AESB1320-17
10-04-18, 9:00
26-Zaal 1

The exam consists of 10 Conceptual Questions (CQs), each valid for 1 point, and 10 Exercises (EXs), each valid for 4 points. The maximum score is 50. The pass score is 29.

Grading rules for numerical exercises:
- correct numeric value and solution: 4 points;
- wrong numeric value, but correct solution (computational mistake): 3 points;
- wrong numeric value, correct intermediate numeric value (exercise half-done): 2 points;
- wrong solution: 0 points.

This is a closed-book exam: only pens, blank paper and non-graphical calculators are allowed.

The formula sheet can be found at the end of the exam.

The exam is structured in two parts:
- Part I: Mechanics 1
- Part II: Mechanics 2

Please answer every part on a separate sheet of paper.

PART I: Mechanics 1.

CQ1

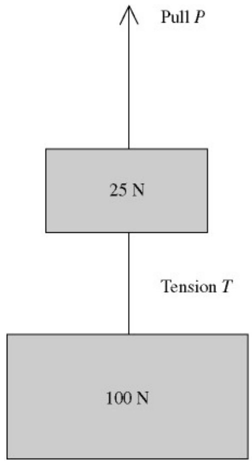
For general projectile motion, when the projectile is at the highest point of its trajectory

A. its acceleration is zero.
B. its velocity is perpendicular to the acceleration.
C. its velocity and acceleration are both zero.
D. the horizontal component of its velocity is zero.
E. the horizontal and vertical components of its velocity are zero.

Answer: B

CQ2

Two weights are connected by a massless wire and pulled upward with a constant speed of 1.50 m/s by a vertical pull P. The tension in the wire is T (see figure). Which one of the following relationships between T and P must be true?



A. $T > P$
B. $T = P$
C. $P + T = 125$ N
D. $P = T + 25$ N
E. $P = T + 100$ N

Answer: D

CQ3

A 3.00-kg ball swings rapidly in a complete vertical circle of radius 2.00 m by a light string that is fixed at one end. The ball moves so fast that the string is always taut and perpendicular to the velocity of the ball. As the ball swings from its lowest point to its highest point

A. the work done on it by gravity and the work done on it by the tension in the string are both equal to -118 J.
B. the work done on it by gravity is -118 J and the work done on it by the tension in the string is +118 J.
C. the work done on it by gravity is +118 J and the work done on it by the tension in the string is 118 J.
D. the work done on it by gravity is -118 J and the work done on it by the tension in the string is zero.
E. the work done on it by gravity and the work done on it by the tension in the string are both equal to zero.

Answer: D

CQ4

Swimmers at a water park have a choice of two frictionless water slides as shown in the figure. Although both slides drop over the same height, h, slide 1 is straight while slide 2 is curved, dropping quickly at first and then leveling out. How does the speed v_1 of a swimmer reaching the end of slide 1 compares with v_2, the speed of a swimmer reaching the end of slide 2?

A. $v_1 > v_2$
B. $v_1 < v_2$
C. $v_1 = v_2$
D. No simple relationship exists between v_1 and v_2 because we do not know the curvature of slide 2.

Answer: C

CQ5

You are standing on a skateboard, initially at rest. A friend throws a very heavy ball towards you. You can either catch the object or deflect the object back towards your friend (such that it moves away from you with the same speed as it was originally thrown). What should you do in order to MINIMIZE your speed on the skateboard?

A. Catch the ball.
B. Deflect the ball.
C. Your final speed on the skateboard will be the same regardless whether you catch the ball or deflect the ball.

Answer: A

CQ6

As you are leaving a building, the door opens outward. If the hinges on the door are on your right, what is the direction of the angular velocity of the door as you open it?

