AESB1320-15
05-07-17, 9:00
CT-IZ 0.96

The exam consists of 10 Conceptual Questions (CQs), each valid for 1 point, and 10 Exercises (EXs), each valid for 2 points. The maximum score is 30.

Important: *Answers to conceptual questions need to be accompanied by a short (1-2 sentences) motivation of the choice. Credits for a conceptual question can only be obtained when both the answer and the motivation are correct.*

Grading rules for numerical exercises:
- correct numeric value and solution: 2 points;
- wrong numeric value, but correct solution (computational mistake): 1.5 points;
- wrong numeric value, correct intermediate numeric value (exercise half-done): 1 point;
- wrong solution: 0 points.

This is a closed-book exam: only pens, blank paper and non-graphical calculators are allowed.

The exam is structured in three parts (conceptual questions, exercises 1-5, exercises 6-10):

Please use three separate sheets of paper, one sheet per part.

PART I: Conceptual questions

CQ1
If an object travels at a constant speed in a circular path, the acceleration of the object is
a) zero.
b) in the same direction as the velocity of the object.
c) larger in magnitude the smaller the radius of the circle.
d) smaller in magnitude the smaller the radius of the circle.
e) in the opposite direction of the velocity of the object.

[**c**: $a_c=v^2/R$, so larger for a smaller R, if v remains the same]

CQ2
A ball is tossed vertically upward. When it reaches its highest point
a) the velocity is zero, the acceleration is directed downward, and the force of gravity acting on the ball is directed downward.
b) the velocity and acceleration reverse direction, but the force of gravity on the ball remains downward.
c) the velocity, acceleration, and the force of gravity on the ball all reverse direction.
d) the velocity is zero, the acceleration is zero, and the force of gravity acting on the ball is directed downward.
e) the velocity is zero, the acceleration is zero, and the force of gravity acting on the ball is zero.

[**a**: the only force acting on the ball is gravity, which is always downward, so also acceleration is always downward; at the highest point, velocity is instantaneously zero.]

CQ3
A string is attached to the rear-view mirror of a car. A ball is hanging at the other end of the string. The car is driving around in a circle, at a constant speed. Which of the following lists gives all of the forces

directly acting on the ball?

a) tension and gravity
b) tension
c) tension, gravity, the centripetal force, and friction
d) tension, gravity, and the centripetal force

[**a**: only tension and gravity, the centripetal force is apparent: just another way to call the vector sum of tension and gravity. There is no friction involved.]

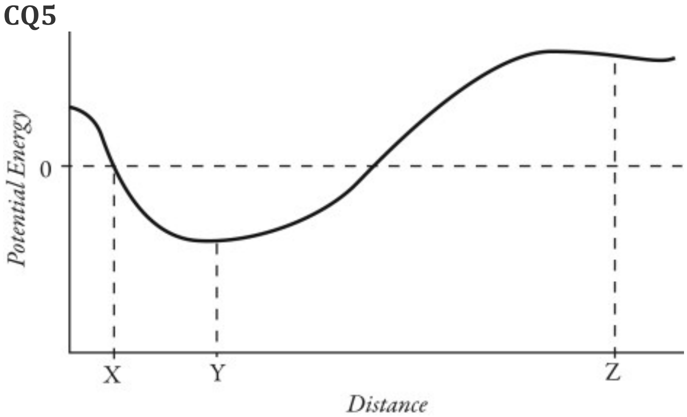
CQ4

Two stones, one of mass *m* and the other of mass 2*m*, are thrown directly upward with the same velocity at the same time from ground level and feel no air resistance. Which statement about these stones is true?

a) At its highest point, the heavier stone will have twice as much gravitational potential energy as the lighter one because it is twice as heavy.
b) The lighter stone will reach its maximum height sooner than the heavier one.
c) Both stones will reach the same height because they initially had the same amount of kinetic energy.
d) The heavier stone will go twice as high as the lighter one because it initially had twice as much kinetic energy.
e) At their highest point, both stones will have the same gravitational potential energy because they reach the same height.

[**a**: At the highest point the ball is still, so kinetic energy is zero, so all mechanical energy is potential. Potential energy is U=Mgh, so double mass means double U.]

CQ5



The plot in the figure shows the potential energy of a particle, due to the force exerted on it by another particle, as a function of distance. At which of the three points labeled in the figure is the magnitude of the force on the particle greatest?

a) point X
b) point Y
c) point Z

[**a**: the force is equal to the derivative of the potential energy. Point X has the steepest curve, so the largest derivative, which means the largest force.]

