AESB1320-15
13-04-16, 9:00
LR-CZ H/J

The exam consists of 10 Conceptual Questions (CQs), each valid for 1 point, and 10 Exercises (EXs), each valid for 2 points. The maximum score is 30.

Important: *Answers to conceptual questions need to be accompanied by a short (1-2 sentences) motivation of the choice. Credits for a conceptual question can only be obtained when both the answer and the motivation are correct.*

Grading rules for numerical exercises:
- correct numeric value and solution: 2 points;
- wrong numeric value, but correct solution (computational mistake): 1,5 points;
- wrong numeric value, correct intermediate numeric value (exercise half-done): 1 point;
- wrong solution: 0 points.

This is a closed-book exam: only pens, blank paper and non-graphical calculators are allowed.

The exam is structured in three parts (conceptual questions, exercises 1-5, exercises 6-10):
Please answer every part on a separate sheet of paper.

PART I: Conceptual questions

CQ1

While an object is in projectile motion (with upward being positive) with no air resistance

a. the vertical component of both its velocity and its acceleration remain constant.
b. the horizontal component of its velocity remains constant and the vertical component of its acceleration is equal to $-g$.
c. the vertical component of its velocity remains constant and the vertical component of its acceleration is equal to -g.
d. the horizontal component of its velocity remains constant and the horizontal component of its acceleration is equal to -g .
e. the horizontal component of its velocity remains constant and the vertical component of its acceleration is equal to zero.

b: *The only force is gravity.*

CQ2

In order to lift a bucket of concrete, you must pull up harder on the bucket than it pulls down on you.

a. True
b. False

False: *The forces between you and the bucket have the same magnitude (and opposite direction), the bucket moves because your force is larger than gravity.*

CQ3

Two bodies P and Q on a smooth horizontal surface are connected by a light cord. The mass of P is greater than that of Q. A horizontal force ***F*** (of magnitude F) is applied to Q as shown in the figure, accelerating the bodies to the right. The magnitude of the force exerted by the connecting cord on body P will be

a. less than F but not zero.
b. equal to F.
c. zero.
d. greater than F.

a: *The cord is only pulling P, while F is pulling both P and Q.*

CQ4

Two objects, one of mass m and the other of mass $2m$, are dropped from the top of a building. When they hit the ground

a. the heavier one will have four times the kinetic energy of the lighter one.
b. the heavier one will have $\sqrt{2}$ times the kinetic energy of the lighter one.
c. both of them will have the same kinetic energy.
d. the heavier one will have twice the kinetic energy of the lighter one.

d: *Because the heavier has double potential energy at the top. / Because double mass implies double force, hence double work by gravity.*

CQ5

Which of the following statements is true?

a. The total mechanical energy of a system is constant only if non-conservative forces act.
b. The total mechanical energy of a system, at any one instant, is either all kinetic or all potential energy.
c. The total mechanical energy of a system is constant only if conservative forces act.
d. The total mechanical energy of a system is equally divided between kinetic and potential energy.

c: *According to the definition of conservative force.*

CQ6

If the mass of the earth and all objects on it were suddenly doubled, but the size remained the same, the acceleration due to gravity at the surface would become

a. 2 times what it now is.
b. the same as it now is.
c. 4 times what it now is.
d. 1/2 of what it now is.
e. 1/4 of what it now is.

a: *$g=GM/R^2$, so doubling M doubles g.*

CQ7

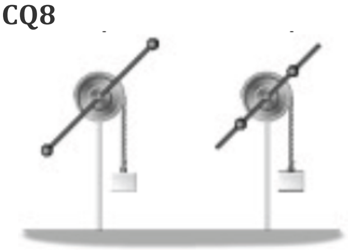
A baseball is thrown vertically upward and feels no air resistance. As it is rising

a. its momentum is not conserved, but its mechanical energy is conserved.
b. both its momentum and its kinetic energy are conserved.
c. its kinetic energy is conserved, but its momentum is not conserved.

d. its gravitational potential energy is not conserved, buts its momentum is conserved.
e. both its momentum and its mechanical energy are conserved.

