Essential University Physics

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6 Work, Energy, and Power

PowerPoint® Lecture prepared by Richard Wolfson

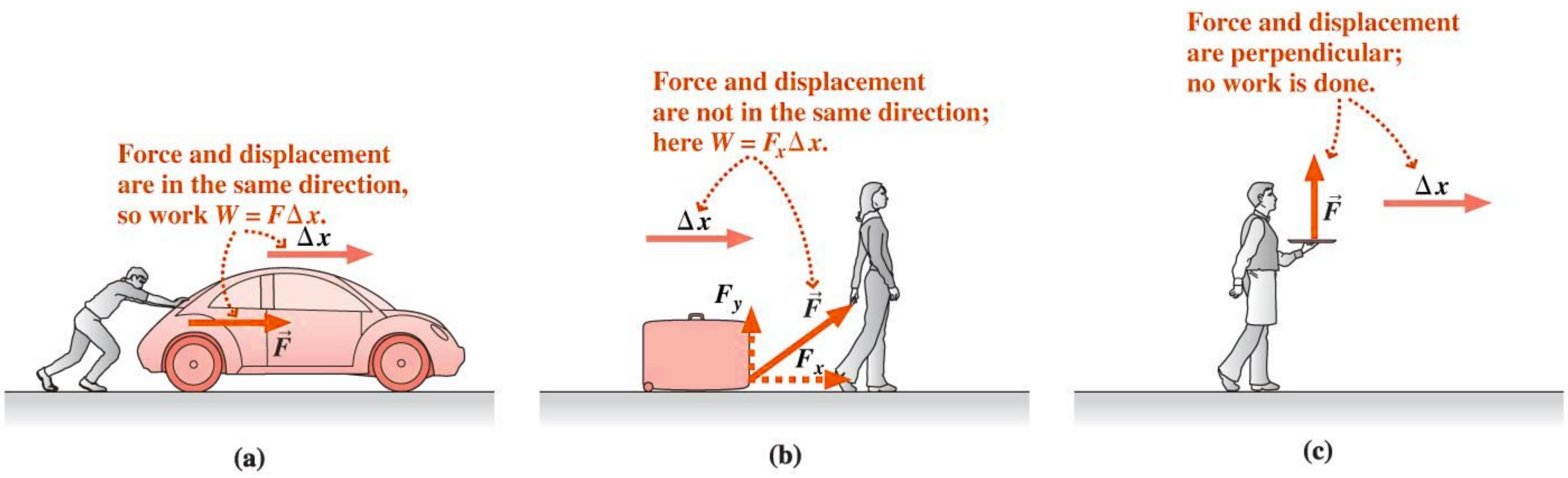
In this lecture you'll learn

- The concept of work
- How to calculate work
  - Done by a constant force
  - Done by a force that varies with position
- The concept of kinetic energy
  - The work-energy theorem
- Power and its relation to energy