CQ6

A baseball is located at the surface of the earth. Which statement about it is correct?

a) The gravitational force on the ball due to the earth is exactly the same as the gravitational force on the earth due to the ball.
b) The gravitational force on the ball is independent of the mass of the ball.
c) The ball exerts a greater gravitational force on the earth than the earth exerts on the ball.
d) The gravitational force on the ball is independent of the mass of the earth.
e) The earth exerts a much greater gravitational force on the ball than the ball exerts on the earth.

[**a**: Third law of Newton. And $F_g=GmM/R^2$, so depending on both masses.]

CQ7

You are standing on a skateboard, initially at rest. A friend throws a very heavy ball towards you. You can either catch the object or deflect the object back towards your friend (such that it moves away from you with the same speed as it was originally thrown). What should you do in order for your final speed (while standing on the skateboard) to be the lowest possible?

a) Deflect the ball.
b) Catch the ball.
c) Your final speed on the skateboard will be the same regardless whether you catch the ball or deflect the ball.

[**b**: Conservation of linear momentum, where the system is composed by you and the ball. Initial momentum is only the one of the ball: let's say, moving right. If you catch it, both you and the ball will go right. If the ball is deflected and goes left, then you need to go right even faster, so that total momentum remains the same.]

CQ8

A tire is rolling along a road, without slipping, with a velocity v. A piece of tape is attached to the tire. When the tape is opposite the road (at the top of the tire), its velocity with respect to the road is

a) v.
b) $1.5v$.
c) $2v$.
d) zero.
e) The velocity depends on the radius of the tire.

[**c**: see Lecture 7, slide 34.
The center of the wheel moves with velocity v. The point of contact with the ground is instantaneously still (otherwise the tire will slip), meaning that, with respect to the center of the wheel, all points have the same linear velocity at the wheel's center of mass. The two velocities are cancelling at the bottom, and are adding together at the top, giving double velocity.
Alternatively: the ground contact point is instantaneously still, so the whole tire is instantaneously rotating around the contact point (all points having the same angular velocity). The top point is twice as far as the middle one, so its velocity is double.]

CQ9

If two vectors are perpendicular to each other, their cross product must be zero.

a) True
b) False

[**b**: cross product depends on the sinus of the angle between the two vectors, which is maximum if they are perpendicular.]

CQ10

As you are leaving a building, the door opens outward. If the hinges on the door are on your right, what is the direction of the angular velocity of the door as you open it?

a) up
b) to your right
c) to your left
d) down
e) forwards

[**d**: right-hand rule.]

PART II: Exercises 1-5

EX1

A hockey puck slides off the edge of a table at point A with an initial velocity of 20.0 m/s and experiences no air resistance. The height of the tabletop above the ground is 2.00 m.

What is the speed of the puck just before it touches the ground? [g=9.8 m/s^2]

[20.96 m/s]

x-direction (horizontal): $v_x=v_i=20.0$ m/s
y-direction (upward, reference at the ground):
$y=y_0+v_{0y}t-g/2\ t^2$, with y=0 m, $y_0=2$ m, $v_{0y}=0$ => $t=\text{sqrt}(2y_0/g)=0.639$ s
$v_y=v_{y0}-gt=-6.26$ m/s
Together: $v=\text{sqrt}(v_x^2+v_y^2)=20.96$ m/s

EX2
A locomotive is pulling 8 freight cars, each of which is loaded with the same amount of weight. The mass of each freight car (with its load) is 37,000 kg. The train is accelerating at 0.25 m/s^2 on a level track.

What is the tension in the coupling between the second and third cars?
(The car nearest the locomotive is counted as the first car, and friction is negligible.)

[5.55 x 10^4 N]
The tension between cars 2 and 3 is equal to the force needed to pull 6 cars (3 through 8), and all cars have the same acceleration.
F=Ma, with M=6 x 37,000 kg and a=0.25 m/s^2.

EX3
A Ferris wheel [in Dutch "reuse rat"] has a diameter of 10 m and makes one revolution in 8.0 seconds. A person weighing 670 N is sitting on one of the benches attached at the rim of the wheel.

What is the normal force exerted on the person by the bench as she passes through the highest point of the wheel?

