## Overview of BSc Course Mechanica (I)

## Mechanica I: TN4110TA

BSc aardwetenschappen, periode 3

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## Boek:

Vanaf cursusjaar 2009-2010 gebruiken we een nieuw boek: 'Essential University Physics' door Richard Wolfson. Voor opgaven zal ondermeer gebruik gemaakt worden van het bijbehorende 'Mastering Physics' pakket. Hoewel voor mechanica alleen volume 1 van het boek nodig is, is het aan te raden om een value-pack te kopen met volume 1 en 2 (wordt in 2 e jaar gebruikt) en het Mastering Physics pakket (met speciale korting bij VSSD te koop [ Euro 36.75] ; www.vssd.nl)

Zorg dat je het boek op tijd hebt; de persoonlijke code in de 'student access kit' van MasteringPhysics is essentieel om aan het eerste (computer) werkcollege te kunnen deeinemen!


[^0]
## Overview of BSc Course Mechanica (I)

## Toetsen:

Het vak wordt afgesloten met een tentamen.
Deze bepaalt 70\% van je eindcijfer.
Het overige deel wordt bepaald door resultaten van toetsen tijdens de hoorcolleges (15\%) an door resultaten van Mastering Physics opgaven (15\%). Resultaten van toetsen in voorgande jaren tellen niet mee.

## College roster 2010/2011:

Hoorcolleges op woensdag: 10:45-12:30 (Aula, zaal D)
Werkcolleges op vrijdag: 10:45-12:30 (CT gebouw,wisselende locaties)
Week van 21-25 februari vakantie - geen colleges

## Essential University Physics

Richard Wolfson


## In this lecture you'll learn

- The different realms of physics, and their applications in both natural and technological systems
- The SI unit system
- How to express and manipulate numbers in physics calculations
- Scientific notation

- Accuracy and significant figures
- Making quick estimates
- A universal strategy for solving physics problems


## Realms of physics

- Physics provides a unified description of basic principles that govern physical reality.
- It's convenient to divide physics into a number of different but related realms.
- Here we consider six distinct realms:



## The SI unit system

- Provides precise definitions of seven fundamental physical quantities
- Length: the meter
- Time: the second
- Mass: the kilogram
- Electric current: the ampere
- Temperature: the kelvin
- Amount of a substance: the mole
- Luminous intensity: the candela
- Supplementary units describe angles
- Plane angle: the radian


## Operational definitions

- Of the three most basic units-length, time, and masstwo are defined operationally, so their definitions can be implemented in any laboratory.
- The meanings of both these definitions will become clearer as you advance in your study of physics:
- The meter is the length of the path traveled by light in vacuum during a time interval of $1 / 299,792,458$ of a second.
- The second is the duration of $9,192,631,770$ periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium-133 atom.
- The standard of mass is less satisfactory:
- The kilogram is defined by the international prototype kilogram kept at the International Bureau of Weights and Measures at Sèvres, France.


## Scientific notation

- The vast range of quantities that occur in physics are best expressed with ordinary-sized numbers multiplied by powers of 10 :
- $31416.5=3.14165 \bullet 10^{4}$
- $0.002718=2.718 \bullet 10^{-3}$
- SI prefixes describe powers of 10 :
- Every three powers of 10 gets a different prefix.
- Examples:
- $3.0 \bullet 10^{9} \mathrm{~W}=3.0 \mathrm{GW}$ (3 gigawatts)
- $1.6 \bullet 10^{-8} \mathrm{~m}=16 \mathrm{~nm}$ (16 nanometers)
- $10^{12} \mathrm{~kg}=1 \mathrm{Pg}$ (1 petagram)

TABLE 1.1 SI Prefixes

| Prefix | Symbol | Power |
| :--- | :---: | :--- |
| yotta | Y | $10^{24}$ |
| zetta | Z | $10^{21}$ |
| exa | E | $10^{18}$ |
| peta | P | $10^{15}$ |
| tera | T | $10^{12}$ |
| giga | G | $10^{9}$ |
| mega | M | $10^{6}$ |
| kilo | k | $10^{3}$ |
| hecto | h | $10^{2}$ |
| deca | da | $10^{1}$ |
| - | - | $10^{0}$ |
| deci | d | $10^{-1}$ |
| centi | c | $10^{-2}$ |
| milli | m | $10^{-3}$ |
| micro | H | $10^{-6}$ |
| nano | n | $10^{-9}$ |
| pico | p | $10^{-12}$ |
| femto | f | $10^{-15}$ |
| atto | a | $10^{-18}$ |
| zepto | z | $10^{-21}$ |
| yocto | y | $10^{-24}$ |

