MECHANICS 2, 12-8-2016

The exam consists of two parts. ***The first part is multiple-choice questions. For this part, you only have to give the answer****. Each correct answer is worth 0.4 points, up to a total of 2 points.* ***The second part consists of open questions. For these questions, you have to motivate your answers, i.e., show all the steps that lead to the answer you provide****. Answers without motivation are considered wrong and bring no points. Answers that show only part of the required steps to reach the correct answer will result in receiving a proportional part of points from the total amount of points given for this answer. (For example, if a subquestion brings 1 point, but you only show half of the path to the correct answer, you will get 0.5 points.) An open question might have alternative paths of reaching the correct answer; all such paths are considered correct. The total number of points for the open questions is 8. If an open question consists of subquestions, translation of the result of a wrong calculation from one subquestion to another subquestion is not judged as an error.*

<u>Multiple choice, 5 assignments, 0.4 points per question</u>

1) A mass *m* is attached to an ideal massless spring. When this system is set in motion, it has a period *T*. What is the period when the mass is doubled to 2*m*?
A) $\sqrt{2}\ T$
B) $T/\sqrt{2}$
C) 2*T*
D) *T/2*
E) 4*T*
Answer: A

2) In designing buildings to be erected in an area prone to earthquakes, what relationship should the designer try to achieve between the natural frequency of the building and the typical earthquake frequency for that zone?
A) The natural frequency of the building should be exactly the same as the typical earthquake frequency.
B) The natural frequency of the building should be very different from the typical earthquake frequency.
C) The natural frequency of the building should be almost the same as the typical earthquake frequency but slightly higher.
D) The natural frequency of the building should be almost the same as the typical earthquake frequency but slightly lower.
Answer: B

3) A transverse wave traveling along a string transports energy at a rate *r*. If we want to double this rate, we could
A) decrease the amplitude of the wave by a factor of 2.
B) decrease the amplitude of the wave by a factor of $\sqrt{2}$.
C) increase the amplitude of the wave by a factor of 2.
D) increase the amplitude of the wave by a factor of $\sqrt{2}$.
E) increase the amplitude of the wave by a factor of 4.

Answer: D

4) Consider the waves on a vibrating guitar string and the sound waves the guitar produces in the surrounding air. The string waves and the sound waves must have the same
A) wavelength.
B) frequency.
C) velocity.
D) amplitude.
Answer: B

5) In solids can propagate both transverse waves and longitudinal waves. To distinguish a longitudinal wave in a heterogeneous solid from a transverse wave, we say that a wave is longitudinal if and only if
A) the mass-flow vector is divergence-free.
B) the mass-flow vector is curl-free.
C) the mass-flow vector is gradient-free.
D) the pressure changes are gradient-free.
E) the pressure changes are curl-free.
Answer: B

Open questions, 6 assignments

6) The velocity of an object vibrating following a simple harmonic motion along the *x*-axis obeys the equation $v(t)$ = -(0.25 m/s) sin[(25 rad/s)t + 0.3rad].
(a) What is the amplitude of the motion of this object (in metres)? ***(0.6 points)***
(b) What is the maximum acceleration of the vibrating object (in metres per second squared)? ***(0.5 points)***
Give the answers till the second digit after the decimal point.
Answer:
(a) The displacement of an object that undergoes simple harmonic motion is $x(t) = A\cos(\omega t + \phi)$. The velocity is the derivative of the displacement with respect to time:
$v(t) = \frac{dx(t)}{dt} = \frac{d(A\cos(\omega t + \phi))}{dt} = -\omega A\sin(\omega t + \phi)$ Comparing this relation with the velocity equation given in the question's description, we see that the angular frequency $\omega = 25\ \frac{\text{rad}}{\text{s}}$. We also see that $\omega A = 0.25\left(\frac{\text{m}}{\text{s}}\right)$. From here, we can calculate the amplitude:
$$A = \frac{0.25}{\omega} = \frac{0.25}{25} = 0.01\ (m).$$
(b) We know that the acceleration is the time derivative of the velocity with respect to time: $a(t) = \frac{dv(t)}{dt} = \frac{d(-\omega A\sin(\omega t))}{dt} = -\omega^2 A\cos(\omega t)$. The acceleration is a maximum when $\cos(\omega t) = 1$. Thus, the maximum acceleration is
$$a_{\max}(t) = \left|-\omega^2 A\right| = 25^2 * 0.01 = 6.25\ \frac{\text{m}}{\text{s}^2}.$$

