^2}$

EXECUTE: $F_x(x) = -\dfrac{d(-Gm_1m_2/x)}{dx} = Gm_1m_2\left[\dfrac{d(1/x)}{dx}\right] = -\dfrac{Gm_1m_2}{x^2}$. The force on m_2 is in the $-x$-direction. This is toward m_1, so the force is attractive.

EVALUATE: By Newton's 3rd law the force on m_1 due to m_2 is Gm_1m_2/x^2, in the $+x$-direction (toward m_2). The gravitational potential energy belongs to the system of the two masses.

7.35. **IDENTIFY:** Apply $F_x = -\dfrac{\partial U}{\partial x}$ and $F_y = -\dfrac{\partial U}{\partial y}$.

SET UP: $r = (x^2 + y^2)^{1/2}$. $\dfrac{\partial(1/r)}{\partial x} = -\dfrac{x}{(x^2+y^2)^{3/2}}$ and $\dfrac{\partial(1/r)}{\partial y} = -\dfrac{y}{(x^2+y^2)^{3/2}}$.

EXECUTE: **(a)** $U(r) = -\dfrac{Gm_1m_2}{r}$. $F_x = -\dfrac{\partial U}{\partial x} = +Gm_1m_2\left[\dfrac{\partial(1/r)}{\partial x}\right] = -\dfrac{Gm_1m_2x}{(x^2+y^2)^{3/2}}$ and

$$F_y = -\frac{\partial U}{\partial y} = +Gm_1m_2\left[\frac{\partial(1/r)}{\partial y}\right] = -\frac{Gm_1m_2y}{(x^2+y^2)^{3/2}}.$$

(b) $(x^2+y^2)^{3/2} = r^3$ so $F_x = -\dfrac{Gm_1m_2x}{r^3}$ and $F_y = -\dfrac{Gm_1m_2y}{r^3}$. $F = \sqrt{F_x^2 + F_y^2} = \dfrac{Gm_1m_2}{r^3}\sqrt{x^2+y^2} = \dfrac{Gm_1m_2}{r^2}$.

(c) F_x and F_y are negative. $F_x = \alpha x$ and $F_y = \alpha y$, where α is a constant, so $\vec{F}$ and the vector $\vec{r}$ from m_1 to m_2 are in the same direction. Therefore, $\vec{F}$ is directed toward m_1 at the origin and $\vec{F}$ is attractive.

EVALUATE: If θ is the angle between the vector $\vec{r}$ that points from m_1 to m_2, then $\dfrac{x}{r} = \cos\theta$ and $\dfrac{y}{r} = \sin\theta$. This gives $F_x = -F\cos\theta$ and $F_y = -F\sin\theta$, our more usual way of writing the components of a vector.

7.36. **IDENTIFY:** Apply Eq.(7.18).

SET UP: $\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right) = -\dfrac{2}{x^3}$ and $\dfrac{d}{dy}\left(\dfrac{1}{y^2}\right) = -\dfrac{2}{y^3}$.

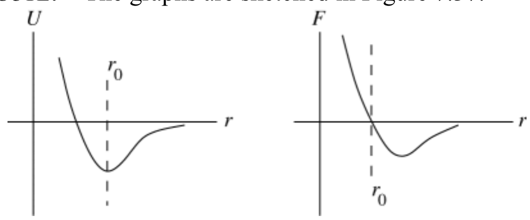
EXECUTE: $\vec{F} = -\dfrac{\partial U}{\partial x}\hat{i} - \dfrac{\partial U}{\partial y}\hat{j}$ since U has no z-dependence. $\dfrac{\partial U}{\partial x} = \dfrac{-2\alpha}{x^3}$ and $\dfrac{\partial U}{\partial y} = \dfrac{-2\alpha}{y^3}$, so

$$\vec{F} = -\alpha\left(\frac{-2}{x^3}\hat{i} + \frac{-2}{y^3}\hat{j}\right) = 2\alpha\left(\frac{\hat{i}}{x^3} + \frac{\hat{j}}{y^3}\right).$$

EVALUATE: F_x and x have the same sign and F_y and y have the same sign. When $x > 0$, F_x is in the $+x$-direction, and so forth.

7.37. **IDENTIFY** and **SET UP:** Use Eq.(7.17) to calculate the force from U. At equilibrium $F = 0$.

(a) EXECUTE: The graphs are sketched in Figure 7.37.



$$U = \frac{a}{r^{12}} - \frac{b}{r^6}$$
$$F = -\frac{dU}{dr} = +\frac{12a}{r^{13}} - \frac{6b}{r^7}$$

Figure 7.37

(b) At equilibrium $F = 0$, so $\frac{dU}{dr} = 0$

$$F = 0 \text{ implies } \frac{+12a}{r^{13}} - \frac{6b}{r^7} = 0$$

$6br^6 = 12a$; solution is the equilibrium distance $r_0 = (2a/b)^{1/6}$

U is a minimum at this r; the equilibrium is stable.

(c) At $r = (2a/b)^{1/6}$, $U = a/r^{12} - b/r^6 = a(b/2a)^2 - b(b/2a) = -b^2/4a$.

At $r \to \infty$, $U = 0$. The energy that must be added is $-\Delta U = b^2/4a$.

(d) $r_0 = (2a/b)^{1/6} = 1.13 \times 10^{-10}$ m gives that

$2a/b = 2.082 \times 10^{-60}$ m^6 and $b/4a = 2.402 \times 10^{59}$ m^{-6}

$b^2/4a = b(b/4a) = 1.54 \times 10^{-18}$ J

$b(2.402 \times 10^{59} \text{ m}^{-6}) = 1.54 \times 10^{-18}$ J and $b = 6.41 \times 10^{-78}$ J$\cdot$m^6.

Then $2a/b = 2.082 \times 10^{-60}$ m^6 gives $a = (b/2)(2.082 \times 10^{-60} \text{ m}^6) =$

$\frac{1}{2}(6.41 \times 10^{-78} \text{ J}\cdot\text{m}^6)(2.082 \times 10^{-60} \text{ m}^6) = 6.67 \times 10^{-138}$ J$\cdot$m^{12}

EVALUATE: As the graphs in part (a) show, $F(r)$ is the slope of $U(r)$ at each r. $U(r)$ has a minimum where $F = 0$.

7.38. **IDENTIFY:** Apply Eq.(7.16).

SET UP: $\frac{dU}{dx}$ is the slope of the U versus x graph.

EXECUTE: **(a)** Considering only forces in the x-direction, $F_x = -\frac{dU}{dx}$ and so the force is zero when the slope of the U *vs* x graph is zero, at points b and d.

(b) Point b is at a potential minimum; to move it away from b would require an input of energy, so this point is stable.

(c) Moving away from point d involves a decrease of potential energy, hence an increase in kinetic energy, and the marble tends to move further away, and so d is an unstable point.

EVALUATE: At point b, F_x is negative when the marble is displaced slightly to the right and F_x is positive when the marble is displaced slightly to the left, the force is a restoring force, and the equilibrium is stable. At point d, a small displacement in either direction produces a force directed away from d and the equilibrium is unstable.

7.39. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the bag and to the box. Apply Eq.(7.7) to the motion of the system of the box and bucket after the bag is removed.

SET UP: Let $y = 0$ at the final height of the bucket, so $y_1 = 2.00$ m and $y_2 = 0$. $K_1 = 0$. The box and the bucket move with the same speed v, so $K_2 = \frac{1}{2}(m_{\text{box}} + m_{\text{bucket}})v^2$. $W_{\text{other}} = -f_k d$, with $d = 2.00$ m and $f_k = \mu_k m_{\text{box}} g$. Before the bag is removed, the maximum possible friction force the roof can exert on the box is $(0.700)(80.0 \text{ kg} + 50.0 \text{ kg})(9.80 \text{ m/s}^2) = 892$ N. This is larger than the weight of the bucket (637 N), so before the bag is removed the system is at rest.

EXECUTE: **(a)** The friction force on the bag of gravel is zero, since there is no other horizontal force on the bag for friction to oppose. The static friction force on the box equals the weight of the bucket, 637 N.

(b) Eq.(7.7) gives $m_{\text{bucket}}gy_1 - f_k d = \frac{1}{2}m_{\text{tot}}v^2$, with $m_{\text{tot}} = 145.0\text{ kg}$. $v = \sqrt{\frac{2}{m_{\text{tot}}}(m_{\text{bucket}}gy_1 - \mu_k m_{\text{box}}gd)}$.

$$v = \sqrt{\frac{2}{145.0\text{ kg}}\left[(65.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) - (0.400)(80.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})\right]}.$$

$v = 2.99\text{ m/s}$.

EVALUATE: If we apply $\sum\vec{F} = m\vec{a}$ to the box and to the bucket we can calculate their common acceleration a. Then a constant acceleration equation applied to either object gives $v = 2.99\text{ m/s}$, in agreement with our result obtained using energy methods.

7.40. **IDENTIFY:** For the system of two blocks, only gravity does work. Apply Eq.(7.5).

SET UP: Call the blocks A and B, where A is the more massive one. $v_{A1} = v_{B1} = 0$. Let $y = 0$ for each block to be at the initial height of that block, so $y_{A1} = y_{B1} = 0$. $y_{A2} = -1.20\text{ m}$ and $y_{B2} = +1.20\text{ m}$. $v_{A2} = v_{B2} = v_2 = 3.00\text{ m/s}$.

EXECUTE: Eq.(7.5) gives $0 = \frac{1}{2}(m_A + m_B)v_2^2 + g(1.20\text{ m})(m_B - m_A)$. $m_A + m_B = 15.0\text{ kg}$.
$\frac{1}{2}(15.0\text{ kg})(3.00\text{ m/s})^2 + (9.80\text{ m/s}^2)(1.20\text{ m})(15.0\text{ kg} - 2m_A)$. Solving for m_A gives $m_A = 10.4\text{ kg}$. And then $m_B = 4.6\text{ kg}$.

EVALUATE: The final kinetic energy of the two blocks is 68 J. The potential energy of block A decreases by 122 J. The potential energy of block B increases by 54 J. The total decrease in potential energy is $122\text{ J} - 54\text{ J} = 68\text{ J}$, and this equals the increase in kinetic energy of the system.

7.41. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

SET UP: $U_1 = U_2 = K_2 = 0$. $W_{\text{other}} = W_f = -\mu_k mgs,$ with $s = 280\text{ ft} = 85.3\text{ m}$

EXECUTE: **(a)** The work-energy expression gives $\frac{1}{2}mv_1^2 - \mu_k mgs = 0$.

$v_1 = \sqrt{2\mu_k gs} = 22.4\text{ m/s} = 50\text{ mph};$ the driver was speeding.

(b) 15 mph over speed limit so \$150 ticket.

EVALUATE: The negative work done by friction removes the kinetic energy of the object.

7.42. **IDENTIFY:** Apply Eq.(7.14).

SET UP: Only the spring force and gravity do work, so $W_{\text{other}} = 0$. Let $y = 0$ at the horizontal surface.

