AESB1320-17 - Solutions

11-04-18, 9:00

TN-TZ 4.25

The exam consists of 10 Conceptual Questions (CQs), each valid for 1 point, and 10 Exercises (EXs), each valid for 4 points. The maximum score is 50. The pass score is 29.

PART I: all students.

CQ1

A pilot drops a package from a plane flying horizontally at a constant speed. Neglecting air resistance, when the package hits the ground the horizontal location of the plane will

A. be in front of the package.
B. be behind the package.
C. depend of the speed of the plane when the package was released.
D. be over the package.

D: there is no force acting in the horizontal direction (air resistance is neglected), so plane and package will keep moving forward with the same velocity.

CQ2

Three people are pushing a car up a hill at constant velocity. The net force on the car is

A. zero.
B. up the hill and greater than the weight of the car.
C. down the hill and greater than the weight of the car.
D. down the hill and equal to the weight of the car.
E. up the hill and equal to the weight of the car.

A: If the car moves with constant velocity, then the net acceleration has to be zero.

CQ3

Which of the following statements is true?

a. The total mechanical energy of a system is constant only if non-conservative forces act.
b. The total mechanical energy of a system, at any one instant, is either all kinetic or all potential energy.
c. The total mechanical energy of a system is constant only if conservative forces act.
d. The total mechanical energy of a system is equally divided between kinetic and potential energy.

c: According to the definition of conservative force.

CQ4

If the mass of the earth and all objects on it were suddenly doubled, but the size remained the same, the acceleration due to gravity at the surface would become

a. 2 times what it now is.
b. the same as it now is.
c. 4 times what it now is.
d. 1/2 of what it now is.
e. 1/4 of what it now is.

a: *$g=GM/R^2$, so doubling M doubles g.*

CQ5
There must be equal amounts of mass on both side of the center of mass of an object.

a. True
b. False

b: The position of the center of mass depends on both the amount of mass and its spatial distribution (i.e., you can have more mass at one side, if it's closer).

CQ6
If two forces of equal magnitude act on an object that is hinged at a pivot, the force acting farther from the pivot must produce the greater torque about the pivot.

A. false
B. true
C. unable to decide without knowing the shape of the object

A: It also depends on the angle between the force and the line between the pivot and the application point of the force itself.

CQ7
When you ride a bicycle, in what direction is the angular velocity of the wheels?

A. to your right
B. to your left
C. backwards
D. up
E. forwards

B: Right-hand rule.

EX1
A hockey puck slides off the edge of a table with an initial velocity of 27.7 m/s and experiences no air resistance. The height of the tabletop above the ground is 2.00 m.
What is the angle below the horizontal of the velocity of the puck just before it hits the ground?

12.7°

Data:
$v_{0x}= v_0=27.7$ m/s; $y_0=h= 2.0$ m; $\theta=?$

Solution:
Note: the question is about instantaneous velocity before hitting the ground, not average quantities.
Horizontal velocity remains unchanged (v_{0x}), vertical velocity (v_y) is the velocity a body in free fall has after falling vertically by 2 meters.
Since you are not asked for the travel time, you can use:
$v_y^2=v_0^2-2g(y-y_0)=2gh \Rightarrow v_y=+/-\ \text{sqrt}(2gh)=-6.26$ m/s (positive sign also ok, if y axis is downward)
Alternatively, you can first solve for time using: $y=y_0+v_{0y}t-1/2gt^2 \Rightarrow t=\text{sqrt}(2h/g)=0.64$ s
And then use: $v_y=v_{0y}-gt \Rightarrow v_y=-gt=-6.27$ m/s
The angle is just given by: $\theta=\arctan(v_y/v_x)=12.7°$ (or -12.7, if you measure angles counter-clockwise).

EX2
You push downward on a box at an angle 25° below the horizontal with a force of 750 N. The box is on a flat horizontal surface for which the coefficient of static friction with the box is 0.64.
What is the mass of the heaviest box you will be able to move?