Work: A measure of force applied over distance

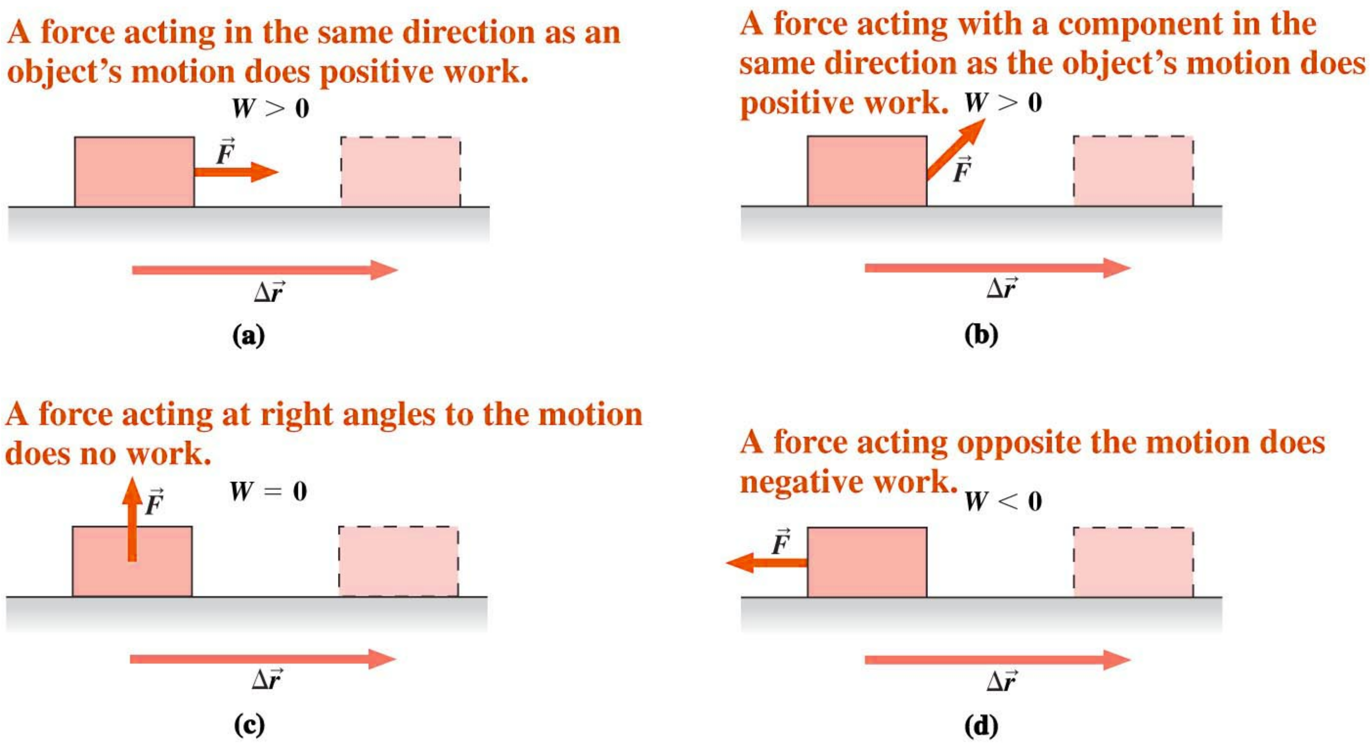
In one dimension: $W = F_x \Delta x$

More generally, work depends on the *component of force in the direction of motion*:



Work can be positive or negative

- Work is positive if the force has a component in the same direction as the motion.
- Work is negative if the force has a component opposite the direction of motion.
- Work is zero if the force is perpendicular to the motion.



The Scalar Product

Work is conveniently characterized using the *scalar product*, a way of combining two vectors to produce a scalar that depends on the vectors' magnitudes and the angle between them.

The scalar product of any two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

where A and B are the magnitudes of the vectors and θ is the angle between them.

With vectors in component form, $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$, the scalar product can be written

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

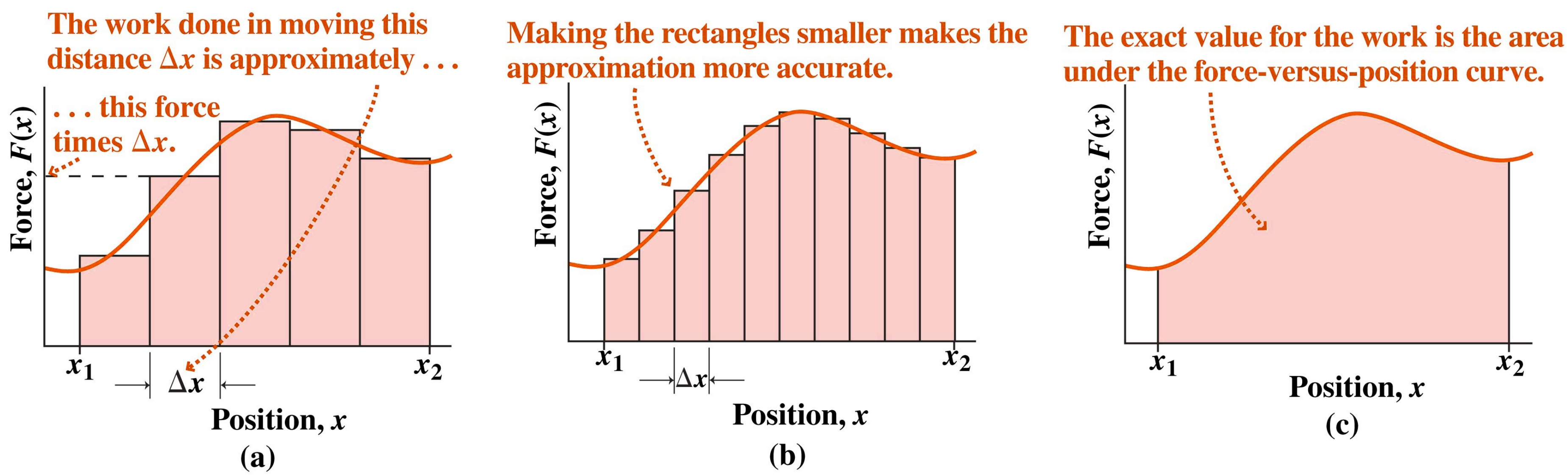
- Work is the scalar product of force with displacement:

$$W = \vec{F} \cdot \Delta\vec{r}$$

Work done by a varying force

When a force varies with position, it's necessary to integrate to calculate the work done.

Geometrically, the work is the area under the force-versus-position curve.



Integration

The *definite integral* is the result of the limiting process in which the area is divided into ever smaller regions.

Work as the integral of the force F over position x is written

$$W = \int_{x_1}^{x_2} F(x)\,dx$$

- Integration is the opposite of differentiation, so integrals of simple functions are readily evaluated. For powers of x, the integral becomes

$$\int_{x_1}^{x_2} x^n\,dx = \left.\frac{x^{n+1}}{n+1}\right|_{x_1}^{x_2} = \frac{x_2^{\,n+1}}{n+1} - \frac{x_1^{\,n+1}}{n+1}$$

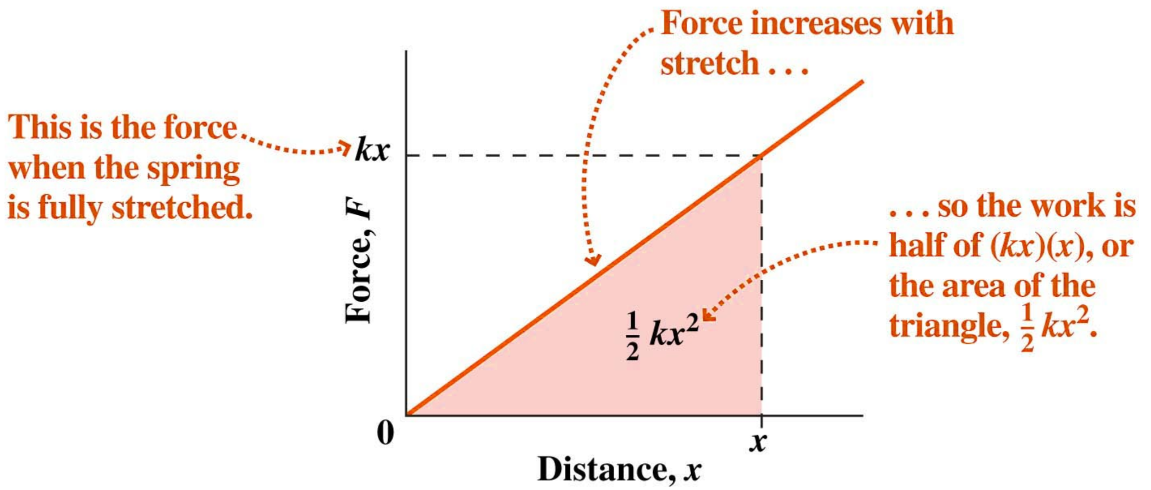
Work done in stretching a spring

A spring exerts a force $F_{\text{spring}} = -kx$.

Therefore the agent stretching a spring exerts a force $F = +kx$, and the work the agent does is

$$W = \int_0^x F(x)\,dx = \int_0^x kx\,dx = \tfrac{1}{2}kx^2 \Big|_0^x = \tfrac{1}{2}kx^2 - \tfrac{1}{2}k(0)^2 = \tfrac{1}{2}kx^2$$

