MECHANICS 2, 18-8-2017

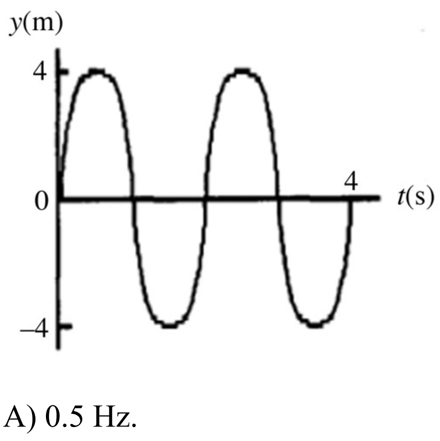
The exam consists of two parts. ***The first part is multiple-choice questions. For this part, you only have to give the answer****. Each correct answer is worth 0.4 points, up to a total of 2 points.* ***The second part consists of open questions. For these questions, you have to motivate your answers, i.e., show all the steps that lead to the answer you provide****. Answers without motivation are considered wrong and bring no points. Answers that show only part of the required steps to reach the correct answer will result in receiving a proportional part of points from the total amount of points given for this answer. (For example, if a subquestion brings 1 point, but you only show half of the path to the correct answer, you will get 0.5 points.) An open question might have alternative paths of reaching the correct answer; all such paths are considered correct. The total number of points for the open questions is 8. If an open question consists of subquestions, translation of the result of a wrong calculation from one subquestion to another subquestion is not judged as an error.*

Multiple choice, 5 assignments, 0.4 point per question

1) In simple harmonic motion, the speed is maximal at that point in the cycle when
A) the potential energy is maximal.
B) the kinetic energy is minimal.
C) the magnitude of the acceleration is maximal.
D) the displacement is maximal.
E) the magnitude of the acceleration is minimal.
Answer: E

2) If we double only the amplitude of a vibrating ideal mass-and-spring system, the mechanical energy of the system
A) increases by a factor of $\sqrt{2}$.
B) increases by a factor of 2.
C) increases by a factor of 4.
D) increases by a factor of 8.
E) does not change.
Answer: C

3) For the wave shown in the figure, the frequency is



A) 0.5 Hz.

B) 1 Hz.
C) 2 Hz.
D) 4 Hz.
E) unable to be determined from the given information.
Answer: A

4) Which one of the following statements is true?
A) The sound intensity can never be negative, but the intensity level (in dB) can be negative.
B) Both the intensity level (in dB) and the sound intensity can never be negative.
C) Both the intensity level (in dB) and the sound intensity can be negative.
D) The intensity level (in dB) obeys an inverse-square distance law, but the sound intensity does not.
E) Both intensity level (in dB) and sound intensity obey inverse-square distance laws.
Answer: A

5) What can we say about the longitudinal waves?
A) the pressure changes are gradient-free.
B) the pressure changes are curl-free.
C) the mass-flow vector is divergence-free.
D) the mass-flow vector is gradient-free.
E) the mass-flow vector is curl-free.
Answer: E

Open questions, 6 assignments

6) An object of mass 1 kg is attached to a vertical ideal massless spring and vibrates up and down between two extreme points A and B following a simple harmonic motion with a period of π s. The distance between A and B is 2 m.
(a) What is the maximum energy of the system (in Joules)? ***(0.8 points)***
(b) What is the maximum acceleration of the vibrating object (in metres per second squared)? ***(0.5 points)***
Give the answers rounded to the closest integer number.
Answer:
(a) The maximum energy of the system is equal to the total energy, which is $E_{max} = \frac{1}{2}kA^2\ (J)$. The amplitude is the maximum displacement from the equilibrium. This means that the amplitude is half the distance between the points A and B, i.e., $A = \frac{1}{2} * 2 = 1\ (m)$. The angular frequency is $\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}\ (\frac{rad}{s})$. From here, we can find that the spring constant is $k = \frac{m*4\pi^2}{T^2} = \frac{1*4\pi^2}{\pi^2} = 4\left(\frac{kg}{s^2}\right)$ From here, we can calculate the maximum energy:
$E_{max} = \frac{1}{2}kA^2 = \frac{1}{2} * 4 * 1 = 2\ (J)$.
(b) We know that the acceleration is the time derivative of the velocity with respect to time: $a(t) = \frac{dv(t)}{dt} = \frac{d(-\omega A \sin(\omega t))}{dt} = -\omega^2 A \cos(\omega t)$. The acceleration is maximal when $\cos(\omega t) = 1$. Thus, $a(t)_{max} = |-\omega^2 A| = \left|-\left(\frac{2\pi}{T}\right)^2 A\right| = 4 * 1 = 4\left(\frac{m}{s^2}\right)$.