76 kg

Data:
$\theta=25°$; $F=750$ N; $\mu=0.64$; $m_{max}=?$

Solution:
Friction will equal the total normal (vertical) force, which is equal to the weight of the box, plus the vertical component of the force pushing down the box.
Vertical component of the pushing force: $F_y=F \sin \theta = 315$ N
Horizontal component of the pushing force: $F_x=F \cos \theta = 679.7$ N
Force of friction: $F_f=\mu n= \mu (F_y+mg)$
So, the maxium mass that can be moved is the one for which the force of friction equals the horizontal component of the pushing force:
$F_f=F_x \Rightarrow \mu (F \sin \theta + mg) = F \cos \theta \Rightarrow m_{max}= F *(\cos \theta - \mu \sin \theta) / (\mu g)= 76$ kg

EX3
A very small 100-g object is attached to one end of a massless 10-cm rod that is pivoted without friction about the opposite end. The rod is held vertical, with the object at the top, and released, allowing the rod to swing. What is the speed of the object at the instant that the rod is horizontal?

1.4 m/s

Data:
m= 100 g; h=0.1 m; v(horiz)=?

Solution:
At the top, all energy is potential: $E_0=mgh=0.098$ J
When horizontal, all energy is kinetic: $E_1=1/2\ mv^2$
Since the only force is gravity, energy is conserved: $E_0=E_1 \Rightarrow v=\mathrm{sqrt}\ (2gh)=1.4$ m/s

Also possible to start from rotational kinetic energy: $E_1=1/2\ I\omega^2=1/2\ mh^2\ (v/h)^2=1/2\ mv^2$

EX4
Three identical very small 50-kg masses are held at the corners of an equilateral triangle, 0.30 m on each side. If one of the masses is released, what is its initial acceleration if the only forces acting on it are the gravitational forces due to the other two masses? ($G= 6.67 \times 10^{-11}$ N m^2/kg^2)

6.4×10^{-8} m/s^2

Data:
m=50 kg; d=0.3 m; $a_i=?$

Solution:
Equilateral triangle: $\theta=60^o$
In horizontal direction: $F_{1x}+F_{2x}=0$
In vertical direction: $F_{1y}=F_{2y}=Gmm/d^2 *\cos (\theta/2) \Rightarrow a=(F_{1y}+F_{2y})/m=2Gm/d^2*\cos 30^o=6.4 \times 10^{-8}$ m/s^2

EX5 (CORRECTED)
A billiard ball traveling at 3.00 m/s collides perfectly elastically with an identical billiard ball initially at rest on the level table. The initially moving billiard ball deflects 30.0° from its original direction. What is the speed of the initially stationary billiard ball after the collision?

1.5 m/s

Total linear momentum (of both balls together) is conserved: it remains constant in the direction of initial motion and zero in the perpendicular direction.
Since the collision is elastic, also energy is conserved.

From example 9.11 of the book, you know that if the first angle is 30 degrees the second one has to be 60 degrees (if the two balls have the same mass and one of them was initially at rest, then the two vectors need to be perpendicular because of conservation of energy).

$p_x=mv_0=mv_{1x}+mv_{2x}=mv_1\cos 30 + mv_2\cos 60 \Rightarrow v_0= v_1\cos 30 + v_2\sin 30$ (note that cos 60 = sin 30)

$p_y=0= mv_{1y}+mv_{2y} =v_1\sin 30 - v_2\sin 60 \Rightarrow v_1=v_2/\tan 30$ (also sin 60 = cos 30)

By substituting v_1 above into the first equation, and solving for v_2 (by writing tan30 in terms of sin and cos, and remembering that $\sin^2\alpha+\cos^2\alpha=1$), you obtain:

$v_2=v_0\sin 30=1.5$ m/s

EX6

An extremely light rod 1.00 m long has a 2.00-kg mass attached to one end and a 3.00-kg mass attached to the other. The system rotates at a constant angular speed about a fixed axis perpendicular to the rod that passes through the rod 30.0 cm from the end with the 3.00-kg mass attached. The kinetic energy of the system is measured to be 100.0 J.
What is the moment of inertia of this system about the fixed axis?