A. up
B. down
C. to your left
D. to your right
E. forwards

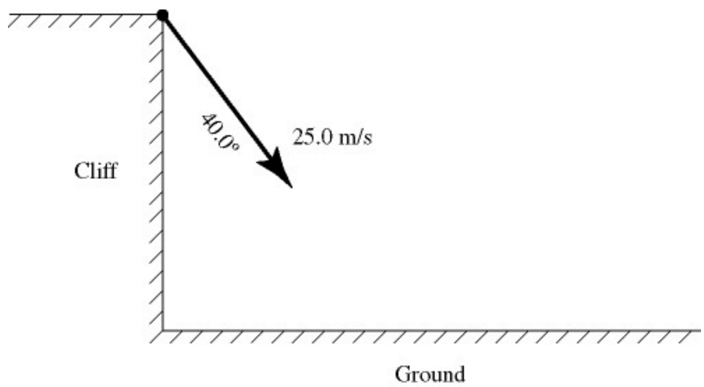
Answer: B

CQ7
2) A heavy boy and a lightweight girl are balanced on a massless seesaw. If they both move forward so that they are one-half their original distance from the pivot point, what will happen to the seesaw? Assume that both people are small enough compared to the length of the seesaw to be thought of as point masses.
A) It is impossible to say without knowing the masses.
B) It is impossible to say without knowing the distances.
C) The side the boy is sitting on will tilt downward.
D) Nothing will happen; the seesaw will still be balanced.
E) The side the girl is sitting on will tilt downward.

Answer: D

EX1

A hiker throws a stone from the upper edge of a vertical cliff. The stone's initial velocity is 25.0 m/s directed at 40.0° with the face of the cliff, as shown in the figure. The stone hits the ground 3.75 s after being thrown and feels no appreciable air resistance as it falls.
What is the height of the cliff?



Answer:
Data: v_0 = 25 m/s; θ = 40°; t = 3.75 s; y_0 = ?

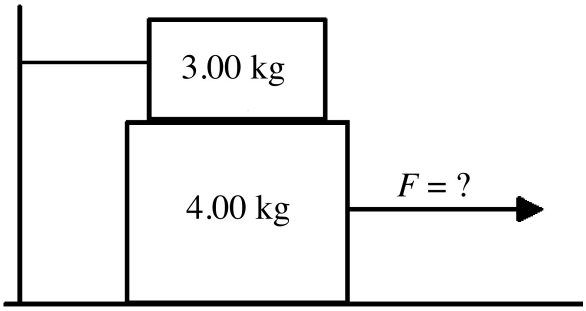
We take the y-axis to point upwards.

$v_y = v_0 \cos\theta = -25 \cos 40° = -19.15$ m/s

$y = y_0 + v_0 t + a/2\, t^2$ => $0 = y_0 - 19.15 * 3.75 - 9.8/2 * 3.75^2$ => y_0 = **142 m**

EX2
A 4.00-kg block rests between the floor and a 3.00-kg block as shown in the figure. The 3.00-kg block is tied to a wall by a horizontal rope. If the coefficient of static friction is 0.800 between each pair of surfaces in contact, what horizontal force *F* must be applied to the 4.00-kg block to make it

move?



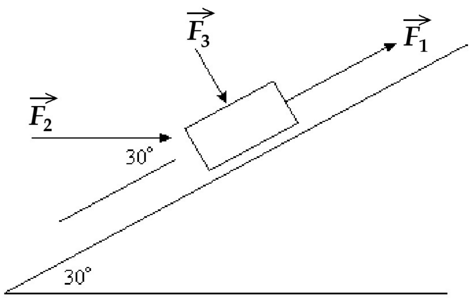
Answer:
Data: m_1 = 3 kg; m_2 = 4 kg; μ = 0.8; F = ?

In the horizontal direction, block 2 (bottom) is subject to three forces: friction from the bottom (F_{fb}), friction for block 1 (top, F_{f1}), and the unknown force F. The maximum static friction is $F_f = \mu n$, with n being the normal force (in this case: $n = mg$). The minimum force F needed to move the bottom block is equal in magnitude and opposite in direction to the sum of the two friction forces.