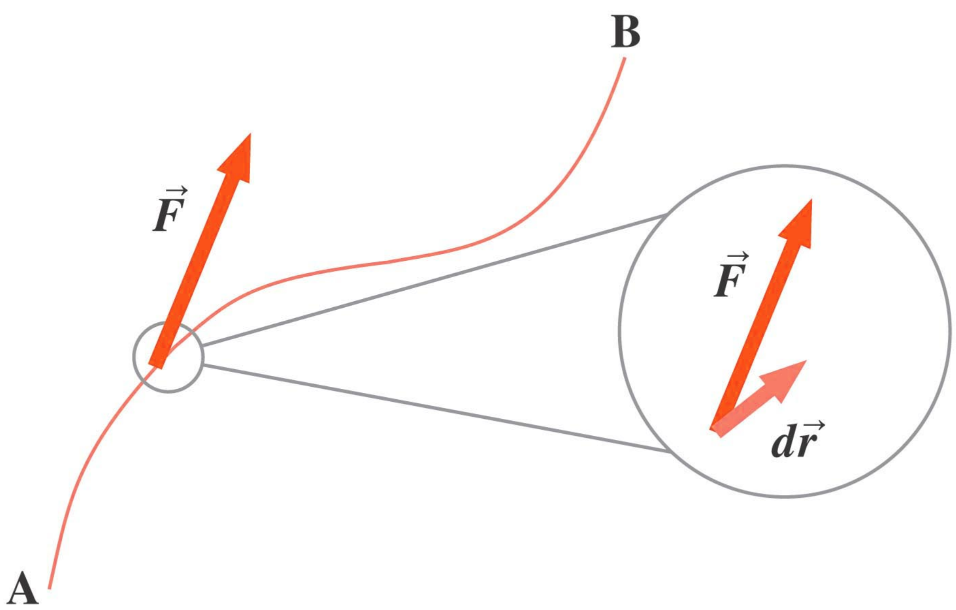
- In this case the work is the area under the triangular force-versus-position curve:



A varying force in multiple dimensions

In the most general case, an object moves on an arbitrary path subject to a force whose magnitude and whose direction relative to the path may vary with position.

In that case the integral for the work becomes a **line integral**, the limit of the sum of scalar products of infinitesimally small displacements with the force at each point.

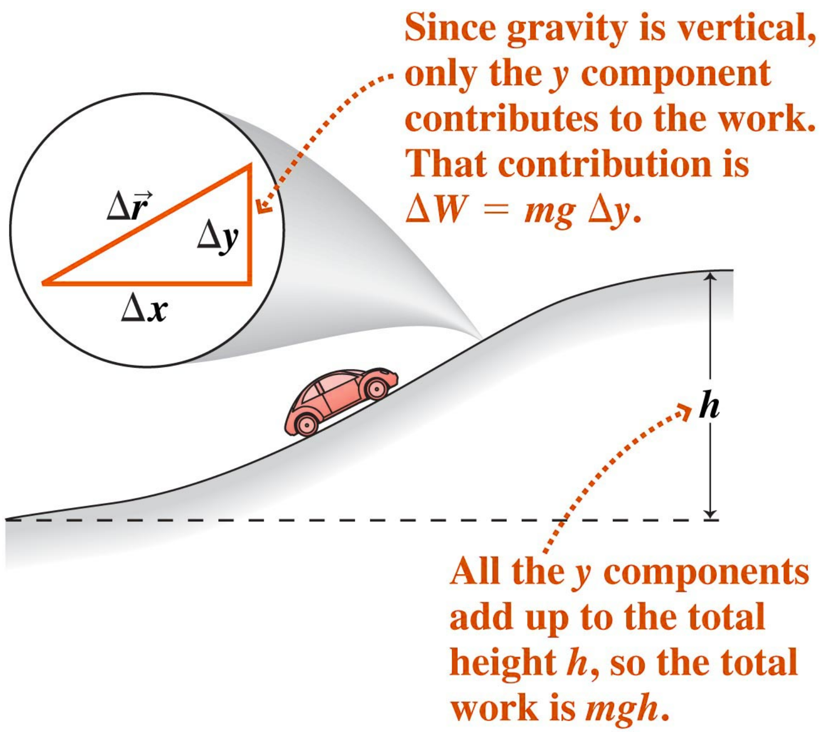


$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

Work done against gravity

The work done by an agent lifting an object of mass m against gravity depends only on the vertical distance h:

$$W = mgh$$



- The work is positive if the object is raised and negative if it's lowered.

