AESB1320-17
4-07-18, 9:00, 26-Hall 1

The exam consists of 10 Conceptual Questions (CQs), each valid for 1 point, and 10 Exercises (EXs), each valid for 4 points. The maximum score is 50. The pass score is 29.

This is a closed-book exam: only pens, blank paper and non-graphical calculators are allowed.

The exam is structured in two parts:
- Part I: old Mechanics 1 (book chapters 2-12);
- Part II: old Mechanics 2 (book chapters 13-14 and wave equation).

Please answer every part on a separate sheet of paper.

PART I

Grading rules for EX1-EX8:
- correct numeric value and solution: 4 points;
- wrong numeric value, but correct solution (computational mistake): 3 points;
- wrong numeric value, correct intermediate numeric value (exercise half-done): 2 points;
- wrong solution: 0 points.

CQ1
Two objects are thrown from the top of a tall building and experience no appreciable air resistance. One is thrown up, and the other is thrown down, both with the same initial speed. What are their speeds when they hit the street?

A. The one thrown up is traveling faster.
B. They are traveling at the same speed.
C. The one thrown down is traveling faster.

B: The object thrown up will pass again by the starting point with the same velocity it was thrown with, just reversed in sign: so exactly the same as the other object. Also possible to use energy.

CQ2
Consider what happens when you jump up in the air. Which of the following is the most accurate statement?

A. When you push down on the earth with a force greater than your weight, the earth will push back with the same magnitude force and thus propel you into the air.
B. Since the ground is stationary, it cannot exert the upward force necessary to propel you into the air. Instead, it is the internal forces of your muscles acting on your body itself that propels your body into the air.
C. You are able to spring up because the earth exerts a force upward on you that is greater than the downward force you exert on the earth.
D. It is the upward force exerted by the ground that pushes you up, but this force cannot exceed your weight.
E. When you jump up the earth exerts a force F1 on you and you exert a force F2 on the earth. You go up because F1 > F2.

A: it is about the force between you and the Earth (equal in magnitude, opposite in sign), which has to be larger than gravity.

CQ3

Three people are pushing a car up a hill at constant velocity. The net force on the car is

A. zero.
B. up the hill and greater than the weight of the car.
C. down the hill and greater than the weight of the car.
D. down the hill and equal to the weight of the car.
E. up the hill and equal to the weight of the car.

A: If the car moves with constant velocity, then the net acceleration has to be zero.

CQ4

A 4.0-kg object is moving with speed 2.0 m/s. A 1.0-kg object is moving with speed 4.0 m/s. Both objects encounter the same constant braking force, and are brought to rest. Which object travels the greatest distance before stopping?

A. the 4.0-kg object
B. the 1.0-kg object
C. Both objects travel the same distance.
D. It is impossible to know without knowing how long each force acts.

C: Both objects have the same initial kinetic energy ($K_{initial}=1/2\ mv^2$), hence they work done to stop them (when $K_{final}=0$) is the same. Since the work done is equal to the force times the travelled distance, and the forces are the same, also the travelled distance will be the same.

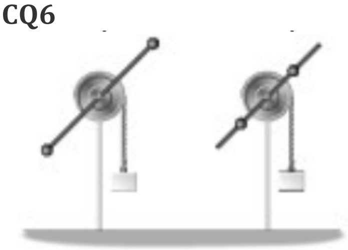
CQ5

A baseball is thrown vertically upward and feels no air resistance. As it is rising

A. its linear momentum is not conserved, but its mechanical energy is conserved.
B. both its momentum and its kinetic energy are conserved.
C. its kinetic energy is conserved, but its momentum is not conserved.
D. its gravitational potential energy is not conserved, buts its momentum is conserved.
E. both its momentum and its mechanical energy are conserved.

A: The is a non-zero external force (gravity), but it is conservative.

CQ6



The two rotating systems shown in the figure differ only in that the two identical movable masses are positioned at different distances from the axis of rotation. If you release the hanging blocks simultaneously from rest, and if the ropes do not slip, which block lands first?

A. The block at the left lands first.
B. The block at the right lands first.
C. Both blocks land at the same time.

B: The right block has smaller I (mass closer to the rotation axis), so it has less rotational kinetic energy in the wheel and more kinetic energy in the falling block.

