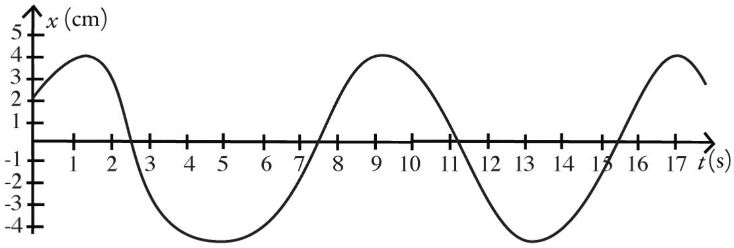
MECHANICS 2, 27-6-2016

The exam consists of two parts. The first part is multiple-choice questions. For this part, you only have to give the answer. Each correct answer is worth 0.25 points, up to a total of 2 points. The second part consists of open questions. For these questions, you have to motivate your answers, i.e., show all the steps that lead to the answer you provide. Answers without motivation are considered wrong and bring no points. Answers that show only part of the required steps to reach the correct answer will result in receiving a proportional part of points from the total amount of points given for this answer. (For example, if a subquestion brings 1 point, but you only show half of the path to the correct answer, you will get 0.5 points.) An open question might have alternative paths of reaching the correct answer; all such paths are considered correct. The total number of points for the open questions is 8. If an open question consists of subquestions, translation of the result of a wrong calculation from one subquestion to another subquestion is not judged as an error.

Multiple choice, 8 assignments, 0.25 point per question

1) The simple harmonic motion of an object is described by the graph shown in the figure. What is the equation for the position $x(t)$ of the object as a function of time t?



A) $x(t) = (8.0\text{ cm})\cos[(2\pi/8.0\text{ s})t - \pi/3.0]$
B) $x(t) = (4.0\text{ cm})\cos[(2\pi/4.0\text{ s})t + \pi/3.0]$
C) $x(t) = (4.0\text{ cm})\cos[(2\pi/8.0\text{ s})t - \pi/3.0]$
D) $x(t) = (4.0\text{ cm})\cos[(2\pi/8.0\text{ s})t + \pi/3.0]$
E) $x(t) = (8.0\text{ cm})\cos[(2\pi/8.0\text{ s})t + \pi/3.0]$
Answer: C

2) The amplitude of a lightly damped harmonic oscillator decreases from 60.0 cm to 40.0 cm in 10.0 s. What will be the amplitude of the harmonic oscillator after another 10.0 s pass?
A) 20.0 cm
B) 16.7 cm
C) 30.0 cm
D) 0.00 cm
E) 26.7 cm
Answer: E

3) If we double only the mass of a vibrating ideal mass-and-spring system, the mechanical energy of the system
A) decreases by a factor of $\sqrt{2}$.
B) decreases by a factor of 2.

C) increases by a factor of 2.
D) increases by a factor of $\sqrt{2}$.
E) does not change.
Answer: E

4) A simple harmonic wave of wavelength 10 cm and amplitude 1.3 cm is propagating along a string in the negative x-direction with a speed of 20 cm/s. Assuming that the displacement at x=0 is a maximum at t=0, what is the mathematical expression describing the displacement y of this wave (in centimeters) as a function of position and time?

A) $y(x,t) = (1.3\text{ cm})\cos\left(\left(0.628\ \frac{1}{\text{cm}}\right)x + \left(12.56\ \frac{1}{\text{s}}\right)t\right)$.

B) $y(x,t) = (1.3\text{ cm})\cos\left(\left(0.628\ \frac{1}{\text{cm}}\right)x - \left(12.56\ \frac{1}{\text{s}}\right)t\right)$.

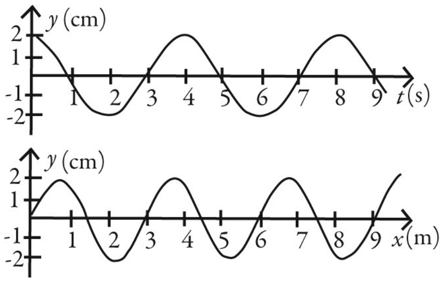
C) $y(x,t) = (2.6\text{ cm})\cos\left(\left(0.628\ \frac{1}{\text{cm}}\right)x - \left(2\ \frac{1}{\text{s}}\right)t\right)$.