a: *The is a non-zero external force (gravity), but it is conservative.*

CQ8



The two rotating systems shown in the figure differ only in that the two identical movable masses are positioned at different distances from the axis of rotation. If you release the hanging blocks simultaneously from rest, and if the ropes do not slip, which block lands first?

a. The block at the left lands first.
b. The block at the right lands first.
c. Both blocks land at the same time.

b: *The right block has smaller I (mass closer to the rotation axis), so it has less rotational kinetic energy in the wheel and more kinetic energy in the falling block.*

CQ9

If ***A*** and ***B*** are nonzero vectors for which their scalar product is zero, it must follow that

a. ***A*** x ***B***=0.
b. |***A*** x ***B***|= *AB.*
c. |***A*** x ***B***|= 1*.*
d. ***A*** is parallel to ***B***.

b: *If the scalar product is zero, the two vectors need to be perpendicular, hence the sinus of the angle in between is 1.*

CQ10

If the torque on an object adds up to zero

a. the object could be accelerating linearly but it could not be turning.
b. the object cannot be turning.
c. the forces on it also add up to zero.
d. the object could be both turning and accelerating linearly.
e. the object is at rest.

d: *A torque only affects angular velocity.*

PART II: Exercises 1-5

EX1

A plane flies directly between two cities, A and B, which are separated by 3700 km. From A to B, the plane flies into a 105 km/h headwind. On the return trip from B to A, the wind velocity is unchanged. The trip from B to A takes 42 minutes less than the trip from A to B. What is the airspeed of the plane, assuming it is the same in both directions?

1059 km/h

Data:
AB=3700 km; V_w=105 km/h; t_{BA}-t_{AB}=-42 min; V_p=?

Solution:
$V_{AB}=V_p-V_w$ & $V_{BA}=V_p+V_w$ => $V_{AB}-V_{BA}= 2V_w = 210$ km/h

$t_{BA}-t_{AB}=AB/ V_{BA} -AB/V_{AB}=AB (V_{AB}-V_{BA})/ (V_{AB}*V_{BA})$ =42 min = 0.7 h => $V_{AB}*V_{BA} =1.11*10^6$ (km/h)^2

$V_{AB}*V_{BA}= V_p^2-V_w^2$ => V_p=sqrt(1.11*10^6 + 105^2)=1059 km/h

EX2
An object attached to an ideal massless spring is pulled across a frictionless surface. If the spring constant is 45 N/m and the spring is stretched by 0.88 m when the object is accelerating at 1.5 m/s^2, what is the mass of the object?

26.4 kg

Data:
k=45 N/m; d=0.88 m; a=1.5 m/s^2; m=?

Solution:
F=ma=kd => m=kd/a=26.4 kg

EX3
A person is dragging a packing crate of mass 100 kg across a rough horizontal floor where the coefficient of kinetic friction is 0.400. He exerts a force ***F*** sufficient to accelerate the crate forward. At what angle above horizontal should his pulling force be directed in order to achieve the maximum acceleration?

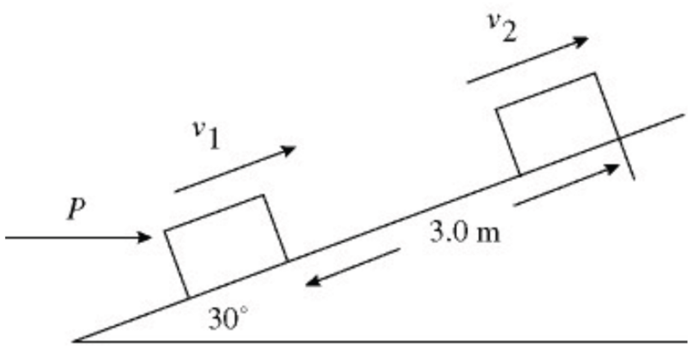
21.8^o

Data:
m=100 kg; μ=0.4; θ=? (measured from the horizontal)

Solution:
Friction: $F_f= \mu F_{normal}= \mu(mg-F \sin\theta)$
Net force: $F_{net}=F \cos\theta -F_f$

You have a maximum when the derivative is zero:
$dF_{net}/d\theta=0$ => $-F \sin\theta + \mu F \cos\theta=0$ => $\tan\theta= \mu$ => $\theta=21.8^o$

EX4



In the figure, a 600-kg crate is on a rough surface inclined at 30°. A constant external force P = 4800 N is applied horizontally to the crate. While this force pushes the crate a distance of 3.0 m up the incline, its velocity along the surface changes from 0.9 m/s to 2.1 m/s. How much work does friction do during this process?