7) On the Earth's surface a frictionless clock with a simple pendulum has a period of 1 s. If the same clock is used to measure the time on the Moon's surface, how many times should the length of the pendulum be shorter, compared to the length of the pendulum on EARTH, to have a period of 2 s? The gravitational acceleration on the moon is 1.62 $\left(\frac{m}{s^2}\right)$. ***(0.8 points)***
Give the answer till the first digit after the decimal point.
Answer:
(a) A simple pendulum will vibrate with an angular frequency of $\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}\ (\frac{rad}{s})$.
At the Earth's surface the period is then $T_{earth} = \frac{1}{2\pi}\sqrt{\frac{L_{earth}}{g_{earth}}} = 1\ (s)$. At the Moon's surface the period is then $T_{moon} = \frac{1}{2\pi}\sqrt{\frac{L_{moon}}{g_{moon}}} = 2\ (s)$. To calculate with how much the length should be shortened, we can take the ration of the two periods: $\frac{T_{moon}}{T_{earth}} = \frac{\frac{1}{2\pi}\sqrt{\frac{L_{moon}}{g_{moon}}}}{\frac{1}{2\pi}\sqrt{\frac{L_{earth}}{g_{earth}}}} = \frac{2}{1}$. From here, we find that $\frac{g_{earth}L_{moon}}{g_{moon}L_{earth}} = 4$. And, thus, the length should be $\frac{L_{earth}}{L_{moon}} = \frac{g_{earth}}{4*g_{moon}} = \frac{9.81}{4*1.62} = \frac{9.81}{6.48} \approx 1.5$ times shorter.

8) A mass/spring system is oscillating in the real world, so the energy loss due to the air resistance and due to the friction cannot be ignored. The constant of the spring is k=7.5 N/m, while the mass weights 300 g. The damping constant for this system is

b=10^{-2} kg/s. At a time t_1, you measure a certain amplitude of the oscillation. After how many oscillations the amplitude will decays to $1/e^2$ of the value at t_1? Assume that the damping force is much smaller than the reactive force. ***(1.9 points)***
Give the answer rounded to the closest integer number.

Answer:

This is a problem nearly identical to problem 65 from Chapter 13.

The equation of damped oscillatory motion is $x(t) = A\,e^{-\frac{bt}{2m}}\cos(\omega t + \phi)$. At time t_1, the amplitude of the oscillation is $A_{damped}(t_1) = Ae^{-\frac{bt_1}{2m}}$. After certain time, the amplitude becomes $A_{damped}(t_2) = Ae^{-\frac{bt_2}{2m}}$. But it is given that at this time, the amplitude is $A_{damped}(t_2) = A_1e^{-2} = Ae^{-\frac{bt_1}{2m}}e^{-2} = Ae^{-\frac{bt_1}{2m}-2}$. Combining the last two equations, we can write $Ae^{-\frac{bt_2}{2m}} = Ae^{-\frac{bt_1}{2m}-2} \Rightarrow \frac{bt_2}{2m} = \frac{bt_1}{2m} + 2$. Thus, the time it took to damp the amplitude from its value at t_1 to the value at t_2 is $t_2 - t_1 = \frac{4m}{b} = \frac{4*0.3}{10^{-2}} = 120\ (s)$.

To calculate how many oscillations (i.e., how many periods) this time difference is equal to, we have to divide this time difference by the period. The period of a damped mass/spring system will depend on the type of motion: weakly damped, underdamped, critically damped, or overdamped. As we can assume that the damping force is much smaller than the reactive force, we are in the regime of weak damping. This means that we can assume that $\omega \approx \sqrt{\frac{k}{m}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$. From here, the period is $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.3}{7.5}} = 2\pi\sqrt{0.04} = 2\pi * 0.2 = 1.26\ (s)$. Thus, $\frac{t_2-t_1}{T} = \frac{120}{1.26} = 95$.

9) A tube that is open at one end and closed at the other end produces a note with a fundamental frequency of 350 Hz. What will be the fundamental frequency if also the closed end is opened? ***(1.1 points)***
Give the answer rounded to the closest integer number.

Answer:

The length of the tube defines the wavelength, and consequently the frequency, of the modes of vibration. We know that the relation for a pipe closed at one end and open at the other is $L = (2m-1)\frac{\lambda_1}{4}\ (m)$. We also know that $v = \lambda_1 f_1\ \left(\frac{m}{s}\right)$. From here, we can find that for the fundamental mode the ratio of the length over the speed of the wave is $\frac{L}{v} = \frac{(2m-1)}{4f_1} = \frac{1}{4*350}\ (s)$.

