Delft University of Technology, EEMCS faculty
Examination Mathematics 2, AESB1210 (test 3)
Tuesday, January 27th, 2015, 10.00-12.00

- It's not allowed to use a calculator or a mathematical table.
- Each answer should be clearly motivated.
- Your grade is obtained by rounding (score+3)/3 to one decimal place.
- Points:

Ex. 1a	$2\frac{1}{2}$	Ex. 2	4	Ex. 3	3	Ex. 4	4	Ex. 5	4	Ex. 6a	3
Ex. 1b	$1\frac{1}{2}$									Ex. 6b	2
Ex. 1c	3										

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1. Let $A = \begin{bmatrix} 1 & 2 & a \\ -2 & 8 & 2 \\ 2 & -2 & 1 \end{bmatrix}$ where $a \in \mathbb{R}$.

a. For what value(s) of a is vector $\underline{p} = \begin{bmatrix} 3 \\ -6 \\ 7 \end{bmatrix}$ in $COL(A)$?

b. For what value(s) of a is vector $\underline{q} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ in $NUL(A)$?

c. Consider matrix $B = \begin{bmatrix} 1 & 2 & b \\ -2 & 4b & 2 \\ b & -2 & 1 \end{bmatrix}$ and find all possible values of $rank(B)$ as b varies.

2. Determine a basis for the subspaces $H = \left\{ \begin{bmatrix} a \\ a \\ b \end{bmatrix} \in \mathbb{R}^3 \middle| a, b \in \mathbb{R} \right\}$ and $H^\perp$ of $\mathbb{R}^3$.

(by $H^\perp$ is meant the orthogonal complement of H in $\mathbb{R}^3$)

3. Find an orthogonal basis for $\mathbb{R}^3$ that includes the vectors $\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 1 \\ 4 \end{bmatrix}$.

p.t.o.

4. Prove that if vector $\underline{u}$ is orthogonal to both the vectors $\underline{v}$ and $\underline{w}$, then $\underline{u}$ is orthogonal to every vector $\underline{h}$ in $H = Span\{\underline{v}, \underline{w}\}$.

5. Consider $\underline{v} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ and subspace $W = Span\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$ of $\mathbb{R}^3$ and decompose $\underline{v}$ into the sum of a vector $\underline{w} \in W$ and a vector $\underline{u} \in W^{\perp}$ (the orthogonal complement of W in $\mathbb{R}^3$).

6. A certain experiment produces the data:

t	0	1	2	3
y	2	3	3	4

a. Find the least-squares curve of the form $y = \alpha + \beta(t-1)^2 + \gamma \sin(\frac{\pi}{2} t)$ to fit the given data.

b. Determine the least-squares error of your approximation.