## Converting units

- Units matter! Measures of physical quantities must always have the correct units.
- Conversion tables (Appendix C of the textbook) give relations between physical quantities in different unit systems:
- Convert units by multiplying or dividing so that the units you don't want cancel, leaving only the units you do want.
- Example: Since $1 \mathrm{ft}=0.3048 \mathrm{~m}$, a 5280 -foot race ( 1 mile ) is equal to

$$
(5280 \mathrm{ft}) \frac{0.3048 \mathrm{~m}}{1 \mathrm{ft}}=1609 \mathrm{~m}
$$

- Example: 1 kilowatt-hour ( kWh , a unit of energy) is equivalent to 3.6 megajoules (MJ, another energy unit). Therefore a monthly electric consumption of 343 kWh amounts to

$$
(343 \mathrm{kWh}) \frac{3.6 \mathrm{MJ}}{1 \mathrm{kWh}}=1.23 \times 10^{3} \mathrm{MJ}=1.23 \mathrm{GJ}
$$

## Significant figures

- The answer to the last example in the preceding slide is 1.23 GJ not 1234.8 MJ or 1.2348 GJ as your calculator would show.
- That's because the given quantity, 343 kWh , has only three significant figures.
- That means we know that the actual value is closer to 343 kWh than to 342 kWh or 344 kWh .
- If we had been given 343.2 kWh , we would know that the value is closer to 343.2 kWh than to 343.1 kWh or 343.3 kWh .
- In that case, the number given has four significant figures.
- Significant figures tell how accurately we know the values of physical quantities.
- Calculations can't increase that accuracy, so it's important to report the results of calculations with the correct number of significant figures.


## Rules for significant figures

- In multiplication and division, the answer should have the same number of significant figures as the least accurate of the quantities entering the calculation.
- Example: $(3.1416 \mathrm{~N})(2.1 \mathrm{~m})=6.6 \mathrm{~N} \cdot \mathrm{~m}$
- Note the centered dot, normally used when units are multiplied (the kWh is an exception).
- In addition and subtraction, the answer should have the same number of digits to the right of the decimal point as the term in the sum or difference that has the smallest number of digits to the right of the decimal point.
- Example: $3.2492 \mathrm{~m}-3.241=0.008 \mathrm{~m}$
- Note the loss of accuracy, with the answer having only one significant figure.


## A strategy for problem solving

- The IDEA strategy consists of four broad steps.
- IDEA is not a "cookbook" but rather a general framework to organizing your path to the solution of a problem.
- The four IDEA steps are
- INTERPRET
- DEVELOP
- EVALUATE
- ASSESS


## INTERPRET

interpret You can't begin a problem unless you're sure what it's about. So the first step is to interpret the problem to be sure you know what it's asking. Then identify the applicable concepts and principles-Newton's laws of motion, the conservation of energy principle, the first law of thermodynamics, Gauss's law for electricity, and so forth. Also identify the players in the situation-the object whose motion you're asked to describe, the forces acting on an object, the thermodynamic system whose heat flow you're going to determine, the charges that produce an electric field, the components in an electric circuit whose power consumption you're after, the light rays that will help you locate an image, and so on.

## DEVELOP

develop The second step is to develop a plan for solving the problem. As part of this step, it's always helpful and often essential to draw a diagram showing the essential aspects of the situation. Your drawing should indicate objects, forces, and other physical entities. Labeling masses, positions, forces, velocities, heat flows, electric or magnetic fields, and other quantities will be a big help. Next, determine the relevant mathematical formulas-namely, those that contain the quantities you're given in the problem as well as the unknown(s) you're solving for. Don't just grab equations-rather, think about how each reflects the underlying concepts and principles that you've identified as applying to this problem. The plan you develop might include calculating intermediate quantities, finding values in a table or in one of this text's several appendices, or even solving a preliminary problem whose answer you need to get your final result.