EXECUTE: **(a)** Equating the potential energy stored in the spring to the block's kinetic energy, $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, or

$$v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{400\text{ N/m}}{2.00\text{ kg}}}(0.220\text{ m}) = 3.11\text{ m/s}.$$

(b) Using energy methods directly, the initial potential energy of the spring equals the final gravitational potential energy, $\frac{1}{2}kx^2 = mgL\sin\theta$, or $L = \frac{\frac{1}{2}kx^2}{mg\sin\theta} = \frac{\frac{1}{2}(400\text{ N/m})(0.220\text{ m})^2}{(2.00\text{ kg})(9.80\text{ m/s}^2)\sin 37.0°} = 0.821\text{ m}.$

EVALUATE: The total energy of the system is constant. Initially it is all elastic potential energy stored in the spring, then it is all kinetic energy and finally it is all gravitational potential energy.

7.43. **IDENTIFY:** Use the work-energy theorem, Eq(7.7). The target variable μ_k will be a factor in the work done by friction.

SET UP: Let point 1 be where the block is released and let point 2 be where the block stops, as shown in Figure 7.43.

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

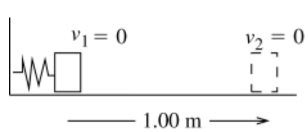


Figure 7.43

Work is done on the block by the spring and by friction, so $W_{\text{other}} = W_f$ and $U = U_{\text{el}}$.

EXECUTE: $K_1 = K_2 = 0$

$U_1 = U_{1,\text{el}} = \frac{1}{2}kx_1^2 = \frac{1}{2}(100\text{ N/m})(0.200\text{ m})^2 = 2.00\text{ J}$

$U_2 = U_{2,\text{el}} = 0,$ since after the block leaves the spring has given up all its stored energy

$W_{\text{other}} = W_f = (f_k\cos\phi)s = \mu_k mg(\cos\phi)s = -\mu_k mgs,$ since $\phi = 180°$ (The friction force is directed opposite to the displacement and does negative work.)

Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$U_{1,\text{el}} + W_f = 0$

$\mu_k mgs = U_{1,\text{el}}$

$$\mu_k = \frac{U_{1,\text{el}}}{mgs} = \frac{200\ \text{J}}{(0.50\ \text{kg})(9.80\ \text{m/s}^2)(1.00\ \text{m})} = 0.41.$$

EVALUATE: $U_{1,\text{el}} + W_f = 0$ says that the potential energy originally stored in the spring is taken out of the system by the negative work done by friction.

7.44. **IDENTIFY:** Apply Eq.(7.14). Calculate f_k from the fact that the crate slides a distance $x = 5.60$ m before coming to rest. Then apply Eq.(7.14) again, with $x = 2.00$ m.

SET UP: $U_1 = U_{\text{el}} = 360$ J. $U_2 = 0$. $K_1 = 0$. $W_{\text{other}} = -f_k x$.

EXECUTE: Work done by friction against the crate brings it to a halt: $U_1 = -W_{\text{other}}$.

$f_k x =$ potential energy of compressed spring, and $f_k = \dfrac{360\ \text{J}}{5.60\ \text{m}} = 64.29$ N.

The friction force working over a 2.00-m distance does work equal to $-f_k x = -(64.29\ \text{N})(2.00\ \text{m}) = -128.6$ J. The kinetic energy of the crate at this point is thus $360\ \text{J} - 128.6\ \text{J} = 231.4$ J, and its speed is found from

$mv^2/2 = 231.4$ J, so $v = \sqrt{\dfrac{2(231.4\ \text{J})}{50.0\ \text{kg}}} = 3.04$ m/s.

EVALUATE: The energy of the compressed spring goes partly into kinetic energy of the crate and is partly removed by the negative work done by friction. After the crate leaves the spring the crate slows down as friction does negative work on it.

7.45. **IDENTIFY:** At its highest point between bounces all the mechanical energy of the ball is in the form of gravitational potential energy.

SET UP: $E = U = mgh$, where h is the height at the highest point of the motion.

EXECUTE: **(a)** $mgh = (0.650\ \text{kg})(9.80\ \text{m/s}^2)(2.50\ \text{m}) = 15.9$ J

(b) The second height is $0.75(2.50\ \text{m}) = 1.875$ m, so the second $mgh = 11.9$ J; it loses $15.9\ \text{J} - 11.9\ \text{J} = 4.0$ J on first bounce. This energy is converted to thermal energy.

(c) The third height is $0.75(1.875\ \text{m}) = 1.40$ m, , so third $mgh = 8.9$ J; it loses $11.9\ \text{J} - 8.9\ \text{J} = 3.0$ J on second bounce.

EVALUATE: In each bounce the ball loses 25% of its mechanical energy.

7.46. **IDENTIFY:** Apply Eq.(7.14) to relate h and v_B. Apply $\sum \vec{F} = m\vec{a}$ at point B to find the minimum speed required at B for the car not to fall off the track.

SET UP: At B, $a = v_B^2/R$, downward. The minimum speed is when $n \to 0$ and $mg = mv_B^2/R$. The minimum speed required is $v_B = \sqrt{gR}$. $K_1 = 0$ and $W_{\text{other}} = 0$.

EXECUTE: **(a)** Eq.(7.14) applied to points A and B gives $U_A - U_B = \frac{1}{2}mv_B^2$. The speed at the top must be at least $\sqrt{gR}$. Thus, $mg(h-2R) > \dfrac{1}{2}mgR$, or $h > \dfrac{5}{2}R$.

(b) Apply Eq.(7.14) to points A and C. $U_A - U_C = (2.50)Rmg = K_C$, so

$$v_C = \sqrt{(5.00)gR} = \sqrt{(5.00)(9.80\ \text{m/s}^2)(20.0\ \text{m})} = 31.3\ \text{m/s}.$$

The radial acceleration is $a_{\text{rad}} = \dfrac{v_C^2}{R} = 49.0\ \text{m/s}^2$. The tangential direction is down, the normal force at point C is horizontal, there is no friction, so the only downward force is gravity, and $a_{\tan} = g = 9.80\ \text{m/s}^2$.

EVALUATE: If $h > \frac{5}{2}R$, then the downward acceleration at B due to the circular motion is greater than g and the track must exert a downward normal force n. n increases as h increases and hence v_B increases.

7.47. **(a) IDENTIFY:** Use work-energy relation to find the kinetic energy of the wood as it enters the rough bottom.

SET UP: Let point 1 be where the piece of wood is released and point 2 be just before it enters the rough bottom. Let $y = 0$ be at point 2.

EXECUTE: $U_1 = K_2$ gives $K_2 = mgy_1 = 78.4$ J.

IDENTIFY: Now apply work-energy relation to the motion along the rough bottom.

SET UP: Let point 1 be where it enters the rough bottom and point 2 be where it stops.

$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$$

EXECUTE: $W_{\text{other}} = W_f = -\mu_k mgs$, $K_2 = U_1 = U_2 = 0$; $K_1 = 78.4\text{ J}$

$78.4\text{ J} - \mu_k mgs = 0$; solving for s gives $s = 20.0\text{ m}$.

The wood stops after traveling 20.0 m along the rough bottom.
(b) Friction does -78.4 J of work.
EVALUATE: The piece of wood stops before it makes one trip across the rough bottom. The final mechanical energy is zero. The negative friction work takes away all the mechanical energy initially in the system.

7.48. **IDENTIFY:** Apply Eq.(7.14) to the rock. $W_{\text{other}} = W_{f_k}$.

SET UP: Let $y = 0$ at the foot of the hill, so $U_1 = 0$ and $U_2 = mgh$, where h is the vertical height of the rock above the foot of the hill when it stops.

EXECUTE: **(a)** At the maximum height, $K_2 = 0$. Eq.(7.14) gives $K_{\text{Bottom}} + W_{f_k} = U_{\text{Top}}$.

$\frac{1}{2}mv_0^2 - \mu_k mg\cos\theta\, d = mgh$. $d = h/\sin\theta$, so $\frac{1}{2}v_0^2 - \mu_k g\cos\theta\frac{h}{\sin\theta} = gh$.

$\frac{1}{2}(15\text{ m/s})^2 - (0.20)(9.8\text{ m/s}^2)\frac{\cos 40°}{\sin 40°}h = (9.8\text{ m/s}^2)h$ and $h = 9.3\text{ m}$.

(b) Compare maximum static friction force to the weight component down the plane.

$f_s = \mu_s mg\cos\theta = (0.75)(28\text{ kg})(9.8\text{ m/s}^2)\cos 40° = 158\text{ N}$. $mg\sin\theta = (28\text{ kg})(9.8\text{ m/s}^2)(\sin 40°) = 176\text{ N} > f_s$, so the rock will slide down.

(c) Use same procedure as in part (a), with $h = 9.3\text{ m}$ and v_B being the speed at the bottom of the hill.

$U_{\text{Top}} + W_{f_k} = K_B$. $mgh - \mu_k mg\cos\theta\frac{h}{\sin\theta} = \frac{1}{2}mv_B^2$ and

$v_B = \sqrt{2gh - 2\mu_k gh\cos\theta/\sin\theta} = 11.8\text{ m/s}$.

EVALUATE: For the round trip up the hill and back down, there is negative work done by friction and the speed of the rock when it returns to the bottom of the hill is less than the speed it had when it started up the hill.

7.49. **IDENTIFY:** Apply Eq.(7.7) to the motion of the stone.

SET UP: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

Let point 1 be point *A* and point 2 be point *B*. Take $y = 0$ at point *B*.

EXECUTE: $mgy_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2$, with $h = 20.0\text{ m}$ and $v_1 = 10.0\text{ m/s}$

$v_2 = \sqrt{v_1^2 + 2gh} = 22.2\text{ m/s}$

EVALUATE: The loss of gravitational potential energy equals the gain of kinetic energy.
(b) IDENTIFY: Apply Eq.(7.8) to the motion of the stone from point *B* to where it comes to rest against the spring.
SET UP: Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, with point 1 at *B* and point 2 where the spring has its maximum compression *x*.

EXECUTE: $U_1 = U_2 = K_2 = 0$; $K_1 = \frac{1}{2}mv_1^2$ with $v_1 = 22.2\text{ m/s}$

$W_{\text{other}} = W_f + W_{\text{el}} = -\mu_k mgs - \frac{1}{2}kx^2$, with $s = 100\text{ m} + x$

The work-energy relation gives $K_1 + W_{\text{other}} = 0$.

$\frac{1}{2}mv_1^2 - \mu_k mgs - \frac{1}{2}kx^2 = 0$

Putting in the numerical values gives $x^2 + 29.4x - 750 = 0$. The positive root to this equation is $x = 16.4\text{ m}$.
EVALUATE: Part of the initial mechanical (kinetic) energy is removed by friction work and the rest goes into the potential energy stored in the spring.
(c) IDENTIFY and **SET UP:** Consider the forces.
EXECUTE: When the spring is compressed $x = 16.4\text{ m}$ the force it exerts on the stone is $F_{\text{el}} = kx = 32.8\text{ N}$. The maximum possible static friction force is

$\max f_s = \mu_s mg = (0.80)(15.0\text{ kg})(9.80\text{ m/s}^2) = 118\text{ N}$.

EVALUATE: The spring force is less than the maximum possible static friction force so the stone remains at rest.

7.50. **IDENTIFY:** Once the block leaves the top of the hill it moves in projectile motion. Use Eq.(7.14) to relate the speed v_B at the bottom of the hill to the speed v_{Top} at the top and the 70 m height of the hill.

