MECHANICS 2, 29-01-2018

The exam consists of two parts. ***The first part is multiple-choice questions. For this part, you only have to give the answer****. Each correct answer is worth 0.4 points, up to a total of 2 points.* ***The second part consists of open questions. For these questions, you have to motivate your answers, i.e., show all the steps that lead to the answer you provide****. Answers without motivation are considered wrong and bring no points. Answers that show only part of the required steps to reach the correct answer will result in receiving a proportional part of points from the total amount of points given for this answer. (For example, if a subquestion brings 1 point, but you only show half of the path to the correct answer, you will get 0.5 points.) An open question might have alternative paths of reaching the correct answer; all such paths are considered correct. The total number of points for the open questions is 8. If an open question consists of subquestions, translation of the result of a wrong calculation from one subquestion to another subquestion is not judged as an error.*

Multiple choice, 5 assignments, 0.4 point per question

1) An object with mass of 5 kg is attached to an ideal massless spring and undergoes simple harmonic oscillations with a period of 1.57 s. What is the spring constant of the spring?
A) 80 N/m
B) 40 N/m
C) 8 N/m
D) 4 N/m
E) 0.4 N/m
Answer: A

2) An ideal harmonic oscillator of mass 0.1 kg has a total mechanical energy of 0.45 J. If the oscillation amplitude is 10.0 cm, what is the oscillation frequency?
A) 4.8 Hz
B) 9.6 Hz
C) 14.4 Hz
D) 24.0 Hz
D) 28.8 Hz
Answer: A

3) Consider a pipe of length L that is open at both ends. What are the wavelengths of the three lowest harmonics produced by this pipe?
A) 4L, 2L, L
B) 2L, L, L/2
C) 2L, L, 2L/3
D) 4L, 4L/3, 4L/5
E) 2L, L, L/2
Answer: C

4) When a rocket is traveling toward a mountain at 100 m/s, the sound waves from this rocket's engine approach the mountain at speed V. If the rocket doubles its speed

to 200 m/s, the sound waves from the engine will now approach the mountain at a speed of
A) 4V.
B) $\sqrt{4}$V.
C) 2V.
D) $\sqrt{2}$V.
E) V.
Answer: E

5) In solids can propagate both longitudinal and transverse waves. Because of this, a wave in homogeneous solids is defined to be longitudinal if and only if
A) the pressure changes are curl-free.
B) the particle velocity is curl-free.
C) the pressure changes are gradient-free.
D) the particle velocity is gradient-free.
E) the particle velocity is divergence-free.
Answer: B

Open questions, 6 assignments

6) A machine part is vibrating along the x-axis in simple harmonic motion with a period of 6 s and a range (from the maximum in one direction to the maximum in the other) of 3 m. At time t = 0 it is at its central position and moving in the +x direction.
(a) What is the position of the machine part when t = 0.5 s? ***(0.8 points)***
(b) What is the maximum speed of the machine part (in m/s)? ***(0.4 points)***
Give the answers till the second digit after the decimal point.
Answer:
(a) A machine part vibrating in harmonic motion is characterized by horizontal displacement $x(t) = Acos(\omega t + \varphi)$. As the machine part is at its central position, then the displacement is 0 m at time 0 s. Thus, we obtain that $0 = cos(\varphi)$, and consequently that $\varphi = -90°$. As we know, $cos(\alpha - 90) = sin(\alpha)$, so the equation of displacement is $x(t) = Asin(\omega t) = Asin\left(\frac{2\pi}{T}t\right)$. The distance between the two maxima is two times the amplitude, so $A = \frac{3}{2} = 1.5\ (m)$. Then, at $f = 0.5\ (s)$, the horizontal displacement is $x(0.5) = 1.5 * sin\left(\frac{2\pi}{6} * 0.5\right) = 1.5 * sin\left(\frac{\pi}{6}\right) = 1.5 * 0.5 = 0.75\ (m)$.
(b) We know that the velocity is the derivative of the horizontal displacement with respect to time: $v(t) = \frac{dx(t)}{dt} = \frac{d(A\sin(\omega t))}{dt} = \omega A\cos(\omega t)$. We are interested in the maximum speed, which is achieved when $\cos(\omega t) = 1$. Thus, $v_{max} = |\omega A * 1| = \frac{2\pi}{6} * 1.5 = 1.57\ \left(\frac{m}{s}\right)$.