D) $y(x,t) = (2.6\text{ cm})\cos\left(\left(0.1\ \frac{1}{\text{cm}}\right)x + \left(2\ \frac{1}{\text{s}}\right)t\right)$.

E) $y(x,t) = (2.6\text{ cm})\cos\left(\left(0.1\ \frac{1}{\text{cm}}\right)x - \left(2\ \frac{1}{\text{s}}\right)t\right)$.

Answer: A

5) The figure shows the displacement *y* of a travelling wave at a given position as a function of time and the displacement of the same wave at a given time as a function of position. Determine the wavelength of the wave.



A) 1.0 m
B) 3.0 m
C) 2.0 m
D) 4.0 m
E) impossible to determine.
Answer: B

6) A wave pulse pointing up and traveling to the right along a thin cord reaches a discontinuity from which point onwards the rope is thinner and lighter. What is the orientation of the reflected and transmitted pulses?
A) The reflected pulse returns pointing up while the transmitted pulse is pointing down.

B) The reflected pulse returns pointing down while the transmitted pulse is pointing up.
C) Both pulses are pointing down.
D) Both pulses are pointing up.
Answer: D

7) Travelling waves are generated on a stretched string by wiggling one end of the string. If the string suddenly starts to wiggle two times more rapidly without appreciably affecting the tension, the waves are moving along the string
A) four times faster than before.
B) two times faster than before.
C) at the same speed as before.
D) two times slower than before.
E) four times slower than before.
Answer: C

8) In the acoustic wave equation for pressure $\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$ the propagation speed c is
A) proportional to the compression modulus and proportional to the density.
B) proportional to the compression modulus and inversely proportional to the density.
C) proportional to the square root of the compression modulus and proportional to the square root of the density.
D) proportional to the square root of the compression modulus and inversely proportional to the square root of the density.
E) inversely proportional to the square root of the compression modulus and inversely proportional to the square root of the density.
Answer: D

Open questions, 4 assignments

9) A 3-kg mass attached to an ideal massless spring with a spring constant of 27 N/m oscillates on a horizontal, frictionless track. At time $t = 0.00$ s, the mass is released from rest at $x = 10.0$ cm. (That is, the spring is stretched by 10.0 cm.)
(a) Find the frequency of the oscillations. *(0.3 points)*
(b) Determine the maximum speed of the mass (in metres per second). At what point in the motion does the maximum speed occur? *(0.7 points)*
(c) Determine the maximum acceleration of the mass (in metres per second squared). At what point in the motion does the maximum acceleration occur? *(0.6 points)*
(d) Determine the total energy of the oscillating system (in Jouls). *(0.4 points)*
Give the answers till the second digit after the decimal point.
Answer:
(a) We know that the angular frequency of the oscillation of the mass-and-spring system is proportional to the square root of the spring constant and inversely proportional to the square root of the mass: $\omega = \sqrt{\frac{k}{m}}$. From here we can calculate the frequency:
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{9} = \frac{3}{2\pi} \text{ Hz } (= 0.48 \text{ Hz}).$$
(b) If the spring is stretched by 10 cm, then the amplitude is 10 cm or 0.1 m. We know that the speed of oscillation is the time derivative of the equation for oscillatory displacement as a function of time: $v(t) = \frac{dx(t)}{dt} = \frac{d(A\cos(\omega t))}{dt} = -\omega A \sin(\omega t)$. The speed is a maximum when $\sin(\omega t) = 1$. Thus, the maximum speed is
$$v_{\max}(t) = |-\omega A| = 3 * 0.1 = 0.3 \ \frac{\text{m}}{\text{s}}.$$
The maximum speed is reached at the equilibrium position, i.e., where the displacement is zero.
(c) We know that the acceleration is the time derivative of the speed as a function of time: $a(t) = \frac{dv(t)}{dt} = \frac{d(-\omega A \sin(\omega t))}{dt} = -\omega^2 A \cos(\omega t)$. The acceleration is a maximum when $\cos(\omega t) = 1$. Thus, the maximum acceleration is
$$a_{\max}(t) = |-\omega^2 A| = 9 * 0.1 = 0.9 \ \frac{\text{m}}{\text{s}^2}.$$
The maximum acceleration is reached at the maximum displacement.
(d) The total energy is the sum of the potential and the kinetic energy of the system. When the kinetic energy is zero, the total energy would be equal to the maximum potential energy. This happens at the maximum displacement. The potential energy is $U(t) = \frac{1}{2}kx^2 = \frac{1}{2}k(A\cos(\omega t))^2$. The potential energy will be at its maximum when $\cos(\omega t) = 1$. Thus, the total energy is
$$E = U_{\max}(t) = \frac{1}{2}kA^2 = \frac{1}{2} * 27 * 0.01 = 0.14 \text{ J}.$$