CQ7

A heavy boy and a lightweight girl are balanced on a massless seesaw. If they both move forward so that they are one-half their original distance from the pivot point, what will happen to the seesaw? Assume that both people are small enough compared to the length of the seesaw to be thought of as point masses.

A. The side the girl is sitting on will tilt downward.
B. The side the boy is sitting on will tilt downward.
C. Nothing will happen; the seesaw will still be balanced.
D. It is impossible to say without knowing the masses.
E. It is impossible to say without knowing the distances

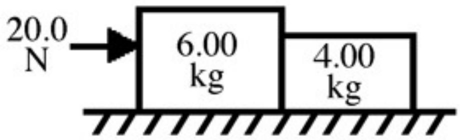
C: Since the torques balanced before and they linearly depend on the distance from the pivot, they will still balance if both distances are halved.

EX1
A soccer ball is released from rest at the top of a grassy incline. After 2.9 seconds, the ball travels 84 meters.
What was the magnitude of the ball's acceleration? (assume it to be constant)

20 m/s^2

$x=a/2\ t^2$ -> $a=2x/t^2=2 * 84/2.9^2=20$

EX2



A 6.00-kg block is in contact with a 4.00-kg block on a horizontal frictionless surface as shown in the figure. The 6.00-kg block is being pushed by a horizontal 20.0-N force as shown. What is the magnitude of the force that the 6.00-kg block exerts on the 4.00-kg block?

8.0 N

$a=F/m$, with $m=(6+4)$ kg -> $a=2$ m/s^2
$F_{6,4}=ma$, with $m=4$ kg -> $F_{6,4}=4*2=8$ N

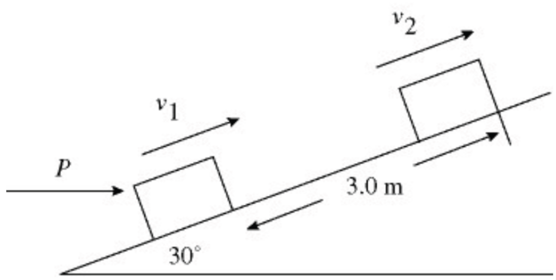
EX3
A 60.0-kg person falls from rest onto a platform supported by an ideal spring. The distance between the initial position of the person and the platform is 1.20 m. The platform drops 6.00 cm before the person comes to rest.
What is the spring constant of the spring? You can neglect the mass of both platform and spring.

4.12 x 10^5 N/m (3 points if 3.92 x 10^5 N/m)

h=1.2 m; d=6 cm
$W_g=mg(h+d)=740.9$ J
$W_s=1/2\ k\ x^2 = -0.0018*k$ [J]
$W_g=-W_s$ -> $k = 740.9/0.018$ N/m = 4.12 x 10^5 N/m

If $W_g=mgh$ -> k=3.92 x 10^5 N/m

EX4



In the figure, a 600-kg crate is on a rough surface inclined at 30°. A constant external force P = 4800 N is applied horizontally to the crate. While this force pushes the crate a distance of 3.0 m up the incline, its velocity along the surface changes from 0.9 m/s to 2.1 m/s. How much work does friction do during this process?

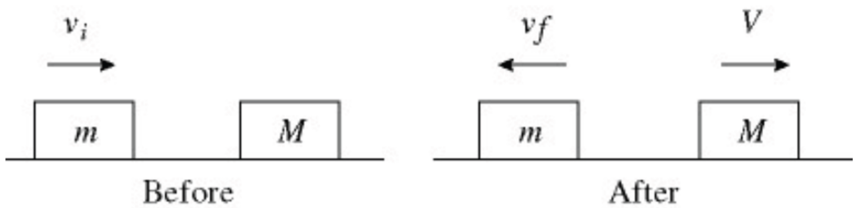
-2570 J

Data:
$m=600$ kg; $\Delta s=3$ m; $P=4800$ N; $V_1=0.9$ m/s; $V_2=2.1$ m/s; $W_f=?$

