

- It's not allowed to use a calculator or a mathematical table.
- Each answer should be clearly motivated.
- Note: Each test takes 1 hour so should be submitted after 60 minutes. After handing in your working out of a test it's allowed to take another test.
- Your grade is obtained by rounding $(\text{score} + 2\frac{1}{2}) / 2\frac{1}{2}$ to one decimal place.
- Points:

Ex. 1	4	Ex. 2	4	Ex. 3	3	Ex. 4a	$5\frac{1}{2}$	Ex. 5	4
						Ex. 4b	2		

1. If C is a 6×6 – matrix and $Nul(C) = \{\mathbf{0}\}$, what can be said about solutions of equations of the form $C\mathbf{x} = \mathbf{b}$ for $\mathbf{b} \in \mathbb{R}^6$?

2. Given $B = \begin{bmatrix} \beta & -2 \\ 4\frac{1}{2} & -\beta \end{bmatrix}$ where $\beta \in \mathbb{R}$. Find the value(s) of β such that $NUL(B) = COL(B)$.

3. Let $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ and $H = \{\mathbf{x} \in \mathbb{R}^4 \mid \mathbf{x} \perp \mathbf{v}\}$. Find a basis for H .

4. Let $A = \begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & 1 & -1 & -2 \\ 2 & -1 & 3 & -2 \\ 2 & 0 & 2 & -1 \end{bmatrix}$ and $\mathbf{d} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$.

- a. Write vector \mathbf{d} as the sum of a vector $\mathbf{v} \in ROW(A)$ and a vector $\mathbf{w} \in ROW(A)^\perp$, the orthogonal complement of $ROW(A)$ in \mathbb{R}^4 .
- b. Find $\dim(NUL(A^T))$, the dimension of $NUL(A^T)$.

5. A certain experiment produces the data:

x	1	4	9	16
y	0	2	4	5

. Find the least-squares curve of the form $y = \alpha \sqrt{x} + \beta \cos(\pi x)$ to fit the given data.