We know that the relation between the length of a pipe that is open at both ends and the wavelength is $L = m\frac{\lambda_2}{2}\ (m)$. For the fundamental mode, the ratio of the length over the speed of the wave is $\frac{L}{v} = \frac{m}{2f_2} = \frac{1}{2f_2}\ (s)$. From here, we can calculate that $f_2 = \frac{1}{2} * \frac{v}{L} = \frac{1}{2} * 4 * 350 = 700\ (Hz)$.

10) You are filming a child playing with a plastic ship in a swimming pool. The child jumps in the pool to pick up the boat, and creates sinusoidal water waves. Analyzing the movie later on, you see that at one point the boat is half the way between the lowest (trough) and highest (crest) points of the wave. The boat is at the right side of a

crest; the wave propagates to the left. You estimate the horizontal distance between these two points to be 25 cm. After 0.4 s, the boat is at the crest. You estimate the speed of the water wave to be 2 m/s. What are the wavelength (in metres) and the frequency (in Hz) of the waves? ***(1 point)***
Give the answer till the second digit after the decimal point.
Answer:
By making a drawing, we can see that the distance between the crest and the middle point between the crest and the trough is $\frac{\lambda}{4}$ (m). From here, we can calculate that the wavelength if $\lambda = 0.25 * 4 = 1$ (m).
Because the water wave travels to the left while the boat is to the right of the crest, the water wave has covered $\frac{3\lambda}{4}$ during time $t = 0.4$ (s). This means that speed of the wave is $v = \frac{\frac{3\lambda}{4}}{t} = \frac{\frac{3*1}{4}}{0.4} = 1.88 \left(\frac{m}{s}\right)$.
We also know that the wave speed is $v = \lambda f$. From here, we can calculate the frequency: $f = \frac{v}{\lambda} = \frac{1.88}{1} = 1.88$ (Hz).

11) A researcher was making an experiment with a spring of mass 600 g, but did not know its spring constant. When the spring was stretched a bit, its tension was 20 N and the researcher measured speed of 5 m/s for the transverse waves propagating along the spring. When the spring was further stretched and became 100-cm long, the transverse waves propagated at 10 m/s. When the researcher decided to make a third experiment for statistical reasons, the spring could not be found. The researcher also realized that the length of the unstretched spring was not noted down. Help the researcher and find the spring constant (in N/m) and the unstretched length of the spring (in metres). ***(1.9 points)***
Give the answers till the second digit after the decimal point.
Answer:
This is a problem nearly identical to problem 58 from Chapter 14.
We know that the speed of waves on a string, and the spring can be seen like a string, is $v = \sqrt{\frac{F}{\mu}} \left(\frac{m}{s}\right)$ with F the tension and μ the mass per unit length. The mass per unit length equals $\mu = \frac{m}{L} \left(\frac{kg}{m}\right)$. From the first experiment, we know the speed and the tension, so we can find μ and consequently the stretched length L_1: $v_1 = \sqrt{\frac{F_1}{\mu}} = \sqrt{\frac{F_1}{\frac{m}{L_1}}} \Longrightarrow L_1 = \frac{(v_1)^2 m}{F_1}$. And thus, $L_1 = \frac{(v_1)^2 m}{F_1} = \frac{(5)^2 * 0.6}{20} = 0.75$ (m).
From the second experiment, we know the stretched length L_2 and the speed, so we can find the tension F_2: $v_2 = \sqrt{\frac{F_2}{\mu}} = \sqrt{\frac{F_2}{\frac{m}{L_2}}} \Longrightarrow F_2 = \frac{(v_2)^2 m}{L_2}$. And thus, $F_2 = \frac{(v_2)^2 m}{L_2} = \frac{(10)^2 * 0.6}{1} = 60$ (N).
We also know that the tension of the spring is in fact the reactive force of the spring, which is equal to the spring constant times the stretching of the spring: $|F| = |k\Delta L|$. Using the result from the first experiment, we find that $F_1 = k\Delta L_1 = k(L_1 - L)$, where L is the unstretched length. We can express the spring constant in terms of the unstretched length: $k = \frac{F_1}{L_1 - L}$. Using the result from the second experiment, we find that $F_2 = k\Delta L_2 = k(L_2 - L)$. We can express the unstretched length in terms of the

spring constant: $L_2 - L = \frac{F_2}{k}$. Substituting the expression for the string constant, we find that $L_2 - L = \frac{F_2}{\frac{F_1}{L_1 - L}} \Rightarrow L = \frac{F_2 L_1 - F_1 L_2}{F_2 - F_1} = \frac{60*0.75-20*1}{60-20} = \frac{25}{40} = 0.63\ (m)$. Substituting this result in the equation for the spring constant, we find that $k = \frac{F_1}{L_1 - L} = \frac{20}{0.75-0.63} = \frac{20}{0.12} = 166.67\ \left(\frac{N}{m}\right)$.