## EVALUATE

evaluate Physics problems have numerical or symbolic answers, and you need to evaluate your answer. In this step you execute your plan, going in sequence through the steps you've outlined. Here's where your math skills come in. Use algebra, trig, or calculus, as needed, to solve your equations. It's a good idea to keep all numerical quantities, whether known or not, in symbolic form as you work through the solution of your problem. At the end you can plug in numbers and work the arithmetic to evaluate the numerical answer, if the problem calls for one.

## ASSESS

ASSESS Don't be satisfied with your answer until you ask whether it makes sense! Are the units correct? Do the numbers sound reasonable? Does the algebraic form of your answer work in obvious special cases, like perhaps "turning off" gravity or making an object's mass zero or infinite? Checking special cases not only helps you decide whether your answer makes sense but can also give you insights into the underlying physics. In worked examples, we'll often use this step to enhance your knowledge of physics by relating the example to other applications of physics.

## Summary

- Together, the different realms of physics provide a unified description of basic principles that govern physical reality.
- The SI unit system provides precise definitions of fundamental physical quantities.
- Those with operational definitions can be reproduced anywhere.
- Handling the numbers that represent physical quantities involves
- Using scientific notation and SI prefixes
- Understanding significant figures
- Estimation
- The IDEA strategy provides a general framework for problem solving in physics.


## Essential University Physics

Richard Wolfson


## In this lecture you'll learn

- The fundamental quantities that describe motion
- Position
- Velocity
- Acceleration
- The difference between average and instantaneous quantities
- The use of calculus to find instantaneous values
- How to solve problems involving constant acceleration in one dimension
- Including the constant acceleration of gravity near Earth's surface



## Position and displacement

- In one dimension, position can be described by a positive or negative number on a number line, also called a coordinate system.
- Position zero, the origin of the coordinate system, is arbitrary and you're free to choose it wherever it's convenient.
- Displacement is change in position.
- For motion along the $x$ direction, displacement is designated $\Delta x$ :

$$
\Delta x=x_{2}-x_{1}
$$

where $x_{1}$ and $x_{2}$ are the initial and final positions, respectively.


## Velocity

- Velocity is the rate of change of position.
- Average velocity over a time interval $\Delta t$ is defined as the displacement divided by the time:

$$
\bar{v}=\frac{\Delta x}{\Delta t}
$$

- Instantaneous velocity is the limit of the average velocity as the time interval becomes arbitrarily short:

$$
v=\lim _{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}=\frac{d x}{d t}
$$

- In calculus, this limiting procedure defines the derivative $d x / d t$.
- Speed is the magnitude of velocity.

- Velocity is the slope of the position-versus-time curve.


## Clicker question

- The figure shows position-versus-time graphs for four objects. Which starts slowly and then speeds up?

A.

B.

C.

D.


## Using calculus to find derivatives

- In calculus, the derivative gives the result of the limiting procedure.
- Derivatives of powers are straightforward:

$$
\frac{d\left(b t^{n}\right)}{d t}=b n t^{n-1}
$$

- Other common derivatives include the trig functions:

$$
\begin{aligned}
& \frac{d(\sin b t)}{d t}=b \cos b t \\
& \frac{d(\cos b t)}{d t}=-b \sin b t
\end{aligned}
$$

## Acceleration

- Acceleration is the rate of change of velocity.
- Average velocity over a time interval $\Delta t$ is defined as the change in velocity divided by the time:

$$
\bar{a}=\frac{\Delta v}{\Delta t}
$$

- Instantaneous acceleration is the limit of the average acceleration as the time interval becomes arbitrarily short:

$$
a=\lim _{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}=\frac{d v}{d t}
$$

- Acceleration is the slope of the velocity-versus-time curve.

(c)


## Position, velocity, and acceleration

- Individual values of position, velocity, and acceleration aren't related.
- Instead, velocity depends on the rate of change of position.
- Acceleration depends on the rate of change of velocity.
- An object can be at position $x=0$ and still be moving.
- An object can have zero velocity and still be accelerating.

