MECHANICS 2, 3-7-2017

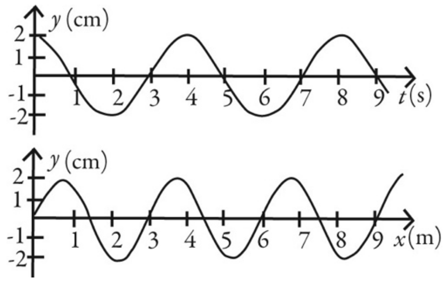
The exam consists of two parts. ***The first part is multiple-choice questions. For this part, you only have to give the answer****. Each correct answer is worth 0.4 points, up to a total of 2 points.* ***The second part consists of open questions. For these questions, you have to motivate your answers, i.e., show all the steps that lead to the answer you provide****. Answers without motivation are considered wrong and bring no points. Answers that show only part of the required steps to reach the correct answer will result in receiving a proportional part of points from the total amount of points given for this answer. (For example, if a subquestion brings 1 point, but you only show half of the path to the correct answer, you will get 0.5 points.) An open question might have alternative paths of reaching the correct answer – all such paths are considered correct. The total number of points for the open questions is 8. If an open question consists of subquestions, translation of the result of a wrong calculation from one subquestion to another subquestion is not judged as an error.*

Multiple choice, 5 assignments, 0.4 points per question

1) If we double only the spring constant of a vibrating ideal mass-and-spring system, the mechanical energy of the system
A) increases by a factor of $\sqrt{2}$.
B) decreases by a factor of $\sqrt{2}$.
C) increases by a factor of 2.
D) decreases by a factor of 2.
E) increases by a factor of 4.
F) decreases by a factor of 4.
Answer: C

2) A frictionless simple pendulum with length L and mass m swings with period T. If both L and m are doubled, what is the new period?
A) $\sqrt{2}T$
B) $2T$
C) $4T$
D) $T/\sqrt{2}$
E) $T/2$
F) $T/4$
Answer: A

3) The figure shows the displacement y of a wave at a given position as a function of time and the displacement of the same wave at a given time as a function of position. What is the frequency of the wave.



A) 4.0 Hz
B) 3.0 Hz
C) 0.25 Hz
D) 0.33 Hz
Answer: C

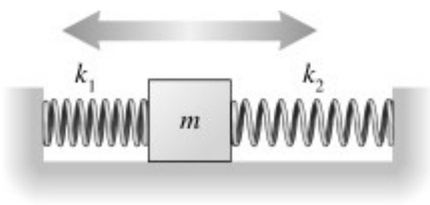
4) A guitar string is fixed at both ends. When you pull the string, you induce a standing wave in a specific mode characterized by a certain frequency and wavelength. You tighten the string to increase its tension and then pull the string again to induce a standing wave in the same mode as before. What can you say about the frequency and the wavelength now?
A) The wavelength increased, but the frequency of the vibrational mode decreased.
B) The wavelength increased, while the frequency of the vibrational mode was not affected.
C) And the wavelength and the frequency of the vibrational mode increased.
D) The frequency of the vibrational mode increased, but the wavelength decreased.
E) The frequency of the vibrational mode increased, while the wavelength was not affected.
F) Neither the frequency of the vibrational mode nor the wavelength were affected.
Answer: E

5) To derive the acoustic wave equation in three dimensions, we used a system of coupled equations. One of the equations is $-\nabla p = \rho_0 \frac{\partial \vec{v}}{\partial t}$. This equation describes what process when the wave passes?
A) Curl.
B) Compression/expansion.
C) Rotation.
D) Translation.
Answer: D

Open questions, 6 assignments

6) The position of an object that is oscillating on an ideal spring following a simple harmonic motion is given by the equation $x(t)$ = (30 cm) $cos[(\pi/3\ s^{-1})t]$. At time t = 0.5 s,
(a) how fast is the object moving (in metres per second)? ***(0.7 points)***
(b) what is the magnitude (absolute value) of the acceleration of the object (in metres per second squared)? ***(0.4 points)***
Give the answers till the second digit after the decimal point.
Answer:
(a) The displacement of an object that undergoes simple harmonic motion is $x(t) = A\cos(\omega t + \phi)$. Comparing this relation with the displacement equation given in the question's description, we see that the amplitude is $A = 0.3\ (m)$ and the angular frequency is $\omega = \frac{\pi}{3}\ (\frac{1}{s})$. The velocity is the derivative of the displacement with respect to time:
$v(t) = \frac{dx(t)}{dt} = \frac{d(A\cos(\omega t))}{dt} = -\omega A\sin(\omega t)$ and we are interested in the magnitude (absolute value) of the velocity, so $|v(t)| = |-\omega A\sin(\omega t)|$ From here, we can calculate how fast the object is moving:
$|v(t)| = \left|-\frac{\pi}{3} * 0.3 * \sin\left(\frac{\pi}{3} * 0.5\right)\right| = \left|\frac{\pi}{10} * \sin\left(\frac{\pi}{6}\right)\right| = 0.314 * 0.5 = 0.16\ (\frac{m}{s})$.
(b) We know that the acceleration is the time derivative of the velocity with respect to time: $a(t) = \frac{dv(t)}{dt} = \frac{d(-\omega A\sin(\omega t))}{dt} = -\omega^2 A\cos(\omega t)$. We are interested in the magnitude (absolute value) of the acceleration, so $|a(t)| = |-\omega^2 A\cos(\omega t)|$. Thus, the magnitude of the acceleration is
$|a(t)| = \left|-\left(\frac{\pi}{3}\right)^2 0.3\cos\left(\frac{\pi}{3} * 0.5\right)\right| = \left|-\frac{\pi^2}{30} * \cos\left(\frac{\pi}{6}\right)\right| = 0.98596 * \frac{\sqrt{3}}{2} = 0.28\ (\frac{m}{s^2})$.