We take the x-axis to point to the right.

$F_f = F_{fb} + F_{f1} = - \mu (m_1 + m_2) g - \mu m_1 g = - \mu (2m_1 + m_2) g = - 0.8 * 10 * 9.8 = - 78.4$ N => $F = -F_f =$ **78.4 N**

EX3
Three forces, F_1 = 20.0 N, F_2 = 40.0 N, and F_3 = 10.0 N act on an object with a mass of 2.00 kg which can move along a frictionless inclined plane as shown in the figure. The questions refer to the instant when the object has moved through a distance of 0.600 m along the surface of the inclined plane in the upward direction. Calculate the amount of work done by F_2.



Answer:
Data: F_2 = 40 N; m = 2 kg; $\theta = 30^o$; d = 0.6 m; W_{F2} = ?

We are only interested in F_2. We take the x-axis along the incline, pointing upwards.
Only the component of F_2 that is parallel to the displacement contributes to the work.

$W = F_x d = F_2 \cos \theta\, d =$ **20.9 J**

EX4
You do 174 J of work while pulling your sister back on a swing, whose chain is 5.10 m long. You start with the swing hanging vertically and pull it until the chain makes an angle of 32.0° with the vertical with your sister is at rest. What is your sister's mass, assuming negligible friction?

Answer:
Data: $W = 174$ J; $L = 5.1$ m; $\theta = 32^{o}$; $m = ?$

Since the force from the swing is perpendicular to the motion, it does not do any work. Hence, the work you do is equal and opposite to the work done by gravity. Since gravity is conservative, the work done only depends in the height difference ($W_g = mgh$). We take the y-axis to point upwards.

$h = L - L \cos\theta = L(1 - \cos\theta) = 5.1 (1 - \cos 32^{o}) = 0.765$ m

$m = W_g/gh = -W/gh = -174 / (-9.8 * 0.765) =$ **23.2 kg**

EX5

A 0.500-kg ball traveling horizontally on a frictionless surface approaches a very massive stone at 20.0 m/s perpendicular to wall and rebounds with 70.0% of its initial kinetic energy. What is the magnitude of the change in momentum of the stone?

Answer:
Data: $m = 0.5$ kg; $v_i = 20$ m/s; $K_f = 0.7 K_i$; $|\Delta p| = ?$

We can use the change in kinetic energy to determine the change in the magnitude of the velocity. When determining the change in momentum, we also have to take into account that the final velocity has opposite sign with respect to the initial velocity. We take the x-axis to point in the direction of the initial velocity.

$K_i = ½ m v_i^2 = 100$ J
$K_f = 0.7 K_i = 70$ J => $v_f = -\text{sqrt}(2K_f/m) = -16.7$ m/s

Alternatively: $K_f = 0.7 K_i$ => $½ m v_f^2 = 0.7 ½ m v_i^2$ => $v_f = -v_i \text{sqrt}(0.7) = -16.7$ m/s

$\Delta p = p_f - p_i = mv_f - mv_i = m(v_f - v_i) = 0.5(-16.7 - 20) = -18.35$ => $|\Delta p| =$ **18.35 kg·m/s**

EX6

A 4.50-kg wheel that is 34.5 cm in diameter rotates through an angle of 13.8 rad as it slows down uniformly from 22.0 rad/s to 13.5 rad/s. What is the magnitude of the angular acceleration of the wheel? [$I = ½ MR^2$]

Answer:
Data: $m = 4.5$ kg; $R = 0.345$ m; $\Delta\theta = 13.8$ rad; $\omega_i = 22$ rad/s; $\omega_f = 13.5$ rad/s; $|\alpha| = ?$

This is actually a kinematic problem (only about motion, not about forces), so neither the mass of the wheel nor the moment of inertia are necessary. You only need the rotational equivalent of the position-velocity-acceleration equation: $v^2 = v_0^2 + 2a(x - x_0)$

$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$ => $\alpha = (\omega_f^2 - \omega_i^2)/(2\Delta\theta) = -10.9$ rad/s^2 => $|\alpha| =$ **10.9 rad/s²**

EX7

A figure skater rotating at 5.00 rad/s with arms extended has a moment of inertia of 2.25 kg·m^2. If the arms are pulled in so the moment of inertia decreases to 1.80 kg·m^2, what is the final angular speed?