SET UP: For the projectile motion, take $+y$ to be downward. $a_x = 0$, $a_y = g$. $v_{0x} = v_{Top}$, $v_{0y} = 0$. For the motion up the hill only gravity does work. Take $y = 0$ at the base of the hill.

EXECUTE: First get speed at the top of the hill for the block to clear the pit. $y = \frac{1}{2}gt^2$. $20\text{ m} = \frac{1}{2}(9.8\text{ m/s}^2)t^2$.

$t = 2.0\text{ s}$. Then $v_{Top}t = 40\text{ m}$ gives $v_{Top} = \dfrac{40\text{ m}}{2.0\text{ s}} = 20\text{ m/s}$.

Energy conservation applied to the motion up the hill: $K_{Bottom} = U_{Top} + K_{Top}$ gives

$\frac{1}{2}mv_B^2 = mgh + \frac{1}{2}mv_{Top}^2$. $v_B = \sqrt{v_{Top}^2 + 2gh} = \sqrt{(20\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(70\text{ m})} = 42\text{ m/s}$.

EVALUATE: The result does not depend on the mass of the block.

7.51. **IDENTIFY:** Apply $K_1 + U_1 + W_{other} = K_2 + U_2$ to the motion of the person.

SET UP: Point 1 is where he steps off the platform and point 2 is where he is stopped by the cord. Let $y = 0$ at point 2. $y_1 = 41.0\text{ m}$. $W_{other} = -\frac{1}{2}kx^2$, where $x = 11.0\text{ m}$ is the amount the cord is stretched at point 2. The cord does negative work.

EXECUTE: $K_1 = K_2 = U_2 = 0$, so $mgy_1 - \frac{1}{2}kx^2 = 0$ and $k = 631\text{ N/m}$.

Now apply $F = kx$ to the test pulls:

$F = kx$ so $x = F/k = 0.602\text{ m}$.

EVALUATE: All his initial gravitational potential energy is taken away by the negative work done by the force exerted by the cord, and this amount of energy is stored as elastic potential energy in the stretched cord.

7.52. **IDENTIFY:** Apply Eq.(7.14) to the motion of the skier from the gate to the bottom of the ramp.

SET UP: $W_{other} = -4000\text{ J}$. Let $y = 0$ at the bottom of the ramp.

EXECUTE: For the skier to be moving at no more than 30.0 m/s; his kinetic energy at the bottom of the ramp can be no bigger than $\dfrac{mv^2}{2} = \dfrac{(85.0\text{ kg})(30.0\text{ m/s})^2}{2} = 38{,}250\text{ J}$. Friction does -4000 J of work on him during his run, which means his combined U and K at the top of the ramp must be no more than $38{,}250\text{ J} + 4000\text{ J} = 42{,}250\text{ J}$. His K at the top is $\dfrac{mv^2}{2} = \dfrac{(85.0\text{ kg})(2.0\text{ m/s})^2}{2} = 170\text{ J}$. His U at the top should thus be no more than $42{,}250\text{ J} - 170\text{ J} = 42{,}080\text{ J}$, which gives a height above the bottom of the ramp of $h = \dfrac{42{,}080\text{ J}}{mg} = \dfrac{42{,}080\text{ J}}{(85.0\text{ kg})(9.80\text{ m/s}^2)} = 50.5\text{ m}$.

EVALUATE: In the absence of air resistance, for this h his speed at the bottom of the ramp would be 31.5 m/s. The work done by air resistance is small compared to the kinetic and potential energies that enter into the calculation.

7.53. **IDENTIFY:** Use the work-energy theorem, Eq.(7.7). Solve for K_2 and then for v_2.

SET UP: Let point 1 be at his initial position against the compressed spring and let point 2 be at the end of the barrel, as shown in Figure 7.53. Use $F = kx$ to find the amount the spring is initially compressed by the 4400 N force.

$K_1 + U_1 + W_{other} = K_2 + U_2$

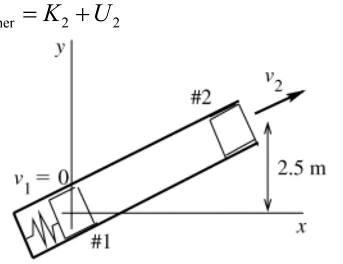


Figure 7.53

Take $y = 0$ at his initial position.

EXECUTE: $K_1 = 0$, $K_2 = \frac{1}{2}mv_2^2$

$W_{other} = W_{fric} = -fs$

$W_{other} = -(40\text{ N})(4.0\text{ m}) = -160\text{ J}$

$U_{1,grav} = 0$, $U_{1,el} = \frac{1}{2}kd^2$, where d is the distance the spring is initially compressed.

$F = kd$ so $d = \dfrac{F}{k} = \dfrac{4400\text{ N}}{1100\text{ N/m}} = 4.00\text{ m}$

and $U_{1,el} = \frac{1}{2}(1100\text{ N/m})(4.00\text{ m})^2 = 8800\text{ J}$

$U_{2,grav} = mgy_2 = (60\text{ kg})(9.80\text{ m/s}^2)(2.5\text{ m}) = 1470\text{ J}$, $U_{2,el} = 0$

Then $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$8800\text{ J} - 160\text{ J} = \frac{1}{2}mv_2^2 + 1470\text{ J}$

$\frac{1}{2}mv_2^2 = 7170\text{ J}$ and $v_2 = \sqrt{\dfrac{2(7170\text{ J})}{60\text{ kg}}} = 15.5\text{ m/s}$

EVALUATE: Some of the potential energy stored in the compressed spring is taken away by the work done by friction. The rest goes partly into gravitational potential energy and partly into kinetic energy.

7.54. **IDENTIFY:** To be at equilibrium at the bottom, with the spring compressed a distance x_0, the spring force must balance the component of the weight down the ramp plus the largest value of the static friction, or $kx_0 = w\sin\theta + f$. Apply Eq.(7.14) to the motion down the ramp.

SET UP: $K_2 = 0$, $K_1 = \frac{1}{2}mv^2$, where v is the speed at the top of the ramp. Let $U_2 = 0$, so $U_1 = wL\sin\theta$, where L is the total length traveled down the ramp.

EXECUTE: Eq.(7.14) gives $\dfrac{1}{2}kx_0^2 = (w\sin\theta - f)L + \dfrac{1}{2}mv^2$. With the given parameters, $\frac{1}{2}kx_0^2 = 248\text{ J}$ and $kx_0 = 1.10\times 10^3\text{ N}$. Solving for k gives $k = 2440\text{ N/m}$.

EVALUATE: $x_0 = 0.451\text{ m}$. $w\sin\theta = 551\text{ N}$. The decrease in gravitational potential energy is only slightly larger than the amount of mechanical energy removed by the negative work done by friction. $\frac{1}{2}mv^2 = 243\text{ J}$. The energy stored in the spring is only slightly larger than the initial kinetic energy of the crate at the top of the ramp.

7.55. **IDENTIFY:** Apply Eq.(7.7) to the system consisting of the two buckets. If we ignore the inertia of the pulley we ignore the kinetic energy it has.

SET UP: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Points 1 and 2 in the motion are sketched in Figure 7.55.

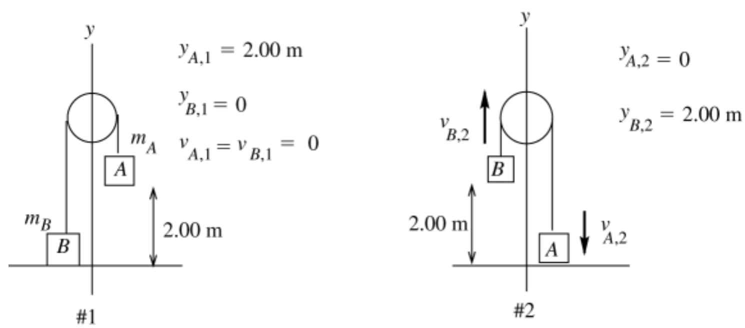


Figure 7.55

The tension force does positive work on the 4.0 kg bucket and an equal amount of negative work on the 12.0 kg bucket, so the net work done by the tension is zero.
Work is done on the system only by gravity, so $W_{\text{other}} = 0$ and $U = U_{\text{grav}}$

EXECUTE: $K_1 = 0$

$K_2 = \frac{1}{2}m_A v_{A,2}^2 + \frac{1}{2}m_B v_{B,2}^2$ But since the two buckets are connected by a rope they move together and have the same speed: $v_{A,2} = v_{B,2} = v_2$.

Thus $K_2 = \frac{1}{2}(m_A + m_B)v_2^2 = (8.00\text{ kg})v_2^2$.

$U_1 = m_A g y_{A,1} = (12.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 235.2\text{ J}$.

$U_2 = m_B g y_{B,2} = (4.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 78.4\text{ J}$.

Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$U_1 = K_2 + U_2$

$235.2\text{ J} = (8.00\text{ kg})v_2^2 + 78.4\text{ J}$

$v_2 = \sqrt{\dfrac{235.2\text{ J} - 78.4\text{ J}}{8.00\text{ kg}}} = 4.4\text{ m/s}$

EVALUATE: The gravitational potential energy decreases and the kinetic energy increases by the same amount. We could apply Eq.(7.7) to one bucket, but then we would have to include in W_{other} the work done on the bucket by the tension T.

7.56. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the rocket from the starting point to the base of the ramp. W_{other} is the work done by the thrust and by friction.

SET UP: Let point 1 be at the starting point and let point 2 be at the base of the ramp. $v_1 = 0$, $v_2 = 50.0$ m/s. Let $y = 0$ at the base and take $+y$ upward. Then $y_2 = 0$ and $y_1 = d\sin 53°$, where d is the distance along the ramp from the base to the starting point. Friction does negative work.

EXECUTE: $K_1 = 0$, $U_2 = 0$. $U_1 + W_{\text{other}} = K_2$. $W_{\text{other}} = (2000\ \text{N})d - (500\ \text{N})d = (1500\ \text{N})d$.

$mgd\sin 53° + (1500\ \text{N})d = \frac{1}{2}mv_2^2$.

$$d = \frac{mv_2^2}{2[mg\sin 53° + 1500\ \text{N}]} = \frac{(1500\ \text{kg})(50.0\ \text{m/s})^2}{2[(1500\ \text{kg})(9.80\ \text{m/s}^2)\sin 53° + 1500\ \text{N}]} = 142\ \text{m}.$$

EVALUATE: The initial height is $y_1 = (142\ \text{m})\sin 53° = 113$ m. An object free-falling from this distance attains a speed $v = \sqrt{2gy_1} = 47.1$ m/s. The rocket attains a greater speed than this because the forward thrust is greater than the friction force.

7.57. **IDENTIFY:** The force exerted by a spring is $F_x = -kx$. The acceleration of the object is given by $F_x = ma_x$. Apply Eq.(7.14) to relate position and speed.

SET UP: Let $+x$ be when the spring is stretched.

EXECUTE: **(a)** $U = \frac{1}{2}kx^2$. Let point 1 be when the spring is initially compressed a distance x_0, so $x_1 = -x_0$. $K_1 = 0$. $W_{\text{other}} = 0$. $\frac{1}{2}kx_0^2 = U_2 + K_2$. The speed is maximum when $x = 0$, so $U_2 = 0$. Then $\frac{1}{2}kx_0^2 = \frac{1}{2}mv_2^2$ and $v_2 = x_0\sqrt{k/m}$ is this maximum speed.