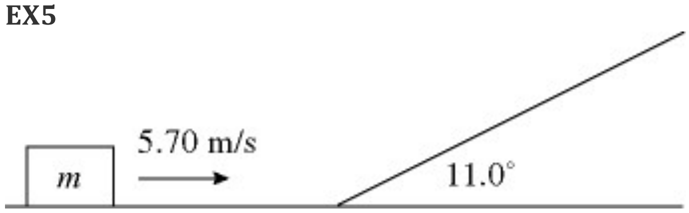
[460 N]
$a_c=\omega^2 R$
$\omega = 2\pi/T$, with T=8 s
Together: $a_c = \frac{4\pi^2}{T^2}R = 3.08\ m/s^2$
Mass of the person: m=W/g=68.37 kg
Apparent weight: $W_{app}=m(g-a_c)$

EX4
A constant horizontal force pulls a sled on a horizontal frictionless ice pond. The sled starts from rest. When the pull acts over a distance *x*, the sled acquires a speed *v* and a kinetic energy *K*.

What are the sled's speed and kinetic energy, if the same pull acts over a distance of 2*x*?

[Speed: v*sqrt(2); Kinetic energy: 2*K*]
If the force is constant, then twice the distance means twice the work, which also means twice the final kinetic energy (no friction, so no other force than the horizontal pull).
Since $K=1/2mv^2$, double *K* implies double v^2, hence an increase in *v* by sqrt(2).

EX5



In the figure, a block of mass *m* is moving along the horizontal frictionless surface with a speed of 5.70 m/s.

How far does the block travel up the incline, if the slope is 11.0° and the coefficient of kinetic friction between the block and the incline is 0.260?

[3.72 m]
At the bottom of the slope, mechanical energy is only kinetic (taking the reference potential to be at the height of the horizontal surface). While going up the hill, the only two forces are gravity (pointing downward) and friction (pointing backward).
$K_0=W_f+U_g$

Let's call L the distance travelled along the slope.

$$\frac{1}{2}mv^2 = \mu\, mg\cos(11^o)\, L + mgh, with\ h = L\sin(11^o)$$

which can be solved for L.

PART III: Exercises 6-10

EX6

The radius of the earth is R and g=9.8 m/s^2.
At what distance above the earth's surface will the acceleration of gravity be 4.9 m/s^2?

[0.41 *R = 2.612 10^6 m*]

At the Earth surface: a_R=g =GM/R^2 => GM=gR^2
At a distance *d* above the surface: a_{R+d}=GM/(R+d)^2=4.9 m/s^2
Substituting GM from the first equation
gR^2/(R+d)^2=4.9 m/s^2 => d=R[sqrt(2)-1]

EX7

A 900-kg car traveling east at 15.0 m/s collides with a 750-kg car traveling north at 20.0 m/s. The cars stick together. Assume that any other unbalanced forces are negligible.

What is the speed of the wreckage just after the collision?

[12.2 m/s]

In a collision, linear momentum is conserved.
East direction: $m_1v_1=(m_1+m_2)v_x$ => $v_x=m_1/(m_1+m_2)v_1$=8.18 m/s
North direction: $m_2v_2=(m_1+m_2)v_y$ => $v_y=m_2/(m_1+m_2)v_2$=9.09 m/s
v_{tot}=sqrt($v_x^2+v_y^2$)

EX8

While spinning down from 500.0 rpm (revolutions per minute) to rest, a solid uniform flywheel does 4.7 kJ of work.

If the radius of the disk is 1.2 m, what is its mass?

[Moment of inertia of a disk about an axis passing through its centre: I=1/2 MR^2]

[4.76 kg]

The initial kinetic energy is equal to the work done on the flywheel to stop it.

$$K_i = \frac{1}{2}I\omega^2 = W, where\ \omega = 500\ rpm = 500\frac{2\pi}{60} = 52.36\ rad/s$$

Substituting the formula for I and solving for m gives

$$M = \frac{4W}{R^2\omega^2}$$

EX9

A figure skater rotating at 5.0 rad/s with arms extended has a moment of inertia of 2.25 kg·m^2.

What is the final angular speed, if the arms are pulled in so the moment of inertia decreases to 1.80 kg·m^2?

[6.25 rad/s]

Conservation of angular momentum: $I_0\omega_0 = I_1\omega_1 \Rightarrow \omega_1 = \frac{I_0}{I_1}\omega_0$

EX10

A light board, 10-m long, is supported by two sawhorses [in Dutch "schragen"], one at one edge of the board and a second at the midpoint. A small 40-N weight is placed between the two sawhorses, 3.0 m from the edge and 2.0 m from the center.

What forces are exerted by the sawhorses on the board?

[F_{edge}=16.0 N; F_{mid}=24.0 N]

Balance of torques about the contact point with the edge sawhorse: -40 x 3 + F_{mid} x 5 = 0 => F_{mid}=24 N
Balance of (vertical) forces: $F_{edge}+F_{mid}$-40 = 0 => F_{edge}=(40-24)N