AESB1320-17
10-04-18, 9:00
26-Zaal 1

PART I: Mechanics 1.

This part of the exam consists of 7 Conceptual Questions (CQs), each valid for 1 point, and 8 Exercises (EXs), each valid for 4 points.

Grading rules for numerical exercises:
- correct numeric value and solution: 4 points;
- wrong numeric value, but correct solution (computational mistake): 3 points;
- wrong numeric value, correct intermediate numeric value (exercise half-done): 2 points;
- wrong solution: 0 points.

The formula sheet can be found after EX8.

CQ1
Jan and Len throw identical rocks off a tall building at the same time. The ground near the building is flat. Jan throws her rock straight downward. Len throws his rock downward and outward such that the angle between the initial velocity of the rock and the horizon is 30°. Len throws the rock with a speed twice that of Jan's rock. If air resistance is negligible, which rock hits the ground first?

A) They hit at the same time.
B) Jan's rock hits first.
C) Len's rock hits first.
D) It is impossible to know from the information given.

CQ2
You swing a bat and hit a heavy box with a force of 1500 N. The force the box exerts on the bat is

A) exactly 1500 N only if the box does not move.
B) exactly 1500 N whether or not the box moves.
C) greater than 1500 N if the box moves.
D) less than 1500 N if the box moves.
E) greater than 1500 N if the bat bounces back.

CQ3
A 4.0-kg object is moving with speed 2.0 m/s. A 1.0-kg object is moving with speed 4.0 m/s. Both objects encounter the same constant braking force, and are brought to rest. Which object travels the greater distance before stopping?

A) the 4.0-kg object
B) the 1.0-kg object
C) Both objects travel the same distance.
D) It is impossible to know without knowing how long each force acts.

CQ4

Is it possible for a system to have negative potential energy?

A) Yes, as long as the kinetic energy is positive.
B) Yes, as long as the total energy is positive.
C) Yes, since the choice of the zero of potential energy is arbitrary.
D) No, because the kinetic energy of a system must equal its potential energy.
E) No, because this would have no physical meaning.

CQ5

A baseball is thrown vertically upward and feels no air resistance. As it is rising

A) both its momentum and its mechanical energy are conserved.
B) its momentum is not conserved, but its mechanical energy is conserved.
C) both its momentum and its kinetic energy are conserved.
D) its kinetic energy is conserved, but its momentum is not conserved.
E) its gravitational potential energy is not conserved, but its momentum is conserved.

CQ6

When a rigid body rotates about a fixed axis, all the points in the body have the same

A) tangential speed.
B) angular acceleration.
C) tangential acceleration.
D) linear displacement.
E) centripetal acceleration.

CQ7

If the torque on an object adds up to zero

A) the forces on it also add up to zero.
B) the object is at rest.
C) the object cannot be turning.
D) the object could be accelerating linearly but it could not be turning.
E) the object could be both turning and accelerating linearly.

EX1

The horizontal coordinates of a frisbee in a strong wind are given by
$x = -12t + 4t^2$ and $y = 10t - 3t^2$, where x and y are in meters, and t is in seconds.
What is the acceleration of the Frisbee? Give a magnitude and a direction, measuring angles from the positive x direction.

The general formula is $x=x_0+v_0t+a/2\ t_2$, so for each component the acceleration is twice the coefficient of the quadratic term.
The total acceleration is the sum of squares of the two components: ***$a = 10\ m/s^2$***
The angle can be determined from the arctan of the y-component over the x-component:
$\theta = -39.9^o = 323.1^o$

EX2

A 10,000-kg rocket blasts off from earth with a uniform upward acceleration of 2.00 m/s^2 and feels no air resistance.
What is the upward thrust force that its engines must provide during this acceleration?

Since the net acceleration is directed upwards, it means that the acceleration due to the engine is larger than that of gravity: $a_e = a_{net} + g = 11.8\ m/s^2$
Hence, the thrust force is: $\boldsymbol{F_e = ma_e = 118\ kN}$

EX3

A traveler pulls on a suitcase strap at an angle 36° above the horizontal.
If 908 J of work are done by the strap while moving the suitcase a horizontal distance of 15 m, what is the tension in the strap?