(b) $F_x = -kx$ and $F_x = ma_x$ give $a_x = -\frac{k}{m}x$. a is maximum when $|x|$ is maximum, so $a = \frac{k}{m}x_0$.

(c) The speed is maximum when $x = 0$, when the spring has returned to its natural length, and the acceleration is maximum when $x = -x_0$, at the initial compression of the spring.

(d) When the spring has maximum extension, $v_2 = 0$. $\frac{1}{2}kx_0^2 = \frac{1}{2}kx^2$ and $x = x_0$. The magnitude of the maximum extension equals the magnitude of the maximum compression.

(e) The machine part oscillates between $x = -x_0$ and $x = +x_0$ and never stops permanently.

EVALUATE: In any real system there are mechanical energy losses, for example due to negative work done by friction, and the object eventually comes to rest.

7.58. **IDENTIFY:** Conservation of energy says the decrease in potential energy equals the gain in kinetic energy.

SET UP: Since the two animals are equidistant from the axis, they each have the same speed v.

EXECUTE: One mass rises while the other falls, so the net loss of potential energy is $(0.500\ \text{kg} - 0.200\ \text{kg})(9.80\ \text{m/s}^2)(0.400\ \text{m}) = 1.176$ J. This is the sum of the kinetic energies of the animals and is equal to $\frac{1}{2}m_{\text{tot}}v^2$, and $v = \sqrt{\frac{2(1.176\ \text{J})}{(0.700\ \text{kg})}} = 1.83$ m/s.

EVALUATE: The mouse gains both gravitational potential energy and kinetic energy. The rat's gain in kinetic energy is less than its decrease of potential energy, and the energy difference is transferred to the mouse.

7.59. **(a) IDENTIFY** and **SET UP:** Apply Eq.(7.7) to the motion of the potato.

Let point 1 be where the potato is released and point 2 be at the lowest point in its motion, as shown in Figure 7.59a.

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

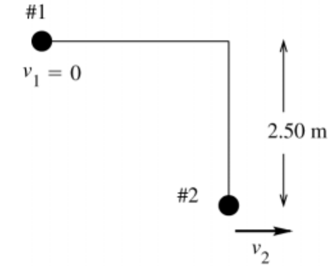


Figure 7.59a

$y_1 = 2.50$ m

$y_2 = 0$

The tension in the string is at all points in the motion perpendicular to the displacement, so $W_T = 0$

The only force that does work on the potato is gravity, so $W_{\text{other}} = 0$.

EXECUTE: $K_1 = 0,\ K_2 = \frac{1}{2}mv_2^2,\ U_1 = mgy_1,\ U_2 = 0$

Thus $U_1 = K_2$.

$mgy_1 = \frac{1}{2}mv_2^2$

$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\text{ m/s}^2)(2.50\text{ m})} = 7.00\text{ m/s}$

EVALUATE: v_2 is the same as if the potato fell through 2.50 m.

(b) IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the potato. The potato moves in an arc of a circle so its acceleration is $\vec{a}_{\text{rad}}$, where $a_{\text{rad}} = v^2/R$ and is directed toward the center of the circle. Solve for one of the forces, the tension T in the string.

SET UP: The free-body diagram for the potato as it swings through its lowest point is given in Figure 7.59b.

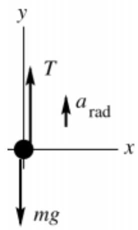


Figure 7.59b

The acceleration $\vec{a}_{\text{rad}}$ is directed in toward the center of the circular path, so at this point it is upward.

EXECUTE: $\sum F_y = ma_y$

$T - mg = ma_{\text{rad}}$

$T = m(g + a_{\text{rad}}) = m\left(g + \frac{v_2^2}{R}\right)$, where the radius R for the circular motion is the length L of the string.

It is instructive to use the algebraic expression for v_2 from part (a) rather than just putting in the numerical value:

$v_2 = \sqrt{2gy_1} = \sqrt{2gL}$, so $v_2^2 = 2gL$

Then $T = m\left(g + \frac{v_2^2}{L}\right) = m\left(g + \frac{2gL}{L}\right) = 3mg$; the tension at this point is three times the weight of the potato.

$$T = 3mg = 3(0.100\text{ kg})(9.80\text{ m/s}^2) = 2.94\text{ N}$$

EVALUATE: The tension is greater than the weight; the acceleration is upward so the net force must be upward.

7.60. IDENTIFY: Eq.(7.14) says $W_{\text{other}} = K_2 + U_2 - (K_1 + U_1)$. W_{other} is the work done on the baseball by the force exerted by the air.

SET UP: $U = mgy$. $K = \frac{1}{2}mv^2$, where $v^2 = v_x^2 + v_y^2$.

EXECUTE: **(a)** The change in total energy is the work done by the air,

$W_{\text{other}} = (K_2 + U_2) - (K_1 + U_1) = m\left(\frac{1}{2}(v_2^2 - v_1^2) + gy_2\right)$.

$W_{\text{other}} = (0.145\text{ kg})\Big((1/2\big[(18.6\text{ m/s})^2 - (30.0\text{ m/s})^2 - (40.0\text{ m/s})^2\big] + (9.80\text{ m/s}^2)(53.6\text{ m})\Big)$.

$W_{\text{other}} = -80.0\text{ J}$.

(b) Similarly, $W_{\text{other}} = (K_3 + U_3) - (K_2 + U_2)$.

$W_{\text{other}} = (0.145\text{ kg})\Big((1/2)\big[(11.9\text{ m/s})^2 + (-28.7\text{ m/s})^2 - (18.6\text{ m/s})^2\big] - (9.80\text{ m/s}^2)(53.6\text{ m})\Big)$.

$W_{\text{other}} = -31.3\text{ J}$.

(c) The ball is moving slower on the way down, and does not go as far (in the x-direction), and so the work done by the air is smaller in magnitude.

EVALUATE: The initial kinetic energy of the baseball is $\frac{1}{2}(0.145\text{ kg})(50.0\text{ m/s})^2 = 181\text{ J}$. For the total motion from the ground, up to the maximum height, and back down the total work done by the air is 111 J. The ball returns to the ground with $181\text{ J} - 111\text{ J} = 70\text{ J}$ of kinetic energy and a speed of 31 m/s, less than its initial speed of 50 m/s.

7.61. IDENTIFY and SET UP: There are two situations to compare: stepping off a platform and sliding down a pole. Apply the work-energy theorem to each.

(a) EXECUTE: Speed at ground if steps off platform at height h:

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$mgh = \frac{1}{2}mv_2^2$, so $v_2^2 = 2gh$

Motion from top to bottom of pole: (take $y = 0$ at bottom)

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$mgd - fd = \frac{1}{2}mv_2^2$

Use $v_2^2 = 2gh$ and get $mgd - fd = mgh$

$fd = mg(d-h)$

$f = mg(d-h)/d = mg(1-h/d)$

EVALUATE: For $h = d$ this gives $f = 0$ as it should (friction has no effect).

For $h = 0$, $v_2 = 0$ (no motion). The equation for f gives $f = mg$ in this special case. When $f = mg$ the forces on him cancel and he doesn't accelerate down the pole, which agrees with $v_2 = 0$.

(b) EXECUTE: $f = mg(1-h/d) = (75\text{ kg})(9.80\text{ m/s}^2)(1 - 1.0\text{ m}/2.5\text{ m}) = 441\text{ N}$.

(c) Take $y = 0$ at bottom of pole, so $y_1 = d$ and $y_2 = y$.

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$0 + mgd - f(d-y) = \frac{1}{2}mv^2 + mgy$

$\frac{1}{2}mv^2 = mg(d-y) - f(d-y)$

Using $f = mg(1-h/d)$ gives $\frac{1}{2}mv^2 = mg(d-y) - mg(1-h/d)(d-y)$

$\frac{1}{2}mv^2 = mg(h/d)(d-y)$ and $v = \sqrt{2gh(1-y/d)}$

EVALUATE: This gives the correct results for $y = 0$ and for $y = d$.

7.62. IDENTIFY: Apply Eq.(7.14) to each stage of the motion.

SET UP: Let $y = 0$ at the bottom of the slope. In part (a), W_{other} is the work done by friction. In part (b), W_{other} is the work done by friction and the air resistance force. In part (c), W_{other} is the work done by the force exerted by the snowdrift.

EXECUTE: (a) The skier's kinetic energy at the bottom can be found from the potential energy at the top minus the work done by friction, $K_1 = mgh - W_f = (60.0\text{ kg})(9.8\text{ N/kg})(65.0\text{ m}) - 10{,}500\text{ J}$, or

$K_1 = 38{,}200\text{ J} - 10{,}500\text{ J} = 27{,}720\text{ J}$. Then $v_1 = \sqrt{\dfrac{2K_1}{m}} = \sqrt{\dfrac{2(27{,}720\text{ J})}{60\text{ kg}}} = 30.4\text{ m/s}$.

(b) $K_2 = K_1 - (W_f + W_{\text{air}}) = 27{,}720\text{ J} - (\mu_k mgd + f_{\text{air}}d)$. $K_2 = 27{,}720\text{ J} - [(0.2)(588\text{ N})(82\text{ m}) + (160\text{ N})(82\text{ m})]$ or

$K_2 = 27{,}720\text{ J} - 22{,}763\text{ J} = 4957\text{ J}$. Then, $v_2 = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(4957\text{ J})}{60\text{ kg}}} = 12.9\text{ m/s}$

(c) Use the Work-Energy Theorem to find the force. $W = \Delta K$, $F = K/d = (4957\text{ J})/(2.5\text{ m}) = 2000\text{ N}$.

EVALUATE: In each case, W_{other} is negative and removes mechanical energy from the system.

7.63. IDENTIFY and SET UP: First apply $\sum \vec{F} = m\vec{a}$ to the skier.

Find the angle α where the normal force becomes zero, in terms of the speed v_2 at this point. Then apply the work-energy theorem to the motion of the skier to obtain another equation that relates v_2 and α. Solve these two equations for α.

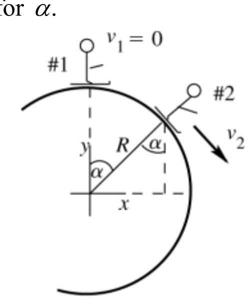


Figure 7.63a

Let point 2 be where the skier loses contact with the snowball, as sketched in Figure 7.63a Loses contact implies $n \to 0$.

$y_1 = R$, $y_2 = R\cos\alpha$

First, analyze the forces on the skier when she is at point 2. The free-body diagram is given in Figure 7.63b. For this use coordinates that are in the tangential and radial directions. The skier moves in an arc of a circle, so her acceleration is $a_{\text{rad}} = v^2/R$, directed in towards the center of the snowball.

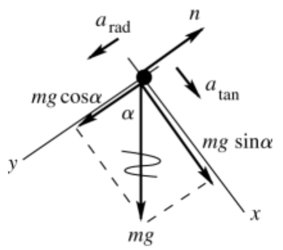


Figure 7.63b

EXECUTE: $\sum F_y = ma_y$

$mg\cos\alpha - n = mv_2^2/R$

But $n = 0$ so $mg\cos\alpha = mv_2^2/R$

$v_2^2 = Rg\cos\alpha$

Now use conservation of energy to get another equation relating v_2 to α:

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

The only force that does work on the skier is gravity, so $W_{\text{other}} = 0$.