7) The angle that a swinging simple pendulum makes with the vertical obeys the equation $\theta(t) = (0.2\ rad)cos\big((3\ rad/s)t + 1.5\big)$.
(a) What is the length of the pendulum? ***(0.6 points)***
(b) What is the mass of the swinging object at the end of the pendulum? ***(0.3 points)***
Give the answer till the second digit after the decimal point.
Answer:
(a) In analogy with a mass/spring system, a simple pendulum undergoing simple harmonic motion is characterized by an $\theta(t) = Acos(\omega t + \varphi)$. Comparing this relation with the given in the description of the problem, we can conclude that the swinging pendulum undergoes simple harmonic motion and, thus, that its angular frequency is $\omega = 3\ (rad)$. For a simple pendulum, we know that $\omega = \sqrt{\frac{g}{L}}$. From here, we can calculate that $L = \frac{g}{\omega^2} = \frac{9.81}{9} = 1.09\ (m)$.
(b) In the description of the question, we are given the equation of angular displacement, which makes known to us the amplitude, angular frequency, and the phase. For a simple pendulum, the mass is not connected to the angular frequency (nor to the phase). We also know that the amplitude of the simple harmonic motion is not connected to the mass either. Thus, we cannot determine the mass from the given parameters.

8) In an ideal world, you are experimenting with an ideal massless spring. You attach to it a mass of 3 kg. You pull the mass from its equilibrium position at x_{eq} =0.00 m to a new position and then release it. You observe that the mass is executing lightly

damped oscillation along the x-axis, and the damping force is proportional to the velocity. You measure the velocity and the acceleration of the mass when they are back at the equilibrium position: they are +4.0 m/s and -4.7 m/s^2, respectively. What is the damping constant b? ***(1.9 points)***
Give the answer till the first digit after the decimal point.

Answer:

(This is a question that was already given during the exam in August 2016.)
Because the damping force is proportional to the velocity, we can write that the displacement $x(t) = Ae^{-\frac{bt}{2m}}cos(\omega t)$.
We know that the velocity is the derivative of the displacement with respect to time:
$v(t) = \frac{dx(t)}{dt} = \frac{d\left(Ae^{-\frac{bt}{2m}}cos(\omega t)\right)}{dt} = -\omega Ae^{-\frac{bt}{2m}}sin(\omega t) - \frac{b}{2m}Ae^{-\frac{bt}{2m}}cos(\omega t)$. When the block returns at $x_{eq} = 0.00$ m, the equilibrium point, the displacement is 0. This means that $cos(\omega t) = 0$ and consequently $sin(\omega t) = 1$. The velocity at the equilibrium point is +4 m/s, so
$v(t) = 4\left(\frac{m}{s}\right) = -\omega Ae^{-\frac{bt}{2m}}sin(\omega t)$. Thus, we find that $e^{-\frac{bt}{2m}} = \frac{4}{-\omega A} = -\frac{4}{\omega A}$.
We know that the acceleration is the derivative of the velocity with respect to time:
$a(t) = \frac{dv(t)}{dt} = \frac{d\left(-\omega Ae^{-\frac{bt}{2m}}sin(\omega t) - \frac{b}{2m}Ae^{-\frac{bt}{2m}}cos(\omega t)\right)}{dt} = -\omega^2 Ae^{-\frac{bt}{2m}}cos(\omega t) + \frac{b}{2m}\omega Ae^{-\frac{bt}{2m}}sin(\omega t) + \frac{b}{2m}\omega Ae^{-\frac{bt}{2m}}sin(\omega t) + \frac{b^2}{4m^2}Ae^{-\frac{bt}{2m}}cos(\omega t)$.
At the equilibrium point, $cos(\omega t) = 0$ and $sin(\omega t) = 1$. The acceleration at the equilibrium point is -4.7 m/s^2, thus
$a(t) = -4.7\left(\frac{m}{s^2}\right) = \frac{b}{m}\omega Ae^{-\frac{bt}{2m}}$. Substituting the expression $e^{-\frac{bt}{2m}} = -\frac{4}{\omega A}$ into the previous equation, we obtain $a(t) = -4.7\left(\frac{m}{s^2}\right) = -\frac{b}{m}\omega A\frac{4}{\omega A}$. Thus, the damping constant is $b = \frac{4.7*m}{4} = \frac{14.1}{4} = 3.5\left(\frac{kg}{s}\right)$.

9) A sound wave propagates through system of flooded caves with a speed of 1500 m/s. At one part of the system, the period of the sound waves is 0.002 s.
(a) What is the distance between two adjacent amplitude maxima of the sound waves at that part of the cave system? ***(0.5 points)***
(b) In another part of the system, the sound waves reach a tube-like cave that is closed at one end (and open at the other). As a result, the sound develops a standing wave. What is the length of the tube-like cave if the frequency of the third higher mode of the standing wave is 3000 Hz? ***(0.7 points)***
Give the answer rounded to the closest integer number.