7) A 3.0-kg block on a table is connected to two ideal massless springs of the same length, which are neither stretched nor compressed. The springs' opposite ends are fixed to two opposite walls, as shown in the figure. The block does not experience friction neither with the air nor with the table. The block is displaced and let oscillate following a simple harmonic motion. What is the angular frequency of the oscillation (in one over second) if the spring constants are $k1$ = 7.0 N/m and $k2$ = 5.0 N/m? ***(1.1 point)***



Give the answer till the second digit after the decimal point.
Answer:
This is a problem nearly identical to problem 59 from Chapter 13.
When the block is at its equilibrium position, the force the block experiences is zero because the springs are neither stretched nor compressed. If the block is displaced to one side by x_1, each spring starts exerting force on the block. Let us suppose that the

displacement is to the left, as shown in the figure. The left spring will exert a force with magnitude $k_1 x_1$, while the second spring – with magnitude $k_2 x_1$. The left spring will be pushing the block to the right, while the right spring will be pulling the block to the right, meaning in the direction opposite to the displacement. Thus, the total force on the block is a restoring force and at any moment is $F = -k_1 x_1 - k_2 x_1 = -(k_1 + k_2)x_1$. Then, the effective spring constant is $k_{eff} = k_1 + k_2$.

Because we have simple harmonic motion, the angular frequency is $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{k_1+k_2}{m}}$. From here, we calculate

$\omega = \sqrt{\frac{k_1+k_2}{m}} = \sqrt{\frac{7+5}{3}} = \sqrt{4} = 2 \left(\frac{1}{s}\right)$.

8) You are investigating a physical pendulum while it is swinging back and forth. You are taking measurements of its potential energy and notice that the potential energy is linearly proportional to the square of the angular displacement, with a proportionality factor of 40. You know the inertia of the pendulum, it is 5 kg.m^2. The measurements are being made under vacuum, so there is no friction with the air. When the pendulum is going to be used in the open air, it is going to experience damped motion. You know that you can counter this by applying a driving force with a certain frequency. But to be sure that you do not enter into resonance, you want to stay away from the resonance frequency. What is the resonance frequency of the physical pendulum (in one over second)? ***(1.8 points)***
Give the answer rounded to the closest integer number.
Answer:
This is a problem nearly identical to problem 80 from Chapter 13.
The problem statement talks about a physical pendulum that swings back and forth, but it does not say that you are dealing with simple harmonic motion.
The potential energy that you observed is $U = 40\theta^2$ (J), where θ is the angular displacement. This describes a parabola. From the lectures, we know that a motion characterized by a parabolic potential curve is in fact a simple harmonic motion.
For a simple harmonic motion of a mass/spring system, we know that the potential energy is $U = \frac{1}{2}kx^2$. From the comparison of a mass/sprint system with a physical pendulum (for small angular displacements), we know that k corresponds to mgL, while x corresponds to θ. This means that the potential energy of a physical pendulum is $U = \frac{1}{2}mgL\theta^2$. Comparing this relation with the relation $U = 40\theta^2$, we see that $mgL = 80$. The angular frequency of a physical pendulum is thus $\omega = \sqrt{\frac{mgL}{I}}$.
From here, we calculate

$\omega = \sqrt{\frac{mgL}{I}} = \sqrt{\frac{80}{5}} = \sqrt{16} = 4 \left(\frac{1}{s}\right)$.

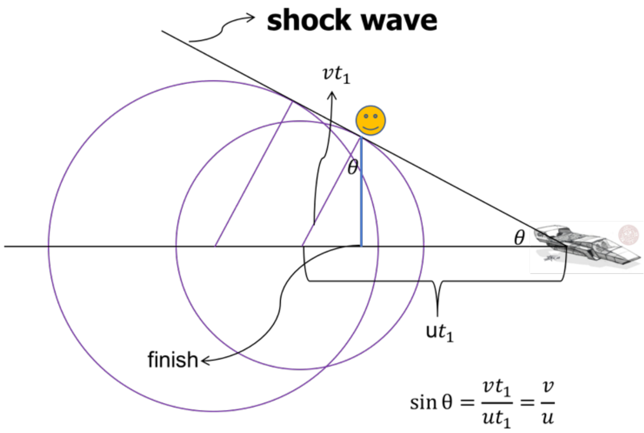
9) A standing wave of frequency 40 Hz is produced on a string that has mass per unit length 0.02 kg/m. With what tension must the string be stretched between two walls if adjacent nodes in the standing wave are to be 0.5 m apart? ***(1 point)***
Give the answer as an integer.
Answer:

Nodes are the points along the string where the amplitude is zero. By making a drawing, for example, we can see that the distance between two adjacent nodes is half the wavelength ($\frac{\lambda}{2}$). The distance between adjacent nodes is 0.5 m, so the wavelength is $\lambda = 2 * 0.5 = 1\ (m)$. The speed of waves is $v = \lambda f$. As we know the frequency of the standing wave, the speed is $v = 1 * 40 = 40\ (\frac{m}{s})$. On the other hand, we can calculate the speed of the waves travelling along the string is the square root of the ratio between the tension and the mass per unit length: $v = \sqrt{\frac{F}{\mu}}$. Combining the two expression for the speed, we can calculate the required tension:
$F = v^2\mu = 1600 * 0.02 = 32\ (N)$.

10) You are experimenting with what you have learned from Mechanics 2. You are standing between two friends that are some distance apart, and form one line. Each of them makes so that the same sound wave with a frequency of 170 Hz is emitted. You start running towards one of them along the line and perceive a beat every 2 s (i.e., 2 $\frac{1}{s}$). How fast are you running? (The speed of sound in the air can be taken to be 340 m/s.) ***(1.2 points)***
Give the answer as an integer.
Answer:
This is a problem identical to problem 74 from Chapter 14.
To experience a beat, the frequencies of the two waves should be slightly different, which comes from the Doppler effect when running towards one of the friends and away from the other. Because we are talking about beats, we are talking about constructive and destructive interference which results in us perceiving a new, resulting wave with maximum amplitude reoccurring with a certain frequency. The amplitude of the new perceived wave is $A_{new} = 2A\cos\left(\frac{1}{2}(\omega_1 - \omega_2)t\right) = 2A\cos\left(\frac{1}{2}2\pi(f_1 - f_2)t\right)$, where A is the amplitude of the emitted waves and f_1 and f_2 are the perceived frequencies of the two waves due to the Doppler effect. The absolute value of the difference of the two perceived frequencies is the beat frequency of 2 Hz ($\frac{1}{s}$). The Doppler effect you perceive is due to a moving observer. This means that the perceived frequencies are $f_{1,2} = f\left(1 \pm \frac{u}{v}\right)$, where f is the frequency of the emitted waves, v is the speed of sound waves in the air, and u is the speed with which you are running. Taking the absolute value of the difference between the two perceived frequencies, we can find the running speed*)*:
$|f_1 - f_2| = \left|f\left(1 + \frac{u}{v}\right) - f\left(1 - \frac{u}{v}\right)\right| = 2f\frac{u}{v} \Rightarrow u = \frac{|f_1 - f_2|v}{2f} = \frac{2*340}{2*170} = 2\ \left(\frac{m}{s}\right)$.

11) You are watching supersonic experiment with a ground vehicle (may be in the near future) that takes place on a special track. You are 680 m perpendicularly away from the finish point on the track. When the ground vehicle is at the finish point, a sound is emitted to mark the reached finish. The wavefront of the shock wave from the vehicle's turbine reaches you 0.5 s after you hear the sound emitted at the finish point. If the speed of sound in the air is 340 m/s, what was the speed of the ground vehicle at the finish? ***(1.8 points)***
Give the answer as an integer.
Answer:

As we talk about shock waves and their fronts, we are also looking at a problem involving Mach number. The Mach number is the ratio of the speed of the vehicle and the speed of sound in the medium. This also equals one over the sinus of the angle between the distance travelled by the sound wave vT and the distance travelled by the vehicle uT (see the drawing below): $M = \frac{u}{v} = \frac{1}{\sin\theta}$.



To find the speed of the ground vehicle at the finish, we need to find $\sin\theta$. From the drawing, $\sin(90-\theta)$ is equal to the ratio between distance between you and the finish and the distance travelled by the sound of the engine through the air: $\sin(90-\theta) = \frac{680\ [m]}{vt_1}$, where t_1 is the time for the turbine sound to reach you. This sound reached you 0.5 s after the sound from the finish signal, where the latter is $t_2 = \frac{680}{340} = 2\ (s)$. From here, we can calculate that $\sin(90-\theta) = \frac{680\ [m]}{vt_1} = \frac{680}{340*(t_2+0.5)} = \frac{680}{850} = 0.8 = \cos\theta$. Knowing the cosine, we can calculate the sinus: $\sin(\theta) = \sqrt{1-(\cos\theta)^2} = \sqrt{1-0.64} = \sqrt{0.36} = 0.6$. And now we can calculate the speed of the ground vehicle using the formula with the Mach angle: $u = \frac{v}{\sin\theta} = \frac{340}{0.6} = 567\ \left(\frac{m}{s}\right)$.

(The current Guinness land-speed record is 341 m/s, which is supersonic!)