1.25 kg m^2

Since the shape of the masses in not given, the fact that the masses are small compared to the length of the rod should have been mentioned (point masses). If this motivation led to the answer "impossible to determine", it will be accepted.

Solution:
The moment of inertia only depends on the position of the two masses about the rotation axis:
$I=m_1r_1^2+m_2r_2^2$

EX7

A 5.0-m radius playground merry-go-round with a moment of inertia of 2000 kg m^2 is rotating freely with an angular speed of 1.0 rad/s. Two people, each having a mass of 60 kg, are standing right outside the edge of the merry-go-round and step on it with negligible speed. What is the angular speed of the merry-go-round right after the two people have stepped on?

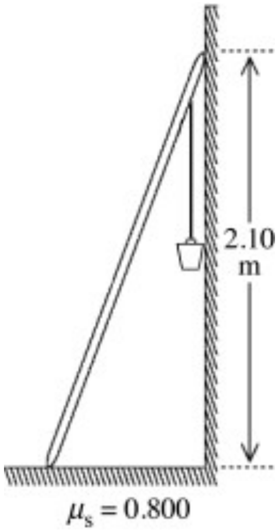
0.4 rad/s

Total angular momentum (merry-go-round) is conserved.

$I_{people}=2m R^2=3000$ km m^2

$L= \omega_0 I_{merry\text{-}go\text{-}round} = \omega_1(I_{merry\text{-}go\text{-}round} + I_{people}) \Rightarrow \omega_1= \omega_0 I_{merry\text{-}go\text{-}round} /(I_{merry\text{-}go\text{-}round} + I_{people})= 0.4$ rad/s

EX8



A 10.0-kg uniform ladder that is 2.50 m long is placed against a smooth vertical wall and reaches to a height of 2.10 m, as shown in the figure. The base of the ladder rests on a rough horizontal floor whose coefficient of static friction with the ladder is 0.800. An 80.0-kg bucket of concrete is suspended from the top rung of the ladder, right next to the wall, as shown in the figure. What is the magnitude of the friction force that the floor exerts on the ladder?

539.5 N

Data:
m_L=10 kg; L_L=2.5 m; h=2.1 m; μ=0.8; m_B=80 kg; F_f=?
Additional: d=sqrt(L_L^2-h^2)=1.36 m

Solution 1 (pivot point at the base of the ladder, point O):

Horizontal forces: $-N_w+F_f=0 \Rightarrow F_f=N_w$
with N_w being the normal force by the wall (horizontal towards the left)

Torque about O: $-m_Bgd+N_wh-m_Lgd/2=0 \Rightarrow N_w=gd(m_B+m_L/2)/h$=539.5 N

Solution 2 (pivot point at the top of the ladder, point P):

Vertical foces: $N_F-m_Lg-m_Bg=0 \Rightarrow N_F=g(m_L+m_B)$
with N_F being the normal force by the floor (vertical upwards)

Torque about P: $F_fh-N_Fd+m_Lgd/2=0 \Rightarrow F_f=gd(m_L+m_B-m_L/2)/h$=539.5 N

PART II: ***only first-year students.***

MECHANICS 2, 11-04-2018

The exam consists of 3 Conceptual Questions (CQs), each valid for 1 point, and 2 Exercises or Open Questions (EXs), each valid for 4 points.
For the CWs, you only have to give the answer.
For the Exs, you have to motivate your answers, i.e., show all the steps that lead to the answer you provide. *Answers without motivation are considered wrong and bring no points. Answers that*

show only part of the required steps to reach the correct answer will result in receiving a proportional part of points from the total amount of points given for this answer. (For example, if a subquestion brings 1 point, but you only show half of the path to the correct answer, you will get 0.5 points.) An EX might have alternative paths of reaching the correct answer; all such paths are considered correct. If an EX consists of subquestions, translation of the result of a wrong calculation from one subquestion to another subquestion is not judged as an error.