$K_1 = 0,\ \ K_2 = \frac{1}{2}mv_2^2$

$U_1 = mgy_1 = mgR,\ \ U_2 = mgy_2 = mgR\cos\alpha$

Then $mgR = \frac{1}{2}mv_2^2 + mgR\cos\alpha$

$v_2^2 = 2gR(1-\cos\alpha)$

Combine this with the $\sum F_y = ma_y$ equation:

$Rg\cos\alpha = 2gR(1-\cos\alpha)$

$\cos\alpha = 2 - 2\cos\alpha$

$3\cos\alpha = 2$ so $\cos\alpha = 2/3$ and $\alpha = 48.2°$

EVALUATE: She speeds up and her a_{rad} increases as she loses gravitational potential energy. She loses contact when she is going so fast that the radially inward component of her weight isn't large enough to keep her in the circular path. Note that α where she loses contact does not depend on her mass or on the radius of the snowball.

7.64. **IDENTIFY:** Use conservation of energy to relate the speed at the lowest point to the speed at the highest point. Use $\sum \vec{F} = m\vec{a}$ to calculate the tension.

SET UP: The rock has acceleration $a_{\text{rad}} = v^2/R$, directed toward the center of the circle.

EXECUTE: If the speed of the rock at the top is v_{t}, then conservation of energy gives the speed v_{b} at the bottom from $\frac{1}{2}mv_{\text{b}}^2 = \frac{1}{2}mv_{\text{t}}^2 + mg(2R)$, R being the radius of the circle, and so $v_{\text{b}}^2 = v_{\text{t}}^2 + 4gR$. The tension at the top and bottom are found from $T_{\text{t}} + mg = \dfrac{mv_{\text{t}}^2}{R}$ and $T_{\text{b}} - mg = \dfrac{mv_{\text{b}}^2}{R}$, so $T_{\text{b}} - T_{\text{t}} = \dfrac{m}{R}(v_{\text{b}}^2 - v_{\text{t}}^2) + 2mg = 6mg = 6w$.

EVALUATE: The tensions T_{t} and T_{b} depend on the speed of the rock and on R, but the difference $T_{\text{b}} - T_{\text{t}}$ is independent of the speed of the rock and the radius of the circle.

7.65. **IDENTIFY** and **SET UP:**

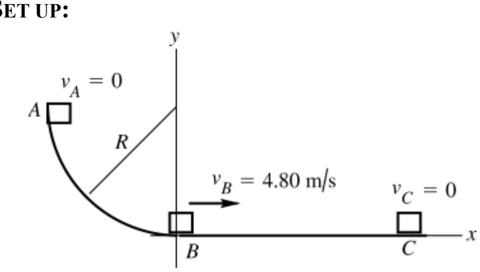


Figure 7.65

$y_A = R$

$y_B = y_C = 0$

(a) Apply conservation of energy to the motion from B to C:

$K_B + U_B + W_{\text{other}} = K_C + U_C$. The motion is described in Figure 7.65.

EXECUTE: The only force that does work on the package during this part of the motion is friction, so

$W_{\text{other}} = W_f = f_{\text{k}}(\cos\phi)s = \mu_{\text{k}} mg(\cos 180°)s = -\mu_{\text{k}} mgs$

$K_B = \frac{1}{2}mv_B^2,\ \ K_C = 0$

$U_B = 0,\ \ U_C = 0$

Thus $K_B + W_f = 0$

$\frac{1}{2}mv_B^2 - \mu_k mgs = 0$

$\mu_k = \frac{\mu_B^2}{2gs} = \frac{(4.80\text{ m/s})^2}{2(9.80\text{ m/s}^2)(3.00\text{ m})} = 0.392$

EVALUATE: The negative friction work takes away all the kinetic energy.

(b) IDENTIFY and **SET UP:** Apply conservation of energy to the motion from A to B:

$$K_A + U_A + W_{\text{other}} = K_B + U_B$$

EXECUTE: Work is done by gravity and by friction, so $W_{\text{other}} = W_f$.

$K_A = 0$, $K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.200\text{ kg})(4.80\text{ m/s})^2 = 2.304\text{ J}$

$U_A = mgy_A = mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(1.60\text{ m}) = 3.136\text{ J}$, $U_B = 0$

Thus $U_A + W_f = K_B$

$W_f = K_B - U_A = 2.304\text{ J} - 3.136\text{ J} = -0.83\text{ J}$

EVALUATE: W_f is negative as expected; the friction force does negative work since it is directed opposite to the displacement.

7.66. **IDENTIFY:** Apply Eq.(7.14) to the initial and final positions of the truck.

SET UP: Let $y = 0$ at the lowest point of the path of the truck. W_{other} is the work done by friction. $f_r = \mu_r n = \mu_r mg\cos\beta$.

EXECUTE: Denote the distance the truck moves up the ramp by x. $K_1 = \frac{1}{2}mv_0^2$, $U_1 = mgL\sin\alpha$, $K_2 = 0$, $U_2 = mgx\sin\beta$ and $W_{\text{other}} = -\mu_r mgx\cos\beta$. From $W_{\text{other}} = (K_2 + U_2) - (K_1 + U_1)$, and solving for x,

$$x = \frac{K_1 + mgL\sin\alpha}{mg(\sin\beta + \mu_r\cos\beta)} = \frac{(v_0^2/2g) + L\sin\alpha}{\sin\beta + \mu_r\cos\beta}.$$

EVALUATE: x increases when v_0 increases and decreases when μ_r increases.

7.67. $F_x = -\alpha x - \beta x^2$, $\alpha = 60.0\text{ N/m}$ and $\beta = 18.0\text{ N/m}^2$

(a) IDENTIFY: Use Eq.(6.7) to calculate W and then use $W = -\Delta U$ to identify the potential energy function $U(x)$.

SET UP: $W_{F_x} = U_1 - U_2 = \int_{x_1}^{x_2} F_x(x)\,dx$

Let $x_1 = 0$ and $U_1 = 0$. Let x_2 be some arbitrary point x, so $U_2 = U(x)$.

EXECUTE: $U(x) = -\int_0^x F_x(x)\,dx = -\int_0^x (-\alpha x - \beta x^2)\,dx = \int_0^x (\alpha x + \beta x^2)\,dx = \frac{1}{2}\alpha x^2 + \frac{1}{3}\beta x^3.$

EVALUATE: If $\beta = 0$, the spring does obey Hooke's law, with $k = \alpha$, and our result reduces to $\frac{1}{2}kx^2$.

(b) IDENTIFY: Apply Eq.(7.15) to the motion of the object.

SET UP: The system at points 1 and 2 is sketched in Figure 7.67.

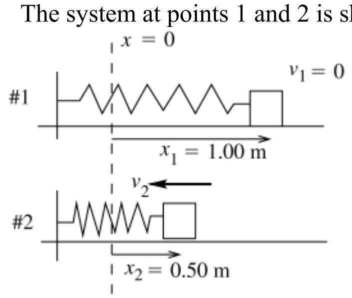


Figure 7.67

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

The only force that does work on the object is the spring force, so $W_{\text{other}} = 0$.

EXECUTE: $K_1 = 0$, $K_2 = \frac{1}{2}mv_2^2$

$U_1 = U(x_1) = \frac{1}{2}\alpha x_1^2 + \frac{1}{3}\beta x_1^3 = \frac{1}{2}(60.0\text{ N/m})(1.00\text{ m})^2 + \frac{1}{3}(18.0\text{ N/m}^2)(1.00\text{ m})^3 = 36.0\text{ J}$

$U_2 = U(x_2) = \frac{1}{2}\alpha x_2^2 + \frac{1}{3}\beta x_2^3 = \frac{1}{2}(60.0\text{ N/m})(0.500\text{ m})^2 + \frac{1}{3}(18.0\text{ N/m}^2)(0.500\text{ m})^3 = 8.25\text{ J}$

Thus $36.0\text{ J} = \frac{1}{2}mv_2^2 + 8.25\text{ J}$

$v_2 = \sqrt{\frac{2(36.0\text{ J} - 8.25\text{ J})}{0.900\text{ kg}}} = 7.85\text{ m/s}$

EVALUATE: The elastic potential energy stored in the spring decreases and the kinetic energy of the object increases.

7.68. **IDENTIFY:** Apply Eq.(7.14). W_{other} is the work done by F.

SET UP: $W_{\text{other}} = \Delta K + \Delta U$. The distance the spring stretches is $a\theta$. $y_2 - y_1 = a\sin\theta$.

EXECUTE: The force increases both the gravitational potential energy of the block and the potential energy of the spring. If the block is moved slowly, the kinetic energy can be taken as constant, so the work done by the force is the increase in potential energy, $\Delta U = mga\sin\theta + \frac{1}{2}k(a\theta)^2$.

EVALUATE: The force is kept tangent to the surface so the block will stay in contact with the surface.

7.69. **IDENTIFY:** Apply Eq.(7.14) to the motion of the block.

SET UP: Let $y = 0$ at the floor. Let point 1 be the initial position of the block against the compressed spring and let point 2 be just before the block strikes the floor.

EXECUTE: With $U_2 = 0, K_1 = 0$, $K_2 = U_1$. $\frac{1}{2}mv_2^2 = \frac{1}{2}kx^2 + mgh$. Solving for v_2 ,

$$v_2 = \sqrt{\frac{kx^2}{m} + 2gh} = \sqrt{\frac{(1900\text{ N/m})(0.045\text{ m})^2}{(0.150\text{ kg})} + 2(9.80\text{ m/s}^2)(1.20\text{ m})} = 7.01\text{ m/s} .$$

EVALUATE: The potential energy stored in the spring and the initial gravitational potential energy all go into the final kinetic energy of the block.

7.70. **IDENTIFY:** Apply Eq.(7.14). U is the total elastic potential energy of the two springs.

SET UP: Call the two points in the motion where Eq.(7.14) is applied A and B to avoid confusion with springs 1 and 2, that have force constants k_1 and k_2 . At any point in the motion the distance one spring is stretched equals the distance the other spring is compressed. Let $+x$ be to the right. Let point A be the initial position of the block, where it is released from rest, so $x_{1A} = +0.150$ m and $x_{2A} = -0.150$ m .

EXECUTE: **(a)** With no friction, $W_{\text{other}} = 0$. $K_A = 0$ and $U_A = K_B + U_B$. The maximum speed is when $U_B = 0$ and this is at $x_{1B} = x_{2B} = 0$, when both springs are at their natural length. $\frac{1}{2}k_1x_{1A}^2 + \frac{1}{2}k_2x_{2A}^2 = \frac{1}{2}mv_B^2$.

$x_{1A}^2 = x_{2A}^2 = (0.150\text{ m})^2$, so $v_B = \sqrt{\frac{k_1 + k_2}{m}}(0.150\text{ m}) = \sqrt{\frac{2500\text{ N/m} + 2000\text{ N/m}}{3.00\text{ kg}}}(0.150\text{ m}) = 5.81\text{ m/s}$.

(b) At maximum compression of spring 1, spring 2 has its maximum extension and $v_B = 0$. Therefore, at this point $U_A = U_B$. The distance spring 1 is compressed equals the distance spring 2 is stretched, and vice versa: $x_{1A} = -x_{2A}$ and $x_{1B} = -x_{2B}$. Then $U_A = U_B$ gives $\frac{1}{2}(k_1 + k_2)x_{1A}^2 = \frac{1}{2}(k_1 + k_2)x_{1B}^2$ and $x_{1B} = -x_{1A} = -0.150$ m . The maximum compression of spring 1 is 15.0 cm.