Work is only done by the horizontal component of the pulling force: $F_x = W/d = 908/15 = 60.5\ N$
Hence, the tension is: $\boldsymbol{T = F_x/\cos\theta = 74.8\ N}$

EX4

A tennis ball is released 2.3 m from the floor and bounces on the floor three times.
If each time it loses 22.0% of its energy due to heating, how high does it rise after the third bounce?

If 22% of energy is lost, 78% is preserved. After bouncing three times, the energy left in the ball is the cube of that: $E_3 = 0.78^3\ E_0 = 0.475\ E_0$.
When the ball is at the highest point, the energy is fully gravitational potential: $U = mgh$.
Hence: $\boldsymbol{h_3 = 0.475\ h_0 = 1.09\ m}$

EX5

A 620-g object traveling at 2.1 m/s collides head-on with a 320-g object traveling in the opposite direction at 3.8 m/s.
If the collision is perfectly elastic, what is the change in the kinetic energy of the 620-g object?

This exercise was a mistake, since the derivation of the necessary formula is rather tedious and you were not supposed to learn it by heart. Hence, it has been considered a bonus (extra points given to those who had solved part/all of it, but no penalty for wrong/missing answers).
$\boldsymbol{\Delta K = -0.224\ J}$

EX6

A 3.45-kg centrifuge takes 100 s to spin up from rest to its final angular speed with constant angular acceleration. A point located 8.00 cm from the axis of rotation of the centrifuge moves with a speed of 150 m/s when the centrifuge is at full speed.
What is the angular acceleration (in rad/s^2) of the centrifuge as it spins up?

The final angular velocity is: $\omega = V_P/R_P = 150/0.08 = 1875\ rad/s$
Since the angular acceleration is constant, the final velocity is given by: $\omega = \omega_0 + \alpha t$.
Inverting the equation, and considering that $\omega_0 = 0$*, gives:* $\boldsymbol{\alpha = \omega / t = 18.75\ rad/s^2}$

EX 7

A record is dropped vertically onto a freely rotating (undriven) turntable. Frictional forces act to bring the record and turntable to a common angular speed.

If the rotational inertia of the record is 0.54 times that of the turntable, what percentage of the initial kinetic energy is lost?

If we take both the record and the turntable as being part of the system, friction is an internal force, hence angular momentum is conserved. That allows to determine the final angular velocity as a function of the initial one. Note that the final rotational inertia is that of turntable+record.

$L_0 = I\omega_i = L_f = (1 + 0.54)\, I\omega_f \Rightarrow \omega_i = 1.54\, \omega_f$

We can determine the change in kinetic energy, expressing it only in terms of initial angular velocity.

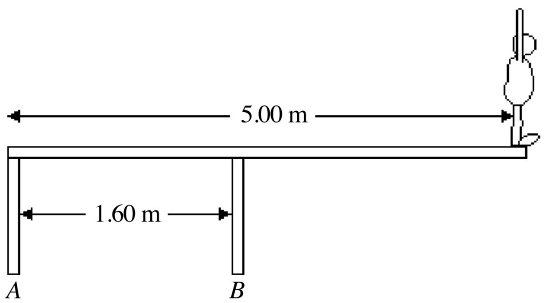
$K_f - K_i = ½\,(1.54\, I)\,(\omega_i /1.54)^2 – ½\, I\, \omega_i = ½\, I\, \omega_i\, (1/1.54 - 1) = ½\, I\, \omega_i\, (-0.35) = -0.35\, K_i$

Hence, the loss in kinetic energy with respect to the initial value is: **$- (K_f - K_i)/K_i = 0.35 = 35\%$**

EX8

An 82.0 kg-diver stands at the edge of a light 5.00-m diving board, which is supported by two narrow pillars 1.60 m apart, as shown in the figure.

Find the magnitude and direction of the force exerted on the diving board by pillar *A*.