Multiple choice, 3 assignments, 1 point per question

1) A restoring force of magnitude F acts on a system with a displacement of magnitude x. In which of the following cases will the system undergo simple harmonic motion?
A) $F \propto \frac{1}{x}$
B) $F \propto x$
C) $F \propto x^2$
D) $F \propto \sqrt{x}$
E) $F \propto sinx$
Answer: B

2) The vertical displacement $y(x, t)$ of a wave traveling along a string stretched along the horizontal x-axis is given by $y(x, t) = (50\ mm)\ cos\left[\left(4.5\ \frac{1}{m}\right)x - \left(8.3\ \frac{rad}{s}\right)t\right]$. How fast does the wave travel?
A) 0.01 m/s.
B) 0.41 m/s.
C) 0.54 m/s.
D) 1.84 m/s.
E) 90 m/s.
Answer: D

3) In the system of coupled equations used to derive the acoustic wave equation in three dimensions, the equation $-\nabla p = \rho_0 \frac{\partial \vec{v}}{\partial t}$ describes
A) rotation.
B) compression
C) translation.
D) expansion.
E) curl.
Answer: C

Open questions, 2 assignments

4) An object of mass 1.5 kg is oscillating in simple harmonic motion at the end of an ideal vertical spring. Its vertical position y as a function of time t is given by $y(t) = (4.5\ cm)\ \cos\left[\left(20\ \frac{rad}{s}\right)t - \frac{\pi}{8}\right]$.
(a) What is the spring constant of the spring? ***(1.5 points)***
(b) What is the maximum speed that the object reaches? ***(1.5 points)***
(c) How long does it take the object to go from its highest point to its lowest point? ***(1 point)***
Give the answers till the second digit after the decimal point.

Answer:

(a) An object vibrating vertically in simple harmonic motion is characterized by vertical displacement $y(t) = A\cos(\omega t + \varphi)$. Comparing this equation with the one given in the question, you can conclude that $\omega = 20\ \left(\frac{rad}{s}\right)$. You also know that $\omega = \sqrt{\frac{k}{m}}$. Thus, you can calculate the spring constant k because the mass m is given:

$k = \omega^2 m = 20^2 * 1.5 = 600\ \left(\frac{kg}{s^2}\right)$.

(b) You know that the velocity is the derivative of the vertical displacement with respect to time: $v(t) = \frac{dy(t)}{dt} = \frac{d(A\cos(\omega t))}{dt} = -\omega A \sin(\omega t)$. You are interested in the maximum speed, which is achieved when $\sin(\omega t) = 1$. From the equation given in the question, you can conclude that the amplitude is $A = 4.5\ (cm)$. Thus, $v_{max} = |\omega A * 1| = 20 * 0.045 = 0.9\ \left(\frac{m}{s}\right)$.

(c) The time to go between the highest and the lowest points is half the period. So, $t = \frac{T}{2} = \frac{\frac{2\pi}{\omega}}{2} = \frac{2*3.14}{20*2} = 0.16\ (s)$.

5) A string with length of 20.0 cm and mass of 10 g is fixed at both ends and is under a tension of 500 N. The vibrating string creates sound waves that propagate in the air. When this string is vibrating in its third OVERTONE, the sound waves caused by this vibration make a nearby pipe start resonating in its third HARMONIC. The pipe is open at both ends. The speed of sound in the air is 340 m/s. How long is the pipe? ***(4 points)***

Give the answers till the second digit after the decimal point.

Answer:

(a) You have a case of a standing wave on a string that is fixed at its both end. For such a case, you know that standing waves exist when the length of the string is an integer multiple of half the wavelength: $L_s = m\frac{\lambda_s}{2}$. The string is vibrating in its third overtone, which means that m=4 As you know the length of the string, you can find the wavelength of the third overtone:

$\lambda_s = \frac{2L_s}{m} = \frac{2*0.2}{4} = 0.1\ (m)$.

You can calculate the speed of the waves travelling along the string, as you know that it is the square root of the ratio between the tension and the mass per unit length:

$v_s = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{500}{\frac{0.01}{0.2}}} = \sqrt{10000} = 100\ \left(\frac{m}{s}\right)$.

Knowing the wave speed and the wavelength, you can calculate the frequency of the vibration of the string:

$f_s = \frac{v_s}{\lambda_s} = \frac{100}{0.1} = 1000\ (Hz)$.

