AESB1320-15 - Solutions
12-04-17, 9:00
TN-TZ 4.25

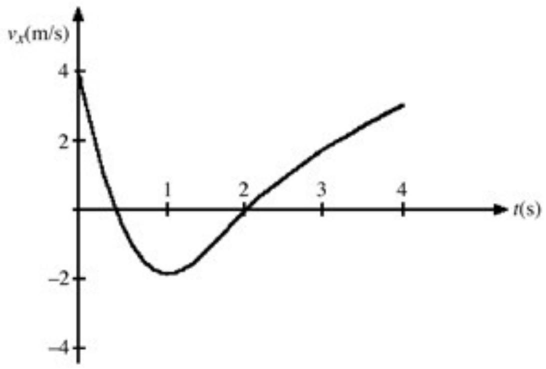
The exam consists of 10 Conceptual Questions (CQs), each valid for 1 point, and 10 Exercises (EXs), each valid for 2 points. The maximum score is 30.

Important: *Answers to conceptual questions need to be accompanied by a short (1-2 sentences) motivation of the choice. Credits for a conceptual question can only be obtained when both the answer and the motivation are correct.*

Grading rules for numerical exercises:
- correct numeric value and solution: 2 points;
- wrong numeric value, but correct solution (computational mistake): 1.5 points;
- wrong numeric value, correct intermediate numeric value (exercise half-done): 1 point;
- wrong solution: 0 points.

PART I: Conceptual questions

CQ1



The figure represents the velocity of a particle as it travels along the x-axis. At what value (or values) of *t(s)* is the instantaneous acceleration equal to zero?

a) t = 0
b) t = 1 s
c) t = 0.5 s and t = 2 s

[b) since there the slope of the curve is zero.]

CQ2

Alice and Tom dive from an overhang into the lake below. Tom simply drops straight down from the edge, but Alice takes a running start and jumps with an initial horizontal velocity of 25 m/s. Neither person experiences any significant air resistance. Compare the time it takes each of them to reach the lake below.

a) Alice reaches the surface of the lake first.
b) Tom reaches the surface of the lake first.
c) Alice and Tom will reach the surface of the lake at the same time.

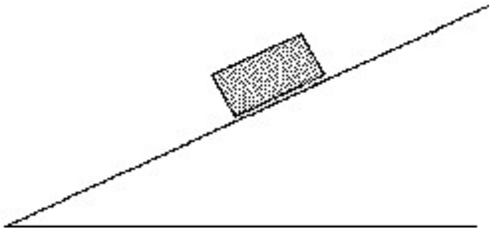
[c) since they both have the same (zero) initial velocity in the vertical direction and the only force acting after the jump is gravity, also in the vertical direction.]

CQ3
A car is being towed at constant velocity on a horizontal road using a horizontal chain. The tension in the chain must be equal to the weight of the car in order to maintain constant velocity.

a) True.
b) False.

[b) is the car is moving with constant velocity, there is no horizontal force, hence no tension in the chain.]

CQ4



A brick is resting on a rough incline as shown in the figure. The friction force acting on the brick, along the incline, is

a) greater than the weight of the brick.
b) zero.
c) less than the weight of the brick.
d) equal to the weight of the brick.

[c) because friction only has to be equal to the component of the gravitational force along the slope, which is less than the total gravitational force (or weight).]

CQ5
A stock person at the local grocery store has a job consisting of the following five segments:

(1) picking up boxes of tomatoes from the stockroom floor
(2) accelerating to a comfortable speed
(3) carrying the boxes to the tomato display at constant speed
(4) decelerating to a stop
(5) lowering the boxes slowly to the floor.

During which of the five segments of the job does the stock person do positive work on the boxes?

a) (1) only
b) (1) and (2)
c) (1), (2), (4), and (5)
d) (2) and (3)
e) (1) and (5)

[b) because in (1) gravitational potential energy increases and in (2) kinetic energy increases.]

CQ6
Two objects are moving at equal speed along a level, frictionless surface. The second object has twice the mass of the first object. They both slide up the same frictionless incline plane. Which object rises to a greater height?

a) Object 1 rises to the greater height because it weighs less.
b) Object 1 rises to the greater height because it possesses a smaller amount of kinetic energy.
c) Object 2 rises to the greater height because it contains more mass.
d) Object 2 rises to the greater height because it possesses a larger amount of kinetic energy.
e) The two objects rise to the same height.

[e) initial kinetic energy, $\frac{1}{2} mv^2$, is converted into gravitational potential energy, mgh, so maximum height is only dependent on initial velocity (and gravity).]