When taking the pivot at the contact point between the diving board and pillar B, and considering as positive torques oriented counter-clockwise, the balance of torques is:

$\tau_{diver} + \tau_{FA} = 0 = -mg\,(5.0 – 1.6) + 1.6\, F_A$

Hence: **$F_A = mg\,(5.0 – 1.6)/1.6 = 1708$ N, directed downwards** *(because it generates a positive torque).*

MECHANICS 2, 3-07-2019

The exam consists of 3 Conceptual Questions (CQs), each valid for 1 point, and 2 Exercises or Open Questions (EXs), each valid for 4 points.
For the CQs, you only have to give the answer*.*
For the Exs, you have to motivate your answers, i.e., show all the steps that lead to the answer you provide*. Answers without motivation are considered wrong and bring no points. Answers that show only part of the required steps to reach the correct answer will result in receiving a proportional part of points from the total amount of points given for this answer. (For example, if a subquestion brings 1 point, but you only show half of the path to the correct answer, you will get 0.5 points.) An EX might have alternative paths of reaching the correct answer; all such paths are considered correct. If an EX consists of subquestions, translation of the result of a wrong calculation from one subquestion to another subquestion is not judged as an error.*

<u>Multiple choice, 3 assignments, 1 point per question</u>

1) 1) The position *x* of an object varies with time *t*. For which of the following equations relating *x* and *t* is the motion of the object simple harmonic motion?
A) $x = 5sin^2(3t)$
B) $x = 8cos(3t^2)$
C) $x = 4tan(2t)$
D) $x = 5sin(3t) + 5cos(3t)$
E) $x = 2cos(3t - 1)$
Answer: E (The equation for simple harmonic motion is $x(t) = Acos(\omega t + \varphi)$.)

2) The lowest-pitch tone to resonate in a pipe of length *L* that is open at both ends is 200 Hz. Which one of the following frequencies will NOT resonate in the same pipe?
A) 400 Hz
B) 600 Hz
C) 1000 Hz
D) 1500 Hz
E) 2000 Hz
Answer: D ($f = \frac{m}{2}\frac{v}{L} \rightarrow \frac{v}{L} = 400\ (Hz)$, because you hear the fundamental mode. The other modes are higher and equal to $\frac{m}{2}400\ (Hz)$.)

3) Because both transverse waves and longitudinal waves can propagate in solids, to distinguish a longitudinal wave in a heterogeneous solid from a transverse wave, we say that a wave is longitudinal if and only if
A) the pressure change is curl-free.
B) the pressure change is gradient-free.
C) the mass-flow vector is gradient-free.
D) the mass-flow vector is curl-free.
E) the mass-flow vector is divergence-free.
Answer: D (See slide 24 from the PPT presentation “Acoustic Wave Equation / Stress, Strain, and Elastic Constants”.)

<u>Open questions, 2 assignments</u>

4) (a) A 0.2-kg block on a horizontal frictionless surface is attached to an ideal massless spring whose spring constant is 500 N/m. The block is pulled from its equilibrium position at *x* = 0.00 m to a displacement *x* = +0.06 m and it is then released. The block then executes simple harmonic motion along the horizontal *x*-axis. When the displacement is *x* = -0.04 m, find the acceleration of the block. ***(3 points)***

(b) To the block of 0.2 kg is attached a second block of 0.8 kg. The two blocks are connected by a very light rigid bar. The two blocks are pulled from their equilibrium position at *x* = 0.00 m to a displacement of *x* = +0.06 m and are released. After one period, the 0.8-kg block separates from the 0.2-kg block, with the latter remaining attached to the spring and continuing to oscillate. What is the amplitude of simple harmonic motion after the separation of the 0.8-kg block? ***(1 point)***

Give the answers as integer numbers.

Answer:

(a) A simple harmonic motion (SHM) is described by the relation $x(t) = Acos(\omega t + \varphi)$. The acceleration is the second derivative of the displacement: $a(t) = \frac{d^2x(t)}{dt^2} = \frac{d(-A\omega sin(\omega t+\varphi))}{dt} = -A\omega^2 cos(\omega t + \varphi)$.