Solution 1 (using energy):
Change in potential energy: $h= \Delta s \sin 30^o=1.5$ m => $\Delta U=mgh= 8820$ J
Change in kinetic energy: $\Delta K=1/2\ m\ (V_2^2-V_1^2)=1080$ J
Work done by force **P**: $W_P=P \cos 30^o = 12471$ J
Work done by friction: $Wf= \Delta U +\Delta K -W_P = -2570$ J

Solution 2 (using forces and work):
$F_{net}=P \cos 30^o-mg \sin 30^o-F_f=ma$
Computing acceleration from kinematics: $v_2=v_1t+at$ & $x=v_1t+a/2\ t^2$ => $a=0.6\ m/s^2$
Friction from above: $F_f= -(P \cos 30^o-mg \sin 30^o-ma)= -857$ N
Work by friction: $W_f=F_f * \Delta s= -2571$ J

EX5



A block of mass m = 8.40 kg, moving on a horizontal frictionless surface with a speed 4.20 m/s, makes a perfectly elastic collision with a block of mass *M* at rest. After the collision, the 8.40-kg block recoils with a speed of 0.400 m/s. In the figure, the blocks are in contact for 0.200 s. The magnitude of the average force on the 8.40-kg block, while the two blocks are in contact, is closest to

193 N

Data:
$M=8.4$ kg; $v_i=4.20$ m/s; $v_f=-0.4$ m/s; $\Delta t=0.2$ s; $F_{av}=?$

Solution:
F_{av} is the force required to change linear momentum within the given time interval:
$P_i=mv_i$ & $P_f=mv_f$ => $F_{av}=(P_f-P_i)/\Delta t=m(v_f-v_i)/\Delta t=-193$ N
The sign is negative because the force points to the left (if the horizontal axis is oriented to the right), but the problem asks for the magnitude, and that is always positive.

EX6
A string is wrapped around a pulley with a radius of 2.0 cm and no appreciable friction in its axle. The pulley is initially not turning. A constant force of 50 N is applied to the string, which does not slip, causing the pulley to rotate and the string to unwind. If the string unwinds 1.2 m in 4.9 s, what is the moment of inertia of the pulley?

0.2 kg m²

Data:
R=0.02 m; F=50 N; Δs=1.2 m; Δt=4.9 s; I=?

Solution:
$\tau=I\alpha=FR$
$\Delta\theta= \Delta s/R=60$ rad
$\Delta\theta=\theta-\theta_0=\omega_0 t+\alpha/2\ t^2= \alpha/2\ t^2 \Rightarrow \alpha=2\ \Delta\theta/\ t^2= 5$ rad/s
$I=FR/\alpha= 0.2$ kg m^2

EX7
A centrifuge takes 100 s to spin up from rest to its final angular speed with constant angular acceleration. A point located 8.00 cm from the axis of rotation of the centrifuge moves with a speed of 150 m/s when the centrifuge is at full speed. How many revolutions does the centrifuge make as it goes from rest to its final angular speed?

1.5 10⁴ rad/s

Both angular acceleration and angular speed are the same for all points of the centrifuge. It's a kinematic problem, so mass is not necessary.

$\omega=v/R=150/0.08=1875$ rad/s

$\omega= \omega_0+\alpha t \Rightarrow \alpha= \omega/t= 18.75$ rad/s^2 [*2 points if correct*]

$\theta= \theta_0+ \omega_0 t + \alpha/2\ t^2 = \alpha/2\ t^2 = 9.4\ 10^4$ rad

$N_{revolutions}= \theta/(2\pi)= 1.5\ 10^4$ rad/s

EX8
Two identical ladders are 3.0 m long and weigh 600 N each. They are connected by a hinge at the top and are held together by a horizontal rope, 1.0 m above the smooth floor forming a symmetric "A" arrangement. The angle between the ladders is 60° and both ladders have their center of gravity at their midpoint. What is the tension in the rope?

281 N

It is enough to balance torques for one ladder (tension is the same at both ends of the rope), about the pivot taken as the hinge where the two ladders connect. Three forces cause torques about the pivot: gravity on the middle of the ladder (down), the normal force from the floor due to the weight of the ladder (up) and the tension of the rope (horizontal, towards the inside).