Answer:

Data: ω_i = 5 rad/s; I_i = 2.25 kg·m^2; I_f = 1.80 kg·m^2; ω_f = ?

Since there are no external forces, angular momentum L = Iω is conserved.

$L_i = I_i \omega_i = L_f = I_f \omega_f$ => $\omega_f = \omega_i I_i/I_f$ = **6.25 rad/s**

EX8

A 30.0-kg child sits on one end of a long uniform beam having a mass of 20.0 kg, and a 40.0-kg child sits on the other end. The beam balances when a fulcrum is placed below the beam a distance 1.10 m from the 30.0-kg child. How long is the beam?

Answer:

Data: m_1 = 30 kg; m_b = 20 kg; m_2 = 40 kg; d_1 = 1.10 m; L = $d_1 + d_2$ = ?

Since $m_1<m_2$ and the beam is uniform, we already know that $d_1>d_2$ (in other words, the CM of the beam must be at the same side as m_1). As usual, we start from balancing the torques. We will see that it is enough to solve the problem, provided we choose a convenient pivot point.
If we take the pivot point to be located at the contact point between the fulcrum and the beam, the only three forces involved are the weights of the two children and of the beam (the latter acting on the beam's centre of mass, CM, located in the middle of the beam).

$\tau_1 + \tau_b - \tau_2 = 0$ (because the beam's CM is at the same side as the first child)

$\tau_1 = m_1 g d_1$
$\tau_2 = m_2 g d_2$
$\tau_b = m_b g (d_1 - L/2) = m_b g [d_1 - (d_1+d_2)/2] = m_b g (d_1 - d_2)/2$

=> $m_1 g d_1 + m_b g (d_1 - d_2)/2 - m_2 g d_2 = 0$ => $d_2 = (m_1 + m_b/2)/(m_2+m_b/2)*d_1$ = **1.98 m**

Formula sheet (actually necessary)

$v = v_0 + at$
$x = x_0 + \frac{1}{2}(v_0 + v)t$
$x = x_0 + v_0 t + \frac{1}{2}at^2$
$v^2 = v_0^2 + 2a(x - x_0)$
$F_f = \mu n$
$I = mr^2$
$K = 1/2\ mv^2$
$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$
$\mathbf{p} = m\mathbf{v}$
$U_g = mgh$
$\Delta U_{AB} = -W_{AB}$
$W_x = F_x \Delta x$
$\tau = rF \sin\theta = I\alpha$

MECHANICS 2, 10-04-2019

The exam consists of 3 Conceptual Questions (CQs), each valid for 1 point, and 2 Exercises or Open Questions (EXs), each valid for 4 points.
For the CQs, you only have to give the answer*.*
For the Exs, you have to motivate your answers, i.e., show all the steps that lead to the answer you provide*. Answers without motivation are considered wrong and bring no points. Answers that show only part of the required steps to reach the correct answer will result in receiving a proportional part of points from the total amount of points given for this answer. (For example, if a subquestion brings 1 point, but you only show half of the path to the correct answer, you will get 0.5 points.) An EX might have alternative paths of reaching the correct answer; all such paths are considered correct. If an EX consists of subquestions, translation of the result of a wrong calculation from one subquestion to another subquestion is not judged as an error.*

Multiple choice, 3 assignments, 1 point per question

CQ8
Which of the following graphs describes simple harmonic motion with amplitude 2.00 cm and angular frequency 2.00 rad/s?