CQ7

Planet Z-34 has a mass equal to 1/3 that of Earth and a radius equal to 1/3 that of Earth. With g representing, as usual, the acceleration due to gravity on the surface of Earth, the acceleration due to gravity on the surface of Z-34 is

a) $g/3$.
b) $3g$.
c) $g/9$.
d) $9g$.
e) $6g$.

[b) because $g=GM/R^2$ and both M and R reduce by a factor of 3.]

CQ8

Jacques and George meet in the middle of a lake while paddling in their canoes. They come to a complete stop and talk for a while. When they are ready to leave, Jacques pushes George's canoe with a force **F** to separate the two canoes. What is correct to say about the final momentum and kinetic energy of the system if we can neglect any resistance due to the water?

a) The final momentum is in the direction opposite of **F** but the final kinetic energy is zero.
b) The final momentum is in the direction of **F** and the final kinetic energy is positive.
c) The final momentum is zero and the final kinetic energy is zero.
d) The final momentum is zero but the final kinetic energy is positive.
e) The final momentum is in the direction of **F** but the final kinetic energy is zero.

[d) the force **F** is internal to the system, so it cannot chance the total momentum, which is initially zero; kinetic energy is the sum of the kinetic energy of each object, so after separating is greater than zero.]

CQ9

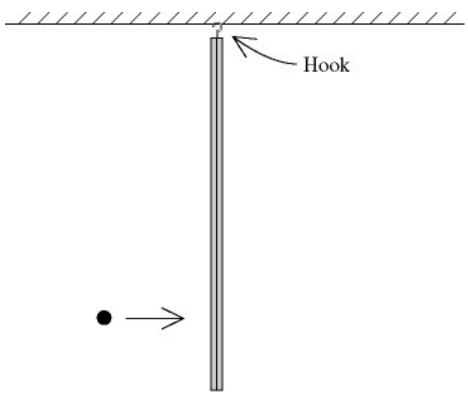
Consider a uniform solid sphere of radius R and mass M rolling without slipping. Which form of its kinetic energy is larger, translational or rotational?

a) Its rotational kinetic energy is larger than its translational kinetic energy.
b) Its translational kinetic energy is larger than its rotational kinetic energy.
c) Both forms of energy are equal.
d) You need to know the speed of the sphere to tell.

[Moment of inertia of a solid sphere about an axis passing through its centre: I=2/5 MR^2]

[b) $K_{trans}=1/2\ Mv^2 > K_{rot}=1/2\ I\omega^2=1/2\ (2/5\ MR^2)\ (v/R)^2= 1/5\ Mv^2$.]

CQ10



A metal bar is hanging from a hook in the ceiling when it is suddenly struck by a ball that is moving horizontally (see figure). The ball is covered with glue, so it sticks to the bar. During the instant of the collision

a) the angular momentum of the system (ball and bar) is conserved about the hook because only gravity is acting on the system.

b) the angular momentum of the system (ball and bar) is conserved about the hook because neither the hook nor gravity exerts any torque on this system about the hook.
c) the angular momentum of the system (ball and bar) is not conserved because the hook exerts a force on the bar.
d) both the angular momentum of the system (ball and bar) and its kinetic energy are conserved.
e) both the linear momentum and the angular momentum of the system (ball and bar) are conserved.

[b) the hook exerts no torque about itself (distance force-pivot is zero), gravity exerts no torque because the force is aligned with the line between pivot and center of gravity (angle is zero).]

PART II: Exercises 1-5

EX1

A ball is thrown at a 60.0° angle above the horizontal across level ground. It is thrown from a height of 2.0 m above the ground with a speed of 25.1 m/s and experiences no appreciable air resistance.

How long does the ball remain in the air before striking the ground? [g=9.81 m/s^2]

[4.52 s]

Only vertical motion is relevant:

$y=y_0 + v_{0y}t + a/2\, t^2$

where

$y=0$ m; $y_0=2$ m; $v_{oy}=v_0 \sin 60^o=21.74$ m/s; $a=g=-9.81$ m/s^2

Note that the other possible solution of the quadratic equation is negative, hence not acceptable.

EX2

A rock is thrown at a window that is located 18.0 m above the ground. The rock is thrown at an angle of 40.0° above horizontal. The rock is thrown from a height of 2.0 m above the ground with a speed of 30.0 m/s and experiences no appreciable air resistance.