If L=3.0 m and W= 600 N:

$\tau_{gravity} = -W\ L/2 \sin 30^o$
$\tau_{floor} = W\ L \sin 30^o$
$\tau_{rope} = -T\ (L \cos 30^o-1)$

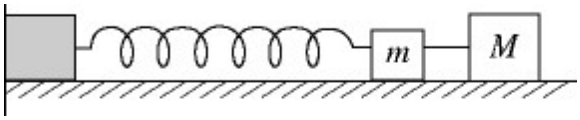
$\tau_{gravity} + \tau_{floor} + \tau_{rope} = 0 \Rightarrow T=1/2\ WL \sin 30^o /(L \cos 30^o -1)= 900/3.2=281$ N

MECHANICS 2, 04-07-2018

The exam consists of 3 Conceptual Questions (CQs), each valid for 1 point, and 2 Exercises or Open Questions (EXs), each valid for 4 points.
For the CQs, you only have to give the answer*.*
For the EXs, you have to motivate your answers, i.e., show all the steps that lead to the answer you provide*. Answers without motivation are considered wrong and bring no points. Answers that show only part of the required steps to reach the correct answer will result in receiving a proportional part of points from the total amount of points given for this answer. (For example, if a subquestion brings 1 point, but you only show half of the path to the correct answer, you will get 0.5 points.) An EX might have alternative paths of reaching the correct answer; all such paths are considered correct. If an EX consists of subquestions, translation of the result of a wrong calculation from one subquestion to another subquestion is not judged as an error.*

Conceptual Questions (Multiple choice), 3 assignments, 1 point per question

1) In the figure, two masses, $M = 16$ kg and $m = 8$ kg, are connected by a very light rigid bar. The connected masses are attached to an ideal massless spring with a spring constant 100 N/m. The system is set into oscillation with an amplitude of 78 cm. At the instant when the acceleration is at its maximum, the 16-kg mass separates from the 8-kg mass, which then remains attached to the spring and continues to oscillate. What will be the amplitude of oscillation of the 8-kg mass?



A. 234 cm.
B. 156 cm.
C. 78 cm.
D. 39 cm.
E. 26 cm.

Answer: C (The amplitude of oscillation does not depend on the mass.)

2) The vertical displacement $y(x,t)$ of a wave traveling along a string stretched along the horizontal *x*-axis is given by $y(x,t) = (50\ mm)\ cos\left[\left(4.5\ \frac{1}{m}\right)x - \left(8.3\ \frac{rad}{s}\right)t\right]$.
What is the speed of the wave along the string?

A. 90 m/s.
B. 1.84 m/s.
C. 0.54 m/s.
D. 0.41 m/s.
E. 0.01m/s.

Answer: B (The wave speed is $v = \frac{\omega}{k} = \frac{8.3}{4.5} = 1.84\ \left(\frac{m}{s}\right)$.)

3) In the acoustic wave equation for pressure $\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 \rho}{\partial t^2}$ the propagation speed c

is

A. inversely proportional to the square root of the compression modulus and inversely proportional to the square root of the density.
B. proportional to the square root of the compression modulus and inversely proportional to the square root of the density.
C. proportional to the square root of the compression modulus and proportional to the square root of the density.
D. proportional to the compression modulus and inversely proportional to the density.
E. proportional to the compression modulus and proportional to the density.

Answer: B (The equation is $c = \sqrt{\frac{K}{\rho}} \left(\frac{m}{s}\right)$.)