At the peak of its trajectory, a ball has zero velocity, but it's still accelerating.


## Constant acceleration

- When acceleration is constant, then position, velocity, acceleration, and time are related by

$$
\begin{aligned}
& v=v_{0}+a t \\
& x=x_{0}+\frac{1}{2}\left(v_{0}+v\right) t \\
& x=x_{0}+v_{0} t+\frac{1}{2} a t^{2} \\
& v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right)
\end{aligned}
$$

where $x_{0}$ and $v_{0}$ are initial values at time $t=0$, and $x$ and $v$ are the values at an arbitrary time $t$.

- With constant acceleration
- Velocity is a linear function of time
- Position is a quadratic function of time

With no


## Clicker question

- A speeding motorist zooms past a stationary police car, which then heads after the speeder. The police car starts with zero velocity and is going at twice the car's velocity when it catches up to the car. So at some intermediate instant the police car must be going at the same velocity as the speeding car. When is that instant?
A. Closer to the time when the police car starts chasing
B. Closer to the time when the police car catches the speeder
C. Halfway between the times when the two cars coincide


## The acceleration of gravity

- The acceleration of gravity at any point is exactly the same for all objects, regardless of mass.
- Near Earth's surface, the value of the acceleration is essentially constant at $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$.
- Therefore the equations for constant acceleration apply:
- In a coordinate system with $y$ axis upward, they read

$$
\begin{aligned}
& v=v_{0}-g t \\
& y=y_{0}+\frac{1}{2}\left(v_{0}+v\right) t \\
& y=y_{0}+v_{0} t-\frac{1}{2} g t^{2} \\
& v^{2}=v_{0}^{2}-2 g\left(y-y_{0}\right)
\end{aligned}
$$



This strobe photo of a falling ball shows increasing spacing resulting from the acceleration of gravity.

## Example: the acceleration of gravity

- A ball is thrown straight up at 7.3 $\mathrm{m} / \mathrm{s}$, leaving your hand 1.5 m above the ground. Find its maximum height and when it hits the floor.
- At the maximum height the ball is instantaneously at rest (even though it's still accelerating). Solving the last equation with $v=0$ gives the maximum height:
$0=v_{0}^{2}-2 g\left(y-y_{0}\right)$
or

$$
y=y_{0}+\frac{v_{0}^{2}}{2 g}=1.5 \mathrm{~m}+\frac{(7.3 \mathrm{~m} / \mathrm{s})^{2}}{(2)\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)}=4.2 \mathrm{~m}
$$

- Setting $y=0$ in the third equation gives a quadratic in time; the result is the two values for the time when the ball is on the floor:
$t=-0.18 \mathrm{~s}$ and $t=1.7 \mathrm{~s}$
- The first answer tells when the ball would have been on the floor if it had always been on this trajectory; the second is the answer we want.



## Clicker question

- Standing on a roof, you simultaneously throw one ball straight up and drop another from rest. Which hits the ground moving faster?
A. The ball dropped from rest
B. The ball thrown straight up


## Summary

- Position, velocity, and acceleration are the fundamental quantities describing motion.
- Velocity is the rate of change of position.
- Acceleration is the rate of change of velocity.

- When acceleration is constant, simple equations relate position, velocity, acceleration, and time.

$$
\begin{aligned}
& v=v_{0}+a t \\
& x=x_{0}+\frac{1}{2}\left(v_{0}+v\right) t
\end{aligned}
$$

- An important case is the acceleration due to gravity near Earth's surface.
- The magnitude of the gravitational acceleration is $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$.


## Essential University Physics

Richard Wolfson

Motion in Two
and Three Dimensions

PowerPoint ${ }^{\circledR}$ Lecture prepared by Richard Wolfson

## In this lecture you'll learn

- To describe position, velocity, and acceleration in three-dimensional space using the language of vectors
- To manipulate vectors algebraically
- To transform velocities to different reference frames
- To solve problems involving constant acceleration in two dimensions

- Including projectile motion due to the constant acceleration of gravity near Earth's surface
- To evaluate acceleration in circular motion


## Vectors

- A vector is a quantity that has both magnitude and direction.
- In two dimensions it takes two numbers to specify a vector.
- In three dimensions it takes three numbers.
- A vector can be represented by an arrow whose length corresponds to the vector's magnitude.
- Position is a vector quantity.
- An object's position is specified by giving its distance from an origin and its direction relative to an axis.
- Here $r_{1}$ describes a point 2.0 m from the origin at a $30^{\circ}$ angle to the axis.