Clicker question

Three forces have magnitudes in newtons that are numerically equal to these quantities: (A) $\sqrt{x}$, (B) x, and (C) x^2, where x is the position in meters. Each force acts on an object as it moves from $x = 0$ to $x = 1$ m. Notice that all three forces have the same values at the two endpoints—namely, 0 N and 1 N. Which force does the most work?

Work and net work

The work *you* do in moving an object involves only the force *you* apply:

But there may be other forces acting on the object as well.

The **net work** is the work done by all the forces acting—that is, the work done by the net force.

Example:

Lift an object at constant speed, and you do work mgh.

But gravity, acting downward, does work $-mgh$.

So the net work in this case is zero.

The work-energy theorem

Applying Newton's second law to the net work done on an object results in the **work-energy theorem:**

$$W_{\text{net}} = \int F_{\text{net}}\, dx = \int ma\, dx = \int m \frac{dv}{dt}\, dx = \int m \frac{dx}{dt}\, dv = \int mv\, dv$$

- Evaluating the last integral between initial and final velocities v_1 and v_2 gives

$$W_{\text{net}} = \int_{v_1}^{v_2} mv\, dv = \tfrac{1}{2} mv^2 \Big|_{v_1}^{v_2} = \tfrac{1}{2} mv_2^2 - \tfrac{1}{2} mv_1^2$$

- So the quantity $\tfrac{1}{2} mv^2$ changes only when net work is done on an object, and the change in this quantity is equal to the net work.

Kinetic energy and the work-energy theorem

- The quantity $\frac{1}{2}mv^2$ is called **kinetic energy**, K. Kinetic energy is a kind of energy associated with motion:

The kinetic energy K of an object of mass m moving at speed v is

$$K = \tfrac{1}{2}mv^2$$

- Then the work-energy theorem states that the change in an object's kinetic energy is equal to the net work done on the object:

$$\Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = W_{net}$$

Power and energy

- **Power** is the *rate* at which work is done or at which energy is used or produced. If work ΔW is done in time Δt, then the **average power** over this time is

$$\overline{P} = \frac{\Delta W}{\Delta t} \quad \text{(average power)}$$

- When the rate changes continuously, the **instantaneous power** is

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

- Power is measured in **watts** (W), with 1 W = 1 J/s.

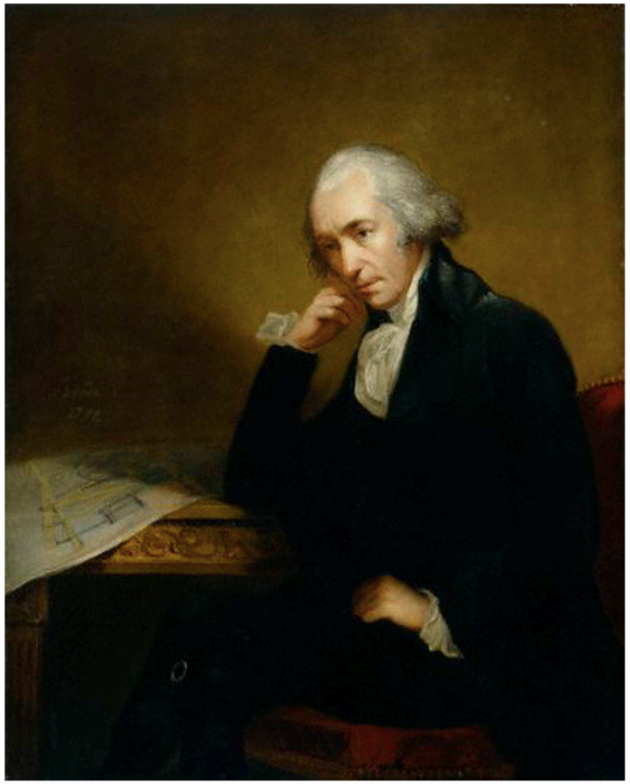
- Total work or energy follows from power by multiplying (for constant power) or integrating (for varying power):

$$W = P\,\Delta t \quad \text{or} \quad W = \int_{t_1}^{t_2} P\,dt$$

SI Units: Work/Energy/Power

James Prescott Joule (1818-1889)

James Watt (1736-1819)



$$J = \frac{kg \cdot m^2}{s^2} = N \cdot m = Pa \cdot m^3 = W \cdot s$$