If the rock strikes the window on its upward trajectory, from what horizontal distance from the window was it released? [g=9.81 m/s^2]

[27.4 m]

$v_{0x}=30 \cos 40^o = 23.0$ m/s

$v_{0y}=30 \sin 40^o = 19.3$ m/s

In the y-direction it works as the previous exercise, with $y=18$ m and $y_0=2$m, giving $t=1.19$ s (the problem states “on its upward trajectory”, so the smallest of the two times has to be chosen).

In the x-direction it is simply: $x=v_{0x}t$.

EX3

In a ballistics test, a 1.50-g bullet is fired through a 28.0-kg block traveling horizontally to the right toward the bullet. In this test, the bullet takes 0.0114 s to pass through the block as it reverses the block's velocity from 1.75 m/s to the right to 1.20 m/s to the left with constant acceleration.

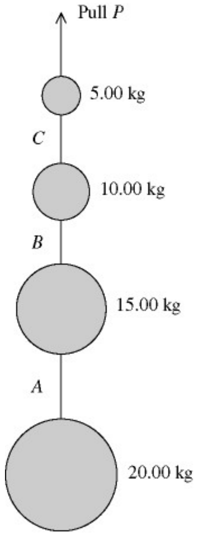
What is the magnitude of the force that the bullet exerts on the block during this ballistics test?

[7246 N]

For the block: $v_1=v_0+at$, with $v_0=1.75$ m/s and $v_1=-1.20$ m/s => $a=-258.8$ m/s^2.

The force magnitude is: $F=m|a|$

EX4



A series of weights connected by very light cords are given an upward acceleration of 4.00 m/s² by a pull *P*, as shown in the figure. *A*, *B*, and *C* are the tensions in the connecting cords.

What is the magnitude of the smallest of the three tensions? [*g*=9.81 m/s²]

[276 N]

All masses have the same acceleration, so the smallest tension will be in A, since that segment is pulling the smallest total mass.

The two forces acting on the lowest weight are tension and gravity, the net force induces the given acceleration.

$F_{net}=ma=T-F_g$ => $T=ma+F_g=m(a+g)$, with $a=4$ m/s² and $m=20$ kg.

EX5

A person pushes horizontally on a heavy box and slides it across the level floor at constant velocity. The person pushes with a 60.0 N force for the first 9.2 m, at which time he begins to tire. The force he exerts then starts to decrease linearly from 60.0 N to 0.0 N across the remaining 9.2 m.

How much total work did the person do on the box?

[828 J]

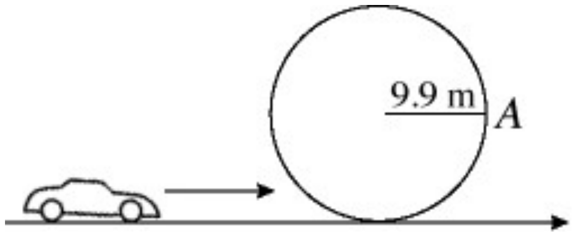
In the first segment, work is: $W_1=dx * F_1= 9.2*60=552$ J

In the second segment, work is (area of a triangle if you plot force versus displacement):

$W_2=dx*(F_1-F_2)/2=9.2*(60-0)/2=276$ J

PART III: Exercises 6-10

EX6



In the figure, a stunt car driver negotiates the frictionless track shown in such a way that the car is barely in contact with the track at the top of the loop. The radius of the track is 9.9 m and the mass of the car is 1800 kg. The car can be treated as a point mass. [*g*=9.81 m/s²]

What is the magnitude of the force between the car and the track when the car is at point *A*?

[35.316 kN]

At the top, energy is only potential: E_{top}=mgh=2mgR (if referred to the bottom)
At point A, energy is both potential and kinetic: E_A=U+K=mgR+1/2 mv_A^2
There is no friction and gravity is conservative, so energy is conserved, which allows to compute the velocity at A:
mgR+1/2 mv_a^2=2mgR => v_A^2=2gR
Knowing the velocity, one can compute the centripetal acceleration in A:
$a_c=v_A^2/R=2g$
At A, the only force between car and track is the centripetal force (gravity is perpendicular):
$F=ma_c=2mg$