EVALUATE: When friction is not present mechanical energy is conserved and is continually transformed between kinetic energy of the block and potential energy in the springs. If friction is present, its work removes mechanical energy from the system.

7.71. **IDENTIFY:** Apply conservation of energy to relate x and h. Apply $\sum \vec{F} = m\vec{a}$ to relate a and x.

SET UP: The first condition, that the maximum height above the release point is h, is expressed as $\frac{1}{2}kx^2 = mgh$. The magnitude of the acceleration is largest when the spring is compressed to a distance x; at this point the net upward force is $kx - mg = ma$, so the second condition is expressed as $x = (m/k)(g + a)$.

EXECUTE: **(a)** Substituting the second expression into the first gives

$$\frac{1}{2}k\left(\frac{m}{k}\right)^2(g + a)^2 = mgh, \quad \text{or} \quad k = \frac{m(g + a)^2}{2gh}.$$

(b) Substituting this into the expression for x gives $x = \dfrac{2gh}{g + a}$.

EVALUATE: When $a \to 0$, our results become $k = \dfrac{mg}{2h}$ and $x = 2h$. The initial spring force is $kx = mg$ and the net upward force approaches zero. But $\frac{1}{2}kx^2 = mgh$ and sufficient potential energy is stored in the spring to move the mass to height h.

7.72. **IDENTIFY:** At equilibrium the upward spring force equals the weight mg of the object. Apply conservation of energy to the motion of the fish.

SET UP: The distance that the mass descends equals the distance the spring is stretched. $K_1 = K_2 = 0$, so $U_1(\text{gravitational}) = U_2(\text{spring})$

EXECUTE: Following the hint, the force constant k is found from $mg = kd$, or $k = mg/d$. When the fish falls from rest, its gravitational potential energy decreases by mgy; this becomes the potential energy of the spring, which is $\frac{1}{2}ky^2 = \frac{1}{2}(mg/d)y^2$. Equating these, $\frac{1}{2}\frac{mg}{d}y^2 = mgy$, or $y = 2d$.

EVALUATE: At its lowest point the fish is not in equilibrium. The upward spring force at this point is $ky = 2kd$, and this is equal to twice the weight. At this point the net force is *mg*, upward, and the fish has an upward acceleration equal to *g*.

7.73. IDENTIFY: Apply Eq.(7.15) to the motion of the block.

SET UP: The motion from *A* to *B* is described in Figure 7.73.

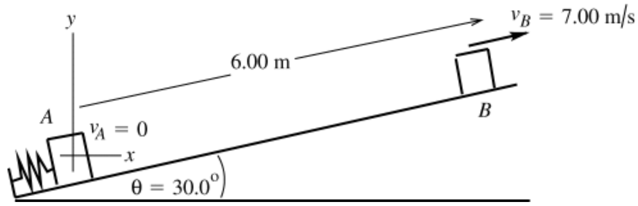


Figure 7.73

The normal force is $n = mg\cos\theta$, so $f_k = \mu_k n = \mu_k mg\cos\theta$.

$y_A = 0;\ \ y_B = (60.0\text{ m})\sin 30.0° = 3.00\text{ m}$

$K_A + U_A + W_{\text{other}} = K_B + U_B$

EXECUTE: Work is done by gravity, by the spring force, and by friction, so $W_{\text{other}} = W_f$ and $U = U_{\text{el}} + U_{\text{grav}}$

$K_A = 0,\ \ K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(1.50\text{ kg})(7.00\text{ m/s})^2 = 36.75\text{ J}$

$U_A = U_{\text{el},A} + U_{\text{grav},A} = U_{\text{el},A}$, since $U_{\text{grav},A} = 0$

$U_B = U_{\text{el},B} + U_{\text{grav},B} = 0 + mgy_B = (1.50\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m}) = 44.1\text{ J}$

$W_{\text{other}} = W_f = (f_k\cos\phi)s = \mu_k mg\cos\theta(\cos 180°)s = -\mu_k mg\cos\theta s$

$W_{\text{other}} = -(0.50)(1.50\text{ kg})(9.80\text{ m/s}^2)(\cos 30.0°)(6.00\text{ m}) = -38.19\text{ J}$

Thus $U_{\text{el},A} - 38.19\text{ J} = 36.75\text{ J} + 44.10\text{ J}$

$U_{\text{el},A} = 38.19\text{ J} + 36.75\text{ J} + 44.10\text{ J} = 119\text{ J}$

EVALUATE: U_{el} must always be positive. Part of the energy initially stored in the spring was taken away by friction work; the rest went partly into kinetic energy and partly into an increase in gravitational potential energy.

7.74. IDENTIFY: Apply Eq.(7.14) to the motion of the package. $W_{\text{other}} = W_{f_k}$, the work done by the kinetic friction force.

SET UP: $f_k = \mu_k n = \mu_k mg\cos\theta$, with $\theta = 53.1°$. Let $L = 4.00\text{ m}$, the distance the package moves before reaching the spring and let *d* be the maximum compression of the spring. Let point 1 be the initial position of the package, point 2 be just as it contacts the spring, point 3 be at the maximum compression of the spring, and point 4 be the final position of the package after it rebounds.

EXECUTE: **(a)** $K_1 = 0$, $U_2 = 0$, $W_{\text{other}} = -f_k L = -\mu_k L\cos\theta$. $U_1 = mgL\sin\theta$. $K_2 = \frac{1}{2}mv^2$, where *v* is the speed before the block hits the spring. Eq.(7.14) applied to points 1 and 2, with $y_2 = 0$, gives $U_1 + W_{\text{other}} = K_2$. Solving for *v*,

$v = \sqrt{2gL(\sin\theta - \mu_k\cos\theta)} = \sqrt{2(9.80\text{ m/s}^2)(4.00\text{ m})(\sin 53.1° - (0.20)\cos 53.1°)} = 7.30\text{ m/s}.$

(b) Apply Eq.(7.14) to points 1 and 3. Let $y_3 = 0$. $K_1 = K_3 = 0$. $U_1 = mg(L+d)\sin\theta$. $U_2 = \frac{1}{2}kd^2$.

$W_{\text{other}} = -f_k(L+d)$. Eq.(7.14) gives $mg(L+d)\sin\theta - \mu_k mg\cos\theta(L+d) = \frac{1}{2}kd^2$. This can be written as

$d^2\dfrac{k}{2mg(\sin\theta - \mu_k\cos\theta)} - d - L = 0.$ The factor multiplying d^2 is 4.504 m^{-1}, and use of the quadratic formula gives $d = 1.06\text{ m}$.

(c) The easy thing to do here is to recognize that the presence of the spring determines *d*, but at the end of the motion the spring has no potential energy, and the distance below the starting point is determined solely by how much energy has been lost to friction. If the block ends up a distance *y* below the starting point, then the block has moved a distance $L+d$ down the incline and $L+d-y$ up the incline. The magnitude of the friction force is the same in both directions, $\mu_k mg\cos\theta$, and so the work done by friction is $-\mu_k(2L+2d-y)mg\cos\theta$. This must be equal to the change in gravitational potential energy, which is $-mgy\sin\theta$. Equating these and solving for *y* gives

$y = (L+d)\dfrac{2\mu_k\cos\theta}{\sin\theta + \mu_k\cos\theta} = (L+d)\dfrac{2\mu_k}{\tan\theta + \mu_k}.$ Using the value of *d* found in part (b) and the given values for μ_k and θ gives $y = 1.32\text{ m}$.

EVALUATE: Our expression for y gives the reasonable results that $y = 0$ when $\mu_k = 0$; in the absence of friction the package returns to its starting point.

7.75. **(a) IDENTIFY** and **SET UP:** Apply $K_A + U_A + W_{\text{other}} = K_B + U_B$ to the motion from A to B.

EXECUTE: $K_A = 0,\ K_B = \frac{1}{2}mv_B^2$

$U_A = 0,\ U_B = U_{\text{el},B} = \frac{1}{2}kx_B^2,$ where $x_B = 0.25$ m

$W_{\text{other}} = W_F = Fx_B$

Thus $Fx_B = \frac{1}{2}mv_B^2 + \frac{1}{2}kx_B^2$. (The work done by F goes partly to the potential energy of the stretched spring and partly to the kinetic energy of the block.)

$Fx_B = (20.0\text{ N})(0.25\text{ m}) = 5.0\text{ J}$ and $\frac{1}{2}kx_B^2 = \frac{1}{2}(40.0\text{ N/m})(0.25\text{ m})^2 = 1.25\text{ J}$

Thus $5.0\text{ J} = \frac{1}{2}mv_B^2 + 1.25\text{ J}$ and $v_B = \sqrt{\dfrac{2(3.75\text{ J})}{0.500\text{ kg}}} = 3.87\text{ m/s}$

(b) IDENTIFY: Apply Eq.(7.15) to the motion of the block. Let point C be where the block is closest to the wall. When the block is at point C the spring is compressed an amount $|x_C|$, so the block is $0.60\text{ m} - |x_C|$ from the wall, and the distance between B and C is $x_B + |x_C|$.

SET UP: The motion from A to B to C is described in Figure 7.75.

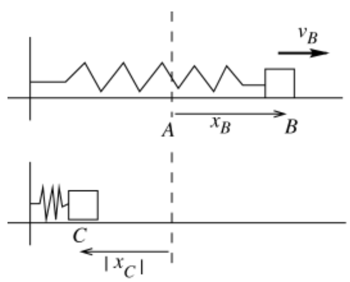


Figure 7.75

$K_B + U_B + W_{\text{other}} = K_C + U_C$

EXECUTE: $W_{\text{other}} = 0$

$K_B = \frac{1}{2}mv_B^2 = 5.0\text{ J} - 1.25\text{ J} = 3.75\text{ J}$ (from part (a))

$U_B = \frac{1}{2}kx_B^2 = 1.25\text{ J}$

$K_C = 0$ (instantaneously at rest at point closest to wall)

$U_C = \frac{1}{2}k|x_C|^2$

Thus $3.75\text{ J} + 1.25\text{ J} = \frac{1}{2}k|x_C|^2$

$|x_C| = \sqrt{\dfrac{2(5.0\text{ J})}{40.0\text{ N/m}}} = 0.50\text{ m}$

The distance of the block from the wall is $0.60\text{ m} - 0.50\text{ m} = 0.10\text{ m}$.

EVALUATE: The work $(20.0\text{ N})(0.25\text{ m}) = 5.0\text{ J}$ done by F puts 5.0 J of mechanical energy into the system. No mechanical energy is taken away by friction, so the total energy at points B and C is 5.0 J.

7.76. **IDENTIFY:** Apply Eq.(7.14) to the motion of the student.

SET UP: Let $x_0 = 0.18$ m , $x_1 = 0.71$ m . The spring constants (assumed identical) are then known in terms of the unknown weight w, $4kx_0 = w$. Let $y = 0$ at the initial position of the student.