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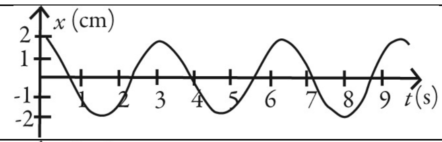
<tr><td>A.</td><td>
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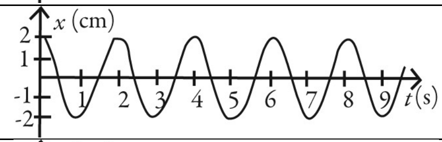
<tr><td>B.</td><td>
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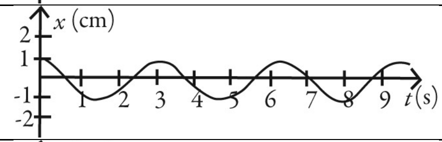
<tr><td>C.</td><td>
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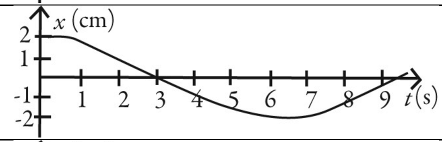
<tr><td>D.</td><td>
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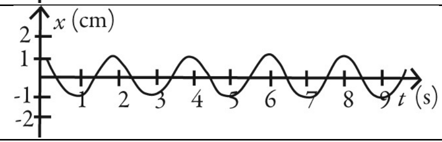
<tr><td>E.</td><td>
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</table>

Answer: A (The amplitude is from 0 to ± 2; the period $T = \frac{2\pi}{\omega} = \frac{2*3.14}{2} = 3.14\ (s)$.)

CQ9
A stationary siren emits sound of frequency 1000 Hz and wavelength 0.343 m. An observer who is moving toward the siren will measure a frequency f and wavelength λ for this sound such that

A. $f > 1000$ Hz and $\lambda > 0.343$ m.
B. $f > 1000$ Hz and $\lambda < 0.343$ m.
C. $f > 1000$ Hz and $\lambda = 0.343$ m.

D. $f < 1000$ Hz and $\lambda = 0.343$ m.
E. $f < 1000$ Hz and $\lambda > 0.343$ m.

Answer: C (The wavelength does not change, but the value of the frequency increases.)

CQ10

Which of the following is an elastic constant as discussed in the lecture on elastic constants?

A. Dilatation.
B. Translation.
C. Axial strain.
D. Shear modulus.
E. Normal stress.

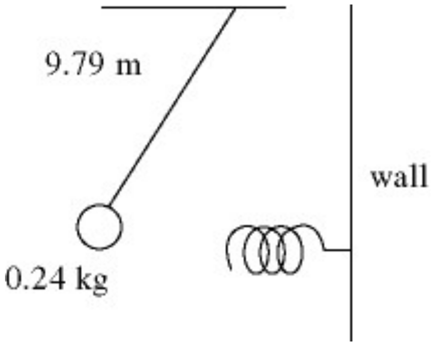
Answer: D

<u>Open questions, 2 assignments</u>

EX9

In the figure, a 0.24-kg ball is suspended from a string 9.79 m long and is pulled slightly to the left. The weight of the string is negligible compared to the weight of the ball; the diameter of the ball is much smaller than the length of the string. As the ball swings without friction through the lowest part of its motion, it encounters an ideal massless spring attached to a wall. The spring pushes against the ball and eventually the ball is returned to its original position to which it was pulled to the left. Find the time for one complete cycle of this motion if the spring constant of the spring is 21 N/m. (Assume that once the ball hits the spring there is no effect due to the vertical movement of the ball.)