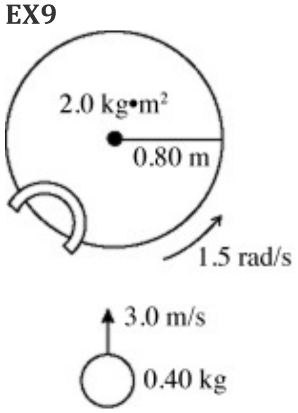
EX7
A plate falls vertically to the floor and breaks up into three pieces, which slide along the floor. Immediately after the impact, a 360-g piece moves along the x-axis with a speed of 2.00 m/s and a 435-g piece moves along the y-axis with a speed of 1.50 m/s. The third piece has a mass of 100 g.
In what direction does the third piece move? Neglect any horizontal forces during the crash.
[222.2° from the x-axis]

There is no initial momentum in the horizontal plane, and horizontal forces during the crash can be neglected. Hence, in the horizontal plane, momentum is conserved and remains zero.
X-direction: $m_1v_1+m_3v_{3x}=0$ => v_{3x}=-7.2 m/s
Y-direction: $m_2v_2+m_3v_{3y}=0$ => v_{3y}=-6.525 m/s
Measuring the angle from the negative x-axis:
$\theta=\tan^{-1}(v_{3y}/v_{3x})=42.2^o$
Measuring from the positive x-axis, 180^o needs to be added. Other reference angles are accepted, as long as it's clear where the angle is measured from.

EX8
A uniform solid sphere starts from rest and rolls without slipping down a 35° incline that is 7.0 m long.
What is the linear speed of the center of the sphere when it reaches the bottom of the incline?
[Moment of inertia of a solid sphere about an axis passing through its centre: I=2/5 MR^2; g=9.81 m/s^2]
[7.5 m/s]

The height of the incline is: h=7*sin35^o=4.02 m
The initial energy is only potential: E_0=Mgh=1.5*9.81*4.02=59.15 J
Final energy is only kinetic, both translational and rotational:
$E_1=K=1/2\ Mv^2 + 1/2\ I\omega^2=1/2\ Mv^2+ 1/2\ *(2/5\ MR^2)*(v/R)^2=7/10\ Mv^2$
Since energy is conserved:
$7/10\ Mv^2$ = Mgh => v=sqrt(10/7 gh)

EX9



A turntable has a radius of 0.80 m and a moment of inertia of 2.0 kg·m^2. The turntable is rotating with an angular velocity of 1.5 rad/s about a vertical axis though its center on frictionless bearings. A very small 0.40-kg ball is projected horizontally toward the turntable axis with a velocity of 3.0 m/s. The ball is caught by a very small and very light cup-shaped mechanism on the rim of the turntable (see figure).

What percent of the initial kinetic energy of the system is lost during the capture of the ball?

[50.6%]

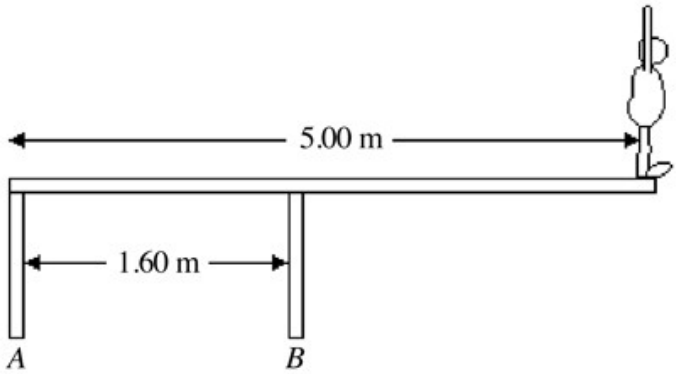
Before impact: $K_0=1/2\ I\omega_0^2+1/2\ mv^2=4.05$ J

Angular momentum is conserved: $L_0= I\omega_0=L_1=(I+mR^2)\ \omega_1 \Rightarrow \omega_1=1.33$ rad/s

After impact: $K_1=1/2\ (I+mR^2)\omega_1^2= 2.0$ J

Percentage of kinetic energy lost: $\%K_{lost}=(K_0-K_1)/K_0$

EX10



An 82.0 kg-diver stands at the edge of a light 5.00-m diving board, which is supported by two narrow pillars 1.60 m apart, as shown in the figure.

I. What is the magnitude of the force exerted on the diving board by pillar *A*?
II. What is the magnitude of the force exerted on the diving board by pillar *B*?

[I: 1709 N; II: 2513 N]

Balance of torques about the contact point between pillar B and board:

$\tau_A+\tau_D=0 \Rightarrow -mg(5-1.6)+F_A*1.6=0 \Rightarrow F_A=1709$ N

Balance of forces in the vertical direction: $F_B=F_A+F_g=1709+ 82*9.81= 2513$ N