The vibration of the string will create a sound wave propagating in the air with a frequency equal to the frequency of the standing wave on the string. The sound wave reaches the pipe and gives rise in it of a standing wave (third harmonic) with a frequency equal to the frequency of the sound wave in the air. Using the value of the speed of waves in the air and the value of the frequency, you can calculate the wavelength of the sound wave, which is also the wavelength of the third harmonic:

$\lambda_p = \frac{v_{air}}{f_s} = \frac{340}{1000} = 0.34\ (m)$.

Because the pipe is open at both ends, the length of the pipe is an integer multiple of half the wavelength: $L_p = m\frac{\lambda_p}{2}$. You are observing the third harmonic, i.e., m=3, which means the second overtone. Finally, the length of the pipe is

$L_p = 3\frac{0.34}{2} = 0.51\ (m)$.

PART III: ***only students who take the resit of Mechanics 1.***

CQ8-b
If two vectors point in opposite directions, their cross product must be zero.

a. True
b. False

A: The magnitude of the cross product depends on *sin (theta)*, and *sin* 180^o=0.

CQ9-b
A box of mass *m* is pulled with a constant acceleration '*a*' along a horizontal frictionless floor by a wire that makes an angle of 15° above the horizontal. If *T* is the tension in this wire, then

a. $T < ma$.
b. $T = ma$.
c. $T > ma$.

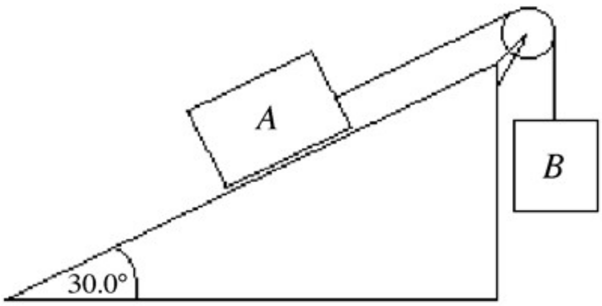
c: You need *ma* to accelerate the box horizontally, plus a vertical component due to the fact that the wire is not horizontal (note that the vertical component does not affect the acceleration).

CQ10-b
If a force always acts perpendicular to an object's direction of motion, that force cannot change the object's kinetic energy.

A. True
B. False

A: The force will not change the magnitude of the velocity, so the kinetic energy remains constant.

EX9-b



Two blocks are connected by a string that goes over an ideal pulley as shown in the figure. Block *A* has a mass of 3.00 kg and can slide over a rough plane inclined 30.0° to the horizontal. The coefficient of kinetic friction between block *A* and the plane is 0.40. Block *B* has a mass of 2.77 kg. What is the acceleration of the blocks?

0.374

The two blocks have almost the same mass, but A is on a small slope, so the force of gravity on block B is larger and block B will fall (and block A move up).
On block A you have the component of gravity pointing down the slope, friction also pointing down the slope (because A moves up), and the tension T pointing up. On block B you only have gravity and tension. The acceleration of the two blocks is the same, since the pulley has no mass.

Block A: $m_A a = -m_A g \sin 30^o - m_A g \cos 30^o \mu + T$ [*0.5 points if correct*]
Block B: $m_B a = m_B g - T$ [*0.5 points if correct*]

Solving for acceleration (a) you obtain:

$a = g [-m_A (\sin 30^o - \cos 30^o \mu)+m_B]/(m_A+m_B)= 9.81 * (0.22/5.77)=0.374 m/s^2$

EX10-b

A ball is thrown upward at an angle with a speed and direction such that it reaches a maximum height of 16.5 m above the point it was released, with no appreciable air resistance. At its maximum height it has a speed of 15.0 m/s. With what speed was the ball released?

23.4 m/s

Only force is gravity (=conservative), which acts in vertical direction.
Conservation of mechanical energy in y direction: initial kinetic is equal to final potential:
$1/2m(v_{0y})^2=mgd$, with d=16.5 m
In x direction v is constant, so it is equal to the value at maximum height: v_{0x}=15 m/s.
Since energy is conserved, the initial velocity is the squared sum of the two components.