-2570 J

Data:
$m=600$ kg; $\Delta s=3$ m; $P=4800$ N; $V_1=0.9$ m/s; $V_2=2.1$ m/s; $W_f=?$

Solution 1 (using energy):
Change in potential energy: $h= \Delta s \sin 30^o=1.5$ m => $\Delta U=mgh= 8820$ J
Change in kinetic energy: $\Delta K=1/2\ m\ (V_2^2-V_1^2)=1080$ J
Work done by force **P**: $W_P=P \cos 30^o = 12471$ J
Work done by friction: $Wf= \Delta U +\Delta K -W_P = -2570$ J

Solution 2 (using forces and work):
$F_{net}=P \cos30^o-mg \sin30^o-F_f=ma$
Computing acceleration from kinematics: $v_2=v_1t+at$ & $x=v_1t+a/2\ t^2$ => $a=0.6$ m/s^2
Friction from above: $F_f= -(P \cos30^o-mg \sin30^o-ma)= -857$ N
Work by friction: $W_f=F_f * \Delta s= -2571$ J

EX5
A very small 100-g object is attached to one end of a massless 10-cm rod that is pivoted without friction about the opposite end. The rod is held vertical, with the object at the top, and released, allowing the rod to swing. What is the speed of the object at the instant that the rod is horizontal?

1.4 m/s

Data:
$m= 100$ g; $h=0.1$ m; v(horiz)=?

Solution:
At the top, all energy is potential: $E_0=mgh=0.098$ J
When horizontal, all energy is kinetic: $E_1=1/2\ mv^2$
Since the only force is gravity, energy is conserved: $E_0=E_1$ => $v=\text{sqrt}\ (2gh)=1.4$ m/s

Also possible to start from rotational kinetic energy: $E_1=1/2\ I\omega^2=1/2\ mh^2\ (v/h)^2=1/2\ mv^2$

PART III: Exercises 6-10

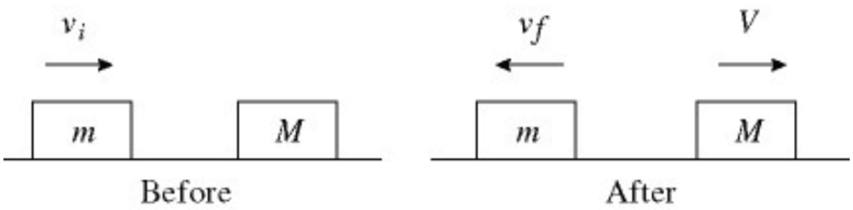
EX6
Three identical very small 50-kg masses are held at the corners of an equilateral triangle, 0.30 m on each side. If one of the masses is released, what is its initial acceleration if the only forces acting on it are the gravitational forces due to the other two masses? ($G= 6.67 \times 10^{-11}$ N m^2/kg^2)

6.4×10^{-8} m/s^2

Data:
$m=50$ kg; $d=0.3$ m; $a_i=?$

Solution:
Equilateral triangle: $\theta=60^o$
In horizontal direction: $F_{1x}+F_{2x}=0$
In vertical direction: $F_{1y}=F_{2y}=Gmm/d^2 *\cos (\theta/2)$ => $a=(F_{1y}+F_{2y})/m=2Gm/d^2*\cos 30^o=6.4 \times 10^{-8}$ m/s^2

EX7



A block of mass m = 8.40 kg, moving on a horizontal frictionless surface with a speed 4.20 m/s, makes a perfectly elastic collision with a block of mass *M* at rest. After the collision, the 8.40-kg block recoils with a speed of 0.400 m/s. In the figure, the blocks are in contact for 0.200 s. The magnitude of the average force on the 8.40-kg block, while the two blocks are in contact, is (closest to):

193 N

Data:
$M=8.4$ kg; $v_i=4.20$ m/s; $v_f=-0.4$ m/s; $\Delta t=0.2$ s; $F_{av}=?$

Solution:
F_{av} is the force required to change linear momentum within the given time interval:
$P_i=mv_i$ & $P_f=mv_f \Rightarrow F_{av}=(P_f-P_i)/\Delta t=m(v_f-v_i)/\Delta t=-193$ N
The sign is negative because the force points to the left (if the horizontal axis is oriented to the right), but the problem asks for the magnitude, and that is always positive.