EXECUTE: **(a)** The speed of the brother at a given height h above the point of maximum compression is then found from $\dfrac{1}{2}(4k)x_1^2 = \dfrac{1}{2}\left(\dfrac{w}{g}\right)v^2 + mgh$, or $v^2 = \dfrac{(4k)g}{w}x_1^2 - 2gh = g\left(\dfrac{x_1^2}{x_0} - 2h\right)$. Therefore,

$v = \sqrt{(9.80\text{ m/s}^2)((0.71\text{ m})^2/(0.18\text{ m}) - 2(0.90\text{ m}))} = 3.13\text{ m/s}$, or 3.1 m/s to two figures.

(b) Setting $v = 0$ and solving for h, $h = \dfrac{2kx_1^2}{mg} = \dfrac{x_1^2}{2x_0} = 1.40$ m, or 1.4 m to two figures.

(c) No; the distance x_0 will be different, and the ratio $\dfrac{x_1^2}{x_0} = \dfrac{(x_1 + 0.53\text{ m})^2}{x_1} = x_1\left(1 + \dfrac{0.53\text{ m}}{x_1}\right)^2$ will be different.

Note that on a planet with lower g, x_1 will be smaller and h will be larger.

EVALUATE: We are able to solve the problem without knowing either the mass of the student or the force constant of the spring.

7.77. **IDENTIFY:** $a_x = d^2x/dt^2$, $a_y = d^2y/dt^2$. $F_x = ma_x$, $F_y = ma_y$. $U = \int F_x dx + \int F_y dy$.

SET UP: $\frac{d}{dt}(\cos\omega_0 t) = -\omega_0 \sin\omega_0 t$. $\frac{d}{dt}(\sin\omega_0 t) = \omega_0 \cos\omega_0 t$. $\int \cos\omega_0 t\ dt = \frac{1}{\omega_0}\sin\omega_0 t$, $\int \sin\omega_0 t\ dt = -\frac{1}{\omega_0}\cos\omega_0 t$.

$v_x = dx/dt$, $v_y = dy/dt$. $E = K + U$.

EXECUTE: **(a)** $a_x = d^2x/dt^2 = -\omega_0^2 x,\ F_x = ma_x = -m\omega_0^2 x.\ \ a_y = d^2y/dt^2 = -\omega_0^2 y,\ \ F_y = ma_y = -m\omega_0^2 y$

(b) $U = -\left[\int F_x dx + \int F_y dy\right] = m\omega_0^2\left[\int xdx + \int ydy\right] = \frac{1}{2}m\omega_0^2(x^2 + y^2)$

(c) $v_x = dx/dt = -x_0\omega_0 \sin\omega_0 t = -x_0\omega_0(y/y_0)$. $v_y = dy/dt = +y_0\omega_0\cos\omega_0 t = +y_0\omega_0(x/x_0)$.

(i) When $x = x_0$ and $y = 0, v_x = 0$ and $v_y = y_0\omega_0$,

$$K = \frac{1}{2}m(v_x^2 + v_y^2) = \frac{1}{2}my_0^2\omega_0^2, U = \frac{1}{2}\omega_0^2 m x_0^2 \text{ and } E = K + U = \frac{1}{2}m\omega_0^2(x_0^2 + y_0^2)$$

(ii) When $x = 0$ and $y = y_0, v_x = -x_0\omega_0$ and $v_y = 0$,

$$K = \frac{1}{2}\omega_0^2 m x_0^2, U = \frac{1}{2}m\omega_0^2 y_0^2 \text{ and } E = K + U = \frac{1}{2}m\omega_0^2(x_0^2 + y_0^2)$$

EVALUATE: The total energy is the same at the two points in part (c); the total energy of the system is constant.

7.78. **IDENTIFY:** Calculate the increase in kinetic energy for the car.

SET UP: The car gets $(0.15)(1.3\times10^8\ \text{J})$ of energy from one gallon of gasoline.

EXECUTE: **(a)** The mechanical energy increase of the car is $K_2 - K_1 = \frac{1}{2}(1500\ \text{kg})(37\ \text{m/s})^2 = 1.027\times10^6\ \text{J}$. Let α be the number of gallons of gasoline consumed. $\alpha(1.3\times10^8\ \text{J})(0.15) = 1.027\times10^6\ \text{J}$ and $\alpha = 0.053$ gallons.

(b) $(1.00\ \text{gallons})/\alpha = 19$ accelerations

EVALUATE: The time over which the increase in velocity occurs doesn't enter into the calculation.

7.79. **IDENTIFY:** $U = mgh$. Use $h = 150$ m for all the water that passes through the dam.

SET UP: $m = \rho V$ and $V = A\Delta h$ is the volume of water in a height Δh of water in the lake.

EXECUTE: **(a)** Stored energy $= mgh = (\rho V)gh = \rho A(1\ \text{m})gh$.

stored energy $= (1000\ \text{kg/m}^3)(3.0\times10^6\ \text{m}^2)(1\ \text{m})(9.8\ \text{m/s}^2)(150\ \text{m}) = 4.4\times10^{12}\ \text{J}$.

(b) 90% of the stored energy is converted to electrical energy, so $(0.90)(mgh) = 1000$ kWh.

$(0.90)\rho Vgh = 1000$ kWh. $V = \dfrac{(1000\ \text{kWh})((3600\ \text{s})/(1\ \text{h}))}{(0.90)(1000\ \text{kg/m}^3)(150\ \text{m})(9.8\ \text{m/s}^2)} = 2.7\times10^3\ \text{m}^3$.

Change in level of the lake: $A\Delta h = V_{\text{water}}$. $\Delta h = \dfrac{V}{A} = \dfrac{2.7\times10^3\ \text{m}^3}{3.0\times10^6\ \text{m}^2} = 9.0\times10^{-4}$ m.

EVALUATE: Δh is much less than 150 m, so using $h = 150$ m for all the water that passed through the dam was a very good approximation.

7.80. **IDENTIFY** and **SET UP:** The potential energy of a horizontal layer of thickness dy, area A, and height y is $dU = (dm)gy$. Let ρ be the density of water.

EXECUTE: $dm = \rho\ dV = \rho A\ dy$, so $dU = \rho Agy\ dy$.

The total potential energy U is

$U = \int_0^h dU = \rho Ag\int_0^h y\ dy = \frac{1}{2}\rho Agh^2$.

$A = 3.0\times10^6\ \text{m}^2$ and $h = 150$ m, so $U = 3.3\times10^{14}\ \text{J} = 9.2\times10^7$ kWh

EVALUATE: The volume is Ah and the mass of water is $\rho V = \rho Ah$. The average depth is $h_{\text{av}} = h/2$, so $U = mgh_{\text{av}}$.

7.81. **IDENTIFY:** Apply $F_x = -\dfrac{\partial U}{\partial x}$, $F_y = -\dfrac{\partial U}{\partial y}$ and $F_z = -\dfrac{\partial U}{\partial z}$.

SET UP: $r = (x^2 + y^2 + z^2)^{1/2}$. $\dfrac{\partial(1/r)}{\partial x} = -\dfrac{x}{(x^2+y^2)^{3/2}}$, $\dfrac{\partial(1/r)}{\partial y} = -\dfrac{y}{(x^2+y^2)^{3/2}}$ and $\dfrac{\partial(1/r)}{\partial z} = -\dfrac{z}{(x^2+y^2)^{3/2}}$.

EXECUTE: **(a)** $U(r) = -\dfrac{Gm_1m_2}{r}$. $F_x = -\dfrac{\partial U}{\partial x} = +Gm_1m_2\left[\dfrac{\partial(1/r)}{\partial x}\right] = -\dfrac{Gm_1m_2x}{(x^2+y^2+z^2)^{3/2}}$. Similarly,

$F_y = -\dfrac{Gm_1m_2y}{(x^2+y^2+z^2)^{3/2}}$ and $F_z = -\dfrac{Gm_1m_2z}{(x^2+y^2+z^2)^{3/2}}$.

(b) $(x^2+y^2+z^2)^{3/2} = r^3$ so $F_x = -\dfrac{Gm_1m_2x}{r^3}$, $F_y = -\dfrac{Gm_1m_2y}{r^3}$ and $F_z = -\dfrac{Gm_1m_2z}{r^3}$.

$F = \sqrt{F_x^2+F_y^2+F_z^2} = \dfrac{Gm_1m_2}{r^3}\sqrt{x^2+y^2+z^2} = \dfrac{Gm_1m_2}{r^2}$.

(c) F_x, F_y and F_z are negative. $F_x = \alpha x$, $F_y = \alpha y$ and $F_z = \alpha z$, where α is a constant, so $\vec{F}$ and the vector $\vec{r}$ from m_1 to m_2 are in the same direction. Therefore, $\vec{F}$ is directed toward m_1 at the origin and $\vec{F}$ is attractive.

EVALUATE: When m_2 moves to larger *r*, the work done on it by the attractive gravity force is negative. Since $W = -\Delta U$, negative work done by gravity means the gravitational potential energy increases.

$U(r) = -\dfrac{Gm_1m_2}{r}$ does increase (becomes less negative) as *r* increases. For an object near the surface of the earth, $U(r) = -\dfrac{Gm_1m_2}{r}$ will be shown in Chapter 12 to be equivalent to $U_{\text{grav}} = mgy$.

7.82. **IDENTIFY:** Calculate the work *W* done by this force. If the force is conservative, the work is path independent.

SET UP: $W = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{l}$.

EXECUTE: **(a)** $W = \int_{P_1}^{P_2} F_y dy = C\int_{P_1}^{P_2} y^2 dy$. *W* doesn't depend on *x*, so it is the same for all paths between P_1 and P_2. The force is conservative.

(b) $W = \int_{P_1}^{P_2} F_x dx = C\int_{P_1}^{P_2} y^2 dx$. *W* will be different for paths between points P_1 and P_2 for which *y* has different values. For example, if *y* has the constant value y_0 along the path, then $W = Cy_0(x_2 - x_1)$. *W* depends on the value of y_0. The force is not conservative.

EVALUATE: $\vec{F} = Cy^2\hat{j}$ has the potential energy function $U(y) = -\dfrac{Cy^3}{3}$. We cannot find a potential energy function for $\vec{F} = Cy^2\hat{i}$.

7.83. $\vec{F} = -\alpha xy^2\hat{j}$, $\alpha = 2.50\ \text{N/m}^3$

IDENTIFY: $\vec{F}$ is not constant so use Eq.(6.14) to calculate *W*. $\vec{F}$ must be evaluated along the path.

(a) SET UP: The path is sketched in Figure 7.83a.

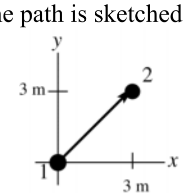


Figure 7.83a

$d\vec{l} = dx\hat{i} + dy\hat{j}$

$\vec{F}\cdot d\vec{l} = -\alpha xy^2\ dy$

On the path, $x = y$ so $\vec{F}\cdot d\vec{l} = -\alpha y^3\ dy$

EXECUTE: $W = \int_1^2 \vec{F}\cdot d\vec{l} = \int_{y_1}^{y_2}(-\alpha y^3)\ dy = -(\alpha/4)\left(y^4\Big|_{y_1}^{y_2}\right) = -(\alpha/4)(y_2^4 - y_1^4)$

$y_1 = 0$, $y_2 = 3.00$ m, so $W = -\tfrac{1}{4}(2.50\ \text{N/m}^3)(3.00\ \text{m})^4 = -50.6\ \text{J}$

(b) SET UP: The path is sketched in Figure 7.83b.