10) You were preparing for your exam on Mechanics 2 and for that were checking if you can determine the gravitational acceleration using a simple pendulum with a

certain length. After letting the pendulum oscillate, you saw that it completed one oscillation in 2 s. In the night, you dreamt that you are on another planet and using the same pendulum you see that it completes one oscillation in 4 s. The length of the pendulum is *L*.
(a) What is the gravitational acceleration of the planet you dreamt about? *(1.2 point)*
(b) What would be the period of oscillations if you use the same pendulum while leaving that planet with a rocket with acceleration of *a*? *(0.8 point)*
Give the answers till the second digit after the decimal point.

Answer:

(a) That the pendulum completes one oscillation in 2 s means that its period $T_e = 2$ s, where the subscript *e* indicates the earth. Knowing the period, we can find the angular frequency of the oscillations: $\omega_e = \frac{2\pi}{T_e}$. So, the angular frequency is

$$\omega_e = \frac{2\pi}{2} = \pi \ \frac{\text{rad}}{\text{s}}.$$

For a simple pendulum, the angular frequency is $\omega = \sqrt{\frac{g}{L}}$. Thus, the length of the pendulum can be obtained from the expression $L = \frac{g_e}{\omega_e^2}$.

Using the same reasoning as above, we find the angular frequency on the dreamt planet: $\omega_p = \frac{2\pi}{T_p} = \frac{2\pi}{4} = \frac{\pi}{2} \ \frac{\text{rad}}{\text{s}}$, where the subscript *p* indicates the dreamt planet.

The gravitational acceleration of the dreamt planet can be obtained from the expression $g_p = \omega_p^2 L$. Substituting in this relation the expression for *L*, we obtain

$g_p = \omega_p^2 \frac{g_e}{\omega_e^2}$ and thus we can calculate that

$$g_p = \frac{\pi^2}{4}\frac{9.81}{\pi^2} = \frac{9.81}{4} \ \frac{\text{m}}{\text{s}^2} \left(= 2.45 \ \frac{\text{m}}{\text{s}^2} \right).$$

(b) (The solution to this part is close to the solution of problem 46 from chapter 13 from your book.)

When accelerating in the rocket to leave the dreamt planet, any object in the rocket will experience an effective acceleration $g_{eff} = g_p + a$ due to the addition of the two forces that act on the objects in the rocket: the gravitational pulling force and the reactive force of the object against the floor. The reactive force with which the object pushes the floor equals *ma* and it points downwards, i.e., in the direction of the gravitational force. We can thus find the period of oscillations from $T = \frac{2\pi}{\omega}$, where

$\omega = \sqrt{\frac{g_{eff}}{L}}$. So, the period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{L}{g_p + a}} \ \text{s}.$$

11) A tiny vibrating source sends waves uniformly in all directions. An area of 3.1 cm^2 on a sphere of radius 2.50 m centred on the source receives energy at a rate of 6.20 J/s.

(a) What is the intensity of the waves at 2.50 m from the source and at 10.0 m from the source? *(1.5 points)*
(b) At what rate is energy leaving the vibrating source of the waves? *(0.5 points)*
Give the answers rounded to the closest integer number.
Answer:
(a) The intensity of waves spreading in two or three dimensions equals the power of the emitted waves divided by the area of the wavefront: $I = \frac{P}{A}$. The source of the waves is said to be sending waves uniformly in all directions. This means that we are dealing with spherical wavefront. Then, $I = \frac{P}{A} = \frac{P}{4\pi r^2}$. Furthermore, because the waves are sent equally in all directions, calculating the intensity for a piece of the sphere would give the same result as calculating the intensity for the complete sphere. So, we can calculate the intensity for the area of the sphere with radius 3.1 cm^2 ($3.1*10^{-4}$ m^2). This area receives energy at a rate of 6.20 J/s, which is the power over this area. Thus,

$$I = \frac{P}{A} = \frac{6.2}{3.1*10^{-4}} = 20000 \ \frac{\text{W}}{\text{m}^2}.$$