The vector $\vec{r}_{1}$ describes the position of this point.


## Adding vectors

- To add vectors graphically, place the tail of the first vector at the head of the second.
- Their sum is then the vector from the tail of the first vector to the head of the second.
- Here $\vec{r}_{2}$ is the sum of $\vec{r}_{1}$ and $\Delta \vec{r}$.



## Vector arithmetic

- To multiply a vector by a scalar, multiply the vector's magnitude by the scalar.
- For a positive scalar the direction is unchanged.
- For a negative scalar the direction reverses.
- To subtract vectors, add the negative of the second vector to the first:

$$
\vec{A}-\vec{B}=\vec{A}+(-\vec{B})
$$

- Vector arithmetic is commutative and associative:



## Unit vectors

- Unit vectors have magnitude 1, no units, and point along the coordinate axes.
- They're used to specify direction in compact mathematical representations of vectors.
- Unit vectors in the $x, y$, and $z$ directions are designated
$i, \vec{j}$ and $\vec{k}$
- Any vector in two dimensions can be written as a linear combination of $\dot{i}$ and $\vec{j}$

- Any vector in three dimensions can be written as a linear combination of

$$
\stackrel{i}{i}, \vec{j} \text { and } \vec{k}
$$

## Vector arithmetic with unit vectors

- To add vectors, add the individual components:
- If $\vec{A}=A_{x} \dot{i}+A_{y} \vec{j}$ and $\vec{B}=B_{x} \dot{i}+B_{y} \vec{j}$
- then $\vec{A}+\vec{B}=\left(A_{x}+B_{x}\right) \dot{i}+\left(A_{y}+B_{y}\right) \vec{j}$
- To multiply by a scalar, distribute the scalar; that is, multiply the individual components by the scalar:
- If $\vec{A}=A_{x} \dot{i}+A_{y} \vec{j}$
- then $c \vec{A}=c A_{x} \dot{i}+c A_{y} \vec{j}$


## Velocity and acceleration vectors

- Velocity is the rate of change of position.
- The average velocity over a time interval $\Delta t$ is the change in the position vector divided by the time.
- Here dividing by $\Delta t$ means multiplying by the scalar $1 / \Delta t$ :

$$
\vec{v}=\frac{\Delta \vec{r}}{\Delta t}
$$

- Instantaneous velocity is the time derivative of position:

$$
\vec{v}=\lim _{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t}=\frac{d \vec{r}}{d t}
$$

- Acceleration is the rate of change of velocity:

$$
\text { Average : } \vec{a}=\frac{\Delta \vec{v}}{\Delta t} \quad \text { Instantaneous : } \vec{a}=\frac{d \vec{v}}{d t}
$$

## Velocity and acceleration in two dimensions

- An acceleration $\vec{a}$ acting for time $\Delta t$ produces a velocity change $\Delta \vec{v}=\vec{a} \Delta t$
- The change adds vectorially to give the new velocity:

$$
\vec{v}=\vec{v}_{0}+\vec{a} \Delta t
$$

- The new velocity depends on the magnitude of the acceleration as well as its direction:
$\dot{a}$ and $\dot{v}$ colinear: only speed changes

$\Delta \vec{v}=\vec{a} \Delta t$
$\dot{a}$ and $v$ perpendicular:
only direction changes


In general:
both speed and direction change

$\vec{v}=\vec{v}_{0}+\vec{a} \Delta t$

## Clicker question

- An object is accelerating downward. Which of the following must be true?
A. The object is moving directly downward.
B. If the object's motion is instantaneously horizontal, it can't continue to be so.
C. The object cannot be moving in a straight line.
D. The object cannot be moving upward.


