MECHANICS 2, 30-11-2017

The exam consists of two parts. ***The first part is multiple-choice questions. For this part, you only have to give the answer****. Each correct answer is worth 0.4 points, up to a total of 2 points.* ***The second part consists of open questions. For these questions, you have to motivate your answers, i.e., show all the steps that lead to the answer you provide****. Answers without motivation are considered wrong and bring no points. Answers that show only part of the required steps to reach the correct answer will result in receiving a proportional part of points from the total amount of points given for this answer. (For example, if a subquestion brings 1 point, but you only show half of the path to the correct answer, you will get 0.5 points.) An open question might have alternative paths of reaching the correct answer; all such paths are considered correct. The total number of points for the open questions is 8. If an open question consists of subquestions, translation of the result of a wrong calculation from one subquestion to another subquestion is not judged as an error.*

Multiple choice, 5 assignments, 0.4 point per question

1) A certain frictionless simple pendulum having a length L and mass M swings with period T. If both L and M are doubled, what is the new period?
A) $4T$
B) $2T$
C) $\sqrt{2}T$
D) $T/\sqrt{2}$
E) $T/4$
Answer: C

2) A 56-kg bungee jumper jumps off a bridge and undergoes simple harmonic motion. If the period of oscillation is 6.28 s, what is the spring constant of the bungee cord, assuming it has negligible mass compared to that of the jumper?
A) 112 N/m
B) 56 N/m
C) 28 N/m
D) 14 N/m
E) 7 N/m
Answer: B

3) A pipe that is 120 cm long resonates to produce sound of wavelengths 480 cm, 160 cm, and 96 cm but does not resonate at any wavelengths longer than these. Thus, one can say:
A) This pipe is open at both ends.
B) This pipe is open at one end and closed at the other end.
C) This pipe is closed at both ends.
D) This pipe is either open at both ends or closed at both ends.
E) It is unclear what is the condition of the ends because the frequency is not given.
Answer: B

4) Two pure tones are being sounded simultaneously. As a result, a particular beat frequency is heard. What happens to the beat frequency if the frequency of one of the tones is increased?
A) It increases.
B) It decreases.
C) It does not change.
D) It becomes zero.
E) Cannot be answered from the given information.
Answer: E

5) For the derivation of the acoustic wave equation in the lecture, what are the two starting equations we used?
A) Newton’s second law and the equation of motion.
B) The equation of motion and Hook’s law for solids.
C) Hook’s law for solids and Newton’s first law.
D) Hook’s law for fluids and the equation of motion.
E) Newton’s second law and Hook’s law for solids.
Answer: D

Open questions, 6 assignments

6) You are observing a torsional oscillator moving around its equilibrium following a simple harmonic motion. The maximum angle that the torsional oscillator makes with the equilibrium is 0.1 rad. Somehow, you estimate the period of oscillation to be π s and the inertia to be 10 kg.m^2.
(a) What is torsional constant of the oscillator (in Nm)? ***(0.5 points)***
(b) What is the maximum energy of the system (in Joules)? ***(0.6 points)***
(c) What is the maximum acceleration of the torsional oscillator (in m/s^2)? ***(0.4 points)***
Give the answers till the first digit after the decimal point
Answer:
(a) We know that for a torsional oscillator the relation for the angular frequency is: $\omega = \sqrt{\frac{\kappa}{I}}$. As the period is known, we can calculate the torsional constant from $\omega = \frac{2\pi}{T} = \sqrt{\frac{\kappa}{I}}$. Thus, $\frac{\kappa}{I} = \frac{4*(\pi)^2}{(\pi)^2} \Longrightarrow \kappa = 10 * 4 = 40\ (Nm)$.
(b) The maximum energy of a mass/spring system in simple harmonic motion is equal to the total energy, which is $E_{max} = \frac{1}{2}kA^2\ (J)$. In analogy with such a system, the maximum energy of the torsional oscillator during simple harmonic motion is $E_{max} = \frac{1}{2}\kappa A^2\ (J)$, where the spring constant is exchanged with the torsional constant. The amplitude is the maximum displacement from the equilibrium or in this case the maximum angle: $A = 0.1\ (rad)$. From here, we can calculate the maximum energy: $E_{max} = \frac{1}{2}\kappa A^2 = \frac{1}{2} * 40 * 0.01 = 0.2\ (J)$.
(c) We know that the acceleration is the second derivative of the angular displacement with respect to time: $a(t) = \frac{d^2\theta(t)}{dt^2} = \frac{d^2(A\cos(\omega t))}{dt^2} = -\omega^2 A\cos(\omega t)$. We are interested in the maximum acceleration, which is achieved when $\cos(\omega t) = 1$. Thus, $a_{max} = |-\omega^2 A * 1| = \left(\frac{2\pi}{\pi}\right)^2 * 0.1 = 0.4\ \left(\frac{m}{s^2}\right)$.