Answer:

(a) The distance between two adjacent amplitude maxima is the wavelength. We know that $v = \lambda f = \frac{\lambda}{T}$ (m/s), where T is the period of the waves. From here, we can calculate the wavelength: $\lambda = vT = 1500 * 0.002 = 3$ (m).
(b) Because the tube-like cave is open at one end and closed at the other, we can write that the length of the tube-like cave is: $L = \frac{\lambda(2m-1)}{4}$ (m), where m is an integer and denotes the harmonic. Because the standing wave formed at its third higher mode, we are dealing with the fourth harmonic and $m = 4$. The frequency of the third higher

mode is known, the velocity of the sound does not change, thus we can calculate the wavelength of the third higher mode: $\lambda_3 = \frac{v}{f_3} = \frac{1500}{3000} = 0.5\ (m)$. Now, we can calculate the length of the tube-like cave: $L = \frac{\lambda_3(2m-1)}{4} = \frac{0.5*(2*4-1)}{4} = 0.875 \approx 1\ (m)$.

10) You are standing by a railway crossing and hear the horn of a cargo train approaching you from the left. You perceive the frequency of the horn as 111 Hz. When the train passed you, you perceive the frequency as 91 Hz. What is the frequency of the horn? And what is the speed of the cargo train? The speed of sound in air is 340 m/s. ***(1 point)***
Give the answer rounded to the closest integer number.
Answer:
Because the source of the sound is moving, you are experiencing the Doppler effect. You are not moving, so you experience the Doppler effect only due to the moving source. For a moving source, you can write that the perceived frequency is $f_{1,2} = \frac{f}{1\pm\frac{u}{v}}$, where f is the frequency of the horn, u is the speed of the cargo train, and $v = 340\ (m/s)$ is the speed of sound. From here, you can write two equations with two unknowns:
$f_1 = \frac{f}{1-\frac{u}{340}}$ for the case when the train approaches you
and
$f_2 = \frac{f}{1+\frac{u}{340}}$ for the case when the train passed you.
Dividing the former by the latter, you obtain $\frac{f_1}{f_2} = \frac{111}{91} = \frac{1+\frac{u}{340}}{1-\frac{u}{340}}$ or
$111 - 111\frac{u}{340} = 91 + 91\frac{u}{340}$, which gives $u \approx 34\ (m/s)$. Knowing the speed of the cargo train, we can calculate the frequency of the horf: $f = f_1\left(1 - \frac{34}{340}\right) = 111 * \frac{9}{10} \approx 100\ (Hz)$.

11) You are using a small vibrating source to emit waves uniformly in all directions. An area of 4.3 dm2 on a sphere that has a radius of 3 m, and is centred on the source, receives energy at a rate of 8.6 J/s.
(a) What is the intensity of the waves at 3 m from the source and at 9 m from the source? ***(1.3 points)***
(b) At what rate is the energy leaving the vibrating source of the waves? ***(0.5 points)***
Give the answers rounded to the closest integer number.
Answer:
(This is a question that was already given during the exam in June 2016.)
(a) The intensity of waves spreading in two or three dimensions equals the power of the emitted waves divided by the area of the (part of the) wavefront: $I = \frac{P}{A}\left(\frac{W}{m^2}\right)$.
Because the source of the waves is said to be sending waves uniformly in all directions, calculating the intensity for a piece of the sphere would give the same result as calculating the intensity for the complete sphere. So, we can calculate the

intensity on the portion of the sphere with area 4.3 cm^2 ($4.3*10^{-2}$ m^2). This area receives energy at a rate of 8.6 J/s, which is the power over this area. Thus,
$I = I_3 = \frac{P}{A} = \frac{8.6}{4.3*10^{-2}} = 200\ \left(\frac{W}{m^2}\right)$.
We are talking about concentric spherical wavefronts. Then, the ratio of the area of a sphere with a radius of 3 m to the area of a sphere with a radius of 9 m will be proportional to the ratio of their radii: $\frac{A_3}{A_9} = \frac{3^2}{9^2} = \frac{1}{9}$. The total power emitted by the source is constant, so
$P = I_3 A_3 = I_9 A_9 \Rightarrow I_9 = \frac{A_3}{A_9} I_3$. Substituting in this equation the quantities we calculated above, we finally obtain
$I_9 = \frac{200}{9} = 22.22 \approx 22\ \left(\frac{W}{m^2}\right)$.
(b) The rate of energy leaving the source is the total power. The total power can be calculated by multiplying the intensity by the area, for example the intensity for the sphere with a radius of 3 m. So, the power of the source is
$P = I_3 A_3 = 200 * 4 * \pi * 3^2 = 22608\ (W)$.