Open questions, 2 assignments

4) A lightly damped harmonic oscillator is characterized by a damping force that is proportional to its speed. At $t = 0\ s$, the amplitude of the damped oscillatory motion is $0.5\ cm$. When $t = 8.2\ s$, the amplitude has become $0.4\ cm$.
(a) At what value of t will the damped oscillations have an amplitude of $0.25\ cm$? Give the answer till the first digit after the decimal point. ***(2.6 points)***
(b) What would be the velocity at that time t if the oscillation frequency at that time would be $f = 2\ Hz$? Give the answers till the third digit after the decimal point. ***(1.4 points)***
Answer:
(a) The problem talking about damped oscillatory motion, so we can write that the displacement is $x(t) = A\, e^{\frac{-bt}{2m}} \cos(\omega t)$. But this is only true because it is stated that the damping force is proportional to the speed. The amplitude of this oscillation is $Amp(t) = Ae^{\frac{-bt}{2m}}$. Because the amplitude at t=0 s is given, you can find A:
$0.5 = Ae^{\frac{-b*0}{2m}} \Rightarrow A = 0.5\ (cm)$.
At $t = 8.2\ s$ the amplitude is also known, so you can find the factor $\frac{b}{2m}$:
$0.4 = 0.5e^{\frac{-b*8.2}{2m}} \Rightarrow \frac{-b*8.2}{2m} = \ln\frac{0.4}{0.5} = -0.223 \Rightarrow \frac{b}{2m} = \frac{0.223}{8.2}\left(\frac{1}{s}\right)$.
Now, you can find the time at which the amplitude is $0.25\ cm$:
$0.25 = 0.5e^{\frac{-bt}{2m}} = 0.5e^{-\frac{0.223}{8.2}t} \Rightarrow -\frac{0.223}{8.2}t = \ln\frac{0.25}{0.5} = -0.693 \Rightarrow t = \frac{0.693*8.2}{0.223} = 25.5\ (s)$.
(b) You know that the velocity is the derivative of the displacement with respect to time:
$v(t) = \frac{dx(t)}{dt} = \frac{d\left(Ae^{-\frac{bt}{2m}}cos(\omega t)\right)}{dt} = -\omega Ae^{-\frac{bt}{2m}}sin(\omega t) - \frac{b}{2m}Ae^{-\frac{bt}{2m}}cos(\omega t)$. You know that $\omega = 2\pi f = 2\pi * 2 = 4\pi\ \left(\frac{rad}{s}\right)$. Making use of the fact that $\cos(2\pi * integer) = 1$ and $\sin(2\pi * integer) = 0$, the velocity is
$v(25.5) = -4\pi * 0.25 * sin(4\pi * 25.5) - \frac{0.223}{8.2} * 0.25 * cos(4\pi * 25.5) = -4\pi * 0.25 * 0 - \frac{0.223}{8.2} * 0.25 * 1 = -0.007\ \left(\frac{cm}{s}\right)$.

5) A person is riding a bicycle at 10 m/s along a straight road that is parallel to and right next to some railroad tracks. The person hears the whistle of a train that is behind. The frequency emitted by the train is 840 Hz, but the frequency the person hears is 800 Hz. Take the speed of sound to be 340 m/s. What is the speed of the train and is it traveling away from or toward the bicycle? ***(4 points)***
Give the answer for the speed till the second digit after the decimal point.
Answer:
You are dealing with a moving observer that hears sound, so you are dealing with a Doppler effect. The observer is moving, so you have the case of a Doppler effect for a moving observer. For this case, you can write that the frequency the person perceives is
$f' = f''\left(1 \pm \frac{u}{v}\right)$,

where f'' is the frequency of the sound from the source, v is the speed of sound in the air, and u is the speed of the moving person. As the frequency perceived by the person on the bicycle is lower than the frequency of the sound from the train, you should take a minus sign. Then, you can calculate f'':

$$f' = f''\left(1 - \frac{u}{v}\right) \Rightarrow f'' = \frac{f'}{1-\frac{u}{v}} = \frac{800}{1-\frac{10}{340}} = \frac{800}{0.971} = 823.89\ (Hz).$$

Comparing this frequency with the frequency of the sound emitted by the train, you see that it is lower. (The description of the question already gives you a tip that the train is moving, so you should expect that.) This means that the train is not stationary but is moving and it is moving AWAY from the person on the bicycle. This also means that you are dealing with a Doppler effect for a moving source. You can then write that the relation between f'' and the frequency of the sound emitted by the train f^T is

$f'' = \frac{f^T}{\left(1\pm\frac{u}{v}\right)} = \frac{f^T}{\left(1+\frac{u}{v}\right)}$. From here, you can calculate the speed of the train:

$$f'' = \frac{f^T}{\left(1+\frac{u}{v}\right)} \Rightarrow 823.89\left(1 + \frac{u}{v}\right) = 840 \Rightarrow \frac{u}{340} = \frac{840}{823.89} - 1 = 0.0196 \Rightarrow u = 0.0196 * 340 = 6.66\ \left(\frac{m}{s}\right).$$