We are talking about concentric spherical wavefront. Then, the ratio of the area of a sphere with a radius of 2.5 m to the area of a sphere with a radius of 10 m will be proportional to the ratio of their radiuses:

$\frac{A_{2.5}}{A_{10}} = \frac{2.5^2}{10^2} = \frac{1}{4^2} = \frac{1}{16}$. The total power emitted by the source is constant, so

$P = I_{2.5}A_{2.5} = I_{10}A_{10} \Rightarrow I_{10} = \frac{A_{2.5}}{A_{10}}I_{2.5}$. Substituting the quantities we calculated above, we finally obtain

$$I_{10} = \frac{20000}{16} = 1250 \ \frac{\text{W}}{\text{m}^2}.$$

(b) The rate of energy leaving the source is the total power. The total power can be calculated by multiplying the intensity by the area, for example the intensity for the sphere with a radius of 2.5 m. So, the power of the source is

$$P = I_{2.5}A_{2.5} = 20000 * 4 * \pi * (2.5)^2 = 20000 * 4 * 6.25 * \pi = 500000 * 3.14$$
$$= 1570000 \text{ W}$$

12) A string with length of 20.0 cm and mass of 10 g is fixed at both ends and is under a tension of 500 N. When this string is vibrating in its third OVERTONE, you observe that it causes a nearby pipe, open at both ends, to resonate in its third HARMONIC. The speed of sound in the room is 340 m/s.
(a) How long is the pipe? *(1.5 points)*
(b) What is the fundamental frequency of the pipe? *(0.5 points)*
Give the answers till the second digit after the decimal point.
Answer:
(a) We have a case of a standing wave on a string that is fixed at its both end. For such a case, we know that standing waves exist when the length of the string is an integer multiple of half the wavelength: $L = m\frac{\lambda}{2}$. The string is vibrating in its third overtone, which means that m=4. As we know the length of the string, we find the wavelength of the third overtone:

$0.2 = 4\frac{\lambda}{2} \Rightarrow \lambda = \frac{0.2}{2} = 0.1\text{ m}.$

We can calculate the speed of the waves travelling along the string, as we know that it is the square root of the ratio between the tension and the mass per unit length:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{500}{\frac{0.01}{0.2}}} = \sqrt{10000} = 100\ \frac{\text{m}}{\text{s}}.$$

Knowing the wave speed and the wavelength, we can calculate the frequency of the vibration of the string:

$$f = \frac{v}{\lambda} = \frac{100}{0.1} = 1000\text{ Hz}.$$

The vibration of the string will create a sound wave propagating in the air with a frequency equal to the frequency of the standing wave on the string. The sound wave reaches the pipe and gives rise in it of a standing wave (third harmonic) as well with a frequency equal to the frequency of the sound wave in the air. Using the value of the speed of waves in the air and the value of the frequency, we can calculate the wavelength of the third harmonic:

$$\lambda = \frac{v}{f} = \frac{340}{1000} = 0.34\text{ m}.$$

Because the pipe is open at both ends, the length of the pipe is an integer multiple of half the wavelength: $L = m\frac{\lambda}{2}$. We are talking about the third harmonic, i.e., $m = 3$, which means the second overtone. Finally, the length of the pipe is

$$L = m\frac{\lambda}{2} = 3\frac{0.34}{2} = 0.51\text{ m}.$$

(b) To find the fundamental frequency of the pipe, we can use the equation $f = \frac{v}{\lambda}$.

The wave speed is the wave speed in the air. We can find the wavelength from the length of the pipe:

$$\lambda = 2\frac{L}{m} = 2\frac{0.51}{1} = 1.02\text{ m}.$$

Then,

$$f = \frac{v}{\lambda} = \frac{340}{1.02}\text{ Hz } (= 333.33\text{ Hz}).$$