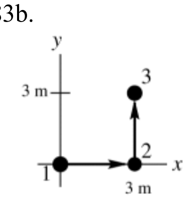


Figure 7.83b

For the displacement from point 1 to point 2, $d\vec{l} = dx\hat{i}$, so $\vec{F}\cdot d\vec{l} = 0$ and $W = 0$. (The force is perpendicular to the displacement at each point along the path, so $W = 0$.)

For the displacement from point 2 to point 3, $d\vec{l} = dy\hat{j}$, so $\vec{F} \cdot d\vec{l} = -\alpha xy^2\ dy$. On this path, $x = 3.00$ m, so

$$\vec{F} \cdot d\vec{l} = -(2.50\ \text{N/m}^3)(3.00\ \text{m})y^2\ dy = -(7.50\ \text{N/m}^2)y^2\ dy.$$

EXECUTE: $W = \int_2^3 \vec{F} \cdot d\vec{l} = -(7.50\ \text{N/m}^2)\int_{y_2}^{y_3} y^2\ dy = -(7.50\ \text{N/m}^2)\frac{1}{3}(y_3^3 - y_2^3)$

$W = -(7.50\ \text{N/m}^2)\left(\frac{1}{3}\right)(3.00\ \text{m})^3 = -67.5$ J

(c) EVALUATE: For these two paths between the same starting and ending points the work is different, so the force is nonconservative.

7.84. **IDENTIFY:** Use $W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{l}$ to calculate W for each segment of the path.

SET UP: $\vec{F} \cdot d\vec{l} = F_x dx = \alpha xy\ dx$

EXECUTE: **(a)** The path is sketched in Figure 7.84.

(b) (1): $x = 0$ along this leg, so $\vec{F} = 0$ and $W = 0$. (2)**:** Along this leg, $y = 1.50$ m, so $\vec{F} \cdot d\vec{l} = (3.00\ \text{N/m})x dx$, and $W = (1.50\ \text{N/m})((1.50\ \text{m})^2 - 0) = 3.38$ J (3) $\vec{F} \cdot d\vec{l} = 0$, so $W = 0$ (4) $y = 0$, so $\vec{F} = 0$ and $W = 0$. The work done in moving around the closed path is 3.38 J.

(c) The work done in moving around a closed path is not zero, and the force is not conservative.

EVALUATE: There is no potential energy function for this force.

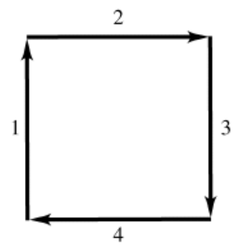


Figure 7.84

7.85. **IDENTIFY:** Use Eq.(7.16) to relate F_x and $U(x)$. The equilibrium is stable where $U(x)$ is a local minimum and the equilibrium is unstable where $U(x)$ is a local maximum.

SET UP: The maximum and minimum values of x are those for which $U(x) = E$. $K = E - U$, so the maximum speed is where U is a minimum.

EXECUTE: **(a)** For the given proposed potential $U(x), -\dfrac{dU}{dx} = -kx + F$, so this is a possible potential function.

For this potential, $U(0) = -F^2/2k$, not zero. Setting the zero of potential is equivalent to adding a constant to the potential; any additive constant will not change the derivative, and will correspond to the same force.

(b) At equilibrium, the force is zero; solving $-kx + F = 0$ for x gives $x_0 = F/k$. $U(x_0) = -F^2/k$, and this is a minimum of U, and hence a stable point.

(c) The graph is given in Figure 7.85.

(d) No; $F_{\text{tot}} = 0$ at only one point, and this is a stable point.

(e) The extreme values of x correspond to zero velocity, hence zero kinetic energy, so $U(x_\pm) = E$, where $x_\pm$ are the extreme points of the motion. Rather than solve a quadratic, note that $\frac{1}{2}k(x - F/k)^2 - F^2/k$, so $U(x_\pm) = E$ becomes $\dfrac{1}{2}k\left(x_\pm - \dfrac{F}{k}\right)^2 - F^2/k = \dfrac{F^2}{k}$. $x_\pm - \dfrac{F}{k} = \pm 2\dfrac{F}{k}$, so $x_+ = 3\dfrac{F}{k}$ $x_- = -\dfrac{F}{k}$.

(f) The maximum kinetic energy occurs when $U(x)$ is a minimum, the point $x_0 = F/k$ found in part (b). At this point $K = E - U = (F^2/k) - (-F^2/k) = 2F^2/k$, so $v = 2F/\sqrt{mk}$.

EVALUATE: As E increases, the magnitudes of x_+ and x_- increase. The particle cannot reach values of x for which $E < U(x)$ because K cannot be negative.

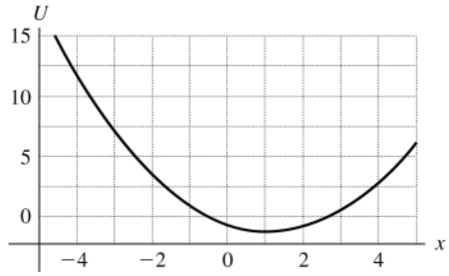


Figure 7.85

7.86. **IDENTIFY:** Use Eq.(7.16) to relate F_x and $U(x)$. The equilibrium is stable where $U(x)$ is a local minimum and the equilibrium is unstable where $U(x)$ is a local maximum.

SET UP: dU/dx is the slope of the graph of U versus x. $K = E - U$, so K is a maximum when U is a minimum. The maximum x is where $E = U$.

EXECUTE: **(a)** The slope of the U *vs.* x curve is negative at point A, so F_x is positive (Eq. (7.16)).

(b) The slope of the curve at point B is positive, so the force is negative.

(c) The kinetic energy is a maximum when the potential energy is a minimum, and that figures to be at around 0.75 m.

(d) The curve at point C looks pretty close to flat, so the force is zero.

(e) The object had zero kinetic energy at point A, and in order to reach a point with more potential energy than $U(A)$, the kinetic energy would need to be negative. Kinetic energy is never negative, so the object can never be at any point where the potential energy is larger than $U(A)$. On the graph, that looks to be at about 2.2 m.

(f) The point of minimum potential (found in part (c)) is a stable point, as is the relative minimum near 1.9 m.

(g) The only potential maximum, and hence the only point of unstable equilibrium, is at point C.

EVALUATE: If E is less than U at point C, the particle is trapped in one or the other of the potential "wells" and cannot move from one allowed region of x to the other.

7.87. **IDENTIFY:** $K = E - U$ determines $v(x)$.

SET UP: v is a maximum when U is a minimum and v is a minimum when U is a maximum. $F_x = -dU/dx$. The extreme values of x are where $E = U(x)$.

EXECUTE: **(a)** Eliminating β in favor of α and $x_0 (\beta = \alpha/x_0)$,

$$U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x} = \frac{\alpha}{x_0^2}\frac{x_0^2}{x^2} - \frac{\alpha}{x_0 x} = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \left(\frac{x_0}{x}\right)\right].$$

$U(x_0) = \left(\dfrac{\alpha}{x_0^2}\right)(1-1) = 0$. $U(x)$ is positive for $x < x_0$ and negative for $x > x_0$ (α and β must be taken as positive). The graph of $U(x)$ is sketched in Figure 7.87a.

(b) $v(x) = \sqrt{-\dfrac{2}{m}U} = \sqrt{\left(\dfrac{2\alpha}{mx_0^2}\right)\left(\left(\dfrac{x_0}{x}\right) - \left(\dfrac{x_0}{x}\right)^2\right)}$. The proton moves in the positive x-direction, speeding up until it reaches a maximum speed (see part (c)), and then slows down, although it never stops. The minus sign in the square root in the expression for $v(x)$ indicates that the particle will be found only in the region where $U < 0$, that is, $x > x_0$. The graph of $v(x)$ is sketched in Figure 7.87b.

(c) The maximum speed corresponds to the maximum kinetic energy, and hence the minimum potential energy. This minimum occurs when $\dfrac{dU}{dx} = 0$, or $\dfrac{dU}{dx} = \dfrac{\alpha}{x_0}3\left[-2\left(\dfrac{x_0}{x}\right)^3 + \left(\dfrac{x_0}{x}\right)^2\right] = 0,$

which has the solution $x = 2x_0$. $U(2x_0) = -\dfrac{\alpha}{4x_0^2}$, so $v = \sqrt{\dfrac{\alpha}{2mx_0^2}}$.

(d) The maximum speed occurs at a point where $\dfrac{dU}{dx} = 0$, and from Eq. (7.15), the force at this point is zero.

(e) $x_1 = 3x_0$, and $U(3x_0) = -\dfrac{2\alpha}{9x_0^2}$.

$$v(x) = \sqrt{\frac{2}{m}(U(x_1) - U(x))} = \sqrt{\frac{2}{m}\left[\left(\frac{-2}{9}\frac{\alpha}{x_0^2}\right) - \frac{\alpha}{x_0^2}\left(\left(\frac{x_0}{x}\right)^2 - \frac{x_0}{x}\right)\right]} = \sqrt{\frac{2\alpha}{mx_0^2}\left(\left(\frac{x_0}{x}\right) - \left(\frac{x_0}{x}\right)^2 - 2/9\right)}.$$

The particle is confined to the region where $U(x) < U(x_1)$. The maximum speed still occurs at $x = 2x_0$, but now the particle will oscillate between x_1 and some minimum value (see part (f)).

(f) Note that $U(x) - U(x_1)$ can be written as

$$\frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \left(\frac{x_0}{x}\right) + \left(\frac{2}{9}\right)\right] = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right) - \frac{1}{3}\right]\left[\left(\frac{x_0}{x}\right) - \frac{2}{3}\right],$$

which is zero (and hence the kinetic energy is zero) at $x = 3x_0 = x_1$ and $x = \frac{3}{2}x_0$. Thus, when the particle is released from x_0, it goes on to infinity, and doesn't reach any maximum distance. When released from x_1, it oscillates between $\frac{3}{2}x_0$ and $3x_0$.

EVALUATE: In each case the proton is released from rest and $E = U(x_i)$, where x_i is the point where it is released. When $x_i = x_0$ the total energy is zero. When $x_i = x_1$ the total energy is negative. $U(x) \to 0$ as $x \to \infty$, so for this case the proton can't reach $x \to \infty$ and the maximum x it can have is limited.

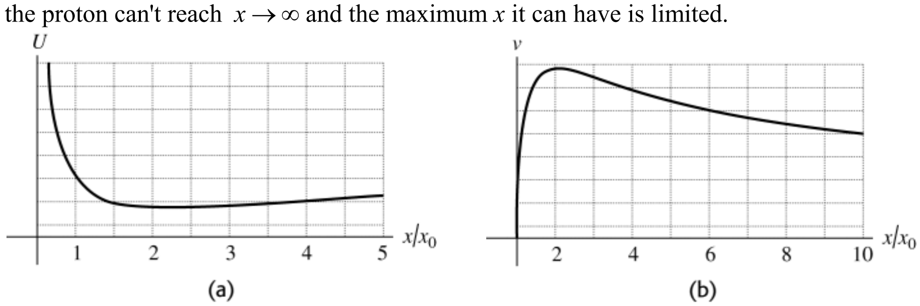


Figure 7.87