7) An object is undergoing simple harmonic motion with frequency f = 5 Hz and an amplitude of 0.12 m. At t = 0.00 s the object is at x = 0.00 m. How long does it take the object to go from x = 0.00 m to x = 0.06 m? ***(0.8 points)***
Give the answer till the second digit after the decimal point.
Answer:
An object undergoes simple harmonic motion is characterized by an $x(t) = Acos(\omega t + \varphi)$. As the objects displacement is 0 m at time 0 s, we obtain that $0 = cos(\varphi)$, and consequently that $\varphi = -90°$. As we know, $cos(\alpha - 90) = sin(\alpha)$, so the equation of displacement is $x(t) = Asin(\omega t) = Asin(2\pi f t)$. As the amplitude and frequency are given, we can write $x(t) = 0.12sin(2\pi * 5 * t)$. At x=0.06 m, $0.06 = 0.12sin(2\pi * 5 * t)$ or $sin(2\pi * 5 * t) = 0.5$. We know that $sin(30°) = 0.5$, and, thus, $2\pi * 5 * t = 30° = \frac{30° * 2\pi}{360°}\ (rad)$. From here, we can calculate that $t = \frac{30*2\pi}{360*5*2\pi} = \frac{1}{12*5} = 0.02$.

8) You are making a scientific experiment with a thin glass rod. You need to draw a graph of its potential energy when one of its ends is clamped, while the other end is bended in the horizontal direction to different points to the left and wright from the

position at rest. This forms a parabola of the form ax^2. Unfortunately, already at the first bending the rod breaks. But you know the mass of the rod, which is 100 g, and its resonant frequency, which is $\frac{10}{2\pi}$ kHz. What is the potential energy at 1 mm and at -2.5 mm? ***(1.7 points)***
Give the answer rounded to the closest integer number.
Answer:
This is a problem actually asks the opposite to what is asked in problem 80 from Chapter 13 and the solution is very close to problem 8 from the exam in July.
The vibrating glass rod can be thought of as a vibrating mass/spring system with a certain effective mass. That does not necessarily mean that we are dealing with simple harmonic motion. But we are told that the potential energy of the vibrating rod forms a parabola, i.e., a graph for which the potential energy is linearly proportional to the square of the displacement. From the lectures, we know that such a motion is in fact a simple harmonic motion.
For a simple harmonic motion of a mass/spring system, we know that the potential energy is $U = \frac{1}{2}kx^2$. To calculate the potential energy at the two required points, we need to estimate the effective k. For a mass/spring system oscillating according to simple harmonic motion, the angular frequency is $\omega = 2\pi f = \sqrt{\frac{k}{m}}$. This f is also the resonance frequency of the glass rod. From here we can calculate the effective k: $k = (2\pi f)^2 m = \left(2\pi \frac{10000}{2\pi}\right)^2 0.1 = 10^7 \left(\frac{N}{m}\right)$.
And now, we calculate the potential energy at the required two points:
$U(0.001) = \frac{1}{2}kx^2 = \frac{1}{2} * 10^7 * (10^{-3})^2 = \frac{1}{2} * 10 = 5\ (J)$.
$U(-0.0025) = \frac{1}{2}kx^2 = \frac{1}{2} * 10^7 * (2.5 * 10^{-3})^2 = \frac{1}{2} * 10 * 6.25 = 31\ (J)$.

9) A wall has a rectangular opening of 2.0 m × 1.0 m. You are standing on one side of the wall and have put a loudspeaker on the other side of the wall. The loudspeaker emits signals with equal power in all directions. The intensity level of the sound entering the opening in the wall from the loudspeaker is 80 dB. Assume that the acoustic energy incident upon the ground is completely absorbed and therefore is not reflected into the opening. The threshold of hearing at 1 kHz (or the reference level) is 1.0 × 10-12 W/m2.
(a) What is the acoustic power (in µW) of the signal entering through the window? ***(0.7 points)***
(b) What is the intensity (in nW/m^2) you would measure at a point 100 m away from the wall? For such a distance, you can assume that the opening is a secondary source that emits sound equally well in all directions. You can also assume that the wall has absorbed all other sound from the loudspeaker except the sound passing through the opening. ***(0.3 points)***
Give the answers rounded to the closest integer number.
Answer:
(a) We know that the sound intensity level is $\beta = 10log\frac{I}{I_0}\ (dB)$, where I is the intensity at the measurement point and I_0 is the threshold of hearing at 1 kHz. Using this relation, we can calculate the intensity level at the measurement point, i.e., at the window: $80 = 10log\frac{I}{10^{-12}}\ (dB)$. From here, $I = 10^{\frac{80}{10}} * 10^{-12} = 10^{-4}\ (s)$. We