Give the answer till the second digit after the decimal point. ***(4 points)***



Answer:

The total cycle consists of a part of an oscillatory motion of a pendulum and a part of an oscillatory motion of a mass/spring system. Because the description states that the string's length is much greater than the ball's diameter and that the ball is much heavier than the string, you can conclude that you are dealing with a simple pendulum. The simple pendulum is moved to the left and released to oscillate. When it reaches the lowest part of its motion, which is the equilibrium point, it would have spent quarter of a period oscillating. Because you are dealing with a simple pendulum, you can calculate the period from the equation for the angular frequency of the oscillatory motion, which is

$$\omega = \sqrt{\frac{g}{L}} \rightarrow T_{pendulum} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} = 2 * 3.14 * \sqrt{\frac{9.79}{9.81}} = 6.28\ (s).$$

When the pendulum is at its lowest point, it hits the spring, which is at its equilibrium position. Due to the kinetic energy of the ball, it starts compressing the spring, and the two together act as a mass/spring system in a simple harmonic motion. At the point of contact of the ball with the spring, the ball will have its maximum velocity. Thus, this will be the equilibrium point of the mass/spring system. After that, the spring will be counteracting the movement of the ball, and thus will be slowing down the ball until it stops, which will be thus the maximum displacement or the amplitude. From that moment on, the spring will start pushing the ball back unit it reaches the equilibrium point again, at which point the ball will have reached again its maximum speed and the lowest point of the oscillatory motion of the pendulum. Thus, the mass/spring system would have been in oscillation for half a period. You can find the period from the equation for the angular frequency:

$$\omega = \sqrt{\frac{k}{m}} \rightarrow T_{mass/spring} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2 * 3.14 * \sqrt{\frac{0.24}{21}} = 0.67\ (s).$$

After the ball leaves the spring, it will continue moving as a simple pendulum to the left until it reaches the left-most point at which it was originally released. This will last again quarter of a period. Thus, the complete cycle will last half a period of the pendulum and half a period of the mass/spring system, i.e.,

$$t = \frac{T_{pendulum}}{2} + \frac{T_{mass/spring}}{2} = 3.14 + 0.34 = 3.48\ (s).$$

EX10

Two strings of identical material and radius are stretched with the same tension with their ends fixed. One string is 8.0 mm longer than the other. Waves on these strings propagate at 420 m/s. The fundamental frequency of the longer string is 630 Hz. What is the beat frequency when each string is vibrating at its fundamental frequency? ***(4 points)***

Give the answer as an integer number.

Answer:

You are dealing with a standing wave that is developing on a string fixed at both ends. This means that for each of the strings its length and the wavelength of the waves propagating on the strings are connected through

$$L = m\frac{\lambda}{2},$$

where m is an integer. The wavelength of the waves on a string can be calculated from $\lambda = \frac{v}{f}$. Thus, you can find the length of the longer string by taking into account that the string is vibrating in its fundamental mode, meaning that $m = 1$:

$$L_{long} = m\frac{\lambda_{long}}{2} = m\frac{\frac{v}{f_{long}}}{2} = 1 * \frac{\frac{420}{630}}{2} = 0.333\ (m).$$

And now, you can find the length of the shorter string: $L_{short} = L_{long} - 0.008 = 0.325\ (m)$. From here, you can find the frequency of the shorted string when it vibrates at its fundamental mode:

$$L_{short} = m\frac{\lambda_{short}}{2} = m\frac{\frac{v}{f_{short}}}{2} \rightarrow f_{short} = m\frac{v}{2L_{short}} = 1 * \frac{420}{2*0.325} = 646\ (Hz).$$

You hear a beat when the sound waves caused by the two strings interfere such that their amplitudes interfere constructively at some points and destructively at others. The equation of the new signal you hear is

$$y(t) = 2A\cos\left(\frac{1}{2}(\omega_1 - \omega_2)t\right)\cos\left(\frac{1}{2}(\omega_1 + \omega_2)t\right),$$

where the beats occur at the difference of the frequencies of the waves on the two strings, i.e., at $|\omega_{short} - \omega_{long}| \rightarrow |f_{short} - f_{long}|$. Thus, the beat frequency is

$$f_{beat} = |f_{short} - f_{long}| = |646 - 630| = 16\ (Hz).$$