EX8
A string is wrapped around a pulley with a radius of 2.0 cm and no appreciable friction in its axle. The pulley is initially not turning. A constant force of 50 N is applied to the string, which does not slip, causing the pulley to rotate and the string to unwind. If the string unwinds 1.2 m in 4.9 s, what is the moment of inertia of the pulley?

0.2 kg m^2

Data:
$R=0.02$ m; $F=50$ N; $\Delta s=1.2$ m; $\Delta t=4.9$ s; $I=?$

Solution:
$\tau=I\alpha=FR$
$\Delta\theta= \Delta s/R=60$ rad
$\Delta\theta=\theta-\theta_0=\omega_0 t+\alpha/2\ t^2= \alpha/2\ t^2 \Rightarrow \alpha=2\ \Delta\theta/\ t^2= 5$ rad/s
$I=FR/\alpha= 0.2$ kg m^2

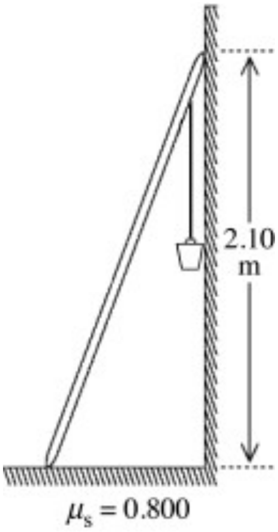
EX9
Three solid, uniform, cylindrical flywheels, each of mass 65.0 kg and radius 1.47 m rotate independently around a common axis. Two of the flywheels rotate in one direction at 7.05 rad/s; the other rotates in the opposite direction at 3.42 rad/s. Calculate the magnitude of the net angular momentum of the system.

750 kg m/s^2 [$I=1/2\ MR^2$]

Data:
$M= 65$ kg; $R=1.47$ m; $\omega_1=\omega_2=7.05$ rad/s; $\omega_3=-3.42$ rad/s; $L_{tot}=?$

Solution:
$L_{tot}=I*(\omega_1+\omega_2+\omega_3)=1/2\ MR^2(2*7.05-3.42)=750$ kg m/s^2

EX10



A 10.0-kg uniform ladder that is 2.50 m long is placed against a smooth vertical wall and reaches to a height of 2.10 m, as shown in the figure. The base of the ladder rests on a rough horizontal floor whose coefficient of static friction with the ladder is 0.800. An 80.0-kg bucket of concrete is suspended from the top rung of the ladder, right next to the wall, as shown in the figure. What is the magnitude of the friction force that the floor exerts on the ladder?

539.5 N

Data:
m_L=10 kg; L_L=2.5 m; h=2.1 m; μ=0.8; m_B=80 kg; F_f=?
Additional: d=sqrt(L_L^2-h^2)=1.36 m

Solution 1 (pivot point at the base of the ladder, point O):

Horizontal forces: $-N_w+F_f=0 \Rightarrow F_f = N_w$
with N_w being the normal force by the wall (horizontal towards the left)

Torque about O: $-m_Bgd+N_wh-m_Lgd/2=0 \Rightarrow N_w=gd(m_B+m_L/2)/h$=539.5 N

Solution 2 (pivot point at the top of the ladder, point P):

Vertical foces: $N_F-m_Lg-m_Bg=0 \Rightarrow N_F=g(m_L+m_B)$
with N_F being the normal force by the floor (vertical upwards)

Torque about P: $F_fh-N_Fd+m_Lgd/2=0 \Rightarrow F_f=gd(m_L+m_B- m_L/2)/h$=539.5 N