7) An object is attached to an ideal massless spring and allowed to hang vertically in the Earth's gravitational field (g=10 m/s^2). The spring stretches 2.5 cm before it reaches its equilibrium position. If this system is allowed to undergo simple harmonic motion, what will be its frequency? ***(1 point)***
Give the answer till the second digit after the decimal point.
Answer:
Just as explained in the book, a mass hanging vertically in the Earth's gravitational field will stretch the spring until the gravitational force is balanced by the reactive force of the spring. That happens at the new equilibrium position. When the two forces are balanced, we can write: $mg - kx_1 = 0$, where x_1 is the stretching of the spring.
For a vertical mass/spring system undergoing simple harmonic motion, the angular frequency is $\omega = \sqrt{\frac{k}{m}}$. From the equation of the balanced forces, we see that we can express the ratio between the spring constant and the mass in terms of the Earth's gravitational acceleration and the spring's stretching: $\frac{k}{m} = \frac{g}{x_1} = \frac{10}{0.025} = 400\left(\frac{1}{\text{s}^2}\right)$.

The frequency is: $f = \frac{\omega}{2\pi}$, $f = \frac{\omega}{2\pi} = \frac{\sqrt{\frac{k}{m}}}{2\pi} = \frac{\sqrt{400}}{2\pi} = \frac{20}{2\pi} = \frac{10}{\pi}$ Hz (= 3.16 Hz).

8) A 5.0-kg block is attached to an ideal massless spring whose spring constant is 125 N/m. The block is pulled from its equilibrium position at $x_{eq} = 0.00$ m to a position at $x = +0.687$ m and is released from rest. The block then executes lightly damped oscillation along the x-axis, and the damping force is proportional to the velocity. When the block first returns to $x_{eq} = 0.00$ m, its velocity is -2.0 m/s and its acceleration is +5.6 m/s^2. What is the damping constant b? ***(1.9 points)***
Give the answer rounded to the closest integer number.
Answer:
Because the damping force is proportional to the velocity, we can write that the displacement $x(t) = Ae^{-\frac{bt}{2m}}\cos(\omega t)$.
We know that the velocity is the derivative of the displacement with respect to time: $v(t) = \frac{dx(t)}{dt} = \frac{d\left(Ae^{-\frac{bt}{2m}}\cos(\omega t)\right)}{dt} = -\omega Ae^{-\frac{bt}{2m}}\sin(\omega t) - \frac{b}{2m}Ae^{-\frac{bt}{2m}}\cos(\omega t)$. When the block returns at $x_{eq} = 0.00$ m, the equilibrium point, the displacement is 0. This means that $\cos(\omega t) = 0$ and consequently $\sin(\omega t) = 1$. The velocity at the equilibrium point is -2 m/s, so $v(t) = -2\ \frac{\text{m}}{\text{s}} = -\omega Ae^{-\frac{bt}{2m}}$. Thus, we find that $e^{-\frac{bt}{2m}} = \frac{-2}{-\omega A} = \frac{2}{\omega A}$.
We know that the acceleration is the derivative of the velocity with respect to time:

$$a(t) = \frac{dv(t)}{dt} = \frac{d\left(-\omega Ae^{-\frac{bt}{2m}}\sin(\omega t) - \frac{b}{2m}Ae^{-\frac{bt}{2m}}\cos(\omega t)\right)}{dt}.$$

$$= -\omega^2 Ae^{-\frac{bt}{2m}}\cos(\omega t) + \frac{b}{2m}\omega Ae^{-\frac{bt}{2m}}\sin(\omega t) + \frac{b}{2m}\omega Ae^{-\frac{bt}{2m}}\sin(\omega t) + \frac{b^2}{4m^2}Ae^{-\frac{bt}{2m}}\cos(\omega t)$$