## Relative motion

- An object moves with velocity $\vec{v}$ relative to one frame of reference.
- That frame moves at $\vec{V}$ relative to a second reference frame.
- Then the velocity of the object relative to the second frame is $v=v^{\prime}+V$
- Example:
- A jetliner flies at 960 $\mathrm{km} / \mathrm{h}$ relative to the air, heading northward. There's a wind blowing eastward at $190 \mathrm{~km} /$ h. In what direction should the plane fly?
- The vector diagram identifies the quantities in the equation, and shows that the angle is $11^{\circ}$.



## Constant acceleration

- With constant acceleration, the equations for onedimensional motion apply independently in each direction.
- The equations take a compact form in vector notation.
- Each equation stands for two or three separate equations.

$$
\begin{aligned}
& \vec{v}=\overrightarrow{v_{0}}+\vec{a} t \\
& \vec{r}=\overrightarrow{r_{0}}+\overrightarrow{v_{0}} t+\frac{1}{2} \vec{a} t^{2}
\end{aligned}
$$

## Clicker question

- An object is moving initially in the $+x$ direction. Which of the following accelerations, all acting for the same time interval, will cause the greatest change in its speed?
A. $10 \vec{j} \mathrm{~m} / \mathrm{s}^{2}$
B. $2 \bar{i}-8 \vec{j} \mathrm{~m} / \mathrm{s}^{2}$
C. $10 \dot{i} \mathrm{~m} / \mathrm{s}^{2}$
D. $10 \dot{i}+5 \vec{j} \mathrm{~m} / \mathrm{s}^{2}$


## Projectile motion

- Motion under the influence of gravity near Earth's surface has essentially constant acceleration $\dot{g}$ whose magnitude is $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$, and whose direction is downward.
- Such motion is called projectile motion.
- Equations for projectile motion, in a coordinate system with $y$ axis vertically upward:

$$
\begin{aligned}
& v_{x}=v_{x 0} \\
& v_{y}=v_{y 0}-g t \\
& x=x_{0}+v_{x 0} t \\
& y=y_{0}+v_{y 0} t-\frac{1}{2} g t^{2}
\end{aligned}
$$

- Horizontal and vertical motions are independent:



## Projectile trajectories

- The trajectory of an object in projectile motion is a parabola, unless the object has no horizontal component of motion.
- Horizontal motion is unchanged, while vertical motion undergoes downward acceleration:

- Equation for the trajectory:

$$
y=x \tan \theta_{0}-\frac{g}{2 v_{0}^{2} \cos ^{2} \theta_{0}} x^{2}
$$



## Projectile trajectories/derivation:



## Uniform circular motion

- When an object moves in a circular path of radius $r$ at constant speed $v$, its acceleration has magnitude

$$
a=\frac{v^{2}}{r}
$$

- The acceleration vector points toward the center of the circle.
- Since the direction of the acceleration keeps changing, this is not constant acceleration.

The velocities
are tangent to


## Nonuniform circular motion

- In nonuniform circular motion, speed and path radius can both change.
- Then the acceleration has components perpendicular and parallel to the velocity.
- The figure shows a car braking as it rounds a curve.

The car is slowing, so
its tangential acceleration $\vec{a}_{t}$ is opposite its velocity.


## Clicker question

- The figure shows velocity vectors for four points on a noncircular path. Choose the correct order, from smallest to largest, of the centripetal accelerations at these points given $v_{1}=v_{4}$ and $v_{2}=v_{3}$.
A. $a_{1}>a_{4}>a_{3}>a_{2}$
B. $a_{2}>a_{3}>a_{4}>a_{1}$
C. $a_{3}>a_{2}>a_{1}>a_{4}$
D. $a_{2}>a_{3}>a_{1}>a_{4}$



## Summary

- In two and three dimensions, position, velocity, and acceleration become vector quantities.
- Velocity is the rate of change of position: $\vec{v}=\frac{d \vec{r}}{d t}$
- Acceleration is the rate of change of velocity: $\quad \vec{a}=\frac{d \vec{v}}{d t}$
- In general, acceleration changes both the magnitude and direction of the velocity.
- Projectile motion results from the acceleration of gravity.
- In uniform circular motion, the acceleration has magnitude $v^{2} / r$ and points toward the center of the circular path.


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