know that the relation between the intensity I and the power P is I= $\frac{P}{A}\left(\frac{W}{m^2}\right)$, where A is the area over which the intensity is measured. As we know the area of the window and the intensity at the window, which we just calculated, we can calculate the power at the window: $P = IA = 10^{-4} * (2 * 1) = 200\ (\mu W)$.
(b) Because the wall opening can be assumed to be emitting sound equally well in all directions, we can use the following relation between the intensity and the power: $I = \frac{P}{4\pi r^2}\left(\frac{W}{m^2}\right)$. The power of the source is what we calculated in (a), so the intensity at 100 m from the wall is:
$I_{100m} = \frac{200}{4*\pi*100^2}\left(\frac{\mu W}{m^2}\right) = 1.6 * 10^{-3}\left(\frac{\mu W}{m^2}\right) = 2\left(\frac{nW}{m^2}\right)$

10) A 2-metre-long gas column is closed at both ends. If has a fundamental frequency for the standing waves of 40 Hz. What is the speed of sound in this gas for the third harmonic? ***(0.9 point)***
Give the answer rounded to the closest integer number.
Answer:
Because the gas column is closed at both ends, the relation between the length L of the column and the wavelength is $L = m\frac{\lambda}{2}\ (m)$, where $m = 1,2,3, \ldots$ Because we talk about the fundamental frequency, we talk about the fundamental mode, so $m = 1$. Then, the wavelength of the fundamental mode is $\lambda = 2 * 2 = 4\ (m)$. We know that the wave speed is $v = \lambda f$. From here, we can calculate the speed of sound in the gas: v= $4 * 40 = 160\ (Hz)$.
The speed of sound does not depend on the harmonic of vibration, so it will be the same for all harmonics.

11) A carousel that is 2 m in radius has a pair of 1000-Hz sirens mounted on posts at opposite ends of a diameter. The carousel rotates with an angular velocity of 1.7 rad/s. A stationary listener is located at a distance from the carousel. The speed of sound is 340 m/s. What is the maximum beat frequency (in Hz) of the sirens the listener will perceive irrespective of his/her position? ***(2.1 points)***
Give the answer rounded to the closest integer number.
Answer:
To perceive a beat, we have to form one line with two sources of sound characterized by a slightly different frequency. Another possibility is to have two sources emitting sound with the same frequency, but we are away from them at a certain distance in a plane, i.e., to have two-dimensional interference. This, though, cannot be the case here as our answers should not depend on the distance from the sources.
The two sirens emit sounds with the same frequency, but because they move, we are dealing with a Doppler effect. When the listener and the two sirens form one line, one of the sirens is actually approaching the listener, while the other is moving away
Thus, the listener will be reached by two signals with frequencies changed due to the Doppler effect of moving source: $f_{1,2} = \frac{f}{1\pm\frac{u}{v}}\ (Hz)$, where v is the speed of sound in air, u is the speed of the source, and f is the frequency of the siren.
From the Mechanics 1 course, we know that the angular ω and the linear speed v are related through the radius r: $v = \omega r\left(\frac{m}{s}\right)$. Using this relation, we calculate the linear speed: $v = 2 * 1.7 = 3.4\left(\frac{m}{s}\right)$. Now we can calculate the frequency of the

approaching source $f_1 = \frac{1000}{1-\frac{3.4}{340}} = \frac{1000}{1-0.01} = 1010\ (Hz)$ and of the receding source $f_2 = \frac{1000}{1+\frac{3.4}{340}} = \frac{1000}{1+0.01} = 990\ (Hz)$.

Because we are talking about beats, we are talking about constructive and destructive interference which results in us perceiving a new, resulting wave with maximum amplitude reoccurring with a certain frequency. The amplitude of the new perceived wave is $A_{new} = 2A\cos\left(\frac{1}{2}(\omega_1 - \omega_2)t\right) = 2A\cos\left(\frac{1}{2}2\pi(f_1 - f_2)t\right)$, where A is the amplitude of the emitted waves and f_1 and f_2 are the perceived frequencies of the two signals from the sirens due to the Doppler effect. The absolute value of the difference of the two perceived frequencies is the beat frequency. Thus, the maximum beat frequency is $f_{beat} = |f_1 - f_2| = |1010 - 990| = 20\ (Hz)$.