At the equilibrium point, $\cos(\omega t) = 0$ and consequently $\sin(\omega t) = 1$. The acceleration at the equilibrium point is +5.6 m/s^2, so $a(t) = 5.6\ \frac{\text{m}}{\text{s}} = 2 * \frac{b}{2m}\omega Ae^{-\frac{bt}{2m}}$. Substituting the expression $e^{-\frac{bt}{2m}} = \frac{2}{\omega A}$, we obtain $a(t) = 5.6\ \frac{\text{m}}{\text{s}} = 2 * \frac{b}{2m}\omega A\frac{2}{\omega A}$. Thus, the damping constant is

$$b = \frac{5.6m}{2} = \frac{28}{2} = 14\ \frac{\text{kg}}{\text{s}}.$$

9) A string is 2 m long and weights 12 grams. The string is stretched between two walls with a tension of 60 N.
(a) What is the speed of waves on the string when it vibrates in its third overtone? ***(0.5 points)***
(b) What is the string's fundamental frequency of vibration? ***(0.6 points)***
Give the answers till the second digit after the decimal point.
Answer:
(a) The speed of the waves on the string is the same for all harmonics (does not depend on the frequency of vibration). We can calculate the speed of the waves

travelling along the string from the square root of the ratio between the tension and the mass per unit length:

$v = \sqrt{\frac{F}{\mu}}$. The mass per unit length is the mass of the string divided by the length:

$\mu = \frac{m}{L} = \frac{0.012}{2} = 0.006 \left(\frac{kg}{m}\right)$. So, we calculate the speed:

$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{60}{0.006}} = \sqrt{10000} = 100 \left(\frac{\text{m}}{\text{s}}\right)$.

(b) To find the fundamental frequency of the string, we can use the equation $f = \frac{v}{\lambda}$.

The wave speed is the wave speed we calculated above. We can find the wavelength from the length of the string. Because the string is attached with its both ends to walls, both ends are clamped. Thus, $L = n\frac{\lambda}{2}$. For the fundamental mode, n=1, so

$\lambda = 2L = 4 \ (\text{m})$. The fundamental frequency is then

$f = \frac{v}{\lambda} = \frac{100}{4} = 25 \text{ Hz}$.

10) A source of sound waves emits waves uniformly in all directions. At a distance of 20 m from the source, the intensity level is 60 db. What is the total acoustic power output of the source, in watts? (The reference intensity I_0 is 1.0 × 10-12 W/m^2.) ***(1.4 points)***

Give the answer till the third digit after the decimal point.

Answer:

The sound intensity level in decibel is

$\beta = 10\log\left(\frac{I}{I_0}\right)$. As the reference intensity I_0 is given, we can calculate the intensity measured at 20 m:

$\log\left(\frac{I}{I_0}\right) = \frac{\beta}{10} = 6, \ \frac{I}{I_0} = 10^6 \Rightarrow I = 10^6 I_0 = 10^6 * 10^{-12} = 10^{-6} \left(\frac{\text{W}}{\text{m}^2}\right)$.

We know that the intensity equals power over area:

$I = \frac{P}{A} = 10^{-6} \left(\frac{\text{W}}{\text{m}^2}\right)$. Because the source emits waves uniformly in all directions, we deal with spherical waves, so $A = 4\pi r^2$. Thus, the total power on a surface of a sphere with 20 m radius is

$P = IA = 10^{-6} * \left(4\pi 20^2\right) = 1.6\pi * 10^{-3} = 0.005 \ (\text{W})$. The total power on the surface of the sphere is the power of the source.

11) The sound from a single source can reach point A by two different paths. The first path is 17 m long and the second path is 19 m long. At point A, the sound interferes destructively and is completely canceled. What is the minimum frequency of the source if the speed of sound is 340 m/s? ***(1.5 points)***

Give the answer till the second digit after the decimal point.

Answer:

This is a problem of interference in two dimensions from the same source. We know that two wave from the same source interfere destructively and cancel each other only when they are at a specific distance from the source. This happens at points where the two distances to the source differ by half a wavelength: $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}, \text{etc}$. To find the frequency, we can use the relation between the speed, frequency, and wavelength:
$v = \lambda f$. As we are looking for the minimum frequency for a fixed point of interference, we have to find the longest wavelength, i.e., we have to use $\frac{\lambda}{2}$. Thus, we have that the difference between the two travel paths is $\frac{\lambda}{2} = 19 - 17 = 2 \ (\mathrm{m})$. So, the longest wavelength is 4 m. Knowing the wavelength, we find the minimum frequency:
$f = \frac{v}{\lambda} = \frac{340}{4} = 85 \ (\mathrm{Hz})$.