Alternative 1:

Because the block is moved from the equilibrium to a new position and released at that position, that position will be the maximum displacement, i.e., the amplitude $A = 0.06\ (m)$. If you choose to use cos to describe SHM, because the time counting starts when the block is at the maximum displacement, $\varphi = 0$. To describe completely the SHM, you also need to know the angular frequency ω. You know that $\omega = \sqrt{\frac{k}{m}}$, where m is the mass of the block and k is the spring constant. Because both the mass and the spring constant are given, you can calculate the angular frequency:

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{500}{0.2}} = \sqrt{2500} = 50\ (rad)$.

Because you are given the displacement at a specific time, you can calculate directly the factor $cos(\omega t)$:

$-0.04 = 0.06cos(\omega t) \rightarrow cos(\omega t) = -\frac{0.04}{0.06} = -\frac{2}{3}$.

Knowing this factor, you can now calculate the acceleration:

$a(t) = -A\omega^2 cos(\omega t + \varphi) = -0.06(50)^2\left(-\frac{2}{3}\right) = 100\ \left(\frac{m}{s^2}\right)$.

Alternative 2:

Comparing the equations for the displacement and for the acceleration, you can see that $a(t) = -\omega^2 x(t)$.

To describe completely the SHM, you also need to know the angular frequency ω. You know that $\omega = \sqrt{\frac{k}{m}}$, where m is the mass of the block and k is the spring constant. Because both the mass and the spring constant are given, you can calculate the angular frequency:

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{500}{0.2}} = \sqrt{2500} = 50\ (rad)$.

Now, you can calculate the acceleration:

$a(t) = -\omega^2 x(t) = -(50)^2(-0.04) = 100\ (\frac{m}{s^2})$.
(b) (This question was given in July 2018 as a conceptual question.) The amplitude of SHM is independent of the mass and frequency, so it stays the same, i.e., $A = 0.06\ (m)$.

5) (a) A transverse wave is traveling on a string stretched along the horizontal *x*-axis. The equation for the vertical displacement $y(x,t)$ of the string is given by $y(x,t) = 0.002cos\big(\pi(15x - 52t)\big)$, where all quantities are in SI units. What is the maximum speed of a particle of the string? ***(2.7 points)***
(b) What is the wavelength and the frequency of the waves propagating along the string? ***(1.3 points)***
Give the answers till the second digit after the decimal point.
Answer:
(a) As you are asked here to calculate the particle velocity, you have to come up with an equation for an oscillatory motion, i.e., $x(t) = Acos(\omega t)$. Then, you can calculate the maximum particle speed by taking the derivative $v(t) = \frac{dx(t)}{dt} = -A\omega sin(\omega t)$, setting $sin(\omega t) = 1$, and taking the absolute value, obtaining $v^{max}(t) = A\omega$.
To obtain an equation for the displacement of a point as a function of time, you can use the equation of the vertical displacement $y(x,t) = Acos(kx \pm \omega t)$ by setting $x = 0$. You can do that as the particle velocity will follow the same oscillatory law at any point, so choosing a smart point simplifies the solution. Comparing this equation with the one you are given in the question statement, So, you can write $y(0,t) = 0.002cos\big(\pi(15 * 0 - 52t)\big) = 0.002cos\big(\pi(-52t)\big) = 0.002cos(\pi 52t)$.
From this equation, you can conclude that the angular frequency $\omega = \pi 52$, while the amplitude is $A = 0.002$. Using these two values, you can now calculate the maximum speed:
$v^{max}(t) = A\omega = 0.002 * \pi * 52 = 0.33\ (\frac{m}{s})$.
(b) Comparing the equation of the vertical displacement $y(x,t) = Acos(kx \pm \omega t)$ with the one you are given in the question statement in (a), you can conclude that the wavenumber is $k = \pi 15$. You know that $k = \frac{2\pi}{\lambda}$, so the wavelength is
$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi 15} = 0.13\ (m)$.
From (a), you already found that $\omega = \pi 52$, so you now calculate the frequency:
$\omega = 2\pi f = \pi 52 \rightarrow f = \frac{\pi 52}{2\pi} = 26\ (Hz)$.